Higher-Order Metaphysics

Lukas Skiba

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Abstract: Subverting a once widely held Quinean paradigm, there is a growing consensus among philosophers of logic that higher-order quantifiers (which bind variables in the syntactic position of predicates and sentences) are a perfectly legitimate and useful instrument in the logico-philosophical toolbox, while neither being reducible to nor fully explicable in terms of first-order quantifiers (which bind variables in singular term position). This article discusses the impact of this quantificational paradigm shift on metaphysics, focussing on theories of properties, propositions, and identity, as well as on the metaphysics of modality.

Keywords: Higher-Order Quantification, Higher-Order Logic, Properties, Propositions, Identity, Modality

1. Introduction: A Paradigm Shift in Metaphysics

To be – W.V.O. Quine famously told us – is to be the value of a variable. By this he meant that our theories are ontologically committed to all and only those entities over which the theory's variables must be taken to range for the theory to be true. If a theory contains the existential quantification '$\exists x$(x is wise)', then, for the theory to be true, the variable 'x' must be taken to range over at least one wise object (in the logician's wide sense of the word 'object' in which anything that can be referred to by a name or other singular term qualifies as one). The theory is thus ontologically committed to a wise object.

Importantly, there are *two* strands to the Quinean paradigm. First, it connects metaphysics (in particular: ontology) to quantification. Second, it takes a stance on the *type* of quantification at issue. For Quine was convinced that the only legitimate type of quantification is *first-order* quantification. This is the type of quantification illustrated by the example above, where the variable bound by the quantifier occupies the syntactic position of a singular term, such as 'Plato'. But first-order quantifiers are not the only type of quantifiers. There also are *higher-order* quantifiers, which allow us to quantify into other syntactic positions, including, importantly, that occupied by predicates. Thus, in '$\exists X$(X(Plato))' we existentially quantify into the position of a predicate, such as '… is wise'. Now, Quine (1970) convinced himself, and many of his peers, that this type of quantification is intelligible only when taken to be a disguised form of first-order quantification and thus shouldn't be regarded as a *self-standing* form of quantification at all. In particular, he took '$\exists X$(X(Plato))' to boil down to the first-order claim that there is a set (a special kind of object) of which Plato is a member. Taking this dismissive attitude towards self-standing higher-order quantification into account, the Quinean paradigm can be expressed more precisely as follows: for Quineans, to be is to be the value of a *first-order* variable.

In recent years, we have seen a shift away from the second strand of the Quinean paradigm. Following the pioneering work of Prior (1971) and Boolos (1975, 1984, 1985) and more recent contributions by Rayo & Yablo (2001), Williamson (2003, 2013) and Wright (2007), there is a growing consensus that one can make good sense of higher-order quantifiers without assimilating them to their first-order cousins. On this conception, higher-order quantification is a *sui generis* form of quantification which is neither reducible to nor explicable in first-order terms. In line with this, the values of higher-order variables are, on this alternative conception, not taken to be objects (not even in the logician's wide sense of that word) but are taken to belong to a distinct logical category of their own. When asked for a natural-language paraphrase of '$\exists X(X(\text{Plato}))$' proponents of this view offer constructions such as 'there is something which Plato is'. And the more complex '$\exists R(R(\text{Socrates, Plato}) \wedge R(\text{Plato, Aristotle}))$' will be paraphrased along the lines of 'Socrates is related to Plato somehow such that Plato is related to Aristotle in that way too'. In offering such paraphrases, proponents of the present conception will stress that the natural language quantificational devices they involve are *non-nominal*, as evidenced by the non-nominal complements of the 'namely'-riders with which they can be amplified, e.g. 'namely, wise' and 'namely, as teacher and pupil' respectively. And they will insist that such non-nominal quantifications are themselves not to be construed as first-order quantifications. Thus, the 'way'-locution in the second paraphrase must not be taken to indicate that we are concerned with a first-order quantification over certain kinds of objects called 'ways'. Rather, the phrase 'in that way' functions there merely as a device for what Rayo and Yablo call *anaphoric cross-indexing* and could be replaced e.g. with 'thus' (see Prior 1971: Ch. 3 for the original appeal to non-nominal quantificational devices in order to interpret higher-order quantification, see Rayo & Yablo 2001 for a development of this idea, and MacBride 2006 for helpful discussion).[1] The anti-Quinean, positive attitude towards self-standing higher-order quantification presently under consideration has a historical pedigree too: it can be seen as following a Fregean paradigm, since it is rooted in Frege's *Begriffsschrift* (1879). Adapting a famous dictum by Boolos, we can contrast it with the Quinean paradigm as follows: for Fregeans, to be is to be the value of a *first-order* variable or the value of a *higher-order* variable.

