FLORENTIN SMARANDACHE

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A UNIFYING FIELD IN LOGICS: NEUTROSOPHIC LOGIC. NEUTROSOPHIC PROBABILITY, NEUTROSOPHIC STATISTICS, NEUTROSOPHIC SET.

(second version)

Introduction.

Let T, I, F be standard or non-standard real subsets $\|^{-}0, 1^{+}\|$,
with $\sup T = t\_sup$, $\inf T = t\_inf$;
$\sup I = i\_sup$, $\inf I = i\_inf$,
$\sup F = f\_sup$, $\inf F = f\_inf$,
and $n\_sup = t\_sup + i\_sup + f\_sup$,
$n\_inf = t\_inf + i\_inf + f\_inf$.

The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single-element, finite, or (either countably or uncountably) infinite; union or intersection of various subsets; etc.

They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points).

This representation is closer to the human mind reasoning. It characterizes/catches the *imprecision* of knowledge or linguistic inexactitude received by various observers (that's why T, I, F are subsets - not necessarily single-elements), *uncertainty* due to incomplete knowledge or acquisition errors or stochasticity (that's why the subset I exists), and vagueness due to lack of clear contours or limits (that's why T, I, F are subsets and I exists; in particular for the appurtenance to the neutrosophic sets).

One has to specify the superior (x_sup) and inferior (x_inf) limits of the subsets because in many problems arises the necessity to compute them.

The real number x is said to be infinitesimal if and only if for all positive integers n one has $x < 1/n$. Let $\varepsilon>0$ be a such infinitesimal number. The *hyper-real number set* is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers $1^+ = 1+\varepsilon$, where "1" is its standard part and "ε" its non-standard part, and $^-0 = 0-\varepsilon$, where "0" is its standard part and "ε" its non-standard part.

Then, we call $\|^-0, 1^+\|$ a non-standard unit interval. Obviously, 0 and 1, and analogously non-standard numbers infinitely small but less than 0 or infinitely small but greater than 1, belong to the non-standard unit interval. Actually, by "^-a" one signifies a monad, i.e. a set of hyper-real numbers in non-standard analysis:

$\mu(^-a)= \{a-x: x\in R^*, x \text{ is infinitesimal}\}$,

and similarly "b^+" is a monad:

$\mu(b^+)= \{b+x: x\in R^*, x \text{ is infinitesimal}\}$.

Generally, the left and right borders of a non-standard interval $\|^-a, b^+\|$ are vague, imprecise, themselves being non-standard (sub)sets $\mu(^-a)$ and $\mu(b^+)$ as defined above.

Combining the two before mentioned definitions one gets, what we would call, a binad of "$^-c^+$":

$\mu(^-c^+)= \{c-x: x\in R^*, x \text{ is infinitesimal}\} \cup \{c+x: x\in R^*, x \text{ is infinitesimal}\}$, which is a collection of open punctured neighborhoods (balls) of c.

Of course, $^-a < a$ and $b^+ > b$. No order between $^-c^+$ and c.

Addition of non-standard finite numbers with themselves or with real numbers:

$^-a + b = {}^-(a + b)$

$a + b^+ = (a + b)^+$

$^-a + b^+ = {}^-(a + b)^+$

$^-a + {}^-b = {}^-(a + b)$ (the left monads absorb themselves)

$a^+ + b^+ = (a + b)^+$ (analogously, the right monads absorb themselves).

Similarly for subtraction, multiplication, division, roots, and powers of non-standard finite numbers with themselves or with real numbers.

By extension let $\inf \|^-a, b^+\| = {}^-a$ and $\sup \|^-a, b^+\| = b^+$.

Łukasiewicz, together with Kotarbiński and Leśniewski from the

Warsaw Polish Logic group (1919-1939), questioned the status of truth: eternal, sempiternal (everlasting, perpetual), or both?

Let's borrow from the modal logic the notion of "world", which is a semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is 'true in all possible worlds' (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth*, Dinulescu-Câmpina (2000) names it 'intangible absolute truth'), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a 'specific' world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each 'world' is dynamic, depending on an ensemble of parameters, we introduce the sub-category 'conjuncture' within it to reflect a particular state of the world.

How can we differentiate <the truth behind the truth>? What about the <metaphoric truth>, which frequently occurs in the humanistic field? Let's take the proposition "99% of the politicians are crooked" (Sonnabend 1997, Problem 29, p. 25). "No," somebody furiously comments, "100% of the politicians are crooked, *even more*!" How do we interpret this "even more" (than 100%), i. e. more than the truth?

One attempts to formalize. For $n \geqslant 1$ one defines the "n-level relative truth" of the statement A if the statement is true in at least n distinct worlds, and similarly "countably-" or "uncountably-level relative truth" as gradual degrees between "first-level relative truth" (1) and "absolute truth" (1^+) in the monad $\mu(1^+)$. Analogue definitions one gets by substituting "truth" with "falsehood" or "indeterminacy" in the above.

In *largo sensu* the notion "world" depends on parameters, such as: space, time, continuity, movement, modality, (meta)language levels, interpretation, abstraction, (higher-order) quantification, predication, complement constructions, subjectivity, context, circumstances, etc. Pierre d'Ailly upholds that the truth-value of a proposition depends on the sense, on the metaphysical level, on the language and meta-language; the auto-reflexive propositions (with reflection on themselves) depend on the mode of representation (objective/subjective, formal/informal, real/mental).

In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the neutrosophic indeterminacy-value $NL_i(A) = 1^+$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

On the other hand, $NL_t(A) = {}^-0$ if A is false in all possible worlds, whereas $NL_t(A) = 0$ if A is false in at least one world; $NL_f(A) = {}^-0$ if A is true in all possible world, whereas $NL_f(A) = 0$ if A is true in at least one world; and $NL_i(A) = {}^-0$ if A is indeterminate in no possible world, whereas $NL_i(A) = 0$ if A is not indeterminate in at least one world.

The ${}^-0$ and 1^+ monads leave room for degrees of super-truth (truth whose values are greater than 1), super-falsehood, and super-indeterminacy.

Here there are some corner cases:

There are tautologies, some of the form "B is B", for which $NL(B) = (1^+, {}^-0, {}^-0)$, and contradictions, some of the form "C is not C", for which $NL(B) = ({}^-0, {}^-0, 1^+)$.

While for a paradox, P, $NL(P) = (1,1,1)$. Let's take the Epimenides Paradox, also called the Liar Paradox, "This very statement is true". If it is true then it is false, and if it is false then it is true. But the previous reasoning, due to the contradictory results, indicates a high indeterminacy too. The paradox is the only proposition true and false in the same time in the same world, and indeterminate as well!

Let's take the Grelling's Paradox, also called the heterological paradox [Suber, 1999], "If an adjective truly describes itself, call it 'autological', otherwise call it 'heterological'. Is 'heterological'

heterological? " Similarly, if it is, then it is not; and if it is not, then it is.

For a not well-formed formula, nwff, i.e. a string of symbols which do not conform to the syntax of the given logic, NL(nwff) = n/a (undefined). A proposition which may not be considered a proposition was called by the logician Paulus Venetus *flatus voci*. NL(*flatus voci*) = n/a.

Let S_1 and S_2 be two (unidimensional) standard or non-standard real subsets, then one defines:

Addition of sets:
$S_1 \oplus S_2 = \{x \mid x=s_1+s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$,
with $\inf S_1 \oplus S_2 = \inf S_1 + \inf S_2$, $\sup S_1 \oplus S_2 = \sup S_1 + \sup S_2$;
and, as some particular cases, we have
$\{a\} \oplus S_2 = \{x \mid x=a+s_2, \text{ where } s_2 \in S_2\}$
with $\inf \{a\} \oplus S_2 = a + \inf S_2$, $\sup \{a\} \oplus S_2 = a + \sup S_2$;
also $\{1\} \oplus S_2 = \{x \mid x=1+s_2, \text{ where } s_2 \in S_2\}$
with $\inf \{1\} \oplus S_2 = 1 + \inf S_2$, $\sup \{1\} \oplus S_2 = 1 + \sup S_2$.

Subtraction of sets:
$S_1 \ominus S_2 = \{x \mid x=s_1-s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$.
For real positive subsets (most of our cases will fall in this range) one gets:
$\inf S_1 \ominus S_2 = \inf S_1 - \sup S_2$, $\sup S_1 \ominus S_2 = \sup S_1 - \inf S_2$;
and, as some particular cases, we have:
$\{a\} \ominus S_2 = \{x \mid x=a-s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{a\} \ominus S_2 = a - \sup S_2$, $\sup \{a\} \ominus S_2 = a - \inf S_2$;
also $\{1\} \ominus S_2 = \{xx=1-s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{1\} \ominus S_2 = 1 - \sup S_2$, $\sup \{1\} \ominus S_2 = 1 - \inf S_2$.

Multiplication of sets:
$S_1 \odot S_2 = \{x \mid x=s_1 s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$.
For real positive subsets (most of the cases will fall in this range) one gets:
$\inf S_1 \odot S_2 = \inf S_1 \cdot \inf S_2$, $\sup S_1 \odot S_2 = \sup S_1 \cdot \sup S_2$;
and, as some particular cases, we have
$\{a\} \odot S_2 = \{x \mid x=as_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{a\} \odot S_2 = a \cdot \inf S_2$, $\sup \{a\} \odot S_2 = a \cdot \sup S_2$;

also $\{1\} \odot S_2 = \{x \mid x = 1 \cdot s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{1\} \odot S_2 = 1 \cdot \inf S_2$, $\sup \{1\} \odot S_2 = 1 \cdot \sup S_2$.

Division of a set by a number:
Let $k \in R^*$, then $S_1 \oslash k = \{x \mid x = s_1/k, \text{ where } s_1 \in S_1\}$,

Let (T_1, I_1, F_1) and (T_2, I_2, F_2) be standard or non-standard triplets of real subsets $\in \|^-0, 1^+\|^3$, then we define:

$(T_1, I_1, F_1) \boxplus (T_2, I_2, F_2) = (T_1 \oplus T_2, I_1 \oplus I_2, F_1 \oplus F_2)$,
$(T_1, I_1, F_1) \boxminus (T_2, I_2, F_2) = (T_1 \ominus T_2, I_1 \ominus I_2, F_1 \ominus F_2)$,
$(T_1, I_1, F_1) \boxdot (T_2, I_2, F_2) = (T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2)$.

NEUTROSOPHIC PROBABILITY is a generalization of the classical probability in which the chance that an event A occurs is t% true - where t varies in the subset T, i% indeterminate - where i varies in the subset I, and f% false - where f varies in the subset F.

One notes $NP(A) = (T, I, F)$.

NEUTROSOPHIC STATISTICS is the analysis of the events described by the neutrosophic probability.

This is also a generalization of the classical statistics.

Neutrosophic Probability Space:

The universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

Let A and B be two neutrosophic events, and $NP(A) = (T_1, I_1, F_1)$, $NP(B) = (T_2, I_2, F_2)$ their neutrosophic probabilities. Then we define:

$NP(A \cap B) = NP(A) \boxdot NP(B)$.
$NP(\neg A) = \{1\} \boxminus NP(A)$.
$NP(A \cup B) = NP(A) \boxplus NP(B) \boxminus NP(A) \boxdot NP(B)$.

1. NP(impossible event) $= (T_{imp}, I_{imp}, F_{imp})$,
where $\sup T_{imp} \leq 0$, $\inf F_{imp} \geq 1$; no restriction on I_{imp}.
NP(sure event) $= (T_{sur}, I_{sur}, F_{sur})$,

where $\inf T_{sur} \geq 1$, $\sup F_{sur} \leq 0$; no restriction on I_{sur}.

NP(totally indeterminate event) = $(T_{ind}, I_{ind}, F_{ind})$;

where $\inf I_{ind} \geq 1$; no restrictions on T_{ind} or F_{ind}.

2. $NP(A) \in \{(T, I, F)$, where T, I, F are real subsets which may overlap$\}$.

3. $NP(A \cup B) = NP(A) \boxplus NP(B) \boxminus NP(A \cap B)$.

4. $NP(A) = \{1\} \boxminus NP(\neg A)$.

C) Applications:

#1. From a pool of refugees, waiting in a political refugee camp in Turkey to get the American visa, a% have the chance to be accepted - where a varies in the set A, r% to be rejected - where r varies in the set R, and p% to be in pending (not yet decided) - where p varies in P.

Say, for example, that the chance of someone Popescu in the pool to emigrate to USA is (between) 40-60% (considering different criteria of emigration one gets different percentages, we have to take care of all of them), the chance of being rejected is 20-25% or 30-35%, and the chance of being in pending is 10% or 20% or 30%. Then the neutrosophic probability that Popescu emigrates to the Unites States is

NP(Popescu) = ((40-60), (20-25)∪(30-35), {10,20,30}), closer to the life.

This is a better approach than the classical probability, where $40 \leq$ P(Popescu) ≤ 60, because from the pending chance - which will be converted to acceptance or rejection - Popescu might get extra percentage in his will to emigration,

and also the superior limit of the subsets sum

$60+35+30 > 100$

and in other cases one may have the inferior sum < 0,

while in the classical fuzzy set theory the superior sum should be 100 and the inferior sum 0.

In a similar way, we could say about the element Popescu that

Popescu((40-60), (20-25)U(30-35), {10,20,30}) belongs to the set of accepted refugees.

#2. The probability that candidate C will win an election is say 25-30% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% or 41% indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Dialectic and dualism don't work in this case anymore.

#3. Another example, the probability that tomorrow it will rain is say 50-54% true according to meteorologists who have investigated the past years' weather, 30 or 34-35% false according to today's very sunny and droughty summer, and 10 or 20% undecided (indeterminate).

#4. The probability that Yankees will win tomorrow versus Cowboys is 60% true (according to their confrontation's history giving Yankees' satisfaction), 30-32% false (supposing Cowboys are actually up to the mark, while Yankees are declining), and 10 or 11 or 12% indeterminate (left to the hazard: sickness of players, referee's mistakes, atmospheric conditions during the game). These parameters act on players' psychology.

D) Remarks:

Neutrosophic probability are useful to those events which involve some degree of indeterminacy (unknown) and more criteria of evaluation - as above. This kind of probability is necessary because it provides a better approach than classical probability to uncertain events.

In the case when the truth- and falsity-components are complementary, i.e. no indeterminacy and their sum is 100, one falls to the classical probability. As, for example, tossing dice or coins, or drawing cards from a well-shuffled deck, or drawing balls from an urn.

An interesting particular case is for $n=1$, with $0 \leq t, i, f \leq 1$, which is closer to the classical probability.

For n_sup=1 and $i=0$, with $0 \leq t, f \leq 1$, one obtains the classical probability.

From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxism, pseudoparadoxism, and tautologism we transfer the "adjectives" to probabilities, i.e. we define the intuitionistic probability (when the probability space is incomplete), paraconsistent probability, faillibilist probability, dialetheist probability, paradoxist probability, pseudoparadoxist probability, and tautologic probability respectively.

Hence, the neutrosophic probability generalizes:

- the intuitionistic probability, which supports incomplete (not completely known/determined) probability spaces (for $0<n\_sup<1$ and $i=0$, $0\leq t,f\leq 1$) or incomplete events whose probability we need to calculate;

- the classical probability (for $n\_sup=1$ and $i=0$, and $0\leq t,f\leq 1$);
- the paraconsistent probability (for $n\_sup>1$ and $i=0$, with both $t,f<1$);
- the dialetheist probability, which says that intersection of some disjoint probability spaces is not empty (for $t=f=1$ and $i=0$; some paradoxist probabilities can be denoted this way);
- the faillibilist probability (for $i>0$);
- the pseudoparadoxism (for $n\_sup>1$ or $n\_inf<0$);
- the tautologism (for $t\_sup>1$).

Compared with all other types of classical probabilities, the neutrosophic probability introduces a percentage of "indeterminacy" - due to unexpected parameters hidden in some probability spaces, and let each component t, i, f be, even boiling, *over* 1 (overflooded) or freezing *under* 0 (underdried).

For example: an element in some tautological probability space may have $t>1$, called "overprobable". Similarly, an element in some paradoxist probability space may be "overindeterminate" (for $i>1$), or "overunprobable" (for $f>1$, in some unconditionally false appurtenances); or "underprobable" (for $t<0$, in some unconditionally false appurtenances), "underindeterminate" (for $i<0$, in some unconditionally true or false appurtenances), "underunprobable" (for $f<0$, in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true ($t>1$, and $f<0$ or $i<0$) and conditionally true appurtenances ($t\leq 1$, and $f\leq 1$ or $i\leq 1$).

NEUTROSOPHIC SET:

Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T,I,F) and belongs to M in the following way:

it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F.

B) Neutrosophic Set Operations:

Let A and B be two neutrosophic sets.

One can say, by language abuse, that any element neutrosophically belongs to any set, due to the percentages of truth / indeterminacy / falsity involved, which varies between 0 and 1 or even less than 0 or greater than 1.

For example: x(50,20,30) belongs to A (which means, with a probability of 50% x is in A, with a probability of 30% x is not in A, and the rest is undecidable); or y(0,0,100) belongs to A (which normally means y is not for sure in A); or z(0,100,0) belongs to A (which means one does know absolutely nothing about z's affiliation with A).

More general, x((20-30), (40-45)∪[50-51], {20,24,28}) belongs to the set A, which means:

- with a probability in between 20-30% x is in A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40-45% or between 50-51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and n_sup = 30+51+28 > 100 in this case.

For example the Schrodinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time, which translated to the neutrosophic set means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory.

In Schroedinger's Equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Don't we better describe, using the attribute "neutrosophic" than "fuzzy" or any others, a quantum particle that neither exists nor non-exists?

How to describe a particle ς in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\varsigma \in P_1$ and $\varsigma \notin P_1$ as a true contradiction, or $\varsigma \in P_1$ and $\varsigma \in \neg P_1$.

Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?

In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.

How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

There are many ways to construct them, in terms of the practical problem we need to simulate or approach. Below there are mentioned the easiest ones:

Let U be a universe.

One notes, with respect to the sets A and B over U,

$x = x(T_1, I_1, F_1) \in A$ and $x = x(T_2, I_2, F_2) \in B$, by mentioning x's neutrosophic probability appurtenance.

And, similarly, $y = y(T', I', F') \in B$.

[The components may be normalized (except for the case of paraconsistent set, intuitionistic set, dialetheist set, paradoxist set) by dividing each of them to their sum.]

Complement of A:

If $x(T_1, I_1, F_1) \in A$,
then $x(1 \ominus T_1, 1 \ominus I_1, 1 \ominus F_1) \in C(A)$.

Intersection:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B$.

Union:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B$.

Difference:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \ominus T_1 \odot T_2, I_1 \ominus I_1 \odot I_2, F_1 \ominus F_1 \odot F_2) \in A \setminus B$,
because $A \setminus B = A \cap C(B)$.

Cartesian Product:

If $x(T_1, I_1, F_1) \in A$, $y(T', I', F') \in B$,
then $(x(T_1, I_1, F_1), y(T', I', F')) \in A \times B$.

M is a subset of N if
$x(T_1, I_1, F_1) \in M \Rightarrow x(T_2, I_2, F_2) \in N$,
where $\inf T_1 \leq \inf T_2$, $\sup T_1 \leq \sup T_2$, and $\inf F_1 \geq \inf F_2$, $\sup F_1 \geq \sup F_2$.

Let $A_1, A_2, \ldots, A_n$ be arbitrary non-empty sets.

A Neutrosophic n-ary Relation R on $A_1 \times A_2 \times \ldots \times A_n$ is defined as a subset of the cartesian product $A_1 \times A_2 \times \ldots \times A_n$, such that for each ordered n-tuple $(x_1, x_2, \ldots, x_n)(T, I, F)$, T represents the degree of validity, I the degree of indeterminacy, and F the degree of non-validity respectively of the relation R.

It is related to the definitions for the Intuitionistic Fuzzy Relation independently given by Atanassov (1984, 1989), Toader Buhăescu (1989), Darinka Stoyanova (1993), Humberto Bustince Sola and P. Burillo Lopez (1992-1995).

In a rough set RS, an element on its boundary-line cannot be classified neither as a member of RS nor of its complement with certainty. In the neutrosophic set a such element may be characterized by x(T, I, F), with corresponding set-values for $T, I, F \subseteq \,]^-0, 1^+[$

NEUTROSOPHIC LOGIC is a logic in which each proposition P has the logical values NL(P) = (T, I, F), where T represents the percentage of truth, I the percentage of indeterminacy, and F the percentage of falsehood.

As an alternative to the existing logic we propose the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction. It is a non-classical logic.

Eksioglu (1999) explains some of them:

"Imprecision of the human systems is due to the imperfection of knowledge that human receives (observation) from the external world. Imperfection leads to a doubt about the value of a variable, a decision to be taken or a conclusion to be drawn for the actual system. The sources of uncertainty can be stochasticity (the case of intrinsic imperfection where a typical and single value does not exist), incomplete knowledge (ignorance of the totality, limited view on a system because of its complexity) or the acquisition errors (intrinsically imperfect observations, the quantitative errors in measures)."

"Probability (called sometimes the objective probability) process uncertainty of random type (stochastic) introduced by the chance. Uncertainty of the chance is clarified by the time or by events' occurrence. The probability is thus connected to the frequency of the events' occurrence."

"The vagueness which constitutes another form of uncertainty is the character of those with contours or limits lacking precision, clearness. [...]For certain objects, the fact to be in or out of a category is difficult to mention. Rather, it is possible to express a partial or gradual membership."

Indeterminacy means degrees of uncertainty, vagueness, imprecision, undefined, unknown, inconsistency, redundancy.

A question would be to try, if possible, to get an axiomatic system for the neutrosophic logic. Intuition is the base for any formalization, because the postulates and axioms derive from intuition.

We use a subset of truth (or indeterminacy, or falsity), instead of a number only, because in many cases we are not able to exactly determine the percentages of truth and of falsity but to approximate them: for example a proposition is between 30-40% true and between 60-70% false, even worst: between 30-40% or 45-50% true (according to various analyzers), and 60% or between 66-70% false.

The subsets are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.) in accordance with the given proposition.

A subset may have one element only in special cases of this logic.

Constants: (T, I, F) truth-values, where T, I, F are standard or non-standard subsets of the non-standard interval $\Vert {}^-0, 1^+ \Vert$.

Atomic formulas: a, b, c, ...

Arbitrary formulas: A, B, C, ...

The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy, and falsehood.

My hypothesis is that **no theory is exempted from paradoxes**, because of the language imprecision, metaphoric expression, various levels or meta-levels of understanding/interpretation that might overlap.

History:

The Classical Logic, also called Bivalent Logic for taking only two values {0, 1}, or Boolean Logic from British mathematician George Boole (1815-64), was named by the philosopher Quine (1981) "sweet simplicity".

Peirce, before 1910, developed a semantics for three-valued logic in an unpublished note, but Emil Post's dissertation (1920s) is cited for originating the three-valued logic. Here "1" is used for truth, "1/2" for indeterminacy, and "0" for falsehood. Also, Reichenbach, leader of the logical empiricism, studied it.

The three-valued logic was employed by Halldén (1949), Körner (1960), Tye (1994) to solve Sorites Paradoxes. They used truth tables, such as Kleene's, but everything depended on the definition of validity.

A three-valued paraconsistent system (LP) has the values: 'true', 'false', and 'both true and false'. The ancient Indian metaphysics considered four possible values of a statement: 'true (only)', 'false (only)', 'both true and false', and 'neither true nor false'; J. M. Dunn (1976) formalized this in a four-valued paraconsistent system as his First Degree Entailment semantics;

The Buddhist logic added a fifth value to the previous ones, 'none of these' (called *catushkoti*).

In order to clarify the anomalies in science, Rugina (1949, 1981) proposed an original method, starting first from an economic point of view but generalizing it to any science, to study the equilibrium and disequilibrium of systems. His Orientation Table comprises seven basic models:

Model M_1 (which is 100% stable)
Model M_2 (which is 95% stable, and 5% unstable),
Model M_3 (which is 65% stable, and 35% unstable),
Model M_4 (which is 50% stable, and 50% unstable),
Model M_5 (which is 35% stable, and 65% unstable),
Model M_6 (which is 5% stable, and 95% unstable),
Model M_7 (which is 100% unstable).

He gives Orientation Tables for Physical Sciences and Mechanics (Rugina 1989), for the Theory of Probability, for what he called Integrated Logic, and generally for any Natural or Social Science (Rugina 1989). This is a Seven-Valued Logic.

The $\{0, a_1, ..., a_n, 1\}$ Multi-Valued, or Plurivalent, Logic was develop by £ukasiewicz, while Post originated the m-valued calculus.

The many-valued logic was replaced by Goguen (1969) and Zadeh (1975) with an Infinite-Valued Logic (of continuum power, as in the classical mathematical analysis and classical probability) called Fuzzy Logic, where the truth-value can be any number in the closed unit interval [0, 1]. The Fuzzy Set was introduced by Zadeh in 1975.

Rugina (1989) defines an anomaly as "a deviation from a position of stable equilibrium represented by Model M_1", and he proposes a Universal Hypothesis of Duality:

"The physical universe in which we are living, including human society and the world of ideas, all are composed in different and changeable proportions of stable (equilibrium) and unstable (disequilibrium) elements, forces, institutions, behavior and value"

and a General Possibility Theorem:

"there is an unlimited number of possible combinations or systems in logic and other sciences".

According to the last assertions one can extend Rugina's Orientation Table in the way that any system in each science is s% stable and u% unstable, with s+u=100 and both parameters $0 \leq s, u \leq 100$, somehow getting to a fuzzy approach.

But, because each system has hidden features and behaviors, and there would always be unexpected occurring conditions we are not able to control - we mean the indeterminacy plays a role as well, a better approach would be the *Neutrosophic Model*:

Any system in each science is s% stable, i% indeterminate, and u% unstable, with s+i+u=100 and all three parameters $0 \leq s, i, u \leq 100$.

Therefore, we finally generalize the fuzzy logic to a transcendental logic, called "neutrosophic logic": where the interval [0, 1] is exceeded, i.e., the percentages of truth, indeterminacy, and falsity are approximated by non-standard subsets - not by single numbers, and these subsets may overlap and exceed the unit interval in the sense of the non-standard analysis; also the superior sums and inferior sum, $n_{sup} = \sup T + \sup I + \sup F \in \| {}^-0, 3^+ \|$, may be as high as 3 or 3^+, while $n_{inf} = \inf T + \inf I + \inf F \in \| {}^-0, 3^+ \|$, may be as low as 0 or ${}^-0$.

Generally speaking, passing from the attribute "classical" (traditional) to the attribute "modern" (in literature, arts, and philosophy today one says today "postmodern") one invalidates many theorems. Voltaire (1694-1778), a French writer and philosopher, asserted that "the laws in arts are made in order to encroach upon them". Therefore, in neutrosophic logic most of the classical logic laws and its properties are not preserved. Although at a first look neutrosophic logic appears counter-intuitive, maybe abnormal, because the neutrosophic-truth values of a proposition A, NL(A), may even be (1,1,1), i.e. a proposition can completely be true and false and indeterminate at the same time, studying the paradoxes one soon observes that it is intuitive.

The idea of tripartition (truth, falsehood, indeterminacy) appeared in 1764 when J. H. Lambert investigated the credibility of one witness affected by the contrary testimony of another. He generalized Hooper's rule of combination of evidence (1680s), which was a Non-Bayesian approach to find a probabilistic model. Koopman in 1940s introduced the notions of lower and upper probability, followed by Good, and Dempster (1967) gave a rule of combining two arguments. Shafer (1976) extended it to the Dempster-Shafer Theory of Belief Functions by defining the Belief and Plausibility functions and using the rule of inference of Dempster for combining two evidences proceeding from two different sources. Belief function is a connection between fuzzy reasoning and probability. The Dempster-Shafer Theory of Belief Functions is a generalization of the Bayesian Probability (Bayes 1760s, Laplace 1780s); this uses the mathematical probability in a more general way, and is based on probabilistic combination of evidence in artificial intelligence.

In Lambert "there is a chance p that the witness will be faithful and accurate, a chance q that he will be mendacious, and a chance 1-p-q that he will simply be careless" [apud Shafer (1986)]. Therefore, three components: accurate, mendacious, careless, which add up to 1.

Van Fraassen introduced the supervaluation semantics in his attempt to solve the sorites paradoxes, followed by Dummett (1975) and Fine (1975). They all tripartitioned, considering a vague predicate which, having border cases, is undefined for these border cases. Van Fraassen took the vague predicate 'heap' and extended it positively to those objects to which the predicate definitively applies and negatively to those objects to which it definitively doesn't apply. The remaining objects border was called penumbra. A sharp boundary between these two extensions does not exist for a soritical predicate. Inductive reasoning is no longer valid too; if S is a sorites predicate, the proposition "$\exists n(Sa_n \& \neg Sa_{n+1})$" is false. Thus, the predicate Heap (positive extension) = true, Heap (negative extension) = false, Heap (penumbra) = indeterminate.

Narinyani (1980) used the tripartition to define what he called the "indefinite set", and Atanassov (1982) continued on tripartition and gave five generalizations of the fuzzy set, studied their properties and applications to the neural networks in medicine:

a) Intuitionistic Fuzzy Set (IFS):

Given an universe E, an IFS A over E is a set of ordered triples <universe_element, degree_of_membership_to_A(M), degree_of_non-membership_to_A(N)> such that $M+N \leq 1$ and $M, N \in [0, 1]$. When $M + N = 1$ one obtains the fuzzy set, and if $M + N < 1$ there is an indeterminacy $I = 1-M-N$.

b) Intuitionistic L-Fuzzy Set (ILFS):

Is similar to IFS, but M and N belong to a fixed lattice L.

c) Interval-Valued Intuitionistic Fuzzy Set (IVIFS):

Is similar to IFS, but M and N are subsets of [0, 1] and $\sup M + \sup N \leq 1$.

d) Intuitionistic Fuzzy Set of Second Type (IFS2):

Is similar to IFS, but $M^2 + N^2 \leq 1$. M and N are inside of the upper right quarter of unit circle.

e) Temporal IFS:

Is similar to IFS, but M and N are functions of the time-moment too.

· This neutrosophic logic is the (first) attempt to unify many logics in a single field. However, sometimes a too large generalization may have no practical impact. Such unification theories, or attempts, have been done in the history of sciences:

a) Felix Klein (1872), in his Erlangen programme, in geometry, has proposed a common standpoint from which various branches of geometries could be re-organized, interpreted, i.e.:

Given a manifold and a group of transformations of the manifold, to study the manifold configurations with respect to those features that are not altered by the transformations of the group (Klein 1893, p. 67; apud Torretti 1999).

b) Einstein tried in physics to build a Unifying Field Theory that seeks to unite the properties of gravitational, electromagnetic, weak, and strong interactions so that a single set of equations can be used to predict all their characteristics; whether such a theory may be developed it is not known at the present (Illingworth 1991, p. 504).

c) Also, one mentions the Grand Unified Theory, which is a unified quantum field theory of the electromagnetic, weak, and strong interactions (Illingworth 1991, p. 200).

But generalizations become, after some levels, "very general", and therefore not serving at much and, if dealing with indeterminacy, underlying the infinite improbability drive. Would the gain of such total generality offset the losses in specificity? A generalization may be done in one direction, but not in another, while gaining in a bearing but loosing in another.

How to unify, not too much generalizing? Dezert (1999) suggested to develop the less limitative possible theory which remains coherent with certain existing theories. The rules of inferences in this general theory should satisfy many important mathematical properties. "Neutrosophic Logic could permit in the future to solving certain practical problems posed in the domain of research in Data/Information fusion. So far, almost all approaches are based on the Bayesian Theory, Dempster-Shafer Theory, Fuzzy Sets, and Heuristic Methods" (Dezert 1999). Theoretical and technical advances for Information Fusion are probability and statistics, fuzzy sets, possibility, evidential reasoning, random sets, neural networks and neuro-mimetic approaches, and logics (Dezert 2000).

The confidence interval <Bel, Pl> in Dempster-Shafer Theory is the truth subset (T) in the neutrosophic set (or logic). The neutrosophic logic, in addition to it, contains an indeterminacy set (say indeterminacy interval) and falsehood set (say in-confidence interval).

An attempt of classification of logics upon the following (among many other) criteria:

a) The way the connectives, or the operators, or the rules of inferences are defined.

b) The definitions of the formal systems of axioms.

c) The number of truth-values a proposition can have: two, three, finitely many-values, infinitely many (of continuum power).

d) The partition of the interval [0, 1] in propositional values: bipartition (in degrees of truth and falsehood), or tri-partition (degrees of truth, falsehood, and indeterminacy).

e) The distinction between conjunctural (relative) true, conjunctural (relative) false, conjunctural (relative) indeterminacy - designed by 1, with respect to absolute true (or super-truth), absolute false (super-falshood), absolute indeterminacy - designed by 1^+. Then, if a proposition is absolute true, it is underfalse ($^-0$), i.e. NL(P)=(1^+, I, $^-0$).

For example, the neutrosophic truth-value of the proposition "The number of planets of the Sun is divisible by three" is 1 because the proposition is necessary *de re*, i.e. relates to an actual individual mentioned since its truth depends upon the number nine, whereas the neutrosophic truth-value of the proposition "The number of planets of the Sun is the number of its satellites" is 1^{+} because the second proposition is necessary *de dicto*, i.e. relates to the expression of a belief, a possibility since its truth is not dependent upon which number in fact that is. The first proposition might not be true in the future if a new planet is discovered or an existing planet explodes in an asteroid impact, while the second one is always true as being a tautology. This is the difference between the truth-value "1" (dependent truth) and the truth-value "1^{+}" (independent truth).

f) The components of the truth values of a proposition summing up to 1 (in boolean logic, fuzzy logic, intuitionistic fuzzy logic), being less than 1 (in intuitionistic logic), or being greater than 1 (in paraconsistent logic, neutrosophic logic). The maximum sum may be 3 in neutrosophic logic, where NL(paradox)=(1,1,1).

g) Parameters that influence the truth-values of a proposition. For example in temporal logic the time is involved. A proposition may be true at a time t_1, but false at a time t_2, or may have some degree of truth in the open interval (0, 1) at a time t_3.

h) Using approximations of truth-values, or exact values.

