The ‘Space’ at the Intersection of Platonism and Nominalism

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Abstract:
This essay explores the use of platonist and nominalist concepts, derived from the philosophy of mathematics and metaphysics, as a means of elucidating the debate on spacetime ontology and the spatial structures endorsed by scientific realists. Although the disputes associated with platonism and nominalism often mirror the complexities involved with substantivalism and relationism, it will be argued that a more refined three-part distinction among platonist/nominalist categories can nonetheless provide unique insights into the core assumptions that underlie spatial ontologies, but it also assists in critiquing alternative uses of nominalism (Field, Arntzenius), platonism (Psillos), and both ontic and epistemic structural realism (French).

In the past several decades, an increasingly popular means of assessing scientific theories has come in the form of the venerable platonism/nominalism dichotomy, an interpretational strategy that, not surprisingly, has often focused on the highly mathematical branch of theories that treat spacetime ontology. Given the current stalemate involving the traditional substantivalist and relationist categories, there would indeed appear to be a need for additional conceptual tools to analyze spacetime theories: specifically, the attempts to capture the ontology of space and spacetime using the conceptual apparatus of substantivalism and relationism—that space is, respectively, either an independently existing entity or a relation among material substances—have proved to be quite problematic, since the sophisticated versions of both substantivalism and relationism seem to be identical in the context of General Relativity (GR), our best theory of the large scale structure of space (see, e.g., Rynasiewicz 1996). But, can platonism and nominalism shed light on the ontological disputes concerning space (spacetime), or, more generally, the realism/anti-realism debate as regards the geometric structures used in scientific theories? Specifically, this essay will examine two interrelated questions: (i) the relationship, if any, between platonism/nominalism and the traditional substantival/relational distinction, as well as (ii) how the spatial/geometric structures utilized by spacetime theorists and scientific realists can benefit from the introduction of platonism/nominalism. Against the backdrop of the spatial ontology controversy, both historically (Leibniz, Newton) and the modern setting (GR, quantum mechanics, quantum gravity), our investigation will demonstrate that a more nuanced version of the traditional platonism/nominalism dichotomy—one which incorporates a distinction, adapted from Pincock (2012), between a fictionalist nominalism and a liberal truth-value nominalism—provides distinctive and important information on various assumptions the undergird the spatial ontology debate, as well as helps to elucidate the scientific realist's commitment to spatial structures. In particular, truth-value nominalism will be shown to be the most versatile and successful hypotheses among the three platonist/nominalist categories in the context of theories of space. Although a resolution of the ontology debate by means of platonist/nominalist notions seems unlikely, the underlying assumptions that motivate substantivalism and relationism gain clarity and context when viewed from the mathematical and metaphysical perspective of platonism and nominalism, thus the introduction of these additional tools for assessment is both warranted and constructive.

In section 1, we will examine the unique challenges that face the application of platonism and nominalism in the spatiotemporal realm, and thereby lay the groundwork for our more refined subdivision of these concepts, namely, immanent realism, fictionalism, and truth-value nominalism. Alongside an historical analysis of Leibniz and Newton's relationship to these issues, section 2 will demonstrate how our new categorization is superior to previous efforts to graft nominalism and platonism to modern spatial ontologies, while section 3 will critique how nominalism and platonism factor into, or could benefit, other approaches that involve the spatiotemporal or geometric components of physical theories: in particular, the conjunction of nominalism and substantivalism (Field, Arntzenius), a platonism coupled to a scientific realist defense of geometric structures (Psillos), and the outright rejection of the relevance of platonism/nominalism for a realist interpretation of these mathematical/geometric structures (French and ontic structural realism). In this last section, one of the members of our new catergorizational scheme, truth-value nominalism, will also be shown to have much in common with the sophisticated form of epistemic structural realism, but it can also assist ontic structural realism.

1. Platonism and Nominalism in A Spatiotemporal Context.

1.1. Platonism and Immanent Realism. Initially, at least, the claim that the platonist/nominalist dispute is similar to, or can assist in the philosophical analysis of, spatial ontology is problematic in various ways. Platonism is often characterized as assuming the existence of non-spatial and non-temporal abstract objects, properties, or facts of some sort (see Lowe 2002, chap. 19). Hence, if platonism requires the non-spatiality of abstract mathematical structures, then platonism will obviously fail to be of service to the spatial ontology dispute. As regards *ante rem* structuralism in the philosophy of mathematics, for example, it is claimed that structures can exist apart from the systems that exemplify them (Shapiro 2000, 263)—but what is the nature of those existing, but uninstantiated, structures? Are they akin to ideal entities, such as Plato's Forms, that reside in a sort of conceptual heaven? If universals and uninstantiated *ante rem* structures do not exist in space and time, then the platonic/nominalist distinction would appear to be of little relevance to both substantivalism and relationism, since one could accept either platonism or nominalism independently of one's stance on the spacetime ontology dichotomy. For example, given the setting of GR's curved spacetime, both the substantivalist and relationist could hold that, say, a flat Newtonian spacetime structure is a platonic uninstantiated entity or property, or both could opt for the nominalist view that Euclidean spacetime structure only exists in the entity (material field for the relationist, unique substance for the substantivalist) that exemplifies that Euclidean structure.

