Cartesianism and The Kinematics Of Mechanisms: Or, How to Find Fixed Reference Frames in a Cartesian Space-Time

EDWARD SLOWIK

Department of Philosophy
University of Alaska Fairbanks

Abstract

In *De gravitatione,* Newton contends that Descartes' physics is fundamentally untenable since the "fixed" spatial landmarks required to ground the concept of inertial motion cannot be secured in the constantly changing Cartesian plenum. Likewise, it is has often been alleged that the collision rules in Descartes' *Principles of Philosophy* undermine the "relational" view of space and motion advanced in this text. This paper attempts to meet these challenges by investigating the theory of connected gears (or "kinematics of mechanisms") for a potential Cartesian method of positing the permanent reference frames necessary to uphold Descartes' conservation law for the quantity of motion. In particular, the insights gained from an examination of the kinematics of mechanisms will provide the Cartesian with much needed resources to counter the two threats posed above.

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EDWARD SLOWIK*

Although much attention has been devoted to developing and refining Newton's absolute conception of space and time, an interest that has not abated over the years, very little recent thought has been dedicated to a similar treatment of the spatial and temporal notions that formed one of the main early competitors of the Newtonian view: namely, Descartes' "relational" theory (i.e., where space and time are regarded as only the relations among bodies, in contrast to the absolutist conception that considers them to be something "over and above" mere relations). Much of the neglect of the Cartesian hypotheses on space and motion stems, of course, from the belief that its internal inconsistencies and underdeveloped concepts render it useless as a foundation for Descartes' theories of bodily interaction, especially as advanced in his most important scientific tract, the *Principles of Philosophy.* Two such problems provide the motivation for the analysis undertaken in this essay: first (1), the alleged inconsistency between Descartes' rules of bodily collision and his professed relational view of motion; and, second (2), Newton's contention, in an early manuscript entitled *De gravitatione,* that the Cartesian plenum (i.e., a universe completely filled with matter) can not provide the "fixed" spatial locations, or frames of reference, necessary to make sense of Descartes' brand of inertial motion (rectilinear uniform speed). Contrary to the popular verdict, this essay will strive to show that Descartes' handling of space and motion is more consistent than has been generally acknowledged; and, in particular, that a "general" Cartesian theory (based on his fundamental laws and concept of relational motion) has the potential to provide a significant and highly effective response to these two problems if examined with the degree of latitude often accorded Newton's essential hypotheses.

Put differently, this essay will attempt to answer the following question: Is it conceivable to construct a Cartesian space-time that eludes Newton's critique while remaining faithful to the essential details of Descartes' physics, i.e., the laws of nature (which include the collision rules) and relational motion? As we shall see, if some sort of "fixed" reference frame or position can be located in the Cartesian plenum, then a more suitable basis for relationally tracking a body's motion can be established, thus dispelling both arguments (1) and (2). This achievement would simultaneously uphold Descartes' all-important conservation law for the quantity of motion (or product of size and speed). On this formulation of Cartesian physics, consequently, our concern will be to posit reference frames that can conserve Descartes' quantity of motion for extended regions of the plenum and for extended temporal periods.[1] Much of the ensuing analysis will draw inspiration from the modern theory of connected gears, also known as the "kinematics of mechanisms." As Mark Wilson has suggested (1993), a Cartesian space-time patterned on the theory of gears may possess the sort of "fixed" landmarks seemingly required by Descartes' physics--a possibility that we shall thoroughly explore.

Admittedly, the Cartesian theory we will develop owes much to recent mathematical and mechanical concepts, which may elicit a number of objections: such as, "How can the modern theory of gears reveal anything about Descartes' seventeenth century theory of motion?", or, "Why bother to investigate Cartesian physics since everyone knows that is obviously wrong?" Although it is certainly true that the forgoing analysis will not deal with Descartes' science in an exclusively seventeenth century setting, the sheer historical importance and, it will be argued, the potential fruitfulness of the basic concepts of Descartes' physics demand a thorough examination--and, if one is to conduct such an investigation, the use of modern conceptual devices can be a particularly effective means of revealing the fundamental strengths and weaknesses of the Cartesian theory. Our use of the kinematics of mechanisms analogy will seem doubly justified, moreover, since Descartes' treatment of the configuration and operation of vortices closely resembles many aspects of the modern theory of connected gears.

Likewise, as noted above, the "Newtonian" world view, broadly conceived, has greatly benefited from the adoption of new mathematical techniques over the preceding three hundred years. The basic laws of motion, the gravitational law, and a (usually weakened) commitment to absolute space have formed the conceptual core of numerous attempts either to consolidate the theoretical advances of a new age, or to resolve specific problems that Newton could not, or did not try to, answer. As an instance of the former, Newton certainly did not foresee the "Full-Newtonian" and "Neo-Newtonian" space-time incorporations of the revolutionary consequences of Einstein's theories;[2] nor could he have clearly anticipated, as an instance of the latter, the development of the analytical mathematical techniques, by Euler, Lagrange, the Bernoullis, etc., which eventually allowed the "Newtonian" theory to eclipse the Cartesian vortex theory.[3] Consequently, not only can the employment of recent mathematical methods in studying Descartes' theory provide possible insights into the formulation of modern relational space-time theories, even those not specifically based on Descartes' ideas, but the use of such techniques may even allow a certain "historical revenge" in that a Cartesian can finally employ the advantages that the "Newtonians" have taken for granted over the last two centuries.

After a brief presentation of the problematical aspects of Descartes' theories of space and motion, in Section 1.1, we will investigate both the Cartesian vortex hypothesis and Newton's main argument against it (Section 1.2). In the second section, we will compare the details of the kinematics of mechanisms theory with several features of Descartes' vortex hypothesis, thereby preparing the way, in Section 3, for a general examination of the viability of constructing a Cartesian theory of space and motion based on the insights gained from the theory of machine parts. In this last section, as well as the conclusion, we will asses the overall success of a suitably altered Cartesian space-time in resolving the purported internal inconsistencies, designated problems (1) and (2) above.

1. The Cartesian Theory and its Problems

1.1. Relational Motion. In the *Principles of Philosophy* (published in 1644), Descartes defines "external place" as the surface of the material bodies contiguous with a given object, which he also describes as the "neighborhood" of the contained body, while motion *"is the transfer of one piece of matter or of one body, from the vicinity of those bodies immediately contiguous to it [or, roughly, external place] and considered at rest, into the vicinity of others."* (Pr II 25)[4] This scheme is relationalist in the following sense: bearing in mind that Descartes' universe is completely packed with matter (a plenum), and provided his definition of "external place" as the surface of the containing bodies (which seems to be identical to the "neighborhood" of the contained body; see, Pr II 25-28), any attempt to regard the surrounding bodies as "at rest" must amount to a purely arbitrary stipulation, since "we cannot conceive of the body AB being transported from the vicinity of the body CD without also understanding that the body CD is transported from the vicinity of the body AB, and that exactly the same force and action is required for the one transference as for the other." (Pr II 29) He therefore infers that "that all the real and positive properties which are in moving bodies, and by virtue of which we say they move, are also found in those [bodies] contiguous to them, even though we consider the second group to be at rest."[5] (Pr II 30) Employing J. Earman's classifications (1989, 12), we can define this relationalist thesis of motion:

R1: All motion is the relative motion of bodies, and consequently, space-time does not have, and cannot have, structures that support "absolute" quantities of motion.

