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Ambiguous Set is a subclass of the Double Refined Indeterminacy Neutrosophic Set, and of the Refined Neutrosophic Set in general

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Abstract: In this short note we show that the so-called Ambiguous Set (2019) is a subclass of the Double Refined Indeterminacy Neutrosophic Set (2017) and is a particular case of the Refined Neutrosophic Set (2013). Also, the Ambiguous Set is similar to the Quadripartitioned Neutrosophic Set (2016), and Belnap's Four-Valued Logic (1975).

Keywords: Double Refined Indeterminacy Neutrosophic Set (DRINS); Refined Neutrosophic Set (RNS); Ambiguous Set (AS); Quadripartitioned Neutrosophic Set (QNS); Belnap's Four-Valued Logic (BFVL).

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1. Introduction

We provide the definitions of the previous five types of sets, and we prove that the Ambiguous Set is a particular case of the Refined Neutrosophic Set (RNS), Quadripartitioned Neutrosophic Set (QNS), and Belnap's Four-Valued Logic (BFVL), and mostly that the Ambiguous Set coincides with the Double Refined Indeterminacy Neutrosophic Set with the distinction that the sum of quadruple components is ≤ 2 for the AS, which makes it a subclass of the DRINS where the sum is any number between 0 and 4.

2. Ambiguous Set

The definition of the Ambiguous Set (AS) according to [1, 2] is given as follows:

Let U = {g} be the universe for any event *g*, which is fixed. An AS Ś for g ∈ U is defined by:

$$Ś = \{g, \Pi t(g), \Pi f(g), \Pi ta(g), \Pi fa(g) \mid g \in U\}$$

where, $\Pi t(g): U \to [0,1]$, $\Pi f(g): U \to [0,1]$, $\Pi ta(g): U \to [0,1]$, and $\Pi fa(g): U \to [0,1]$ are called the true membership degree (TMD), false membership degree (FMD), true-ambiguous membership degree (TAMD), and false-ambiguous membership degree (FAMD), respectively.

Where Πt(g), Πf(g), Πta(g) and Πfa(g) must satisfy the following condition as:

$$0 \leq \Pi t(g) + \Pi f(g) + \Pi ta(g) + \Pi fa(g) \leq 2$$

3. Double Refined Indeterminacy Neutrosophic Set (DRINS)

The definition of Double Refined Indeterminacy Neutrosophic Set is given in [3] as follows:

Let X be a space of points (objects) with generic elements in X denoted by x.

A Double Refined Indeterminacy Neutrosophic Set (DRINS) A in X is characterized by four components:

truth membership function $T_A(x)$, indeterminacy leaning towards truth membership function $I_{TA}(x)$,

indeterminacy leaning towards falsity membership function $I_{FA}(x)$, and falsity membership function $F_A(x)$.

For each generic element x ∈ X, there are $T_A(x), I_{TA}(x), I_{FA}(x), F_A(x) \in [0, 1]$,

and $0 \leq T_A(x)+I_{TA}(x)+I_{FA}(x)+ F_A(x) \leq 4$.

Therefore, a DRINS A can be represented by

$A = \{\langle x, T_A(x), I_{TA}(x), I_{FA}(x), F_A(x)\rangle \mid x \in X\}$.

4. Ambiguous Set vs. Double Refined Indeterminacy Neutrosophic Set

Let's compare the two definitions.

The definition of Ambiguous Set, as presented by Singh, Huang, & Lee [1, 2] in 2019 and in 2023, coincides with that of Double Refined Indeterminacy Neutrosophic Set introduced by Ilanthenral & Smarandache [3] in 2017, ahead of them.

They only renamed:

the indeterminacy leaning towards truth membership function $I_{TA}(x)$, as true-ambiguous membership degree (TAMD),

and the indeterminacy leaning towards falsehood membership function $I_{TA}(x)$, as false-ambiguous membership degree (FAMD).

The only distinction between AS and DRINS is that:

the sum of AS quadruple components is restricted to be ≤ 2, while the sum of DRINS quadruple components is ≤ 4 (no restriction), which means that one can take any number between 0 and 4, in the particular case they took the number 2, whence AS is a subclass of the DRINS.

5. Refined Neutrosophic Set

The Definition of Refined Neutrosophic Set is the following.

Let X be a space of points (objects) with generic elements in X denoted by x.