Consequently, the best option for a platonist-inspired conception of space is to drop the requirement that abstract objects are non-spatial. Henceforth, our analysis will refer to "immanent realism" as the hypothesis that abstract objects, such as mathematical structures or universals, only exist within spatiotemporal entities, broadly construed, whether that entity is a substance (substantivalism) or a physical field/object/event (relationism). It should be noted that immanent realism can be categorized as a rival to platonism, and, since material things house abstract objects, there is an obvious affinity between immanent realism and nominalism (see, e.g., Balaguer 2009, who allows immanent realism to be classified as a form of nominalism). Yet, since both platonism and immanent realism accept the existence of abstract objects, such as mathematical or geometric structures, but only differ on where they exist, there is nonetheless a close relationship between immanent realism and platonism as well—indeed, since all of the other contenders in the realm of mathematical ontology reject the existence of abstract (or mathematical) objects, there is a good justification for accepting immanent realism as a surrogate for the standard (non-spatial) conception of platonism in the assessment of spatial ontology.

Nevertheless, there are tricky conceptual difficulties connected with immanent realism that may ultimately render it an unattractive option in the spacetime ontology debate. In particular, whereas it seems (relatively) unproblematic to claim that the abstract object "sphere" is located in a particular spherical body in spacetime, how does one make sense of the claim that space or spacetime *itself* is located or exemplified in a substance (substantivalism) or a physical body/field (relationism)? Take, for instance, metric-field substantivalism and its identification of the metric-field *g* with spacetime (e.g., Hoefer 1996, a view that will henceforth be labeled "sophisticated substantivalism"): since an abstract object provides the defining essence of a substance, the abstract object *g* makes a particular substance, call it *x*, the spacetime substance *g*. But, since *x* lacks all spatial properties or attributes prior to its exemplification of the abstract object *g*, how can it situate or locate the abstract object *g* at all? There may be strategies available to the immanent realist to forestall these difficulties, such as a non-spatial or non-topological haecceity for *x* (i.e., *x* possess a "bare numerical difference" that secures its identity and ability to exemplify universals like *g*). Yet, it is interesting to note that Armstrong, one of the chief advocates of immanent realism, ultimately supports a position that takes spatiotemporal relations, conceived as abstract objects or universals, as constitutive of spacetime, but not themselves located in spacetime: "Where are external spatiotemporal relations located? Here, it seems to me, we could cheerfully concede, if we wanted to, that they are not located, yet not place them 'outside space and time'. . . . So, if they help to constitute space-time, then it is no objection to their spatiotemporality that they are not located in space-time" (Armstrong 1988, 111-112; see also Magalhães 2006).

1.2. Fictionalist Nominalism versus Truth-Value Nominalism. Just as the ontological domain of abstract objects justifies a multifaceted conception of platonism, a complex subdivision of nominalist interpretations will likewise be of use in our investigation. First, a strict eliminativist approach to nominalism is evident in Field's (1980) effort to provide a mathematically anti-realist version of Newtonian gravitation theory via a substantivalist commitment to the reality of the points of the manifold. This form of nominalism, dubbed "fictionalism", strives to eliminate the mathematical ontology in favor of an isomorphic physical/substantival ontology, with the latter providing the meaning of the former (e.g., the point manifold, being isomorphic to $\mathfrak{R}_4$, can provide the real number structure essential for the mathematics of classical gravitation theories). Fictionalist nominalism is thus a close analogue of the strict, eliminativist branch of relationism, for both replace structure, mathematical and spacetime, respectively, with existing physical entities—that is, just as "eliminative relationism", as we will call it, requires a congruence of spatiotemporal relations and extant physical relations, fictionalism requires a congruence of mathematical structure and physical or substantival structure.

Turning to the "sophisticated" versions of relationism, such as Sklar (1974), Teller (1991), Huggett (1999), since these hypotheses sanction a modality that can transcend the spatial relationships exhibited by the actually existing physical objects, a different form of nominalism is required (e.g., Teller allows a rotating body in an otherwise empty universe to instantiate inertial structure; 1991, 396). There are a number of less stringent nominalist theories that would stand as the mathematical equivalent of sophisticated relationism; e.g., Chihara (2004), Azzouni (2004), Lewis' (1993) mereological approach, or Hellman's (1989) *in re* structuralism. These "truth-value nominalists", to adapt Pincock's (2012) term for our purposes, do not allow structures to exist independently of physical systems, yet they do not believe that those structures can be eliminated in favor of an underlying physical ontology that is isomorphic to the real numbers or some other mathematical structure. Specifically, what links these similar truth-value nominalist strategies is the rejection of both fictionalism and the existence of the platonist's abstract objects, and a realist commitment to the truth-values of mathematical statements and their associated modality.

Given our analysis of platonism and nominalism above, the philosophy of mathematics thus offers a three-part distinction: immanent realism, fictionalist nominalism, and truth-value nominalism. This three-part division is advanced by Pincock (2012), who explains that fictionalists, like Field, "are not truth-value realists, while the standard nominalists remain realists about truth-values, even if they are not realists about abstract objects. Platonists are realists about both truth-values and abstract objects" (15). However, Pincock's three-part distinction among platonist and nominalist categories differs in various ways from our usage, most conspicuously in that our analysis replaces platonism with immanent realism, but there are other important differences as well.[1]

Overall, it is the eliminativist component of fictionalism that is central to the case advanced in this essay, and not the claim that mathematical statements are false (but useful fictions). Given the final theory of physics (which provides the final ontology of space and spacetime), it might turn out that the physical structure and the mathematical structure are indeed isomorphic: "truth-value nominalism", as defined in this essay, is compatible with that outcome, but it does not require it, unlike Field's eliminativist conception.