For example, the probabilistic logic, interval-valued fuzzy logic, interval-valued intuitionistic fuzzy logic, possibility logic (Dubois, Prade) use approximations.

The boolean logic uses exact values, either 0 or 1.

i) Studying the paradoxes or not.

In the neutrosophic logic one can treat the paradoxes, because NL(paradox)=(1,1,1), and in dialetheism. In fuzzy logic FL(paradox)=(1,0) or (0,1)? Because FL(paradox)≠(1,1), due to the fact that the sum of the components should be 1 not greater.

j) The external or internal structure of propositions: Sentential (or Propositional) Calculus, which is concerned with logical relations of propositions treated only as a whole, and Predicate (or Functional) Calculus which is concerned besides the logical relations treated as a whole with their internal structure in terms of subject and predicate.

k) Quantification: First-Order (or Lower) Predicate Calculus (quantification is restricted to individuals only, and predicates take only individuals as arguments), Second-Order Predicate Calculus (quantification over individuals and over some classes as well), Higher-Order Calculus (n-predicates take, and quantifiers bind, order n-1 predicates as arguments, for n>1).

l) In proof-theoretic terms:

- Monotonic Logic: let Γ be a collection of statements, $v_1, v_2, \ldots, v_n$, and ϖ, φ other statements; if $\Gamma \vdash \varphi$ then also $(\Gamma, \varpi) \vdash \varphi$.

- Non-Monotonic Weak Logic: For some Γ, ϖ, φ one has
$\Gamma \vdash_{NML} \varphi$ but from (Γ, ϖ) does not $\vdash_{NML} \varphi$;

- Non-Monotonic Strong Logic: For some Γ, ϖ, φ, where Γ and $\Gamma \wedge \varphi$ are consistent, one has
$(\Gamma, \varpi) \vdash_{NML} \neg\varphi$.

m) From a traditional standpoint: Classical or Non-classical.

n) Upon inclusion or exclusion of empty domains (and defining the logical validity accordingly), there are Inclusive Predicate Logic, and (Standard) Predicate Logic respectively.

o) Upon the number of arguments the predicates can take, there are Monadic Predicate Logic (predicates take only one argument), Dyadic Predicate Logic (predicates take two arguments), Polyadic Predicate Logic or Logic of Relations (predicates take n>1 arguments).

p) Upon formalization again: Formal Logic, and Informal Logic.

q) Upon types of formalization, there are: Number-Theoretic Predicate Calculus (system with function symbols and individual constants), Pure Predicate Calculus (system without function symbols nor individual constants).

r) Upon standardization: Standard Logic, and Non-Standard Logic.

s) Upon identity: Predicate Logic With Identity (with the axiom $(\forall x)(x=x)$, and the axiom schema $[(x=y)\rightarrow(A\rightarrow A')]^c$, where A' is obtained from A by replacing any free occurrence of x in A with y, and B^c is an arbitrary closure of B), Predicate Logic Without Identity.

t) According to the *ex contradictione quodlibet* (ECQ) principle, from contradictory premises follows anything, there are:

- Explosive logics, which validates it (classical logic, intuitionistic logic);

- Non-Explosive Logics, which invalidate it (paraconsistent logic, neutrosophic logic).

u) According to the Law of Excluded Middle (LEM), either A or ¬A, there are:

- Constructive Logic, which invalidates it (intuitionistic logic, paraconsistent logic, neutrosophic logic);

- Non-Constructive Logics (classical logic).

The criteria are not exhausted. There are sub-classifications too.

Let's take the Modal Logic which is an extension of the Propositional Calculus but with operators that express various modes of truth, such as: necessarily A, possible A, probably A, it is permissible that A, it is believed that A, it has always been true that A. The Modal Logic comprises:

-Alethic Logic (which formalizes the concepts of pertaining to truth and falsehood simultaneously, such as *possibly true* and *necessarily true*); only for this case there are more than two hundred systems of axioms!

- Deontic Logic (which seeks to represent the concepts of *obligatoriness* and *permissibility*); it is sub-divided into:

- Standard Deontic Logic, which has two monadic operators added to the classical propositional calculus: "O" = it ought to be that, and "P" = it is permissible that;

- Dyadic Deontic Logic, which has two similar dyadic operators added to the classical propositional calculus: "O(/)" = it ought to be that …, given that …, and P(/) = is it permissible that …, given that … ;

- Two-sorted Deontic Logic (Castañeda 1975), which distinguishes between *propositions* (which bear truth-values) and *practitions* (which content imperatives, commands, requests). The deontic operators in this case are: Oi = it is obligatory I that, Pi = it is permissible i that, Wi = it is wrong I that, and Li = it is optional i that. A deontic operator applied to a practition yields a proposition.

- Epistemic Logic (which seeks to represent to concepts of *knowledge, belief*, and *ignorance*);

- and Doxastic Logic (which studies the concept of *belief*); it is included in the Epistemic Logic, which is the investigation of epistemic concepts, the main ones being: knowledge, reasonable belief, justification, evidence, certainty.

Dynamic Logic (1970), as a generalization of the modal logic, has a category of expressions interpretable as *propositions* and another category of expressions interpretable as *actions*, with two operators:

$[\alpha]A$ = after every terminating computation according to α it is the case that A;

$<\alpha>A$ = after some terminating computation according to α it is the case that A,

and it is used in the verification of the computer programs.

Combinatory Logic (Schoenfinkel, Haskell Curry, 1920s) is a system for reducing the operational notation of logic, mathematics, or functional language to a sequence of modifications to the input data structure.

Temporal Logic is an extension of Predicate Calculus that includes notation for arguing about when (at what time) statements are true, and employs prefix operators such as:

$\bigcirc x$ = x is true at the next time;

$\Box x$ = x is true from now on;

$\Diamond x$ = x is eventually true;

or infix operators such as:

xUy = x is true until y is true;

xPy = x precedes y;

xWy = x is weak until y is true.

Temporal Logic studies the Linear Time, which considers only one possible future, and Branching Time, which has two extra operators:

"A" = all futures,

and "E" = some futures.

Default Logic (Raymond Reiter 1980) is a formal system with two default operators:

P:MQ/Q = if P is believed, and Q is consistent with this believe, then Q must be believed;

P:M¬Q/¬Q = if P is believed, and Q is not consistent with this believe, then Q must not be believed.

Tense Logic (Arthur Prior 1967), which is related to the Modal Logic, introduces in the classical logic two operators:

P = it was the case that … (past tense);

F = it will be the case that … (future tense).

The truth-value is not static as in classical logic, but changing in time.

Deviant Logics are logics which treat the same classical logic subjects, but in a different way (either by interpreting the connectives and quantifiers non-classically, or rejecting some classical laws): intuitionistic logic, paraconsistent logic, free logic, multi-valued logic.

Free Logic is a system of quantification theory which allows non-denoting singular terms (free variables and individual constants).

In Webster's dictionary (1988) denotation of a term means the class of all particular objects to which the term refers, and connotation of a term means the properties possessed by all the objects in the term's extension.

Erotetic Logic is the logic of questions, answers, and the relations between them. There are (1) imperative approaches (A. Åqvist, J. Hintikka, et al.), epistemic sentences embedded in an imperative sentence system, and (2) interrogative approaches (N. Belnap, T. Kubiński, and others), system of interrogative expressions and their answers.

Relational Logic (Pierce 1870, 1882) is a formal study of the properties of the (binary) relations and the operations on relations.

Because the neutrosophic logic is related to intuitionistic logic, paraconsistent logic, and dialetheism we'll focus more in these types of logics.

Intuitionistic Logic (Brouwer 1907) is a deviant logic from the classic, where the Law of Excluded Middle of Aristotle ($A \vee \neg A$) is invalidated. In this logic: a proof of existence, $\exists x P(x)$, does not count unless a method/algorithm of constructing a such x is giving (the interpretation of 'there exists' as 'we can construct' distinguishes between classical *mathematics* and *constructive mathematics* respectively); and a proof of $A \vee B$ counts only if a proof of A exhibits or a proof of B. Similarly (Bridges 1997), a proof of $A \wedge B$ counts if both a proof of A and a proof of B exibit, a proof of $A \rightarrow B$ counts if an algorithm is constructed that converts a proof of A into a proof of B, a proof of $\neg A$ means to show that A implies a contradiction, and a proof of $\forall x P(x)$ means to construct an algorithm that applied to any x proves that P(x) holds. As a consequence, the axiom of choice also fails. Brouwer considered some unsolved problem from number theory as proposition A, which is not – with our present knowledge – proved true, neither $\neg A$ is proved true. Thus, neutrosophically $NL_t(A \vee \neg A) < 1$, $NL_f(A \vee \neg A) < 1$, and $NL_t(A \vee B) < 1$, $NL_f(A \vee B) < 1$, for some propositions A, B.

Paraconsistent Logic is a logic in which the principle that anything follows from contradictory premises, for all formulas A and B one has

$A\wedge\neg A\supset B$, fails. Therefore, $A\wedge\neg A$ is not always false, i.e. for some A $NL_t(A\wedge\neg A)>0$ or $NL(A) = (t, i, f)$ where $t+f>1$. It is motivated by dialetheists who support the idea that some contradictions are true, by automated reasoning (information processing) due to inconsistent data stored in computers, and by the fact that people impart opposite beliefs. There are four types of propositional paraconsistent logics (Priest and Tanaka, 1996):

- Non-Adjunctive Systems (Jaskowski's discussive logic), where the inference $\{A, B\}\supset A\wedge B$ fails; in a discourse a participant's opinion A may be inconsistent with other participant's opinion B on the same subject;
- Non-Truth-Functional Logics (da Costa), which maintains the mechanism of positive logics (classical, intuitionistic) but the value of the negation, $\neg A$, is interpreted independently of that of A;
- Many-Valued Systems (Asenjo), many-valued logic which allows both A and $\neg A$ to be designated (to function as the analogue of truth in a two-valued logic); for example a three-valued paraconsistent system (LP) has the values: 'true', 'false', and 'both true and false', while in a four-valued system (J. M. Dunn 1976) one adds another value 'neither true nor false';
- Relevance Logic (or Relevant Logic) (Wilhelm Ackermann 1956, Alan Anderson and Nuel Belnap 1959-1974) promulgates that the premises of a valid inference must be relevant to the conclusion. The disjunctive syllogism, which states that 'if $A\vee B$ and $\neg A$ are true then so is B', is not admitted in relevance logic, neither in neutrosophic logic. However, Ackermann's rule Gamma, that 'if $A\vee B$ and $\neg A$ are theses then so is B', is admitted.

<u>Dialetheism</u> asserts that some contradictions are true, encroaching upon the Aristotle's Law of Non-Contradiction (LNC) that not both A and $\neg A$ are true. The dialetheism distinguishes from the <u>trivialism</u>, which views all contradictions as being true. Neither neutrosophic logic is trivialist.

There is a duality (Mortensen 1996) between paraconsistency and intuitionism (i.e. between inconsistency and incompleteness respectively), the Routley * operation (1972) between inconsistent theories and incomplete theories.

<u>Linear Logic</u> (J. Y. Girard 1987) is a resource sensitive logic that emphasizes on state. It employs the central notions of truth from classical

logic and of proof construction from intuitionistic logic. Assumptions are considered resources, and conclusions as requirements; A implies B means that the resource A is spent to meet the requirement B. In the deductions there are two structural rules (Scedrov 1999), that allow us to discard or duplicate assumptions (distinguishing linear logic from classical and intuitionistic logics): *contraction*, which stipulates that any assumption once stated may be reused as often as desired, and *weakening* which stipulates that it's possible to carry out a deduction without using all the assumptions. They are replaced by explicit modal logical rules such as: "storage" or "reuse" operator, !A, which means unlimited creation of A, and its dual, ?B, which means unlimited consumption of B.

How to adopt the Gödel-Gentzen negative translation, which transforms a formula A of a language L into an equivalent formula A' with no $\vee$ or $\exists$, in the neutrosophic predicate logic?

In the Boolean logic a contingent statement is a statement which is true under certain conditions and false under others. Then a neutrosophic contingent statement is a statement which has the truth value (T_1, I_1, F_1) under certain conditions and (T_2, I_2, F_2) under others.

The Medieval paradox, called Buridan's Ass after Jean Buridan (near 1295-1356), is a perfect example of complete indeterminacy. An ass, equidistantly from two quantitatively and qualitatively heaps of grain, starves to death because there is no ground for preferring one heap to another.

The neutrosophic value of ass's decision, NL = (0, 1, 0).

In a two-valued system one regards all the designated values as species of truth and all the anti-designated values as species of falsehood, with truth-value (or falsehood-value) gaps between designated and anti-designated values. In the neutrosophic system one stipulates the non-designated values as species of indeterminacy and, thus, each neutrosophic consequence has degrees of designated, non-designated, and anti-designated values.

Of course, the Law of Excluded Middle (a proposition is either true or false) does not hold in a neutrosophic system.

The Contradiction Law, that no <A> is <Non-A> does not hold too. NL(<A>) may be equivalent with NL(<Non-A>) and often they at least overlap. Neither the law of *Reductio ad absurdum* (or method of indirect proof): $(A \supset \neg A) \supset \neg A$ and $(\neg A \supset A) \supset A$.

Some tautologies (propositions logically necessary, or *true in virtue of form*) in the classical logic might not be tautologies (absolute truth-value propositions) in the neutrosophic logic and, *mutatis mutandis*, some contradictions (propositions logically impossible, or *false in virtue of form*) in the classical logic might not be contradictions (absolute falsehood-value propositions) in the neutrosophic logic.

The mixed hypothetical syllogism Modus Ponens,

If P then Q
P
———————
Q

The mixed hypothetical syllogism Modus Tollens,

If P then Q
Non Q
———————
Non P

The Inclusive (Weak) Disjunctive Syllogism:

If (P or Q)
Non P
———————
Q

The Exclusive (Strong) Disjunctive Syllogism:

If (either P or Q)
Non P
———————
Q

Hypothetical Syllogism,

If P then Q
If Q then R

If P then R

Constructive Dilemma,

P or Q
If P then R
If Q then R

R

Destructive Dilemma,

P or Q
Non P

Q

The Polysyllogism, which is formed by many syllogisms such that the conclusion of one becomes a premise of another,

and the Nested Arguments, a chainlike where the conclusion of an argument forms the premise of another where intermediate conclusions are typically left out,

are not valid anymore in the neutrosophic logic, but they acquire a more complex form.

Also, the classical *entailment*, which is the effect that a proposition Q is a necessary consequence of another proposition P, $P \rightarrow Q$, partially works in the neutrosopohic logic. Neither the informal ***fish-hook*** symbol, $\rightarrow$, use to show that a proposition Q is an accidental consequence of a proposition P, $P \rightarrow Q$, works.

Is it possible in the neutrosophic predicate calculus to transform each formula into an equivalent in prenex form one using the prenex operations?

Prenex (normal) form means a formula formalized as follows:

$(Qx_1)(Qx_2)\ldots(Qx_n)S$,

where "Q" is a universal or existential quantifier, the variables x_1, x_2, ..., x_n are distinct, and S is an open sentence (a well-formed expression containing a free variable). Prenex operation is any operation which transforms any well-formed formula into equivalent in prenex form formula; for example, $(\exists x)Ax \rightarrow B \equiv (\forall x)(Ax \rightarrow B)$.

In the classical predicate calculus any well-formed formula can be transformed into a prenex form formula.

The double negation, $\neg(\neg A) \equiv A$, which is not valid in intuitionistic logic, is not valid in the neutrosophic logic if one considers the negation operator $\eta_1(A)=1 \ominus NL(A)$, but it is valid for the negation operator $\eta_2(A)=(F, I, T)$, where $NL(A)=(T, I, F)$.

Neutrosophic Logic admits non-trivial inconsistent theories.

In stead of saying "a sentence holds (or is assertible)" as in classical logic, one extends to "a sentence p% holds (or is p% assertible)" in neutrosophic logic. In a more formalized way, "a sentence (T, I, F)% holds [or is (T, I, F)% assertible]".

A neutrosophic predicate is a vague, incomplete, or not well known attribute, property or function of a subject. It is a kind of three-valued set function. If a predicate is applied to more than one subject, it is called neutrosophic relation.

An example: Andrew is tall.

The predicate "tall" is imprecised. Andrew is maybe tall according to Linda, but short in Jack's opinion, however his tallness is unknown to David. Everybody judges him in terms of his/her own tallness and acquaintance of him.

A paradox within a sorites paradox: a frontal bald man, with a hair high density on the remaining region of his head, may have more hairs on

The neutrosophic set and logic attempt to better model the non-determinism. They try:

- to represent the paradoxical results even in science, not talking in the humanistic where the paradox is very common;
- to evaluate the peculiarities;
- to illustrate the contradictions and conflicting theories, each true from a specific point of view, false from another one, and perhaps indeterminate from a third perspective;
- to catch the mysterious world of the atom, where the determinism fails; in quantum mechanics we are dealing with systems having an infinite number of degrees of freedom;
- to study submicroscopic particles which behave non-Newtonianly, and some macroscopic phenomena which behave in nearly similar way.

In physics, the light is at once a wave and a particle (photon). Two contradictory theories were both proven true:

The first one, Wave Theory (Maxwell, Huygens, Fresnel), says that light is a wave due to the interference: two beams of light could cross each other without suffering any damage.

The second one, Particle Theory (Newton, Hertz, Lenard, Planck, Einstein), says that light is corpuscular, due to the photoelectric effect that ultraviolet light is able to evaporate electrons from metal surfaces and to the manner in which light bounces off electrons.

De Broglie reconciled both theories proving that light is a matter wave! Matter and radiation are at the same time waves and particles.

Let L1(x) be the predicate: "X is of corpuscular nature", and L2(x) the predicate: "X is of wave nature".

L2(x) is the opposite of L1(x), nonetheless L1(light) = true and L2(light) = true simultaneously.

Also, there exist four different Atom Theories: of Bohr, Heisenberg, Dirac, and Schrödinger respectively, each of them plausibly true in certain conditions (hypotheses).

Another example, from Maxwell's equations an electron <u>does</u> radiate energy when orbits the nucleus, from Bohr's theory an electron <u>does not</u> radiate energy when orbits the nucleus, and both propositions are proved true with our today's knowledge.

Falsehood is infinite, and truthhood quite alike; in between, at different degrees, indeterminacy as well.

In the neutrosophic theory:
between being and nothingness
existence and nonexistence
geniality and mediocrity
certainty and uncertainty
value and nonvalue
and generally speaking <A> and <Non-A>
there are infinitely many transcendental states.
And not even 'between', but even beyond them.
An infinitude of infinitudes.
They are degrees of neutralities <Neut-A> combined with <A> and <Non-A>.

In fact there also are steps:
between being and being
existence and existence
geniality and geniality
possible and possible
certainty and certainty
value and value
and generally speaking between <A> and <A>.

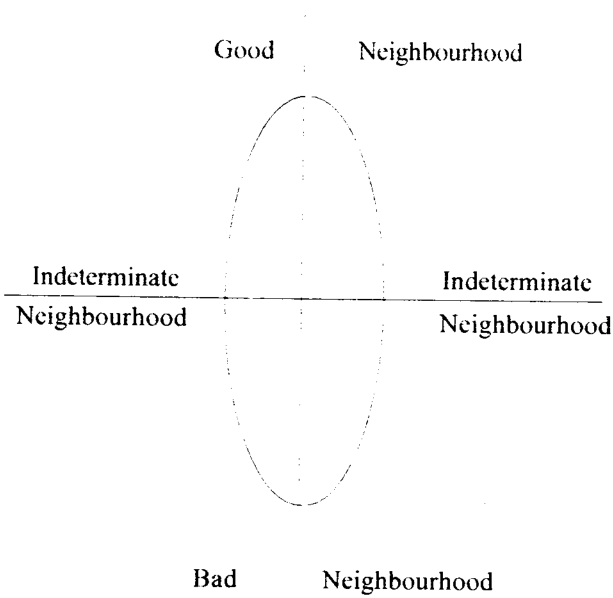
The notions, in a pure form, last in themselves only (intrinsicalness), but outside they have an interfusion form.

Infinitude of shades and degrees of differentiation:
between white and black there exists an unbounded palette of colors resulted from thousands of combinations among them.

All is alternative: progress alternates with setback, development with stagnation and underdevelopment.

In between objective and subjective there is a plurality of shades.
In between good and bad...
In between positive and negative...
In between possible and impossible
In between true and false...
In between "A" and "Anti-A"...

As a neutrosophic ellipse:



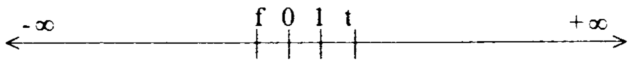
Everything is g% good, i% indeterminate, and b% bad, where g varies in the real subset G, i varies in the real subset I, and b varies in the real subset B.

Besides Diderot's dialectics on good and bad ("Rameau's Nephew", 1772), any act has its "good", "indeterminate", and of "bad" as well incorporated.

Rodolph Carnap said:

"Metaphysical propositions are neither true nor false, because they assert nothing, they contain neither knowledge nor error (...)".

Hence, there are infinitely many states between "Good" and "Bad", and generally speaking between "A" and "Anti-A" (and even beyond them), like on the real number line:



False True
Bad Good
Non-sense Sense
Anti-A A

f is the absolute falsity (f<0), t the absolute truth (t>0). In between each oppositing pair, normally in a vicinity of 0.5, are being set up the neutralities.

There exist as many states in between "True" and "False" as in between "Good" and "Bad". Irrational and transcendental standpoints belong to this interval.

Even if an act apparently looks to be only good, or only bad, other hidden sides might be sought. The ratios

$$\frac{\text{Anti-A}}{\text{A}}, \frac{\text{Non-A}}{\text{A}}$$

vary indefinitely. They may be transcendental.

If a statement is 30%T (true) and 60%I (indeterminate), then it is 15%F (false). This is somehow alethic, meaning to simultaneously pertain to truthhood and falsehood, or to truthhood and indeterminacy, or to falsehood and indeterminacy, or even to all three components.

More general, if a statement is 30%T and 60%I, it may be between 5-20%F or 25%F.

In opposition to Fuzzy Logic, if a proposition <A> is t% true, doesn't necessarily mean it is (100-t)% false. A better approach is t% true, f% false, and i% indeterminate, where $t \in T$, $i \in I$, $f \in F$, and the sum t+i+f as well as t, i, f may be any real numbers - not necessarily between 0 and 1.

One considers subsets of truth, indeterminacy, and falsity instead of single numbers because of imprecision, uncertainty, and vagueness.

The neutrosophic logical value of <A> is noted by NL(A) = (T,I,F). On components one writes:

for the true value $NL_t(A) = T$;
for the indeterminacy value $NL_i(A) = I$;
for the falsity value $NL_f(A) = F$.

Neutrosophic Logic means the study of neutrosophic logical values of the propositions.

There exist, for each individual one, PRO parameters, CONTRA parameters, and NEUTER parameters which influence the above values.

Indeterminacy results from any hazard which may occur, from unknown parameters, or from new arising conditions.

This resulted from practice.

Applications:

Neutrosophic logic is useful in the real-world systems for designing control logic, and may work in quantum mechanics.

The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

This proposition is, say, 20-25% true (percentage of people voting for him), 35-45% false (percentage of people voting against him), and 40% or 50% indeterminate (percentage of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Tomorrow it will rain.

This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, between 20-30% false according to today's very sunny and droughty summer, and 40% undecided.

This is a heap.

As an application to the sorites paradoxes, we may now say that this proposition is 80% true, 40% false, and 25-35% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective). Vagueness plays here an important role.

We are not able to distinguish the difference between yellow and red as well if a continuum spectrum of colors is painted on a wall imperceptibly changing from one into another.

We would be able to say at a given moment that a section is both yellow and red in the same time, or neither one!

his head that another man who is not bald but the skin surface of his head and the hair density are smaller than the previous one.

Definition of Neutrosophic Logical Connectives:

The connectives (rules of inference, or operators), in any non-bivalent logic, can be defined in various ways, giving rise to lots of distinct logics. For example, in three-valued logic, where three possible values are possible: true, false, or undecided, there are 3072 such logics! (Weisstein, 1998) A single change in one of any connective's truth table is enough to form a (completely) different logic.

The rules are hypothetical or factual. How to choose them? The philosopher Van Fraassen (1980) [see Shafer, 1986] commented that such rules may always be controvertible "for it always involves the choice of one out of many possible but nonactual worlds". There are general rules of combination, and ad hoc rules.

For an applied logic to artificial intelligence, a better approach, the best way would be to define the connectives recursively (Dubois, Prade), changing/adjusting the definitions after each step in order to improve the next result. This might be comparable to approximating the limit of a convergent sequence, calculating more and more terms, or by calculating the limit of a function successively substituting the argument with values closer and closer to the critical point. The recurrence allows evolution and self-improvement.

Or to use *greedy algorithms*, which are combinatorial algorithms that attempt at each iteration as much improvement as possible unlike myopic algorithms that look at each iteration only at very local information as with steepest descent method.

As in non-monotonic logic, we make assumptions, but we often err and must jump back, revise our assumptions, and start again. We may add rules which don't preserve monotonicity.

In bio-mathematics Heitkoetter and Beasley (1993-1999) present the *evolutionary algorithms*, which are used "to describe computer-based problem solving systems which employ computational models of some of the known mechanisms of evolution as key elements in their design and implementation". They simulate, via processes of selection, mutation, and reproduction, the evolution of individual structures. The major evolutionary algorithms studied are: genetic algorithm (a model of machine learning based on genetic operators), evolutionary programming (a stochastic optimization strategy based on linkage between parents and their offspring; conceived by L. J. Fogel in 1960s), evolution strategy, classifier system, genetic programming.

Pei Wang devised a Non-Axiomatic Reasoning System as an intelligent reasoning system, where intelligence means working and adopting with insufficient knowledge and resources.

The inference mechanism (endowed with rules of transformation or rules of production) in neutrosophy should be non-monotonic and should comprise ensembles of recursive rules, with preferential rules and secondary ones (priority order), in order to design a good expert system. One may add new rules and eliminate old ones proved unsatisfactory. There should be strict rules, and rules with exceptions. Recursivity is seen as a computer program that learns from itself. The statistical regression method may be employed as well to determine a best algorithm of inference.

Non-monotonic reasoning means to make assumptions about things we don't know. Heuristic methods may be involved in order to find successive approximations.

In terms of the previous results, a default neutrosophic logic may be used instead of the normal inference rules. The distribution of possible neutrosophic results serves as an orientating frame for the new results. The flexible, continuously refined, rules obtain iterative and gradual approaches of the result.

A comparaison approach is employed to check the result (conclusion) p by studying the opposite of this: what would happen if a non-p conclusion occurred? The inconsistence of information shows up in the result, if not eliminated from the beginning. The data bases should be stratified. There exist methods to construct preferable coherent sub-bases within incoherent bases. In Multi-Criteria Decision one exploits the complementarity of different criteria and the complementarity of various sources.

For example, the Possibility Theory (Zadeh 1978, Dubois, Prade) gives a better approach than the Fuzzy Set Theory (Zadeh 1965) due to self-improving connectives. The Possibility Theory is proximal to the

Fuzzy Set Theory, the difference between these two theories is the way the fusion operators are defined.

One uses the definitions of neutrosophic probability and neutrosophic set operations.

Similarly, there are many ways to construct such connectives according to each particular problem to solve; here we present the easiest ones:

One notes the neutrosophic logical values of the propositions A_1 and A_2 by $NL(A_1) = (T_1, I_1, F_1)$ and $NL(A_2) = (T_2, I_2, F_2)$.

Negation:

$NL(\neg A_1) = (1, 1, 1) \boxminus NL(A_1) = (1, 1, 1) \boxminus (T_1, I_1, F_1) = (1 \ominus T_1, 1 \ominus I_1, 1 \ominus F_1)$.

Conjunction:

$NL(A_1 \wedge A_2) = NL(A_1) \boxdot NL(A_2) = (T_1, I_1, F_1) \boxdot (T_2, I_2, F_2) = (T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2)$.

(And, in a similar way, generalized for n propositions.)

Weak or inclusive disjunction:

$NL(A_1 \vee A_2) = NL(A_1) \boxplus NL(A_2) \boxminus NL(A_1) \boxdot NL(A_2) = (T_1, I_1, F_1) \boxplus (T_2, I_2, F_2) \boxminus (T_1, I_1, F_1) \boxdot (T_2, I_2, F_2) = (T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2)$.

(And, in a similar way, generalized for n propositions.)

Strong or exclusive disjunction:

$$NL(A_1 \underline{\vee} A_2) =$$
$$(T_1 \odot (1 \ominus T_2) \oplus T_2 \odot (1 \ominus T_1) \ominus T_1 \odot T_2 \odot (1 \ominus T_1) \odot (1 \ominus T_2),$$
$$I_1 \odot (1 \ominus I_2) \oplus I_2 \odot (1 \ominus I_1) \ominus I_1 \odot I_2 \odot (1 \ominus I_1) \odot (1 \ominus I_2),$$
$$F_1 \odot (1 \ominus F_2) \oplus F_2 \odot (1 \ominus F_1) \ominus F_1 \odot F_2 \odot (1 \ominus F_1) \odot (1 \ominus F_2)).$$

(And, in a similar way, generalized for n propositions.)

Material conditional (implication):

$$NL(A_1 \mapsto A_2) = (1 \ominus T_1 \oplus T_1 \odot T_2,\ 1 \ominus I_1 \oplus I_1 \odot I_2,\ 1 \ominus F_1 \oplus F_1 \odot F_2).$$

Material biconditional (equivalence):

$$NL(A_1 \leftrightarrow A_2) = ((1 \ominus T_1 \oplus T1 \odot T2) \odot (1 \ominus T_2 \oplus T_1 \odot T_2),$$
$$(1 \ominus I_1 \oplus I_1 \odot I_2) \odot (1 \ominus I_2 \oplus I_1 \odot I_2),$$
$$(1 \ominus F_1 \oplus F_1 \odot F_2) \odot (1 \ominus F_2 \oplus F_1 \odot F_2)).$$

Sheffer's connector:

$$NL(A_1 \mid A_2) = NL(\neg A_1 \vee \neg A_2) = (1 \ominus T_1 \odot T_2,\ 1 \ominus I_1 \odot I_2,\ 1 \ominus F_1 \odot F_2).$$

Peirce's connector:

$$NL(A_1 \downarrow A_2) = NL(\neg A_1 \wedge \neg A_2) =$$
$$= ((1 \ominus T_1) \odot (1 \ominus T_2), (1 \ominus I_1) \odot (1 \ominus I_2), (1 \ominus F_1) \odot (1 \ominus F_2)).$$

Comparison between Fuzzy Logic and Neutrosophic Logic:

The neutrosophic connectives have a better truth-value definition approach to the real-world systems than the fuzzy connectives. They are defined on triple subsets, not on double or triple numbers, with no restrictions on the subsets nor on their superior or inferior limits; while the components of a fuzzy proposition should sum up to 1 and be greater than or equal to 0.

Neutrosophical Modal Logic:

In modal logic, the primitive operators 'it is possible that' and 'it is necessary that' can be defined by:

$t\_inf(\diamond A) > 0$,

and, because $\Box A$ could be regarded as $\neg(\diamond \neg A)$,

$t\_sup(\Box A) < 1$.

The sufficient reason principle (Aristotle, Leibniz), which asserts that every statement has a grounding, partially works in this logic.

Also, identity principle, that $A \leftrightarrow A$ is true, partially works, because

if say $NL_t(A) = 0.3$ then $NL_t(A \leftrightarrow A) = 0.6241$,
the only cases when $NL_t(A \leftrightarrow A) = 1$ are for $NL_t(A) = 0$ or 1.

Same thing for the principles of bivalence (a statement is either true or false), and of excluded middle (a statement with its negation is always true).

The principle of noncontradiction (a statement and its negation may not both be true) functions only if $NL_t(A)$ is straight 0 or 1, otherwise $NL_t(A \wedge \neg A) \neq 0$.

Neutrosophy shows that a philosophical idea, no matter if proven true by ones or false by others, may get any truth-value depending on the referential system we are reporting it to.

Let $t \equiv NL_t$.

The conjunction is well defined, associative, commutative, admits a unit element U with $t(U) = 1^+$, but no element whose truth-component is different from 1, inversable.

The conjunction is not absorbent, i.e. $t(A \wedge (A \wedge B)) \neq t(A)$, except for the cases when $t(A) \leq 0$, or $t(A) = t(B) \geq 1$.

The disjunction is well-defined, associative, commutative, admits a unit element O with $t(O) = {}^-0$, but no element, whose truth-component is different from 0, inversable.

The disjunction is not absorbent, i.e. $t(A \vee (A \vee B)) \neq t(A)$, except for the cases when one of $t(A) \geq 1$, or $t(A) = t(B) \leq 0$.

None of them is distributive with respect to the other.

De Morgan laws do not apply either.

Therefore $(NL, \wedge, \vee, C)$, where NL is the set of neutrosophic logical propositions, is not an algebra.

Nor $(P(]^-0, 1^+[), \cap, \cup, C)$, where $P(]^-0, 1^+[)$ is the set of all subsets of $]^-0, 1^+[$, and $C(A)$ is the neutrosophic complement of A.

One names a set N, endowed by two associative unitary internal laws * and #, which are not inversable except for their unit elements respectively, and not distributive with respect to each other, Ninversity.

If both laws are commutative, then N is called a Commutative Ninversity.

For a better understanding of the neutrosophic logic one needs to study the commutative ninversity.

One defines a Neutrosophic Topology on $]^{-}0,1^{+}[$, considering all subsets (a, b) of this non-standard interval, where a, b are standard or non-standard numbers.

The whole set $]^{-}0,1^{+}[$, the empty set $\phi = (0,0)$, and the above ones are open sets. They are closed under set union and finite intersection.

The union is defined as:

$(a_1,b_1) \cup (a_2,b_2) = (a_1+a_2-a_1a_2,\ b_1+b_2-b_1b_2)$,

and the intersection as:

$(a_1,b_1) \cap (a_2,b_2) = (a_1a_2,\ b_1b_2)$.

The complementary of (a,b) is $(1^{+}-b,\ 1^{+}-a)$ wich is a closed set.

