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Generalized Neutrosophic Sampling Strategy for Elevated estimation of Population Mean

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Abstract: One of the disadvantages of the point estimate in survey sampling is that it fluctuates from sample to sample due to sampling error, as the estimator only provides a point value for the parameter under discussion. The neutrosophic approach, pioneered by Florentin Smarandache, is an excellent tool for estimating the parameters under consideration in sampling theory since it yields interval estimates in which the parameter lies with a very high probability. As a result, the neutrosophic technique, which is a generalization of classical approach, is used to deal with ambiguous, indeterminate, and uncertain data. In this investigation, we suggest a new general family of ratio and exponential ratio type estimators for the elevated estimation of neutrosophic population mean of the primary variable utilizing known neutrosophic auxiliary parameters. For the first degree approximation, the bias and Mean Squared Error (MSE) of the suggested estimators are computed. The neutrosophic optimum values of the characterizing constants are determined, as well as the minimum value of the neutrosophic MSE of the suggested estimator is obtained for these optimum values of the characterizing scalars. Because the minimum MSE of the classical estimators of population mean lies inside the estimated interval of the neutrosophic estimators, the neutrosophic estimators are better than the equivalent classical estimators. The empirical investigation, which used both real and simulated data sets, backs up the theoretical findings. For practical utility in various areas of applications, the estimator with the lowest MSE or highest Percentage Relative Efficiency (PRE) is recommended.
Keywords: Classical Ratio Estimators, Neutrosophic Estimators, Bias, MSE, PRE, Simulation.

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1. Introduction

Due to time and financial constraints, sampling becomes unavoidable when the population is big. The most apt estimator for the parameter under consideration is the corresponding statistic and so is the sample mean ($\bar{y}$) for the population mean ($\bar{Y}$) of main variable Y . Although $\bar{y}$ is an unbiased estimator of $\bar{Y}$, its sampling variance is rather high, hence the sampling distribution of $\bar{y}$ will not be very close to the genuine $\bar{Y}$. Therefore, we look for a population mean estimator that is even biased yet has a sampling distribution closer to the true $\bar{Y}$. The employment of an auxiliary variable (X) having a high degree of positive or negative association with $\bar{Y}$ achieves the goal of finding efficient estimators. The use of supplementary information to elevate the effectiveness of the estimators of the parameters under consideration is well established in sampling theory. For elevated estimation of $\bar{Y}$ using positively and negatively correlated auxiliary information with main variable, respectively, ratio and product technique of estimation processes are utilized with the condition that the line of regression pass through origin. If the line does not cross through the origin, the regression method of estimation is favored above the ratio and product approaches. The ratio method is preferred in real-world applications due to its broad applicability; for example, area and production in crop yield applications, income and investment in business and economics, hospital infrastructure and health are some examples of applications, where ratio estimators are used to estimate $\overline{Y}$. As a result, the current research focuses on estimating $\overline{Y}$ using known positively associated auxiliary variable.

1.2. Estimation under Classical Sampling Theory

In estimation methods of classical sampling theory, the data utilized for elevated estimation of $\overline{Y}$ using ratio, product, or regression type estimators are known and produced by crisp numbers. In classical statistics, various authors worked on numerous estimators of $\overline{Y}$ in the presence of known X and suggested various ratio type estimators. In classical sampling theory, [1] introduced the conventional ratio estimator of $\overline{Y}$ using the positively correlated X . As an auxiliary parameter, he utilized the known population mean ($\overline{X}$) of X . Various authors later used well-known auxiliary parameters like coefficient of variation (CV), coefficient of skewness, coefficient of kurtosis, standard deviation, quartiles, and so on to improve the estimation of $\overline{Y}$. [2] worked on a modified ratio estimator of $\overline{Y}$ utilizing the known CV of X . For the elevated estimation of $\overline{Y}$, [3] proposed the exponential ratio estimator employing a known X . [4] proposed two ratio estimators for more efficient estimation of $\overline{Y}$, utilizing known coefficient of kurtosis and the CV of X . [5] focused on improving $\overline{Y}$ estimate utilizing known population correlation coefficient between Y and X , and their results outperformed rival estimators. For increased estimate of $\overline{Y}$, [6] suggested the modifications on ratio estimator of $\overline{Y}$, that makes the use of known coefficient of kurtosis of X . [7] proposed several modified ratio estimators of $\overline{Y}$ based on known information on some well-known auxiliary parameters. [8] suggested two ratio type estimators of $\overline{Y}$ utilizing known skewness and kurtosis of X , which outperformed rival estimators. [9] presented an increased estimation approach for population mean using auxiliary parameters on characteristic. [10] worked in the direction of improving a family of ratio and product estimators of $\overline{Y}$ with known parameters of X and [11] worked on a generic family of estimators of $\overline{Y}$ using transformed X . [12] proposed a generalized family of dual to ratio-cum-product $\overline{Y}$ estimators with known auxiliary parameters. [13] developed a new ratio estimator for $\overline{Y}$ utilizing linear transformation of X as minimum and maximum values. Using auxiliary parameters, [14] provided several efficient estimators for $\overline{Y}$. [15] introduced a new family of $\overline{Y}$ estimators based on the main variable's known population median and shown improvement over the estimators in competition. [16] proposed a new modified ratio type estimator based on an auxiliary variable's exponential parameter. [17] proposed an improved family of $\overline{Y}$ estimators utilizing known parameters of Y and X for improving the efficiency of the estimators, [18] used some well-known traditional and non-traditional auxiliary parameters. Many more authors have attempted to improve $\overline{Y}$ estimation using known data on traditional and non-traditional, robust and non-robust auxiliary parameters in classical sampling theory.

1.3. Estimation under Neutrosophic Sampling Theory

The data in classical sampling theory is mostly deterministic with no uncertainty in the measurements of the observations for the characteristics under investigation, however, we frequently encounter difficulties in everyday life where the data for the attributes under examination are not determined, for instance the measurement of temperature at any place along with other applications including information technology, information systems, decision support systems, financial data set detection, new economy growth, decline analysis, and more. In such cases, we seek alternate ways for dealing with undetermined data, and the fuzzy logic pioneered by Prof. Lofti A. Zadeh in 1965 gives a solution for dealing with such data when exact measurements of the variable under examination are unavailable. Although fuzzy statistics deals with ambiguous, unclear, or imprecise data, it does not take into account the indeterminacy measurements. Neutosophic logic, further, is a generalized fuzzy logic that measures indeterminacy together with the determinate component of the observations and is utilized to analyze when the observations are imprecise or ambiguous, [19, 20]. [21] utilized the fuzzy logic in decision making for more precise decisions. Later different procedures using fuzzy logic have been developed and utilized extensively for making decisions in different areas of applications, [22-26]. [27] mentioned that the complex fuzzy sets are the advanced fuzzy sets and its generalization is the complex neutrosophic set. [28] suggested a diagram of fuzzy sets along with the generalizations of the sets and utilized the interval-valued neutrosophic sets for making decisions.