This paradigm-shift is of great importance for many fundamental questions in metaphysics, since it comes with a drastic shift in perspective on the structure of reality. For the Quinean foe of sui generis higher-order quantification, reality is flat: there are objects and objects alone. For the Fregean friend of sui generis higher-order quantification, reality is hierarchically structured: there is a bottom layer of objects –the values of first-order variables– followed by a layer of ways for these objects to be (and to relate) –the values of second-order variables which belong to a different logical type– followed by a layer of ways for such ways to be (and to relate) –the

[1] Whether such, or indeed any, natural language constructions allow us to produce sentences *strictly synonymous* with formulas of second-order logic is, however, unclear. Thus notice that while we are quantifying into the position of a *full predicate*, such as '… is wise', in '$\exists X(X(\text{Plato}))$', we are quantifying into the position of a *copula-free adjective*, such as 'wise', in 'there is something which Plato is'. Presumably, we will take the two types of quantification to be semantically equivalent only if we believe, controversially, that '… is wise' and 'wise' are semantically equivalent (see Trueman 2021: Ch. 7, §3 for discussion). This is not to say, however, that the paraphrases discussed are pointless, should we reject the controversial equivalence and come to believe that, in general, there are no natural language sentences which *fully capture* the content of higher-order quantifications. In this case, we will come to understand the language of higher-order logic only as a result of what Williamson (2003: 459) calls the 'direct method': by immersing ourselves in its use, rather than by translating it into a (more) familiar language (see also Williamson 2013: Ch. 5, §9). In fact, it is generally accepted among higher-orderists that we will have to resort to this direct method at *some* point: even if natural language can capture second-order quantification, it doesn't contain the resources to capture, say, fourteenth-order quantification. So, sooner or later, we need to take a leap of faith and plunge into the practise of speaking *higher-orderese*. Since the paraphrases at any rate *approximate* the content of some higher-order quantifications they still play an important role in preparing us for the plunge.

values of third-order variables which belong to yet another logical type– and so on (ad infinitum).[2] These different outlooks have a knock-on effect on how Quineans and Fregeans approach a large variety of questions in metaphysics: when the Quinean is asked, for instance, whether there are properties, then this question is, from her perspective, tantamount to that of whether there are properties *among the objects*. When the Fregean is asked the same question, she will approach it differently: she *may* answer positively because she thinks that some objects are properties. But she *may also* (and will more likely) answer positively because she thinks that the inhabitants of some *other* level(s) of the hierarchy should be regarded as properties.

Over the last decade, more and more philosophers have become enthralled by the project of *Higher-Order Metaphysics*, the project of investigating questions and debates in metaphysics against the backdrop of the Fregean, rather than the Quinean, paradigm regarding higher-order quantification. This paper discusses some of the most important developments in this quickly emerging field, focussing on higher-order conceptions of properties (§2), propositions and facts (§3), identity (§4), and modality (§5).

2. Higher-Order Conceptions of Properties

First-order metaphysicians trying to fit properties into their ontology have no choice but to regard them as special types of objects. This has a threefold effect on how they think and talk about them: (i) In line with the characteristic mark of objects (on our broad understanding), they will regard singular terms such as 'wisdom' or 'the property of being wise' as the linguistic devices tailor-made to facilitate property discourse. As a result, they (ii) will take constructions such as 'Plato instantiates wisdom' as the most perspicuous form of ascribing properties to (other) objects and (iii) will consider first-order quantifications such as '$\exists x$(Plato instantiates x)' as faithful expressions of their property realism.

