Introduction to the Complex Refined Neutrosophic Set

Florentin Smarandache[1]

[1]University of New Mexico, Gallup, NM 87301, USA
smarand@unm.edu

Abstract

In this paper, one extends the single-valued complex neutrosophic set to the subset-valued complex neutrosophic set, and afterwards to the subset-valued complex refined neutrosophic set.

Keywords

single-valued complex neutrosophic set, subset-valued complex neutrosophic set, subset-valued complex refined neutrosophic set.

1 Introduction

One first recalls the definitions of the single-valued neutrosophic set (SVNS), and of the subset-value neutrosophic set (SSVNS).

Definition 1.1.

Let X be a space of elements, with a generic element in X denoted by x. A *Single-Valued Neutrosophic Set (SVNS) A* is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$, where for each element $x \in X$, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 1.2.

Let X be a space of elements, with a generic element in X denoted by x. A *SubSet-Valued Neutrosophic Set (SSVNS)* A [3] is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$, where for each element $x \in X$, the subsets $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$,

with $0 \le \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \le 3$.

2 Complex Neutrosophic Set

Ali and Smarandache [1] introduced the notion of single-valued complex neutrosophic set (SVCNS) as a generalization of the single-valued neutrosophic set (SVNS) [2].

Definition 2.1.

Let X be a space of elements, with a generic element in X denoted by x. A *Single-Valued Complex Neutrosophic Set (SVCNS) A* [1] is characterized by a truth membership function $T_{1_A}(x)e^{iT_{2A}(x)}$, an indeterminacy membership function $I_{1_A}(x)e^{iI_{2A}(x)}$, and a falsity membership function $F_{1_A}(x)e^{iF_{2A}(x)}$, where for each element $x \in X$, single-valued numbers $T_{1_A}(x), I_{1_A}(x), F_{1_A}(x) \in [0,1]$, $0 \le T_{1_A}(x) + I_{1_A}(x) + F_{1_A}(x) \le 3$, $i = \sqrt{-1}$,

and the single-valued numbers $T_{2_A}(x), I_{2_A}(x), F_{2_A}(x) \in [0, 2\pi]$,

with $0 \le T_{2_A}(x) + I_{2_A}(x) + F_{2_A}(x) \le 6\pi$.

$T_{1_A}(x), I_{1_A}(x), F_{1_A}(x)$ represent the real part (or amplitude) of the truth membership, indeterminacy membership, and falsehood membership respectively; while $T_{2_A}(x), I_{2_A}(x), F_{2_A}(x)$ represent the imaginary part (or phase) of the truth membership, indeterminacy membership, and falsehood membership respectively.

Definition 2.2.

In the previous Definition 2.1., if one replaces the single-valued numbers with subset-values, i.e. the subset-values $T_{1_A}(x), I_{1_A}(x), F_{1_A}(x) \subseteq [0,1]$, $i = \sqrt{-1}$, and the subset-values $T_{2_A}(x), I_{2_A}(x), F_{2_A}(x) \subseteq [0, 2\pi]$,

with $0 \le \sup(T_{1_A}(x)) + \sup(I_{1_A}(x)) + \sup(F_{1_A}(x)) \le 3$,

and $0 \le \sup(T_{2_A}(x)) + \sup(I_{2_A}(x)) + \sup(F_{2_A}(x)) \le 6\pi$,

one obtains the *SubSet-Valued Complex Neutrosophic Set (SSVCNS).*

3 Refined Neutrosophic Set

Smarandache introduced the refined neutrosophic set [4] in 2013.

Definition 3.1.