According to [29], Neutosophic statistics are used when data has some indeterminacy. Neutrosophic statistics is the extended form of classical statistics and are applied when the observations in the population or sample are imprecise, indeterminate, or vague. Further he mentioned that the methods of Neutrosophic statistics are utilized to analyze Neutrosophic data, which is indeterminate to some degree and the sample size may not be an exact number. In their works, [30] and [19] argue that neutrosophic statistics are particularly useful and acceptable for use in the system with the uncertainty. [31] used neutrosophic statistics to analyze the effect on scale and anisotropy for neutrosophic numbers of rock joint roughness coefficient. [20] focused on a Neutrosophic analysis of variance for university student data. [32] used a neutrosophic soft matrix (NSM) and relative weights of experts to develop an algorithmic strategy for group decision making (GDM) challenges. [33] used neutrosophic statistics to examine data from diabetes patients who had undergone a new diagnosis test. [34] worked on the estimation of the ratio of a crisp variable and a neutrosophic variable and shown improvement over the classical ratio method of estimation. [35] employed NEWMA chart and recurrent sampling to monitor road traffic crashes using neutrosophic statistics and in his research, [36] used neutrosophic statistics to develop a new goodness of fit test utilizing unclear parameters. In a study of skewness and kurtosis estimators of wind speed distributions under indeterminacy, [37] employed neutrosophic statistics. [38] devised a decision-making approach for determining the best fit of those damages in a neutrosophic environment, with the badly damaged machine receiving preference. [39] developed several new single-valued neutrosophic graph (SVNG) concepts, stating that the fuzzy set and the neutrosophic set are two effective instruments for dealing with the uncertainties and ambiguity of any real-world scenario.

When dealing with the uncertainties of a real-life scenario, the neutrosophic set outperforms the fuzzy set. [40] used neutrosophic parameterized hypersoft set theory to develop a decision-making application. They first conceptualized the neutrosophic parameterized hypersoft set, as well as some of its basic features and operations, and then used this theory to construct a decision-making-based method. For both one and two sample hypothesis testing situations, [41] suggested a modified Sign test that takes into account the indeterminate condition and true data form. They evaluated the suggested improved Sign test using two real data sets: covid-19 reproduction rate and covid-positive daily cases in ICU in Pakistan, and found that the suggested methodologies are appropriate for the problems of nonparametric in decision-making involving interval-valued data. To handle medical diagnostics and decision-making difficulties, [42] worked on algorithms for a generalization of multipolar neutrosophic soft set with measures of information. They proposed a general multipolar neutrosophic soft set, complete with operations and fundamental features. Later, they extended it to tackle decision-making problems by introducing various information measures for the generalization of multipolar neutrosophic soft set, such as distance, similarity, and correlation coefficient. [43] mentioned that in traditional survey sample studies where data is definite, certain, and unambiguous, the estimates are a single valued crisp results that may be incorrect, overestimated, or understated, which might be a disadvantage. There are a variety of scenarios where data is neutrosophic in nature, and this is when Neutrosophic statistics is used instead of traditional approaches.

Uncertain and ambiguous values of the variables, non-clear contentions, and imprecise interval values are examples of neutrosophic data. As a result, data from trials or populations may be interval-valued neutrosophic numbers. The factual observation, that was ambiguous at the time of collecting, was thought to be a value within that range. There are more indeterminate data than definite data available in real life. As a result, more statistical techniques that are neutrosophic are needed. In real life, there are so many research variables that gathering information is quite costly, especially when the information is confusing. Thus, using traditional methods for indeterminate data to determine the unknown real value of the parameter will be dangerous and costly. After a thorough review of the literature, no study in sample surveys for ratio method of estimation for $\overline{Y}$ utilizing known X under neutrosophic data has been found. There are not enough promising articles in this subject of statistics yet. There was no available solution to tackle the issue using ratio estimation when Y and X were neutrosophic in nature. As a result, [43] presented a neutrosophic ratio-type estimation approach as the initial step in this direction. Further [43] mentioned that Neutrosophic Statistical analysis aids in the study of data with a degree of indeterminacy or insufficient knowledge, as well as conflicting beliefs. For the problem of indeterminacy, traditional statistics unsucceeded to analyze the data since certain observations were presented in a range of unknown values with the possibility of including a factual measurement within that range. As a result, in an uncertain environment, neutrosophic statistics is used, which is a more flexible alternative to and generalization of classical statistics. There have been numerous studies in the field of sample surveys under the Neutrosophy, where the method of ratio estimation is still new and necessitates a great deal of attention to the uncertain data system. For instance, the measurements of a machine product such as nuts or bolts may have slight measurement or manufacturing errors, and we may accept such product if it falls within the specified measurement range. Marks in grade system and health parameters through different testing procedures may be the areas of applications where neutrosophic statistics may be a better choice than the traditional one. Thus it is clear that in many situations, discussed above, the Neutrosophic estimators are used for improved estimation of population mean over the classical estimators where the observations of the study variable are not deterministic rather these are nondeterministic.

In this investigative work, we suggest a novel generalized neutrosophic ratio estimator for enhanced estimation of $\overline{Y}$ utilizing the known parameters of X . The sampling properties of the suggested estimator are studied for the first degree of approximation. The complete manuscript is being presented in different sections from introduction to the references.

1.4. Observations in Neutrosophic Environment and Notations

Quantitative neutrosophic data, where a number may lie in an uncertain interval [a, b], is one sort of observation in the neutrosophic environment, [30]. Neutosophic numbers' interval value can be represented in a variety of ways. [43] have defined neutrosophic interval values as $Z_N = Z_L + Z_U I_N$, where, $I_N \in [I_L, I_U]$. We also use the same notations of [43] for the considered neutrosophic data, which are in the interval form as $Z_N \in [Z_L, Z_U]$, where Z_L and Z_L are the lower and upper values of the neutrosophic variable Z_N. Let the neutrosophic population consists of N distinct units $(P_1, P_2, ..., P_N)$ and a neutrosophic random sample of size $n_N \in [n_L, n_U]$ is taken from the above population using simple random sampling without replacement (srswor) technique. Let $y_N(i)$ be the observation on the ith unit of the sample for the neutrosophic data under consideration for the main variable y_N, of the form $y_N(i) \in [y_L, y_U]$ and by the same way for the auxiliary variable $x_N(i) \in [x_L, x_U]$. Let $\bar{y}_N(i) \in [\bar{y}_L, \bar{y}_U]$ be the sample mean for the neutrosophic study variable y_N and $\bar{x}_N(i) \in [\bar{x}_L, \bar{x}_U]$ be sample mean for the neutrosophic x_N which is correlated with y_N. Further let $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$ be the population means for the neutrosophic variables y_N and x_N respectively, which are the overall averages of the neutrosophic data set. The neutrosophic coefficients of variation of y_N and x_N are given as $C_{yN} \in [C_{yNL}, C_{yNU}]$ and $C_{xN} \in [C_{xNL}, C_{xNU}]$ respectively. The correlation coefficient between the neutrosophic variables y_N and x_N is represented as $\rho_{yxN} \in [\rho_{yxNL}, \rho_{yxNU}]$. The neutrosophic coefficients of skewness and kurtosis for x_N are given by $\beta_{1(x)N} \in [\beta_{1(x)NL}, \beta_{1(x)NU}]$ and $\beta_{2(x)N} \in [\beta_{2(x)NL}, \beta_{2(x)NU}]$ respectively. The neutrosophic quartiles of x_N are given by $Q_{iN} \in [Q_{iNL}, Q_{iNU}]$, $i = 1,3$ and the neutrosophic median of auxiliary variable as $M_{dN} \in [M_{dNL}, M_{dNU}]$.