Although Descartes' views, thus far, resemble closely the relational theories of motion of his Scholastic forebears, his *Principles of Philosophy* puts forth a series of natural laws on motion that appear to undermine this relational view. Consider the second law: "all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle which they are describing." (Pr II 39) According to (R1), however, motion (or the lack their of) can only be meaningfully understood relative to some arbitrarily chosen body or reference frame. Hence, there can be no "absolute" or "actual" determinations of an individual body's state of rest or motion, since the body will be assigned numerous conflicting values of motion relative to these varying perspectives. Likewise, because trajectories are determined relative to each observer, and all observers are in relative motion (as above), it is not possible to determine the "actual" or "absolute" path of each individual body. Thus, Descartes' second law of nature would appear to violate his professed relationalism; as do many of the seven collision rules that constitute the particular instances of his third natural law (for the conservation of the total "quantity of motion," or product of size and the scalar quantity speed, $m|v|$, of all material bodies).

In fact, Descartes' collision rules (which explicate his third natural law) depict bodily impact in a decidedly non-relational fashion, as is most prominently demonstrated in the case of collision rules four and five. The fourth rule involves the interaction of a small moving body with a larger resting body. On his estimation, the larger object possesses a resisting force that deflects the smaller moving body back along its path, guaranteeing that the larger body will remain in a state of rest during the collision.

> Fourth, if the body C were entirely at rest, . . . and if C were slightly larger than B; the latter could never move C, no matter how great the speed at which B might approach C. Rather, B would be driven back by C in the opposite direction: . . . . (Pr II 49)

Despite the unintuitive nature of this outcome, the real problem for the relationalist concerns the conjunction of Descartes' fourth rule with his fifth rule:

> Fifth, if the body C were at rest and smaller than B; then, no matter how slowly B might advance toward C, it would move C with it by transferring to C as much of its motion as would permit the two to travel subsequently at the same speed. (Pr II 50)

In the collisions comprising this law, Descartes' reckons that a large body will always move a smaller stationary object, transferring to it as much quantity of motion as is required to allow both bodies to travel at the same speed and along the same line while conserving the total quantity of motion.

For the relationalist, the contradiction in Descartes' analysis is painfully evident--and constitutes the first, (1), problematical aspect of this theory as outlined in the Introduction. From the perspective of a relational theory, rules four and five constitute the *same* type of collision, or an equivalent state of affairs, but Descartes draws completely different conclusions from each: in the fourth rule, one of the bodies reverses its direction after impact, while in the fifth rule, the collision brings about the joint motion of both bodies along the same course. Put another way, since a relationalist is confined to a relative notion of speed (as above), both rules present an identical scenario to the relationalist. Therefore, because they represent the same physical events, a consistent relational theory must derive the same results from the situations outlined in collision rules four and five.

The failure to foresee this simple relational fact must give the Cartesian cause to question Descartes' confessed devotion to his brand of Aristotelian/Scholastic relationalism. In fact, as Daniel Garber has convincingly argued, such developments would seem to indicate that Descartes did not accept the "strong" form of relationalism defined above; which Garber dubs the "reciprocity of transfer", and we have labeled as (R1).[6] Descartes' treatment of "rest" and "motion" as *different states* of bodies also seems to corroborate Garber's contention, since this thesis would effectively assign individual component velocities to Cartesian bodies, in direct violation of (R1).[7] (See, Pr II 37)

If we bear in mind that Descartes unequivocally rejected the possibility of a vacuum in the physical world (Pr II 16-18), then a more fruitful form of Cartesian relationalism immediately avails itself. We can define this alternative relationalist scheme (Earman 1989, 12) as follows:

R2: Spatiotemporal relations among bodies are direct; that is, they are not parasitic on relations to a substantival space that underlies bodies.

Essentially, (R2) rejects the view that space is "substantival", or "absolute"; i.e., that it is a form of substance or entity that exists independently of matter. As a result, (R2) can now permit the introduction of reference frames into the relational space-time (for the determination of motion) without violating relationalist tenets. More specifically, *relative* to a reference frame or set of frames, the individual components of motion among moving bodies can now be meaningfully determined (although the individual state of motion of a single reference frame, or among several such frames, depends on which space-time structure is adapted to the (R2) relationalism).[8] This form of relationalism has the advantage of nicely accommodating Descartes' actual treatment of motion, which, as we have seen, violates (R1), alongside his repeated denials that space and time (and hence motion) are anything more than relations among bodies: e.g., "the names 'place' or 'space' do not signify a thing different from the body which is said to be in the place." (Pr II 13; see also, Pr II 8-18) In effect, (R2) replaces substantival space or vacuum with a sort of "absolute" reference frame--a frame attached to a material body or point which serves as the basis for ascertaining the motions of all other bodies. Yet, if Descartes' treatment of motion tacitly presumes this kind of notion, then where in the Cartesian plenum are they located, and can they be identified? Before exploring these issues, however, we will need to examine our second Cartesian "problem".

1.2. The Cartesian Vortex and Newton's De gravitatione Argument. Due in part to his acceptance of a plenum and relational motion, Descartes was convinced that a treatise on natural philosophy could incorporate Copernican astronomy without running afoul of the Church's ban on theories of terrestrial motion. Descartes hoped to avoid this problem by placing the earth within a vortex of minute particles circling the sun, and by further demanding that the earth not change its neighborhood of contiguous bodies, or "external place" (as defined above). Through this ingenious bit of reasoning, Descartes could then claim that the earth does not move, since motion is a change of external place, and yet maintain the Copernican hypothesis that the earth orbits the sun (see, Pr III 24-30). On Descartes' scheme, the universe operates as a network or series of separate interlocking vortices, with each vortex housing an individual planetary system or celestial body. In our solar system, for example, the particulate matter within the vortex has formed into a set of stratified bands, each lodging a planet, that circle the sun at varying speeds.