The non-standard interval $]^{-}0,\ 1^{+}[$, endowed with this topology, forms a neutrosophic topological space.

References:

[1] Albee, Ernest, History of English Utilitarianism, *Collier Books, Crowell-Collier Publ. Co., N.Y., 1962.*

[2] Ashbacher, Charles D., Exploring Some Specific Functional Mappings To Create Operators In The Neutrosophic Logic, *mss., 1999.*

[3] Association for Symbolic Logic, Urbana, IL, USA, *New Book Anouncement of Interest to Logicians,*
http://www.aslonline.org/links/html.

[4] Atanassov, K., Burillo, P., Bustince, H., On the intuitionistic fuzzy relations, *Notes on Intuitionistic Fuzzy Sets, Vol. 1 (1995), No. 2, 87 - 92.*

[5] Atanassov, K., Bustince, H., Burillo, P., Mohedano, V., A method for inference in approximate reasoning for the one-dimensional case based on normal intuitionistic fuzzy sets, *Proceedings of the VI IFSA World Congress, Sao Paulo, Brazil, July 1995, Vol. 1, 149-152.*

[6] Atanassov, K., Stoyanova, D., Cartesian products over intuitionistic fuzzy sets, Methodology of Mathematical Modelling, *Vol. 1, Sofia, 1990, 296-298.*

[7] Atanassov, K., Stoyanova, D., Remarks on the intuitionistic fuzzy sets. II, Notes on Intuitionistic Fuzzy Sets, *Vol. 1 (1995), No. 2, 85 - 86.*

[8] Audi, Robert, general editor, The Cambridge Dictionary of Philosophy, *Cambridge University Press, 1995.*

[9] Ayer, A. J., Logical Positivism, *The Free Press of Glencoe, New York, 1958.*

[10] Bailey, Cyril, The Greek Atomists and Epicurus, *Russell & Russell, Inc., New York, 1964.*

[11] Barbu, Marian, Aspecte al romanului românesc contemporan, *Meridian Publ. House, Craiova, 1995.*

[12] Berlin, Isaiah (ed.), The Empiricists: John Locke, George Berkeley, David Hume, *Dolphin Books, Doubleday & Company, Inc., Garden City, N.Y., 1961.*

[13] Blunden, Andy, *A New Logic: Neutrosophic Logic*, Hegel by HyperText Page, http://werple.net.au/~andy/email.htm.

[14] Bogolubov, N. N., Logunov, A. A., Todorov, I. T., Introduction to Axiomatic Quantum Field Theory, *Translated from Russian by Stephen A. Fulling and Ludmila G. Popova, W. A. Benjamin, Inc., Reading, Massachusetts, 1975.*

[15] Bouvier, Alain, George, Michel, Dictionnaire des Mathématiques, *sous la direction de François Le Lionnais, Presses Universitaire de France, Paris, 1979.*

[16] Bouwsma, W. J., The Culture of Renaissance Humanism, *American Historical Association, Washington, 1973.*

[17] Bridges, Douglas, *Constructive Mathematics*, Stanford Encyclopedia of Philosophy, editor Edward N. Zalta,
http://plato.stanford.edu/mathematics-constructive/, 1997.

[18] Buhaescu, T., On an order relation between fuzzy numbers and fuzzy functions convexity, *Itinerant seminar on functional equations, approximation and convexity, Cluj-Napoca, 1987, 85-90.*

[19] Buhaescu, T., On quasicontinuity of fuzzy functions, *Analele Universității din Galați, Matematica, Fizica, Mecanica Teoretica, Fascicula II, Anul VI (XI), 1988, 5-7.*

[20] Buhaescu, T., On the convexity of intuitionistic fuzzy sets, *Itinerant Seminar on Functional Equations, Approximation and Convexity, Cluj-Napoca, 1988, 137-144.*

[21] Buhaescu, T., Some observations on intuitionistic fuzzy

rerelations, *Itinerant Seminar on Functional Equations, Approximation and Convexity, Cluj-Napoca, 1989, 111-118.*

[22] Buhaescu, T., Intuitionistic fuzzy numbres, *Analele Universităţii "Dunarea de Jos", Galaţi, Fascicula II, Anul VIII 1990, 47-53.*

[23] Buhaescu, T., Nombres fuzzy intuitionistiques, *Analele Universitatii Galati, fascicola II, 1990-1991, 1-2.*

[24] Buhaescu, T., Interval valued real numbers, *Sesiunea de comunicări ştiintifice, Constanţa, 6-8 iunie 1991, 34.*

[25] Buhaescu, T., Convex structures on the fuzzy sets class, *Fuzzy Systems & AI, Vol. 2, No. 3, 1993, 15-20.*

[26] Buhaescu, T., Linear programming with intuitionistic fuzzy objective, *Notes on Intuitionistic Fuzzy Sets, Vol. 1 (1995), No. 2, 130-131.*

[27] Buhaescu, T., Linear programming with intuitionistic fuzzy objective, *International Colloquy the Risk in Contemporary Economy, Galati, Romania, Nov. 10-11, 1995, 29-30.*

[28] Burillo, Lopez P., Bustince Sola H., Entropy on intuitionistic fuzzy sets and on interval-values fuzzy sets, *Fuzzy Sets and Systems, Vol. 78 (1996), No. 3, 305-316.*

[29] Burillo, P., Bustince, H., Mohedano, V., Some definitions of intuitionistic fuzzy number. first properties, *Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.),* Sofia, Sept. 28-30, 1994, 53-55.

[30] Burillo, P., Bustince, H., Algebraic structures for intuitionistic fuzzy sets, *Fifth Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 5, 1994, Preprint MRL-MFAIS-10-94, Sofia, 1994, 1-13.*

[31] Burillo, P., Bustince, H., Isoentropic methods for construction of IVFS, *Proc. of the 4-th International Workshop CIFT'94, Trento, June 1-3, 1994, 57-60.*

[32] Burillo, P., Bustince, H., Numerical measurements of information on intuitionistic fuzzy sets and interval-valued fuzzy sets (Φ - fuzzy), *Fifth Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 5, 1994, Preprint MRL-MFAIS-10-94, Sofia, 1994, 14-26.*

[33] Burillo, P., Bustince, H., Two operators on interval-valued intuitionistic fuzzy sets: Part I, *Comptes rendus de l'Academie Bulgare des Sciences, Tome 47, 1994, No. 12, 9-12.*

[34] *Burillo, P., Bustince, H.*, Informational energy on intuitionistic fuzzy sets and on interval-values intuitionistic fuzzy sets (Φ -fuzzy). Relationship between the measures of information, *Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.), Sofia, Sept. 28-30, 1994, 46-49.*

[35] *Burillo, P., Bustince, H.*, Numeros Intuicionistas Fuzzy, *IV Congreso de la Asociacion Espanola de logica y Tecnologia Fuzzy, 1994, 97-103.*

[36] *Burillo, P., Bustince, H.*, Orderings in the referential set induced by an intuitionistic fuzzy relation, *Notes on Intuitionistic Fuzzy Sets, Vol. 1 (1995), No. 2, 93-103.*

[37] *Burillo, P., Bustince, H.*, Two operators on interval-valued intuitionistic fuzzy sets: Part II, *Comptes rendus de l'Academie Bulgare des Sciences, Tome 48, 1995, No. 1, 17-20.*

[38] *Burillo, P., Bustince, H.*, Metodos intuicionistas fuzzy, *V Congreso de la Asociacion Espanola de logica y Tecnologia Fuzzy*, 1995, 147-153.

[39] *Burillo, P., Bustince, H.*, Intuitionistic fuzzy relations. Part I, *Mathware and Soft Computing, Vol. 2 (1995), No 1, 5-38.*

[40] *Burillo, P., Bustince, H.*, Intuitionistic fuzzy relations. Part II, *Mathware and Soft Computing, Vol. 2 (1995), No 2, 117-148.*

[41] *Burillo, P., Bustince, H.*, Construction theorems for intuitionistic fuzzy sets, *Fuzzy Sets and Systems, Vol. 84, 1996, No. 3, 271-281.*

[42] *Burnet, John*, Greek Philosophy: Thales to Plato, *St. Martin's Press, Inc., New York, 1962.*

[43] *Bustince, H.*, Numerical information measurements in intervalvalued intuitionistic fuzzy sets (IVIFS), *Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.), Sofia, Sept. 28-30, 1994, 50-52.*

[44] *Bustince, Sola H.*, Conjuntos Intuicionistas e Intervalo-valorados Difusos: Propiedades y Construccion. Relaciones Intuicionistas y Estructuras, *Ph. D., Univ. Publica de Navarra, Pamplona, 1994.*

[45] *Bustince, H.*, Correlation of interval-valued intuitionistic fuzzy sets, *Fifth Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, October 5, 1994, Preprint MRL-MFAIS-10-94, Sofia, 1994, 27-35.*

[46] *Bustince, H.*, Handling multicriteria fuzzy decision making

problems based on intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets, Vol. 1, No. 1, 1995, 42-47.*

[47] Bustince, H., Burillo P., A theorem for constructing interval valued intuitionistic fuzzy sets from intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets, Vol. 1, No. 1, 1995, 5-16.*

[48] Bustince, H., Burillo, P., Antisymmetrical ituitionistic fuzzy relation - Order on the referential set induced by an intuitionistic fuzzy relation, *BUSEFAL Vol. 62, 1995, 17-21.*

[49] Bustince, H., Burillo, P., Correlation of interval-valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems, Vol. 74 (1995), No.2, 237-244.*

[50] Bustince, H., Burillo, P., Mohedano, V., A method for inference in approximate reasoning based on normal intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets, Vol. 1, No. 1, 1995, 51-55.*

[51] Bustince, H., Burillo, P., Vague sets are intuitionistic fuzzy sets, *Fuzzy Sets and Systems, Vol. 79, 1996, No. 3, 403-405.*

[52] Bustince, H., Mohedano, V., About the intuitionistic fuzzy set generators, *Proceedings of the First International Conference on Intuitionistic Fuzzy Sets (J. Kacprzyk and K. Atanassov Eds.), Sofia, Oct 18-19, 1997; Notes on Intuitionistic Fuzzy Sets, Vol. 3 (1997), No. 4, 21-27.*

[53] Bustince, H., Mohedano, V., About the complement in intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets, Vol. 3 (1997), No. 1, 12-19.*

[54] Carré, M. H., Realists and Nominalists, *Oxford University Press, Fair Lawn, NJ, 1946.*

[55] Copleston, Frederick, Arthur Schopenhauer, Philosopher of Pessimism, *Barnes and Noble Books, New York, 1975.*

[56] Dempster, A. P., Upper and Lower Probabilities Induced by a Multivalued Mapping, *Annals of Mathematical Statistics, 38, 325-339, 1967.*

[57] Dezert, J., Autonomous navigation with Uncertain Reference points using the PDAF, *In Multitarget-Multisensor Tracking : Applications and Advances, Volume 2, Yaakov Bar-Shalom Editor, pp 271-324, 1992.*

[58] Dezert, Jean, E-mails to the Author, 1999-2000.

[59] Dezert, Jean, On a Problem of Autonomous Navigation of an

Engine Car *(approximate title), Ph. D. Thesis, ONERA, Paris, 1990.*

[60] Dezert, J., Vers un nouveau concept de navigation autonome d'engin; Un lien entre la théorie de l'évidence et le filtrage à association probabiliste de données, *Ph. D. Thesis, no 1393, University Paris 11, Orsay, France, Sept. 1990.*

[61] Didero, Daniele, Dictionaries and Encyclopedias, Italy, http://lgxserver.uniba.it/lei/dionary/dizlink.htm.

[62] Dimitrov, D., Atanassov, K., Shannon, A., Bustince, H., Kim, S.-K., Intuitionistic fuzzy sets and economic theory, *Proceedings of The Second Workshop on Fuzzy Based Expert Systems* FUBEST'96 (D. Lakov, Ed.), Sofia, Oct. 9-11, 1996, 98-102.

[63] Dinulescu-Câmpina, Gheorghe, The Intangible Absolute Truth, *"Smarandache Notions Journal", 142-143, 2000.*

[64] Dubois, D., Prade, H., *Révision mise à jour, et action,* http://www.irit.fr/ACTIVITES/RPDMP/RMF.html/.

[65] Dudau, Victor, *A better use of neutrosophic probability*, Sci.Math Archives Topics, 8 July 1999, http://forum.swarthmore.edu/epigone/sci.math/genverqua.

[66] Dummett, M., Wang's paradox, *Synthese, 30, 301-324, 1975.*

[67] Dunn, J. M., Intuitive Semantics for First Degree Entailment and Coupled Trees, *Philosophical Studies, Vol. XXIX, pp. 149-68, 1976.*

[68] Eksioglu, Kamil Murat, *Imprecision, Uncertainty & Vagueness: a reply* (from his Ph. D. Dissertation), 1999, http://www.dbai.tuwien.ac.at/marchives/fuzzy-mail99/0819.html.

[69] Fine, K., Vagueness, truth and logic, *Synthese, 30, 265-300, 1975.*

[70] Fisch, Max, and Turquette, Atwell, Pierce's Triadic Logic, *Transactions of the Charles S. Peirce Society, 11, 71-85, 1966.*

[71] Gilbert, John, Institute for Logic at the University of Vienna, Austria, Abstract, http://www.logic.univie.ac.at/cgi-bin/abstract/.

[72] Girard, Jean-Yves, Linear logic, *Theoretical Computer Science, 50:1-102, 1987.*

[73] Goldberg, Samuel, Probability / An Introduction, *Prentice-Hall, Inc., Englewood Cliffs, NJ, 1960.*

[74] Goguen, J. A., The Logic of Inexact Concepts, *Synthese, 19, 325-375, 1969.*

[75] Grmela, Ales, E-mails to C. T. Le, August 1999.

[76] Guinnessy, Paul; Gilbert, John, Proceedings on the Neutrosophic Logic and Their Applications in Neural Networks, Computer Programming, and Quantum Physics, *Institute of Physics, editors: Kenneth Holmlund, Mikko Karttunen, Ghenther Nowotny,* http://physicsweb.org/TIPTOP/FORUM/BOOKS/describebook.phtml?entry_id=116.

[77] Hallden, S., The Logic of Nonsense, *Uppsala Universitets Arsskrift, 1949.*

[78] Hammer, Eric M., *Pierce's Logic*, Stanford Encyclopedia of Philosophy, edited by Edward N. Zalta, URL=http://plato.stanford.edu/entries/pierce-logic/, 1996.

[79] Heitkoetter, Joerg; David Beasley, David, The Hitch-Hiker's Guide to Evolutionary Computing, *Encore,* http://surf.de.uu.net/encore/, ftp://gnomics.udg.es/pub/encore/EC/FAQ/part2, 1993-1999.

[80] Hassing, Richard F., Final Causality in Nature and Human Affairs, *The Catholic University of America Press, Baltimore, 282p., 1997.*

[81] Hegel, G.W.F., The Phenomenology of Spirit, *trans., A.V.Miller, Clarendon Press, Oxford, 1977.*

[82] Hellerstein, N. S., DELTA, Á Paradox Logic, *World Scientific, Singapore, New Jersey, London, Hong Kong, 1999.*

[83] Hobbes, Thomas, Body, Man and Citizen, *Collier Books, Crowell-Collier Publishing Co., New York, 1962.*

[84] Hoffmann, Banesh, The strange Story of the Quantum, An account for the general reader of the growth of the ideas underlying our present atomic knowledge, *Dover Publications, Inc., New York, 1959.*

[85] Hogg, Robert V., Craig, Allen T., Introduction to Mathematical Statistics, *Second edition, The Macmillan Company, New York, 1969.*

[86] Howe, Denis, *Neutrosophic Logic (or Smarandache Logic)*, On-Line Dictionary of Computing,
http://foldoc.doc.ic.ac.uk/foldoc/foldoc.cgi?Smarandache+logic.
and FOLDOC Australian Mirror - Australia's Cultural Network,
http://www.acn.net.au/cgi-bin/foldoc.cgi?Smarandache+logic,
http://www.acn.net.au/foldoc/contents/S.htm.

[87] Howe, Denis, Neutrosophic Probability, *On-Line Dictionary of Computing, England.*

http://foldoc.doc.ic.ac.uk/foldoc/foldoc.cgi?neutrosophic+probability.

[88] Howe, Denis, Neutrosophic Set, *On-Line Dictionary of Computing, England,* http://foldoc.doc.ic.ac.uk/foldoc/foldoc.cgi?neutrosophic+set.

[89] Howe, Denis, Neutrosophic Statistics, *On-Line Dictionary of Computing, England,* http://foldoc.doc.ic.ac.uk/foldoc/foldoc.cgi?neutrosophic+statistics.

[90] Howe, Denis, Neutrosophy, *On-Line Dictionary of Computing,* http://foldoc.doc.ic.ac.uk/foldoc/foldoc.cgi?neutrosophic.

[91] Hyde, Dominic, *Sorites Paradox*, Stanford Encyclopedia of Philosophy, URL=http://plato.stanford.edu/entries/sorites-paradox/, edited by Edward N. Zalta, 1996.

[92] Illingworth, Valerie, The Penguin Dictionary of Physics, *second edition, Penguin Books, 1991.*

[93] Iorga, Nicolae, Cugetări, *edited by Elisabeta Jurca-Pod, The Yellow Bird Publ., Chicago, 1991.*

[94] Jasper, Paul, Abstract Service for Mathematical Logic, Institute for Logic at the University of Vienna, http://www.logic.univie.ac.at/cgi-bin/abstract/.

[95] Jasper, Paul; Le, C. T. (editors), 'A Unifying Field In Logics. Neutrosophy: Neutrosophic Probability, Set, and Logic' by Florentin Smarandache, *book review, Institute of Physics, London, England, editors: Kenneth Holmlund, Mikko Karttunen, Ghenther Nowotny,* http://physicsweb.org/TIPTOP/FORUM/BOOKS/describebook.phtml?entry_id=117.

[96] Jaspers, K., Nietzsche: An Introduction to the Understanding of His Philosophical Activity, *University of Arizona Press, Tucson, 1965.*

[97] Jaspers, Karl, General Psychopathology, *translated by J. Hoenig and Marian W. Hamilton, Introduction by Paul McHugh, The John Hopkins University Press, Baltimore, Vol. I and II.*

[98] Kant, Immanuel, Critique of Pure Reason, *St. Martin's Press, New York, 1965.*

[99] Kasabov, N., Foundations of neural networks, fuzzy systems and knowledge engineering, *MIT Press, 1996.*

[100] Kathwaroon, Maggie, Society for Technical Communications, Montreal Links Page, Canada, http://www.stc-montreal.org/links.html.

[101] Kenny, A., Aquinas, *Hill and Wang, Inc., New York, 1980.*

[102] Klein, Felix, Vergleichende Betrachtungen öber neuere geometrische Forschungen, *Mathematische Annalen, 43, 63-100, 1893.*

[103] Kockelmans, J.L., Phenomenology: The Philosophy of Edmund Husserl and Its Interpretation, *Doubleday and Company, Inc., Garden City, N.Y., 1967.*

[104] Korner, S., The Philosophy of Mathematics, *Hutchinson, London, 1960.*

[105] Lambert, J. H., Neues Organon, *Leipzig, 1764.*

[106] Lambert, J. H., Philosophische Schriften, *Vol. I & II, reprinted by Olms, 1985.*

[107] Le, Charles T., *Neutrosophic logic used in neural networks,* CIO Communications, Inc.,

http://wellengaged.com/engaged/cio.cgi?c=connection&f=0&t=255.

[108] Le, Charles T. Le, *Software for neutrosophic logical operators,* Networking by Industry, Inc. Online,

http://www.inc.com/bbs/show/4/935277052-999.

[109] Le, Charles T., The Smarandache Class of Paradoxes, *in "Journal of Indian Academy of Mathematics", Bombay, India, No. 18, 53-55, 1996.*

[110] Leff, Gordon, Medieval Thought from St. Augustine to Ockham, *Penguin Books, Inc., Baltimore, 1962.*

[111] Lin, T. Y., E-mails to C. T. Le, August 1999.

[112] Mackey, George W., Mathematical Foundations of Mechanics / A Lecture-Note Volume, *The Benjamin/Cummings Publishing Company, Reading, Massachusetts, 1980.*

[113 Mathematical Logic Around The World, University of Bonn, Germany, http://www.uni-bonn.de/logic/world.html.

[114] Marcel, Gabriel, Man against Mass Society, *Henry Regnery Co., Chicago, 1962.*

[115] Marcuse, Herbert, Reason and Revolution: Hegel and the Rise of Social Theory, *Beacon Press, Boston, 1960.*

[116] McKeon, Richard P., An Introduction to Aristotle, *Random House, Inc., New York, 1947.*

[117] McNeil, Martin, F., Thro, Ellen, Fuzzy Logic / A Practical Approach, *Foreword by Ronald R. Yager, Academic Press, 1994.*

[118] Mehta, J. L., Martin Heidegger: The Way and the Vision,

University of Hawaii Press, Honolulu, 1976.

[119] Mendenhall, William, Introduction to Probability and Statistics, *Fourth edition, Duxbury Press, North Scltuate, Massachusetts, 1975.*

[120] Mortensen, Chris, *Inconsistent Mathematics*, Stanford Encyclopedia of Philosophy, editor Edward N. Zalta,

http://plato.stanford.edu/entries/mathematics-inconsistent/, 1996.

[121] Moschovakis, Joan, *Intuitionistic Logic*, Stanford Encyclopedia of Philosophy, editor Edward N. Zalta,

http://plato.stanford.edu/contents.html#1.

[122] Moschovakis, Joan, E-mails to C. T. Le, August 1999.

[123] Munshi, K. M., Diwakar, R.R. (gen. eds.), Introduction to Vedanta, *by P. Nagaraja Rao, Bhavan's Book University, Chowpatty, Bombay, India, 1966.*

[124] Narinyani, A., Indefinite sets - a new type of data for knowledge representation, *Preprint 232, Computer Center of the USSR Academy of Sciences, Novosibirsk, 1980 (in Russian).*

[125] Nobre, Farley Simon M., E-mails to M. Perez, August 1999.

[126] Perez, Minh, Neutrosophy - book review , *Sci.Math Archives Topics, 8 July 1999,*

http://forum.swarthmore.edu/epigone/sci.math/lelswoboi.

[127] Perez, M. L., New Books and Books Reviews, *Institute of Physics, London, England, editors: Kenneth Holmlund, Mikko Karttunen, Ghenther Nowotny,*

http://physicsweb.org/TIPTOP/FORUM/BOOKS/newbooks.phtml.

[128] Petrescu, M. I., Institute for Logic at the University of Vienna, Austria,

http://www.logic.univie.ac.at/cgi-bin/abstract/.

[129] Petrescu, M. I., Le, Charles T., Neutrosophic Logic, a generalization of the fuzzy logic, *7/30 July 1999,*

http://forum.swarthmore.edu/epigone/sci.math/shelkelwhim.

[130] Peirce, C. S., Essays in the Philosophy of Science, *The Liberal Arts Press, Inc., New York, 1957.*

[131] Piwinger, Boris, Institute for Logic at the University of Vienna, Austria,

http://www.logic.univie.ac.at/cgi-bin/abstract/author.shtml.

[132] Popa, Constantin M., The Paradoxist Literary Movement, *Xiquan Publ., Hse., Phoenix, 1992.*

[133] Popescu, Titu, Estetica paradoxismului, *Tempus Publ. Hse., Bucharest, 1995.*

[134] Pollett, Phil, The Probability Web / Miscellaneous Probability links, Australia,
http://www.maths.uq.oz.au/~pkp/probweb/misc.html.

[135] Priest, Graham; Tanaka, Koji, Paraconsistent Logic, *Stanford Encyclopedia of Philosophy, editor Edward N. Zalta,*
http://plato.stanford.edu/entries/logic-paraconsistent/.

[136] Priest, Graham, Dialetheism, Stanford Encyclopedia of Philosophy, editor Edward N. Zalta,
http://plato.stanford.edu/entries/dialetheism/.

[137] Quine, W. V., What price bivalence?, *Journal of Philosophy, 77, 90-95, 1981.*

[138] Robinson, A., Non-Standard Analysis, *Princeton University Press, Princeton, NJ, 1996.*

[139] Rousseau, Jean-Jacques, On the Social Contract, *trans. Judith R. Masters, St. Martin's Press, Inc., New York, 1978.*

[140] Routley, Richard and Val, The Semantics of First Degree Entailment, *Nous, Vol. 6, 335-359, 1972.*

[141] Rugina, Anghel N., Geldtypen und eldordnungen. Fundamente für eine echte allgemeine Geld und Wirtschaftstheorie, *W. Kohhammer Verlag, Stuttgard, Germany, 1949.*

[142] Rugina, Anghel N., What is the Alternative for the West, *International Journal of Social Economics, Vol. 8, No. 2, 1981.*

[143] Rugina, Anghel N., Principia Methologica 1: A Bridge from Economics to all Other Natural Sciences / Towards a Methodological Unification of all Sciences, *MCB University Press Ltd., 1989.*

[144] Rugina, Anghel N., Prolegomena to any Future Study in Economics, Finance and Other Social Sciences: The Road to a Third Revolution in Economic, Financial, Social, Ethical, Logical and Political Thinking, *<International Journal of Social Economics>, Vol. 25, No. 5, 1998.*

[145] Russell, Bertrand, Introduction to Mathematical Philosophy, *Dover Publications, Inc., New York, 1993.*

[146] Ryle, Gilbert, The Concept of Mind, *Barnes and Noble, Inc., New York, 1950.*

[147] Sartre, Jean-Paul, Existentialism and Human Emotions, *Philosophical Library, Inc., New York, 1957.*

[148] Scedrov, Andre, *Linear Logic and Computation: A Survey*, http://www.cs.cmu.edu/~fp/mdorf93.pdf, 1999.

[149] Schiffel, Jeffrey; Petrescu, M. I., *Neutrosophic logic*, Sci.Math Archives Topics, 7-8 July 1999,

http://forum.swarthmore.edu/epigone/sci.math/ploutoxshol.

[150] Scruton, Roger, A Short History of Modern Philosophy / From Descartes to Wittgenstein, *Routledge, London, 1992.*

[151] Shafer, Glenn, A Mathematical Theory of Evidence, *Princeton University Press, NJ, 1976.*

[152] Shafer, Glenn, The Combination of Evidence, *International Journal of Intelligent Systems, Vol. I, 155-179, 1986.*

[153] SIGART, ACM Organization, *A generalization of the fuzzy logic, the neutrosophic logic*,

http://www.sigart.acm.org/Announcements/Other/Louise-99.Announcement.

[154] Smarandache, Florentin, *Á Unifying Field in Logics: Neutrosophic Logic. / Neutrosophic Probability, Neutrosophic Set*, Preliminary report, Western Section Meeting, Santa Barbara, CA, USA, Meeting # 951 of the American Mathematical Society, March 11-12, 2000,

http://www.ams.org/amsmtgs/2064_presenters.html and

http://www.ams.org/amsmtgs/2064_program_saturday.html.

[155] Smarandache, Florentin, Collected Papers, Vol. II, *University of Kishinev Press, Kishinev, 1997.*

[156] Smarandache, Florentin, Distihuri paradoxiste, *Dorul, Norresundby, 1998.*

[157] Smarandache, Florentin, Linguistic Paradoxists and Tautologies, *Libertas Mathematica, University of Texas at Arlington, Vol. XIX, 143-154, 1999.*

[158] Smarandache, Florentin, Neutrosophic Logic, A Generalization of the Fuzzy Logic,

http://www.gallup.unm.edu/~smarandache/NeutLog.txt.

[159] Smarandache, Florentin, *Neutrosophic Probability, A Generalization of the Classical Probability*,

http://www.gallup.unm.edu/~smarandache/NeutProb.txt.

[160] Smarandache, Florentin, Neutrosophic Set, A Generalization of the Fuzzy Set,

http://www.gallup.unm.edu/~smarandache/NeutSet.txt.

[161] Smarandache, Florentin, Neutrosophy, A New Branch of Phylosophy,

http://www.gallup.unm.edu/~smarandache/NeutroSo.txt.

[162] Smarandache, Florentin, Neutrosophy: Neutrosophic Probability, Set, and Logic, *American Research Press, Rehoboth, USA, 105p., 1998.*

[163] "The Florentin Smarandache Papers" Special Collection, Archives of American Mathematics, Center for American History, SRH 2.109, University of Texas at Austin, TX 78713, USA.

[164] "The Florentin Smarandache Papers" Special Collection, Arizona State University, Hayden Library, Tempe, AZ 85287, USA.

[165] Soare, Ion, Un Scriitor al Paradoxurilor: Florentin Smarandache, *Almarom, Rm. Vâlcea, 1994.*

[166] Sonnabend, Thomas, Mathematics for Elementary Teachers, *Second Edition, Saunders College Publishing, 1997.*

[167] Song, Feijun, E-mail to C. T. Le, August 1999.

[168] Stephens, J., Francis Bacon and the Style of Science, *University of Chicago Press, Chicago, 1975.*

[169] Stojmenovic, Ivan, editor, Many-Valued Logic, on-line journal, E-mails to C. T. Le, August 1999.

[170] Stoyanova, D., A variant of a cartesian product over intuitionistic fuzzy sets, *Second Sci. Session of the ''Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, 1990, Prepr. IM-MFAIS-1-90, 43-45.*

[171] Stoyanova, D., Algebraic structures of intuitionistic fuzzy sets, *Third Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, June 12, 1990, Preprint* IM-MFAIS-2-90, Part 1, 19-21.

[172] Stoyanova, D., Algebraic structures of fuzzy sets, *Third Sci. Session of the ''Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, June 12, 1990, Preprint IM-MFAIS-2-90, Part 1, 15-18.*

[173] Stoyanova, D., Sets from (α,β)—level generated by an intuitionistic fuzzy sets, *Ninetieth Session of the Nat. Seminar of Informatics of the Union of Bulg. Mathematicians and Fourth Scientific Session of the "Mathematical Foundation Artificial Intelligence" Seminar, Sofia, Nov. 5, 1990, Preprint IM-MFAIS-5-90, Sofia, 1990, 40 - 42.*

[174] Stoyanova, D., Atanassov K., Relations between operators, defined over intuitionistic fuzzy sets, *Second Sci. Session of the "Mathematical Foundation of Artificial Intelligence" Seminar, Sofia, March 30, 1990, Prepr. IM-MFAIS-1-90, 46-49.*

[175] Stoyanova, D., Compositions of intuitionistic fuzzy relations, *BUSEFAL Vol. 54, 1993, 21-23.*

[176] Stoyanova, D., More on Cartesian products over intuitionistic fuzzy sets, *BUSEFAL Vol. 54, 1993, 9-13.*

[177] Suber, Peter, *Glossary of First-Order Logic*, Philosophy Department, Earlham College, http://www.earlham.edu/~peters/courses/logsys/glossary.htm, 1999.

[178] Tabirca, Sabin, Some Recent Results Concerning the Smarandache Type Notions, *conference held at the Department of Mathematics, Manchester University, England, 15 May 2000.*

[179] TeSelle, E., Augustine the Theologian, *Herder & Herder, Inc., 1970.*

[180] Troelstra, Anne S., Lectures on Linear Logic, *CSLI Lecture Notes 29, Center for the Study of Language and Information, Stanford, California, 1992.*

[181] Vasiliu, Florin, Paradoxism's Main Roots, *Ed. Haiku, Bucharest, 1994.*

[182] Veatch, H. B., A Contemporany Appreciation, *Indiana University Press, Bloomington, 1974.*

[183] Vlastos, Gregory, The Philosophy of Socrates, *Anchor Books, Garden City, New York, 1971.*

[184] Tanaka, Koji, Philosophy Department, University of Queensland, Brisbane, Australia, E-mails to Minh Perez, USA, August 1999.

[185] Torretti, Roberto, *Nineteenth Century Geometry*, in Stanford Encyclopedia of Philosophy, edited by Edward N. Zalta, URL=http://plato.stanford.edu/entries/geometry-19th/#1, 1999.

[186] Tye, M., Sorites Paradoxes and the Semantics of Vagueness, *in Philosophical Perspectives: Logic and Language, edited by J. Tomberlin, Ridgeview, Atascadero, USA, 1994.*

[187] Van Fraassen, B. C., The Scientific Image, *Clarendon Press, 1980.*

[188] Vinueza, Adam, *Introduction to Logic / Philosophy 1440,*

University of Colorado at Boulder, Fall 1999,
http://stripe.colorado.edu/~vinueza/1440/validforms.html.

[189] Weisstein, Eric W., CRC Concise Encyclopedia of Mathematics, *CRC Press, Boca Raton, p. 1806, 1998.*

[190] Wittgenstein, L., Tractatus Logico-Philosophicus, *Humanitas Press, New York, 1961.*

[191] Zadeh, Lotfi A., Fuzzy Logic and Approximate Reasoning, *Synthese, 30, 407-428, 1975.*

[192] Zadeh, Lotfi A., Reviews of Books (A Mathematical Theory of Evidence. Glenn Shafer, Princeton University Press, Princeton, NJ, 1976), *The AI Magazine, 81-83, 1984.*

NEUTROSOPHIC PROBABILITY, SET, AND LOGIC

(first version)

Abstract.

This project is a part of a National Science Foundation interdisciplinary project proposal. Starting from a new viewpoint in philosophy, the neutrosophy, one extends the classical "probability theory", "fuzzy set" and "fuzzy logic" to <neutrosophic probability>, <neutrosophic set> and <neutrosophic logic> respectively.

They are useful în artificial intelligence, neural networks, evolutionary programming, neutrosophic dynamic systems, and quantum mechanics.

1) NEUTROSOPHY, A NEW BRANCH OF MATHEMATICAL PHILOSOPHY

A) **Etymology**:

Neutro-sophy [French *neutre* < Latin *neuter*, neutral, and *Greek sophia*, skill/wisdom] means knowledge of neutral thought.

B) **Definition**:

Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

C) **Characteristics**:

This mode of thinking:

- proposes new philosophical theses, principles, laws, methods, formulas, movements;
- interprets the uninterpretable;
- regards, from many different angles, old concepts, systems: showing that an idea, which is true in a given referential system, may be false in another one, and vice versa;
- measures the stability of unstable systems,

and instability of stable systems.