1.5. Flow Chart of the Study

The graph given below represents the flow chart of the suggested study using neutrosophic numbers. The following chart is a recreated flow chart suggested by [43].

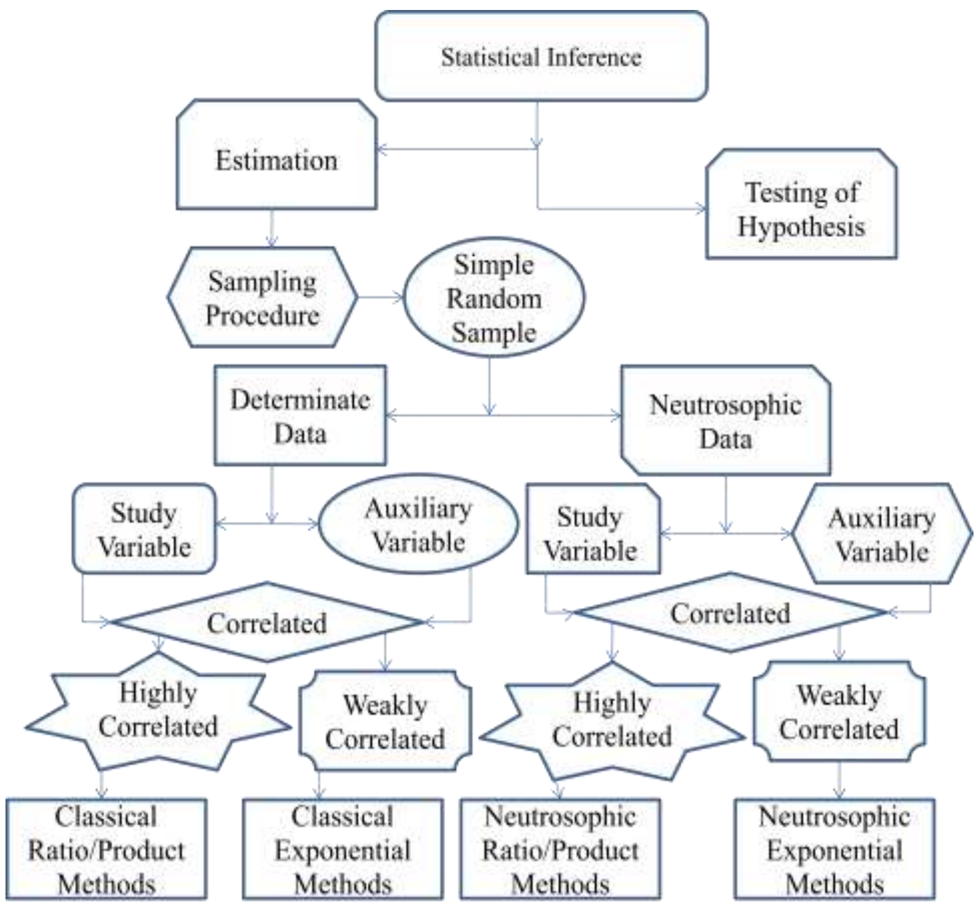

Figure-1: Flow chart of the study

1.6. Standard Approximations

Following are some standard approximations used for the sampling properties of the neutrosophic estimators, suggested by [43] as,

Let $\bar{e}_{yN} \in [\bar{e}_{yL}, \bar{e}_{yU}]$ and $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$ be the mean errors for the study and the auxiliary neutrosophic variables along with $\bar{e}_{yN}(i) = \bar{y}_N(i) - \bar{Y}_N$ and $\bar{e}_{xN}(i) = \bar{x}_N(i) - \bar{X}_N$ respectively. The expectations of these errors for different orders are defined as;

$E(\bar{e}_{yN}) = E(\bar{e}_{xN}) = 0$ and,

$E(\bar{e}_{yN}^2) = \theta_N \bar{Y}_N^2 C_{yN}^2$, $E(\bar{e}_{xN}^2) = \theta_N \bar{X}_N^2 C_{xN}^2$, $E(\bar{e}_{yN}\bar{e}_{xN}) = \theta_N \bar{X}_N \bar{Y}_N \rho_{yxN} C_{yN} C_{xN}$

Where,

$\bar{e}_{yN} \in [\bar{e}_{yL}, \bar{e}_{yU}]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$, $\bar{e}_{yN}\bar{e}_{xN} \in [\bar{e}_{yL}\bar{e}_{xL}, \bar{e}_{yU}\bar{e}_{xU}]$, $\bar{e}_{yN}^2 \in [\bar{e}_{yL}^2, \bar{e}_{yU}^2]$, $\bar{e}_{xN}^2 \in [\bar{e}_{xL}^2, \bar{e}_{xU}^2]$,

$C_{xN}^2 = \dfrac{\sigma_{xN}^2}{\bar{X}_N^2}$, $C_{yN}^2 = \dfrac{\sigma_{yN}^2}{\bar{Y}_N^2}$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $\rho_{yxN} = \dfrac{\sigma_{yxN}}{\sigma_{yN}\sigma_{xN}}$, $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$,

$\theta_N = \dfrac{1 - f_N}{n_N}$, $\theta_N \in [\theta_L, \theta_U]$, $n_N \in [n_L, n_U]$, $\sigma_{xN}^2 \in [\sigma_{xL}^2, \sigma_{xU}^2]$, $\sigma_{yN}^2 \in [\sigma_{yL}^2, \sigma_{yU}^2]$, $\sigma_{yxN} \in [\sigma_{yxL}, \sigma_{yxU}]$

On the basis of the errors of the neutrosophic variables, bias and the Mean Squared Error (MSE) of the introduced and the competing estimators are obtained for an approximation of order one. The Bias and the MSE in neutrosophic environment are defined as, $Bias(\bar{y}_N) \in [Bias_L, Bias_U]$ and $MSE(\bar{y}_N) \in [MSE_L, MSE_U]$. Further the correlated auxiliary variables are used for the elevated estimation of $\bar{Y}_N$ and neutrosophic ratio type estimators are applied when there is indeterminacy in the data.

