DESCARTES, SPACE-TIME, AND RELATIONAL MOTION*

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ABSTRACT

This paper examines Descartes' problematic relational theory of motion, especially when viewed within the context of his dynamics, the Cartesian natural laws. The work of various commentators on Cartesian motion is also surveyed, with particular emphasis placed upon the recent important texts of D. Garber and D. Des Chene. In contrast to the methodology of most previous interpretations, however, this essay employs a modern "space-time" approach to the problem. By this means, the role of dynamics in Descartes' theory, which has often been neglected in favor of kinematic factors, is shown to be central to finding a solution to the puzzle of Cartesian motion.

1. Introduction. There are few topics in Cartesian natural philosophy more problematic than Descartes' hypotheses on space and motion, a treatment that has often been classified as "relationalist"; i.e., which rejects that space and motion are, or require, anything over and above the relations among bodies. From Descartes' time down to our own, his alleged advocacy of a relational hypothesis has prompted a number of negative critical responses and lengthy denunciations, both from the "absolutists" who deny relationalism, such as Newton, and from fellow relationalists committed to his general, if not exact, goals, such as Leibniz. Most of these difficulties, of course, center upon the incompatibility of Descartes' espoused relationalism with his proto-Newtonian series of natural laws. Recently, however, some commentators, notably Daniel Garber and Dennis Des Chene, have presented strong arguments purporting to show that Descartes' theory of motion has been largely misunderstood by his critics. Descartes did not advocate a strong form of relational motion in the modern sense, they claim, and any attempt to read such a theory into his natural philosophy misconstrues the historical context in which it was developed.

Although it is sensible to be wary of the implicit dangers of over relying on modern notions, an interesting, if unintentional, question is thus raised: If Descartes' theory has been mistaken for a crude form of early relationalism, then which type of relationalism was it assumed to be? As John Earman has persuasively argued (1989), there are several discernible variations on a relationalist theme, thus it seems legitimate to inquire into the precise details of the *type* of relational theory of motion that Descartes' allegedly employed. Our investigation, consequently, will explore the problem of Cartesian motion in the light of contemporary work on the structure of theories of space, time, and motion--an analysis which, to the best of my knowledge, has largely neglected Descartes in favor of his later rival, Newton. Consequently, despite the historian's misgivings, it will be assumed that modern conceptual tools, if exercised judiciously, can be quite effective in revealing hidden facets of such historical theories. In fact, Descartes' theory of motion will be shown to not exactly fit the strict relationalism of popular belief, just as recent scholarship has charged.

Nevertheless, our modern approach to the problem of Cartesian motion will also reveal the shortcomings of many previously attempted solutions, including both Garber's and Des Chene's (sections 3 and 4). In the final section of this paper (section 5), a deeper, underlying reason for the limited success of these earlier readings will be disclosed: namely, commentators have neglected the dynamics of Descartes' system (that pertains to bodily motions under the actions of forces) in favor of the kinematics (that concerns bodily motion per se). More specifically, whereas most commentators have attempted to treat the kinematical problems of Cartesian physics before dealing with its numerous dynamical inconsistencies, a more serviceable solution to the problem of Descartes' kinematics might take the dynamics as primary, and thus reverse the course of the standard interpretation. As will be demonstrated, the (dynamic) Cartesian collision rules can play an essential role in picking out the privileged reference frames needed to secure a consistent relational account of (kinematic) Cartesian motion.

2. The Cartesian Theory. Descartes' hypotheses on space run roughly along Aristotelian lines, for he unequivocally rejected the existence of a vacuum and of a space separate from body (Pr II 16). Indeed, one of the most well-known principles of Cartesian metaphysics is that "the extension in length, breadth, and depth which constitutes the space occupied by a body, is exactly the same as that which constitutes the body", (Pr II 10)[1] a novel solution which thus allowed Descartes to take a nominalist stance on the concept of "space", which he also dubs "internal place":

> we attribute a generic unity to the extension of the space [of a body], so that when the body which fills the space has been changed, the extension of the space itself is not considered to have been changed or transported but to remain one and the same; as long as it remains of the same size and shape and maintains the same situation [*situm*] among certain external bodies by means of which we specify that space. (Pr II 10F)

Consequently, by employing a set of external bodies as a reference frame to delimit the general extension of a part of the plenum (i.e., a world completely packed with matter), one can secure an "abstract" notion of space.

That Descartes' analysis of motion has been regarded as rather blatantly relationalist is understandable given Part II of the *Principles of Philosophy,* the work which presents his most extensive discussion of this phenomena. In this work, Descartes defines motion (*motus*) as "*the transfer [translatio] of one piece of matter or of one body, from the neighborhood [vicinia] of those bodies immediately contiguous to it and considered at rest, into the neighborhood of others."* (Pr II 25) As noted by most commentators, Descartes deliberately wants to distinguish his "proper" conception of motion as change of "neighborhood" (which roughly corresponds to "external place", see Garber, 346, fn. 11) from the common or "vulgar" conception of motion as a change of "place" (*locus*); where, as above, "(internal) place" is defined as the situation of the body relative to some arbitrary set of, usually resting, distant bodies. (see, Pr II 10-15, 24-28) On this scheme, relational motion seems to arise in the following manner: in Descartes' plenum universe, any attempt to regard the surrounding bodies "at rest" amounts to a mere stipulation, since "we cannot conceive of the body AB being transported from the vicinity of the body CD without also understanding that the body CD is transported from the vicinity of the body AB"; (Pr II 29) and thus, "all the real and positive properties which are in moving bodies, and by virtue of which we say they move, are also found in those [bodies] contiguous to them, even though we consider the second group to be at rest." (Pr II 30)

Following Earman's useful classificatory scheme (12), Descartes' statements seem to advocate the following relationalist conception of motion:

R1: All motion is the relative motion of bodies, and consequently, space-time does not have, and cannot have, structures that support "absolute" quantities of motion.