D) **Methods of Neutrosophic Study**:

mathematization (neutrosophic logic, neutrosophic probability and statistics, duality), generalization, complementarity, contradiction, paradox, tautology, analogy, reinterpretation, combination, interference, aphoristic, linguistic, multidisciplinarity.

E) **Formalization**:

Let's note by <A> an idea or theory or concept, by <Non-A> what is not <A>, and by <Anti-A> the opposite of <A>. Also, <Neut-A> means what is neither <A>, nor <Anti-A>, i.e. neutrality in between the two extremes. And <A'> a version of <A>.

<Non-A> is different from <Anti-A>.

For example:

If <A> = white, then <Anti-A> = black (antonym),

but <Non-A> = green, red, blue, yellow, black, etc. (any color, except white), while <Neut-A> = green, red, blue, yellow, etc. (any color, except white and black), and <A'> = dark white, etc. (any shade of white).

<Neut-A> $\equiv$ <Neut-(Anti-A)>, neutralities of <A> are identical with neutralities of <Anti-A>.

<Non-A> $\supset$ <Anti-A>, and <Non-A> $\supset$ <Neut-A> as well,

also

<A> $\cap$ <Anti-A> = $\varnothing$

<A> $\cap$ <Non-A> = $\varnothing$

<A>, <Neut-A>, and <Anti-A> are disjoint two by two.

<Non-A> is the completitude of <A> with respect to the universal set.

F) **Main Principle**:

Between an idea <A> and its opposite <Anti-A>, there is a continuum-power spectrum of neutralities <Neut-A>.

G) **Fundamental Thesis**:

Any idea <A> is t% true, i% indeterminate, and f% false, where t+i+f = 100.

H) **Main Laws**:

Let <α> be an attribute, and $(a, i, b) \in [0, 100]^3$, with a+i+b = 100. Then:

- There is a proposition <P> and a referential system <R>, such that <P> is a% <α>, i% indeterminate or <Neut-α>, and b% <Anti-α>.

- For any proposition <P>, there is a referential system <R>, such that <P> is a% <α>, i% indeterminate or <Neut-α>, and b% <Anti-α>.

- <α> is at some degree <Anti-α>, while <Anti-α> is at some degree <α>.

2) NEOTROSOPHIC PROBABILITY AND NEUTROSOPHIC STATISTICS

Let's first generalize the classical notions of "probability" and "statistics" for practical reasons.

A) Definitions:

Neutrosophic Probability studies the chance that a particular event E will occur, where that chance is represented by three coordinates (variables): t% true, i% indeterminate, and f% false, with $t+i+f = 100$ and $f, i, t \in [0, 100]$.

Neutrosophic Statistics is the analysis of such events.

B) Neutrosophic Probabi l ity Space:

The universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

C) Applications:

1) The probability that candidate C will win an election is say 25% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Dialectic and dualism don't work in this case anymore.

2) Another example, the probability that tomorrow it will rain is say 50% true according to meteorologists who have investigated the past years' weather, 30% false according to today's very sunny and droughty summer, and 20% undecided (indeterminate).

3) NEUTROSOPHIC SET

Let's second generalize, in the same way, the fuzzy set.

A) Definition:

Neutrosophic Set is a set such that an element belongs to the set with a neutrosophic probability, i.e. t% is true that the element is in the set, f% false, and i% indeterminate.

B) **Neutrosophsc Set Operations**:

Let M and N be two neutrosophic sets.

One can say, by language abuse, that any element neutrosophically belongs to any set, due to the percentage of truth/indeterminacy/falsity which varies between 0 and 100.

For example: $x(50,20,30) \in M$ (which means, with a probability of 50% x is in M, with a probability of 30% x is not in M, and the rest is undecidable) $y(0,0,100) \in M$ (which normally means y is not for sure in M), or $z(0,100,0) \in M$ (which means one doesn't know absolutely anything about z's affiliation with M).

Let $0 \leq t_1, t_2, t' \leq 1$ represent the truth-probabilities,
$0 \leq i_1, i_2, i' \leq 1$ the indeterminacy-probabilities, and
$0 \leq f_1, f_2, f' \leq 1$ the falsity-probabilities of an element x to be in the set M and in the set N respectively, and of an element y to be in the set N, where $t_1 + i_1 + f_1 = 1$, $t_2 + i_2 + f_2 = 1$, and $t' + i' + f' = 1$.

One notes, with respect to the given sets,

$x = x(t_1, i_1, f_1) \in M$ and $x = x(t_2, i_2, f_2) \in N$,

by mentioning x's neutrosophic probability appurtenance.
And, similarly, $y = y(t', i', f') \in N$.

Also, for any $0 \leq x \leq 1$ one notes $1-x = x'$.

Let $W(a,b,c) = (1-a) / (b+c)$ and $W(R) = W(R(t), R(i), R(f))$ for any tridimensional vector $R = (R(t), R(i), R(f))$.

Complement of M:

Let $N(x) = 1-x = x'$. Therefore:
if $x(t_1, i_1, f_1) \in M$,
then $x(N(t_1), N(i_1)W(N), N(f_1)W(N)) \in C(M)$.

Intersection:

Let $C(x,y) = xy$, and $C(z_1,z_2) = C(z)$ for any bidimensional vector

$z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$,
then $x(C(t), C(i)W(C), C(f)W(C)) \in M \cap N$.

Union:

Let $D1(x,y) = x+y-xy = x+\bar{x}y = y+x\bar{y}$, and $D1(z_1,z_2) = D1(z)$ for any bidimensional vector $z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$,
then $x(Dl(t), Dl(i)W(Dl), D1(f)W(D1)) \in M \cup N$.

Cartesian Product:

if $x(t_1, i_1, f_1) \in M$, $y(t', i', f') \in N$,
then $(x(t_1, i_1, f_1), y(t', 1', f')) \in M \times N$.

Difference:

Let $D(x,y) = x-xy = x\bar{y}$, and $D(z_1,z_2) = D(z)$ for any bidimensional vector $z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$, then $x(D(t), D(i)W(D), D(f)W(D)) \in M \setminus N$, because $M \setminus N = M \cap C(N)$.

C) <u>**Applications**</u>:

From a pool of refugees, waiting in a political refugee camp to get the America visa of emigration, a% are accepted, r% rejected, and p% in pending (not yet decided), a+r+p=100. The chance of someone in the pool to emigrate to USA is not a% as in classical probability, but a% true and p% pending (therefore normally bigger than a%) - because later, the p% pending refugees will be distributed into the first two categories, either accepted or rejected.

Another example, a cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (i.e. there are separated water drops, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud).

We are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That's why the percent of indeterminacy is required: for a more organic, smooth, and especially accurate estimation.

4) NEUTROSOPHIC LOGIC, A GENERALIZATION OF FUZZY LOGIC

A) **Introduction**:

One passes from the classical {0, 1} Bivalent Logic of George Boole, to the Three-Valued Logic of Reichenbach (leader of the logical empiricism), then to the $\{0, a_1, ..., a_n, 1\}$ Plurivalent one of £ukasiewicz (and Post's m-valued calculus), and finally to the [0, 1] Infinite Logic as in mathematical analysis and probability: a Transcendental Logic (with values of the power of continuum), or Fuzzy Logic.

Falsehood is infinite, and truthhood quite alike; in between, at different degrees, indeterminacy as well.

Everything is G% good, I% indeterminate, and B% bad, where $G + I + B = 100$.

Besides Diderot's dialectics on good and bad ("Rameau's Nephew", 1772), any act has its percentage of "good", "indeterminate", and of "bad" as well incorporated.

Rodolph Carnap said:

"Metaphysical propositions are neither true nor false, because they assert nothing, they contain neither knowledge nor error (...)". Hence, there are infinitely many statuses in between "Good" and "Bad", and generally speaking in between "A" and "Anti-A", like on the real number segment:

[0,	1]
False	True
Bad	Good
Non-sense	Sense
Anti-A	A

0 is the absolute falsity, 1 the absolute truth. In between each oppositing pair, normally in a vicinity of 0.5, are being set up the neutralities.

There exist as many states in between "True" and "False" as in between "Good" and "Bad". Irrational and transcendental standpoints belong to this interval.

Even if an act apparently looks to be only good, or only bad, the other haded side should be sought. The ratios

$$\frac{\text{Anti-A}}{\text{A}}, \qquad \frac{\text{Non-A}}{\text{A}}$$

vary indefinitely. They are transfinite.

If a statement is 30%T (true) and 6.0%I (indeterminate), then it is 10%F (false). This is somehow alethic, meaning pertaining to truthhood and falsehood in the same time.

In opposition to Fuzzy Logic, if a statement is 30%T doesn't involve it is 70%F. We have to study its indeterminacy as well.

B) **Definition of Neutrosophic Logic**:

This is a generalization (for the case of null indeterminacy) of the fuzzy logic.

Neutrosophic logic is useful in the real-world systems for designing control logic, and may work in quantum mechanics.

If a proposition P is t% true, doesn't necessarily mean it is 100-t% false as in fuzzy logic. There should also be a percent of indeterminacy on the values of P.

A better approach of the logical value of P is f% false, i% indeterminate, and t% true, where $t+i+f=100$ and $t, i, f \in [0, 100]$, called neutrosophic logical value of P, and noted by $n(P)=(t,i,f)$.

Neutrosophic Logic means the study of neutrosophic logical values of the propositions.

There exist, for each individual event, PRO parameters, CONTRA parameters, and NEUTER parameters which influence the above values. Indeterminacy results from any hazard which may occur, from unknown parameters, or from new arising conditions.

This resulted from practice.

C) **Aplications**:

1) The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

This proposition is, say, 25% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

2) Tomorrow it will rain.

This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, 30% false according to today's very sunny and droughty summer, and 20% undecided.

3) This is a heap.

As an application to the sorites paradoxes, we may now say this proposition is t% true, f% false, and i% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective).

We are not able to distinguish the difference between yellow and red as well if a continuum spectrum of colors is painted on a wall imperceptibly changing from one into another.

D) Definition of Neutrosophic Logical Connectors:

One uses the definitions of neutrosophic probability and neutrosophic set.

Let , $0 \leq t_1, t_2 \leq 1$ represent the thrut-probabilities,

$0 \leq i_1, i_2 \leq 1$ the indeterminacy-probabilities, and

$0 \leq f_1, f_2 \leq 1$ the falsity-probabilities of two events P_1 and P_2 respectively, where $t_1 + i_1 + f_1 = 1$ and $t_2 + i_2 + f_2 = 1$. One notes the neutrosophic logical values of P_1 and P_2 by

$n(P_1) = (t_1, i_1, f_1)$ and $n(P_2) = (t_2, i_2, f_2)$ ·

Also, for any $0 \leq x \leq 1$ one notes $1\text{-}x = x$.

Let $W(a,b,c) = (1\text{-}a) / (b+c)$ and $W(R) = W(R(t), R(i), R(f))$ for any tridimensional vector $R = (R(t), R(i), R(f))$.

Negation:

Let $N(x) = 1\text{-}x = x$. Then:

$n(\neg P_1) = (N(t_1), N(i_1)W(N), N(f_1)W(N))$.

Conjunction: Let $C(x,y) = xy$, and $C(z_1,z_2) = C(z)$ for any bidimensional vector $z = (z_1, z_2)$. Then:

$n(P_1 \wedge P_2) = (C(t), C(i)W(C), C(f)W(C))$.

(And, in a similar way, generalized for n propositions.)

Weak or inclusive disjunction:

Let $D1(x,y) = x+y\text{-}xy = x+\bar{x}y = y+x\bar{y}$, and $D1(z_1,z_2) = D1(z)$ for any bidimensional vector $z = (z_1,z_2)$. Then:

$n(P_1 \vee P_2) = (D1(t), D1(i)W(D1), D1(f)W(D1))$.
(And, in a similar way, generalized for n propositions.)

Strong or exclusive disjunction:
Let $D2(x,y) = x(1-y)+y(1-x)-xy(1-x)(1-y) = x\bar{y}+\bar{x}y-xy\bar{x}\bar{y}$, and $D2(z_1,z_2) = D2(z)$ for any bidimensional vector $z = (z_1, z_2)$. Then:
$n(P1 \underline{\vee} P2) = (D2(t), D2(i)W(D2), D2(f)W(D2))$.
(And, in a similar way, generalized for n propositions.)

Material conditional (implication):
Let $I(x,y) = 1-x+xy = \bar{x}+xy = 1-x\bar{y}$, and $I(z_1,z_2) = I(z)$ for any bidimensional vector $z = (z_1,z_2)$. Then:
$n(P_1 \rightarrow P_2) = (I(t), I(i)W(I), I(f)W(I))$.

Material biconditional (echivalence):
Let $E(x,y) = (1-x+xy)(1-y+xy) = (\bar{x}+xy)(\bar{y}+xy) = (1-x\bar{y})(1-\bar{x}y)$, and $E(z_1,z_2) = E(z)$ for any bidimensional vector $z = (z_1,z_2)$.
$n(P \leftrightarrow Q) = (E(t), E(i)W(E), E(f)W(E))$.

Sheffer's connector:
Let $S(x,y) = 1-xy$, and $S(z_1,z_2) = S(z)$ for any bidemensional vector $z = (z_1,z_2)$.
$n(P \mid Q) = n(\neg P \vee \neg Q) = (S(t), S(i)W(S), S(f)W(S))$.

Peirce's connector:
Let $P(x,y) = (1-x)(1-y) = \bar{x}\bar{y}$, and $P(z_1,z_2) = P(z)$ for any bidimensional vector $z = (z_1,z_2)$.
$n(P \downarrow Q) = n(\neg P \wedge \neg Q) = (P(t), P(i)W(P), P(f)W(P))$.

E) **Properties of Neutrosophic Logical Connectors**:

Let's note by t(P) the truth-component of the neutrosophic value n(P), and $t(P) = p$, $t(Q) = q$.

a) Conjunction:
$t(P \wedge Q) \min\leq \{p,q\}$.

$$\bigwedge_{k=1}^{\infty} t(P) = 0 \text{ if } t(P) \neq 1$$

b) Weak disjunction:
$t(P \vee Q) \geq \max\{p,q\}$.

$$\bigvee_{k=1}^{\infty} t(P) = 1 \text{ if } t(P) \neq 0$$

c) Implication:
$t(P \rightarrow P) = 1$ if $t(P) = 0$ or 1, and $\rangle$ p otherwise.

$$\lim_{t(P) \to 0} t(P \rightarrow Q) = 1$$

$$\lim_{t(Q) \to 1} t(P \rightarrow Q) = 1$$

$$\lim_{t(P) \to 1} t(P \rightarrow Q) = q$$

$$\lim_{t(Q) \to 0} t(P \rightarrow Q) = 1\text{-}p$$

d) Equivalence:
$t(P \leftrightarrow Q) = t(Q \leftrightarrow P) = t(\neg P \leftrightarrow \neg Q)$

$$\lim_{\substack{t(P) \to 0 \\ t(Q) \to 0}} t(P \leftrightarrow Q) = 1$$

$$\lim_{\substack{t(P) \to 1 \\ t(Q) \to 1}} t(P \leftrightarrow Q) = 1$$

$$\lim_{\substack{t(P)\to 0\\ t(Q)\to 1}} t(P \leftrightarrow Q) = 0$$

$$\lim_{\substack{t(P)\to 1\\ t(Q)\to 0}} t(P \leftrightarrow Q) = 0$$

$$\lim_{t(P)\to 0} t(P \leftrightarrow Q) = 1-q$$

$$\lim_{t(P)\to 1} t(P \leftrightarrow Q) = q$$

Let $q \neq 0, 1$ be constant, and one notes

$P_{max}(q) = (q^2-3q+1)/(2q^2-2q)$. Then:

$\max_{0\leq t(P)\leq 1} t(P \leftrightarrow Q)$ occurs when:

$P = P_{max}(q)$ if $p_{max}(q) \in [0, 1]$,
or $p = 0$ if $p_{max}(q) < 0$,
or $p = 1$ if $p_{max}(q) > 1$,

because the equivalence connector is described by a parabola of equation

$e_q(p) = (q^2-q)p^2 + (-q^2+3q-1)p + (1-q)$,

which is concave down.

5) NEOTROSOPHIC TOPOLOGY

A) <u>**Definition**</u>:

Let's construct a **Neutrosophic Topology** on NT = [0, 1], considering the associated family of subsets (0, p), for $0 \leq p \leq 1$, the whole set [0 , 1], and the empty set $\emptyset$ = (0 , 0), called open sets, which is closed under set union and finite intersection. The union is defined as $(0, p) \cup (0, q) = (0, d)$, where $d = p+q-pq$, and the intersection as $(0, p) \cap (0, q) = (0, c)$, where $c = pq$. The complementary of (0, p) is (0, n), where $n = 1-p$, which is a closed set.

B) **Neutrosophic Topological Space**:

The interval NT, endowed with this topology, forms a neutrosophic topological space.

C) **Isomorphicity**:

Neutrosophic Logical Space, Neutrosophic Topological Space, and Neutrosophic Probability Space are all isomorphic.

A method of Neutrosophy is the:

6) **TRANSDISCIPLINARITY**:

A) **Introduction**:

Transdisciplinarity means to find common features to uncommon entities: <A> $\cap$ <Non-A> $\neq \emptyset$, even if they are disjunct.

B) **Multi-Structure and Multi-Space**:

Let S_1 and S_2 be two distinct structures, induced by the group of laws L which verify the axiom groups A_1 and A_2 respectively, such that A_1 is strictly included in A_2.

One says that the set M, endowed with the properties:

a) M has an S_1-structure,

b) there is a proper subset P (different from the empty set, from the unitary element, and from M) of the initial set M which has an S_2-structure,

c) M doesn't have an S_2-structure,

is called an **S_1-structure with respect to the S_2-structure.**

Let $S_1, S_2, \ldots, S_k$ be distinct space-structures.

We define the **Multi-Space** (or **k-structured-space**) as a set M such that for each structure S_i, $1 \leq i \leq k$, there is a proper (different from $\emptyset$ and from M) subset M_i of it which has that structure. The $M_1, M_2, \ldots, M_k$ proper subsets are different two by two.

Let's introduce new terms:

C) **Psychomathematics**:

A discipline which studies psychological processes in connection with mathematics.

D) **Mathematical Modeling of Psychological Process**:

Weber's law and Fechner's law on sensations and stimuli are improved.

E) **Psychoneutrosophy**:

Psychology of neutral thought, action, behavior, sensation, perception, etc. This is a hybrid field deriving from theology, philosophy, economics, psychology, etc.

For example, to find the psychological causes and effects of individuals supporting neutral ideologies (neither capitalists, nor communists), politics (not in the left, not in the right), etc.

F) **Socioneutrosophy**:

Sociology of neutralities.

For example the sociological phenomena and reasons which determine a country or group of people or class to remain neuter in a military, political, ideological, cultural, artistic, scientific, economical, etc. international or internal war (dispute).

G) **Econoneutrosophy**:

Economics of non-profit organizations, groups, such as: churches, philanthropic associations, charities, emigrating foundations, artistic or scientific societies, etc.

How they function, how they survive, who benefits and who loses, why are they necessary, how they improve, how they interact with for-profit companies.

These terms are in the process of development.

References :

[1] McNeil, F. Martin, Thro, Ellen, "Fuzzy Logic / A Practical Approach", Foreword by Ronald R. Yager, Academic Press, Florida, 1994.

[2] Smarandache, Florentin, "Collected Papers", Vol. II, University of Kishinev, Kishinev, 1997.

[3] "The Florentin Smarandache papers", Special Collection, Center for American History,,Archives of American Mathematics, University of Texas at Austin, TX.

[4] "The Florentin Smarandache papers", Special Collection, Hayden Library, Arizona State University, Tempe, AZ.

["Proceedings of the Second Symposium", Editor Şerban C. Andronescu, Romanian Academy of Scientists, American Branch, City University of New York, American Institute for Writing Research, New York, 81-92, 1999.]

ON RUGINA'S SYSTEM OF THOUGHT

I) Introduction.

Coming across Rugina's System of Thought, in his published books and articles [3-6], I learned about the *connection* between classic and modern. It is not a contradiction, but a complementarity from the part of modern with respect to the classic; and always the new 'modern' will have something to bring to the old knowledge.

In a similar way we may talk on the complementarity between theory and practics, rather than their contradiction.

In economics, Rugina negated Marx's social justice for the mass and Keynes's involuntary unemployment. His methology in science tries to unite all scientific fields, preserving however independence in thinking and judgement.

Einstein worked in the last period of his life on the Unified Field Theory (a single general theory in physics), but didn't succeed. At the present, his supposition that the speed of light is a barrier in the universe is also being denied.

The economical systems are characterized by free market or centrally-planned and controlled economy. I think each system has a mixture of the previous, where a part of the market is free and another is centrally-planned and controlled.

Rugina's Universal Hypothesis of Duality:

The physical universe is composed of stable and unstable elements arranged in various proportions, may be completed with unknown elements, a strip border between stable and unstable, which are continously changing from the state of equilibrium to disequilibrium and vice-versa, and which therefore are giving the dynamics of the universe.

Unknown may be: anomalies, relativities, uncertainties, revolution risks, hidden parameters.

The internal parameters are involved in Rugina's Universal Law of Natural Parameter (NaPa):

Any system in order to reach and maintain a position of stable equilibrium must have a very strong natural parameter (center of weight).

Whereas the external parameters are involved in Rugina's Universal Law of General Consistency:

Any system produces and maintains a position of stable equilibrium if there is a suitable space-time frame work.

Leon Walras's Economics of Stable Equilibrium and Keynes's Economics of Disequilibrium are combined in Rugina's Orientation Table in systems which are s% stable and 100-s% unstable, where s may be 100, 95, 65, 50, 35, 5, and 0.

The Classical Logic and Modern Logic are united in Rugina's Integrated Logic, and then generalized in the Neutrosophic Logic.

II) Theory of Paradoxes

How did I get to the Theory of Paradoxes?

I have observed that: what's good for someones, may be bad for others - and reciprocally. There are peoples who are considered therorists by their enemies, and patriots by their friends. All of them are right and wrong in the same time. If one changes the referential system, the result is different.

Nice paradoxes can be seen in [1], the first chapter.

III) ON RUGINA'S ORIENTATION TABLE

Starting from a new viewpoint in philosophy, the neutrosophy, one extends the classical 'probability theory', 'fuzzy set' and 'fuzzy logic' to <neutrosophic probability>, <neutrosophic set> and <neutrosophic logic> respectively.

They are useful in artificial intelligence, neural networks, evolutionary programming, neutrosophic dynamic systems, quantum theory, and decision making in economics.

With the neutrosophic logic help one explores Rugina's Orientation Table, a remarkable tool of study, at the micro- and macro-level, of problems in all sciences.

III. RUGINA'S ORIENTATION TABLE

In order to clarify the anomalies in science, Rugina (1989, 1998) proposes an original method, starting first from an economic point of view but generalizing it to any science, to study the equilibrium and disequilibrium of systems. His Table comprises seven basic models:

Model M_1 (which is 100% stable),
Model M_2 (which is 95% stable, and 5% unstable),
Model M_3 (which is 65% stable, and 35% unstable),
Model M_4 (which is 50% stable, and 50% unstable),
Model M_5 (which is 35% stable, and 65% unstable),
Model M_6 (which is 100% unstable).

He gives Orientation Tables for Physical Sciences and Mechanics (Rugina, 1989, p. 18), for the Theory of Probability, for Logic, and generally for any Natural or Social Science (Rugina, 1989, pp. 286-288).

"An anomaly can be simply defined as a deviation from a position of stable equilibrium represented by Model M_1" (Rugina, 1989, p. 17).

Rugina proposes the Universal Hypothesis of Dualit :

"The physical universe in which we are living, including human society and the world of ideas, all are composed in different and changeable proportions of stable (equilibrium) and unstable (disequilibrium) elements, forces, institutions, behavior and value"

and the General Possibility Theorem:

"there is an unlimited number of possible combinations or systems in logic and other sciences".

According to the last assertations one can extend Rugina's Orientation Table in the way that any system in each science is s% stable and u% unstable, with s+u=100 and both parameters $0 \leq s, u \leq 100$, somehow getting to a fuzzy approach.

But, because each system has hidden features and behaviors, and there would always be unexpected occuring conditions we are not able to control - we mean the indeterminacy plays a role as well, a better approach would be the *Neutrosophic Model*:

Any system in each science is s% stable, i% indeterminate, and u% unstable, with s+i+u=100 and all three parameters $0 \leq s,i,u \leq 100$.

EXAMPLE OF MODEL M3 IN RUGINA'S ORIENTATION TABLE:

The Paradoxis Geometry (actually the percentage of instability is between 20-35): in [7], the paper "Paradoxist Mathematics".

FIRST EXAMPLE OF MODEL M7 IN RUGINA'S ORIENTATION TABLE:

The Non-Geometry (the percentage of instability is 100): in [7], the paper "Paradoxist Mathematics".

SECOND EXAMPLE OF MODEL M7 IN RUGINA'S ORIENTATION TABLE:

The Counter-Projective Geometry (the percentage of instability is 100): in [7], the paper "Paradoxist Mathematics".

THIRD EXAMPLE OF MODEL M7 IN RUGINA'S ORIENTATION TABLE:

The Anti-Geometry (the percentage of instability is 100 - even... more, this is the geometry of total chaos!): in [7], the paper Paradoxist Mathematics.

FOURTH EXAMPLE OF MODEL M7 IN RUGINA'S ORIENTATION TABLE:

The Inconsistent System of Axioms, and The Contradictory Theory (the percentage of instability is 100 - even... more, this is the system of chaos!): in [7], the paper "Paradoxist Mathematics".

References:

[1] Andronescu, Şerban C., "Analogii estetice / Teoria spaţiului-timp în arte şi literatură", Editura Fundaţiei <România de Mâine>, Bucharest, 1998.

[2] McNeil, F. Martin, Thro, Ellen, "Fuzzy Logic / A Practical Approach", Foreword by Ronald R. Yager, Academic Press, Florida 1994.

[3) Rugina, Anghel N., "A Long Journey to the Third Revolution in Economic Thinking", <International Journal of New Ideas>, Vol. III, No. 1/2, 1994.

[4] Rugina, Anghel N., "Principia Methologica 1: A Bridge from

Economics to all other Natural Sciences / Towards a Methodological Unification of all Sciences", MCB University Press Ltd., 1989.

[5] Rugina, Anghel N., "The Problem of Values and Value Judgements in Science and a Positive Solution", <International Journal of Sociology and Social Policy>, Vol. 4, No. 3, 1984.

[6] Rugina, Anghel N., "Prolegomena to any Future Study in Economics, Finance and Other Social Sciences: The Road to a Third Revolution in Economic, Financial, Social, Ethical, Logical and Political Thinking", <International Journal of Social Economics>, Vol. 25, No. 5, 1998.

[7) Smarandache, Florentin, "Collected Papers", Vol. II , University of Kishinev, Kishinev, 1997.

[8] Smarandache, Florentin, "Neutrosophy: Neutrosophic Probability, Set, and Logic", American Research Press, Rehoboth 1998.

[9] "The Florentin Smarandache papers", Special Collection , Center for American History, Archives of American Mathematics, University of Texas at Austin, TX.

[10] "The Florentin Smarandache papers", Special Collection , Hayden Library, Arizona State University, Tempe, AZ.

["Octogon", Braşov, Vol. 7, No. 2, 89-93, 1999.]

FLORENTIN SMARANDACHE

SPECIAL ALGEBRAIC STRUCTURES

Abstract.

New notions are introduced in algebra in order to better study the congruences in number theory. For example, the <special semigroups> make an important such contribution.

Introduction.

By <proper subset> of a set A we consider a set P included in A, and different from A, different from the empty set, and from the unit element in A - if any.

We rank the algebraic structures using an order relationship:
we say that the algebraic structures S1 « S2 if:

- both are defined on the same set;
- all S1 laws are also S2 laws;
- all axioms of an S1 law are accomplished by the corresponding S2 law;
- S2 laws accomplish strictly more axioms than S1 laws, or S2 has more laws than S1.

For example: semigroup « monoid « group « ring « field,
or semigroup « commutative semigroup, ring « unitary ring, etc.

We define a GENERAL SPECIAL STRUCTURE to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is an SN structure, where SM « SN.

1) The SPECIAL SEMIGROUP is defined to be a semigroup A, different from a group, such that a proper subset of A is a group (with respect to the same induced operation).

For example, if we consider the commutative multiplicative group SG = $\{18^2, 18^3, 18^4, 18^5\}$ (mod 60) we get the table:

x	24	12	36	48
24	36	48	24	12
12	48	24	12	36
36	24	12	36	48
48	12	36	48	24

Unitary element is 36.

Using the algorithm [Smarandache 1972] we get that 18^2 is congruent to 18^2 (mod 60).

Now we consider the commutative multiplicative semigroup SS = {18^1, 18^2, 18^3, 18^4, 18^5} (mod 60) and we get the table:

x	18	24	12	36	48
18	24	12	36	48	24
24	12	36	48	24	12
12	36	48	24	12	36
36	48	24	12	36	48
48	24	12	36	48	24

Because SS contains a proper subset SG, which is a group, then SS is a Special Semigroup. This is generated by the element 18. The powers of 18 form a cyclic sequence: 18, 24, 12, 36, 48, 24, 12, 36, 48, ...

Similarly are defined:

2) The SPECIAL MONOID is defined to be a monoid A, different from a group, such that a proper subset of A is a group (with respect with the same induced operation).

3) The SPECIAL RING is defined to be a ring A, different from a field, such that a proper subset of A is a field (with respect with the same induced operations).

We consider the commutative additive group M={0,18^2,18^3,18^4,18^5} (mod 60) [using the module 60 residuals of the previous powers of 18], M={0,12,24,36,48}, unitary additive unit is 0.

(M,+,x) is a field.

While (SR,+,x)={0,6,12,18,24,30,36,42,48,54} (mod 60) is a ring whose proper subset {0,12,24,36,48} (mod 60) is a field.

Therefore (SR,+,x) (mod 60) is a Special Ring.

This feels very nice.

4) The SPECIAL SUBRING is defined to be a Special Ring B which is a proper subset of a Special Ring A (with respect to the same induced

operations).

5) The SPECIAL IDEAL is defined to be an ideal A, different from a field, such that a proper subset of A is a field (with respect to the same induced operations).

6) The SPECIAL LATTICE is defined to be a lattice A, different from a Boolean algebra, such that a proper subset of A is a Boolean algebra (with respect to the same induced operations).

7) The SPECIAL FIELD is defined to be a field (A,+,x), different from a K-algebra, such that a proper subset of A is a K-algebra (with respect to the same induced operations, and an external operation).

8) The SPECIAL R-MODULE is defined to be an R-MODULE (A,+,x), different from an S-algebra, such that a proper subset of A is an S-algebra (with respect to the same induced operations, and another "x" operation internal on A), where R is a commutative unitary ring and S is its proper subset field.

9) The SPECIAL K-VECTORIAL SPACE is defined to be a K-vectorial space (A,+,.), different from a K-algebra, such that a proper subset of A is a K-algebra (with respect to the same induced operations, and another "x" internal operation on A), where K is a commutative field.

1973

References:

[1] Smarandache, Florentin, "A generalization of Euler's Totient Function", manuscript, 1972 .

In this mean time the following papers, inspired by this subject/ paper, have been published or presented to international conferences;

[2] Castillo, J., "The Smarandache Semigroup", <International Conference on Combinatorial Methods in Mathematics>, II Meeting of the project 'Algebra, Geometria e Combinatoria', Faculdade de Ciencias da Universidade do Porto, Portugal, 9-11 July 1998.

[3] Padilla, Raul, "Smarandache Algebraic Structures", <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 17E, No. 1, 119-121, 1998.

[4] Padilla, Raul, "Smarandache Algebraic Structures", <Smarandache Notions Journal>, USA, Vol. 9, No. 1-2, 36-38, Summer 1998; http://www.gallup.unm.edu/-smarandache/alg-s-tx.txt;

Presented to the <International Conference On Semigroups>, Universidade do Minho, Braga, Portugal, 18-23 June, 1999.

About the characteristic function of the set [1]

In our paper we give a method, based on characteristic function of the set, of resolving some difficult problem of set theory found in high school study.

Definition:Let be $A \subset E \neq \theta$ (a universal set), then the $f_A : E \to \{0, 1\}$, where the function $f_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A, \end{cases}$

is named the characteristic function of the set A.

Theorem 1. Let $A, B \subset E$. In this case $f_A = f_B$ if and only if $A=B$.

Proof.

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A = B \\ 0, & \text{if } x \notin A = B \end{cases} = f_B(x)$$

Reciprocally: In case of any $x \in A$, $f_A(x) = 1$, but $f_A = f_B$ and for that $f_B(x) = 1$, namely $x \in B$ from where $A \subset B$. The same way we prove that $B \subset A$, namely $A = B$.

Theorem 2. $f_{\tilde{A}} = 1 - f_A$, where $\tilde{A} = C_E A$.

Proof.

$$f_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x \in \tilde{A} \\ 0, & \text{if } x \notin \tilde{A} \end{cases} = \begin{cases} 1, & \text{if } x \notin A \\ 0, & \text{if } x \in A \end{cases}$$

$$= \begin{cases} 1 - 0, & \text{if } x \notin A \\ 1 - 1, & \text{if } x \in A \end{cases} = 1 - \begin{cases} 0, & \text{if } x \notin A \\ & \text{if, } x \notin A \end{cases} = 1 - f_A(x).$$

Theorem 3. $f_{A \cap B} = f_A * f_B$

Proof.

$$f_{A \cap B}(x) = \begin{cases} 1, & \text{if } x \in A \cap B \\ 0, & \text{if } x \notin A \cap B \end{cases} = \begin{cases} 1, & \text{if } x \in A \text{ and } x \in B \\ 0, & \text{if } x \notin A \text{ or } x \notin B \end{cases}$$

$$= \begin{cases} 1, & \text{if } x \in A, x \in B \\ 0, & \text{if } x \in A, x \notin B \\ 0, & \text{if } x \notin A, x \in B \\ 0, & \text{if } x \notin A, x \notin B \end{cases} = \left(\begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \right) \cdot \left(\begin{cases} 1 & \text{if } x \in B \\ 0 & \text{if } x \notin B \end{cases} \right)$$

$= f_A(x) f_B(x)$

The theorem can be generalized by induction:

Theorem 4. $$f_{\bigcap_{k=1}^{n} A_k} = \prod_{k=1}^{n} f_{A_k}$$

[1] Together with Mihály Bencze

Consequence. For any $n \in N^*$ $f_M^n = f_M$

Proof. In the previous theorem we write $A_1 = A_2 = ... = A_n = M$.