1.7. Review of Neutrosophic Estimators

The most appropriate neutrosophic estimator for the neutrosophic $\bar{Y}_N$ of Y is the corresponding neutrosophic sample mean and is given by,

$t_0 = \bar{y}_N$

The variance of the neutrosophic sample mean for the first degree of approximation is,

$$V(t_0) = \theta_N \bar{Y}_N^2 C_{yN}^2 \tag{1}$$

Where, $t_{0N} \in [t_{0L}, t_{0U}]$

Using [1], [43] suggested the usual neutrosophic ratio estimator of $\bar{Y}_N$ using the known neutrosophic population mean of X as,

$$t_{RN} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N} \right)$$

The bias and MSE of the neutrosophic ratio estimator t_{RN}, for an approximation of degree one respectively are,

$\text{Bias}(t_{RN}) = \theta_N \bar{Y}_N [C_{xN}^2 - C_{yxN}]$, where, $C_{yxN} = \rho_{yxN} C_{yN} C_{xN}$

$$\text{MSE}(t_{RN}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + C_{xN}^2 - 2C_{yxN}] \tag{2}$$

Where, $t_{RN} \in [t_{RL}, t_{RU}]$

Motivated by [2], [43] suggested the following neutrosophic ratio estimator using CV of neutrosophic variable X as,

$$t_{1N} = \bar{y}_N \left(\frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}} \right)$$

The bias and MSE of the neutrosophic ratio estimator t_{1N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{1N}) = \theta_N \bar{Y}_N [\lambda_{1N}^2 C_{xN}^2 - \lambda_{1N} C_{yxN}]$$

$$MSE(t_{1N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{1N}^2 C_{xN}^2 - 2\lambda_{1N} C_{yxN}] \quad (3)$$

Where, $\lambda_{1N} = \dfrac{\bar{X}_N}{\bar{X}_N + C_{xN}}$ and $t_{1N} \in [t_{1L}, t_{1U}]$

Based on [3], [43] proposed the following neutrosophic exponential ratio estimator as,

$$t_{2N} = \bar{y}_N \exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N} \right)$$

The bias and MSE of the neutrosophic exponential ratio estimator t_{2N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{2N}) = \theta_N \bar{Y}_N \left[\frac{3}{8} C_{xN}^2 - \frac{1}{2} C_{yxN} \right]$$

$$MSE(t_{2N}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{C_{xN}^2}{4} - C_{yxN} \right] \quad (4)$$

Where, $t_{2N} \in [t_{2L}, t_{2U}]$

Motivated by [4], the two neutrosophic ratio estimators using CV and coefficient of kurtosis of X may be given as,

$$t_{3N} = \bar{y}_N \left(\frac{C_{xN} \bar{X}_N + \beta_{2(x)N}}{C_{xN} \bar{x}_N + \beta_{2(x)N}} \right)$$

$$t_{4N} = \bar{y}_N \left(\frac{\beta_{2(x)N} \bar{X}_N + C_{xN}}{\beta_{2(x)N} \bar{x}_N + C_{xN}} \right)$$

The biases and MSEs of the neutrosophic ratio estimators t_{3N} and t_{4N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{3N}) = \theta_N \bar{Y}_N [\lambda_{3N}^2 C_{xN}^2 - \lambda_{3N} C_{yxN}]$$

$$\text{Bias}(t_{4N}) = \theta_N \bar{Y}_N [\lambda_{4N}^2 C_{xN}^2 - \lambda_{4N} C_{yxN}]$$

$$MSE(t_{3N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{3N}^2 C_{xN}^2 - 2\lambda_{3N} C_{yxN}] \quad (5)$$

$$MSE(t_{4N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{4N}^2 C_{xN}^2 - 2\lambda_{4N} C_{yxN}] \quad (6)$$

Where, $\lambda_{3N} = \dfrac{C_{xN} \bar{X}_N}{C_{xN} \bar{X}_N + \beta_{2(x)N}}$, $\lambda_{4N} = \dfrac{\beta_{2(x)N} \bar{X}_N}{\beta_{2(x)N} \bar{X}_N + C_{xN}}$ and $t_{3N} \in [t_{3L}, t_{3U}]$, $t_{4N} \in [t_{4L}, t_{4U}]$

Motivated by [5], the neutrosophic ratio estimator t_{5N}, using known population coefficient of correlation may be given as,

$$t_{5N}=\bar{y}_N\left(\frac{\bar{X}_N+\rho_{yxN}}{\bar{x}_N+\rho_{yxN}}\right)$$

The bias and MSE of the neutrosophic ratio estimator t_{5N}, for an approximation of order one respectively are,

$$\mathrm{Bias}(t_{5N})=\theta_N\bar{Y}_N[\lambda_{5N}^2C_{xN}^2-\lambda_{5N}C_{yxN}]$$

$$MSE(t_{5N})=\theta_N\bar{Y}_N^2[C_{yN}^2+\lambda_{5N}^2C_{xN}^2-2\lambda_{5N}C_{yxN}] \quad (7)$$

Where, $\lambda_{5N}=\dfrac{\bar{X}_N}{\bar{X}_N+\rho_{yxN}}$ and $t_{5N}\in[t_{5L},t_{5U}]$

[43] suggested the following neutrosophic ratio estimator by adapting the estimator by [6], using coefficient of kurtosis of X as,

$$t_{6N}=\bar{y}_N\left(\frac{\bar{X}_N+\beta_{2(x)N}}{\bar{x}_N+\beta_{2(x)N}}\right)$$

The bias and MSE of the neutrosophic ratio estimator t_{6N}, for an approximation of order one respectively are,

$$\mathrm{Bias}(t_{6N})=\theta_N\bar{Y}_N[\lambda_{6N}^2C_{xN}^2-\lambda_{6N}C_{yxN}]$$

$$MSE(t_{6N})=\theta_N\bar{Y}_N^2[C_{yN}^2+\lambda_{6N}^2C_{xN}^2-2\lambda_{6N}C_{yxN}] \quad (8)$$

Where, $\lambda_{6N}=\dfrac{\bar{X}_N}{\bar{X}_N+\beta_{2(x)N}}$ and $t_{6N}\in[t_{6L},t_{6U}]$

Motivated by [44], the two neutrosophic ratio estimators using first and third quartiles of X may be given as,

$$t_{7N}=\bar{y}_N\left(\frac{\bar{X}_N+Q_{1N}}{\bar{x}_N+Q_{1N}}\right)$$

$$t_{8N}=\bar{y}_N\left(\frac{\bar{X}_N+Q_{3N}}{\bar{x}_N+Q_{3N}}\right)$$

The biases and MSEs of the neutrosophic ratio estimators t_{7N} and t_{8N}, for an approximation of order one respectively are,

$$\mathrm{Bias}(t_{7N})=\theta_N\bar{Y}_N[\lambda_{7N}^2C_{xN}^2-\lambda_{7N}C_{yxN}]$$

$$\mathrm{Bias}(t_{8N})=\theta_N\bar{Y}_N[\lambda_{8N}^2C_{xN}^2-\lambda_{8N}C_{yxN}]$$

$$MSE(t_{7N})=\theta_N\bar{Y}_N^2[C_{yN}^2+\lambda_{7N}^2C_{xN}^2-2\lambda_{7N}C_{yxN}] \quad (9)$$

$$MSE(t_{8N})=\theta_N\bar{Y}_N^2[C_{yN}^2+\lambda_{8N}^2C_{xN}^2-2\lambda_{8N}C_{yxN}] \quad (10)$$

