A Layperson's Understanding Of Bell's Theorem

By Vaclav Skutil

The modern notion of physicality was precisely defined by Albert Einstein in his 1948 letter to Max Born: that physics should adhere to "the requirements for the independent existence of the physical reality present in different parts of space"[1] Einstein's definition may be called Non-Contextual Local Realism. By 'Realism' we mean that stuff has definite measurable and observable properties.

By non-contextual, we mean that those properties exist even when not observed or measured, In his 1935 paper which he co authored with Nathan Rosen and Boris Podolsky (EPR), Einstein claimed that "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to that quantity."[2]

By 'Local', we mean that stuff cannot influence other stuff, faster than the speed of light, as stipulated by Special Relativity.

Realism of the non-contextual and local variety is also known as Classical Realism.

Einstein was unhappy with quantum particles having indefinite values prior to being measured. Quantum theory stated that unobserved particles are in a superposition of all their possible states described by their wave function.

Prior to measurement one could only calculate a probability that the particle would be in one state or the other. Once measured, this wave function collapses to a definite value, which could not be definitely predicted.

---

[1] A 1948 letter of Einstein to Born, in Born, M. 1971. The Born-Einstein Letters (New York:Walker).
[2] Einstein,A., Podolsky, B., and Rosen, N. 1935. "Can a quantum-mechanical description of physical reality be considered complete?' Physical Review 47: 777-80.

The EPR paper titled: ‘Can Quantum-Mechanical Description of Physical Reality be Considered Complete?’[3], argues that quantum theory is incomplete, and proposes that quantum particles, unknown to us, in fact have information about their states. This hypothetical information came to be known as ‘hidden variables’

EPR sets up a thought experiment to elucidate the conflict quantum theory has with Classical Realism. This conflict is called the EPR paradox.

EPR imagines a pair of particles being prepared in an entangled state, then separated at an arbitrarily far distance. Even when separated, the measurement of the same properties of particle A and B, must yield negatively correlated values, so as to comply with conservation rules.

For example if particle A is displaced at velocity +V to the right, then particle B must move at velocity -V in the opposite direction, to the left, so as to conserve momentum, so that +V - V = 0. If particle A spins clockwise (+) along axis Z, then particle B must spin counterclockwise (-) along the same axis Z, so that +spin(Z) + -spin(Z) = 0, so as to conserve angular momentum.

Imagine, particles A and B fly off in opposite directions to detectors A and B respectively. The detectors are 10 light seconds apart. In flight prior to being measured by the detectors, both particles A and B are in a superposition of clockwise and counterclockwise spin along the Z axis.

Quantum theory predicts that if detector A measures clockwise spin along particle A’s Z axis, collapsing its counterclockwise spin possibility, then, to preserve angular momentum, we know that particle B’s clockwise spin possibility on its Z axis must collapse instantly, revealing a counterclockwise spin measurement.

This instantaneous collapse of particle A and B’s wavefunctions, violates locality, since it cannot take a signal less than 10 seconds to pass

---

[3] Einstein, A; B Podolsky; N Rosen, 1935."Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" Physical Review. 47 (10): 777–780. Bibcode:1935PhRv...47..777E. doi:10.1103/PhysRev.47.777.

between detectors A and B. Einstein was unhappy with this interpretation, which he called ‘spooky action at a distance’.

He reasoned that the superposition of spin states in the wave function was an artefact of our ignorance, and that both particles were really encoded with a definite spin prior to separation, and which they maintained in flight prior to being measured, so that, there is no wave function collapse and particle A was measured to have clockwise spin because it had it all along.

In 1935 no one could think of a test that could decide between local hidden variables, and quantum state superposition. The problem with my simplified account is, that if you only test for spin along the Z axis, then you will always, by necessity of angular momentum conservation, measure the spins as negatively correlated.

How would you tell the difference between A ‘really’ spinning clockwise and B ‘really’, spinning counterclockwise from A ‘really’ spinning counterclockwise and B clockwise? There is no indexical sign pointing one way or the other..