Theorem 5.

$f_{A \cup B} = f_A + f_B - f_A f_B$.

Proof. $f_{A \cup B} = f_{\overline{\overline{A \cup B}}} = f_{\overline{\bar{A} \cap \bar{B}}} = 1 - f_{\bar{A} \cap \bar{B}} = 1 - f_{\bar{A}} f_{\bar{B}} =$
$= 1 - (1 - f_A)(1 - f_B) = f_A + f_B - f_A f_B$.

Can be generalized by induction:

Theorem 6. $f_{\cup_{k=1}^n A_k} = \sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < ... < i_k \le n} (-1)^{k-1} f_{A_{i_1}} f_{A_{i_2}} ... f_{A_{i_k}}$

Theorem 7. $f_{A-B} = f_A(1 - f_B)$

Proof. $f_{A-B} = f_{A \cap \bar{B}} = f_A f_{\bar{B}} = f_A(1 - f_B)$.

Can be generalized by induction :

Theorem 8. $f_{A_1 - A_2 - ... - A_n} = \sum_{k=1}^{n} (-1)^{k-1} f_{A_{i_1}} f_{A_{i_2}} ... f_{A_{i_k}}$.

Theorem 9. $f_{A \Delta B} = f_A + f_B - 2 f_A f_B$

Proof. $f_{A \Delta B} = f_{A \cup B - A \cap B} = f_{A \cup B}(1 - f_{A \cap B}) =$
$= (f_A + f_B - f_A f_B)(1 - f_A f_B) = f_A + f_B - 2 f_A f_B$.

Can be generalized by induction:

Theorem 10.

$F_{\Delta_{k=1}^n A_k} = \sum_{k=1}^{n} (-2)^{k-1} \sum_{1 \le i_1 < ... < i_k \le n} f_{A_{i_1} A_{i_2} ... A_{i_k}}$.

Theorem 11. $f_{A \times B}(x, y) = f_A(x) f_B(y)$

Proof. If $(x,y) \in AXB$, then $f_{AXB}(x,y) = 1$ and $x \in A$, namely $f_A(x) = 1$ and $y \in B$, namely $f_B(y) = 1$, so $f_A(x) f_B(y) = 1$. If $(x,y) \notin AXB$, then $f_{AXB}(x,y) = 0$ and $x \notin A$, namely $f_A(x) = 0$ or $y \notin B$, namely $f_y(B) = 0$ so $f_A(x) f_B(y) = 0$. Can be generalized by induction.

Theorem 12.

$$f_{\times_{k=1}^n A_k}(x_1, x_2, ..., x_n) = \prod_{k=1}^{n} f_{A_k}(x_k).$$

Theorem 13. (De Morgan) $\overline{\bigcup_{k=1}^{n} A_k} = \bigcap_{k=1}^{n} \bar{A}_k$.

Proof. $f_{\overline{\bigcup_{k=1}^n A_k}} = 1 - f_{\bigcup_{k=1}^n A_k} =$

$1 - \sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < ... < i_k \le n} f_{A_{i_1}} f_{A_{i_2}} ... f_{A_{i_k}} =$

$\prod_{k=1}^{n} (1 - f_{A_k}) = \prod_{k=1}^{n} f_{\bar{A}_k} = f_{\cap_{k=1}^n \bar{A}_k}$.

We prove in the same way the following theorem:

Theorem 14. (De Morgan) $\overline{\bigcap_{k=1}^{n} A_k} = \bigcup_{k=1}^{n} \bar{A}_k$.

Theorem 15.

$$\left(\bigcup_{k=1}^{n} A_k\right) \cap M = \bigcup_{k=1}^{n} \left(A_k \cap M\right) .$$

Proof. $f_{\left(\bigcup_{k=1}^{n} A_k\right) \cap M} = f_{\bigcup_{k=1}^{n} A_k} \; f_M =$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1}} f_{A_{i_2}} \ldots f_{A_{i_k}} f_M =$$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1}} f_{A_{i_2}} \ldots f_{A_{i_k}} f_M^k =$$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1} \cap M} f_{A_{i_2} \cap M} \ldots f_{A_{i_k} \cap M} = f_{\bigcup_{k=1}^{n} (A_k \cap M)} .$$

In the same way we prove that:

Theorem 16. $\left(\bigcap_{k=1}^{n} A_k\right) \cup M = \bigcap_{k=1}^{n} \left(A_k \cup M\right)$.

Theorem 17. $\left(\Delta_{k=1}^{n} A_k\right) \cap M = \Delta_{k=1}^{n} \left(A_k \cap M\right)$.

Application.

$\left(\Delta_{k=1}^{n} A_k\right) \cup M = \Delta_{k=1}^{n} \left(A_k \cup M\right)$ if and only if $M = \phi$.

Theorem 18.

$$MX\left(\bigcup_{k=1}^{n} A_k\right) = \bigcup_{k=1}^{n} \left(MXA_k\right) .$$

Proof. $f_{MX\left(\bigcup_{k=1}^{n} A_k\right)}(x,y) = f_M(y) \, f_{\bigcup_{k=1}^{n} A_k}(x) =$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1}}(x) \, f_{A_{i_2}}(x) \ldots f_{A_{i_k}}(x) \, f_M(y) =$$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1}}(x) \, f_{A_{i_2}}(x) \ldots f_{A_{i_k}}(x) \, f^k_M(y) =$$

$$\sum_{k=1}^{n} (-1)^{k-1} \sum_{1 \le i_1 < \ldots < i_k \le n} f_{A_{i_1} XM}(x,y) \ldots f_{A_{i_k} XM}(x,y) = f_{\bigcup_{k=1}^{n} (MXA_k)}$$

In the same way we prove that:

Theorem 19. $MX\left(\bigcap_{k=1}^{n} A_k\right) = \bigcap_{k=1}^{n}\left(MXA_k\right)$.

Theorem 20.

$MX(A_1 - A_2 - \dots - A_n) = (MXA_1) - (MXA_2) - \dots - (MXA_n)$.

Theorem 21. $(A_1-A_2) \cup (A_2-A_3) \cup \dots \cup (A_{n-1}-A_n) \cup (A_n-A_1) = \bigcup_{k=1}^{n} A_k - \bigcap_{k=1}^{n} A_k$.

Proof 1. $f_{(A_1-A_2)\cup\dots\cup(A_n-A_1)} =$

$$\sum_{k=1}^{n}(-1)^{k-1} \sum_{1\le i_1<\dots<i_k\le n} f_{A_{i_1}-A_{i_2}} \dots f_{A_{i_k}-A_{i_1}} =$$

$$\sum_{k=1}^{n}(-1)^{k-1} \sum_{1\le i_1<\dots<i_k\le n} (f_{A_{i_1}} - f_{A_{i_2}} f_{A_{i_1}} f_{A_{i_2}}) \dots (f_{A_{i_k}} f_{A_{i_1}} f_{A_{i_k}} f_{A_{i_1}}) =$$

$$\sum_{k=1}^{n}(-1)^{k-1} \sum_{1\le i_1<\dots<i_k\le n} f_{A_{i_1}} \dots f_{A_{i_k}}\left(1 - \prod_{p=1}^{n} f_{A_p}\right) =$$

$$f_{\bigcup_{k=1}^{n} A_k}\left(1 - f_{\bigcap_{k=1}^{n} A_k}\right) = f_{\bigcup_{k=1}^{n} A_k - \bigcap_{k=1}^{n} A_k}.$$

Proof 2. Let $x \in \bigcup_{i=1}^{n}(A_i - A_{i+1})$, (where $A_{n+1} = A_1$), then there exists k such that $x \in (A_k - A_{k+1})$, namely $x \notin (A_k \cap A_{k+1}) \subset A_1 \cap A_2 \cap \dots \cap A_n$, namely $x \notin A_1 \cap \dots \cap A_n$ and

$x \in \bigcup_{k=1}^{n} A_k - \bigcap_{k=1}^{n} A_k$.

Now we prove the inverse statement:

Let $x \in \bigcup_{k=1}^{n} A_k - \bigcap_{k=1}^{n} A_k$, we show that there exists k such that $x \in A_k$ and $x \notin A_{k+1}$. On the contrary it would result that for any $k \in \{1,2,\dots,n)$, $x \in A_k$ and $x \in A_{k+1}$ namely $x \in \bigcup_{k=1}^{n} A_k$, it results

that there exists p such that $x \in A_p$, but from the previous reasoning it result that $x \in A_{p+1}$, and using this we consequently obtain that $x \in A_k$ for $k=\overline{p,n}$. But from $x \in A_n$ we get that $x \in A_1$ using consequently, it results that $x \in A_k$, $k = \overline{1,p}$, from where $x \in A_k$, $k = \overline{1,n}$, namely $x \in A_1 \cap \dots \cap A_n$, that is a contradiction. Thus there exists r such that $x \in A_r$

and $x \notin A_{r-1}$, namely $x \in (A_r - A_{r-1})$ and so $x \in \bigcup_{k=1}^{n} (A_k - A_{k-1})$.

In the same way we prove the following theorem:

Theorem 22. $(A_1 \Delta A_2) \cup (A_2 \Delta A_3) \cup \ldots \cup (A_{n-1} \Delta A_n) = \bigcup_{k=1}^{n} A_k - \bigcap_{k=1}^{n} A_k$.

Theorem 23. $(A_1 X A_2 X \ldots X A_k) \cap (A_{k+1} X A_{k+2} X \ldots X A_{2k}) \cap (A_n X A_1 X \ldots X A_{k-1}) = (A_1 \cap A_2 \cap \ldots \cap A_n)^k$.

Proof. $f_{(A_1 x \ldots x A_k) \cap \ldots \cap (A_n x A_1 x \ldots x A_{k-1})}(x_1, \ldots, x_n) =$
$f_{A_1 x \ldots x A_k}(x_1, \ldots, x_n) \ldots f_{A_n x \ldots x A_{k-1}}(x_1, \ldots, x_n) =$
$(f_{A_1}(x_1) \ldots f_{A_k}(x_k)) \ldots (f_{A_n}(x_n) \ldots f_{A_{k-1}}(x_{k-1}) =$
$f^k_{A_1}(x_1) \ldots f^k_{A_n}(x_n) = f^k_{A_1 \cap \ldots \cap A_n}(x_1, \ldots, x_n) =$
$f_{(A_1 \cap \ldots \cap A_n)^k}(x_1, \ldots, x_n)$.

Theorem 24. $(P(E), \cup)$ is a commutative monoid.

Proof. For any A, B $\in$ P(E); $A \cup B \in P(E)$, namely the intern operation. Because $(A \cup B) \cup C = A \cup (B \cup C)$ is associative, $A \cup B = B \cup A$ commutative, and because $A \cup \phi = A$ then ϕ is the neutral element.

Theorem 25. $(P(E), \cap)$ is a commutative monoid.

Proof. For any A, B $\in$ P(E); $A \cap B \in P(E)$ namely intern operation. $(A \cap B) \cap C = A \cap (B \cap C)$ associative, $A \cap B = B \cap A$, commutative $A \cap E = A$, E is the neutral element.

Theorem 26. $(P(E), \Delta)$ is an abellan group.

Proof. For any A, B $\in$ P(E); $A \Delta B \in P(E)$, namely the intern operation. $A \Delta B = B \Delta A$ commutative. The proof of associativity is in the XII class manual as a problem. We prove it, using the characteristic function of the set.

$f_{(A \Delta B) \Delta C} = 4 f_A f_B f_C - 2 f_A f_B + f_B f_C + f_C f_A + f_A + f_B + f_C = f_{A \Delta (B \Delta C)}$

Because $A \Delta \phi = A$, ϕ is the neutral element and because $A \Delta A = \phi$; A is the symmetric element itself.

Theorem 27. $(P(E), \Delta, \cap)$ is a commutative Boole ring with divisor of zero.

Proof. Because of the previous theorem it satisfies the commutative ring axioms. Now we prove that it has a divisor of zero. If $A \neq \phi$ and $B \neq \phi$ are two disjoint sets, then $A \cap B = \phi$, thus it has divisor of zero. From Theorem 17 we get that it is distributive for n = 2. Because for any A $\in$ P(E); $A \cap A = A$ and $A \Delta A = \phi$ it also satisfies the Boole-type axioms.

Theorem 28. Let be $H = \{ f \mid f : E \to \{0, 1\} \}$, then $(H, \oplus)$ is an Abelian group, where $f_A \oplus f_B = f_A + f_B - 2 f_A f_B$ and $(P(E), \Delta) \cong (H, \oplus)$.

Proof. Let $F : P(E) \to H$, where $F(A) = f_A$, then from the previous theorem we get that it is bijective and because

$F(A\Delta B) = f_{A\Delta B} = F(A) \oplus F(B)$ it is compatible.

Theorem 29. $card(A_1\Delta A_n) \leq card(A_1\Delta A_2)+$

$+card(A_2\Delta A_3)+ \cdots + card(A_{n-1}\Delta A_n)$

Proof. By induction. If $n = 2$, then it is true, we show that for $n = 3$ it is also true. Because $(A_1 \cap A_2) \cup (A_2 \cap A_3) \subseteq A_2 \cup (A_1 \cap A_3)$;

$card((A_1 \text{ cap} A_2) \cup (A_2 \cap A_3)) \leq card(A_2 \cup (A_1 \cap A_3))$ but

$card(M \cup N) = cardM + cardN - card(M \cap N)$ and thus

$cardA_2 + card(A_1 \cap A_3) - card(A_1 \cap A_2) - card(A_2 \cap A_3) \geq 0$ can be writen as $cardA_1 + cardA_3 - 2card(A_1 \cap A_3) \leq$

$(cardA_1+cardA_2-2card(A_1 \cap A_2))+(cardA_2+cardA_3-2card(A_2 \cap A_3))$. But because of $(M\Delta N) = cardM + cardN - 2card(M \cap N)$ then $card(A_1 \Delta A_3) \leq card(A_1\Delta A_2) + card(A_2\Delta A_3)$. The proof of this step of the induction relies on the above method.

Theorem 30. $(P^2(E), card(A\Delta B))$ is a metric space.

Proof. Let $d(A,B) = card(A\Delta B) : P(E)xP(E) \to R$.

1. $d(A, B) = 0 \Leftrightarrow card(A\Delta B) = 0 \Leftrightarrow card((A - B) \cup (B - A)) = 0$ but because $(A - B) \cap (B - A) = \phi$ we get $(A - B) + card(B - A) = 0$ and because $(A - B) = 0$ and $card(B - A) = O$, then $A - B = \phi$, $B - A = \phi$ and $A = B$.

2. $d(A, B) = d(B,A)$ results from $A\Delta B = B\Delta A$.

3. In consequence of the previous theorem

$d(A, C) \leq d(A, B) + d(B, C)$.

As result of the above three properties it is a metric space.

PROBLEMS

Problem 1.

Let $A = B \cup C$ and $f : P(A) \to P(A)XP(A)$, where

$f(x) = (X \cup B, X \cup C)$. Prove that f is injective if and only if $B\cap C=\phi$.

Solution 1. If f is injective. Then

$f(\phi) = (\phi\cup B, \phi\cup C) = (B,C) = ((B\cap C)\cup B, (B\cap C)\cup C) = f(B \cap C)$ from where $B \cap C = \phi$. Now reciprocally: Let $B \cap C = \phi$, then $f(x) = f(Y)$, it result, that $X \cup B = Y \cup B$ and $X \cup C = Y \cup C$ or $X = X\cup\phi = X\cup(B\cap C) = (X\cup B)\cap(X\cup C)=(Y\cup B)\cap(Y\cup C) = Y \cup (B \cap C) = Y \cup \phi = Y$ namely it is injective.

Solution 2. Let $B \cap C = \phi$ passing over the set function $f(x) = f(Y)$ if and only if $X \cup B = Y \cup B$ and $X \cup C = Y \cup C$, namely $f_{X\cup B} = f_{Y\cup B}$ and

$f_{X\cup C} = f_{Y\cup C}$ or $f_X + f_B - f_X f_B = f_Y + f_B - f_Y f_B$ and

$f_X + f_C - f_X f_C = f_Y + f_C - f_Y f_C$ from where

$(f_X - f_Y)(f_B - f_C) = 0$. Because $A = B \cup C$ and $B \cap C = \phi$ therefore

$$(f_B - f_C)(u) = \begin{cases} 1, & \text{if } u \in B \\ -1, & \text{if } u \in C \end{cases} \neq 0$$

therefore $f_X - f_Y = 0$, namely $X = Y$ and thus it is injective.

Generalization. Let $M = \bigcup_{k=1}^{n} A_k$ and $f : P(A) \to P^n(A)$, where

$f(X) = (X \cup A_1, X \cup A_2, \ldots, X \cup A_n)$. Prove that f is injective if and only if $A_1 \cap A_2 \cap \ldots \cap A_n = \phi$.

Problem 2. Let $E \neq \phi$ and $A \in P(E)$ and

$f : P(E) \to P(E)xP(E)$, where $f(X) = (X \cap A, X \cup A)$.

a. Prove that f is injective

b. Prove that $\{f(x), x \in P(E)\} = \{(M,N) \mid M \subset A \subset N \subset E\} = K$.

c. Let $g : P(E) \to K$, where $g(X) = f(X)$. Prove that g is bijective and compute its inverse.

Solution.

a. $f(X) = f(Y)$, namely $(X \cap A, X \cup A) = (Y \cap A, Y \cup A)$ and so

$X \cap A = Y \cap A$, $X \cup A = Y \cup A$, from where $X \Delta A = Y \Delta A$ or

$(X\Delta A)\Delta A = (Y\Delta A)\Delta A$, $X\Delta(A\Delta A) = Y\Delta(A\Delta A)$, $X\Delta\phi = Y\Delta\phi$ and thus $X = Y$, namely f is injective.

b. $\{f(X), X \in P(E)\} = f(P(E))$. We show that $f(P(E)) \subset K$. For any $(M,N) \in f(P(E))$, $\exists X \in P(E) : f(X) = (M,N)$;

$(X \cap A, X \cup A) = (M, N)$. From here $X \cap A = M$, $X \cup A = N$, namely $M \subset A$ and $A \subset N$ thus $M \subset A \subset N$ and so $(M, N) \in X$. Now we show that $K \subset f(P(E))$, for any $(M, N) \in K$, $\exists X \in P(E)$ so that $f(X) = (M, N)$.$f(X) = (M, N)$, namely $(X \cap A, X \cup A) = (M, N)$ from where $X \cap A = M$ and $X \cup A = N$, namely

$X\Delta A = N - M$, $(X\Delta A)\Delta A = (N - M)\Delta A$, $X\Delta\phi = (N - M)\Delta A$,

$X = (N-M)\Delta A$, $X = (N\cap\bar{M})\Delta A$, $X = ((N\cap\bar{M}) - A) \cup (A - (N\cap\bar{M})) =$

$((N\cap\bar{M})\cap\bar{A}) \cup (A\cap(\overline{N\cap\bar{M}})) = (N\cap(\bar{M}\cap\bar{A})) \cup (A\cap(\bar{N}\cap M)) =$

$(N\cap\bar{A}) \cup ((A\cap\bar{N}) \cup (A\cap M)) = (N\cap\bar{A}) \cup (\phi \cup M) = (N-A) \cup M$.

From here we get the unic solution:

$X = (N-A) \cup M$.

We test $f((N-A)\cup M)=(((N-A)\cup M)\cap A,((N-A)\cup M)\cup A)$ but
$((N-A)\cup M)\cap A=((N\cap\bar{A})\cup M)\cap A=((N\cap\bar{A})\cap A)\cup(M\cap A)=$
$(N\cap(\bar{A}\cap A))\cup M=(N\cap\phi)\cup M=\phi\cup M=M$ and
$((N-A)\cup M)\cup A=(N-A)\cup(M\cup A)=(N-A)\cup A=$
$(N\cap\bar{A})\cup A=(N\cup A)\cap(\bar{A}\cup A)=N\cap E=N$, $f((N-A)\cup M)=(M,N)$. Thus $f(P(E))=K$.

c. From point a. we get g is injective, from point b. we get g is surjective, thus g is bijective.The inverse function is :

$g^{-1}(M,N)=(N-A)\cup M$.

Problem 3. Let $E\neq\phi$, $A, B\in P(E)$ and

$f: P(E)\to P(E)XP(E)$, where $f(X)=(X\cap A, X\cap B)$.

a. Give the necessary and sufficient condition such that f is injective.

b. Give the necessary and suffcient condition such that f is surjective.

c. Supposing that f is bijective, compute its inverse.

Solution.

a. Suppose f is injective. Then: $f(A\cup B)=$
$((A\cup B)\cap A,(A\cup B)\cap B)=(A,B)=(E\cap A,E\cap B)=f(E)$, from where $A\cup B=E$, Now we suppose that $A\cup B=E$, it results that
$X=X\cap E=X\cap(A\cup B)=(X\cap A)\cup(X\cap B)=(Y\cap A)\cup(Y\cap B)=Y\cap(A\cup B)=Y\cap E=Y$, namely from $f(X)=f(Y)$ we get that
$X=Y$, namely f is injective.

b. Suppose f is surjective, for any $M,N\in P(A)XP(B)$, there exists $X\in P(E)$, $f(X)=(M,N)$,$(X\cap A,X\cap B)=(M,N)$,$X\cap A=M$,$X\cap B=N$. In special cases $(M,N)=(A,\phi)$, there exists $X\in P(E)$, from $X\supset A$, $\phi=X\cap B\supset A\cap B$,$A\cap B=\phi$. Now we suppose that$A\cap B=\phi$ and show that it is surjective. Let $(M,N)\in P(A)XP(B)$ then $M\subset A$, $N\subset B$ and $M\cap B\subset A\cap B=\phi$ and$N\cap A\subset B\cap A=\phi$ namely $M\cap B=\phi$, $N\cap A=\phi$ and $f(M\cup N)=((M\cup N)\cap A,\ (M\cup N)\cap B)=((M\cap A)\cup(N\cap A),\ (M\cap B)\cup(N\cap B))=(M\cup\phi,\ \phi\cup N)=(M,N)$, for any (M,N) there exists $X=M\cup N$ such that $f(X)=(M,N)$, namely f is surjective.

c. We show that $f^{-1}((M,N))=M\cup N$.

Observation. In the previous two problems we can use the characteristic function of the set as in the first problem. This method we leave to the readers.

Application. Let $E\neq\phi$, $A_k\in P(E)(k=1,...,n)$ and

$f: P(E) \to P^n(E)$, where $f(X) = (X \cap A_1, X \cap A_2, \ldots, X \cap A_n)$.
Prove that f is injective if and only if $\bigcup_{k=1}^{n} A_k = E$.

Application. Let $E \neq \phi, A_k \in P(E)(k = 1,\ldots,n)$ and
$f: P(E) \to P^n(E)$, where $f(X) = (X \cap A_1, X \cap A_2, \ldots, X \cap A_n)$.
Prove that f is surjective if and only if $\cap_k^n \complement A_k = \phi$.

Problem 4. We name the set M convex if for any $x,y \in M$
$tx + (1 - t)y \in M$, for any $t \in [0, 1]$.

Prove that if $A_k(k= 1,\ldots,n)$ are convex sets, then $\bigcap_{k=1}^{n} A_k$ is also convex.

Problem 5. If $A_k(k = 1, \ldots, n)$ are convex sets, then $\bigcap_{k=1}^{n} A_k$ is also convex .

Problem 6. Give the necessary and sufficient condition such that if A, B are convex /concave sets then $A \cup B$ is also convex /concave. Generalization for n set.

Problem 7. Give the necessary and sufficient condition such that if A, B are convex /concave sets then $A\Delta B$ is also convex /concave. Generalization for n set.

Problem 8. Let $f,g : P(E) \to P(E)$, where $f(X) = A\text{-}X$ and $g(X) = A\Delta X$, $A \in P(E)$. Prove that f, g are bijective and compute their inverse functions.

Problem 9. Let

$A \circ B = \{(x,y) \in R\times R \mid \exists z \in R : (x,z) \in A \text{ and } (z,y) \in B\}$. In a particular case let $A = \{(x, \{x\}) \mid x \in R\}$ and $B = \{(\{y\},y) \mid y \in R\}$.
Represent the $A \circ A$, $B \circ A$, $B \circ B$ cases.

Problem 10.

i. If $A \cup B \cup C = D, A \cup B \cup D = C, A \cup C \cup D = B$,
$B \cup C \cup D = A$, then $A = B = C = D$.

ii. Are there different A, B, C, D sets such that
$A \cup B \cup C = A \cup B \cup D = A \cup C \cup D = B \cup C \cup D$?

Problem 11. Prove that $A\Delta B = A \cup B$ if and only if $A \cap B = \phi$.

Problem 12. Prove the following identity.

$$\bigcap_{i,j=1,i<j}^{n} A_k \cup A_j = \bigcup_{i=1}^{n} \left(\bigcap_{j=1,\, j\neq i}^{n} A_j \right).$$

Problem 13. Prove the following identity.
$(A\cup B)\text{-}(B\cap C)=[A\text{-}(B\cap C))\cup(B\text{-}C) = (A - B) \cup (A - C) \cup (B - C)$ and

$A - [(A \cap C) - (A \cap B)] = (A - \bar{B}) \cup (A - C)$.

Problem 14. Prove that $A \cup (B \cap C) = (A \cup B) \cap C = (A \cup C) \cap B$ if and only if $A \subset B$ and $A \subset C$.

Problem 15. Prove the following identities:

$(A-B)-C = (A-B)-(C-B)$,

$(A \cup B)-(A \cup C) = B-(A \cap C)$,

$(A \cap B)-(A \cap C)=(A \cap B)-C$.

Problem 16. Solve the following system of equations:

$$\begin{cases} A \cup X \cup Y = (A \cup X) \cap (A \cup Y) \\ A \cap X \cap Y = (A \cap X) \cup (A \cap Y). \end{cases}$$

Problem 17. Solve the following system of equations:

$$\begin{cases} A \Delta X \Delta B = A \\ A \Delta Y \Delta B = B. \end{cases}$$

Problem 18. Let $X, Y, Z \subseteq A$.

Prove that: $Z = (X \cap \bar{Z}) \cup (Y \cap \bar{Z}) \cup (\bar{X} \cap Z \cap \bar{Y})$ if and only if $X=Y=\phi$.

Problem 19. Prove the following identity:

$$\bigcup_{k=1}^{n} [A_k \cup (B_k - C)] = \left(\bigcup_{k=1}^{n} A_k\right) \cup \left[\left(\bigcup_{k=1}^{n} A_k\right) - C\right].$$

Problem 20. Prove that: $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$.

Problem 21. Prove that:

$(A \Delta B) \Delta C = (A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C) \cup (A \cap B \cap C)$.

References:

[1] Mihály Bencze, F.Popovici, Permutaciok, Matematikai Lapok, Kolozsvar, 7-8/1991.

[2] Pellegrini Miklós, Egy ujabb kiserlet, a retegezett halmaz, M.L., Kolozsvar, 6/1978.

[3] Halmazokra vonatkozo egyenletekrol, Matematikai Lapok, Kolozsvar, 6/1970.

[4] Alkalmazasok a halmazokkal kapcsolatban, Matematikai Lapok, Kolozsvar, 3/1970.

[5] Ion Savu, Produsul elementelor într-un grup finit comutativ, Gazeta Matematică Perf., 1/1989.

[6] Nicolae Negoescu, Principiul includerii-excluderii, RMT 2/ 1987.

[7] F. C. Gheorghe, T. Spiru, Teorema de prelungire a unei

probabilităţi, dedusă din teorema de completare metrică, Gazeta Matematica-A., 2/1974.

[8] C. P. Popovici: Funcţii Boolene, Gazeta Matematică-A, 1/1973.

[9] Algebra tankonyv IX oszt., Romania.

[10] Năstăsescu stb., Exerciţii şi probleme de algebră pentru clasele IX-XII., România.

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About Very Perfect Numbers[1]

A natural number n is named very perfect if $\sigma(\sigma(n))=2n$ (see [1]).

Theorem. The square of an odd prime number can't be very perfect number.

Proof. Let be $n=p^2$, where p is an odd prime number, t en $\sigma(n)=1+p+p^2$, $\sigma(\sigma(n)) = \sigma(1+p+p^2) = 2p^2$. We decompose $\sigma(n)$ in canonical form, from where $1+p+p^2= p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$. Because $p(p+1)+1$ is odd, in the canonical decompose must be only odd primes.

$$\sigma(\sigma(n)) = (1+p_1+\dots+p_1^{\alpha_1})\dots(1+p_k+\dots+p_k^{\alpha_k}) = \frac{p_1^{\alpha_1+1}-1}{p_1-1}\dots\frac{p_k^{\alpha_k+1}-1}{p_k-1} = 2p^2.$$

Because $\frac{p_1^{\alpha_1+1}-1}{p_1-1} > 2, \dots, \frac{p_k^{\alpha_k+1}-1}{p_k-1} > 2$,

one gets that $2p^2$ can't be decomposed in more than two factors, so each one > 2, therefore $k \le 2$.

Case 1. For $k=1$ we find $\sigma(n) = 1+p+p^2 = p_1^{\alpha_1}$, from where one gets $p_1^{\alpha_1+1} = p_1(1+p+p^2)$ and

$$\sigma(\sigma(n)) = \frac{p_1^{\alpha_1+1}-1}{p_1-1} = 2p^2,$$

$p_1(1+p+p^2)-1=2p^2(p_1-1)$, from where $p_1-1 = p(pp_1-2p-p_1)$. The right side is divisible by p, thus p_1-1 is a p multiple. Because $p_1 > 2$ it results $p_1 \ge p+1$ and

$$p_1^2 \ge (p+1)^2 > p^2+p+1 = p_1^{\alpha_1},$$

thus $\alpha_1=1$ and $\sigma(n)=p^2+p+1=p_1$, $\sigma(\sigma(n)) = \sigma(p_1) = 1+p_1$. If n is very perfect then $1+p_1 = 2p^2$ or $p^2+p+2 = 2p^2$. The solutions of the equation are $p=-1$ and $p=2$ which is a contradiction.

Case2. For $k=2$ we have $\sigma(n)=p^2+p+1 = p_1^{\alpha_1} p_2^{\alpha_2}$.

$$\sigma(\sigma(n)) = (1+p_1+\dots+p_1^{\alpha_1})(1+p_2+\dots+p_2^{\alpha_2}) = \frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1} = 2p^2.$$

Because $\frac{p_1^{\alpha_1+1}-1}{p_1-1} > 2$ and $\frac{p_2^{\alpha_2+1}-1}{p_2-1} > 2$

it results

$$\frac{p_1^{\alpha_1+1}}{p_1-1} = p \text{ and } \frac{p_2^{\alpha_2+1}-1}{p_2-1} = 2p$$

(or inverse),

thus $p_1^{\alpha_1+1}-1 = p(p_1-1)$, $p_2^{\alpha_2+1}-1 = 2p(p_2-1)$,

then $p_1^{\alpha_1+1}p_2^{\alpha_2+1} - p_1^{\alpha_1+1} - p_2^{\alpha_2+1} + 1 = 2p^2(p_1-1)(p_2-1)$,

[1]Together with Mihály Bencze and Florin Popovici

thus $\sigma(n) = p^2+p+1 = p_1^{\alpha_1-1}p_2^{\alpha_2-1}$
and $p_1p_2(p^2 + p + 1) = 2p^2(p_1-1)(p_2-1) + p_1^{\alpha_1+1} + p_2^{\alpha_2+1} - 1$
or $p_1p_2p(p+1) + p_1p_2 - 1 = 2p^2(p_1-1)(p_2-1) + (p_1^{\alpha_1+1} - 1) + (p_2^{\alpha_2-1} - 1) =$
$2p^2(p_1 - 1)(p_2-1) + p(p_1-1) + 2p(p_2-1)$ accordingly p divides p_1p_2-1, thus $p_1p_2 > p+1$ and $p_1^2 p_2^2 \geq (p+1)^2 > p^2 + p + 1 = p_1^{\alpha_1} p_2^{\alpha_2}$ Hence:
Π_1) If $\alpha_1 = 1$ and $n = 2p^2$, then $\sigma(n) = p^2+p+1 = p_1p_2^{\alpha_2}$

and $\dfrac{p_1^2 - 1}{p_1 - 1} = p$ and $\dfrac{p_2^{\alpha_2+1} - 1}{p_2 - 1} = 2p,$

thus $p_1 + 1 = p$ wich is a contradiction.

Π_2) If $\alpha_2 = 1$ and $n = 2p^2$, then $\sigma(n) = p^2+p+1 = p_1^{\alpha_1} p_2$

and $\dfrac{p_1^{\alpha_1+1} - 1}{p_1 - 1} = p$ and $\dfrac{p_2^2 - 1}{p_2 - 1} = 2p,$

thus $p_2 + 1 = 2p$, $p_2 = 2p - 1$ and $\sigma(n) = p^2+p+1 = p_1^{\alpha_1}(2p+1)$,
from where $4\sigma(n)=(2p-1)(2p+3)+7= 4p_1^{\alpha_1}(2p-1)$, accordingly 7 is divisible by $2p-1$ and thus p is divisible by 4 which is a contradiction.

Reference :

I Suryanarayama, Elemente der Mathematik, 1969.

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Inequalities For The Integer Part Function[1]

In this paper we prove some inequalities for the integer part function and we give some applications in the number theory.

Theorem 1. For any x, y > 0 we have the inequality

(1) $[5x] + [5y] \geq [3x+y] + [3y+x]$, where $[\cdot]$ means the integer part function.