Where, $\lambda_{7N}=\dfrac{\bar{X}_N}{\bar{X}_N+Q_{1N}}$, $\lambda_{8N}=\dfrac{\bar{X}_N}{\bar{X}_N+Q_{3N}}$ and $t_{7N}\in[t_{7L},t_{7U}]$, $t_{8N}\in[t_{8L},t_{8U}]$

Motivated by [8], the two neutrosophic ratio estimators using coefficients of skewness and kurtosis of X may be represented as,

$$t_{9N} = \bar{y}_N \left(\frac{\bar{X}_N + \beta_{1(x)N}}{\bar{x}_N + \beta_{1(x)N}} \right)$$

$$t_{10N} = \bar{y}_N \left(\frac{\beta_{1(x)N} \bar{X}_N + \beta_{2(x)N}}{\beta_{1(x)N} \bar{x}_N + \beta_{2(x)N}} \right)$$

The biases and MSEs of the neutrosophic ratio estimators t_{9N} and t_{10N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{9N}) = \theta_N \bar{Y}_N [\lambda_{9N}^2 C_{xN}^2 - \lambda_{9N} C_{yxN}]$$

$$\text{Bias}(t_{10N}) = \theta_N \bar{Y}_N [\lambda_{10N}^2 C_{xN}^2 - \lambda_{10N} C_{yxN}]$$

$$\text{MSE}(t_{9N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{9N}^2 C_{xN}^2 - 2\lambda_{9N} C_{yxN}] \quad (11)$$

$$\text{MSE}(t_{10N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{10N}^2 C_{xN}^2 - 2\lambda_{10N} C_{yxN}] \quad (12)$$

Where, $\lambda_{9N} = \frac{\bar{X}_N}{\bar{X}_N + \beta_{1(x)N}}$, $\lambda_{10N} = \frac{\beta_{1(x)N} \bar{X}_N}{\beta_{1(x)N} \bar{X}_N + \beta_{2(x)N}}$ and $t_{9N} \in [t_{9L}, t_{9U}]$, $t_{10N} \in [t_{10L}, t_{10U}]$

Motivated by [45], the two neutrosophic ratio estimators using median and coefficients of variation of X, we may define as,

$$t_{11N} = \bar{y}_N \left(\frac{\bar{X}_N + M_{d(x)N}}{\bar{x}_N + M_{d(x)N}} \right)$$

$$t_{12N} = \bar{y}_N \left(\frac{C_{xN} \bar{X}_N + M_{d(x)N}}{C_{xN} \bar{x}_N + M_{d(x)N}} \right)$$

The biases and MSEs of the neutrosophic ratio estimators t_{11N} and t_{12N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{11N}) = \theta_N \bar{Y}_N [\lambda_{11N}^2 C_{xN}^2 - \lambda_{11N} C_{yxN}]$$

$$\text{Bias}(t_{12N}) = \theta_N \bar{Y}_N [\lambda_{12N}^2 C_{xN}^2 - \lambda_{12N} C_{yxN}]$$

$$\text{MSE}(t_{11N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{11N}^2 C_{xN}^2 - 2\lambda_{11N} C_{yxN}] \quad (13)$$

$$\text{MSE}(t_{12N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{12N}^2 C_{xN}^2 - 2\lambda_{12N} C_{yxN}] \quad (14)$$

Where, $\lambda_{11N} = \frac{\bar{X}_N}{\bar{X}_N + M_{d(x)N}}$, $\lambda_{12N} = \frac{C_{xN} \bar{X}_N}{C_{xN} \bar{X}_N + M_{d(x)N}}$ and $t_{1N} \in [t_{11L}, t_{11U}]$, $t_{12N} \in [t_{12L}, t_{12U}]$

Motivated by [46], the neutrosophic ratio estimator t_{13N}, using known population coefficient of correlation may be given as,

$$t_{13N} = \bar{y}_N \left(\frac{\bar{X}_N + n_N}{\bar{x}_N + n_N} \right)$$

The bias and MSE of the neutrosophic ratio estimator t_{13N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{13N}) = \theta_N \bar{Y}_N [\lambda_{13N}^2 C_{xN}^2 - \lambda_{13N} C_{yxN}]$$

$$\text{MSE}(t_{13N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{13N}^2 C_{xN}^2 - 2\lambda_{13N} C_{yxN}] \tag{15}$$

Where, $\lambda_{13N} = \dfrac{\bar{X}_N}{\bar{X}_N + n_N}$ and $t_{13N} \in [t_{13L}, t_{13U}]$

Motivated by [47], [43] suggested the following neutrosophic modified exponential ratio estimator as,

$$t_{14N} = \bar{y}_N \exp\left(\frac{(a\bar{X}_N + b) - (a\bar{x}_N + b)}{(a\bar{X}_N + b) + (a\bar{x}_N + b)} \right)$$

where, a and b are the neutrosophic auxiliary parameters.

The bias and MSE of the neutrosophic exponential ratio estimator t_{14N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{14N}) = \theta_N \bar{Y}_N \left[\frac{3}{8} \lambda_{14N}^2 C_{xN}^2 - \frac{1}{2} \lambda_{13N} C_{yxN} \right]$$

$$\text{MSE}(t_{14N}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + \lambda_{14N}^2 C_{xN}^2 - 2\lambda_{14N} C_{yxN}] \tag{16}$$

Where, $\lambda_{14N} = \dfrac{a\bar{X}_N}{2(a\bar{X}_N + b)}$ and $t_{14N} \in [t_{14L}, t_{14U}]$

Motivated by [48], [43] proposed the following generalized neutrosophic exponential ratio estimator as,

$$t_{15N} = \bar{y}_N \exp\left[\alpha \left(\frac{\bar{X}_N^{\frac{1}{h}} - \bar{x}_N^{\frac{1}{h}}}{\bar{X}_N^{\frac{1}{h}} + (a-1)\bar{x}_N^{\frac{1}{h}}} \right) \right]$$

where, α and h are the real known constants with $-\infty < \alpha < \infty$ and $h > 0$. The characterizing scalar a ($a \neq 0$) is determined so that the MSE of t_{15N} is minimum.

