A review on possible physical meaning of elastic-electromagnetic mathematical equivalences

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Abstract

It is known, despite special theory of relativity has been widely accepted, in our recent draft submitted to this journal it is shown that some experiments have been carried out suggesting superluminal wave propagation, which make Minkowski lightcone not valid anymore. Therefore, it seems worth to reconsider the connection between elastic wave and electromagnetic wave equations, as in their early development. In this paper we will start with Maxwell-Dirac isomorphism, then we will find its connection with elastic wave equations.

Keywords: elasto-electromagnetic wave equations, realism interpretation, Maxwell-Dirac isomorphism.

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Introduction

In its earliest development, electromagnetic equations of Maxwell have elastic wave properties. As Ikelle noted :

> "Ether was held to be invisible, without odour, and of such a nature that it did not interfere with the motions of bodies through space. The concept was intended to connect the elastic wave theory with Maxwell's field theory. However, all attempts to demonstrate the existence of ether, most notably the experiment reported by Michelson & Morley (1887), produced negative results and stimulated a vigorous debate that was not ended until the special theory of relativity, proposed by Einstein (1905), became accepted."

Despite special theory of relativity has been widely accepted, in our recent draft submitted to this journal, it is shown that some experiments have been carried out suggesting superluminal wave propagation, which make Minkowski lightcone not valid anymore:

$$ds^2 = dr^2 - c^2.dt^2 \quad (1)$$

setting ds=0, we got:

$$dr^2 = c^2.dt^2 \quad (2)$$

then

$$dr/dt = c \quad (3)$$

while experiments showing superluminal propagation are abound. This result may be not surprising, along with some discussions on the meaning of empty spacetime [5], but it leads us to consider again the connection between elastic wave equations and electromagnetic wave equations.

An expression of Maxwell-Dirac isomorphism

First of all, let us point out that there are some papers in literature which concerned with the formal connection between classical electrodynamics and wave mechanics, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author will review a derivation of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik.

Sallhofer's method

Summing up from one of Sallhofer's papers[1], he says that under the sufficiently general assumption of periodic time dependence the following connection exists between source-free electrodynamics and wave mechanics:

$$\sigma \cdot \begin{bmatrix} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ div\varepsilon E = 0 \\ div\mu H = 0 \end{bmatrix}_{divE=0} \equiv \left[(\gamma \cdot \nabla + \gamma^{(4)}\partial_4)\Psi = 0 \right] \tag{4}$$

In words: Multiplication of source-free electrodynamics by the Pauli-vector yields wave mechanics.[1] In simple terms, this result can be written as follows:

$$P \cdot M = D\,, \tag{5}$$

Where:

P = Pauli vector,

M = Maxwell equations,

D = Dirac equations.

We can also say: Wave mechanics is a solution-transform of electrodynamics. Here one has to bear in mind that the well-known circulatory structure of the wave functions, manifest in Dirac's hydrogen solution, is not introduced just by the Pauli-vector.[1]

Possible connection with elastic wave equations

Now it would be certainly more interesting to connect the above equations further to elastic wave equations, as discussed by Ikelle [4]:

$$\begin{aligned} -\Delta^{+}_{ijkl}\partial_j \tilde{H}_{kl}(x,t,x_s) + \partial_t D_i\,(x,t,x_s) &= -J_i(x,t,x_s), \\ \Delta^{-}_{nmpq}\partial_p \tilde{E}_q(x,t,x_s) + \partial_t \tilde{B}_{nm}(x,t,x_s) &= -\tilde{K}_{nm}(x,t,x_s). \end{aligned} \tag{6a-b}$$

Assuming that equation (6a-b) are equivalent to elastic wave equations, then we can write :

$$\sigma \cdot \begin{bmatrix} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ div\varepsilon E = 0 \\ div\mu H = 0 \end{bmatrix}_{divE=0} \equiv \begin{bmatrix} -\Delta^{+}_{ijkl}\partial_j \tilde{H}_{kl}(x,t,x_s) + \partial_t D_i\,(x,t,x_s) = -J_i(x,t,x_s) \\ \Delta^{-}_{nmpq}\partial_p \tilde{E}_q(x,t,x_s) + \partial_t \tilde{B}_{nm}(x,t,x_s) = -\tilde{K}_{nm}(x,t,x_s) \end{bmatrix} \qquad (7)$$