Proof. We use the notations $x_1 = [x]$, $y_1 = [y]$, $u = \{x\}$, $v = \{y\}$, $x_1, y_1 \in N$ and $u, v \in [0, 1)$. We can write the inequality (1) as

$x_1 + y_1 + [5u] + [5v] \geq [3u+v] + [3v+u]$. We distinguish the following cases:

α) Let $u \geq v$. If $u \leq 2v$, then $5v \geq 3v+u$ and $[5v] \geq [3v+u]$, analogously $5u \geq 3u+v$ and $[5u] \geq [3u+v]$, from where by addition we obtain (1). If $u>2v$ and $5u=a+b$, $5v=c+d$, $a,c \in N$, $0 \leq b < 1$, $0 \leq d < 1$, then we have to prove the following inequality

$$a + c + x_1 + y_1 \geq \left[\frac{3a+c+3b+d}{5}\right] + \left[\frac{3c+a+3d+b}{5}\right] \quad (2).$$

But, considering that $1 > u > 2v$, we get $5 > 5u > 10v$, from where $5 > a+b > 2c+2d$, thus $a+b < 5$ and $a \leq 4$. If $a < 2c$, then $a \leq 2c - 1$ and $a + 1 - 2c \leq 0$, thus $a+b-2c < 0$; contradiction with $a+b-2c>2d$, thus $4 \geq a$, $a \geq 2c$ and $3b+d<4$, $3d+b<4$. From $4 \geq a \geq 2c$ we have the cases from the table and in each of the nine cases is verified the inequality (2).

a	4	4	4	3	3	2	2	1	0
c	2	1	0	0	1	1	0	0	0

Application 1. For any m, n ∈ N, (5m)!(5n)! is divisible by m!n!(3m+n)!(3a+m)!.

Proof. If p is a prime number, the power exponent of p in decomposition of m! is $\left[\frac{m}{p}\right] + \left[\frac{m}{p^2}\right] + \ldots$ It is sufficient to prove that

$$\left[\frac{5m}{r}\right] + \left[\frac{5n}{r}\right] \geq \left[\frac{m}{r}\right] + \left[\frac{n}{r}\right] + \left[\frac{3m+n}{r}\right] + \left[\frac{3n+m}{r}\right]$$

for any $r \in N$, $r \geq 2$. If $m = rm_1+x$, $n = rn_1+y$, where $0 \leq x < r$, $0 \leq y < r$, m, n ∈ Z, is sufficient to prove that

$\left[\frac{5x}{r}\right] + \left[\frac{5y}{r}\right] \geq \left[\frac{3x+y}{r}\right] + \left[\frac{3y+x}{r}\right]$, but this inequality verifies the theorem 1.

Remark. If x, y > 0 , then we have the inequatity

$[5x]+[5y] \geq [x]+(y)+[3x+y]+[3y+x]$.

[1] Together with Mihály Bencze and Florin Popovici.

Theorem 2. (Szilárd András). If $x, y, z \geq 0$, then we have the inequality $[3x]+[3y]+[3z] \geq [x]+(y]+[z]+[x+y]+[y+z]+[z+x]$.

Application 2. For any $a,b,c \in N$, $(3a)!(3b)!(3c)!$ is divisible by $a!b!c!(a+b)!(b+c)!(c+a)!$.

Proof. Let k_1,k_2,k_3 be the biggest power for which $p^{k_1} \mid (3a)!$, $p^{k_2} \mid (3b)!$, $p^{k_3} \mid (3c)!$ respectively, and r_i ($i \in \{1, 2, 3\ 4, 5, 6\}$) the biggest power for which $p^{r_1} \mid a!$, $p^{r_2} \mid b!$, $p^{r_3} \mid c!$, $p^{r_4} \mid (a+b)!$, $p^{r_5} \mid (b+c)!$, $p^{r_6} \mid (c+a)!$ respectively, then

$$k_1+k_2+k_3=\left(\left[\frac{3a}{p}\right]+\left[\frac{3a}{p^2}\right]+...\right)+\left(\left[\frac{3b}{p}\right]+\left[\frac{3b}{p^2}\right]+...\right)+\left(\left[\frac{3c}{p}\right]+\left[\frac{3c}{p^2}\right]+...\right)$$

and

$$\sum_{i=1}^{6} r_i \left(\left[\frac{a}{p}\right]+\left[\frac{a}{p^2}\right]+...\right)+\left(\left[\frac{b}{p}\right]+\left[\frac{b}{p^2}\right]+...\right)+\left(\left[\frac{c}{p}\right]+\left[\frac{c}{p^2}\right]+...\right)$$

$$+\left(\left[\frac{a+b}{p}\right]+\left[\frac{a+b}{p^2}\right]+...\right)+\left(\left[\frac{b+c}{p}\right]+\left[\frac{b+c}{p^2}\right]+...\right)+\left(\left[\frac{c+a}{p}\right]+\left[\frac{c+a}{p^2}\right]+...\right).$$

We have to prove that $k_1+k_2+k_3 \geq \sum_{i=1}^{6} r_i$, but this inequality reduces to theorem 2.

Theorem 3. If $x, y, z \geq 0$, then we have the inequality

$[2x]+[2y]+[2z] \leq [x]+[y]+[z]+[x+y+z]$.

Application 3. If $a,b,c \in N$, then $a!b!c!(a+b+c)!$ is divisible by $(2a)!(2b)!(2c)!$.

Theorem 4. If $x, y \geq 0$ and $n, k \in N$ so that $n \geq k \geq 0$, then we have the inequality $[nx] + [ny] \geq k[x] + k[y] + (n - k)[x + y]$.

Application 4. If $a, b, n, k \in N$ and $n \geq k$, then $(na)!(nb)!$ is divisible by $(a!)^k(b!)^k((a+b)!)^{n-k}$.

Theorem 5. If $x_k \geq 0$ ($k = 1, 2, .., n$), then we have the inequality

$$2\sum_{k=1}^{n} [2x_k] \geq 2 \sum_{k=1}^{n} [x_k] + [x_1+x_2] + [x_2+x_3] + ... + [x_n+x_1].$$

Application 5. If $a_k \in N$ ($k = 1, 2, ..., n$), then $\prod_{k=1}^{n} ((2a_k)!)^2$ is divisible by $\prod_{k=1}^{n} (a_k!)^2 (a_1+a_2)!(a_2+a_3)! ... (a_n+a_1)!$.

Theorem 6. If $x_k \geq 0$ ($k = 1, 2, ..., n$), then we have the inequality

$$m\sum_{k=1}^{n} [2x_k] + n \sum_{p=1}^{m} [2x_p] \geq m\sum_{k=1}^{n} [x_k] + n \sum_{p=1}^{m} [x_p] + \sum_{k=1}^{n}\sum_{p=1}^{m} [x_k+x_p].$$

Application 6. If $a_k \in N$ ($k = 1, 2, ..., n$), then

$$\prod_{k=1}^{n} (2a_k!)^m \prod_{p=1}^{m} (2a_p!)^n \text{ is divisible by } \prod_{k=1}^{n} (a_k!)^m \prod_{p=1}^{m} (a_p!)^n \prod_{k=1}^{n}\prod_{p=1}^{m} ((a_k+a_p)!).$$

Theorem 7. If $x,y \geq 1$, then we have the inequality

$$\left[\sqrt{x}\right]+\left[\sqrt{y}\right]+\left[\sqrt{x+y}\right]\geq\left[\sqrt{2x}\right]+\left[\sqrt{2y}\right].$$

Proof. By the concavity of the square root function

$$\sqrt{x+y}=\sqrt{\frac{2x+2y}{2}}\geq\frac{1}{2}\sqrt{2x}+\frac{1}{2}\sqrt{2y}\geq\left[\frac{1}{2}\sqrt{2x}\right]+\left[\frac{1}{2}\sqrt{2y}\right],$$

it follows that $\left[\sqrt{x+y}\right]\geq\left[\frac{1}{2}\sqrt{2x}\right]+\left[\frac{1}{2}\sqrt{2y}\right]$.

Therefore it is sufficient to show that $\left[\sqrt{x}\right]+\left[\frac{1}{2}\sqrt{2x}\right]\geq\left[\sqrt{2x}\right]$ for $x\geq 1$. The identity $[x]+\left[x+\frac{1}{2}\right]$ has a straightforward proof. We use it to replace $\left[\frac{1}{2}\sqrt{2x}\right]$ with $\left[\sqrt{2x}\right]-\left[\frac{1}{2}\sqrt{2x}+\frac{1}{2}\right]$.

This yields $\left[\sqrt{x}\right]\geq\left[\frac{1}{2}\sqrt{2x}+\frac{1}{2}\right]$ for $x\geq 1$. This last inequality followed by notice that $x\geq 4$ implies $\left(2-\sqrt{2}\right)\sqrt{x}>1$ or $\left[\sqrt{x}\right]>\left[\frac{1}{2}\sqrt{2x}+\frac{1}{2}\right]$ and $1\leq x<4$ implies $\frac{1}{2}\sqrt{2x}+\frac{1}{2}<2$.

Application 7. If $a,b\in N$, then $a!b!\left\lfloor\sqrt{a^2+b^2}\,!\right\rfloor$ is divisible by $\left\lfloor a\sqrt{2}\right\rfloor!\,\left\lfloor b\sqrt{2}\right\rfloor!$

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About Bernoulli's Numbers[1]

Many methods to compute the sum of the same powers of the first n natural numbers (see ([4]) are well-known.

In this paper we present a simple proof of the method from [3].

The Bernoulli's numbers are defined by

(1) $$B_n = \frac{-1}{n+1}\left(C_{n+1}^{0}B_0 + C_{n+1}^{1}B_1 + \dots + C_{n+1}^{n-1}B_{n-1}\right),$$

where $B_0 = 1$. It is known that $B_{n+1} = 0$ if $n \geq 1$. By calculation we find that

(2) $B_1 = -1/2$, $B_2 = 1/6$, $B_4 = -1/30$, $B_6 = 1/42$, $B_8 = -1/30$, $B_{10} = 5/66$, $B_{12} = -691/2730$, $B_{14} = 7/6$, $B_{16} = -3617/510$, $B_{18} = 43867/798$, $B_{20} = -174611/330$, $B_{22} = 854513/138$, $B_{24} = -236364091/2730$ etc.

Let $S_n^k = 1^k + 2^k + \dots + n^k$ sum of the first n natural numbers which have the same power.

Theorem.

(3) $$S_n^k = \frac{1}{k+1}\left(n^{k+1} + \frac{1}{2}C_{k+1}^{1}n^k + C_{k+1}^{2}B_2 n^{k-1} + \dots + C_{k+1}^{k}B_n n\right)$$

Proof. (1) can be written as:

(4) $$\sum_{i=0}^{n} C_{n+1}^{i}B_i = 0, \quad n \geq 1.$$

If $$P(x) = \sum_{i=0}^{k} C_{k+1}^{i}B_i x^{k+1-i},$$ then P(n+1) - P(n) =

$$= \sum_{i=0}^{k} C_{k+1}^{i}B_i\left((n+1)^{k+1-i} - n^{k+1-i}\right) =$$

$$= \sum_{i=0}^{k} C_{k+1}^{i}B_i\left(\sum_{j=1}^{k+1-i} C_{k+1-i}^{j}n^{k+1-i-j}\right).$$

Let A_t be the coeficients of n^{k-t}, where $t \in \{0, 1, \dots, k\}$.

$$A_t = \sum_{i=0}^{t} C_{k+1}^{i}C_{k+1-i}^{t+1-i}B_i = C_{k+1}^{t+1}\left(\sum_{i=0}^{t} C_{t+1}^{i}B_i\right).$$

If $t \geq 1$, then $A_t = 0$, only $A_0 = C_{k+1}^{1}$. On behalf of these

$P(n+1) - P(n) = C_{k+1}^{1}n^k$. Using this

$$\sum_{i=0}^{n-1} i^k = \frac{1}{k+1}\sum_{i=0}^{n-1}\left(P(i+1) - P(i)\right) = \frac{1}{k+1}P(n),$$

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because P(0) = 0. Then $S_n^k = \frac{1}{k+1}P(n) + n^k$. From here one gets (3).

Note. From the previous result we can also find the formula $S_n^k = \frac{1}{k+1}P(n+1)$. Using the previsious, we find the next equalities:

$S_n^0 = n$, $S_n^1 = \frac{1}{2}n(n+1)$, $S_n^2 = \frac{1}{6}n(n+1)(2n+1)$, $S_n^3 = \frac{1}{4}n^2(n+1)^2$,

$$S_n^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$$

$$S_n^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$$

$$S_n^6 = \frac{1}{42}n(n+1)(2n+1)(3n^4+6n^3-3n+1)$$

$$S_n^7 = \frac{1}{24}n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)$$

$$S_n^8 = \frac{1}{90}n(n+1)(2n+1)(5n^6+15n^5+5n^4-15n^3-n^2+9n-3)$$

$$S_n^9 = \frac{1}{20}(2n^{10}+10n^9+15n^8-14n^6+10n^4-3n^2)$$

$$S_n^{10} = \frac{1}{66}(6n^{11}+33n^{10}+55n^9-66n^7+66n^5-33n^3+5n)$$

$$S_n^{11} = \frac{1}{24}(2n^{12}+12n^{11}+22n^{10}-33n^8+44n^6-33n^4+10n^2)$$

$$S_n^{12} = \frac{1}{2730}(210n^{13}+1365n^{12}+3630n^{11}-4935n^9+115n^8+9640n^7$$

$$+1960n^6-5899n^5+35n^4+4550n^3+1382n^2-691n) \text{ etc.}$$

Problems.

1). Using the mathematical induction on the base of (1), we prove that $B_{2n+1} = 0$, if $n \geq 1$.

2). Prove that S_n^k is divisible by n(n + 1).

3). Prove that S_n^{2k+1} is divisible by $n^2(n+1)^2$.

4). Determine those natural numbers n, k for which S_n^{2k} is divisible n(n + 1)(2n + 1).

5). Detach in parts the sums S_n^9, S_n^{10} S_n^{11}, S_n^{12}.

6). Using (2), (3), compute the sums $S_n^{13}, \ldots, S_n^{21}$.

References:

[1] M. Kraitchik, Recherches sur la théorie des nombres, Paris, 1924.

[2] Mihály Bencze, Osszegekrol, A Matematika Tanitasa, 1/1983.

[3] Z. I. Borevici, I. R. Safarevici, Teoria Numerelor, Ed. Ştiinţifică şi Enciclopedică, Bucureşti, România, 1985.

[4]. Sándor József: Veges osszegekrol, Matematikai Lapok, Cluj-Napoca, 9/1987, Romania.

[5] Mihály Bencze: A Bernoulli szamok egyik alkalmazasa, Matematikai Lapok, Kolozsvar 7/ 1989, pp. 237-238, Romania.

["Octogon", Braşov, Vol. 7, No. 1, 151-3, 1999.]

The Solution Of OQ.102[1]

In the "Octogon" Vol.5, No.2, Zoltán Blázik, in the open problem OQ.102, asked if there exists a polynomial $P(x,y)$ of at most second degree such that on the set $\{1,2,3\}x\{1,2,3\}$ it takes the values 1, 2, 3, 4, 5, 6, 7, 8, 10 , each of them exactly once. We show that doesn't exist such a polynom. Let $P(x,y)=Ax^2+Bxy+Cy^2+Dx+Ey+F$ be a such polynom. It results that $P(1,1)-2P(1,2)+P(1,3)-2P(2,1)+4P(2,2)-2P(2,3)+P(3,1)-2P(3,2)+P(3,3)=0$. In this sum there are only integer numbers, and each coefficient divided by 3 give one remainder. From this one gets that

$0=P(1,1)-2P(1,2)+P(1,3)-2P(2,1)+4P(2,2)-2P(2,3)+P(3,1)-2P(3,2)+P(3,3) \equiv P(1,1)+P(1,2)+P(1,3)+P(2,1)+P(2,2)+P(2,3)+P(3,1)+P(3,2)+P(3,3)=1+2+3+4+5+6+7+8+10\equiv46(mod3)$ and this is a contradiction.

Next we propose the following open question:

Is there a polynomial $P(x_1,x_2,...,x_n)$ of at most degree n such that on the set $\{1,2,...,n,n+1\}x\{1,2,...,n,n+1\}x...x\{1,2,...,n,n+1\}$ (the braces are repeated n times) it takes the values $1,2,3, ...,n,...,(n+1)^2-2,(n+1)^2-1,(n+1)^2+1$ exactly once?

["Octogon", Vol.6, No.1, 81, 1998.]

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CONJECTURES ON PRIMES' SUMMATION

A) Any odd integer n can be expressed as a combination of three primes as follows:

1) As a sum of two primes minus another prime:

$n = p+q-r$, where p, q, r are all prime numbers.

Do not include the trivial solution: $p = p+q-q$ when p, q are prime.

For example: $1 = 3+5-7 = 5+7-11 = 7+11-17 = 11+13-23 = \dots$;

$3 = 5+5-7 = 7+19-23 = 17+23-37 = \dots$;

$5 = 3+13-11 = \dots$;

$7 = 11+13-17 = \dots$;

$9 = 5+7-3 = \dots$;

$11 = 7+17-13 = \dots$.

a) Is this conjecture equivalent to Goldbach's Conjecture (any odd integer >=9 is the sum of three primes)?

b) Is the conjecture true when all three prime numbers are different?

c) In how many ways can each odd integer be expressed as above?

2) As a prime minus another prime and minus again another prime:

$n = p-q-r$, where p, q, r are all prime numbers.

For example: $1 = 13-5-7 = 17-5-11 = 19-5-13 = \dots$;

$3 = 13-3-7 = 23-7-13 = \dots$;

$5=13-3-5=\dots$;

$7 = 17-3-7 = \dots$;

$9 = 17-3-5 = \dots$;

$11 = 19-3-5 = \dots$.

a) Is this conjecture equivalent to Goldbach's Conjecture ?

b)Is the conjecture true when all three prime numbers are different?

c) In how many ways can each odd integer be expressed as above?

B) Any odd integer n can be expressed as a combination of five primes as follows:

3) $n = p+q+r+t-u$, where p, q, r, t, u are all prime numbers, and $t \neq u$.

For example: $1 = 3+3+3+5-13 = 3+5+5+17-29 = \dots$;

3 = 3+5+11+13-29 = ... ;
5 = 3+7+11+13-29 = ... ;
7 = 5+7+11+13-29 = ... ;
9 = 5+7+11+13-29 = ... ;
11 = 5+7+11+17-29 = ...

a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

4) n = p+q+r-t-u, where p, q, r, t, u are all prime numbers, and t, u ≠ p, q, r.
For example: 1 = 3+7+17-13-13 = 3+7+23-13-19 = ... ;
3 = 5+7+17-13-13 = ... ;
5 = 7+7+17-13-13 = ... ;
7 = 5+11+17-13-13 = ... ;
9 = 7+11+17-13-13 = ... ;
11 = 7+11+19-13-13 = ...

a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

5) n = p+q-r-t-u, where p, q, r, t, u are all prime numbers, and r,t,u≠p,q.
For example: 1 = 11+13-3-3-17 = ... ;
3 = 13+13-3-3-17 = ... ;
5 = 3+29-5-5-17 = ... ;
7 = 3+31-5-5-17 = ... ;
9 = 3+37-7-7-17 = ... ;
11 = 5+37-7-7-17 = ...

a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

6) n = p-q-r-t-u, where p, q, r, t, u are all prime numbers, and q,r,t,u≠p.
For example: 1 = 13-3-3-3-3 = ... ;
3 = 17-3-3-3-5 = ... ;
5 = 19-3-3-3-5 = ... ;
7 = 23-3-3-5-5 = ... ;
9 = 29-3-5-5-7 = ... ;
11 = 31-3-5-5-7 = ...

a) Is the conjecture true when all five prime numbers are different?

b) In how many ways can each odd integer be expressed as above?

GENERAL CONJECTURE:

Let $k \geq 3$, and $1 < s < k$, be integers. Then:

i) If k is odd, any odd integer can be expressed as a sum of k-s primes (first set) minus a sum of s primes (second set) [such that the primes of the first set is different from the primes of the second set].

a) Is the conjecture true when all k prime numbers are different?

b) In how many ways can each odd integer be expressed as above?

ii) If k is even, any even integer can be expressed as a sum of k-s primes (first set) minus a sum of s primes (second set) [such that the primes of the first set is different from the primes of the second set].

a) Is the conjecture true when all k prime numbers are different?

b) In how many ways can each even integer be expressed as above?

Reference:

[l] Smarandache, Florentin, "Collected Papers", Vol. II, Kishinev University Press, Kishinev, article <Prime Conjecture>, p. 190, 1997.

["Math Power", Pima Community College, Tucson, AZ, USA, Vol.5, No.9, 2-4, September 1999;

"Octogon", Braşov, Vol.8, No.1, 189-91, 2000.]

CONJECTURES WHICH GENERALIZE ANDRICA'S CONJECTURE

Five conjectures on paires of consecutive primes are listed below with examples in each case.

1) The equation $p_{n+1}^{x} - p_{n}^{x} = 1,$ (1)
where p_n is the n-th prime, has a unique solution situated in between 0.5 and 1. Checking the first 168 prime numbers (less than 1000), one gets that:

- the maximum occurs of course for n=1, i.e.
$3^x - 2^x = 1$ when x=1.
- the minimum occurs for n=31, i.e.
$127^x - 113^x = 1$ when $x = 0.567148... = a_0$. (2)

Thus, Andrica's Conjecture

$$A_n = \sqrt{p_{n+1}} - \sqrt{p_n} < 1,$$

is generalized to

2) $B_n = p_{n+1}^{a} - p_n^{a} < 1, \quad where \quad a < a_0.$ (3)

It is remarkable that the minimum x doesn't occur for
$11^x - 7^x = 1$
as in Andrica's Conjecture the maximum value, but in (2).

Also, the function B_n in (3) is falling asymptotically as A_n in (2). Look at these prime exponential equations solved with a TI-92 Graphing Calculator (approximately: the bigger the prime number gap is, the smaller solution x for the equation (1);
for the same gap between two consecutive primes, the larger the primes, the bigger x):

$3^x - 2^x = 1$, has the solution x = 1.000000.
$5^x - 3^x = 1$, has the solution x ≈ 0.727160.
$7^x - 5^x = 1$, has the solution x ≈ 0.763203.
$11^x - 7^x = 1$, has the solution x ≈ 0.599669.

$13^x - 11^x = 1$, has the solution $x \approx 0.807162$.
$17^x - 13^x = 1$, has the solution $x \approx 0.647855$.
$19^x - 17^x = 1$, has the solution $x \approx 0.826203$.
$29^x - 23^x = 1$, has the solution $x \approx 0.604284$.
$37^x - 31^x = 1$, has the solution $x \approx 0.624992$.
$97^x - 89^x = 1$, has the solution $x \approx 0.638942$.
$127^x - 113^x = 1$, has the solution $x \approx 0.567148$.
$149^x - 139^x = 1$, has the solution $x \approx 0.629722$.
$191^x - 181^x = 1$, has the solution $x \approx 0.643672$.
$223^x - 211^x = 1$, has the solution $x \approx 0.625357$.
$307^x - 293^x = 1$, has the solution $x \approx 0.620871$.
$331^x - 317^x = 1$, has the solution $x \approx 0.624822$.
$497^x - 467^x = 1$, has the solution $x \approx 0.663219$.
$521^x - 509^x = 1$, has the solution $x \approx 0.666917$.
$541^x - 523^x = 1$, has the solution $x \approx 0.616550$.
$751^x - 743^x = 1$, has the solution $x \approx 0.732706$.
$787^x - 773^x = 1$, has the solution $x \approx 0.664972$.
$853^x - 839^x = 1$, has the solution $x \approx 0.668274$.
$877^x - 863^x = 1$, has the solution $x \approx 0.669397$.
$907^x - 887^x = 1$, has the solution $x \approx 0.627848$.
$967^x - 953^x = 1$, has the solution $x \approx 0.673292$.
$997^x - 991^x = 1$, has the solution $x \approx 0.776959$.

If $x > a_0$, the difference of x-powers of consecutive primes is normally grater than 1. Checking more versions:

$3^{0.99}$	-	$2^{0.99}$	≈ 0.981037.
$11^{0.99}$	-	$7^{0.99}$	≈ 3.874270.
$11^{0.60}$	-	$7^{0.60}$	≈ 1.001270.
$11^{0.59}$	-	$7^{0.59}$	≈ 0.963334.
$11^{0.55}$	-	$7^{0.55}$	≈ 0.822980.
$11^{0.50}$	-	$7^{0.50}$	≈ 0.670873 .
$389^{0.99}$	-	$383^{0.99}$	≈ 5.596550.
$11^{0.599}$	-	$7^{0.599}$	≈ 0.997426.
$17^{0.599}$	-	$13^{0.599}$	≈ 0.810218.
$37^{0.599}$	-	$31^{0.599}$	≈ 0.874526.
$127^{0.599}$	-	$113^{0.599}$	≈ 1.230100.

$997^{0.599} - 991^{0.599} \approx 0.225749.$

$127^{0.5} - 113^{0.5} \approx 0.639282.$

3) $C_n = p_{n+1}^{1/k} - p_n^{1/k} < 2/k$, where p_n is the n-th prime, and $k \geq 2$ is an integer.

$11^{1/2} - 7^{1/2} \approx 0.670873.$
$11^{1/4} - 7^{1/4} \approx 0.1945837251.$
$11^{1/5} - 7^{1/5} \approx 0.1396211046.$
$127^{1/5} - 113^{1/5} \approx 0.060837.$
$3^{1/2} - 2^{1/2} \approx 0.317837.$
$3^{1/3} - 2^{1/3} \approx 0.1823285204.$
$5^{1/3} - 3^{1/3} \approx 0.2677263764.$
$7^{1/3} - 5^{1/3} \approx 0.2029552361.$
$11^{1/3} - 7^{1/3} \approx 0.3110489078.$
$13^{1/3} - 11^{1/3} \approx 0.1273545972.$
$17^{1/3} - 13^{1/3} \approx 0.2199469029.$
$37^{1/3} - 31^{1/3} \approx 0.1908411993.$
$127^{1/3} - 113^{1/3} \approx 0.191938.$

4) $D_n = p_{n+1}^a - p_n^a < 1/n$, (4)

where $a < a_0$ and n big enough, $n = n(a)$, holds for infinitely many consecutive primes.

a) Is this still available for $a < 1$?

b) Is there any rank n_0 depending on a and n such that (4) is verified for all $n \geq n_0$?

A few examples:

$5^{0.8} - 3^{0.8} \approx 1.21567.$
$7^{0.8} - 5^{0.8} \approx 1.11938.$
$11^{0.8} - 7^{0.8} \approx 2.06621.$
$127^{0.8} - 113^{0.8} \approx 4.29973.$
$307^{0.8} - 293^{0.8} \approx 3.57934.$
$997^{0.8} - 991^{0.8} \approx 1.20716.$

5) $P_{n+1}/P_n \leq 5/3$, (5)

the maximum occurs at n=2.

{The ratio of two consecutive primes is limited, while the

difference $p_{n+1} - p_n$ can be as big as we want!}
However, $1/p_n - 1/p_{n+1} \leq 1/6$, and the maximum occurs at n=1.

Reference:

[1] Sloane, N. J. A., Sequence A001223/M0296 in <An On-Line Version of the Encyclopedia of Integer Sequences>.

["Octogon", Braşov, Vol.7, No.1, 173-6, 1999.]

A Generalization Of The Leibniz Theorem[1]

In this paper we show a generalization of Leibniz's theorem and an application of this.

Leibniz's theorem. Let M be an arbitrary point in the plane of the ABC triangle, then $MA^2+MB^2+MC^2 = 1/3(a^2+b^2+c^2)+3MG^2$, where G is the centroid ofthe triarigle. We generalize this theorem:

Theorem. Let $A_1,A_2,\dots,A_n$ be arbitrary points in the space and G the centroid of this points system; then for an arbitrary M point of the space is valid the following equation

$$\sum_{i=1}^{n} MA_i^2 = \frac{1}{n}\sum_{1\le i<j\le n} A_iA_j^2 + n\cdot MG^2.$$

Proof. First, we interpret the centroid of the n points system in a recurrent way. If n =2, then is the midpoint of the segment. If n = 3, then it is the centroid of the triangle. Suppose that we found the centroid of the n-1 points created system. Now we join each of the n points with the centroid of the n-1 points created system; so we obtain n bisectors of the sides. It is easy to show that these n medians are concurrent segments. In this way we obtain the centroid of the n points created system. Denote G_i the centroid of the A_k,k = 1,2,...,i-1,i+1,...,n points created system. It can be showed that $(n-1)A_iG=GG_i$. Now by induction we prove the theorem. If n=2 the $MA_1^2 + MA_2^2 = \frac{1}{2}A_1A_2^2 + 2MG^2$ or $MG^2 = \frac{1}{4}\left(2\left(MA_1^2 + MA_2^2\right)\right)$, where G is the midpoint of the A_1A_2 segment. The above formula is the side bisector's formula in the MA_1A_2 triangle. The proof can be done by Stewart's theorem, cosines theorem, generalized theorem of Pythagoras or can be done vectorially. Suppose that the assertion of the theorem is true for n = k. If $A_1,A_2,\dots,A_k$ are arbitrary points in the space, G_0 is the centroid of this points system, then we have the following relation $\sum_{i=1}^{k} MA_i^2 = \frac{1}{k}\sum_{1\le i<j\le k} A_iA_j^2 + k\cdot MG_o^2$.

Now we prove for n=k+1. Let $A_{k+1} \notin \{A_1,A_2,\dots,A_k, G_o\}$ an arbitrary point in the space and let G be the centroid of the $A_1,A_2,\dots,A_k,A_{k+1}$, points system. Taking into account that G is on the $A_{k+1}G_o$ segment and

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$k \cdot A_{k+1}G = GG_o$, we apply Stewart's theorem to the M, G_o, G, A_{k+1} points, from where

$$MA_{k+1}^2 \cdot GG_o + MG_o^2 \cdot GA_{k+1} - MG^2 \cdot A_{k+1}G_o = GG_o \cdot GA_{k+1} \cdot A_{k+1}G_o.$$

According to the previous observation $A_{k+1}G = \frac{k}{k+1} A_{k+1}G_o$ *and* $GG_o = \frac{1}{k+1} A_{k+1}G_o$.

Using these, the above relation becomes

$$MA_{k+1}^2 + k \cdot MG_o^2 = \frac{k}{k+1} A_{k+1}G_o^2 + (k+1)MG^2.$$

From here

$$k \cdot MG_o^2 = \sum_{i=1}^{k} MA_i^2 - \frac{1}{k} \sum_{1 \le i < j \le k} A_iA_j^2.$$

From the supposition of the induction with $M \equiv A_{k+1}$ as substitution we get $\sum_{i=1}^{k} A_iA_j^2 = \frac{1}{k} \sum_{1 < i < j \le k} A_iA_j^2 + k \cdot A_{k+1}G_o^2$ and thus

$$\frac{k}{k+1} A_{k+1}G_0^2 = \frac{1}{k+1} \sum_{i=1}^{k} A_iA_{k+1}^2 - \frac{1}{k(k+1)} \sum_{1 \le i < j \le k} A_iA_j^2.$$

Substituting this in the above relation we obtain that

$$\sum_{i=1}^{k+1} MA_i^2 = \left(\frac{1}{k} - \frac{1}{k(k+1)}\right) \sum_{1 \le i < j \le k} A_iA_j^2 + \frac{1}{k+1} \sum_{i=1}^{k} A_iA_{k+1}^2$$

$$+ (k+1)MG^2 = \frac{1}{k+1} \sum_{1 \le i < j \le k+1} A_iA_j^2 + (k+1)MG^2.$$

With this we proved that our assertion is true for $n = k+1$. According to the induction it is true for every $n \ge 2$ natural numbers.

1. Application. If the $A_1, A_2, \ldots, A_n$ points are on the sphere with the center O and radius R, then using in the theorem the substitution $M \equiv O$ we get the identity $OG^2 = R^2 - \frac{1}{n^2} \sum_{1 \le i < j \le n} A_iA_j^2$

In case of a triangle $OG^2 = R^2 - \frac{1}{9}\left(a^2 + b^2 + c^2\right)$.

In case of a tetrahedron $OG^2 = R^2 - \frac{1}{16}\left(a^2 + b^2 + c^2 + d^2 + e^2 + f^2\right)$.

2. Application. If the $A_1, A_2, \ldots, A_n$ points are on the sphere with the

center O and radius R, then $\sum_{1\le i<j\le n} A_iA_j^2 \le n^2R^2$.

Equality holds if and only if G≡O. In case of a triangle $a^2+b^2+c^2 \le 9R^2$, in case of a tetrahedron $a^2+b^2+c^2+d^2+e^2+f^2 \le 16R^2$.

3. Application. Using the arithmetic and harmonic mean inequality, from the previous application results the following inequality results: $\sum_{1\le i<j\le n} \frac{1}{A_iA_j^2} \ge \frac{(n-1)^2}{4R^2}$.

In case of a triangle $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \ge \frac{1}{R^2}$, in case of a tetrahedron

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}+\frac{1}{e^2}+\frac{1}{f^2} \ge \frac{9}{4R^2}.$$

4. Application. Considering the Cauchy-Buniakowski-Schwarz inequality from the application 2 we obtain the following inequality

$$\sum_{1\le i<j\le n} A_iA_j \le nR\sqrt{\frac{n(n-1)}{2}}.$$

In case of a triangle $a+b+c\le 3\sqrt{3}R$, in case of a tetrahedron $a+b+c+d+e+f \le 4\sqrt{6}R$.