When drafting his early essay, *De gravitatione,* Newton was fully cognizant of the specific features of the Cartesian vortex hypothesis. In a powerful argument, one of many directed against the overall concept of Cartesian motion, he criticizes Descartes' relational theory on the grounds that it cannot supply the absolute spatial positions deemed necessary to explicate bodily motion, especially the concept of inertial motion often utilized in the *Principles* (see, Section 1.1). Without a notion of "same spatial position over time," Newton believed that Descartes' universe could not coherently define rectilinear uniform motion, since there would exist no means of comparing a body's change in position over time. To demonstrate his point, Newton appealed to details of Descartes' vortex theory:

> I say that thence it follows that a moving body has no determinate velocity and no definite line in which it moves. . . . But that this may be clear, it is first of all to be shown that when a certain motion is finished it is impossible, according to Descartes, to assign a place in which the body was at the beginning of the motion; And the reason is that according to Descartes the place cannot be defined or assigned except by the position of the surrounding bodies, and after the completion of a certain motion the position of the surrounding bodies no longer stays the same as it was before. For example, if the place of the planet Jupiter a year ago be sought, by what reason, I ask, can the Cartesian philosopher define it? Not by the positions of the particles of the fluid matter, for the positions of these particles have greatly changed since a year ago. Nor can he define it by the positions of the Sun and the fixed stars. For the unequal influx of subtle matter through the poles of the vortices towards the central stars (Part III, Art. 104), the undulation (Art. 114), inflation (Art. 111) and absorption of the vortices, and other more true causes, . . . , change both the magnitude and positions of the stars so much that perhaps they are only adequate to designate the place sought with an error of several miles; and still less can the place be accurately defined and determined by their help, as a Geometer

> would require. . . . And so, reasoning as in the question of Jupiter's position a year ago, it is clear that if one follows Cartesian doctrine, not even God himself could define the past position of any moving body accurately and geometrically now that a fresh state of things prevails, since in fact, due to the changed positions of the bodies, the place does not exist in nature any longer.
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> . . . So it is necessary that the definition of places, and hence of local motion, be referred to some motionless thing such as extension alone or space in so far as it is seen to be truly distinct from bodies. (Newton 1962a, 129-130)

As Newton correctly points out, the Cartesian vortex is a system whose constituent bodies and particles are in a constant state of flux. For instance, Descartes maintains that a vortex may undergo a "shrinking" or collapsing stage, a process which relinquishes the matter of the vortex to its adjacent neighbors. Descartes drafted this complex hypothesis in an effort to reconcile his vortex theory with the irregular motions of comets: "It can also happen that an entire vortex that contains some such star [at the center of the vortex] is absorbed by the other surrounding vortices and that its star, snatched into one of these vortices, becomes a Planet or a Comet." (Pr III 115) Hence, due to the instability of the plenum, Descartes' concept of place apparently cannot sustain fixed spatial locations for any indefinitely long period of time. If one were to attempt to utilize a body in the plenum, or the contiguous particles surrounding a body, as a relational means of securing the fixed frames of reference needed to determine velocity, via (R2) above, the flux of the plenum would consistently thwart this process by dislocating and disintegrating the relational reference frames. (This problem with Descartes' theory we labeled (2) in the Introduction.)

As revealed in the above quotation, Newton's argument contains a strong epistemological component. That is, he seems intent on demonstrating that the Cartesian theory of place and motion can only provide, at best, a very inaccurate approximation of the positions, and hence velocities, of bodies over time: e.g., "[the positions of the stars] are only adequate to designate the place [of Jupiter] with an error of several miles." Since the stars are likely to alter their relative positions due to the ceaseless flux of the plenum, they cannot furnish reliable estimations of place and motion. However, it is important to note that Newton seems to be offering, or combining, two separate arguments against the Cartesian theory: (1) the epistemological criticism just noted, which centers upon the plenum's inability to secure accurate measurements of a body's place and motion, and (2) an ontological problem that "due to the changed positions of the [moving] bodies, the place does not exist in nature any longer." In other words, when a given body moves, its place no longer exists, since "(external) place" is defined as the common surface between the contained and containing surface--a surface which is irrevocably lost once the displaced body takes on a new set of contiguous neighbors. This form of reasoning, in my opinion, is best interpreted as an ontological criticism of Descartes' theory; for it claims that the very *concept or meaning* of velocity (speed) is not definable given the Cartesian doctrine of external place.

Of course, regardless of whether we adopt an epistemological or ontological interpretation of Newton's argument (which is the only argument from the *De gravitatione* against Cartesian motion that we shall consider), his conclusion still seems to involve the notion of a substance-like entity independent of matter, i.e., "absolute space" (ibid., 132)--a concept that a relationalist, and even a Neo-Newtonian, may have good reason to reject. Nevertheless, Newton's insight, that a plenum without absolute space constitutes an environment hostile to a relational determination of speed, velocity, acceleration, etc., is of singular importance, especially when translated into the context of our search for an (R2) reference frame.

2. The Kinematics of Mechanisms and Cartesian Space-Time.

In this section, we will begin to explore the possibility of constructing a Cartesian space-time that can overcome problems (1) and (2), from Section 1 above, while conserving a relational quantity of motion, or size times speed,[9] by means of fixed reference frames. This space-time will also need to closely adhere to the spirit of Cartesian physics by taking into account the kind of harmonious, interconnected motions typical of a Descartes' plenum. In order to examine these issues more closely, it will be fruitful to correlate the Cartesian program with a more recent mechanical theory that investigates many aspects of equivalent problems. Entitled the "kinematics of mechanisms" (**KM**), this branch of physics analyzes systems of rigid mechanical linkages, such as an array of connected gears. On the whole, many of the worries that would motivate an engineer in constructing an elaborate series of gears relate directly to the obstacles encountered in attempting to comprehend motion in the Cartesian plenum, especially with respect to vortex motion.

2.1. Vortices and Gears Compared. The theory of machine parts and Cartesian physics are similar in many ways. In an elaborate set up of gears, for example, the movement of one cogwheel entails a determinate motion of all the other cogwheels connected to the system. This parallels a similar situation confronted in the Cartesian universe where the displacement of one particle inevitably results in the vast (circular) movements of others: "It has been shown . . . that all places are full of bodies . . . . From this it follows that no body can move except in a complete circle of matter or ring of bodies which all move at the same time." (Pr II 33) Circular motion is necessary for Descartes because there exist no empty spaces for a moving object to occupy. If the motion were not circular, the movement of a single body in his "indefinitely" large universe would result in an equally indefinite material displacement, an outcome that may violate the Cartesian conservation law. Consequently, on Descartes' own admission, the predominate interactions among the material constituents of a plenum, as well as for an assembly of gears, would seem to be some form of interlocking uniform motion, and not the direct impact as spelled out in his collision rules.

Descartes' concept of an "inclination" towards motion fits naturally into this comparative project, moreover. In attempting to achieve the goal of a kinematically-oriented description of material body interactions, Descartes located a body's "tendency" or "inclination" to move, which can be roughly conceived as its "force" of motion (measured by quantity of motion--Pr II 43), at the level of individual instants; while conceiving motion as a process that only occurs over a finite temporal interval. "No movement is accomplished in an instant; yet it is obvious that every moving body, at any given moment in the course of its movement, is inclined to continue that movement in some direction in a straight line." (Pr II 39)[10] The Cartesian objective, in simplest terms, is to analyze dynamic interactions, which seem to involve bodily forces, while confined to the language of the relative motions of particles, a purely kinematic approach that is directly analogous to the "interactions" of connected gears (see, Garber 1992, 218-221, for an extended discussion of Cartesian "tendency"). Descartes illustrates the underlying affinity or kinship of these two programs in a very revealing passage from the *Principles* concerning the "strivings" of secondary element matter:

> When I say that these little globules strive, {or have some inclinations}, to recede from the centers around which they revolve, I do not intend that there be attributed to them any thought from which this striving might derive; I mean only that they are so situated, and so disposed to move, that they will in fact recede if they are not restrained by any other cause. (Pr III 56)

If "not restrained by any other cause", bodies will thus move because "they are so situated, and so disposed." When we conjoin this hypothesis with his belief that inclinations or strivings are instantaneous, the following picture of Cartesian "inclinations" begins to emerge: on each time slice, the configuration of all plenum bodies--their relative disposition and situation--determines how a single body in the system *can* move; that is, how the arrangement of the whole affect or inhibits the motion of a single body. Much like machine parts, the possibilities for movement of a single body are governed by the interconnections of all material objects, since a given body can only move if the resulting displacement of bodies is harmonious and does not lock-up the system (which explains the reason for mass circular motions noted above). Or, to put it differently, the configuration of all bodies determines how the motion of one body instantaneously affects all the others (as with connected gears). Finally, if one envisions a succession of several instants, the inclination of a body at each separate instant gives rise to its speed, and thus the measurement of its quantity of motion; since, as Descartes states, speed is a higher-level phenomenon that is manifest over a span of time.

Gears and plenums likewise must be designed so that the motion of their constituent parts are compatible and harmonious, and will not lock or jam. A "lockup" can occur in machine parts when the motion of single gear is prevented through its connection to two oppositely rotating cogwheels. For the Cartesians, this complication would translate into a collision, or "blending," of particles from several divergently rotating vortices. Overall, Descartes was well aware of the need for an effective and harmonious positioning of neighboring vortices: "No matter how these individual vortices were moved in the beginning, they must now be arranged in harmony with one another so that each one is carried along in the direction in which the movements of all the remaining surrounding ones least oppose it." (Pr III 65) In Part III of the *Principles,* Descartes presents a number of hypotheses on the mutual ordering of vortices that reveal a deep understanding of the problems that can beset arranging mechanical systems. Even though many of these constraints on the ordering of vortices are intended to satisfy his hypothesis on the flow of subtle matter, the above quotation is clearly aimed at forestalling the collision or interference of adjacent vortex rotations. Within these Articles, Descartes describes in painful detail the configuration that is necessary to prevent the "opposition" or clashing of the rotational motion of four neighboring vortices:

> The laws of nature are such that the movement of each body is easily turned aside by encounter with another body. Accordingly, if we suppose that the first vortex, the center of which is S, is rotated from A through E toward I, the other vortex near to it, the center of which is F, must be rotated from A through E toward V if no other nearby vortices prevent this; for thus are their movements most compatible. And in the same way, the third vortex, which has its center, not on the plane SAFE, but above it (forming a triangle with the centers S and F), and which is joined to the other two vortices AEI and AEV on the line AE, must be rotated from A through E upward. (Pr III 65--see Figure 1)

Figure 1. This is a simplified illustration of the harmonious configuration of Descartes' vortices in the *Principles,* Part III, Art. 65 (Plate VI). The third vortex, which is suppressed in Descartes' original figure, lies above the plane of the other two.

2.2. The Kinematics of Mechanisms Theory and Cartesian Space-Time. In devising a Cartesian space-time[11] according to **KM**, certain requirements need to be met. (The technical details of this section owe much to the discussion in Wilson 1993, 216-218) First, a time function must be established that partitions the events in space-time into simultaneity classes. This is usually depicted as the carving of space-time into a series of "time slices" or spatial planes, with each slice representing all the existing material bodies at a particular temporal instant. Second, the spatial geometry on each slice must be three-dimensional and, probably (but not essentially), Euclidean[12]; and a time metric is imposed on the space-time to uniquely order the time-slices. So far, these conditions can also be found in Newtonian space-time, but not enough structure has been added to make it obviously "absolutist." Since we want to insure that particles can be tracked through time

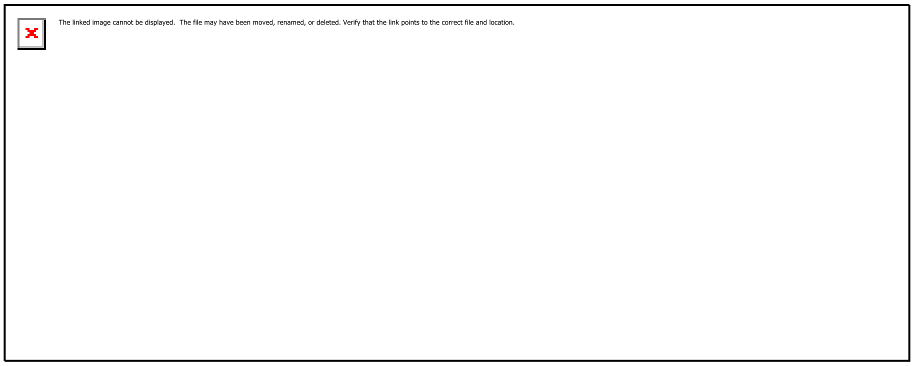

Figure 2.

while preserving the topology of their local connections, the space-time of our connected gears requires a device or function that will identify the same material particles or points between time slices. This is accomplished by defining a map Ψ from the material points on a reference set of machine parts (or bodies) to the same points on each slice. The mapping Ψ ensures that our mechanical gears remain rigid over time by maintaining the same distance relations among their various parts and material points (see Figure 2). In addition, all the machine parts located on a time slice must be interconnected via some mechanical process that prevents the slipping of gears at their contact points (i.e., the point where two meshed gears touch). Suppose, for instance, that two material points, p and q, that are situated on separate mechanical gears are in contact at a time t_0. Next, divide the relative arc length displacement $\bar{d}_p$ of p away from the previous contact point (on t_0) by the change in time Δt (i.e., $t_1 - t_0$) to obtain the value $\bar{d}_p/\Delta t$. One can avert the slipping of gears, consequently, by demanding that $\bar{d}_p/\Delta t$ varies smoothly and equals $\bar{d}_q/\Delta t$ as the limit of Δt approaches zero. (The relative displacement of points on rotating gears can also be determined by fixing three points. See, Zimmerman 1962, 29)

Another important component of **KM** is the notion of a "fixed space" or landmark between temporal slices. Ideally, if one wishes to employ a *relative* velocity function in our space-time, it will be necessary to establish temporally fixed reference frames so that the map Ψ can assign a relative displacement, and hence velocity, to all material particles. This can be accomplished in a number of ways: one can simply tie the reference frame to a

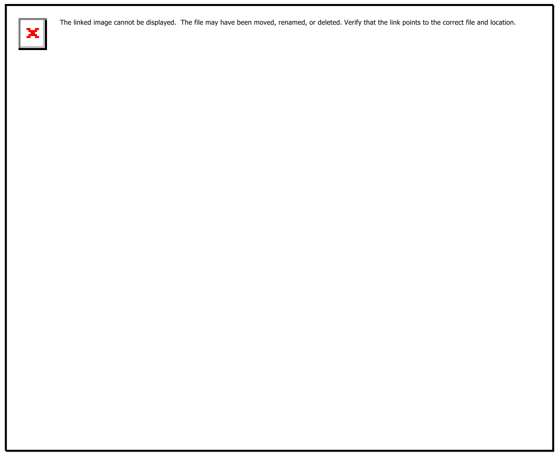

Figure 3.