Motion, as conceived according to (R1), is relational in the strict sense, since Descartes seems to assert in his phrase, "considered at rest" (*tanquam quiescentia spectantur*), that the choice of which bodies are at rest or in motion is purely arbitrary, or relative to different frames of reference. Hence, since different perspectives will assign conflicting values to the very same body, there can be no "actual" or "absolute" determinations of an individual body's state of motion. Most commentators regard (R1) as Descartes' preferred hypothesis of motion, albeit with definite qualms about his consistent application of it (Dugas, 178-179; Westfall, 57-58; Shea, 322-323; Earman, 41; Barbour, 449-450; to name only a few), while others have suggested that Descartes' avowal of (R1) is, at least in part, a sop to church censorship in order to advance his Copernican brand of planetary vortex theory (e.g., Koyré, 265; Blackwell, 227). Garber aptly dubs this (R1) view of motion, the "reciprocity of transfer" (167), and it is not difficult to comprehend why it has generated such controversy over the centuries. Not only does this hypothesis appear to run afoul of the very criticisms Descartes leveled at the "vulgar" conception of motion, i.e., rendering motion hopelessly relative to conflicting perspectives (see, Pr II 24-28), but (R1) motion is also inconsistent with the laws of motion he advanced in the *Principles,* such as the second: "all movement is, of itself, along straight lines. . . ." (Pr II 39) Unfortunately, since all trajectories are determined relative to each observer given (R1), and all observers are in relative motion, any effort to fix the unique path of a particular moving body will result in a host of conflicting measurements, none of which can lay claim to its "actual" path. Consequently, Descartes' second law would appear to transgress his adopted relationalism; as do many of the collision rules that spell out his third natural law (which conserves the "quantity of motion," or product of size and the scalar quantity speed, $m|v|$, of all bodies).[2] Newton, in his early *De gravitatione,* gave one of the best known extended critiques of the inconsistencies in Descartes' theory, arguing that "it follows indubitably that Cartesian motion is not motion." (Newton, 131) Newton insists that "as it is impossible to pick out the place in which a motion began, for this place no longer exists after the motion is completed [due to the constant change of relative body position in Descartes' plenum], so the space passed over, having no beginning, can have no length; and hence, since velocity depends upon the distance passed over in a given time, it follows that the moving body can have no velocity [given the Cartesian definition of motion as change of (external) place] (131)".

3. Cartesian Relationalism. In this section, we will investigate the problem of an (R1) construal of Cartesian motion, and thus set the stage for the introduction of alternative formulations of his relationalism.

3.1. Relational Space-Time. Since its inception, many critics have charged Descartes' ostensible (R1) relationalism with undermining the very reality of motion. Leibniz argued that, "if there is nothing more in motion than this reciprocal change, it follows that there is no reason in nature to ascribe motion to one thing rather than to others. The consequence of this will be that there is no real motion." (Leibniz, 393) Likewise, T. Prendergast insists that "if we are to take this text seriously the reality of rest and motion is destroyed." (Prendergast, 104) Presumably, these commentators hold that if a property, such as rest or motion, cannot be posited unambiguously to a single body, then the "reality" of motion as a physical process is endangered. But, this form of reasoning misconstrues basic facts of relational space-times: for, although individual assignments of rest or motion to bodies are relative to a choice of perspective, this does not undermine the reality of motion if conceived as a relative difference in motion among bodies, that is, as "relative speed (velocity, acceleration)". According to (R1), only absolute quantities of motion, which are relative to a fixed, absolute space-time structure, are ruled out; not the reality of motion itself.

More specifically, the Cartesian space-time we are investigating forms a member of the larger class of Leibnizian space-times (Earman, chap. 2), which possess a Euclidean spatial metric on the planes of absolute simultaneity, or "time-slices" of the space-time manifold, and also a time metric to uniquely order these time-slices. The invariant quantities of this structure are, of course, the "relative speeds, etc."; and, as in our earlier discussion, these relative motions are ambiguous with regard to the assignment of *individual* component motions: if, for example, one calculates a speed difference of 10 knots among two bodies, the "actual" determination of each individual's state of rest or motion is just not possible--yet, the 10 knot "speed difference" between the bodies is assuredly real and will be agreed to by all. The primary reason for regarding Cartesian motion as favoring the Leibnizian structure, as opposed to the more sparse Machian space-time (which lacks a time metric), is due to Descartes' penchant for utilizing a robust sense of the concept "speed" in the *Principles* (as will be discussed below). Relative speed is not an invariant of Machian space-time and hence cannot be meaningfully ascertained; although if construed as a change in the instantaneous spatial separation among bodies (when compared across time-slices), then a very weak form of invariant motion could be tolerated by the Machian models.

3.2. Garber and Des Chene on Relational Motion. Based in part on their tacit understanding of Descartes' hypothesis of motion as a change of contiguous neighborhood (section 2), both Garber and Des Chene reach the identical conclusion that motion is an objective feature of the Cartesian plenum which cannot be simply transformed away by switching to a different reference frame. Using the terminology of "vulgar" motion or rest, which is measured relative to an arbitrarily assigned "(internal) place", and "proper" motion or rest as change of neighborhood, Garber asserts with respect to a motion between two bodies, A and B:

> Even though it is arbitrary whether or not B is at [vulgar]-rest, once we *consider* B at [vulgar]-rest, it is *not* a matter of arbitrary choice whether or not to consider A as being in [vulgar] motion; if A is *really* in motion in [Descartes'] proper sense of the term, if it is *really* separating from its neighborhood B, then no mere change of perspective will allow us to set A at rest in the *proper sense.* (169)

Des Chene argues along the same lines, stating that "the nonrelational facts upon which judgments about motion are based are reciprocal facts of touching and not touching" (271), and thus "motion is always entirely actual, the instantaneous rupture of a body from its neighbors" (256). In expressing this objectivity of motion, however, Des Chene employs the notion of a "physical trajectory", which he defines as "a series of places, in the generic sense, which joined together constitute a tubelike virtual body" (270). That is, as a relative translational displacement occurs between a body A and its neighborhood B, a generic place is left behind such that, "if we imagine the trajectory to be generated by a succession of [these] translations, each of them adding to its increment, then the trajectory, like its parts, will be defined absolutely" (270). The complications inherent in this idea we will postpone discussing until section 4, but, for our present purposes, it is important to note that, like Garber, Des Chene's physical trajectory is an attempt to capture the objectivity, or "absoluteness", of motion in the Cartesian plenum even granting its reciprocal character.