5. Application. Using the arithmetic and harmonic mean inequality, from the previous application we get the following inequality

$$\sum_{1\le i<j\le n} \frac{1}{A_iA_j} \ge \frac{(n-1)\sqrt{n(n-1)}}{2R\sqrt{2}}.$$

In case of a triangle $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge \frac{\sqrt{3}}{R}$, in case of a tetrahedron

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}+\frac{1}{f} \ge \frac{3}{R}\sqrt{\frac{3}{2}}.$$

6. Application. Considering application 3, we obtain the following inequality: $\frac{n^2(n-1)^2}{4} \le \left(\sum_{1\le i<j\le n} A_iA_j^k\right)\left(\sum_{1\le i<j\le n} \frac{1}{A_iA_j^k}\right) \le$

$$\le \begin{cases} \frac{(M+m)^2 n^2(n-1)^2}{16M\cdot m} \text{ if } \frac{n(n-1)}{2} \text{ is even}, \\ \frac{(M+m)^2 n^2(n-1)^2 - 4(M-m)^2}{16M\cdot m} \text{ if } \frac{n(n-1)}{2} \text{ is odd}, \end{cases}$$

where $m = \min\{A_iA_j^k\}$ and $M = \max\{A_iA_j^k\}$ In case of a triangle

$$9 \le (a^k+b^k+c^k)(a^{-k}+b^{-k}+c^{-k}) \le \frac{2M^2+5M\cdot m+2m^2}{M\cdot m},$$

in case of a tetrahedron

$$36 \le \left(a^k + b^k + c^k + d^k + e^k + f^k\right)\left(a^{-k} + b^{-k} + c^{-k} + d^{-k} + e^{-k} + f^{-k}\right) \le \frac{9(M+m)^2}{M \cdot m}.$$

7. Application. Let $A_1, A_2, \ldots, A_n$ be the vertexes of the polygon inscribed in the sphere with the center O and radius R. First we interpret the orthocenters of the $A_1A_2\ldots A_n$ inscribable polygon. For three arbitrary vertexes, one orthocenter corresponds. Now we take four vertexes. In the obtained four orthocenters of the triangles we construct the circles with radius R, which has one common point. This will be the orthocenter of the inscribable quadrilateral. We continue in the same way. The circles with radius R that we construct in the orthocenters of the n-1 sides inscribable polygons have one common point. This will be the orthocenter of the n sides, inscribeable polygon. It can be shown that O, H, G are collinears and n·OG=OH. From the first application

$$OH^2 = n^2R^2 - \sum_{1 \le i < j \le n} A_iA_j^2 \quad \text{and} \quad GH^2 = (n-1)R^2 - \left(1 - \frac{1}{n}\right)^2 \sum_{1 \le i < j \le n} A_iA_j^2.$$

In case of a triangle $OH^2 = 9R^2 - (a^2+b^2+c^2)$ and $GH^2 = 4R^2 - \frac{4}{9}\left(a^2 + b^2 + c^2\right)$.

8. Application. In the case of an $A_1A_2\ldots A_n$ inscribable polygon $\sum_{1 \le i < j \le n} A_iA_j^2 = n^2R^2$ if and only if O≡H≡G. In case of a triangle this is equivalent with an equilateral triangle.

9. Application. Now we compute the length of the midpoints created by the $A_1, A_2, \ldots, A_n$ space points system. Let S={1,2,...,i-1, i+1,...,n} and G_0 be the centroid of the A_k, $k \in S$, points system. By substituting $M \equiv A_i$ in the theorem, for the length of the midpoints we obtain the following relation $A_iG_0^2 = \frac{1}{n-1}\sum_{k \in S} A_iA_k^2 - \frac{1}{(n-1)^2}\sum_{u,v \in S,\, u \ne v} A_uA_v^2$.

10. Application. In case of a triangle $m_a^2 = \frac{2\left(b^2 + c^2\right) - a^2}{4}$ and its permutations. From here

$$m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}\left(a^2 + b^2 + c^2\right),$$

$$m_a^2 + m_b^2 + m_c^2 \le \frac{27}{4}R^2, \quad m_a + m_b + m_c \le \frac{9}{2}R.$$

11. Application. In case of a tetrahedron $m_a^2 = \frac{1}{9}\left(3\left(a^2 + b^2 + c^2\right) - \left(d^2 + e^2 + f^2\right)\right)$ and its permutations. From here $\sum m_a^2 = \frac{4}{9}\left(\sum a^2\right), \sum m_a^2 \le \frac{64}{9}R^2, \sum m_a \le \frac{16}{3}R.$

12. Application. Denote $m_{a,f}$ the length of the segments which join

midpoint of the a and f skew sides of the tetrahedron (bimedian). In the interpretation of the application

$$9m_{a,f}^2 = \frac{1}{4}\left(b^2 + c^2 + d^2 + e^2 - a^2 - f^2\right)$$

and its permutations. From here

$$m_{a,f}^2 + m_{b,d}^2 + m_{c,e}^2 = \frac{1}{4}\left(\sum a^2\right)$$

$$m_{a,f}^2 + m_{b,d}^2 + m_{c,e}^2 \leq 4R^2, \qquad m_{a,f} + m_{b,d} + m_{c,e} \leq 2R\sqrt{3}.$$

References:

1. Hajós Gy., Bevezetés a geometriába, Tankönyvkiadó, Bp. 1966.
2. Kazarinoff N.D., Geometriai egyenlötlenségek, Gondolat, 1980.
3. Stoica Gh., Egy ismert maximum feladatról, Matematikai Lapok, Kolozsvár, 9/1987, pp. 330-332.
4. Caius Jacob, Lagrange egyik képletéröl és ennek kiterjesztéséröl, Matematikai Lapok, Kolozsvár, 2/1987, pp. 50-56.
5. Sándor József, Geometriai egyenlötlenségek, Kolozsvár, 1988.

["Octogon", Vol.6, No.1, 67-70, 1998.]

A Solution of OQ. 128[1]

In "Octogon", vol. 6, Nr. 1, April 1998, Mihály Bencze proposed the following open question:

"Let $A_1 A_2 \ldots A_n$ be a convex polygon and $\{B_1\} = A_1A_3 \cap A_2A_n$, $\{B_2\} = A_2A_4 \cap A_1A_3$, ..., $\{B_n\} = A_1A_{n-1} \cap A_2A_n$. Prove that

$$\frac{A_1B_1}{B_1A_2} \cdot \frac{A_2B_2}{B_2A_3} \cdot \ldots \cdot \frac{A_{n-1}B_{n-1}}{B_nA_1} = 1".$$

Let:

$x_1 = m(A_2A_1B_1)$, $x_2 = (A_1A_2B_1)$, $x_3 = m(A_3A_1A_2)$,
$x_4 = m(A_2A_3B_2)$, ..., $x_{k+1} = m(A_{k+1}A_kB_k)$,
$x_{k+2} = m(A_kA_{k+1}B_k)$, ..., $x_{2n-1} = m(A_1A_nB_n)$,
$x_{2n} = m(A_nA_1B_n)$ and $A_kA_{k+1} = a_k$ $(k = 1, 2, ..., n)$.

Using the sinus theorem in the triangle $A_kB_kA_{k+1}$, we obtain:

$$(1) \qquad \frac{A_kB_k}{B_kA_{k+1}} = \frac{\sin x_{k+1}}{\sin x_k}.$$

Using again the sinus theorem in the triangle $A_kA_{k+1}A_{k+2}$, we obtain:

$$(2) \qquad \frac{A_kA_{k+1}}{A_{k+1}A_{k+2}} = \frac{a_k}{a_{k+1}} = \frac{\sin x_{k+3}}{\sin x_k}.$$

From (1) and (2), by multiplication, we obtain the proposed relation.

["Octogon", Vol. 7, No.1, 183-4, 1999.]

[1] Together with Marian Dinca, Mihály Bencze, and Szilárd András.

NEGĂRI ALE POSTULATULUI V AL LUI EUCLID

Postulatul V al lui Euclid se enunţă sub forma: dacă o dreaptă, care intersectează două drepte, formează unghiuri interioare de aceeaşi parte mai mici decât două unghiuri drepte, aceste drepte, prelungite la infinit, se întâlnesc în partea unde unghiurile interioare sunt mai mici decât două unghiuri drepte.

Însă el este mai cunoscut sub forma: printr'un punct exterior unei drepte se poate duce o paralelă şi numai una la acea dreaptă.

În acest articol vom prezenta cele două negări clasice (Lobacevski-Bolyai-Gauss şi Riemann), plus altă negare parţială (combinând, totuşi, negările anterioare).

Postulatul V al lui Euclid (315? - 255? î.C.) este recunoscut, de toată lumea, consistent (logic) în sine, dar şi împreună cu celelalte patru postulate formează un sistem axiomatic consistent.

Întrebarea, care s-a pus din antichitate, era dacă al cincilea postulat este dependent de celelalte patru? Pentru că un sistem axiometic, în viziune clasică, trebuie să fie:

1) **consistent** (axiomele să nu fie contradictorii între ele: adică unele să afirme ceva, iar celelalte opusul);

* 2) **independent** (o axiomă să nu fie o consecinţă rezultată din celelalte axiome prin aplicarea unor reguli, teoreme, leme, metode valabile în acel sistem;
dacă o axiomă se dovedeşte a fi dependentă (rezultată din) de altele, se elimină din sistem;
sistemul trebuie să fie minimal);

3) **complet** (axiomele să dezvolte întreaga teorie, nu doar parţialităţi).

Deci, geometrii au crezut că postulatul (= axioma) V se deduce din primele patru postulate ale lui Euclid. Însuşi Euclid a incitat la aceste cercetări. Deci, că sistemul propus de Euclid, care a pus bazele geometriei, n'ar fi independent.

În acel caz, postulatul V ar fi putut să fie eliminat, fără a altera deloc dezvoltarea geometriei.

Au fost numeroase încercări de-a "demonstra" această "dependenţă", desigur nereuşite. Aşadar, postulatul 5 are importanţă istorică fiindcă mulţi s'au ocupat de el.

Atunci, s'a trecut la negarea postulatului 5, şi constituirea unui sistem axiomatic din primele patru postulate euclidiene neschimbate plus negaţia postulatului 5. S'a observat că se obţin geometrii total dfiferite, bizare, curioase, aparent rupte de practică.

a) Lobacevski (1793-1856), rus, primul a negat astfel:
"Printr'un punct exterior unei drepte se pot duce o infinitate de paralele la acea dreaptă",
care s'a numit **Geometrie Lobacevskiană** sau hiperbolică.

După el, independent, au făcut acelaşi lucru: Bolyai (1802-1860), ungur din Transilvania, şi Gauss (1777-1855), neamţ. Dar Lobacevski a publicat primul.

Beltrami (1835-1900), italian, a găsit şi un model (= construcţie geometrică şi convenţii în definirea noţiunilor de spaţiu, dreaptă, paralelism) la această geometrie hiperbolică, marcând un progres şi dând o importanţă ei. Analog Pointcaré (1854-1912), francez.

b) Riemann (1826-1866), neamţ, a urmat cu o altfel de negaţie:
"Printr'un punct exterior unei drepte nu se poate duce nici o paralelă la acea dreaptă",
care s'a numit **Geometrie Riemanniană** sau eliptică.

c) Smarandache (n. 1954) a negat parţial postulatul V:
"Există drepte şi puncte exterioare lor astfel încât prin acele puncte exterioare se puteau duce la acele drepte:

1) numai o singură paralelă - într'o anumită zonă a spaţiului geometric [deci, aici funcţiona Geometria Euclidiană];

2) mai multe paralele, dar în număr finit - în altă zonă a spaţiului;

3) un număr infinit de paralele, dar numărabile - în altă zonă a spaţiului;

4) un număr infinit de paralele, dar nenumărabile - în altă zonă a spaţiului [deci, aici funcţiona Geometria Lobacevski];

5) nici o paralelă - în altă zonă a spaţiului [deci, aici funcţiona Geometria Riemann].

Adică, întregul spaţiu era împărţit în cinci regiuni (zone), iar fiecare zonă funcţiona diferit. Eram elev, ideea mi-a venit în 1969. De ce? Fiindcă am observat că în practică spaţiile nu sunt pure, omogene, ci un melanj. Aşa am unit cele trei geometrii legate de postulatul V, şi le-am chiar extins (cu alte 2 zone alăturate).

Problema era: cum conectezi un punct dintr'o zonă, cu un punct din altă zonă diferită (trecerea peste "frontiere")?

În "Bulletin of Pure and Applied Science" (Delhi, India), apoi în prestigioasa revistă germană care recenzează articole de matematică "Zentralblatt für Mathematik" (Berlin) există patru variante de **Geometrii Neeuclidiene Smarandache** [urmând tradiția: Geometria lui Euclid (cea clasică, tradițională), Geometria Lobacevski, Geometria Riemann, Geometrii Smarandache]. E bine să lăsăm și noi, românii, urme prin științe și arte - ca să nu ne mai desconsidere atâta occidentalii. Mă obsedează acest lucru... Eu caut să citez mereu români în tot ce fac - pentru promovare.

Referințe:

[1] Ashbacher, Charles, "Smarandache Geometries", <Smarandache Notions Journal>, Vol. 8, No. 1-2-3, Fall 1997, pp. 212-215.

[2] Brown, Jerry L., "The Smarandache Counter-Projective Geometry", <Abstracts of Papers Presented to the American Mathematical Society Meetings>, Vol. 17, No. 3, Issue 105, 595, 1996.

[3] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Anti-Geometry", <Bulletin of Pure and Applied Sciences>, Delhi India, Vol. 17E No. 1, 103-114 1998.

[4] Chimienti, Sandy P., Bencze, Mihaly, "Smarandache Anti-Geometry", <Smarandache Notions Journal>, Vol. 9, No. 1-2, 49-63, 1998.

[5] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Counter-Projective Geometry", <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 17E, No. 1, pp. 117-118, 1998.

[6] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Counter-Projective Geomefry", <Smarandache Notions Joumal>, Vol. 9, No, 1-2, 47-48, 1998.

[7] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Non-Geometry", <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 17E, No. 1, 115-116, 1998.

[8] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Non-Geometry", <Smarandache Notions Journal>, Vol. 9, No. 1-2, 45-6, 1998.

[9] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Paradoxist Geometry", <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 17E, No. 1, 123-1124, 1998.

[10] Chimienti, Sandy P., Bencze, Mihály, "Smarandache Paradoxist

Geometry", <Smarandache Notions Journal>, AZ, USA, Vol. 9, No. 1-2, 43-44, 1998.

[11] Mudge, Mike, "A Paradoxist Mathematician, His Function, Paradoxist Geometry, and Class of Paradoxes", <Smarandache Notions Journal>, Vol. 7, No. 1-2-3, August 1996, pp. 127-129;
reviewed by David E. Zitarelli, <Historia Mathematica>, USA, Vol, 24, No. 1, p. 114, #24.1.119, 1997.

[12] Popescu, Marian, "A Model for the Smarandache Paradoxist Geometry", <Abstracts of Papers Presented to the American Mathematical Society Meetings>, Vol. 17, No. 1, Issue 103, 265, 1996.

[13] Popov, M. R, "The Smarandache Non-Geometry", <Abstracts of Papers Presented to the American Mathematical Society Meetings>, Vol. 17, No. 3, Issue 105, 1996, p. 595.

[14] Smarandache, Florentin, "Collected Papers" (Vol. II), University of Kishinev Press, Kishinev, pp. 5-28, 1997.

[15] Smarandache, Florentin, "Paradoxist Mathematics" (lecture), Bloomsburg University, Mathematics Department, PA, USA, November 1995.

[16] Torretti, Roberto, Universidad de Chile, Santiago, "A Model for the Smarandache's Anti-Geometry',
http://www.gallup.unm.edu/~smarandache/Torretti.htm and
http://www. gallup.unm.edu/~smarandache/AntiGeomTorretti.pdf .

One Application Of Wallis Theorem[1]

Theorem 1. (Wallis, 1616-1703)

$$\int_0^{\pi/2} \sin^{2n+1} x dx = \int_0^{\pi/2} \cos^{2n+1} x dx = \frac{2.4\ldots(2n)}{1.3\ldots(2n+1)}.$$

Proof. Using integration by parts, we obtain

$$I_n = \int_0^{\pi/2} \sin^{2n+1} x dx = \int_0^{\pi/2} \sin^{2n} x \sin x dx = -\cos x \cdot \sin^{2n} x \Big|_0^{\pi/2} +$$

$$+ 2n \int_0^{\pi/2} \sin^{2n-1} x \left(1 - \sin^2 x\right) dx = 2nI_{n-1} - 2nI_n$$ from where $I_n = \frac{2n}{2n+1} I_{n-1}$.

By multiplication we obtain the statement. We prove in the same way for cos x.

Theorem 2. $\int_0^{\pi/2} \sin^{2n} x dx = \int_0^{\pi/2} \cos^{2n} x dx = \frac{1.3\ldots(2n-1)}{2.4\ldots(2n)} \cdot \frac{\pi}{2}$.

Proof. Same as the first theorem.

Theorem 3. If $f(x) = \sum_{k=0}^{\infty} a_{2k} x^{2k}$, then

$$\int_0^{\pi/2} f(\sin x) dx = \int_0^{\pi/2} f(\cos x) dx = \frac{\pi}{2} a_0 + \frac{\pi}{2} \sum_{k=1}^{\infty} a_{2k} \frac{1.3\ldots(2k-1)}{2.4\ldots(2k)}.$$

Proof. In the $f(x) = \sum_{k=0}^{\infty} a_{2k} x^{2k}$ function we substitute x by $\sin x$ and then integrate from 0 to $\pi/2$, and we use the second theorem.

Theorem 4. If $g(x) = \sum_{k=0}^{\infty} a_{2k+1} x^{2k+1}$, then

$$\int_0^{\pi/2} g(\sin x) dx = \int_0^{\pi/2} g(\cos x) dx = a_1 + \sum_{k=1}^{\infty} a_{2k+1} \frac{2.4\ldots(2k)}{1.3\ldots(2k+1)}.$$

Theorem 5. If $h(x) = \sum_{k=0}^{\infty} a_k x^k$, then

$$\int_0^{\pi/2} h(\sin x) dx = \int_0^{\pi/2} h(\cos x) dx = \frac{\pi}{2} a_0 + a_1 + \sum_{k=1}^{\infty} \left(\frac{\pi}{2} a_{2k} \frac{1.3\ldots(2k-1)}{2.4\ldots(2k)} + a_{2k+1} \frac{2.4\ldots(2k)}{1.3\ldots(2k+1)} \right).$$

Application 1.

$$\int_0^{\pi/2} \sin(\sin x) dx = \int_0^{\pi/2} \sin(\cos x) dx = \sum_{k=0}^{\infty} (-1)^k \frac{1}{1^2 3^2 \ldots (2k+1)^2}.$$

Proof. We use that $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$.

Application 2.

$$\int_0^{\pi/2} \cos(\sin x) dx = \int_0^{\pi/2} \cos(\cos x) = \frac{\pi}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{4^k (k!)^2}.$$

Proof. We use that $\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$.

Application 3.

$$\int_0^{\pi/2} sh(\sin x) dx = \int_0^{\pi/2} sh(\cos x) dx = \sum_{k=0}^{\infty} \frac{1}{1^2 3^2 \ldots (2k+1)^2}$$

Proof. We use that $sh\ x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$

[1] Together with Mihály Bencze.

Application 4. $\int_0^{\pi/2} ch(\sin x)dx = \int_0^{\pi/2} ch(\cos x)dx = \frac{\pi}{2}\sum_{k=0}^{\infty}\frac{1}{4^k (k!)^2}$.

Proof. We use that $ch\ x = \sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)\ !}$.

Application 5.

$\sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}$.

Proof. In the $\arcsin x = x + \sum_{k=1}^{\infty}\frac{1.3.....(2k-1)x^{2k+1}}{2.4.....(2k)(2k+1)}$ expression we substitute x by sin x, and use theorem 4. It results that $\pi^2/8 = \sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}$. Because $\sum_{k=1}^{\infty} 1/k^2 = \sum_{k=0}^{\infty}\frac{1}{(2k+1)^2} + \frac{1}{4}\sum_{k=1}^{\infty} 1/k^2$

we get $\sum_{k=1}^{\infty} 1/k^2 = \pi/6$.

Application 6.

$$\int_0^{\pi/2}\sin x\ \ ctg(\sin x)dx = \int_0^{\pi/2}\cos x\ \ ctg(\cos x)dx = \frac{\pi}{2} - \frac{\pi}{2}\sum_{k=1}^{\infty}\frac{B_k}{(k!)^2}$$

where B_k is the k-th Bernoulli type number (see[1]).

Proof. We use that $x ctg\ x = 1 - \sum_{k=1}^{\infty}\frac{4^k B_k}{(2k)\ !}x^{2k}$.

Application 7.

$$\int_0^{\pi/2} arctg\ (\sin x)dx = \int_0^{\pi/2} arctg\ (\cos x)dx =$$
$$= 1 + \sum_{k=1}^{\infty}(-1)^k\frac{2.4.....(2k)}{1.3.....(2k-1)(2k+1)^2} .$$

Proof. We use that $arctg\ x = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$.

Application 8.

$$\int_0^{\pi/2}\arg th(\sin x)dx = \int_0^{\pi/2}\arg th(\cos x)dx = 1 + \sum_{k=1}^{\infty}\frac{2.4....(2k)}{1.3....(2k-1)}\frac{1}{(2k+1)^2} .$$

Proof. We use that $\arg th\ x = \sum_{k=0}^{\infty}\frac{x^{2k+1}}{2k+1}$.

Application 9.

$$\int_0^{\pi/2}\arg sh(\sin x)dx = \int_0^{\pi/2}\arg sh(\cos x)dx = \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)^2} .$$

Proof. We use that $\arg sh\ x = \sum_{k=0}^{\infty}(-1)^k\frac{1.3....(2k-1)x^{2k+1}}{2.4....(2k)(2k+1)}$.

Application 10.

$$\int_0^{\pi/2} tg(\sin x)dx = \int_0^{\pi/2} tg(\cos x)dx = \sum_{k=1}^{\infty}\frac{2^{2k-1}(4^k-1)B_k}{1^2 3^2 ..(2k-1)^2 k} .$$

Proof. We use that
$$tg\ x = \sum_{k=1}^{\infty}\frac{2^{2k}(4^k-1)B_k}{(2k)\ !}x^{2k-1} .$$

Application 11.

$$\int_0^{\pi/2} \frac{\sin x}{\sin(\sin x)}dx = \int_0^{\pi/2} \frac{\cos x}{\sin(\cos x)}dx = \frac{\pi}{2} + \pi \sum_{k=1}^{\infty} \frac{\left(2^{2k-1}-1\right)B_k}{2^{2k}(k!)^2} .$$

Proof. We use that $\frac{x}{\sin x} = 1 + 2\sum_{k=1}^{\infty} \frac{\left(2^{2k-1}-1\right)B_k}{(2k)!}x^{2k}$.

Application 12.

$$\int_0^{\pi/2} \frac{\sin x}{sh(\sin x)}dx = \int_0^{\pi/2} \frac{\cos x}{sh(\cos x)}dx = \frac{\pi}{2} + \pi \sum_{k=1}^{\infty} \frac{\left(2^{2k-1}-1\right)B_k}{2^{2k}(k!)^2} .$$

Proof. We use that $\frac{x}{sh\ x} = 1 + 2\sum_{k=1}^{\infty} (-1)^k \frac{\left(2^{2k-1}-1\right)B_k x^{2k}}{(2k)!}$.

Application 13.

$$\int_0^{\pi/2} \sec(\sin x)dx = \int_0^{\pi/2} \sec(\cos x)dx = \frac{\pi}{2} + \pi \sum_{k=1}^{\infty} \frac{E_k}{2^{2k+1}(k!)^2} ,$$

where E_k is the k-th Euler type number (see([1]).

Proof. We use that $\sec x = 1 + \sum_{k=1}^{\infty} \frac{E_k}{(2k)!}x^{2k}$.

Application 14.

$$\int_0^{\pi/2} \operatorname{sech}(\sin x)dx = \int_0^{\pi/2} \operatorname{sech}(\cos x)dx = \frac{\pi}{2} + \pi \sum_{k=1}^{\infty} (-1)^k \frac{E_k}{2^{2k-1}(k!)^2} .$$

Proof. We use that

$$\operatorname{sech} x = 1 + \sum_{k=1}^{\infty} (-1)^k \frac{E_k}{(2k)!}x^{2k} .$$

References:

[1] Octav Mayer: Teoria funcţiilor de o variabilă complexă, Ed. Academică, Bucureşti, 1981.

[2] Mihály Bencze, About Taylor-formula (manuscript).

["Octogon", Vol.6, No.2, 117-20, 1998.]

LINGUISTIC PARADOXES AND TAUTOLOGIES

Abstract.

Classes of linguistic paradoxes are introduced with examples and explanations. They are part of the author's work on the Paradoxist Philosophy based on mathematical logic.

The general cases exposed below are modeled on the English language structure in a rigid way. In order to find nice particular examples of such paradoxes one grammatically adjusts the sentences.

Let <N>, <V>, <A> be some noun, verb, and attribute respectively, and <Non-N>, <Non-V>, <Non-A> respectively their antonyms. For example, if <A> is <small> then <Non-A> is <big> or <large>, etc.

Also let <N'>, <N " >, etc. represent synonyms of <N> or just <N>, and so <V'>, <V " >, etc.

or <A'>, <A " >, etc.

Let <NV> represent a noun-ed verb, and <NV'> a synonym, etc. Then, one defines the following classes of linguistic paradoxes and semi-paradoxes:

1. All is <A>, the <Non-A> too.

Examples:

All is possible, the impossible too.
All is real, the unreal too.
All is justice, the injustice too.
All life is complex, the simple life too.
All people are actors, the non-actors too.
All can be happy, even the unhappy.
It's so near, but yet so far away.
All is weird, the natural too.
All is joyful, the sorrow too.

2. <Non-N> is a better <N>.
<Non-A> is a better <A>.
<Non-V> is a better <V>.

Examples:

Not to speak is sometimes a better speech.
Not to complain is a better complain.
Unattractive is sometimes better than attractive.
Slow is sometimes better than fast.
No government is a better government.
A non-ruler is a better ruler.
No news is good news.
Not to stare is sometimes better to look.
Not to love is a better love.
Not to move is sometimes a better move.
Impoliteness is a better politeness.
Not to hear is better than not listening.
No reaction is sometimes the best reaction.
Not to show kindness is a better kindness [welfare].
She is better than herself.
No fight is a better fight [i.e. to fight by non-violent means].

3. Only <N> is truly a <Non-N>.
Only <A> is truly a <Non-A>.
Examples:
Only a rumor is truly a gossip.
Only a fiction is truly a fact.
Only normal is truly not normal.
Nobody is truly a 'somebody'.
Only fiction is truly real.
The friend is the most dangerous hidden enemy.
Only you are truly not you [=you act strangely].
Only mercy can be truly merciless.
If you spit at the sky, it will fall in your face.
Only gentleness is truly wild.

4. This is so <A>, that it looks <Non-A>.
Examples:
This is so true, that it looks false!
This is so ripe, that it looks spoilt.
This is so friendly, that it looks hostile.
He seemed so trustworthy, that he looks untrustworthy.

This is so fake, it looks real!
This is so proper, that it looks improper.
This is so beautiful, that it looks unreal.
This is so simple, that it looks difficult.
The story was so real, that it looked fiction.
Can't see the trees for the forest.

5. There is some <N> which is <A> and <Non-A> at the same time.

Examples:

There are events which are good and bad at the same time.
There are laws which are good and bad at the same time.
There are some news which are real and wrong at the same time.
There are some insects which are helpful and dangerous at the same time. [like the spider]
There are men who are handsome and ugly at the same time.
There are classes that are fun and boring at the same time.
There are some ministers which are believers and mis-believers at the same time.
There are moments that are sweet and sour.
There are games which are challenged and not competitive at the same time.
Food which are simultaneously hot and cold.
The game was exciting, yet boring [because we were losing].
People are smart and foolish at the same time [i.e., smart at something, and foolish at other thing].

6. There is some <N> which <V> and really <Non-V> at the same time.

Examples:

There are people who trick and do not really trick at the same time.
There are some children who play and don't play at the same time. Some of life's experiences are punishments and rewards at the same time.
Exercise is exhausting but also invigorating.
There are children who listen and do not really listen at the same time.
There are teachers who teach and don't teach at the same time.
There are students who spell and misspell at the same time.
Nice and rough men concomitantly.

Politicians who lie and tell the truth all the time!

7. To <V>, even when <Non-V>.

Examples:

A sage thinks even when he doesn't think.
I exist when I don't exist.
A clown is funny even when he isn't being funny.
To die of thirst surrounded by water. [saltwater]
To be a poet and not know it.
A mother worries even when she doesn't worry.
To believe even when you don't believe
Is matching even when not-matching.
I sleep even when I am awake.
Always running around.
To dream, even when not sleeping.

8. This <N> is enough <Non-N>.

Examples:

This silence is enough noise.
This vacation from work is hard work. [when you come back!]
The superiority brings enough inferiority. [=listlessness]
This day is my night.
This diary is enough non-diary.
This sleep is enough awake.
This sweet truthfulness is enough sarcasm.
This table of four is enough for six people.
I had enough.
This job is enough recreation. [when enjoying the job]

9. <Non-V> sometimes means <V>.

Examples:

Not to speak sometimes means to speak.
Not to touch sometimes means to touch.
The preserve peace sometimes means going to war.
To destroy life (as in viruses) sometimes means to preserve life. Not to listen sometimes means to listen.
Two feet forward sometimes means standing still.

Not to litter sometimes means to litter.
Speeding is sometimes not speeding [in case of emergency].
Not to show anger is sometimes to show anger.

10. <N> without <N>.

Examples:
Hell without hell.
The style without style.
The rule applied: there were no rule!
Our culture is our lack of culture.
Live without living.
Some people are so afraid of death, that they do not live.
Work without work.
Can't live with them, can't live without them.
Death without death. [for a Christian dying is going on to eternal life]
Guilt without guilt. [sometimes is guilty but doesn't feel guilty]

11. a) <N> inside/within the <Non-N>.

Examples:
Movement inside the immobility.
Silence within the noise.
Slavery within the freedom.
Loneliness within a crowd.
A circle within a circle.
The wrestling ring inside a squared section.
To find wealth in poverty [i.e., happiness and love].

b) <Non-N> in the <N>.

Examples:
Immobility inside the movement.
Noise inside the silence.
The eye of the storm.
Government. Bureaucracies.
Inequality inside the equality.
Single inside the marriage.
Anger inside the happiness.
Warmth in the cold.

Cold in the heat.
Laughing without being happy.
Has not gotten anywhere.
Poverty in wealth. [no poverty or love in a wealth family]

12. The <A> of the <Non-A>.

Examples:

The shadow of the light.
Music of silence.
Relaxing of exercise effect.
The restrictions of the free.
Life through death.
The sound/loudness of the silence.
I can see the light of the tunnel.
The slave of freedom. [someone who couldn't give up his freedom, even in marriage]

13. <V> what one <Non-V>.

Examples:

To see what one can't see.
To heat what one can't hear.
To taste what one can't taste.
To accept what on can't understand.
To say what one can't say. [to tell a secret]
To wait patiently when one doesn't know how to wait.
To breath what one can't breath.
To feel what one can't feel.
To appreciate what one dis-appreciates.
To believe what one can't believe. [faith]
To smell what one can't smell.

14. Let's <V> by <Non-V>.

Examples:

Let s strike by not striking. [=Japanese strike]
Let's talk by not talking: [means to think].
To vote by not voting at all.
To help someone by not helping. [using experience as a teacher]

Let's justify by not justifying.
Let's win by not winning.
Let's strip by not stripping. [to make bare or clear]
Let's fight by not fighting. [Ghandi's Motto]

15. <N> of the <Non-N>.
Examples:
The benefits we get from non-benefits.
The smoke we got from non-smokers.
The easy work we get from hard work.
The service we get from non-service.
The good that comes from bad.
The pleasure we get from the pain.

16. <Non-A> is <A>.
Examples:
The bad is good. [because makes you try harder]
The good is bad. [because doesn't leave any room for improvement]
Work is a blessing.
The poor is spiritually rich.
Sometimes ugly is beauty. [because beauty is in the eyes of the beholden]
You have to kiss a lot of frogs before you find a prince.
Hurt is healing.
"There is no absolute" is an absolute.
Not to commit any error is an error.

17. A <Non-N> <N>.
Examples:
A positive negative. [which means: a failure enforces you to do better]
A sad happiness.
An impossible possibility.
Genuine imitation leather.
A loud whisper.
A beautiful disaster. [which means beauty can be found anywhere] A bitter sweet.
A harsh gentleness. [a gentleness that is very firm with you]
A guiltless sinner. [someone who doesn't regret sinning]

18. Everything has an <A> and a <Non-A>.

Examples:

Everything has a sense and a non-sense.
Everything has a truthful side and a wrong side.
Everything has a beginning and an ending.
Everything has a birth and a death.
Everything has its time and a non-time.
Everything has an appearance and a non-appearance.
Everything around you resolves and also dissolves.
Everybody has a good side and a bad side.
Everyone has a right and a wrong.

19. <V> what <Non-V>.

Examples:

To be what you are not.
One needs what one doesn't need.
Expect the unexpected!
Culture exists by its non-existence.
No matter how rich we are, we never make enough money.
One purchases what one doesn't purchase.
To work when we are not working.
To die might mean to live for ever. [= for an artist]
One wants because one doesn't want. [sometimes one wants something only because someone else likes it]

Linguistic Tautologies:

A tautology is a redundacy, a pleonasm, a needless repetition of an idea, according to the "Webster's New World Dictionary", Third College Edition, 1988.
However, the following classes of tautologies - using repetition - go to a deeper meaning, and even changes the sense. A double assertation reverses to a negation.
One also may play with the synonyms.

20. Mirror semi-paradox:
<N> of the <N'>.

<N> of the <N'> of the <N" >...

Examples:

Best of the best.
Worst of the worst of the worst.
Mother of the mother of the mother... [the maternal grand grandmother]
Follower of the followers.
The rows of the rows. [lots of rows]

21. This is not an <N>, this is an <N'>.

Examples:

This is not a teacher, this is a professor.
This is not a car, this is a Wolswagen.
This is not a truck, this is a Chevy.
This is not noise, this is music.
This is not music, this is noise.
This is not a cedar tree, this is a <gad>. [gad = Navaho name for cedar tree]
This is not me, this is I.
This is not a sword, this is a saber.
This is not a problem, this is an exercise. [= easier]
Practice makes you practice.
This is not a girl, this is Katie.
This is not a horse, this is a pony.

22. <N> is not enough <N'>.
<A> is not enough <A'>.