material point picked out on each time slice by the mapping Ψ ; or one can choose an enduring geometric feature of the overall configuration of gears as the preferred reference point.[13] On the latter procedure, a contact point between two gears would naturally serve the role of a fixed space in our theory, since the only permanent locations on the contact surface of moving machine parts are the places where they touch. Hence, provided Ψ and the mapping Φ of a unique contact point across temporal slices, the velocity of a point p (on a gear A) is easily obtained by measuring the difference in relative arc length displacements $\bar{d}_p$ of p between slices (see Figure 3--the mapping of p from the reference set Ψ has been suppressed in this illustration). It should be noted, furthermore, that there is no limitation on the temporal duration of the Ψ and Φ mappings (although $\bar{d}_p/\Delta t$ is instantaneously defined, of course, as the limit of Δt as it approaches zero). The temporal extent of these mappings depends on the stability of the gear system. However, in Section 3, we will explore the possibility that the intrinsic instability of the Cartesian plenum may confine these mappings to a very brief temporal duration (perhaps infinitesimal, just as long as it can allow the limit of $\bar{d}_p/\Delta t$ to be established).

In many respects, the details of the **KM** program correlate nicely with our earlier characterization of Descartes' plenum theory. For instance, via the natural laws (as above), Cartesian space-time invokes a notion of rectilinear uniform speed, or velocity,[14] that necessitates a comparison of "information" across time slices, a concept that also features in the theory of gears. More specifically, the motion of both a Cartesian body and a point on a gear can only be determined over a series of successive temporal slices. One must examine the difference in a body's displacement between slices (relative to a reference frame)--the "information" is simply the displacement on each slice--in order to attribute this property to the body on a single time slice. Of course, **KM**, like all modern dynamic theories, has a perfectly meaningful concept of instantaneous velocity defined at an instant; namely, the derivative of the position function. But calculating the derivative of a particle trajectory requires a span of time: as defined above, it is the limit of the function as the change in time approaches zero. The motion of a Cartesian body, if not incorporating infinitesimal quantities of this exact sort, is nevertheless characterized in a similar way; i.e., as requiring some means of *transmitting information across temporal slices* on a body's displacement (although Descartes does not employ these sophisticated mathematical techniques). Moreover, even if Descartes' understanding of motion cannot be isolated from duration or a succession of time slices ("no movement is accomplished in an instant," (Pr II 39)), his conservation law for the quantity of motion does appear to implicate instantaneous processes or magnitudes (e.g., his account of "strivings", or "the force to continue in motion", Pr III 121-122; see Section 2.1). Thus, Descartes' theory is much closer to the instantaneous magnitudes of the modern formalism than has been often acknowledged.[15]

If Cartesian speed can only subsist over a series of time slices, many tenets of **KM** may look appealing to a relationalist engaged in reworking Descartes' laws of motion. Foremost among these properties of the theory of gears is the use of the landmark mapping Φ to ground the measurements of relative velocity, since these landmarks would essentially guarantee the determination of relational motion as mandated in Section 1. If some form of analogous *enduring* landmark can be established in the Cartesian plenum, a relationalist can provide a coherent system for determining relative speed according to the tenets of (R2) by simply constructing a relatively "fixed" reference frame for an extensive region of Descartes' plenum, possibly even the entire cosmos. From these landmarks, the *individual* motions of all material bodies will be measured, thus securing a means of resolving relational contradiction between the fourth and fifth collision rules--our problem (1) above from Section 1.1. More carefully, given a meaningful measurement of the individual states of motion of the impacting bodies, the types of collisions depicted in rules four and five are now relationally different or unique. In rule four, the larger body really is at rest and the smaller body really is in motion (relative to the reference frame, of course), and visa-versa for rule five.

However, before the Cartesians can endorse the landmark mapping Φ as a means of resolving the inconsistency in Descartes' handling of the collision rules, problem (1), they will need to refute Newton's allegations concerning plenum instability, our problem (2). As translated into the language of **KM**, Newton's argument contends that a landmark or Φ based conception of linear uniform speed ultimately fails due to the lack of a constant Φ in Descartes' universe. Unlike our series of connected gears, there exist no immutable landmarks or contact points in the Cartesian plenum to ground the computations of these motions. Consequently, if one accepts the view that Cartesian space-time resembles a series of connected gears, Newton would have to insist that nature persistently conspires to detach the connections among the machine parts, thus dispelling the contact points. In fact, the constant flux of the primary particles in Descartes' universe nicely demonstrates the mutability of the connections among the "gears" in our Cartesian kinematics of mechanisms analogy: just as the particles in the plenum consistently change their relative position, a similar process must transpire as regards the gears in our Cartesian machine-part universe. That is, the gears (particles) will partake in a continuous shifting of their relative positions and a changing of their mutual connections, meshing and unmeshing with a host of different particles (gears).

Alternatively, one can interpret Newton's argument in this context as a straightforward denial of the smoothness of the connections among the particles (gears) required for $\partial_p/\Delta t$. Likewise, because there are no completely impenetrable or indestructible bodies in Descartes' plenum--all material elements can shatter under sufficient impact (see, e.g., Pr II 20, and II 48)--rigidity cannot be guaranteed. As a direct consequence, either the slipping of the particles (gears), or their lack of rigidity (i.e., the Ψ mapping cannot maintain a fixed separation among the parts), can be viewed as primarily responsible for the lack of a workable landmark-based measure of motion. Regardless of the particular cause of this instability, a world without permanent contact points is a world that still requires a permanent notion of uniform speed (velocity). As presented in the *De gravitatione,* Newton's "Jupiter" example was intentionally designed to demonstrate this very point: although the Cartesian plenum exhibits no enduring landmarks, it is clearly the case that the trajectory of Jupiter is determinable.

3. Developing a Kinematics of Mechanisms Cartesian Space-Time

3.1. Locating Φ*-Landmarks in the Cartesian Plenum.* But not all hope is lost for the Cartesian, for we have yet to examine the available options for locating the elusive fixed landmarks in Descartes' plenum that can evade our problem (2), and thus obtain a means of resolving problem (1). In fact, as previously argued, a close inspection of the Cartesian vortex theory can reveal many hidden facets of Descartes' understanding of mechanical systems; an awareness of the complexity of vortex ordering that may suggest possible methods of devising Φ-landmarks.

Among the basic operating principles of the vortex, the movement of the first particles of matter (or subtle matter) figures prominently. Briefly, Descartes reckons that a significant amount of subtle matter perpetually flows between adjacent vortices: as the matter travels out of the equator of one vortex, it passes into the poles of its neighbor. This hypothesis is an integral component in his story of vortex "collapse" (Pr III 115-120). Under normal conditions, particles of subtle matter flow from the poles into the center of the vortex (i.e., the sun); then, due to centrifugal force, the particles "press out" against the surrounding secondary globules as they begin their advance towards the equator (ibid., 120-121). However, since the adjacent vortices also possess the same tendency to swell or increase in size, centrifugal force prevents the encroachment of neighboring vortices by setting up a balance of mutual expansion forces. On occasion, a debilitating condition of the sun (identified as sun spots) may conspire to prevent the incoming flow of first element matter from the poles. Gradually, as all the first matter is expelled at the equator, the sun can no longer press against the secondary globules, and the vortex is engulfed by its expanding neighbors.