2. Platonism/Nominalism and Spatiotemporal Philosophies.

In this section, the three-part distinction advanced above—immanent realism, truth-value nominalism, and fictionalist nominalism—will be put to use in evaluating some of the other combinations of platonism/nominalism and substantivalism/relationism in the contemporary literature, namely, Field and Arntzenius.

2.1. Nominalism: Field versus Leibniz. Interestingly, the claim that the platonism/nominalism divide in mathematics can benefit the analysis of substantivalism and relationism was briefly mentioned in Sklar (1974, 165), one of the seminal works in the modern approach to the philosophy of space and time. Sklar regarded nominalism as an exclusively relationist standpoint, a verdict that is in keeping with relationism's historical disdain for absolute space, with the latter suggesting, perhaps mistakenly, a platonist's commitment to abstract objects. Nevertheless (and leaving aside Slowik 2005), the combination of nominalism and substantivalism is a central element in Field's program (1980), and has recently been advocated by Arntzenius (2112, 213-268). In short, Field reasons that spacetime physics requires spacetime points, which relationism cannot tolerate, so substantivalism is the only remaining option for the nominalist (1980, 34). Field examines two types of relationism, which he ultimately rejects: "eliminative relationalism", which is the view that space can be eliminated in favor of a purely physical account of these structures (and is identical with our use of the same term); and "reductive relationalism", which corresponds to our "sophisticated relationism", i.e., spatiotemporal structures are grounded in, or can be reduced to, physical entities, although these structures cannot be eliminated given their important function within the overall theory. Field concludes that "[i]t is clear that reductive relationalism is unavailable to the nominalist: for according to this form of relationalism, points and regions of space-time are mathematical entities, and hence entities that the nominalist has to reject" (34).

Field's reference to spacetime points as "mathematical entities" prompts a number of questions, in particular, whether truth-values, abstract objects, or both, are incorporated into his conception. Besides granting a number of concessions to the more sophisticated brands of relationism in his later (1989), he nonetheless insists that relationism can only succeed as an alternative to substantivalism if it takes "the relation between physical things and numbers to be a brute fact, not explainable in other terms" (1989, 186). This view—which he dubs, "heavy duty platonism" (hereafter HDP)—would seem to denote our truth-value nominalist conception, since a "brute fact", if interpreted minimally, has (or is) a truth-value. Furthermore, he states that HDP "is a much more radical position than mere platonism, i.e., the mere acceptance of mathematical objects" (185), an admission that suggests that HDP's brute facts are a separate issue from the question of abstract objects. Whether or not abstract objects are a necessary ingredient of Field's HDP remains unclear, but henceforth HDP will be taken as synonymous with truth-value nominalism alone (and not abstract objects). Overall, Field does conclude that, if one accepts HDP, then a relationist can successfully account for, say, acceleration in Newtonian physics: "one could simply take co-ordinate functors defined on points of matter as primitive, and define an acceleration functor from them in the usual way, without ever having to appeal to unoccupied regions" (191). Hence, Field reasons that sophisticated modal relationist proposals, such as Mundy (1983), tacitly endorse HDP (199), even though HDP "violates the whole spirit of relationalism" (192).

Yet, from a philosophy of mathematics perspective, Field's (1989) case against the relationist's use of HDP begs the question, for there have been numerous nominalist formulations that forsake the existence of abstract objects but defend a conception of mathematical truth and modality that goes beyond fictionalism: besides some mentioned above, such as Azzouni 2004, one can add Salmon 1998, Priest 2005, and Bueno 2005. Accordingly, why must relationists forsake HDP (= truth-value nominalism) if many philosophers of mathematics embrace hypotheses that are either identical to, or closely aligned with, truth-value nominalism?

Does truth-value nominalism (i.e., Field's HDP) specifically flout relationist doctrine? Besides the numerous modern proposals that uphold the non-eliminative brands of sophisticated relationism consistent with truth-value nominalism (such as by Sklar, Teller, Huggett, and many more), an important precedent for admitting the truth-values associated with spatiotemporal structures, but not their existence as an entity, can be found in Leibniz' nominalist account of space at the level of material bodies.[2] While space is not "an absolute being" (15; L.III.5), Leibniz holds that it still represents "real truth" (47; L.V.47), even in the absence of matter: "[t]ime and space are of the nature of eternal truths, which equally concern the possible and the actual" (1996, II.xiv.26). But, since "truth and reality are grounded in God, like all eternal truths" (II.xiii.17), these God-grounded spatial truths also determine the range of possibilities for the arrangement of bodies in space. On the numerous ways that the world could be filled with matter, Leibniz states that "there would be as much as there possibly can be, given the capacity of time and space (that is, the capacity of the order of possible existence); in a word, it is just like tiles laid down so as to contain as many as possible in a given area" (1989, 151). Truth-value nominalism best describes Leibniz' approach, moreover, especially when the difficulties associated with a fictionalist interpretation of the space-God relationship are taken into account. In the case of Leibniz, fictionalist nominalism would require the elimination of spatial structure and its replacement by an analogous isomorphic structure in God—but that would imply, also in keeping with fictionalism, that the ontology of space is really God. Yet, Leibniz rebuffs any attempt to link matter/space to his non-spatial God, preferring to state that "space is an order but that God is the source" while simultaneously rejecting the claim that "space is God or that it is only an order or relation" (1996, II.xiii.17). On a fictionalist construal, conversely, it really would be true to say that space is God, since space would constitute a fictional order/relation that can be replaced by that ontology (God)—this parallels Field's scheme, where mathematical structures are really the substantivalist's point manifold and only a fictional order/relation that can be eliminated in favor of that ontology. Leibniz does claim that God has the property of "immensity" (II.xv.2), but he explicitly denies that God's immensity is spatial: "the property of God is immensity but . . . space (which is often commensurate with bodies) and God's immensity are not the same things" (2000, 44; L.V.36). Likewise, since "space cannot be in God because it has parts" (48; L.V.51), it is rather difficult to envisage a structure preserving isomorphism between space, which is extended and has parts, and God, who is neither extended nor has parts. In short, given the mature Leibniz' claim that God is non-spatial, his appeal to the "truths" of spatial geometry appears inconsistent with fictionalism.