The upshot of all this is that, having grasped onto an objective, non-relational facet of Cartesian motion, Garber and Des Chene believe they are now free to dismiss some of the long-standing doubts about the consistency of the Cartesian natural laws, especially the collision rules. As hinted at above, there is a distinct non-relational undercurrent to Descartes' analysis of impact, as is most evident in the case of rules four and five: briefly, in the fourth rule, a large object remains at rest during impact with a smaller moving body, and simply deflects the smaller body back along its path (Pr II 49); whereas in the fifth rule, a large body will move a smaller stationary object, "transferring to [the smaller body] as much of its motion as would permit the two to travel subsequently at the same speed" (Pr II 50). From a relational standpoint, however, rules four and five constitute the same type of collision, since they both involve the interaction of a small and large body with the same relative motion (or speed difference) between them. Therefore, because they represent an identical scenario, a consistent relational theory demands an identical outcome, which, unfortunately, Descartes does not provide. Garber renounces this conclusion, nevertheless:

> For Descartes, the case in which [a body] is in motion is physically distinct from the case in which it is at rest. And so, for him, the situations described in [rule four] and [rule five] are not mere redescriptions of one another; one cannot arbitrarily designate which of two bodies in relative motion is in motion and which is at rest. (241)

Equipped with a means of discerning rest from motion--translation of neighborhood or no translation--the fourth and fifth rules can thus be individuated without transgressing relationalist tenets (Des Chene makes the same point, 297-298).

The problem with this line of reasoning, to put it bluntly, is that it only works if one presupposes that the two bodies are approaching one another, which is a property of the impact phenomenon not captured by sole reference to the neighborhood of each individual body. Even if there is relative motion between a body and its neighborhood of contiguous bodies, which we will dub, "local translation", it is still not possible to determine which collision rule the impact will fall under, or if the bodies will even collide at all, unless some reference frame is referred to that can compute the *motion of both bodies relative to one another.* Suppose, for instance, that a spatial interval separates two bodies of which one is, and the other is not, in local translation. Given this scenario, it is not possible to determine if; (i) the "translating" body is approaching the "non-translating" body, or (ii) the spatial interval between them remains fixed and the translating body simply undergoes a change of neighborhood (i.e., the neighborhood moves relative to a stationary body). Consequently, Garber's and Des Chene's appeal to local translation underdetermines the outcome of bodily collisions, as well as the capacity to apply, and make predictions from, the Cartesian collision rules. Without recourse to a reference frame, one presumably would have to wait until after impact to retrospectively assess, by means of the outcomes of the collision rules, if the local translations amounted to a motion of the body or a motion of the neighborhood--but, if you exploit the (usually) unique post-impact behavior of Cartesian bodies in order to determine which bodies moved, then the local translation story has served no useful purpose.

On the whole, one might hold that Des Chene's "physical trajectory" interpretation of Cartesian motion is immune to these underdetermination problems. In the dilemma posed above, for example, since local translation gives rise to a trajectory, a shrinking spatial distance between the two bodies will eliminate case (i) from consideration. Nevertheless, information on the paths of bodily motion is not sufficient in itself to rescue Cartesian physics from the clutches of underdetermination. To illustrate, consider the example that accompanies the third collision rule: "if [body] B had initially been traveling at six degrees of speed, and [body] C at four degrees of speed, both would subsequently move towards the left at five degrees of speed" (Pr II 48). This sort of physical explanation will continue to create havoc with our various relationalist schemes, but, for the moment, attention should be focused upon Descartes' concept "degree of speed". Overall, it is difficult to see how the impact represented in this example, as in all the others, could be understood by appeal to anything less than a reference frame for measuring the bodies' speeds relative to each other. If speed, as employed in the collision rules, were a measure of local translation alone, then odd situations would arise wherein the approach speeds of bodies differed from their local translation speeds: so, returning to Descartes' example, although body B only approaches at, say, one degree of speed, the "rate of neighborhood change" due to a joint motion of B and its neighborhood results in a local translation speed of six, matching the example--and what would happen in such situations? If the outcome of the collision were dictated by the local translation speeds, than the respective neighborhoods of our two bodies would need to instantaneously harmonize their divergent motions to guarantee the outcome as mandated by rule three; and such long-range, let alone mysterious, cooperation would be asking a lot of the Cartesian plenum, even by Descartes' standards. Finally, and more importantly, the context of the collision rules seems to strongly favor the common notion of approach speed, and not rate of local translation, as is evident throughout Descartes' exegesis: e.g., the fourth rule, where B could never move C "no matter how great the speed at which B might approach C" (Pr II 49).

Despite the best efforts of Garber and Des Chene, therefore, Descartes' analysis of motion remains largely incompatible with his laws of nature--and this is the major weakness of their analysis. Garber concedes this basic point, observing that "without a common framework in which to conceive of the relative motions of more than one body, it is difficult to see how we could give an adequate treatment of the phenomenon of impact" (171). Yet, if this difficulty is acknowledged, then Garber's aforementioned claim that impact rules four and five are not inconsistent (241) is technically correct, but rather misleading. Basically, our commentators seem to be providing the following argument: because it is a fact that a body is, or is not, translating with respect to its neighborhood, there is a real distinction between rules four and five, thus resolving the difficulty. This is only half of what is needed, unfortunately. Although everyone will agree when a body is at rest, or not translating, not everyone will agree on which body moves once a translation occurs. In short, despite the reality of Cartesian motion, a relative motion is ambiguous as regards the individual components of motion, and the collision rules *require* such individual determinations for their correct *application.* As mentioned in section 3.1, Leibnizian space-time is the relational theory best suited to Descartes' actual handling of motion, since the collision rules employ a strong notion of speed (as in the example from rule three). Yet, Leibnizian space-time can only countenance determinations of "relative speed", and not, without installing reference frames, individual components of speed. The following dilemma is, accordingly, imposed on our Cartesian space-time: if reference frames are employed to measure the individual speeds of bodies, then the fact that our space-time is relational means that any reference frame is admissible, which would result in numerous conflicting estimations of bodily speeds, and hence applications of the collision rules; if, however, a privileged class of frames is designated, and the impact rules which conserve "quantity of motion" are only valid relative to them, the space-time would then appear to be endowed with a structure over and above what relationalism permits.