Higher-order metaphysicians may disagree on all three points and still think of themselves as property realists. This is because, in their ontology, the most natural place for properties is not among the objects but among the values of higher-order variables. Accordingly, they (i) will regard predicates such as '… is wise' to be the linguistic vehicles of choice when it comes to property talk, (ii) will take simple predications such as 'Plato is wise' to be the most perspicuous form of ascribing properties to objects and (iii) will consider higher-order quantifications such as '$\exists X(X(\text{Plato}))$' as faithful expression of their property realism. A higher-order conception of properties along these lines is advocated e.g. by Williamson 2013, Jones

[2] There is, of course, something misleading about this way of describing the higher-orderists' metaphysical outlook (and similar remarks apply to many other passages in this paper): for instance, by using expressions of the *same* logico-syntactical type, e.g. 'the values of n^{th}-order of variables', for the inhabitants of each layer of the hierarchy we are belying what we then go on to say about them, namely that they belong to *different* such types. And by using, in particular, (plural) noun phrases to refer to them, we inappropriately assimilate the inhabitants of all layers to objects. Higher-orderists are well aware of such tensions which, to some extent, may be inevitably when we are trying to communicate their view in English rather than in the language of higher-order logic (see n. 1). The latter would involve using a first-order quantifier (binding variables in the position of singular terms) to postulate the inhabitants of the bottom layer, a second-order quantifier (binding variables in the position of first-level predicates, i.e. predicates that form sentences when combined with singular terms) to postulate inhabitants for the second layer, a third-order quantifier (binding variables in the position of second-level predicates, i.e. predicates that form sentences when combined with first-level predicates) to postulate inhabitants for the third layer, and so on.

2018, Bacon 2020, Skiba 2020, Trueman 2021. We can discern three major types of advantages cited in favour of it by its proponents.[3]

First, the higher-order conception delivers a purely logical justification for property realism unavailable to first-order realists. All that higher-orderists need in order to get from an entirely uncontroversial claim, e.g. 'Plato is wise', to a testimony of their property realism, '$\exists X(X(\text{Plato}))$', is a simple application of the inference rule of (second-order) existential generalisation.[4] In contrast, first-orderists will additionally have to endorse some principle which, applied to the case at hand, guarantees the transition from 'Plato is wise' to 'Plato instantiates wisdom'. Only then can they apply (first-order) existential generalisation to infer '$\exists x$(Plato instantiates x)'. Whatever the status of the additional principle; it is not going to be a *purely logical* principle. So, higher-orderists can, while first-orderists cannot, regard their property realism as justified by the laws of logic alone.

Second, the higher-order conception promises a *resolution* of a large variety of disputes regarding the nature of properties that seem to have become intractable against the background of the first-order conception. For instance, let's focus on polyadic properties (aka *relations*) and consider the question of whether non-symmetric relations, such as the *is taller than* relation, should be distinguished from their converses, in this case, the *is shorter than* relation. In the first-order setting, in which this debate is traditionally pursued, it remains hotly contested (with e.g. Russell (1903) and Paul (2012) in the 'yes' and Williamson (1985) and Fine (2000) in the 'no' camp). When the debate is transposed into the higher-order setting, however, it admits of a straightforward resolution. While there is some disagreement on how identifications of higher-order entities are to be understood *exactly* (see §4), it is universally accepted that co-extensionality is a necessary condition for higher-order identity. This means that dyadic predicates, 'F' and 'G', stand for the same higher-order relation only if '$\forall x \forall y(Fxy \leftrightarrow Gxy)$' holds. Since it is uncontroversial that '$\forall x \forall y$(x is taller than $y \leftrightarrow x$ is shorter than y)' is false, it is equally uncontroversial, that 'is taller than' and 'is shorter than' stand for distinct higher-order relations (see Trueman 2021: Ch. 10, §4). Other questions that have been taken to be resolved by the higher-order conception (not necessarily with the same degree of obviousness), include the question whether properties are universals, rather than tropes, ('yes' argues Jones 2018) and the question whether fundamental properties are freely recombinable ('yes' argues Bacon 2020). Quite generally, the influential idea that a higher-order conception of properties can help to clarify and resolve a number of important disputes in modal metaphysics (see e.g. Stalnaker 2012, Williamson 2013, Fritz & Goodman 2017a), can be subsumed under this heading, but will be discussed in more detail in §5.

Third, the higher-order conception is also taken to *dissolve* certain disputes that have become grid-locked in the first-order setting.[5] In contrast to a resolution, which presupposes the possibility of transposing a given question into the higher-order setting where it is then taken to receive a (more) straightforward response, a dissolution is taken to occur if a certain question

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[3] An important additional motivation for higher-orderism about properties and related entities stems from its potential to allow for absolute generality without running into versions of Russell's paradox. See Williamson 2003 for this line ofargument for higher-orderism and Florio 2014 for an overview of the ensuing debate.