Invoking Leibniz as a counter-example to Field's conception of nominalism is particularly warranted, moreover, since one of his arguments mirror Clarke's case, in the correspondence with Leibniz, that geometric ratios or proportions are not quantities. Calling it "the problem of quantities for the relationalist" (1989, 196), Field brings up the well-known fact that spatial ratios or proportions, which are presumed to be as the only acceptable form of spatial relationships for a relationist, underdetermine the quantitative (distance, metric) relationships among things unless one accepts HDP (185). Leibniz' "order of coexistences" account of space (2000, 14; L.III.4) provokes the same argument from Clarke: "the distance, interval, or quantity of time or space . . . is entirely a distinct thing from the situation or order and does not constitute any quantity of situation of order; the situation or order may be the same when the quantity of time or space intervening is very different" (73; C.V.54). Yet, Leibniz insists that his conception of spatial relationships includes quantity, thereby sanctioning the type of "brute fact" reading of quantity (cf. Field's HDP) that constitutes truth-value nominalism: "As for the objection that space and time are quantities, or rather things endowed with quantity, and that situation or order are not so, I answer that order also has its quantity: there is in it that which goes before and that which follows; there is distance or interval. Relative things have their quantity as well as absolute ones" (50; L.V.54). Contra Field, the modality associated with truth-value nominalism has long been favored by spatial relationists, as well as by Leibniz, even though the latter's views differ from modern relationism in significant ways (see Slowik 2012).

Before leaving the historical aspect of our investigation, it would be useful to contrast Leibniz' nominalist-inspired approach to space with Newton's conception, specifically as it relates the platonist/nominalist distinction. While Newton denies that space is a substance (Newton 2004, 21-22), a careful perusal of his writings reveals that absolute space is closely akin to God's property: God contains "all other substances in Him as their underlying principle and place" (Newton 1978, 123). Can a case be made for a platonist designation of Newton's spatial ontology, especially immanent realism (where abstract objects exist in space and time)? In the *De grav*, the presence in space of Euclidean geometry is revealed in his claim that "the delineation of any material figure is not a new production of that figure with respect to space, but only a corporeal representation of it, so that what was formerly insensible in space now appears before the senses" (Newton 2004, 22). This claim appears consistent with immanent realism, but it also rules out Armstrong's brand of constitutive platonism, where universals constitute space but are not themselves in space. Yet, to qualify as a version of immanent realism, abstract objects or universals would need to be responsible for the very structure of space—and, while possible, there is simply no evidence that Newton supports such a view. Rather, it is God's very existence that brings about space: e.g., "space is eternal in duration and immutable in nature because it is the emanative effect of an eternal and immutable being" (26). In the passage from *De grav* cited above (22), consequently, bodies reveal the geometric structure that is instantiated or exemplified in God's being or substance, since God is actually present in space. The *De grav*'s "determined quantities of extension" thesis, where bodies are depicted as movable properties in God's infinite spatial extension, is the most straightforward presentation of this God-space relationship (27-28). In short, the evidence of the texts supports a general nominalist stance whereby space is instantiated by God's own being.

Nevertheless, given the theological obstacles connected with the identification of space with God's being *per se*, fictionalism is almost certainly not the correct form of nominalism. That is, fictionalist nominalism would demand the replacement of the spatial ontology with God's ontology, but Clarke, who was likely advised by Newton in his correspondence with Leibniz, straightforwardly rejects this view: "Space is not a being, an eternal and infinite being, but a property or a consequence of the existence of an infinite and eternal being. Infinite space is immensity, but immensity is not God; and therefore infinite space is not God" (2000, 19; C.III.3). Newton accepts this line of argument as well: e.g., space is "an emanative effect of God" (Newton 2004, 21), which is a type of description that seems to put some ontological room between God's being *per se* and space, so that God brings about space but is not literally identical to space. On a fictionalist construal of this particular ontology, conversely, it really would be true to say that space is God, since space would constitute a fictional order/relation that can be replaced by that ontology (God)—this parallels Field's scheme, where mathematical structures are really the substantivalist's point manifold and only a fictional order/relation that can be eliminated in favor of that ontology. Therefore, given the obstacles that face the immanent realist and fictionalist construals of Newton's spatial ontology, truth-value nominalism is the most plausible, as well as the only remaining, option.