No matter how unsavory the consequences may be, this last option is apparently the only alternative for the union of a committed (R1) relationalist and the Cartesian natural laws, for it is difficult to perceive how Descartes' physics could be rendered consistent without a set of select view points. Invoking such frames would seem to mark a departure from a strict (R1) reading, nevertheless. In essence, the relationalist could make the following argument: despite retaining Leibnizian structure, the conjunction of this space-time with the Cartesian natural laws privileges a class of material-based reference frames from which the Cartesian conservation law is upheld. This modified formulation of (R1), which we will denominate, (R1*), thus retains the prohibition on "absolute" quantities of motion (relative to absolute space-time structure) but allows, via the privileged frames and the Cartesian conservation law, the determination of individual body speeds from the invariant relational quantity "relative speed". Overall, this sort of "bootstrapping" technique would allow Descartes' "quantity of motion" to pick out the special material-based frames that actually conserve this quantity through measurement, and thus secure agreement on the outcomes of the collision rules. There remains a certain suspicion, however, that such a strategy merely masks an underlying inertial structure in a relationally palatable fashion. That is, although the space-time is nominally committed to the non-inertial Leibnizian structure (which does not possess devices for determining the straight-line continuation of paths, like the "covariant derivative" of Newtonian space-times, often labeled $\nabla_V V$, or, $d^2x^\alpha/d\lambda^2 + \Gamma^\alpha_{\mu\nu}(dx^\mu/d\lambda)(dx^\nu/d\lambda)$, in the coordinate frame), the coupling of this relational space-time with the Cartesian natural laws essentially constitutes, or presupposes, inertial structure--and inertial structure violates (R1), and by implication (R1*). If this line of argument sounds convincing, then the Cartesian will most likely want to fall back upon (R2), the relationalist thesis which merely denies space-time substantivalism (i.e., the view that space-time is a form of substance; Earman, 12). As long as space and time are regarded as somehow contingent upon matter (possibly as a sort of "emergent" entity or property of matter?), the Cartesian would thus seem free to invoke as much structure as deemed necessary to explicate Descartes' natural laws, even inertial structure like $\nabla_V V$. Whether or not (R2) is a consistent relationalist position is the main question, needless to say; and though it is beyond the scope of this essay (see Earman, chap. 6), we will briefly return to this issue in the next section.

4. Relationalist Strategies: Reference Frames and Absolute Structure. Given the results of section 3, the Cartesian appears to be led inexorably away from the austere (R1) relationalism to a more moderate variety, whether construed as (R1*) or (R2). One of the most exploited methods of supplying a non-(R1) foundation is to postulate preferred reference frames, either permanently or temporarily fixed, from which the measurements of individual body motions can be conducted. The more fixed these frames become, however, the more they resemble a form of absolute space, as will be discussed. Possibly the first, and most likely the best, of such proposals is Huygens' center-of-mass (-gravity) reference frame.

Huygens, who like most Cartesians was inclined towards an (R1) relationalism (Elzinga, 96), attempted to correct Descartes' collision rules by extending the scope of the first rule, which is the only verifiable hypothesis of the set, to govern the collisions of bodies of all types and sizes. Descartes' first rule asserts that two equally sized bodies moving at identical speed (in opposite directions) will rebound "without having lost any of their speed," (Pr II 46) and thus conserve their quantity of motion. Recognizing what today we would call the Principle of Galilean Relativity, Huygens sought to remedy the deficiencies of Descartes' impact theory by utilizing an identical analysis for the remaining six cases treated in the *Principles,* which would guarantee the conservation of quantity of motion for all interactions. With the discovery of a colliding system's center-of-mass reference frame, Huygens found a means of generalizing Descartes' first collision rule: "if a larger body A strikes a smaller body B, but the velocity of B is to the velocity of A reciprocally as the magnitude [size] A to B, then each will rebound with the same speed with which it came." (*Oeuvres Complètes,* vol. 16, 92) As viewed from the origin of that frame, where bodies preserve their initial speeds after rebound, the ratio of their speeds is reciprocal to the ratio of their sizes. When one body strikes another, irrespective of their size and speed, an observer situated at the center-of-mass perspective will perceive both bodies to recoil in the opposite direction while retaining their initial speeds, and thus quantity of motion. Of the various "reference frame" proposals for mending Descartes' physics, Huygens' project probably comes closest to what we have labeled (R1*), since it is a forthright endeavor to maintain (R1) by using the Cartesian conservation law to isolate a preferred set of plenum-based reference frames; which, in this case, are temporary frames limited to each bodily collision (see, Slowik 1997; and among modern attempt to provide such frames via (R1*), see Wilson 1993, which draws upon the modern theory of connected gears).

Aside from the collision rules that detail the third natural law, a coherent means of grounding the second law is also necessitated, of course. One interesting possibility is to rely on the definition of the second law to single out the frames from which straight-line motion recedes. Specifically, Descartes' second natural law states: "all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle which they are describing." (Pr II 39) In Part III, Articles 57-58, of the *Principles,* the exact meaning of the passage "move away from the center of the circle" is made clear, for it not along the tangent to the circle that the body strives to recede, as in Newtonian mechanics, but along a radial line directed outward from the center point. Circular motion is the rule and not the exception, moreover, since Descartes believes that all plenum motion ultimately involves a ring of circling bodies, with each body in the ring moving simultaneously into the (internal?) place of the one ahead (Pr II 33; supposedly to avoid the creation of a vacuum). If one correlates these circular motions with the second law, therefore, the tendency towards straight-line motion evidenced by all moving (= circular moving) Cartesian bodies can be referred back, at least in part, to a particular reference frame, i.e., the center point of the circle from which they attempt to flee. For every ring of synchronously circling bodies, there will be a reference frame located at the center point that serves as the perspective for measuring the individual body's "quantity of motion" (which is the measure of this "tendency" or "force" to recede; Pr II 43). Unfortunately, as is often the case with Descartes' physical hypotheses, there is no indication that he remained faithful in the *Principles* to the idea that all bodies must partake in such circular motions; which is especially true of his lengthy discussion of Terrestrial phenomena in Part IV, where circular motion is hardly mentioned. Likewise, even if this strategy were to supply a system of privileged reference frames for the second law and, presumably, the third law, it would still not remedy the inherent incompatibility of simultaneous plenum motion and a set of collision rules; for, as W. Anderson, has succinctly put it, "the necessary conditions for translation [motion] are contrary to the necessary conditions for collision" (Anderson, 220). If bodies can only participate in synchronous circular motions, when, in fact, are the impact rules ever actualized?