Then, if put the same equivalence from (7) to Sallhofer's version of Dirac-Maxwell isomorphism, then we obtained:

$$\left[(\gamma\cdot\nabla + \gamma^{(4)}\partial_4)\Psi = 0\right] \equiv \begin{bmatrix} -\Delta^{+}_{ijkl}\partial_j \tilde{H}_{kl}(x,t,x_s) + \partial_t D_i\,(x,t,x_s) = -J_i(x,t,x_s) \\ \Delta^{-}_{nmpq}\partial_p \tilde{E}_q(x,t,x_s) + \partial_t \tilde{B}_{nm}(x,t,x_s) = -\tilde{K}_{nm}(x,t,x_s) \end{bmatrix} \qquad (8)$$

The above equations suggest that there is not only mathematical correspondence between Dirac-Maxwell equations but also between elastic wave and Maxwell equations. Their implications should be investigated by experiments.

Discussion: The Intriguing Dance: Maxwell and Dirac in the Realm of Quaternions

The Maxwell equations and the Dirac equation, cornerstones of electromagnetism and quantum mechanics respectively, seem like disparate entities. One governs the classical realm of light and fields, the other the dance of spin-half particles like electrons. Yet, a fascinating possibility emerges when we step into the arena of quaternions, revealing a potential "exact correspondence" between these seemingly distinct equations.

Let's delve into this captivating waltz:

From Electromagnetism to Quaternions:

The Maxwell equations, in their familiar form, deal with vectors and their derivatives, painting a picture of electric and magnetic fields weaving through space-time. But this beauty can be recast in the language of quaternions, those curious number-like objects that combine scalars and vectors. This reformulation, known as the "biquaternionic representation," opens a new window into the electromagnetic world.

Dirac's Equation: A Quantum Twist:

The Dirac equation, on the other hand, describes the behavior of a spin-half fermion – an electron, for instance. It employs a four-component spinor field, imbued with the essence of both particle and antiparticle. This equation, too, can be clothed in the quaternionic garb, revealing a surprising connection to the biquaternionic Maxwell equations.

The Bridge: A Mathematical Elegance:

The key to the correspondence lies in the intricate dance of the mathematical structures. By carefully manipulating the quaternionic representations of both equations, physicists have shown that under specific conditions, they become mathematically equivalent. This means that solutions of one equation can be mapped precisely to solutions of the other.

Physical Implications:

While the mathematical elegance is undeniable, the physical implications of this correspondence are still under debate. Some argue that it offers a deeper understanding of the connection between quantum and classical realms, hinting at a unified framework for electromagnetism and matter. Others caution that the equivalence might be limited to specific situations and doesn't necessarily erase the fundamental differences between particles and fields.

Open Questions and Future Steps:

The journey has just begun. Unraveling the physical meaning behind this quaternionic bridge requires further exploration. Can we use this correspondence to derive new insights into quantum electrodynamics? Does it shed light on the nature of spin and its relation to electromagnetism? These are just a few of the tantalizing questions that beckon further investigation.

Concluding remarks

Despite its wide acceptance, special theory of relativity is not without problems, like the meaning of empty spacetime. Does empty spacetime have their own mechanical properties?

Last but not least, allow us to write a few remark as follows the Maxwell-Dirac correspondence in the realm of quaternions offers a glimpse into a hidden connection between the classical and quantum worlds. While the full implications remain to be unraveled, it serves as a testament to the power of mathematical abstraction and the beauty of unexpected bridges that emerge in the vast landscape of physics.

In this paper we discuss link between elasto-electromagnetic wave equations, then we link them with Sallhofer's approach of Maxwell-Dirac isomorphism.

This paper was inspired by an old question: Is there a consistent and realistic description of wave function, both classically and quantum mechanically?

It can be expected that the above discussions will shed some lights on such an old problem especially in the context of physical meaning of quantum wave function. This is reserved for further investigations.

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