2.2. A Nominalist Counter-Argument. Returning to Field's critique of non-fictionalist relationism, the reasons he offers for linking substantivalism to nominalism also reveal his unique, and controversial, conception of modern field theories in physics. As he explains, "a field is usually described as an assignment of some property, or some number or vector or tensor, to each point of space-time", but, on Field's construction of Newtonian gravitation theory, "it does without the properties or the numbers or vectors or tensors, but it does not do without the space-time points" (1980, 35). A host of objections have been raised against this conception of spacetime points as physical entities: unlike ordinary objects, spacetime points do not endure through time, have no mass, cannot move or be empirically detected, and have no location (since they comprise location; see

Resnik 1985). Furthermore, Field's ontology is committed to spacetime regions as much as spacetime points (1980, 36-37), and this helps to explain his later admission that both the substantivalist and relationalist can construct spacetime points from a mereological postulate, so that points become minimal parts of a spacetime region (1989, 172-174). Field concedes, essentially, that the conception of the metric-field as a physical entity, called "metric-field relationism" in the recent literature, is as acceptable as his substantivalist rendering of the metric-field: it is for these reasons that Field claims that relationism must admit the possibility of unoccupied spatial regions (174), for only on this scenario can a distinction be drawn between a nominalism based on substantivalism as opposed to one based on relationism. Yet, since the metric-field of GR is defined over the entire manifold, there are no, as it were, "metrical voids" (i.e., a region of the manifold lacking a metrical value); and, since the metric alone (absent non-gravitational stress-energy) has the potential to induce physical effects via its dual role as the gravitational-field, a matter-less metric-field in GR fits the definition of a nominalist relationism as much as it does a nominalist substantivalism (see, e.g., Earman and Norton 1987, 519, who detail some of the physical consequences of gravity waves). Consequently, not only is Field's case for a substantivalist nominalism predicated on an outdated theory, Newtonian gravity, but our best theory of the large scale structure of spacetime, GR, does not admit the type of void space that he requires to distinguish nominalist substantivalism from nominalist relationism.

The difference between an ontological commitment to points versus a commitment to extended regions, so that points become parts of a larger entity, is also evident in Arntzenius' case for a nominalist substantivalism (2012), a work that strives to extend

Field's project in order to encompass the curved spacetime of GR and its mathematics built upon differential geometry. However, like Field's substantival conception of spacetime points, Arntzenius puts forward an approach to nominalism that simply "substantivalizes" those mathematical entities, i.e., he treats mathematical entities as substances and thereby secures by fiat a nominalist classification. For instance, Arntzenius argues that, given "the need to quantify over functions from spacetime points to real numbers, the obvious strategy to consider is to enrich the ontology in such a way as to provide nominalistic surrogates for such functions. . . . [W]e will do this by positing, for each spacetime point, a miniature one-dimensional space—a 'scalar value line'—endowed with a rich structure making it in effect a copy of the real line" (243-244). Whereas truth-value nominalism can secure the real line structure via, e.g., the possible divisions of an extended body (à la Aristotle), Arntzenius' fictionalism not only demands "collections of entities isomorphic to the real numbers", but "[i]t also contains some entities we might think of as uniquely natural candidates to *be* the real numbers; namely, the constant scalar fields" (246). Ultimately, besides suggesting "fibre-bundle substantivalism" (185), the application of Arntzenius' nominalism to quantum theory leads to "Hilbert space substantivalism" (269). With regard to the latter, Arntzenius raises the concern that treating a mathematical structure (in this case, a Hilbert space) as a substance would seem quite removed from traditional nominalism (269). Nevertheless, his strategy does fit the minimal definition of nominalism advanced in this investigation, namely, the rejection of abstract objects.

A major problem with Arntzenius' scheme, as well as Field's, is that it fits the definition of relationism as equally as it does substantivalism. A component of his nominalist strategy is to show “how all the ‘mixed’ vocabulary of some platonistic physical theory can be eliminated in favour of ‘pure’ predicates all of whose arguments are concrete physical entities” (215). But, the elimination of spacetime structure in favor of “concrete physical entities” could describe relationism as well as substantivalism. Therefore, as with Field, Arntzenius’ project would seem equally suited to those sophisticated relationists who conceive mathematical fields as physical entities, such as metric-field relationism. Moreover, the absolutist/substantivalist standpoint has traditionally regarded space as lacking the ability to interact causally with bodies or other fields (e.g., Newton), but the type of fibre-bundle structures that Arntzenius develops includes the metric of GR (213-270), which can partake in these mutual interactions; likewise for Field, whose approach to field theories is motivated by the idea that spacetime points have causal powers (“space-time points or regions are full-fledged causal agents”; 1980, 114). Therefore, by bestowing a causal capacity upon space, Field and Arntzenius are open to the charge that they are appropriating a strategy of “elimination of spatiotemporal structure in favor of physical entities with causal powers” that one would normally associate with relationism.