In 1964, when it was still beyond the experimental capability to set up physical experiments to confirm or falsify Einstein’s notion of realism, the physicist “John Bell proposed an inequality that would test the concepts of locality and realism posited by” Einstein. “Bell’s inequality essentially stated that if the world operated under local realism (where physical properties exist before measurement and no influence can travel faster than light), then the results of certain experiments would be limited in a specific way. If these limits were violated, it would mean that either locality or realism (or both) would have to be abandoned”.[4]

Bell’s insight was to increase the degrees of freedom, across which multiple measurements could be made to reveal a statistical pattern. Applying this to the above example, imagine spins are measured across 3 different axes, XYZ. Also imagine that at each detector a random decision will be made along which axis to measure the particles spin.

---

[4] Introduction To The Bell Inequality For The Perplexed (quantumzeitgeist.com)

Through such a random mixing, spin pair correlations need not always be negative. May this set up an indexical sign pointing one way or another?

Here I will use and extend physicist Dr Brian Greene's rendering of David Mermin's simplified version of Bell's theorem.[5]

Imagine 2 entangled particles A and B, fired in opposite directions to their respective detectors. They may each be measured in 2 possible states : spin up ↑ or spin down ↓ along one of 3 axes: 1, 2, and 3. While the particles are in flight, the axis along which the particle's spin will be measured is separately and randomly chosen at each detector.

Assuming Einstein is right, then particles A and B are not in a superposition of all 8 spin states (2 possible spin directions along 3 independent axes: so 2x2x2 = 8) simultaneously, but are in fact encoded with definite spin direction on all 3 axes. Each particle pair is therefore in only one of the 8 possible configurations as follows:

Particle A :		1	2	3	Particle B:	1	2	3
Configuration	i	↑	↑	↑		↓	↓	↓
	ii	↑	↑	↓		↓	↓	↑
	iii	↑	↓	↓		↓	↑	↑
	iv	↑	↓	↑		↓	↑	↓
	v	↓	↓	↓		↑	↑	↑
	vi	↓	↓	↑		↑	↑	↓
	vii	↓	↑	↑		↑	↓	↓
	viii	↓	↑	↓		↑	↓	↑

[5] Greene, B.,2020. Your Daily Equation #21: Bell's Theorem and the Non-locality of the Universe:The World Science Festival (WSF): https://www.worldsciencefestival.com, https://www.youtube.com/watch?v=UZiwtfrisTQ&list=PLKy-B3Qf_RDVL6Z_CmgKf0tAbpXTua9mV&index=30&t=1746s

Particle pairs in configurations i and v, will be expected to be negatively correlated across all 3 of the randomly chosen axes. For example if spin on axis 1 is measured for particle A, and spin on axis 3 is measured for particle B, no difference in spin correlation would occur if the respective axes were changed to 2 and 3. This is not to be expected for the other configurations.

If axis settings are randomly chosen at each detector, then we should expect 9 pairs of axis settings, and their corresponding spin directions, for each of the 8 possible configurations.

For example, particles A,B in configuration ii : A↑↑↓ (123) B↓↓↑ (123) have the following spin direction, paired with the chosen axis measured :

11 ↑↓	**21**↑↓	31↓↓
12 ↑↓	**22** ↑↓	32 ↓↓
13 ↑↑	23 ↑↑	**33** ↓↑

Notice that the axis pairs in **red** have negatively correlated spins, while the rest are positively correlated. All configurations with mixed up and down spins, yield a 5/9 proportion of particle spin pairs as negatively correlated, and 4/9 as positively correlated.

Configurations i and v, have all their spin pairs negatively correlated.

If local hidden variables exist, then we should expect to observe negative spin correlations at least 5 out of 9 times (55.56%) and as high as 100%, in cases of uniform spin up and down configurations.

A Bell type inequality for local hidden variables may be set up as follows:

Imagine a particle generator that shoots particles A and B in opposite directions. The particles are randomly generated. Each shot generates a pair of particles in one of the possible 8 configurations. One could, for example, conduct 10 trials of 100 particle spin pair measurements in each trial run. For each trial sequence of 100 measurements you could measure the proportion of the 100 that are negatively correlated. Since particle generation is random one may expect that some trial runs will contain more particles configured in a uniform spin sequence relative to mixed ones and in other trial runs there may be less. Trial runs containing more will have a higher negative correlation percentage and those containing less will have a lower negative correlation percentage.