[4] See Jones 2018 for a clear statement of this argument. It presupposes a classical or negative free higher-order logic (which guarantees that 'Fa' logically entails '$\exists X(Xa)$'). For discussions of various free higher-order logics, including positive free higher-order logic, see Besson 2009, Bacon et al. 2016, Skiba 2020.

[5] The 'resolution' vs. 'dissolution' terminology is due to Jones 2018.

can simply no longer be posed in the higher-order setting. To illustrate, consider the question of whether properties are spatiotemporally located. From the perspective of the first-orderists, who take properties to be the referents of singular terms, this question makes perfect sense. It therefore demands a response (what the right response is, is, again, hotly contested, with e.g. Paul (2006) and Effingham (2015) in the 'yes' camp and Russell (1912: Ch. 9) being the figurehead of the 'no' camp). In contrast, from the perspective of the higher-orderists, who take predicates to be the right linguistic vehicles to facilitate talk about properties, there is no intelligible question to be asked here: the expression '... is spatiotemporally located' and other locative vocabulary is most naturally construed as combinable *only* with singular terms, not with predicates: 'is wise is spatiotemporally located' is simply ungrammatical. So, from the perspective of higher-orderists, the question cannot even be intelligibly formulated, which absolves them of any obligation to even engage with it (see Jones 2018). Further questions to be eschewed in this way include the question of whether higher-order entities are sets (Jones 2019: 172), once more the question whether properties are tropes or universals (which Skiba 2020 marks out for dissolution, rather than resolution), and the question whether properties need to be connected to objects by an instantiation relation, a question which famously serves as the starting point for Bradley's Regress (Trueman 2021: Ch. 10, §2).

3. Higher-Order Conceptions of Propositions and Facts

So far, we have focussed on quantification into the position of predicates ('… is wise') and its deployment in theories of properties. But the remit of higher-order quantification is broader than that. It also covers sentential quantification, as in '$\exists P(P \vee \neg P)$', where we quantify into the position of an assertoric sentence, such as 'Plato is wise'. Sentential quantification naturally leads to a higher-order conception of propositions.

Like the higher-order conception of properties, this conception is unavailable to first-order metaphysicians who are, once more, compelled to regard propositions as a special type of object (see e.g. Künne 2003, King et al. 2014, Hanks 2015). Accordingly, they treat quantification over propositions as first-order quantification, formalizing, e.g. 'Plato believes everything he says' as '$\forall x$(Plato says $x \rightarrow$ Plato believes x)'. This requires a construal of propositional attitude reports as containing a singular term standing for a proposition, which is achieved by taking propositions to be the referents of that-clauses and declaring those as singular terms. As a result, first-orderists take attitude ascriptions to contain a dyadic predicate flanked by a singular term on either side:

(FO-Attitudes) [Plato] [believes] [that snow is white]

Higher-order metaphysicians, in contrast, find inspiration in Prior's suggestion (1971: Ch. 2) to recast the logical form of attitude reports as follows:

(HO-Attitudes) [Plato] [believes that] [snow is white]

Here, we have a singular term and an assertoric sentence flanking what can be called, following Künne (2003), a *prenective*, i.e. an operator which behaves like a *pre*dicate on the left and like a sentential con*nective* on the right. On such an account, quantification over propositions is naturally treated as sui generis quantification into sentential position, so that 'Plato believes everything he says' can be formalized as '$\forall P$(Plato says that $P \rightarrow$ Plato believes that P)'.

Those higher-orderists about propositions who follow, in varying degrees of proximity, the Priorian suggestion (Rosefeldt (2008), Trueman (2018, 2020 2021), Jones (2019)) have claimed two advantages over the first-order conception.

First, the higher-order conception of propositions avoids the *substitution problem*, one of the most troubling consequences of first-order accounts (see Rosefeldt 2008): according to first-orderists, 'the proposition that snow is white' and 'that snow is white' are co-referring singular terms. If this were correct, one would expect the two expressions to be smoothly inter-substitutable in attitude reports. In fact, however, a perfectly well-formed sentence, 'Plato hopes that snow is white', becomes *ungrammatical* as the result of such a substitution: 'Plato hopes the proposition that snow is white' is not a well-formed sentence (see e.g. Moltmann 2003, 2013: Ch. 4, §3.1). While first-orderists will have to (at least) complicate their account in some way or another in order to address this problem (see e.g. King 2007: 137-63, Nebel 2019), higher-orderists flat-out avoid it.