Transferring the context of substantivalism to the realm of quantum mechanics, via Arntzenius’ Hilbert space substantivalism, likewise opens the door to another nominalist critique. If, as Field insists, relationism must admit the possibility of a void space, then, to be fair to the relationist, a closely related scenario should also apply to the substantivalist—namely, the existence of spatial structure in the complete absence of all matter or physical fields. Yet, would Arntzenius’ Hilbert space substantivalist accept the existence of QM’s Hilbert space, an abstract vector space, given the non-existence of the quantum mechanical wavefunction that is an element of that vector space? If an affirmative answer is given, as would seem necessary to render Hilbert space substantivalism truly substantivalist in the customary sense that we associate with the possibility of universal vacuum state in Newtonian theory, then Arntzenius would be placed in the unenviable position of having to defend the existence of a sort of Pythagorean bare mathematical entity; i.e., an abstract vector space without the wavefunction. As a counter-reply, the committed substantivalist may invoke an analogy between GR's metric-field and QM's wavefunction, insisting that, just like metric-field substantivalism, Hilbert space substantivalism also posits a congruence of the physical component (gravitational-field in GR, wavefunction in QM) and the spatial component (metric in GR, Hilbert space in QM). Put differently, a Hilbert space substantivalist can claim that there are no "wavefunction voids" in QM just as there are no metrical voids in GR. So, the relationist's claim that substantivalism must allow for the possibility of an empty space (i.e., entirely void of matter/fields) simply fails to fit the details of these contemporary theories, GR and QM.[3] However, this response, while perfectly correct, only demonstrates the weakness of Field's contention that relationism must also admit a void—i.e., given the congruence of the physical and geometric/mathematical components in each of these theories, GR's metric-field and QM's wavefunction, the problematic potential for a truly energy-less void space is ruled out on both the substantivalist and the relationist interpretations. Finally, it should be noted that the current debate on the interpretation of the wavefunction in QM (see Ney and Albert 2013) neither supports substantivalism nor relationism, since that controversy pertains to the non-reductive or non-eliminative character of the mathematical structures in QM: in other words, either configuration space or Hilbert space, or possibly another space, is a feature of QM that cannot be reduced to other structures, nor eliminated in favor of these other structures—yet, whether a *mathematical* structure is reducible or non-reducible no more supports substantivalism than it does relationism, but it does impact the disputes among the different factions that make up nominalism, truth-value versus fictionalist, as we have seen. Indeed, despite Wallace and Timpson's defense of the view they call "spacetime state realism" (as a rival to wavefunction realism), they concede that both substantivalist and relationist interpretations are admissible (2010, 700, n.2).

3. Scientific Realism and Platonism/Nominalism.

To conclude our investigation, it will be worthwhile to examine the confluence of platonism/nominalism and contemporary scientific realism, in particular, the contrasting approaches to spatial or geometric structures in the recent work of Psillos and French, for our more fine-grained three-part platonist/nominalist scheme can also be of service in evaluating their respective philosophies.

3.1. Platonism and Scientific Realism. In several works, Psillos defends a platonist conception of abstract objects in opposition to the fictional nominalist position dubbed "nominalistic scientific realism" (NSR), an approach that strives to rid theories of mathematical objects by drawing on Field's fictionalist program. Because Psillos equates NSR with anti-realism and its commitment to the empirical adequacy of theories, he concludes that non-causal abstract objects should be embraced by scientific realists in much the same way that they accept unobservable theoretical entities. Among these abstract objects, Psillos lists several items familiar to spacetime ontologists, "like phase spaces, vector spaces, and groups" (2010, 951), as well as inertial frames (2011, 4). Leaving aside Psillos' justifiable critique of a fictionalist interpretation of scientific theories, his embrace of platonism should raise a number of concerns for scientific realists. First of all, it is unclear what Psillos means by platonism: Does he endorse Plato's notion, where abstract objects exist outside of space and time, or immanent realism, where abstract objects exist in material objects? Since it is difficult to envision a scientific realist embracing an hypothesis whose domain is akin to Plato's heaven of non-spatiotemporal Forms, immanent realism would seem the best option, although Psillos does leave open the possibility that his "physical abstract entities", as he calls them, lack spatiotemporal location (2010, 950). Regardless of the domain quandary concerning abstract objects, the main obstacle confronting a platonist scientific realism is that it would seem committed to an immense number of abstract objects due to the well-known fact that modern theories in foundational physics can be given numerous different mathematical formulations involving different abstract objects. A full treatment of this topic is beyond the bounds of this essay, but a commonly cited example from the history of classical gravitation theories is the choice between the standard spacetime formulation of GR, with its non-flat spacetime structure representing one type of abstract object, as opposed to the Newton-Cartan formulation, with its flat spacetime structure representing another brand of abstract object (see Pooley 2006, 87-88).

While platonists may happily tolerate this profusion of abstract objects, a better option for a scientific realist would seem to lie in a commitment to the truth-values associated with abstract objects (truth-value nominalism), as opposed to their existence as entities (immanent realism). That is, admitting a plethora of truth-values linked to the different formulations of a given scientific theory and its *single* ontology seems much more congenial to scientific realism than admitting an ontology populated by a plethora of different, undetectable abstract objects for each of its many possible formulations. Besides, Psillos' case for platonism is largely centered on the perceived need to uphold the truth of mixed physical-mathematical scientific statements—and, since he concedes that "we are ignorant" of the manner by which "the concrete [physical] and the abstract co-operate to render mixed statements true" (2012, 80), it follows that the realist stance favored by truth-value nominalism as regards the truth-values of possible structures should constitute a solution to Psillos' quest. In all fairness, it should be further noted that some of the truth-value nominalist positions described in section 1.2, such as Lewis and Hellman, are occasionally categorized as platonist due to their realist approach to truth-values (see Colyvan 2001, 142), so it is reasonable to conclude that truth-value nominalism would meet Psillos' anti-NSR requirements.

3.2. Nominalism and Structural Realism. Finally, the platonist/nominalist dichotomy is relevant for assessing ontic structural realism (OSR), which accepts a realist ontology of structure, and epistemic structural realism (ESR), which sides with a realism focused on the epistemology of structure. In what follows, we will confine our attention to the work of Steven French, for he has specifically commented upon the relationship between his version of OSR and structuralism in the philosophy of mathematics, i.e., *ante rem* (platonist) and *in re* (nominalist) structuralism (see Shapiro 2000). In brief, French is skeptical of the applicability of mathematical structuralism, claiming that "the classification of '*ante rem*' vs. '*in re*' structuralism may not be appropriate in the context of OSR, where the world-as-structure is neither abstract, in the sense of being non-causal nor a system that is structured, in the sense that there is a system that is ontologically prior to the structure" (2012, 25; and where "world-as-structure" signifies a non-individuals, holistic ontology of quantum particles; see Ladyman and Ross 2007, 158). Rather than view the group structure of QM as representing the relations among autonomous individual quantum particles, OSR offers an "entirely structural" account, whereby physical objects, such as electrons or quarks, are "reduced to mere 'nodes' of the structure, or 'intersections' of the relevant relations, in some sense" (French 2006, 171, 183).