The bias and MSE of the neutrosophic generalized exponential ratio estimator t_{15N}, for an approximation of order one respectively are,

$$\text{Bias}(t_{15N}) = \theta_N \bar{Y}_N \left[\frac{\alpha C_{xN}^2}{ah^2} - \frac{\alpha C_{xN}^2}{a^2 h^2} + \frac{\alpha^2 C_{xN}^2}{2a^2 h^2} - \frac{\alpha C_{yxN}}{ah} \right]$$

$$\text{MSE}(t_{15N}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{\alpha^2 C_{xN}^2}{a^2 h^2} - \frac{2\alpha C_{yxN}}{ah} \right] \tag{17}$$

The optimum value of the characterizing constant a is obtained by minimizing $\text{MSE}(t_{15N})$ and the optimum value is,

$$a_{opt} = \frac{\alpha C_{xN}^2}{h C_{yxN}} \tag{18}$$

The minimum value of the $\text{MSE}(t_{15N})$ for the optimum value of a_{opt} is,

$$\text{MSE}_{\min}(t_{15N}) = \theta_N \bar{Y}_N^2 C_{yN}^2 (1 - \rho_{yxN}^2) \tag{19}$$

2. Material and Methods

Motivated by [49], we suggest a ratio cum exponential ratio class of neutrosophic main variable using the neutrosophic auxiliary parameters as,

$$t_{pN}=\kappa_1\bar{y}_N\left(\frac{a\bar{X}_N+b}{a\bar{x}_N+b}\right)+\kappa_2\bar{y}_N\exp\left(\frac{(a\bar{X}_N+b)-(a\bar{x}_N+b)}{(a\bar{X}_N+b)+(a\bar{x}_N+b)}\right)$$

Where, κ_1 and κ_2 are the characterizing scalars to be determine such that the MSE of t_{pN} is minimum. It is worth notable that,

(i) If $\kappa_2=0$, then the introduced estimator t_{pN} reduces to [49] ratio type estimators having different estimators by different authors as its special cases.

(ii) If $\kappa_2=0$ and $\kappa_1=1$, the introduced estimator t_{pN} reduces to ratio type estimators having different estimators by different authors as its special cases.

(iii) If $\kappa_1=0$, then the suggested class of estimators t_{pN} reduces to [49] exponential ratio type estimators having different estimators by different authors as its special cases.

(iv) If $\kappa_1=0$ and $\kappa_2=1$, the suggested family of estimators t_{pN} reduces to exponential ratio type estimators having different estimators by different authors as its special cases.

Expressing the introduced estimator in terms of $\bar{e}_{yN}$ and $\bar{e}_{xN}$, we have

$$t_{pN}=\kappa_1\bar{Y}_N(1+\bar{e}_{yN})\left(\frac{a\bar{X}_N+b}{a\bar{X}_N(1+\bar{e}_{xN})+b}\right)+\kappa_2\bar{Y}_N(1+\bar{e}_{yN})\exp\left(\frac{(a\bar{X}_N+b)-(a\bar{X}_N(1+\bar{e}_{xN})+b)}{(a\bar{X}_N+b)+(a\bar{X}_N(1+\bar{e}_{xN})+b)}\right)$$

$$=\bar{Y}_N(1+\bar{e}_{yN})\left[\kappa_1\left(\frac{a\bar{X}_N+b}{a\bar{X}_N(1+\bar{e}_{xN})+b}\right)+\kappa_2\exp\left(\frac{(a\bar{X}_N+b)-(a\bar{X}_N(1+\bar{e}_{xN})+b)}{(a\bar{X}_N+b)+(a\bar{X}_N(1+\bar{e}_{xN})+b)}\right)\right]$$

$$=\bar{Y}_N(1+\bar{e}_{yN})\left[\kappa_1(1+\lambda\bar{e}_{xN})^{-1}+\kappa_2\exp\left(-\frac{\lambda\bar{e}_{xN}}{2}(1+\frac{\lambda\bar{e}_{xN}}{2})^{-1}\right)\right]$$

Expanding the terms on the right hand side and simplifying and retaining the terms for the first degree of approximation, we get

$$t_{pN}=\bar{Y}_N[\kappa_1(1+\bar{e}_{yN}-\lambda\bar{e}_{xN}-\lambda\bar{e}_{yN}\bar{e}_{xN}+\lambda^2\bar{e}_{xN}^2)+\kappa_2(1+\bar{e}_{yN}-\frac{\lambda\bar{e}_{xN}}{2}-\frac{\lambda\bar{e}_{yN}\bar{e}_{xN}}{2}+\frac{3}{8}\lambda^2\bar{e}_{xN}^2)]$$

Subtracting $\bar{Y}_N$ on both sides of the above equation, we have

$$t_{pN}-\bar{Y}_N=\bar{Y}_N[\kappa_1(1+\bar{e}_{yN}-\lambda\bar{e}_{xN}-\lambda\bar{e}_{yN}\bar{e}_{xN}+\lambda^2\bar{e}_{xN}^2)+\kappa_2(1+\bar{e}_{yN}-\frac{\lambda\bar{e}_{xN}}{2}-\frac{\lambda\bar{e}_{yN}\bar{e}_{xN}}{2}+\frac{3}{8}\lambda^2\bar{e}_{xN}^2)-1] \quad (20)$$

Taking expectations on both sides of (20) and putting values of different expectations, we get the bias of t_{pN} as,

$$Bias(t_{pN})=\bar{Y}_N[\kappa_1(1-\lambda\theta_N\bar{Y}_N\bar{X}_NC_{yxN}+\lambda^2\theta_N\bar{X}_N^2C_{xN}^2)+\kappa_2(1-\frac{\lambda}{2}\theta_N\bar{Y}_N\bar{X}_NC_{yxN}+\frac{3}{8}\lambda^2\theta_N\bar{X}_N^2C_{xN}^2)-1] \quad (21)$$

Squaring on both sides of (20), simplifying for the first degree of approximation, we get

$$(t_{pN}-\bar{Y}_N)^2=\bar{Y}_N^2\left\{\begin{array}{l}1+\kappa_1^2(1+\bar{e}_{yN}^2+3\lambda^2\bar{e}_{xN}^2-4\lambda\bar{e}_{yN}\bar{e}_{xN})+\kappa_2^2(1+\bar{e}_{yN}^2+\lambda^2\bar{e}_{xN}^2-2\lambda\bar{e}_{yN}\bar{e}_{xN})\\-2\kappa_1(1+\lambda^2\bar{e}_{xN}^2-\lambda\bar{e}_{yN}\bar{e}_{xN})-2\kappa_2(1+\frac{3}{8}\lambda^2\bar{e}_{xN}^2-\frac{\lambda}{2}\bar{e}_{yN}\bar{e}_{xN})\\+2\kappa_1\kappa_2(1+\bar{e}_{yN}^2+\frac{15}{8}\lambda^2\bar{e}_{xN}^2-3\lambda\bar{e}_{yN}\bar{e}_{xN})\end{array}\right\}$$