Other submissions for a Cartesian reference frame have been both less practical and more metaphysically abstract, yet are worth investigating since they exhibit definite (R2) leanings. Representative of one major theme is K. Hübner's proposal, which simply declares God to be the framework from which motions are discerned: *"Descartes' laws of impact describe fundamental occurrences of nature as if seen from the standpoint of God"* (Hübner, 130). This explanation, despite its somewhat incongruous tone, is ostensibly latent in all occasionalist renditions of Descartes' natural philosophy; which holds, roughly, that God is the direct cause of all bodily phenomena (Garber, 299-305). Hübner is led to this conclusion since he believes that the relativity of translation implies that "motion" is merely a mode of thought, and not of extension (128-130). Nevertheless, even if one interprets Descartes' occasionalist-sounding claims, such as "God is the primary cause of motion" (Pr II 36), in a literal fashion, and it is unclear that one should, Hübner's tactics are fundamentally implausible in that they procure an ontological foundation for Descartes' natural laws at the expense of the epistemological. When Descartes listed the factors that complicate the application of his impact rules, such as lack of solidity and plenum interference (Pr II 45), he did not further add that only God could provide the correct calculations. Possibly sensing this dilemma, Hübner's solution is rather Platonic: "the true changes [in bodies] are, namely, those given in divine revelation, and not the merely apparent ones given through the senses or determined in accordance with some arbitrary relativity as a [mode of thought]" (130). Despite the essential implausibility of appealing to "divine revelation" to ground Descartes' physics, we will return to a more promising basic thread in Hübner's argument in section 5.

Furthermore, as the reference frames postulated by these commentators become more and more permanent, the Cartesian space-time grows inevitably more "absolutist". This is nicely demonstrated in the case of R. Dugas, who entertained the notion that the material world prior to the formulation of the three material elements and planetary vortices, i.e., before God imparted a conserved quantity of motion, serves as the sole preferred frame (Dugas, 196; for the Cartesian cosmological creation story, see Pr III 46). Des Chene's notion of a "physical trajectory", explained above, also appears to invoke this richer space-time structure, particularly in his repeated denials that Cartesian motion obligates "a local frame of reference in relation to which motion is defined" (262). Des Chene repudiates such arbitrary frameworks for measuring motion, arguing instead that Descartes "is trying to explain what is true of a thing at each moment of its *absolute* motion" (272). A view characterized by "the instantaneous, punctual *translatio* from the vicinity of one set of bodies to another, and the thought that by gluing together the spaces successively occupied by a body one can construct a physical trajectory to which the relative conception of place is irrelevant" (272). Ironically, in a later section, Des Chene cautions against thinking of Descartes' theory "as being like Newtonian absolute space" (374); yet his "trajectory" hypothesis follows, practically verbatim, Newton's reasoning in the *De gravitatione* (section 2), which rejects Cartesian motion for lacking this absolute structure! For both Newton and Des Chene, motion requires a space-time that can identify the same spatial position across time, thus ensuring a coherent groundwork for measurements of speed, trajectory, etc.; and only the Full-Newtonian space-time has the requisite structure to accomplish this task (being thus equipped with a space-time "rigging" to identify such positions; see Friedman 1983, 74).[3]

The preceding analysis should not be taken as necessarily denouncing Des Chene's hypothesis, however, for there is much in Descartes' approach to the problem of space and motion which suggests a commitment to absolute space, if only to its structure via (R2) and not to its ontology. In fact, Des Chene's interpretation helps to belie the knee-jerk tendency to categorize historical theories of space and motion as either clearly in the strict (R1) relationalist camp or clearly in the substantivalist camp. If Descartes' espousal of a relationalist theory is viewed more as a repudiation of the idea that space is an entity independent of matter, as certain of his predecessors like Patrizi believed, then it is possible to read Descartes as having endowed space with a more robust structure than has been commonly accepted. As presented in Grant (192), there are historical precedents prior to Descartes for the belief that space (or vacuum), although different from matter, can only exist as long as matter exists; namely, John Philoponus. Also, in common with his Scholastic predecessors, Descartes' theory of motion as change of (external) place rules out the possibility that the entire plenum could, in unison, translate or rotate: since the plenum as a whole has no "contiguous neighborhood", it cannot be said to move. The line of thought leading from Patrizi, through More to Newton, on the other hand, clearly allows for such "global" motions of the material world, for space is a "thing" (not necessarily either substance or property) *independent* of matter. With respect to global material motion, consequently, Descartes is a straightforward relationalist, but Des Chene's position intimates that he may have endorsed an even richer structure for space, possibly as potent as Newtonian space-time, just as long as it is acknowledged that it cannot exist apart from matter.

Unfortunately, this hypothesis, which Earman has dubbed the "property view of space" (14),[4] does not appear to be corroborated by the evidence from the texts, since Descartes repeatedly claims that "the names 'place' or 'space' do not signify a thing different from the body which is said to be in the place" (Pr II 13). Overall, Descartes' view is quite nominalist, for he seems to regard space as a sort of conceptual abstraction, much in the way that numbers are abstractions: "quantity [extension, volume] does not in fact differ from the extended substance except insofar as our conception of it is concerned; similarly, number does not differ from the thing which is numbered" (Pr II 8). A conceptual abstraction does not amount to a property theory of space, needless to say, nor does it presumably have the capacity to ground the individual motions of bodies. Thus, unless textual evidence is forthcoming that can verify that Descartes equated his conceptual abstractions with some form of higher-level structural feature of matter (that is capable of grounding motion), this theory must remain merely an intriguing suggestion.