Second, the higher-order conception of propositions is taken to allow for a preferable account of representational content. On the first-order conception, for Plato to make cognitive contact with the world surrounding him (e.g. by representing it as being such that snow is white) is for him to stand in an object-to-object relation to a certain first-order proposition. Now it seems that this account is incomplete until the proposition in question is *itself* ensured to have the right representational content (viz. to represent the world as being such that snow is white). How are first-orderists to account for this representational content of propositions? Higher-orderists (Trueman 2018, 2021; Jones 2019) argue that any plausible account available to first-orderists will ultimately employ an object-to-higher-order-entity relation connecting the first-order proposition that snow is white to the semantic value of the sentence 'snow is white'. But this, they point out, is just to give a Priorian account of the representational content of propositions. Why then, they ask, not cut out the middleman and take the representational content of attitudes to be specified *directly* on the Priorian model, as per (HO-Attitudes)?

Not all recent advocates of higher-order propositions are motivated by considerations such as the above. As with properties, e.g. Stalnaker (2012), Williamson (2013), Fritz & Goodman (2017b) instead locate the main advantage of the higher-order conception of propositions in its potential to advance debates concerning the logic and metaphysics of modality (on which see §5 below).

A further important line of thought (manifested e.g. in Fine 2012; Dorr 2016; Correia & Skiles 2019; Fritz 2019, 2021) is that the employment of sui generis sentential quantification is beneficial for the formulation of theories of ground. On this approach, the canonical form of grounding claims, '$S<Q$', involves a sentential connective (which connects the sentence 'Q', stating the grounded, and the sentence 'S', stating its full grounds, and which can be read as 'Q because S'). Grounding principles, such as e.g. the irreflexivity of ground, then take the form of a sentential quantification: '$\forall P \neg(P<P)$'. On this view, grounding is a relation not among special types of objects, but among potential values of higher-order, viz. sentential, variables. In the grounding literature, we often hear these higher-order relata of grounding being described as higher-order *facts* rather than higher-order *propositions.* It is not clear, however, whether this marks a genuine difference in the notion that the respective theorists are trying to capture. In fact, it has been argued that the higher-order setting collapses any distinction between 'representational' propositions and 'worldly' facts (see Trueman 2020), which, if correct, would suggest that we are concerned here only with a verbal difference.

4. Higher-Order Conceptions of Identity

Higher-order conceptions of properties and propositions require a higher-order conception of identity, because our standard notion of identity applies exclusively to first-order entities. Identity statements, such as 'Hesperus is Phosphorus', are routinely formalized in first-order logic as 'h=p'. And the relational predicate '=' of first-order logic can be flanked only by singular terms (and first-order variables). Yet we clearly need a way of saying that two predicates/sentences stand for the same (or distinct) higher-order entities. In other words: we need a notion of higher-order identity. Following Dorr we will use '≡' for this purpose. For instance, we can express the identity of the higher-order property of being a lawyer with that of being an attorney thus: 'L≡A' (see Dorr 2016: §3 for two different formalisms generalizing this use of '≡'). Several approaches have been pursued in order to provide an informative account of such higher-order identifications. They agree that higher-order identity is to obey structural principles similar to those governing first-order identity. In particular, ≡ is taken to be a reflexive, symmetric and transitive relation, and subject to a higher-order version of Leibniz' Law (which may be restricted, e.g. to non-opaque contexts; see Dorr (2016: 43-46), Correia & Skiles (2019: 645)). Beyond this basal agreement, however, we encounter two major dimensions of dispute.