Putting values of different expectations after taking expectation on both sides, we get the MSE of t_{pN} as,

$$MSE(t_{pN}) = \bar{Y}_N^2 \left\{ \begin{array}{l} 1+\kappa_1^2(1+\theta_N \bar{Y}_N^2 C_{yN}^2 + 3\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 4\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN}) \\ +\kappa_2^2(1+\theta_N \bar{Y}_N^2 C_{yN}^2 + \lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 2\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN}) \\ -2\kappa_1(1+\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - \lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN}) \\ -2\kappa_2(1+\frac{3}{8}\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - \frac{\lambda}{2}\theta_N \bar{Y}_N \bar{X}_N C_{yxN}) \\ +2\kappa_1\kappa_2(1+\theta_N \bar{Y}_N^2 C_{yN}^2 + \frac{15}{8}\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 3\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN}) \end{array} \right\} \tag{22}$$

$$MSE(t_{pN}) = \bar{Y}_N^2[1+A\kappa_1^2 + B\kappa_2^2 - 2C\kappa_1 - 2D\kappa_2 + 2F\kappa_1\kappa_2] \tag{23}$$

Where,

$$A = (1+\theta_N \bar{Y}_N^2 C_{yN}^2 + 3\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 4\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN})$$

$$B = (1+\theta_N \bar{Y}_N^2 C_{yN}^2 + \lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 2\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN})$$

$$C = (1+\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - \lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN})$$

$$D = (1+\frac{3}{8}\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - \frac{\lambda}{2}\theta_N \bar{Y}_N \bar{X}_N C_{yxN})$$

$$F = (1+\theta_N \bar{Y}_N^2 C_{yN}^2 + \frac{15}{8}\lambda^2\theta_N \bar{X}_N^2 C_{xN}^2 - 3\lambda\theta_N \bar{Y}_N \bar{X}_N C_{yxN})$$

The optimum values of the characterizing constants κ_1 and κ_2 which minimizes the MSE of the suggested estimator t_{pN} respectively are,

$\kappa_{1(opt)} = \frac{(DF-BC)}{(F^2-AB)}$ and $\kappa_{2(opt)} = \frac{(CF-AD)}{(F^2-AB)}$

The minimum value of the MSE of t_{pN} for these optimum values of κ_1 and κ_2 is,

$$MSE_{\min}(t_{pN}) = \bar{Y}_N^2 \left\{ 1 - \frac{\begin{bmatrix} 2C(DF-BC)(F^2-AB) + 2D(CF-AD)(F^2-AB) \\ -2F(DF-BC)(CF-AD) - (DF-BC) - (CF-AD) \end{bmatrix}}{(F^2-AB)^2} \right\} \tag{24}$$

$$MSE_{\min}(t_{pN}) = \bar{Y}_N^2 \left\{ 1 - \frac{P}{Q} \right\} \tag{25}$$

Where,

$$P = \begin{bmatrix} 2C(DF-BC)(F^2-AB) + 2D(CF-AD)(F^2-AB) \\ -2F(DF-BC)(CF-AD) - (DF-BC) - (CF-AD) \end{bmatrix}$$

$$Q = (F^2-AB)^2$$

3. Theoretical Efficiency Comparison

Under this section, we have compared the introduced neutrosophic estimator with the competing neutrosophic estimators of $\overline{Y}$ using the neutrosophic auxiliary parameters. The efficiency of the introduced estimator has been compared in terms of MSEs and the efficiency condition of the introduced estimator to be more efficient than the competing one is obtained.

The suggested estimator t_{pN} is more efficient than t_0 for the condition if,

$V(t_0)-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N C_{yN}^2-\left\{1-\frac{P}{Q}\right\}>0$$

The introduced estimator t_{pN} has lesser MSE than estimator t_{RN} for the following condition.

$\mathrm{M}SE(t_{RN})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N[C_{yN}^2+C_{xN}^2-2C_{yxN}]-\left\{1-\frac{P}{Q}\right\}>0$$

The suggested estimator t_{pN} is better than the estimator t_{1N} by [43] under the restriction if,

$\mathrm{M}SE(t_{1N})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N[C_{yN}^2+\lambda_{1N}^2C_{xN}^2-2\lambda_{1N}C_{yxN}]-\left\{1-\frac{P}{Q}\right\}>0$$

The suggested estimator t_{pN} is better than the exponential ratio type estimator t_{2N} by [43] for the condition if,

$\mathrm{M}SE(t_{2N})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N\left[C_{yN}^2+\frac{C_{xN}^2}{4}-C_{yxN}\right]-\left\{1-\frac{P}{Q}\right\}>0$$

The introduced estimator t_{pN} performs better than the estimator t_{3N} if,

$\mathrm{M}SE(t_{3N})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N[C_{yN}^2+\lambda_{3N}^2C_{xN}^2-2\lambda_{3N}C_{yxN}]-\left\{1-\frac{P}{Q}\right\}>0$$

The introduced estimator t_{pN} has lesser MSE than the estimator t_{4N} if it satisfies,

$\mathrm{M}SE(t_{4N})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N[C_{yN}^2+\lambda_{4N}^2C_{xN}^2-2\lambda_{4N}C_{yxN}]-\left\{1-\frac{P}{Q}\right\}>0$$

The proposed estimator t_{pN} is better than the estimator t_{5N} if,

$\mathrm{M}SE(t_{5N})-MSE_{\min}(t_{pN})>0$ or,

$$\theta_N[C_{yN}^2+\lambda_{5N}^2C_{xN}^2-2\lambda_{5N}C_{yxN}]-\left\{1-\frac{P}{Q}\right\}>0$$

The suggested estimator t_{pN} performs better than the ratio estimator t_{6N} by [43] if,

$$\mathrm{MSE}(t_{6N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{6N}^2 C_{xN}^2 - 2\lambda_{6N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The introduced estimator t_{pN} is better than the ratio estimator t_{7N} under the condition if,

$$\mathrm{MSE}(t_{7N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{7N}^2 C_{xN}^2 - 2\lambda_{7N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The proposed estimator t_{pN} has lesser MSE than the ratio estimator t_{8N} for the condition if,

$$\mathrm{MSE}(t_{8N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{8N}^2 C_{xN}^2 - 2\lambda_{8N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The introduced estimator t_{pN} performs better than the estimator t_{9N} if,

$$\mathrm{MSE}(t_{9N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{9N}^2 C_{xN}^2 - 2\lambda_{9N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The introduced estimator t_{pN} has lesser MSE than the estimator t_{10N} if,

$$\mathrm{MSE}(t_{10N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{10N}^2 C_{xN}^2 - 2\lambda_{10N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The proposed estimator t_{pN} has lesser MSE in comparison to the ratio estimator t_{11N} under the condition if,

$$\mathrm{MSE}(t_{11N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{11N}^2 C_{xN}^2 - 2\lambda_{11N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The introduced estimator t_{pN} is better than the ratio estimator t_{12N} for the condition if,

$$\mathrm{MSE}(t_{12N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{12N}^2 C_{xN}^2 - 2\lambda_{12N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The proposed estimator t_{pN} perform better the estimator t_{13N} if,

$$\mathrm{MSE}(t_{13N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{13N}^2 C_{xN}^2 - 2\lambda_{13N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The suggested estimator t_{pN} has lesser MSE with that of the ratio estimator t_{14N} under the condition if,

$$\mathrm{MSE}(t_{14N}) - MSE_{\min}(t_{pN}) > 0 \text{ or,}$$

$$\theta_N [C_{yN}^2 + \lambda_{14N}^2 C_{xN}^2 - 2\lambda_{14N} C_{yxN}] - \left\{1 - \frac{P}{Q}\right\} > 0$$