For our purposes, an analysis of these "intervortex" relationships is important because it may provide a suitable candidate for an invariant or unchanging contact point. Although the possibility of vortex collapse effectively dashes any hope of locating a permanent landmark within a vortex, it does not automatically dismiss the potential existence of such landmarks *between vortices.* In particular, since Descartes envisions vortex collapse as a gradual expansion process, the contact points will be continuously maintained between the remaining adjacent vortices. These mutual connections may gradually shift or alter position as the rotating masses increase in size, but the enormous forces that lock the vortices together can, it is hoped, ensure that the contacts are not dissolved by separation, and that any displacement or dislocation of these points will *generally* occur smoothly without slipping (as required to supply the measurements of velocity). In contrast to a more random, chaotic configuration of vortices, the enormous pressure exerted between the interlocked vortices will, on this reading of Descartes' project, greatly decrease the potential for the slipping of particles (gears). Of course, this strategy cannot completely guarantee that only smooth connections will prevail among

Descartes' vortices; and, hence, it can only diminish or mitigate the force of the Newtonian argument. Nevertheless, a Cartesian can go a long way towards resolving Newton's problem by employing these intervortex contacts as the basis for a mapping Φ. This mapping would effectively equip his space-time with a quasi-fixed reference frame or class of such frames--the intervortex contact points--for the (R2) relational determination of bodily motions within each vortex (as noted in Section 1). Finally, it should be noted that the idea of a "contact point" in a Cartesian plenum is somewhat of a misnomer, since only the three primary elements of matter would retain the necessary rigidity to provide anything remotely close to a point-like connection between surfaces. Since Descartes views most, if not all, macroscopic bodies as "elastic" (due to the presence of bodily pores which can contract under pressure; see, Pr IV 132), the contact between vortices would more resemble an extended two-dimensional surface or band, rather than a point.[16]

Besides the ever present threat of non-smooth connections, Newtonians may dispute the legitimacy of this stratagem on other grounds, however. They would most likely insist that Descartes never envisioned the contact points among vortices as a prerequisite structure for his concept of speed (velocity). He defined motion, you will recall, as the transfer of contiguous bodies *within* a vortex; a process, moreover, that would not seem to necessitate the existence of any intervortex relationships. Yet, as argued in Section 1, modern "Newtonians" (of either the Full-, or Neo-, variety) are not in a favorable position to criticize others for taking conceptual liberties with historical space-time theories, since the product of their efforts are a far cry from Newton's account of these issues. For example, would a reemergent Newton accept the Neo-Newtonian metric tensor, **g**(**u**,**v**), and covariant derivative, $\nabla_{\mathbf{V}}\mathbf{V}$ (or $d^2x^\alpha/d\lambda^2 + \Gamma^\alpha_{\mu\nu}(dx^\mu/d\lambda)(dx^\nu/d\lambda)$ in the coordinate frame), as an updated reflection of his conception of absolute space and time (which was as embedded in theological concerns as much as it was in physical problems)? In essence, there is much to be learned about the structure and limitations of *both* the Newtonian and Cartesian theories when set within the modern mathematical formalism, hence the modern Cartesian should be allowed equal access to these techniques.

As a second difficulty with the intervortex contact points, even if one permits a Cartesian to exploit this method, is securing a consistent basis for velocity in a potential world consisting of only one vortex (or none)? Inasmuch as contact points require at least two vortices, one could not locate a coherent notion of velocity in such a universe, while any Cartesian efforts to exclude this scenario from the domain of possible evolutionary states of the plenum would appear an unjustified and ad hoc restriction. Interestingly, in the *Principia,* where the main assault on the Cartesian vortex theory appears (in Book II), Newton contends that the inherent instability of the stratified layers in a planetary vortex, such as our solar system, would inevitably lead to their "blending" and certain destruction (by possibly forming one large undifferentiated universal vortex?--Newton, 1962b, 391).

In response to these troubling possibilities, a Cartesian may fall back on the text of the *Principles* and point out that these scenarios are simply ruled out as possible states of Descartes' universe; or, in other words, since Descartes did not discuss it, it must not be possible in his plenum. Despite the unappealing tone of this form of response, Newton's allegation of vortex instability does force one issue to the forefront of our attempts to locate a Φ-landmark: a Cartesian may decide in the light of these difficulties to abandon the search for a permanent landmark, while concentrating instead on the class of transitory or temporary contact points between vortices. Given the details of Descartes' cosmology, wherein he assumes that there will always exist numerous vortices, temporary contact points (or surfaces) will always exist. Moreover, many such points also exist in a planetary system, since each planet is transported around the sun in its own circling band of secondary particles. Hence, the centrifugal force exerted by these rotating bands will guarantee the availability of an entire class of potential "intravortex" landmarks. The circling bands that comprise a vortex will be locked together by their mutual expansion, thus (hopefully) providing an array of smooth connections (notwithstanding the difficulties mentioned above). Although the fluctuating nature of the plenum dictates that these contact points will endure only temporarily, possibly only infinitesimally, it is equally true that there will always be a landmark available. These landmarks will change over the course of time, and may be of infinitesimal duration, but every time slice will have recourse to at least one Φ-based frame for determining motion. Therefore, adapting Descartes' natural laws to a set of temporary intervortex, or intravortex, landmarks might serve as the Cartesian's last line of defense against the Newtonians.

A Newtonian may willingly grant the Cartesian this procedure for establishing landmarks, since such reference frames are only temporary and not permanently fixed. Nevertheless, it remains unclear if such methods can successfully provide the foundation for a coherent notion of speed (velocity); that is, if it can resolve the relational contradiction between collision rules four and five, which we defined as problem (1). In the next section, we will examine problem (1) updated to the context of **KM**, for several important insights into the nature of Cartesian space-time can be gleaned from such a setting.

3.2. Dead Points and the Collision Rules. There are various ways of interpreting the nature of problems (1) and (2). One method of construing problem (2), for instance, is to regard Newton's argument as pertaining to the lack of a suitable method of comparing information on the status of bodies, particularly, position, between time-slices. Due to plenum instability, the ties between time slices, Φ, that serve as the means of comparing information on the positions of bodies relative to a Φ-based frame, are constantly severed; which brings about a corresponding loss of information on the positions, and hence motions of bodies (or "striving", if we consider Descartes' quasi-infinitesimal quantities). Yet, similar "under-determination" difficulties, as we may dub this information-transfer debacle, also arise even if we grant the Cartesian a regular system; that is, even if we ignore Newton's instability argument (problem (2)). To demonstrate this point we will need to return to our analysis of machine parts.