<u>4. Dynamical Considerations.</u> In the previous two sections, interpretations of Cartesian motion that fit the (R1) category, as well as some non-(R1) formulations, have been shown to be unsatisfactory. As in the cases of both Garber and Des Chene, the failure to take into account the demands of Cartesian dynamics (i.e., natural laws) is the source of most of the difficulties (although some interesting hypotheses are simply inconsistent

with Descartes' texts). This section, on the other hand, will present a resolution to the problem of Cartesian motion which draws its support from Descartes' dynamics, especially from a current debate centered upon its ontological status of "force". In common with the natural laws, Cartesian force has generally not been considered a means of resolving the troubling issues surrounding the relativity of motion. Rather, force has traditionally been discussed by commentators after having grappled with the pure "kinematic" concepts of motion; i.e., translation, reciprocity, neighborhood, etc.. As in both Garber's and Des Chene's texts, the examination of force is largely separate from the kinematical exegesis. There are many reasons for this general approach, most notably Descartes' general avoidance of the relativity issue in his scientific texts. Nevertheless, there are good reasons for reversing the course of this "normal" interpretation of Cartesian physics, and thus viewing Descartes' dynamics as primary or essential to a proper understanding of his kinematics.

Before proceeding further, however, we need to reexamine the fourth and fifth impact rules to discuss the apparent distinction between a force of resistance and force of motion. In rule five, the larger body B transfer as much speed to the smaller stationary C in order to allow them both to move in B's direction at the same speed. This suggests that B's force for "proceeding", or motion, is simply the product of its size and speed, as one would expect given the conserved quantity of motion. But, Descartes also postulates a force of "resistance": "a body which is at rest puts up more resistance to high speed than to low speed; and this resistance increases in proportion to the speed" (Pr II 49). This force becomes manifest in the fourth rule, where the larger resting C deflects the smaller moving B back along its path. Since C is at rest, its force of resistance is presumably measured by the size of C and the speed of B, for only then is its force greater than the moving B's force of motion.

With respect to bodily force, moreover, commentators are divided as to its ontological status: some, such as M. Gueroult and A. Gabbey, conceive force, in at least one important sense, as being *in* the objects themselves as a feature or mode of body; while others, like Garber and Des Chene, relegate it to a sort of derived phenomenal effect of God's sustenance of bodily speed and size, and thus not present within extension. Regardless of which side of the dispute one favors, a potential resolution of the kinematical puzzle of relative motion is contained in this deliberation on the dynamic status of quantity of motion.

If one sides with the Gueroult/Gabbey interpretation, for instance, then Descartes' handling of the conserved quantity of motion in the collision rules, which seems to posit the separate forces of rest and motion, demonstrates that these forces "are immanent in 'nature' or extension . . . and can be calculated at each instant for each body, according to the formula mv" (Gueroult, 198). Likewise, Gabbey reckons "at the practical level of physical investigation, forces--whether of motion or rest--are real causes in their own right and distinct from motion and rest" (Gabbey, 238). Interestingly, by placing the force in the body, whether as a mode or some aspect of extension, this maneuver effectively locates the long sought after reference frame for determining the individual motions of impacting bodies--the frame is an intrinsic feature of bodily forces, principally rest. Since a resting body puts up a force of resistance comparable to its size and the *speed of the approaching body,* its internal force constitutes a built-in method of determining not only its state of motion, but the speed of the second impacting body. Thus, a resting body's force acts as a reference frame for overcoming the relativity of Descartes' reciprocity of transfer hypothesis: regardless of which bodies are deemed to move once a translation occurs, and given the existence of non-translating bodies in the Cartesian plenum (which a safe assumption), these resting bodies have an intrinsic capacity to determine the individual speeds of the bodies they meet on impact, and this capacity constitutes a platform that, at least hypothetically, can be extended as a reference frame to determine the individual states of motion of all plenum bodies. In this context, a "resting" body is to be understood according to the concept of "proper-rest" described in section 3: i.e., as *non-translation* with respect to the *contiguous neighboring bodies,* which is an invariant (non-relative) feature of the space-time. This eliminates, of course, the problem that "vulgar rest" (a.k.a. Galilean relativity) would pose for this thesis. Since, if manifestations of the rest force were relative to any and all inertial perspectives, then a host of conflicting determinations of the force would be ascribed to the same body. Once again, the non-(R1) nature of Descartes' theory is in evidence. Nevertheless, by utilizing this "rest force" process, one can secure a foundation for the measurement of the collisions rules, and hence natural laws.

Moreover, even if one rejects these bodily forces as actual properties of bodies, and adopts the Garber interpretation instead, it is still the case that resting bodies behave as if they possess forces of rest, and hence they will retain the reference frame ability detailed above. More carefully, Garber (283) and Des Chene (327) both contend that God sustains a certain quantity of motion in the world according to the plan of Descartes' natural laws, and thus there is nothing in bodies except their extension. Yet, God's maintenance of the Cartesian laws ensures that resting bodies (which are not problematic on the local translation scheme of section 3) will have the capacity to measure the approaching speeds of impacting bodies, and this is all that is needed to resolve the problem of relative motion. Since God is doing the work, one might be tempted to claim, along with Hübner, that God is now the preferred reference frame; and while this may be superficially correct, it is also true that, under the ideal conditions stipulated in the

Principles, Descartes assumes that the individual motions of bodies are determinable using normal empirical means, without recourse to divine revelation.