Returning to the platonism/nominalism distinction, French's comment that OSR rejects abstract structure thereby rules out platonism, whether of the standard or immanent realist variety, but perhaps truth-value nominalism as well, since the latter accepts a realism concerning the truth-values related to these non-causal abstract objects (although not to their existence as entities). On the other hand, fictionalist nominalism is much closer to French's interpretation of OSR—specifically, fictionalism takes an eliminativist stance on mathematical structure, a position that accords nicely with French's claim that the world-as-structure is neither an abstract object nor "a system [of individual particles] that is ontologically prior to the [group-theoretic] structure". The support for a fictionalist-based interpretation of French's philosophy is, moreover, quite compelling; e.g., "the idea is that instead of conceiving our ontology in terms of objects, and then having to face the dilemma of whether to regard them as individuals or not, we focus on the relevant group-theoretical structures underpinning quantum statistics and effectively re-conceptualize (or eliminate) our putative objects in terms of these structures" (2011, 217). Group-theoretic structure is not the underlying ontology; rather, the underlying ontology is a physical structure that is precisely described using group-theoretic mathematical structure. This response, incidentally, is exactly what fictionalists would counsel, for they take mathematical structure to be a replaceable means of describing an isomorphic physical ontology. Furthermore, unlike platonism, immanent realism, and truth-value nominalism, causality is incorporated into French's version of OSR (2012, 25), a feature that he shares with Field and Arntzenius' fictionalism (as noted in section 2.2)

Nevertheless, in a recent work (2011), French has suggested that his world-as-structure ontology is "multi-featured", a development that would appear to undermine a fictionalist classification for his brand of OSR. Since the physical ontology and its mathematical representation are structurally isomorphic for a fictionalist, it follows that different mathematical formulations of a theory correspond to a different structured ontologies, thereby inducing the same kind of underdetermination problem that inspires the anti-realist's case against the existence of theoretical entities (i.e., the pessimistic meta-induction). Therefore, if truly fictionalist in the manner of Field and Arntzenius' project, French's OSR would be equally susceptible to an ontological underdeterminism resulting from an underdeterminism of structure. Bueno has demonstrated this point as regards the different structures required for QM's overall mathematical representation: while both group-theoretic and Hilbert space structures are essential, he concludes that "when we combine both mathematical frameworks, no clear picture of what is going on at the quantum level emerges" (Bueno 2012, 94). French (2011) responds to the general underdetermination challenge by seeking a "common structure" that lies beneath the different mathematical formulations, a common structure that, besides his long standing preference for a group-theoretic account, may require the addition of dynamical (symplectic) structure for its full characterization. French concludes:

> Hence, the structuralist still has some work to do in supplementing the 'object structure' with the relevant dynamical structure and fleshing out the 'world structure' as multi-featured. In effect what we have is an appropriately complex ontology that includes both the group-theoretically characterized structure underlying the particles-as-individuals and particles-as-non-individuals packages, and the common symplectic structure underlying the Hamiltonian and Lagrangian formulations (2011, 219).

Consequently, since there are now several mathematical structures that track his "complex ontology" from different mathematical perspectives, the fictionalist prescribed isomorphism between mathematical and physical structures is no longer assured; i.e., the same physical ontology may not be capable of accommodating these different mathematical structures in a coherent fashion, as Bueno's case makes clear.

Furthermore, despite French's causation-based misgivings about the relevance of mathematical structuralism for OSR, philosophers of mathematics will insist, quite correctly, that all theories that posit a relationship between mathematics and physics fall within the scope of traditional platonist/nominalist categories, hence an evaluation of OSR's on these grounds is justified. Among the categories in our tripartite scheme, truth-value nominalism would appear to be the best, and maybe only, option: that is, leaving aside his rebuff of platonism's abstract objects, the central point of French's "multi-featured" OSR is his hypothesis that different mathematical structures can be used to characterize the underlying physical ontology, and this overall conception is best captured by truth-value nominalism (and not fictionalism), as argued above. The revelation that the world-as-structure ontology is multi-featured also places OSR fairly close to ESR, especially the sophisticated or liberal forms of ESR, save for OSR's allegiance to a particles-as-non-individuals QM holism. In short, there is tension between

French's claim that world-as-structure is not "a system that is ontologically prior to the [group-theoretic] structure", and the fact that (i) a particles-as-individuals interpretation is compatible with the group-theoretic account of QM, and (ii) the recognition that QM's "complex ontology" necessitates supplementary structures beyond the group-theoretic for a more accurate characterization (in other words, group structure alone cannot disclose QM's complex ontology). Provided (i) and (ii), a more accurate category for French's theory would thus seem to lie in the liberal form of ESR, dubbed ESR_2 in French and Ladyman (2012), where the invariance of structure (perhaps approximate structure) over theory change fails to disclose the full nature of the underlying ontology; i.e., whether the ontology is only structure, only particles, or both particles and structure on a par. Yet, Ladyman and French (2012, 27) reject ESR_2 and posit instead a particles-as-non-individuals interpretation, a particular metaphysical reading of an acknowledged complex, multi-faceted ontology. As a direct result, this OSR strategy must inevitably prompt underdetermination anxieties (such as the anti-realist's pessimistic meta-induction), especially in an environment where a projected theory of quantum gravity may upend the prevailing mathematical and ontological conceptions of the micro realm.