A Refined Neutrosophic Set (RNS) A in X is characterized by *n* sub-components:

sub-truth membership functions $T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x)$;

sub-indeterminacy membership functions $I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x)$;

and sub-falsehood membership functions $F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x)$;

where p, r, s ≥ 0 are integers, and $p + r + s = n \geq 2$, such that at least one of p, r, s is ≥ 2 for assuring the refinement of at least one neutrosophic component amongst T, I, or F.

For each generic element x ∈ X, the functions

$T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x), I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x), F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x) \in [0, 1]$,

with their sum

$$0 \leq T_{1A}(x) + T_{2A}(x) + \dots + T_{pA}(x) + I_{1A}(x) + I_{2A}(x) + \dots + I_{rA}(x) + F_{1A}(x) + F_{2A}(x) + \dots + F_{sA}(x) \leq n$$

Therefore, a RNS A can be represented by

$$A_{RNS} = \{\langle x, T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x), I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x), F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x)\rangle, \mid x \in X\}.$$

The Ambiguous Set is a particular case of the Refined Neutrosophic Set, since one takes

p = 1 (only one true membership);

r = 2 (two types of indeterminacy memberships,

I_1 = true-ambiguous membership degree (TAMD),

and

I_2 = false-ambiguous membership degree (FAMD);

s = 1 (only one false membership).

Therefore, the Ambiguous Set is a particular case of the Refined Neutrosophic Set.

In the same way it is proven that the Double Refined Indeterminacy Neutrosophic Set is a particular of the Refined Neutrosophic Set.

6. Ambigous Set vs. Refined Neutrosophic Set

Both, the so-called Ambiguous Set and the Double Refined Indeterminacy Neutrosophic Set are particular cases of the Refined Neutrosophic Set [4] introduced by Smarandache in 2013.

7. Quadripartitioned Neutrosophic Set

The Definition of single-valued Quadripartitioned Neutrosophic Set [5]

Let X be a non-empty set. The Quadripartitioned single-valued Neutrosophic Set (QNS) A over X characterizes each element x in X by a truth-membership function T_A , a contradiction membership function C_A , an ignorance–membership function U_A and a falsity membership function F_A such that:

for each $x \in X$ one has $T_A(x), C_A(x), U_A(x), F_A(x) \in [0,1]$ and

$0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \in [0,1] \le 4$.

When X is discrete, A is represented as

$$A = \sum_{i=1}^{n} < T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i) > / x_i, x_i \in X.$$

However, when X is continuous, A is represented as:

$$\int_X < T_A(x), C_A(x), U_A(x), F_A(x) > / x, x \in X.$$

It is clear the Quadripartitioned Neutrosophic Set (no matter if it is single-valued, interval-valued, or set-valued in general) is a particular case of the Refined Neutrosophic Set, of the form T, F, and indeterminacy I is split into two parts: I_1 = C (contradiction-membership) and I_2 = U (ignorance-membership).

While the Ambiguous Set is similar with the Quadripartitioned Neutrosophic Set, where the two types of sub-indeterminacies I_1 and I_2 are named differently: true-ambiguous membership and respectively false-ambiguous membership.

Surely, one can rename the sub-indeterminacies I_1 and I_2 in many ways, since there are many types of indeterminacies / uncertainties / vagueness / conflicting informations etc.

8. **Belnap's Four-Valued Logic**

In 1975 Belnap has considered a logic of four values: true, false, both (true and false), and neither (neither true, nor false). We can denote them by T (true), F (false), C (true and false = contradiction), U (neither true not false = ignorance) respectively and we see that the Ambiguous Set and Quadripartitioned Neutrosophic Set are similar to Belnap's Logic

Further on, the *Belnap's 4-valued Logic* is a particular case of the *Refined Neutrosophic n-valued Logic* that has types of truths T_1, T_2, …, T_p, types of indeterminacies I_1, I_2, …, I_r, and types of falsehoods: F_1, F_2, …, F_s.

9. **Conclusion**

We proved that the so-called Ambiguous Set coincides with the Double Refined Indeterminacy Neutrosophic Set with respect their quadruple structures, while, with respect to the sum of components, AS is a subclass of the DRINS.

Also, AS is similar with the Quadripartitioned Neutrosophic Set and Belnap Four-Valued Logic as well.

Further on, we proved that the AS, DRINS, QNS and BFVL are particular cases of the Refined Neutrosophic Set / Logic respectively.

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