This last ingredient--the ability to gauge the speed of the approaching body, as well as the body's own state of motion--is what has been needed all along to secure a foundation for the collision rules free of the relativity problem, and it looks as if Descartes has assumed it as a basic constituent of Cartesian bodies all along. Consequently, Garber and Des Chene are correct in denying that Descartes favored a strong (R1) relationalist theory; but, at least on this estimation, they have reached the right conclusion while simultaneously neglecting the source of the main difficulty, namely the need to supply the privileged reference frames to secure a consistent relational theory of both kinematics and dynamics. (In all fairness, Des Chene's absolute trajectory hypothesis did try to meet this challenge.) Given the discussion in section 3, Garber's and Des Chene's accurate interpretation of Descartes' "reciprocity of transfer" hypothesis is insufficient to resolve all the difficulties associated with relational motion, particularly as regards the collision rules (and they both seem to admit this point in numerous asides throughout their texts). Oddly enough, the indispensable mechanism for determining the relative speeds of bodies, as mandated by the impact rules, appears to be embodied within (resting) Cartesian bodies as a result of Descartes' own handling of his conservation law. Put differently, one way of summarizing the pitfalls of most, if not all, earlier examinations of Cartesian motion is that commentators have (quite naturally) tried to resolve the kinematic problem of the reciprocity of transfer before addressing the inherent dynamic weaknesses of Descartes' physics; whereas a more adequate solution may lie in attending to the dynamical aspects prior to, or in conjunction with, the kinematical details of Descartes' theory.

One major problem with this explanation, unfortunately, is that it ostensibly undermines the purpose and importance of the reciprocity of transfer thesis. If the conservation law can ground the individual motions of bodies on its own, then there is no apparent reason for Descartes' local translation story; unless, of course, it is simply to caution against regarding motion as something over and above extension, i.e., a dynamic property *in* bodies, which is a reading of its intended meaning that would seem to favor the Garber/Des Chene interpretation above. In other words, motion and rest, considered apart from a conserved quantity of motion, are merely relations between a body and its contiguous neighborhood which are both symmetric and kinematic. Once God introduces the (dynamic) element of a conserved quantity of motion, however, the symmetry of the kinematic relations is broken, and a preferred manner of viewing bodily motions is imposed on the space-time observer.

Moreover, if we concede the consistency of (R1*), then the reference system described above would appear to vindicate this form of relational structure, since it is the conserved quantity of motion that guarantees the existence of the reference frames for determining the individual states of bodily motion. The conserved quantity of motion, as a whole, "bootstraps" the privileged reference frames required to give a coherent relational measure of that quantity, as well as the specific outcomes of the seven collision rules. Consequently, it is not necessary to view the reference frame process described in this section as mandating a special bodily rest force. In fact, an alternative rendition of our reference frame technique could forego the need for the resting force to instate the frames, altogether; and, instead, simply declare that the conjunction of the Cartesian conservation law and the "relative speeds" among all impacting bodies always selects a unique perspective for determining their individual motions. In short, rather than relying on some nearby resting body to measure motion, and thus seemingly invoke a force inside bodies, the conserved force could directly individuate bodily motions relative to unique reference frames which may, or may not, be attached to one of the colliding bodies. This version of our reference frame strategy is hence closely akin to Huygens' center-of-mass frame method of upholding the Cartesian conservation law and collision rules. Yet, whereas Huygens utilized the specific outcome of the first rule as the basis of his reconstruction of Cartesian dynamics, the story outlined above only uses the conserved quantity of motion to directly select the chosen perspectives to maintain the quantity: it does not attempt to rework the precise predictions of the impact rules, as was one of Huygens' main intentions.

As for textual evidence supporting our dynamics-based "rest force" maneuver, the very details of the collision rules straightforwardly confirm that resting Cartesian bodies have this capacity (i.e., to untangle the relational difficulties by measuring approach speeds, as above). The impact rules thereby provide strong indirect evidence for the general claim that Cartesian dynamics can resolve the kinematic problem; but a more direct instance of dynamical consideration taking precedence over the kinematical would greatly strengthen the arguments of this section. While discussions of this sort are extremely rare in the Cartesian corpus, a challenge by Henry More prompted Descartes to defend his relationalist views by means of a remarkable hypothetical case (which, moreover, has been largely neglected in the literature). If transfer of a body is merely reciprocal, More asked how one should regard the case of wind blowing through a tower window (5 March 1649, AT V 312): Is this motion reciprocal as well, so that the air can be viewed at rest and the tower in motion? Rather than answer More's question as kinematically presented, Descartes tries to resolve the problem by invoking a different, dynamics-based example. This example involves the force or "strength" (*vires*) of two men attempting to free a grounded boat; with one on board pushing against the shore, and the other on shore pushing against the boat:

> If the force (*vires*) of the men is identical, the effort of the man on the shore, who is thus connected to the land, contributes no less to the boat's motion that the effort of the man on the boat, who is transported along with it. Therefore it is obvious that the action by which the boat recedes from the shore is equally in the shore as in the boat. (15 April 1649, AT V 346)

In essence, Descartes' "boat" example, as we will call it, applies a *dynamical* solution to an original *kinematical* problem: the reciprocity of the transfer of the boat and shore, which is a rather troubling kinematical consequence of his espoused relationalism, is reinterpreted as a sort of "reciprocity of force" between two possible dynamic sources of the motion, namely the push of each man.

One might be tempted to agree with Shea (1991, 323) that Descartes' explication of the "boat" example unwittingly conflates the kinematic and dynamic elements of his theory. This may be true, but it does not undermine the importance of the above passage for our present purposes. As *literally* interpreted (which is always a virtuous appeal by a commentator), Cartesian dynamics would appear, at the very least, intertwined with Cartesian kinematics, if not (in this case) taking precedence over the latter. The reciprocity of transfer "process", broadly construed, now appears to encompass force as much as it involves kinematics. In fact, Descartes' reply to More seems to tacitly acknowledge that it is "really" the boat that is in motion, and not the shore, which is the motivation underlying the switch to a different example: i.e., it is the reciprocal nature of the force (of pushing) that causes or generates the individual motion of the boat, which is a relationally "legitimate" state-of-affairs not present in More's original case (or so Descartes believes). Of course, Descartes is not implementing a rest force to break the symmetry of the kinematical transfer (as argued above)--but he is appealing to the dynamics of the situation *in general* to resolve this relational difficulty. Thus, in the letter to More, Descartes' resolution of the kinematical "reciprocity of transfer" problem *falls back upon the dynamics* of his theory of motion in the exact same way that his kinematical collision rules employ an intrinsic rest force (to measure approach speeds). In the "boat" example, however, the primary role of dynamics (in solving kinematical puzzles) is much more directly presented and evident (if possibly more confused) than in its analogous use in the collision rules. The importance of Descartes' "boat" example should not be underestimated, moreover: not only does it have a chronological advantage over earlier work (being one of his last discussions of natural philosophy; spring, 1649), but it is one of the only known instances where he was actually pressed for the details of his brand of relationalism (by way of a counter-argument).[5]