In constructing an array of connected linkages, engineers strive to eliminate what are deemed "dead points" from their chosen configuration. Briefly, a gear reaches a "dead point" when its future motion is not determined by the instantaneous actions or motions of the other gears located on the same time slice. Two options are generally presented at such points: either the linkage can proceed forward or reverse its direction. A "dead point" in the motion of two connected gears is often presented in the scenario depicted in Figure 4 (see, Wilson 1989, 509-512). Assuming the left wheel is driven counter-clockwise, when the linkage reaches the position at time t_0 illustrated in (i), it has two options: It can proceed along the same circular route (ii), or it can reverse its direction (iii) resulting in one of the two depicted scenarios at time t_1. In either case, the configuration of the gears cannot, by itself, determine the unique evolution of the system beyond t_0 (i). Yet, this outcome is not constrained in any way by the current dispositions of the surrounding gears. One must bring forth additional information or methods in order to determine the future course of the machine parts. "In practice, dead points must be avoided or external means provided to carry the mechanism past a dead point." (Zimmerman 1962, 123) Often, the method or

Figure 4.

means of carrying a gear past a dead point is its inertial motion. The unique evolution of the gear assembly is determined by the inertia retained by the flywheel, an "absolutist property" that prevents the gear from reversing its direction at the point in question.

If the problem of the relational incompatibility of Descartes' collision rules, i.e., problem (1), is viewed in this context, then the phenomenon of dead points bears a striking resemblance to the relationalist puzzle of Descartes' handling of rules four and five. In the gear analogy, the configuration and motions of the gears is exactly the same in cases (ii) and (iii) *before* the dead point is reached, and after, or at, this point their behavior diverges radically. Likewise, in Descartes' plenum, the relational configuration and motions of the bodies in both rules four and five is exactly the same *before* their collisions, but the outcomes of these collisions differ considerably; which, given their identical relational scenarios, should not be the case. Hence, both the mechanical engineer and the Cartesian physicist are faced with an under-determination problem; an under-determination that occurs despite the *stability* of their respective systems (since problem (2) is not in effect). Just as the gear behavior after the dead point is unknowable given the setup, so the behavior of Descartes' colliding bodies is unknowable given his formulation of natural philosophy. The combination of Descartes' collision rules and relational motion has resulted in a corresponding inability to determine what occurs after a specific type of bodily impact (between a smaller and larger body): if we accept the account in the collision rules, then relational motion is violated; and if we adhere to his relationalism, then the outcome of the collisions must be the same (which they are not). Basically, problem (1) amounts to a Cartesian version of **KM** dead points; although, of course, Descartes' under-determination is rooted in the contradictory *theoretical* or *conceptual* demands he has made on his physical system, whereas the gear under-determination stems from an unfortunate, but avoidable, material arrangement of the gears.

The mechanical engineer can overcome the dead point difficulty by merely allowing the inertia of the gear, or flywheel, to carry it past the dead point, thus eliminating outcome (iii). In other words, the directed motion of the gears in (i) is preserved in (iii) without the sudden stopping and reversing, deceleration and acceleration, exhibited in (ii). For a Cartesian, the means of overcoming the relational quandary of collision rules four and five, as detailed in Section 2, is to posit fixed reference frames that can provide the individual components of motion of the colliding bodies (and thus delineate instances of rule four from rule five). If the Φ-based reference frames endure only temporarily, that is, for finite periods of time rather than infinitesimally, the existence of one of these frames throughout the time spanned by the collisions will allow the Cartesian to distinguish instances of the two collision rules, and hence eliminate the sort of theoretical under-determination implicit in Descartes' handling of the cases. On the other hand, an infinitesimal rendition of the Φ mapping will fail to resolve the problem of dead points, most notably if the frame is established at t_0. A frame attached to an infinitesimal Φ mapping at the instant of collision would seem to be incapable (by its very instant-based construction) of tracking the motion of the bodies past their impact, and thereby removing the inconsistency of rules four and five.[17]

The possibility of installing Φ-based reference frames leaves the Cartesian with one more major decision, however: a single privileged Φ reference frame, or set of such frames (presumably at rest relative to each other), must be selected. Likewise, since these frames are temporary, as above, and will be in operation only as long as the Φ-landmark endures, a new privileged frame will need to be selected with each demise of the old select landmark. A major obstacle would present itself to the Cartesian if this choice of an a exclusive reference frame were not made; for although the frames can distinguish instances of, say, rule four from rule five, which collisions become categorized as falling under one rule or the other depends on where you position the reference frame in Descartes' plenum. If the frame is placed alongside, or on, the smaller object in rule four, and this frame moves in tandem with the body, from its perspective the body will remain at rest. This means that it will view the collision with a larger stationary body as a collision with a larger moving one, or, as an instance of rule five (small resting body vs. large moving body), and not rule four (small moving body vs. large resting body). Other frames not attached to, or moving with, the smaller body will, obviously, view the smaller body as at rest, i.e., as an instance of rule five. (A similar situation would arise in our gear analogy if a Φ-landmark were attached to a fixed material point on the edge of the smaller gear in (iii), since its motion backtracks upon reaching the dead point.) Consequently, the relational problem reemerges once again, but this time at the level of the reference frames, rather than with the impacting bodies. Unless the Cartesian is willing to admit conflicting ascriptions of the collision rules to the very same collision--an outcome that would render the application of Descartes' natural laws disastrously relative--they will need to privilege one particular frame, or a certain set of frames among the set of all possible frames, thus guaranteeing a unique or equivalent determination of motion for the duration of that frame. (This problem is essentially the same as the difficulty faced by proponents of (R1) relationalism, as described in Section 1.1.)

4. Conclusions.

In this paper, we have attempted to formulate a modern version of Descartes' relationalist physical theory that, in the process of providing the foundation for his conservation law, does not run afoul of Newton's *De gravitatione* argument, problem (2), while also resolving the relational inconsistency in Descartes' treatment of his collision rules, problem (1). In order to achieve this goal, we have examined the kinematics of mechanisms theory for a means of obtaining fixed, or temporarily fixed, reference frames necessary to ground relational measurements of a body's speed, and hence quantity of motion, according to (R2). In this setting, both the implications of Newton's argument and the problems of Cartesian relationalist dynamics/kinematics are exhibited in a new light. As frequently noted, the similarity in detail between **KM** and Cartesian dynamics is quite striking, especially given Descartes' hypothesis on the configuration of vortices. Both theories advance a view of material interactions that is strongly kinematic, a system or world comprised of rigid bodies locked in harmonious circular movements. On the whole, and despite the obstacles encountered, utilizing a **KM**-type method of positing reference frames, in order to solve our problem (1), must rank as one the more successful, and relationally palatable, interpretations or reconstructions of Cartesian natural philosophy--that is, depending on whether or not (R2) is a consistent relationalist strategy (see note 7). It cannot resolve, of course, all the problems raised by Newton's argument, i.e., problem (2): in particular, it was necessary to abandon the search for permanently fixed Φ-landmarks and concentrate, instead, on establishing reference frames at the Φ-landmarks that only endure for finite temporal intervals, or even infinitesimally (although these latter landmarks proved more problematic; see Section 3.2).