The first dimension concerns the *fineness of grain* with which higher-order entities are to be distinguished. It is relatively uncontroversial that intensional equivalence is a *lower* limit for higher-order identity. Higher-order properties are to be identified only if they necessarily apply to the same objects, and sentences are to stand for the same higher-order entity only if their truth-values necessarily coincide. There is also wide agreement regarding its *upper* limit: the Russell-Myhill paradox (Russell 1903, Myhill 1958) is widely taken to show that there is a limit on how much structure higher-order entities can be taken to exhibit (see Dorr 2016, Goodman 2017a; see Fritz 2019 for a technique of still assigning 'proxy' structure to them). In particular, it is taken to show that some complex sentences correspond to the same higher-order entity despite being built up from components standing for entities that themselves are *not* pairwise higher-order identical. What is contested is where higher-order identity is to be located exactly on the resulting interval. Three important approaches have emerged:

(i) A prominent option is to regard intensional equivalence as a necessary *and sufficient* condition for higher-order identity (Stalnaker 2012, Williamson 2013, Rayo 2013, Trueman 2021). This has the advantage of simplicity but treats, e.g., all logically equivalent sentences as corresponding to the same higher-order proposition.

(ii) The second option strengthens intensional equivalence to ground-theoretic equivalence. On this approach, sentences correspond to the same higher-order proposition just in case they can safely be intersubstituted in statements of ground (Correia (2010, 2016), Correia & Skiles (2019)). Correia's logic GI (Generalized Identity) allows, e.g., to distinguish the intensionally equivalent sentences 'S' and 'S∨(Q∧¬Q)' since they differ in their grounding profile (the former being a plausible ground for the latter, but not vice versa).

(iii) The third option is motivated by a desire to avoid potentially vicious forms of circularity. Thus Dorr (2016) treats higher-order identity as classical equivalence subject to a no-circularity requirement: Following Prior (1964), the guiding idea behind his logic OLC (Only Logical Circles) is to reject higher-order identities 'A≡B(A)' whenever 'A' occurs as a proper component of '$B(A)$' whose remainder 'B' contains some non-logical expressions, (yielding e.g. the verdict that '$P \equiv P \vee P$' is false whenever 'P' is not purely logical).

The second dimension of dispute concerns the intelligibility of *identifications across orders*. Higher-order theories of properties and propositions are commonly assumed to take the form of Standard Type Theory (STT).[6] In type theories, the logical type of an expression is made explicit via a superscript (making additional typographical distinctions superfluous). For instance, the application of a first-level predicate constant to a term, previously '*Fa*', becomes '$f^1(a^0)$'. STT, which goes back to Russell & Whitehead's *Principia Mathematica* (1910-1913), imposes strict limits on what counts as meaningful predication: it regards a predication of the form '$b^\beta(a^\alpha)$' as well-formed iff α immediately precedes β (i.e. $\beta=\alpha+1$). In line with this, STT treats identifications as well-formed iff they combine two expressions of the same type with an identity predicate of the next higher type. While, according to STT, identifications such as '$a^0 =^1 b^0$' and '$f^1 \equiv^2 g^1$' are thus meaningful, any attempt to express an *order-straddling identity* between, say, an entity of type 0 (i.e. an object, a potential value of a first-order variable) and an entity of type 1 (i.e. a higher-order property/proposition, a potential value of a second-order variable), e.g. with a string of symbols such as '$a^0 =^1 f^1$' or '$a^0 \equiv^2 f^1$', is ill-formed.[7] In contrast, a number of higher-orderists have recently shown sympathy for some form of Cumulative Type Theory (CTT) as the background framework for higher-order theorizing (Williamson 2003, 2013; Linnebo & Rayo 2012; Krämer 2017; Florio & Jones 2021).[8] CTT is more relaxed about what counts as meaningful, regarding predications '$b^\beta(a^\alpha)$' as well-formed not only when β is directly succeeds α but also when it is strictly greater than α (i.e. $\beta>\alpha$). As a result, CTT allows for the definition of a type-straddling identity predicate '$\approx$' with the help of which entities of distinct types can be identified, as in '$a^0 \approx f^1$'.[9]

What hinges on the choice between STT and CTT and how are we to make it? The main argument in favour of CTT, due to Linnebo & Rayo (2012), runs, in rough outline, as follows: First, (a precisification of) the claim that each formal language can be provided with a semantic theory of a desirable form (*Optimism*) is motivated.[10] Second, it is argued that this requires allowing for languages with expressions of *infinite types* (e.g. expressions of type ω with $\omega>n$ for any natural number n). Third, it is pointed out that only CTT sits naturally with infinite types (for '$b^\omega(a^\alpha)$' to be well-formed according to the standards of STT, α would have to immediately precede ω, which is impossible since ω is an infinite type; the requirement that $\omega>\alpha$, in contrast, is unproblematic). The argument's success remains contested. Button & Trueman (Ms: §6) point to the observation, made by Florio & Shapiro (2014: 162-3), that *Optimism* is inconsistent with another prima facie attractive principle, according to which, for any formal languages you have, there is a union language comprising them all into one (*Union*).[11] Button & Trueman then