5. Conclusion.

At the outset of our investigation, two questions were posed concerning the application of the platonism/nominalism dichotomy to the spatial ontology debate, the first pertaining to its relationship with substantivalism and relationism, and the second involving the benefits that accrue from platonism and nominalism. As regards the first, a nominalist approach to spatial structure would appear to be the view favored by substantivalists and relationists, both historically and in the contemporary literature, whereas platonism and immanent realism find little support. The substantivalist position advocated by Field and Arntzenius offers an obvious example of a contemporary nominalist strategy to spacetime, but, in section 2.1, we have also argued that truth-value nominalism reveals salient features about the respective God-based spatial ontologies of Newton and Leibniz, the two natural philosophers often presumed to be the most important representatives of substantivalism and relationism. The problem with platonism and immanent realism is likely due to their dependence on abstract objects, whether conceived as non-spatial (platonism) or as a property of spatially-situated entities (immanent realism). In short, explaining spatial structure by way of an abstract object does not appear to be an attractive option for spacetime theorists.

Turning to the second question, the lack of interest in an abstract object conception of space also reveals that the platonism/nominalism issue is mainly centered on the choice between the reduction or elimination of spatial structure, i.e., whether the mathematical/geometric structure of space is isomorphic (fictionalism) or non-isomorphic (truth-value nominalism) to the entity that underlies space. And, while other investigations have questioned the modality implicit in sophisticated versions of relationism (e.g., Belot 2000), Field's case against these types of relational theories depends crucially on his argument against the conjunction of truth-value nominalism and relationism (the ban on HDP, examined in section 2), thereby providing another example of the relevance of nominalism for the investigation of spatial ontology. As noted at the outset, this inference does not imply that the introduction of the platonism/nominalism dichotomy is required to resolve the spatial ontology debate, but it does demonstrate the close similarity between, on the one hand, substantivalism and relationism in spacetime ontology, and on the other, platonism (immanent realism) and nominalism in the philosophy of mathematics and metaphysics.

Furthermore, given our three-part division among platonist and nominalist concepts, the most appealing and versatile options for understanding the relationship between the underlying spatial ontology and its mathematical/geometrical structure would seem to reside in truth-value nominalism, a non-eliminativist conception of spatial structure that is open to substantivalist and relationist alike. Part of the reason for truth-value nominalism's success may stem from the fact that it represents a sort of intermediate approach between the extremes of abstract objects (platonism/immanent realism) and fictionalism, a middle way that is analogous to the position that the sophisticated forms of both substantivalism and relationism take with respect to manifold substantivalism and a strictly eliminative relationism. In addition, Field's effort to block a relationist from espousing truth-value nominalism begs the question and runs counter to the historical record, while his and Arntzenius' fictionalist strategy is no more or less amenable to substantivalism than to strict eliminative relationism in the context of GR and QM.

As presented in section 3, truth-value nominalism can also aid Psillos' case against a fictionalist-based scientific realism, but without requiring a realism about abstract objects or a commitment to substantivalism. Likewise, French's conception of OSR, with its multi-faceted world-as-structure ontology, could benefit from truth-value nominalism, especially given both his rejection of abstract objects and the obstacles associated with linking fictionalism to a diverse multi-layered mathematical treatment of a complex ontology. Yet, it would seem that ESR and truth-value nominalism form a more natural pair: in contrast to most conceptions of OSR, ESR can easily admit alternative mathematical constructions, since the epistemological invariance of a given structure does not fix the underlying ontology to that structure alone. Rather, alternative mathematical formalisms may reveal additional epistemological invariants that either track different aspects of the ontology or simply describe the same ontology differently. More importantly, since ESR is epistemologically based, and truth is an epistemological notion, it naturally follows that truth-value nominalism aligns with ESR.

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1 In particular, Pincock (2013, 271) has doubts concerning the hypothesis entitled, "heavy duty platonism" (see below), although our analysis treats this concept as equivalent to truth-value nominalism. The problem probaby resides in the difficulty in determinng just what Field intends by this concept, as will be further explained below.

2 One of the most straightforward declarations of nominalism can be found in the *New Essays,* where numbers and spatial extension are compared: "[I]n conceiving several things at once one conceives something in addition to the number, namely the things numbered; and yet there are not two pluralities, one of them abstract (for the number) and the other concrete (for the things numbered). In the same way, there is no need to postulate two extensions, one abstract (for space) and the other concrete (for body)" (1996, II.iv.5).

3 The analogy developed here is a bit loose, as a referee has pointed out. Since a wavefunction is an element of a Hilbert space, and is thus not defined on a Hilbert space in the same way that a metric is defined on a manifold, a more correct analogy would be between a wavefunction as an element of a Hilbert space and a manifold point as an element of a manifold. However, substituting a wavefunction/Hilbert space analogy with a point/manifold, rather than using a wavefunction/Hilbert space analogy with a metric/manifold, works just as well for our purposes. On a related note, Dieks correctly points out that, as regard non-relativistic QM, "the Hilbert space formalism does not start from a space-time manifold in which particles are located. The quantum state is given by a vector in Hilbert space, and has in general no special relation to specific space-time points. Rather, 'position' is treated in the same way as 'spin' or other quantities that are direct particle properties: all the quantities are 'observables', represented by Hermetian operators in Hilbert space" (Dieks 2001, 16).