Finally, the overall results of this section should be placed within the historical context of another relationalist strategy not yet surveyed in this essay. By holding motion to be generally relational, but that individual determinations of motion are an intrinsic fact of bodies, this interpretation of Descartes' places him close to Leibniz's view of space and time, or at least to one of Leibniz' many themes on these issues. In the "Discourse on Metaphysics", he states:

> For considering only what it means narrowly and formally, that is, a change of place, motion is not something entirely real; when a number of bodies change their position with respect to each other, it is impossible, merely from a consideration of these changes, to determine to which of the bodies motion ought to be ascribed. . . . But the force or the immediate cause of these changes is something more real, and there is a sufficient basis for ascribing it to one body rather than another. This, therefore, is also the way to learn to which body the motion preferably belongs. (Leibniz 1970, 315)

Earman concludes (132) that this form of reasoning falls within the (R2) class, for it requires a structure for space-time richer than (R1) relationalism allows (but not (R1*), since, if we grant its consistency, Leibnizian space-time structure would be preserved, as above). The theory of Cartesian motion developed in this section closely follows the

Leibnizian argument, as well. Like Leibniz, Descartes' theory would entail that motion, kinematically conceived, is relational (as in the reciprocity of transfer), but that the dynamical aspects of bodies can assist in breaking the relational symmetry in order to reveal which ones are truly in motion. Whether regarded as *in* bodies, according to Leibniz or Gueroult/Gabbey, or deemed to be the phenomenal product of God's sustaining act, according to Garber/Des Chene, bodies have a capacity *born of conserved forces* (i.e., the rest force) to reveal individual states of motion, contra (R1). As an interesting aside, L. Sklar's theory of "absolute acceleration", as an internal feature of bodies without need of substantival space, would also seem to correspond to this view (230). But this analogy should not be pushed too far, since, unlike Leibniz and Sklar, force is not required to be an intrinsic property of Cartesian bodies. Notwithstanding these differences, with this last set of references it has been thus surreptitiously demonstrated that nearly every modern hypothesis of space and time, from strict relationalism to absolute space-time structure, has an analogous interpretive counterpart in the philosophy of Cartesian space and motion.

5. Conclusion. In retrospect, it is fair to say that Cartesian physics has been largely neglected, if not outright ignored, by twentieth century philosophers of space and time. While there have been no shortage of attempts to encapsulate Newton's natural philosophy in the modern mathematical formalism (starting with the pioneering work of Cartan), Descartes' theories have not been as fortunate. One of the goals of this paper has been to partially reverse this trend by demonstrating the added insights that the modern formalism can provide the Cartesian researcher. In particular, the preceding analysis has strived to clarify what most commentators have often incompletely or vaguely perceived; i.e., that a strict (R1) relationalism is incompatible with the Cartesian natural laws.

Nonetheless, most commentators have been even less successful in perceiving that there are "weaker" formulations of relationalism that may adequately ground Descartes' physics. This realization prompted the second goal of our investigation, namely the search for the elusive privileged reference frames that can provide a sound basis for a relational (R1*) or (R2) space-time. Specifically, most commentators have also overlooked the fundamental importance of Cartesian dynamics in resolving the kinematic obstacles of the "reciprocity of transfer" hypothesis--a startling feature of Descartes' theory directly supported in his letter to More. Moreover, as argued, Descartes' handling of the collision rules strongly suggests that Cartesian bodies have a capacity, via the conservation law, for determining the individual motions of both themselves and their co-impacting partners, thus establishing the groundwork for the privileged reference frames as mandated by (R1*) or (R2) relationalism.

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FOOTNOTES

[1] Descartes, 1983. Translations from the *Principles* are based on Miller and Miller but are checked against the Adam and Tannery edition of the *Oeuvres de Descartes* (1976). I will identify passages according to the standard convention: thus, Article 15, Part II, of the *Principles* will be labeled "Pr II 15." Passages from the French translation of 1647 will be prefaced by "Fr". Other translations that are based on the Adam and Tannery will be marked, "AT", followed by volume and page number.

[2] Furthermore, Descartes' pronouncements on the bodily "modes" of motion and rest (Pr II 27) also run afoul of relationalist doctrine, for he seems to presume that they are both distinct and opposing bodily states, a qualitative difference that cannot be captured by any strict relationalist means: i.e., "rest is the opposite of movement, and nothing moves by virtue of its own nature towards its opposite or its own destruction." (Pr II 37)

[3] Des Chene somewhat confuses the issue by stating that "the distance between two points (or space-time points) is likewise an invariant" (270, fn. 13), after discussing the (supposedly?) metric invariants of space-times that license Galilean (Neo-Newtonian) and Special Relativistic kinematics. But, spatial distance is not an invariant between non-simultaneous (event) locations in Neo-Newtonian and Special Relativistic space-times; in fact, it is not even a meaningful concept in such space-times. Only Full-Newtonian space-time has a well-defined notion of spatial separation between non-simultaneous event locations (Sklar 1974, 207), as Des Chene appears to be assuming in his physical trajectory concept.

[4] Technically, the property view of space violates both absolutism and (R2), since (R2) also contends that bodies cannot have individual properties of spatiotemporal location.

Yet, in rejecting substantivalism and the independence of space from matter, it is much closer in spirit to relationalism, and will thus be included within the (R2) category.

[5] The "boat" example also raises interesting questions about the epistemology of force (which are beyond the scope of this essay, however). Although the conservation of force in the totality of matter is grounded in God's "immutability", and thus has a claim to "a priori" status (Pr II 36-37), Descartes' example apparently suggests that the forces involved in particular motions (the boat) can be identified and delimited, to some degree, through observation (the reciprocal push of the men). See, Normore (1993) and Nadler (1990), for some recent work on the overall epistemological status of the conservation principle and natural laws.