[6] Also known as 'Strict Type Theory', see Florio & Jones 2021.
[7] Given the type-indicating superscripts, the typographical distinction between '=' and '≡' is no longer necessary; it is retained for the sake of consistency with the preceding discussion.
[8] There are some important differences between the versions of CTT proposed by these authors; see Button & Trueman (Ms: §7) for discussion.
[9] See Linnebo & Rayo (2012: 282) and Krämer (2017: 516) on how to define a type-straddling identity predicate in slightly different settings (for which they use the sign '≡' which we have already reserved for type-restricted higher-order identity). While they, as well as Button & Trueman (Ms: §1.2), all take type-straddling identity to be a genuine notion of identity, Florio & Jones (2021: 56 n. 15) voice some concerns over this.
[10] This is Linnebo & Rayo's 'Principle of Semantic Optimism' (2012: 276)
[11] This is Florio & Shapiro's 'Principle of Naive Union' (2014: 163), not Linnebo & Rayo's 'Principle of Union' (2012: 276), which is a restriction of the former and consistent with their Principle of Semantic Optimism.

argue that Linnebo & Rayo's attempt to preserve *Optimism* (which leads to CTT and requires restricting *Union*) is but one natural reaction to the conflict. An alternative, which they argue to be preferable, is to preserve *Union* by restricting *Optimism,* which they show can be done in a way that is compatible with STT (and well-motivated from its perspective).

The choice between STT and CTT is bound up, in intricate ways, with the question whether absolute generality (AG) – quantification over absolutely everything – is possible (see Florio 2014 for an introduction to AG). Linnebo & Rayo's semantic argument presupposes an AG-principle and Florio & Jones (2021) argue that their version of CTT is as compatible with AG as STT is. In contrast, Krämer (2017) and Button & Truman (Ms: §7) argue that only STT allows for genuine AG (a claim from which they draw, however, opposite conclusions: while Krämer takes it to tell against the possibility of AG, Button & Trueman take it to speak against CTT.)

A different type of consideration in favour of STT is provided in Skiba (forthcoming a): in §2 we saw arguments from dissolution to be an important motivation for looking at metaphysical disputes through the lens of higher-order languages. Skiba takes these arguments to also support this lens being cut in the shape of STT, rather than CTT. Put briefly, the point is that adopting CTT would threaten to undermine dissolution arguments, since it would, for instance, re-introduce the possibility of asking after the location of properties *even* when those are conceived of as higher-order entities (by allowing to ask whether they are identical, in the sense of '≈', with some object that is located.)

5. Higher-Order Metaphysics of Modality

Since the seminal work by Saul Kripke and Ruth Barcan-Marcus, results in first-order modal logic have routinely informed our views on the metaphysics of modality. In contrast, interest in the metaphysical lessons to be drawn from higher-order modal logic, which studies the interaction of modal notions with higher-order quantifiers and higher-order identity, has increased only in the recent wake of Williamson's milestone contribution *Modal Logic as Metaphysics* (2013). The book's central claim is that the best systems of higher-order modal logic lend support to *necessitism*: the radical view that necessarily all objects, including, e.g., Plato, exist necessarily (formally: $\Box\forall x\Box\exists y(x=y)$). Despite its historical pedigree (versions of necessitism can be found in Bolzano, Wittgenstein and Ramsey) many find this doctrine simply unbelievable. They instead favour contingentism and think that, had Plato's parents never met, Plato would simply not have been there at all, rather than carving out an existence as a non-concrete, merely possible person, as necessitists would have us believe.