The introduced estimator t_{pN} is better than that of the estimator t_{15N} if,

$\mathrm{MSE}_{\min}(t_{15N}) - MSE_{\min}(t_{pN}) > 0$ or,

$$\theta_N C_{yN}^2 (1 - \rho_{yxN}^2) - \left\{1 - \frac{P}{Q}\right\} > 0$$

4. Simulation Study

To verify the theoretical efficiency conditions and evaluate the efficiencies of the suggested and competing neutrosophic estimator of $\bar{Y}$ utilizing known auxiliary parameters, we have simulated a neutrosophic data set using the same parameters of [43]. To generate the neutrosophic data, we have considered that the neutrosophic main and auxiliary random variables Y_N and X_N follow the neutrosophic normal distributions. Thus $Y_N \sim NN(\mu_{yN}, \sigma_{yN}^2)$; $Y_N \in (Y_L, Y_U)$, $\mu_{yN} \in (\mu_{yL}, \mu_{yU})$, $\sigma_{yN}^2 \in (\sigma_{yL}^2, \sigma_{yU}^2)$ and $X_N \sim NN(\mu_{xN}, \sigma_{xN}^2)$; $X_N \in (X_L, X_U)$, $\mu_{xN} \in (\mu_{xL}, \mu_{xU})$, $\sigma_{xN}^2 \in (\sigma_{xL}^2, \sigma_{xU}^2)$. For the numerical illustration, we have taken $Y_N \sim NN([76.0, 84.9], [(12.9)^2, (17.2)^2])$, where, $\mu_{yN} \in (76.0, 84.9)$, $\sigma_{yN} \in (12.9, 17.2)$ and $X_N \sim NN([171.2, 180.4], [(5.8)^2, (6.7)^2])$, where, $\mu_{xN} \in (171.2, 180.4)$, $\sigma_{xN} \in (5.8, 6.7)$ and generated 1000 normal random observation for both the variables. The descriptive statistics for the simulated data is presented in Table 1.

Table 1. Descriptive statistics of the simulated data for the neutrosophic data

Parameter	Neutrosophic Value	Parameter	Neutrosophic Value
N_N	$[1000, 1000]$	C_{xN}	$[0.0332, 0.0369]$
n_N	$[20, 20]$	$\beta_{1(x)N}$	$[0.0020, 0.0051]$
μ_{yN}	$[76.20, 85.63]$	$\beta_{2(x)N}$	$[3.0227, 2.9539]$
μ_{xN}	$[171.08, 180.34]$	$Q_{1(x)N}$	$[167.3941, 176.1144]$
σ_{yN}	$[12.79, 17.37]$	$M_{d(x)N}$	$[170.9067, 180.3451]$
σ_{xN}	$[5.67, 6.65]$	$Q_{3(x)N}$	$[174.9269, 184.7586]$
C_{yN}	$[0.1679, 0.2028]$	ρ_{yxN}	$[0.01933, 0.00703]$

The Table 2 is representing the neutrosophic MSEs of different competing along with the suggested estimator of population mean.

Table 2. Neutrosophic MSEs of different competing and suggested estimator

SR. No.	Estimators	MSE
1.	t_0	[8.019213, 14.77799]
2.	t_{RN}	[17.39673, 27.98680]
3.	t_{1N}	[17.39674, 27.98681]
4.	t_{2N}	[8.066852, 14.8812]
5.	t_{3N}	[17.39709, 27.98701]

6.	t_{4N}	[17.39674, 27.98681]
7.	t_{5N}	[17.39674, 27.98681]
8.	t_{6N}	[17.3978, 27.98741]
9.	t_{7N}	[17.42703, 28.00546]
10.	t_{8N}	[17.4277, 28.00592]
11.	t_{9N}	[17.39673, 27.98681]
12.	t_{10N}	[17.4517, 28.01563]
13.	t_{11N}	[17.42734, 28.00569]
14.	t_{12N}	[17.45602, 28.02323]
15.	t_{13N}	[17.40314, 27.99058]
16.	t_{14N}(a=1,b=0)	[17.42736, 28.00569]
17.	t_{14N}(a=1,b=1)	[17.42754, 28.00579]
18.	t_{15N}	[8.016216, 14.77726]
19.	t_{pN}	**[7.864525, 13.821846]**

5. Results and Discussion

From Table 2, it may clearly be observed that the estimator t_0 of $\overline{Y}_N$ has its neutrosophic sampling variance as [8.019213, 14.77799] and the neutrosophic MSE of the exponential ratio estimator t_{2N} is [8.066852, 14.8812] while the neutrosophic MSEs of all the mentioned ratio type estimators lie in the interval [17.45602, 28.02323]. The neutrosophic ratio type estimators have high MSEs than the neutrosophic estimator t_0 because of the low neutrosophic correlation between neutrosophic y and x. The neutrosophic MSE of the suggested class of estimators is [7.864525, 13.821846], which is the minimum among the group of all neutrosophic estimators of $\overline{Y}_N$ in competition.

6. Conclusion

In this scripture, we have suggested a novel family of neutrosophic estimators of $\overline{Y}_N$ for the elevated estimation of neutrosophic $\overline{Y}_N$ using the known neutrosophic auxiliary parameters. We studied the neutrosophic sampling properties mainly bias and MSE of the proposed family of estimators for the approximation of degree one. The neutrosophic optimum values of the characterizing scalars of the introduced estimator are obtained and the neutrosophic minimum MSE of the suggested estimator has also been obtained for these neutrosophic optimum values of the characterizing scalars. The introduced estimator has been compared with the neutrosophic competing estimators theoretically and the efficiency condition over the competing estimators have been obtained. These efficiency conditions are verified using a neutrosophic simulated data set. The results in Table-2 are showing that the suggested estimator is most efficient among the class of all neutrosophic competing estimators of $\overline{Y}_N$. Thus the introduced class of estimators may be recommended for elevated estimation of neutrosophic $\overline{Y}_N$ in different areas of applications. It is to be mentioned here that the neutrosophic estimators are most suitable for improved estimation of population mean for the situations where the observations of the study variable are nondeterministic but for the situation where its observations are deterministic, it may be inferior to the classical estimators.

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