Examples:

Sufficient is not enough sufficient [which means: to do more than "sufficient"].
Punishment is not enough punishment.
Health is not enough wealth.
Clean is not enough clean.
Studying is not enough studying. [which means to do more than just getting by, i.e. to do research]
Extravagant is not enough extravagant.
Time is not enough time.
The more you have, the more you want.

Attention is not enough attention. [some people need action too]

23. More <A> than <A'>.

Examples:

Better than better. [=perfection]
Worst than worst. [=evil]
Sweeter than sweeter. [=honey]
More life than life. [=spirituality]
More depressed than depressed.
Foster than foster.
More beautiful than pretty.
More ugly than ugly. [really ugly]
Smarter than smart. [like a genius]

24. How <A> is an <A'> <N>?

Examples:

How democratic is a so called democratic society.
How republican is a so called republic an society?
How civilized is a so called civilized person?
How free is a free country?
How commanding is a so called commanding officer?
How Pop Culture is a so called Pop Culture?
How strong is a strong man?
How lone is a lone ranger? [not very, he has tanto]

25. No <A> is really <A'>.

Examples:

No friend is really a friend. [s/he betrays you when you don t even expect!]
No luck is really a luck.
No original is really original.
No husband is really a husband. [you learn to depend on yourself] No tomboy is really a tomboy. [girl considered boyish]
No work is really less work.
No true Marxist is really a true Marxist. [they contradict their own beliefs]
No magic is really magic [all is only a trick].

26. I would rather prefer <A>, than <A'>.

Examples:

I would rather prefer pretty, than prettier.
I want that, not that.
I would rather prefer this, than this.
I would rather be old, than *old*.
I would rather prefer great than big.
I would rather be crazy than crazy. [crazy like foolish, than crazy like insane]

27. More <A> than <A'>.

Examples:

Prettier than pretty.
More real than real.
More advantage than advantage.
More help than help.
More smiles than smiles. [she didn't psychically smile, but there were smiles written over her face]
He earns more than himself.
More suspicious than suspected.
More cries than cries.
More meters than kilometers.
Make everyday a rainbow day.

28. <V> those who <V'> you.

Examples:

Ignore those who ignore you.
Criticize those who criticize you.
Defend those who defend you.

29. <V>, because <V'>.

Examples:

I want because I want.
I think because I think.
I hear because I listen.
I see because I look.
I need because I need.

I know because I know.
I live because I live.
I believe what is unbelievable. [faith]
I am happy because I am happy. [there is no reason for my happiness]

30. <V> the <NV'>.
Examples:
I hate the haters. [therefore I hate myself!]
I envy the enviers. [therefore I envy myself]
I am strange to strangers.
I cheat the cheaters. [therefore I cheat myself]
I lie to liars. [therefore I lie to myself]
I kick the kickers. [therefore I kick myself]
I love the lovers.

Exercises for readers:

Try to construct a general scheme - using <N>, <V>, <A>, etc. notations as above - and then give particular cases for each of the following paradoxes or semi-paradoxes:

- Dream the impossible dream.
- It is not a question of what we are, but more of who we are.
- Only a small dream/output is really a big dream/output.
- One vote is enough to make a difference; and yet one vote isn't often enough to make a difference.
- Good times come and good times go, but memories last for lifetime.
- The dark and light of infinity.
- The sense we get from non-sense.
- Think before you think.
- Sometimes less is more.
- Enjoy life today, tomorrow may never come.
- Make it happen, by making it happen.
- Less is more.
- My shoes are cleaner than my feet.
- No matter how hard it seems, it will get easier.
- See the things as they are not. [see their hidden spot]
- Quitters never win, and winners never quit.

- My needs exist for needs.
- Bad things happen for a good reason.

Look at this Funny Law example:
A Paradoxist Government:
Suppose you have two cows. Then the government kills them and milks you!

A poem:

Sometimes in life we see but do not have sight
or don't see what we should see
we hear but do not listen
we speak but do not communicate
we live but do not know how to live
we love but do not love
And then we die but we've already have been dead

The list of such invented linguistic paradoxes can be indefinitely extended. It is specific to each language, and it is based on language expressions and types of sentence and phrase constructions and structures.

One can also play with antonymic/synonymic adverbs, prepositions, etc. to construct other categories of linguistic paradoxes.

References:

[1] Ashbacher, Charles, "The Most Paradoxist Mathematician of the World', by Charles T. Le", review in <Journal of Recreational Mathematics>, USA, Vol. 28(2), 130, 1996-7.

[2] Begay, Anthony, "The Smarandache Semantic Paradox", <Humanistic Mathematics Network Journal>, Harvey Mudd College, Claremont, CA, USA, Issue # 17, 48, May 1998.

[3] Le, Charles T., "The Smarandache Class of Paradoxes , <Bulletin of the Transilvania University of Braşov>, Vol. 1 (36), New Series, Series B, 7-8, 1994.

[4] Le, Charles T., "The Smarandache Class of Paradoxes , <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 14 E (No. 2), 109-

110, 1995.

[5] Le, Charles T., "The Most Paradoxist Mathematician of the World: Florentin Smarandache", <Bulletin of Pure and Applied Sciences>, Delhi, India, Vol. 15E (Mathematics & Statistics), No. 1, 81-100, January-June 1996.

[6] Le, Charles T., "The Smarandache Class of Paradoxes", <Journal of Indian Academy of Mathematics>, Indore, Vol. 18, No. 1, 53-55, 1996.

[7] Le, Charles T., "The Smarandache Class of Paradoxes / (mathematical poem)", <Henry C. Bunner / An Anthology in Memoriam>, Bristol Banner Books, Bristol, IN, USA, 94, 1996.

[8] Mitroiescu, I., "The Smarandache Class of Paradoxes Applied in Computer Sciences", <Abstracts of Papers Presented to the American Mathematical Society>, New Jersey, USA, Vol. 16, No. 3, 651, Issue 101, 1995.

[9] Mudge, Michael R., "A Paradoxist Mathematician: His Function, Paradoxist Geometry, and Class of Paradoxes", <Smarandache Notions Journal>, Vail, AZ, USA, Vol. 7, No. 1-2-3, 127-129, 1996.

[10] Popescu, Marian, "A Model of the Smarandache Paradoxist Geometry", <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 1, Issue 103, 96T-99-15, 265, 1996.

[11] Popescu, Titu, "Estetica Paradoxismului", Editura Tempus, Bucharest, 26, 27-28, 1995.

[12] Rotaru, Ion, "Din nou despre Florentin Smarandache", <Vatra>, Tg. Mureş, Romania, Nr. 2 (299), 93-94, 1996.

[13] Seagull, Larry, "Clasa de Paradoxuri Semantice Smarandache" (in translation), <Abracadabra>, Salinas, CA, USA, Anul 2, Nr. 20, 2, iunie 1994.

[14] Smarandache, Florentin, "Mathematical Fancies & Paradoxes", <The Eugene Strens Memorial on Intuitive and Recreational Mathematics and its History>, University of Calgary, Alberta, Canada, 27 July - 2 August, 1986.

[15] Vasiliu, Florin, "Paradoxism's main roots", Translated from Romanian by Ştefan Benea, Xiquan Publishing House, Phoenix, USA, 64 p., 1994;
reviewed in <Zentralblatt für Mathematik>, Berlin, No. 5, 830 - 17, 03001, 1996.

[16] Tilton, Homer B., "Smarandache's Paradoxes", <Math Power>, Tucson, AZ, USA, Vol. 2, No. 9, 1-2, September 1996.

[17] Weisstein, Eric W, "CRC Concise Encyclopedia of Mathematics", <Smarandache Paradox>, CRC Press, USA, 1998.

[18] Zitarelli, David E., "Le, Charles T. / The Most Paradoxist Mathematician of the World", <Historia Mathematica>, PA, USA, Vol. 22, No. 4, # 22.4.110, 460, November 1995.

[20] Zitarelli, David E., "Mudge, Michael R. / A Paradoxist Mathematician: His Function, Paradoxist Geometry, and Class of Paradoxes", <Historia Mathematica>, PA, USA, Vol. 24, No. 1, 24.1.119, 114, February 1997.

["Libertas Mathematica", editor C. Corduneanu, University of Texas at Arlington, USA, Vol. XIX, 143-154, 1999.]

FUNNY PROBLEMS

In this paper we present original or collected recreational mathematical problems.

1) Prove that 2 = 1.

Solution:
2 pints = 1 quart.

2) A man weights the following weights on the following dates. How is it possible?

6/1/70	150 lbs.
6/3/70	0 lbs.
6/5/70	25 lbs.
6/7/70	0 lbs.
6/9/70	145 lbs.

Solution:

Man is astronaut who went to Moon and back.
Outerspace weightlessness: 0 lbs.
$\frac{1}{6}$ of his Earth Gravity, or Gravity of Moon: 25 lbs.

3) If you have a couple of three's and divided them in half, why do you end up with 4 pieces?

Solution:
~~3 3~~

4) How 70 > 3 = LOVƐ?

Solution:
Move the characters up or dow, or reverse them.

5) 10 - 1 = 0.

Solution:
If you have a stick (1) and egg (0) and you give away the stick (1), you still have the egg (0) left.

7) Twelve minus one is egual to two.

Solution:
12 - 1 = 2 (take di git 1 out from 12).

8) 7 + 7 = $\square$

Solution:
Take the four sticks from the 7's, rearrange them to form a rectangle.

9) 3 x 2578 = hɛLL

Solution:
Read your calculator upside down: 7734 (the product of the first two numbers) becomes hɛLL (aproximatively).

10) An earthworm is cut down the middle. How many halves are there?

Solution:
One,
because the other half can still be one whole earthworm !

11) From two false hypotheses get a true statement.

Examples:

a) Grass is edible. False
Edible things are green. False
Therefore, grass is green. True

b) All dogs are poodles.
Spot is a dog.
Thus, spot is a poodle.

12) How can you add 3 with 3 and get 8 ?

Solution:

Ɛ3 = 8

13) If 10 trees fall down, and no one is around to hear them falling,

how many of the trees fall ?

<u>**Solution**</u>:
Ten.

14) When algebraically 1 = 0 ?

<u>**Solution**</u>:
In a null ring, wich is a set with one element only, and one binary operation. If we take for "+" and for "·" in the same time this operation, we get a commutative, unitary ring.
In this case the unitary element for "·" (wich normally is noted by "1") and the null element for "+" (wich normally is noted by "0") coincide.

15) When is it possible to hace : 1 + 1 = 10 ?

<u>**Solution**</u>:
In base 2.

16) Another logic:
How can we have ten divided by two egual to zero ?

<u>**Answer**</u>:
Ten cookies divided by two kids are eaten and nothing is remained !

17) You are lost and walking down a road. You want to get to town and know the road leads to town but don't know wich direction. You meet two twin boys. You know one boy always tells the truth and one always lies. The boys know the direction to town. You cannot tell the boys apart and can only ask one question to one boy to find out the direction to town. What question would you ask?

<u>**Solution**</u>:
Ask either boy what the other boy would say is the direction to town. This would be a lie because if you were asking the dishonest boy, he would tell you a lie. If you were asking the honest boy he would tell you the truth about what the dishonest boy would say (wich would be a lie) so he would give you the wrong direction. Town wold be in the opposite direction.

18) Why are manhole covers round?

You know, the manholes on the streets, is there a reason why they made them round or could they be square or triangular?

Solution:

Manhole covers are round because a circle cannot fall inside of itself. If they were square, triangular or some other shape they could be dropped into the hole, wich would be dangerous to traffic.

19) You have eleven lines. How can you move five lines and still have nine?

| | | | | | | | | | |

Solution:

| | | | | | | | |

<-move-->

to form:

|\| | |\| E

20) You have a cannon and two identical cannon balls. You take the cannon to a large open location that is perfectly flat and you adjust the cannon barrel so that it is perfectly level. You load one of the cannon balls into the cannon and you hold the other cannon ball at the same height as the barrel. You fire the cannon and drop the other cannon ball at the same time. Wich cannon ball will hit the ground first?

Solution:

Both cannon balls should hit the ground at the same time, since gravity acts equally on two objects having the same mass. The cannon barrel was leveled and the cannon ball would begin to fall as it moved forward out of the barrel at the same rate as the cannon ball that dropped by hand. They would hit at the same time but the cannon ball fired from the cannon would hit the ground far away.

21) I am invisible but can be mesured. I affect everyone and everything that is anything. I span the universe and change from place to place. What am I?

Solution:

I am “gravity”.

22) The Moon rotates at a rate of one rotation to every 27.3 Earth days and revolves around the Earth at a rate of one revolution to every 27.3 Earth days. This seems to be a strange coincidence. How does it relate to our perception of the Moon as veiwed from Earth?

Solution:

People on Earth only see one side of the Moon because the same side is always facing us. If you lived on the far side of the Moon you would never see Earth. Man first saw the far or “dark side of the Moon” in the 1960’s.

23) A semantic puzzle:

GEOMETRY IS THE MEASUREMENT OF THE WORLD, THE GEO, THE SAME GEO WE PICTURE OR GRAPH IN GEOGRAPHY. THESE ARE EASY AND SENSIBLE. THE ONE I COULD NEVER MAKE HEADS OR TAILS OF, THOUGH IS

TRI ... GON (O) ... METRY

METRY IS MEASURE AND TRI IS THREE. BUT WHAT THE HECK’S A GON-(O) THAT ONE HAS TO HAVE THREE OF IT TO METRY?

EXPONENTIAL SILLINESS . . .

24) What is a hungry man’s multiplication factor?

Solution:

8 x 8.

25) Spell out the number NINE!

Solution:

| | | | | | | | | | | |
(cleven bars!)

26) There are two 24 x 24 corrals. In each corral there are 6 steers. The farmer expects to produce a calf from each steer. How many calves will be produced?

Solution:
Zero! (Steers can't produce calves.)

27) How would a mathematician measure the intensity of an earthquake on a meter as in the movie Armageddon?

Solution:
It is impossible to have an earthquake on a meteor!

28) 15 Hunters Went Bear Hunting. One Killed 2 Bears. How Many Bears Have One Killed?

Solution:
Two. ("One" is the name of one of the hunters.)

29) w /2 = u. Find a logic for this equality.

Solution:
Double "u" divided by 2 is "u".

HOW TO CREATE A PROGRAM ON T.I.-92 CALCULATORS

Here it is a program created on T.I.-92 Graphing Calculator to simplify a given fraction. The program has two inputs: N (the numerator) and D (the denominator) of the initial fraction, and two outputs: a (the numerator) and b (the denominator) of the simplified fraction. Also, the program tells you if a fraction is undefined.

The steps are the following:

- press APPS (applications);
- move the menu bar down and select 7: Program Editor;
- press Enter;
- move the new menu bar down and select 3: New (new program);
- press Enter;
- move this other menu bar down (using) to: Variable and

type your program's name, say FRACT (fraction);

- press Enter twice;

the first two lines of your program and the last line of the program are displayed on the screen ; type n, d on the first line in between the empty parentheses of the title, i.e. FRACT (n,d) ,

where n,d are parameters of the program

(n is the numerator, d is the denominator);

- press F2 and select 9: © (which meens comment :

it is ignored by the calculator, but is useful to someone reading the program);

type: This program simplifies a fraction.

- press Enter to move down to the next line;
- press F3 and select 2: Disp (display);
- press Enter ; hence Disp will be pasted up in the program;
- type on the same line "N=", n , "D=", d , where N is the numerator and D the denominator of the fraction;
- press Enter for moving again to the next line;
- press F2 and select 2: If ... then ;

press Enter and select 2: If ... then ... else ... endif,

press Enter again ;

on the screen you will get three new lines (related to IF instruction);

type, after If, d = 0
- press F3 and select 2: Disp ;
press Enter ;
- type "undefined fraction";
- use the ↓ arrow to move to the next line after **Else**
- type n/(gcd (n,d)) → a ;
you get gcd from 2nd MATH , then select 1 : Number , then select C: gcd(; gcd is the greatest common divisor ;
you get → , wich means store, from STO→ ;
- similar thing; type on the next line: d/(gcd (n,d)) → b
- press Enter ;
- press F3 , select 2: Disp , press Enter ;
type "Simplified fraction is" ;
- on the next line: press F3 , select 2: Disp , press Enter ;
- type a, "/", b
- type 2nd Quit to exit the program.

*

The program will look on the screen in the following way :

```
: fract (n,d)
: Prgm
:© This program simplifies a fraction.
: Disp "N=", "D=", d
: If d=o Then
: Disp "Undefined fraction"
: Else
: n/(gcd (n,d)) →a
: d/(gcd(n,d))→b
: Disp "Simplified fraction is"
: Disp a, "/", b
: EndIf
: EndPrgm
```

*

Now, **to call the program** , in the home page, type :
FRACT (8,0) and press Enter .

The answer, you'll get, is: Undefined fraction , because $\frac{8}{0}$ is undefined. Press 2nd Quit to exit the Prgm IO 's page.
Try again by typing, for example :
FRACT (42,54) and press Enter .
The new answer on the screen is :

```
N=
4 2
D=
5 4
Simplified fraction is
7
/
9
```

i.e. $\frac{42}{54} = \frac{7}{9}$.

A PROGRAM (AN IN-OUT MACHINE) ON T.I.-83 CALCULATORS

This program takes an input number, performs some rule on it, and shows the output number that results.

Steps to Follow	Comments
1. Press PRGM, highlight NEW, and press ENTER.	This is how you start to write a new program.
2. Type INOUT, and press ENTER.	INOUT will be the name of the program.
3. Press PRGM, highlight I/O, highlight Disp, and press ENTER.	Disp is used for showing something on the screen.
4. Press 2nd ALPHA.	The blinking A means you are using the letters above the keys.
5. Type "INPUT NUMBER", and press ENTER.	The blank spacer is the "⊔" symbol.
6. Press PRGM, highlight I/O, highlight Input, and press ENTER.	Input is used for getting a value from the program user and storing it.
7. Press ALPHA, and type I, and press ENTER.	The input number is stored in a bin labeled with the letter I.
8. Press 2nd ALPHA, and type "APPLY RULE:", and press ENTER.	This line will act as a marker for the rule which follows.
9. Press ALPHA, and type I.	You start your rule with the value given as the input.
10. Press each of the following keys: + 4 STO→ ALPHA O, and press ENTER.	The rule is to add 4 to the input value and store (→) the result in a bin labeled with the letter O.
11. Press PRGM, highlight I/O, highlight Disp, and press ENTER.	
12. Press 2nd ALPHA, type "OUTPUT NUMBER", and press ENTER.	
13. Press PRGM, highlight I/O, highlight Disp, press ENTER, press ALPHA, type O, and press ENTER.	The value that is stored in the bin labeled with the O is shown on the screen.
14. Press QUIT.	This will get you out of the programming area and back to the home screen.

To use the program:

Press PRGM, highlight EXEC, highlight the program number, and press ENTER. The screen will show the name of the program. If this is correct, press ENTER. Type a number as an input value, and you will get the corresponding output value. The program will execute again by pressing ENTER.

To end the program, press QUIT.

NOTES ON USING T.I.-82 CALCULATORS TO FIND SUMS

Definition: The set of integers is denoted *Z*. (After the German word for number, *Zahl*).

Definition: *A sequence* is the range of a function whose domain is a subset of *Z*.

Example: Define $f(n) = 2n + 1$ for n in the set $\{1,2,3,4,5,...\}$. To determine the sequence produced by f just plug some of the first few values of the domain.

$f(1)=3, f(2)=5, f(3)=7, f(4)=9,\ldots$ So *f* produces all of the odd integers beginning with 3.

We say that the sequence is the set $\{3, 5, 7, 9,..\}$.

A common kind of question that you have probably seen in your past math classes and on many kinds of standardized tests is "find the next term in the sequence...".

For example find the next term in the sequence $\{-1, 2, 5, 8, \_\_\_,...\}$

Find the number was easy. A little harder is coming up with a formula for that sequence, give that a whirl, clearly state what the domain is.

We will use the calculator to produce the numbers in a sequence. Then we can do things like add up all of those numbers. Here's how it works:

The sequence above could be written $f(n) = 3n - 4$. Suppose we want to produce all of the values in the sequence for n = 1 to 50. That sequence would be $\{-1, 2, \ldots, 146\}$. To get atl of these numbers in your calculator carry out the following steps:

LIST
[2nd] [STAT]

You will now see this OPS MATH

1: Sort A(
2: Sort D(
3: dim

4: Fill (

5: seq(

Select 5: (press the 5 key)

You will now see seq(on the screen. Enter the following:

the formule

seq(3x- 1,x,1,50,1).

the "step" or increment for x

specifies the variable

ending value for x

starting value for x

Now press ENTER . The sequence should appear on your screen. You won't be able to see any of the numbers past '14', unfortunately, but they are in the catculator. To get better access to the numbers in the sequence we need to put them in a better place.

Re-enter seq(3x - 1, x, 1, 50, 1), press STO ▸ and then 2nd 1 (**L1**).

You should see this on your calculator:

seq (3z - 1, x, 1, 50, 1) → **L1**... Now press ENTER .

The entire sequence is stored safely in Ll, (List 1). To view Ll do the following:

STAT Then, in the menu that appears on the screen, select 1: Edit

You should see this	L1	L2	L3
	-1		
	2		
	5		

(There might be some numbers in L2 or L3 from previous work you have done.)

Finding Sums of the terms in a sequence.

Suppose we want to find the sum of all the numbers in List 1: S= -1+2+5+ ... +149.

Symbolically we write this as $S = \sum_{n=1}^{50} (3n - 4)$. (The symbol Σ is the capital Greek letter *sigma*.)

To do this on the calculator, do the following:

LIST

2nd STAT In the resulting menu, use the ► key to select MATH.
From that menu select 5: sum. You will now see sum at the top of the screen; put L1 after it - sum L1 - and press ENTER. The answer should be 3625.

THE A.M.A.T.Y.C. COMPETITION

The American Mathematical Association of Two-Year Colleges organizes each year a mathematical competition.

The Student Mathematical League was founded in 1970 by Nassau Community College in New York. In 1981 the AMATYC assumed sponsorship of it.

Now the competition involves more than 120 colleges in more than 20 states, involving more than 3000 students.

Each U.S. or Canada College may enter a team of 5 students.

They are 3 year's rounds :

Round One : November 7 - November 22, 1997 ;
Round Two : Ianuary 30 - February 16, 1998 ;
Round Three : March 27 - April 11, 1998 .

Each exam contains 20 multi-choice questions, up to College Algebra level.

For the first time at UNM - Gallup the AMATYC Competition has been introduced in the Fall 1997 by Dr. Florentin Smarandache, a new hired tenure track assistant professor of mathematics, who is the moderator in charge of grading the three exams and sending the results to the director Glenn Smith (Santa Fe Community College, Gainesville, Florida) and to Mrs. Sharon MacKendirek (NMSU-Grants, coordinator for the State of New Mexico). Dr. Smarandache is known for a few notions in Numbers Theory that bear his name: Smarandache type Functions Smarandache type Sequences, and he is the author of 30 books of proposed problems and articles of mathematics, of poetry, drames, novels, essays, philosophy.

The other two full-time mathematics instructors, Dr. Mark Wilson (Math & Science Department Chair) and Dr. Val Shirley help as well in mobilizing the students.

This university year the UNM-Gallup student team participating for the first time) occuped the first place in the state of New Mexico, and eleventh place in the South west region.

The best results were obtained by the following students: LUKE W. BULTHNIS, BRAN COX, DAVID YAZZIE, BEN GORDON, BRIAN WEEKS, MARKOS CHAVEZ, HEATHER ESCUDERO, MIKE SHIRLEY.

The first five of them will be awarded.

Luke W. Bulthnis got the nineth place, individual standing, in the Southwest region.

Fall 1997

"CAIETUL DE INFORMARE MATEMATICĂ" LA 20 DE ANI

Şi iată că timpul trece...

Mi-amintesc zilele când luam întâia dată contact cu revista Liceului "N. Grigorescu" din Câmpina, pe când eram în ţară, student la Facultatea din Craiova, în 1979.

Pe urmă, proaspăt încă absolvent, şi analist-programator la Întreprinderea de Utilaj Greu, apoi profesor la Bălceşti şi'n Maroc (Lycée Sidi El Hassan Lyoussi), ţineam legătura cu remarcabilul animator al revistei, profesorul Gane Policarp.

După tăcerea lagărului turcesc, 1988-1990, odată cu emigrarea în America şi deschiderea României către lume, am putut relua corespondenţa cu amabilul editor, l-am şi sunat la telefon de câteva ori, dar niciodată nu l-am întâlnit personal, deşi mi-o dorisem. În scurtele peripluri prin ţară, când traseul meu trecea prin Oltenia ca să-mi văd rudele şi prietenii din copilărie şi adolescenţă, îmi lipsea timpul necesar să m'abat şi pe Valea Prahovei...

Am avut plăcerea să colaborez la CIM cu probleme şi note matematice, şi mai ales s'o depun (revista) în două biblioteci americane: de la Arizona State University (Tempe) şi University of Texas (Austin), alături de alte publicaţii româneşti pe care le-am donat (literatură, matematică) - pentru a îmbogăţi fondul de carte românească în străinătate. Autorii, interesaţi în răspândirea publicaţiilor lor, mi le pot expedia pe adresa Universităţii New Mexico.

Mare mi-a fost bucuria când, după trimiterea "Caietului" către editorul David E. Zitarelli, de la Pennsylvania State University, responsabil cu recenzarea articolelor de istorie matematică, să citesc în revista internaţională HISTORIA MATHEMATICA, Vol. 24, No. 1, p. 464, din noiembrie 1997, că articolele "Aniversări în 1995" şi "100 ani de la înfiinţarea «Gazetei matematice»" semnate de Gane Policarp în CIM, Nr. 34, 1995, au fost recenzate de I. Erdelyi (din Philadelphia).

Am ataşat copia paginii respective şi am expediat-o la Câmpina.

Aştept în continuare să primesc noi numere ale "Caietului de Informare Matematică".

ENSEIGNEMENT ET RECHERCHE SCIENTIFIQUE (DANS LES MATHEMATIQUES)

De nos jours on met un accent puissant sur la correlation de l'enseignement avec la recherche et la production. Entre ces deux domaines il y a d'ailleurs une liaison étroite ("osmose") une union dialectique, mais chacun d'eux maintenant sa personalité.

L'enseignement doit se développer en concordance avec les besoins et les exigences de la révolution technique-scientifique.

L'intégration de l'enseignement avec la recherche et la production signifie l'introduction des facultés au milieu de la production et de la recherche (de la projection), et aussi l'introduction de la production et de la recherche dans les unités scolaires; ainsi, on tient compte que les projets de diplôme des élèves et des étudiants soient utilisés immédiatement dans la production; c'est à l'école que revient la tâche de préparer et de former les futurs spécialistes dans toutes les branches de production.

Aux conditions dans lesquelles nous assistons à une explosion informatinnelle dans tous les domaines d'activité, on remarque un effort soutenu de la part de l'enseignement pour s'adapter aux exigences augmentant sans cesse de la société, pour tenir le pas avec les nouvelles conquêtes de la science et de la technique. Et dans le cadre de ces conquêtes scientifiques les mathématiques occupent une place centrale "reine des sciences", comme les a surnommées Gauss.

Les mathématiques donnent, à ceux qui les étudient, la précision des formules et des expressions, une discipline intellectuelle, discrétion, modestie, désintérêt, mesure, abnégation, sensibilité artistique. À notre époque, celles-ci ont beaucoup évolué, se transformant d'une science des nombres et des quantités (comme on les appelait dans l'Antiquité) dans une science des structures essentielles. Des nouvelles branches des mathématiques ont fait leur apparition (beaucoup d'entre elles grâce à son interpénétration avec les autres "sciences") et même des branches comme: linguistique mathématique, poétique mathématique (dans cette dernière discipline ayant une contribution remarquable le professer universitaire dr. Solomn Marcus à l'Université de Bucarest). (La linguistique mathématique, ayant pour point de départ les modèles logiques de la langue naturelle et développant une grammaire algebrique, étudie d'une manière simplifiée les phénomènes des langues naturelles).

"(...) les mathématiques n'ont pas de limites, come l'espace qu'elles trouvent trop réduit pour leurs aspirations; les possibilités des mathématiques sont aussi illimitées que celles des mondes qui ne cessent plus d'augmenter de l'astronomie ; les mathématiques ne porraient être restreintes à des limites précises où réduites à des définitions valables, éternellement, comme la conscience, la vie, qui semble sommeiller en chaque nonade, chaque atome de matière, chaque feuille, chaque bouton de fleur et en chaque cellule et qui est toujours prête à faire explosions sous les nouvelles formes de l'éxistence animale et végetale" (James-Joseph Sylvester, mathématicien anglais).

On observe la pénétration de plus en plus prégnante des mathématiques dans les autres science. Nous disons qu'il s'agit de leur mathématisation. Toutes ces sciences ne pouvaient progresser si elles n'étaient pas mathématisées. Ainsi, toute une série de découvertes n'auraient pas eu lieu si l'on n'avait pas connu certains procédés mathématiques, si les mathématiques n'avaient pas possédé une certaine quantité de connaissance (par exemple, Einstein n'aurait pas découvert la théorie de la relativité si l'on n'avait pas découvert avant lui le calcul tensoriel). D'autres découvertes ont été faites tout d'abord par des calculs mathématiques et ultérieurement prouvées expérimentalement (le physicien Maxwell a généralisé la conception du champ de forces électromagnetiques, en précisant que même s'il s'agit d'une champ électrique, celui-ci se propage à la distance par des ondes avec la vitesse de la lumière).

Les mathématiques se mettent aussi, toujours, à la disposition de la technique, en résolvant certains problèmes qui surgissent dans les processus de production.

L'abstractisation très grande des mathématiques n'empêche pas son applicabilité immédiate dans la practique et il en serait à remarquer quelques exemples:

- le géomètre romain Gh. Țițeica a fait des découvertes en matière de géométrie différentiélle, mais il a constaté à peine 20 ans plus tard qu'elle pouvaient être appliquées dans la théorie de la relativité généralisée;

- Cayley a découvert les matrices, découverte appliquée 87 ans plus tard par Heisenberg à la mécanique des quanta;

- le mathématicien anglais George Boole découvre vers le milieu du XIX-e siècle l'algèbre qui porte son nome; pendant longtemps elle a été envisagée comme une "curiosité" mathématique; c'est à peine 100 ans plus tard qu'on a trouvé sa place bien méritée dans le logiciel des calculateurs électroniques.

Une interessante corrélation existe entre les mathématiques, et les arts: musique, peinture, sculpture, architecture, et poésie.

L'art est l'expression pure du "sentiment", tardis que les mathématiques sont l'expression cristalline de la "raison" pure. L'art, en partant du sentiment, est plus chaud, plus humain, les mathématiques en partant de la raison, sont plus froides, mais brillantes. Une intéressante correlation entre l'art (littérature, en special) et les mathématiques essaie de faire Solomon Marcus, à la Faculté de Mathématiques et à celle de Philologie. Mais il montrait la supériorité du language artistique par rapport à celui scientifique: tandis que le langage scientifique a un sens unique, celui littéraire a une infinité. Et, d'ailleurs, dans la science est éliminé le langage ambigu.

En rappelant ce "point lumineux où la géométrie rencontre la poésie", comme disait le mathématicien et le poète Dan Barbilian alias Ion Barbu, notons aussi l'idée suivante: "La poésie de l'avenir, la poésie sublime par excellence, sera empruntée de la science." (Pierre-Jules-César Janssen).

En parlant de la recherche en général, il faut mentionner aussi les risques que l'homme de science peut courir:

- il peut trouver des résultats déjà connus (mais cela ne doit pas être désillusion, mais aussi satisfaction);

- ses recherses peuvent ne mener à aucun résultat, ou peuvent mener à des résultats suggestifs (il faut avoir la patience, il faut persévérer);

- il peut commetre des erreurs dans ses démonstrations (déductions) - (presque tous les mathématiciens ont commis des erreurs).

Ces risques sont dus an fait que dans le travail de recherche la découverte n'est pas une illumination subite sans travail cérébral, sans beaucoup, beaucoup de travail.

THERE IS NO SPEED BARRIER IN THE UNIVERSE

In this short paper one promotes the hypothesis that: There is no speed barrier in the universe, and one asks if it's possible to have an infinite speed?

What's new in science (physics)? According to reseaschers from the University of Innsbruck in Austria (December 1997):

- photon is a bit of light, the quantum of electromagnetic radiation (quantum is the smallest amount of energy that a system can gain or lose);
- polarization refers to the direction and characteristics of the light wave vibration;
- if one uses the entanglement phenomenon, in order to transfer the polarization between two photons, then: whatever happens to one is the opposite of what happens to the other; hence, their polarizations are opposite of each other;
- in quantum mechanics, objects such as subatomic particles do not have specific, fixed characteristic at any given instant in time until they are measured - suppose a certain physical process produces a pair of entangled particles A and B (having opposite or complementary characteristics), which fly off into space in the opposite direction and, when they are billions of miles apart, one measures particle A; because B is the opposite , the act of measuring A instantaneously tells B what to be; therefore those instructions would somehow have to travel between A and B faster than the speed of light; hence, one can extend the Einstein-Podolsky-Rosen paradox and Bell's inequality and assert that the light speed is not a speed barrier in the universe. We even promote the scientific hypothesis that: THERE IS NO SPEED BARRIER IN THE UNIVERSE, which would theoretically be proved by increasing, in the previous example, the distance between particles A and B as much as the universe allows it, and then measuring particle A.

Now an Open Question: If the space is infinite, is the maximum speed infinite? We say yes.

[Early versions presented at the University of Blumenau, Brazil, in May 1993, and at the University of Kishinev, in a Scientific Conference chaired by Professors Gheorghe Ciocan, Ion Goian, and Vasile Marin, in December 1994.]

References:

[1] Illingworth, Valerie, editor, "Dictionary of Physics", The Penguin, London, New York, Victoria, Toronto, 1990.

[2] Smarandache Florentin, "Eternal Life at Infinite Speed", Arizona State University Special Collection, Hayden Library, Tempe, 1993.

[3] Suplee, Curt, "'Beaming Up' No Longer Science Fiction", <Albuquerque Journal>, December 11, 1997.

["Bulletin of Pure and Applied Sciences", Delhi, India, Vol. 17D (Physics), No. 1, 61, January-June 1998.]

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