The percentage should move between 2 boundary limits. Trials containing only mixed configurations should yield 55.56% and trials containing only uniform configurations should yield 100%

Where % NCSP stands for percentage of Negatively Correlated Spin Pairs, the following inequality can be set up:

$$55.56 \leq \% \text{ NCSP} \leq 100$$

If the observed proportions of NCSP lie within this range, satisfying the inequality, then we may infer that the particles were in determined spin configurations, supporting the notion of hidden variables and Einstein’s claim to a locally determined reality.

The problem is, that in fact, negative spin correlations are observed only 50% of the time, violating the inequality and suggesting that the world may not be locally determined.

Alternatively, if Einstein wants to keep hidden variables which give particles their predetermined spin configurations, he must give up locality and assume the hidden variables are non local.

To unpack further, what this result means is that the actually observed proportions of NCSP were less than they should have been, so assuming no proportion greater than unity, the proportion of positively correlated spin pairs(PCSP) actually observed were higher than expected, if local determinism is true.

This is like throwing a die thousands of times, which lands on either 3 or 5, 3/6 of the time, rather than the expected 2/6 times. Reasoning like Einstein, we may suspect that the die has a hidden bias (hidden variable).

Normally we would look for the bias, locally in the die and suspect that the die is weighted on sides 3 and 5. Imagine our surprise if we find no such bias in the die.

Unlike the die we are familiar with, each 'throw'(separation of 2 entangled particles) generates a die in one of 8 possible spin configurations. Each configuration is separated into 2 mirror images (to preserve angular momentum). The 2 images 'land' at separate detectors. Each image dice has 9 possible 'sides' it can land on. Due to the necessary correlations between the 2 images, to preserve angular momentum, the die images are not expected to land on each side with equal probability 1/9.

As outlined above: The expected probability (Pe) of the die images landing on a NCSP= 0.5556, so conversely (Pe) PCSP = 0.4444.

Actual observations reveal a bias in favour of PCSP at 50%. How to account for this?

If particle spins were set at definite values at the moment of separation, wouldn't a higher proportion of PCSP violate angular momentum conservation?

PCSP's themselves do not violate angular momentum. They only have expected proportions in a predetermined configuration. However, if those predetermined configurations do not exist, and spin values emerge at the point of measurement, then PCSP and NCSP have no meaningful Pe in the classical sense.

In setting up his Inequality, Bell assumed measurement or statistical independence. This assumes that the choice along which axis to measure particle spin, made at detectors A and B are independent from any potential hidden variable. Also known as the ‘freedom of choice’ assumption. The choice of axis measurement settings at each detector cannot be determined in advance, for the inequality to be a meaningful test.

Although independence between settings made at detection devices A and B are made randomly, to eliminate correlation bias, one cannot test for global determinism.

Nobel Prize in Physics winner Gerard 't Hooft discussed this loophole with John Bell in the early 1980s:

“I raised the question: Suppose that also Alice's and Bob's decisions have to be seen as not coming out of free will, but being determined by everything in the theory. John said, well, you know, that I have to exclude. If it's possible, then what I said doesn't apply. I said, Alice and Bob are making a decision out of a cause. A cause lies in their past and has to be included in the picture". [6]

According to the physicist Anton Zeilinger, if superdeterminism is true, some of its implications would bring into question the value of science itself by destroying falsifiability:

The freedom of the experimentalist is always implicitly assumed. “This fundamental assumption is essential to doing science. If this were not true, then, I suggest, it would make no sense at all to ask nature questions in an experiment, since then nature could determine what our questions are, and that could guide our questions such that we arrive at a false picture of nature”[7]

Such superdeterminism would be a loophole, allowing for local hidden variables, without the need for superluminal signalling. It also violates criterion 3 and 5 as listed on pg 28. Since it is a difference that makes no difference, because it lacks empirical content, it is an unfalsifiable metaphysical assumption.

---

[6] https://www.scientificamerican.com/blog/critical-opalescence/does-some-deeper-level-of-physics-underlie-quantum-mechanics-an-interview-with-nobelist-gerard-e28099t-hooft/

[7] https://books.tarbaweya.org/static/documents/uploads/pdf/Anton%20Zeilinger%20Dance%20of%20the%20Photons%20From%20Einstein%20to%20Quantum%20Teleportation%20%202010.pdf p299

Bell's inequality also makes another classical realist assumption that all the spin values of the premeasured particle configurations have non contextual values, which should be independently definable.