Finally, returning to a point raised at the beginning of Section 3.2, it was revealed that one of the implicit assumptions in Newton's argument is a stipulation for some process of conveying information on the states of bodies across time (e.g., position, displacement, velocity, or even "strivings"). If conceived in this fashion, the exploitation of temporary Φ-based reference frames clearly satisfies Newton's conception of the minimum structure required of a theory of space and time. In this version of Cartesian physics, the temporary frames serve as the method of linking across time the information concerning bodily states that exists on the individual time slice. Thus, in a sense, the Φ-based frames are assuming the role normally occupied by substantival space in the Newtonian picture of the world, as was noted in our previous discussion of the properties of (R2) relationalism. Like substantival space, the temporary reference frames in this relationalist theory allow information on the states of bodies to be meaningfully compared across time.

Department of Philosophy
University of Alaska Fairbanks

ENDNOTES

* I would like to thank Mark Wilson, Ron Laymon, Calvin Normore, Bob Batterman, and two referees from *Noûs,* for their helpful comments and suggestions on the drafts of this paper.

1 Although outside the scope of this paper, various methods of salvaging Cartesian dynamics can be devised. For example, one may seek to employ some of Descartes' collision rules as the basis of the reconstruction, as Huygens' had attempted. (see, Slowik 1997)

2 See, e.g., Earman 1989, chapter 2; and, Friedman 1983.

3 One of the best recountings of this story is in, Aiton 1972.

4 Descartes 1983. Translations are based on Miller and Miller but are checked against the Adam and Tannery edition of the *Oeuvres de Descartes* (Paris: Vrin, 1976). I will identify passages according to the standard convention: thus, Article 15, Part II, of the *Principles* will be labeled "Pr II 15." Brackets, {}, represent additions to the French translation of 1647. Passages from the Adam and Tannery will be marked, "AT", followed by volume and page number.

5 There are numerous problems with Descartes' theory of motion which are beyond the intended scope of this essay. For instance, Descartes' concepts of relative motion and "place" seem to entail that, say, the occupants of a speeding car are at rest, since they do not change their relative "place" (i.e., the occupants remain stationary relative to each other within the car). Such "unintuitive" results generated much debate

among both Descartes' supporters and detractors. See, G. W. Leibniz, "Critical Thoughts on the General Part of the Principles of Descartes," in *Leibniz: Philosophical Papers and Letters* (Dordrecht: D. Reidel, 1969), for an exhaustive critical analysis of the problematic features of Descartes' theory. See also footnote 17.

[6] See, D. Garber 1992, 162-172.

[7] That is, by declaring these states intrinsically or fundamentally "opposite" or "contrary", Descartes reasons that motion and rest are mutually exclusive phenomenon that cannot transform or change into the one another when isolated from external influences. For a complete discussion of role of this Scholastic form of reasoning in Descartes' natural philosophy, known as the logic of contraries, see, P. Damerow, et al., *Exploring the Limits of Preclassical Mechanics.* (New York: Springer-Verlag, 1992) 82-91.

[8] See, Slowik 1997, for a more detailed analysis of the relation between (R1) and (R2), as well as the debate on the consistency of this relationalist variant. The move to a Cartesian space-time that allows the (pseudo-)fixed reference frames just discussed, moreover, presupposes a structure which admits the invariant quantity "relative speed (velocity, acceleration, etc.)", such as the Leibnizian space-time in Earman (1989, 30-36), yet (R2) would seem to allow even richer structures if so desired. In this essay, however, we shall be attempting to develop a relational (R2) space-time limited to the invariants of the Leibnizian class of models, but which sanctions privileged reference frames for determining Descartes' quantity of motion. In, Slowik 1997, this strategy is labeled, (R1*), although it may only be a special case of (R2).

[9] The Cartesian notion of 'size' as it appears in the conservation law is rather complex problem. See, Slowik 1996, for an attempt to untangle its various intricate meanings.

[10] 10. As R. S. Westfall has pointed out, there is a great deal of ambiguity in Descartes' use of such terms as, 'inclination', 'tendency', 'agitation', etc.; all of which seem to describe "the force to continue in its motion". Thus, for example, Westfall agrees that the term 'agitation' appears to signify 'momentum' (or quantity of motion, if it is a scalar property) in the articles on the motions of stars in Part III of the *Principles.* See, R. S. Westfall, *The Concept of Force in Newton's Physics* (London: MacDonald, 1971), pp. 61-62.

[11] The relationalist Cartesian space-time which we will develop essentially forms a member of the broader class of Leibnizian space-times, as mentioned in note 7.

[12] Descartes clearly viewed space as three-dimensional (ibid., 41-46), but it is unclear if he regarded space as Euclidean. Given the lack of any known alternatives at that time, it is almost certain that he did; yet, Francesco Patrizi and Newton seem to be the only natural philosophers of the period who explicitly described space as Euclidean. See, Grant 1981, 232-234.

[13] The concept of an "enduring geometric feature" is somewhat of a misnomer in our Cartesian space-time, of course, as will be argued at greater length in the remainder of Section 2 and 3. In fact, as mentioned above, there are many ways of mapping Ψ. One could, for instance, select a particle located inside a gear (body) for the mapping and reference frame, although long-term plenum instability will eventually bring the particle to the gear contact surface. Or, a reference frame may be linked to a contact point on an initial time slice, but the frame can follow one of the points on the gears on each successive slice rather then remain with the contact point. See, Dyson 1969, 38-39.

[14] Although only the non-vectorial quantity speed figures in the conservation law for the quantity of motion, Descartes' natural laws stipulate *directed* uniform motions. Thus a modern Cartesian may attempt to develop a Cartesian concept of velocity, as long as only the scalar property speed figures in the conservation law.

[15] In fact, Descartes' early treatment of the principle of "virtual work" (although not fully equivalent to the modern concept) specifically calls for an infinitesimal measurement of "size (weight) $\times$ distance". The virtual work principle is closely related to the quantity of motion, size $\times$ speed, and it could be argued that

the former is a special instance of the latter (but this would require more argumentation than can be given here). See, AT II 233-234, and especially, II 352-355.

16 Furthermore, Descartes' definition of "external place", which he defines as the surface of the containing bodies, employs his "surface" concept, which he further reasons must be an abstract notion since "is not a part of one body more than of the other" (Pr II 15). Therefore, it is possible that the contact point between two vortices is an equally abstract concept (since it is the point, or surface, where two such bodies meet).

17 In fact, a much greater problems lies just underneath the surface, here; and it is largely due to Descartes' "discontinuous" treatment of the motion of impacting bodies. He argues, AT III 592-593, that bodies do not gradually acquire or lose speed during collision; rather, they achieve their post-impact speeds instantaneously. Needless to say, such a view spells trouble for any attempt to establish our instantaneous $\bar{d}_{p}/\Delta t$ concept of velocity. In addition, this problem also nicely demonstrates the advantages of a principle of continuity, like Leibniz's, that can apparently evade these difficulties.

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