We can discern two important routes along which higher-order modal logic is taken to lead to necessitism (for a third one, see Goodman 2017b). Both proceed via higher-order necessitism, the view that necessarily all higher-order properties, relations and propositions exist necessarily (a formal instance of which is: $\Box\forall X\Box\exists Y(X\equiv Y)$). The first route motivates higher-order necessitism by appeal to *theoretical virtues.* Thus Williamson (2013, Ch. 6) maintains that it falls out of the higher-order modal logic to strike the best balance between simplicity, strength and uniformity. The second route motivates it by an appeal to *modal discourse* (Williamson 2013, Ch. 7; Fritz & Goodman 2017a). The central idea here is that contingentists, who reject mere possibilia, struggle to account for a range of modalized quantifications including e.g. 'there are four possible knives that can be made out of these two blades and two handles' and 'most possible persons will never be born'. While *some* of these can be provided with paraphrases on weaker assumptions (see Rosefeldt 2017), contingentists will have to embrace higher-order necessitism, so the argument goes, if they want to account for the *full range* of modalized quantifications. In particular, they will have to assume the necessary existence of *haecceities* (higher-order properties such as *being identical to Plato*) in order to make sense of the second example claim along the lines of 'most haecceities that can be instantiated by a person will in fact never be instantiated by anything that is born'. (We cannot here do justice to the subtleties of the two versions of this route, nor to the differences between them. See Williamson 2016 and Goodman 2016 for further discussion.)

The two routes then converge on the *haecceities argument* (Williamson 2013: Ch. 6, §2; Fritz & Goodman 2017a: §3.2; see Fine 1985, Menzel 1990 for some precursors of this type of argument) intended to show that, in the end, higher-order necessitism requires necessitism. The main idea here is that one would otherwise have to accept that Plato's haecceity could exist without Plato, which is taken to constitute an unacceptable *explanatory embarrassment*: how could Plato's haecceity still single out Plato as its target when he is absent? There are two broad strategies contingentist can pursue in response to these arguments. First, they can try to resist the arguments for higher-order necessitism and defend what we may call uniform contingentism (the combination of first-order and higher-order contingentism).[12] Versions of uniform contingentism are developed in Fine 1977 and Stalnaker 2012, and further refined and critically examined in detail by Fritz & Goodman (2016, 2017a) and Fritz (2018a, 2018b). Second, they can try to resist the haecceities argument, proposing what we may call hybrid contingentism (the combination of first-order contingentism and higher-order necessitism); for instances of this strategy see Pérez Otero 2013 and Skiba forthcoming b.

In addition to these debates, which may be regarded as pertaining to the *extent* of possibility and necessity, higher-order resources have also begun to be employed in debates concerning the *nature* of modality. Thus Dorr (2016: 68-70) considers analysing metaphysical modality in terms of higher-order identity by understanding a claim of the form '$\Box P$' as '$P \equiv \top$' (see Dunaway 2013 and Roberts 2020 for further uses of higher-order quantification to elucidate the nature of modality). Finally, we are starting to see work incorporating both projects, with Rayo (2020) providing an argument for contingentism, partially based on a higher-order analysis of modal notions.

6. Conclusion

As illustrated by its application to theories of properties, propositions, identity, and modality, the project of higher-order metaphysics is geared towards a major reshaping of a comprehensive theoretical landscape. Against the backdrop of the rehabilitation of sui generis higher-order quantification, it aims to transmogrify a large variety of well-trodden areas of debate into more

[12] Here, the distinction between resolving and dissolving philosophical issues from §2 may find a further useful application: thus, we can think of necessitists as appealing to the argument from modal discourse in order to resolve (in their favour) the dispute between necessitism and contingentism. One way for contingentists to respond to this argument consists in maintaining that, contrary to first impressions, the problematic modalized quantifications lack well-defined content, thereby dissolving the problem of how they should account for them. For such a dissolution to be successful, they would have to find a way to block the necessitists' arguments (see Williamson 2013: Ch. 7) that contingentists must take the relevant claims to have content, on account of their similarity to other claims which contingentists accept as contentful. Thanks to an anonymous reviewer for helpful discussion on this matter.

fruitful metaphysical climes. In this regard, it is comparable to the research projects that have resulted from the rehabilitation of modal discourse (following the subversion of a another Quinean orthodoxy) and, more recently, the rehabilitation of notions of metaphysical ground. Qua large-scale transformational project, the overall success of higher-order metaphysics is unlikely to depend on the accomplishments of any given application but rather on the theoretical progress it is taken to yield in aggregate. Since the project is in full process, with new terrain charted and old ground dug over by the hour, it is too early to reach a definite verdict. Extrapolating from the advances it has already made, however, it is timely to attest its potential to have a major impact on the shape of metaphysics in the coming decades.

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