Let X be a space of elements, with a generic element in X denoted by x. A *Single-Valued Refined Neutrosophic Set (SVRNS) A* is characterized by p sub-truth membership functions $T_{1_A}(x), T_{2_A}(x), \dots, T_{p_A}(x)$, r sub-indeterminacy membership functions $I_{1_A}(x), I_{2_A}(x), \dots, I_{r_A}(x)$, and s sub-falsity membership functions $F_{1_A}(x), F_{2_A}(x), \dots, F_{s_A}(x)$, where for each element $x \in X$, the single-valued numbers

$$T_{1_A}(x), T_{2_A}(x), \dots, T_{p_A}(x), I_{1_A}(x), I_{2_A}(x), \dots, I_{r_A}(x), F_{1_A}(x), F_{2_A}(x), \dots, F_{s_A}(x) \in [0,1],$$

$$0 \le T_{1_A}(x) + T_{2_A}(x) + \dots + T_{p_A}(x) + I_{1_A}(x) + I_{2_A}(x) + \dots + I_{r_A}(x) + F_{1_A}(x) + F_{2_A}(x) + \dots$$
$$\dots + F_{s_A}(x) \le p + r + s,$$

and the integers $p, r, s \ge 0$, with at least one of p, r, s to be ≥ 2.

In other words, the truth membership function $T_A(x)$ was refined (split) into p sub-truths $T_{1_A}(x), T_{2_A}(x), \dots, T_{p_A}(x)$, the indeterminacy membership function $I_A(x)$ was refined (split) into r sub-indeterminacies $I_{1_A}(x), I_{2_A}(x), \dots, I_{r_A}(x)$, and the falsity membership function $F_A(x)$ was refined (split) into s sub-falsities $F_{1_A}(x), F_{2_A}(x), \dots, F_{s_A}$.

Definition 3.2.

In the previous Definition 3.1., if one replaces the single-valued numbers with subset-values i.e., the subset-values $T_{1_A}(x), T_{2_A}(x), \dots, T_{p_A}(x), I_{1_A}(x), I_{2_A}(x), \dots, I_{r_A}(x), F_{1_A}(x), F_{2_A}(x), \dots, F_{s_A}(x) \subseteq [0,1]$, and

$$0 \le \sup(T_{1_A}(x)) + \sup(T_{2_A}(x)) + \dots + \sup(T_{p_A}(x)) + \sup(I_{1_A}(x)) + \sup(I_{2_A}(x)) + \dots$$
$$\dots + \sup(I_{r_A}(x)) + \sup(F_{1_A}(x)) + \sup(F_{2_A}(x)) + \dots + \sup(F_{s_A}(x)) \le p + r + s,$$

one obtains the *SubSet-Valued Refined Neutrosophic Set (SSVRNS)*.

4 Complex Refined Neutrosophic Set

Now one combines the complex neutrosophic set with refined neutrosophic set in order to get the complex refined neutrosophic set.

Definition 4.1.

Let X be a space of elements, with a generic element in X denoted by x. A *Single-Valued Complex Refined Neutrosophic Set (SVCRNS) A* is characterized by *p* sub-truth membership functions

$T_{11_A}(x)e^{iT_{21_A}(x)}, T_{12_A}(x)e^{iT_{22_A}(x)}, \ldots, T_{1p_A}(x)e^{iT_{2p_A}(x)}$, *r* sub-indeterminacy membership functions $I_{11_A}(x)e^{iI_{21_A}(x)}, I_{12_A}(x)e^{iI_{22_A}(x)}, \ldots, I_{1r_A}(x)e^{iI_{2r_A}(x)}$, and *s* sub-falsity membership functions $F_{11_A}(x)e^{iF_{21_A}(x)}, F_{12_A}(x)e^{iF_{22_A}(x)}, \ldots, F_{1s_A}(x)e^{iF_{2s_A}(x)}$, and $i = \sqrt{-1}$, where for each element *x* ∈*X*, the single-valued numbers (sub-real parts, or sub-amplitudes)

$$T_{11_A}(x), T_{12_A}(x), \ldots, T_{1p_A}(x), I_{11_A}(x), I_{12_A}(x), \ldots, I_{1r_A}(x), F_{11_A}(x), F_{12_A}(x), \ldots, F_{1s_A}(x) \in [0,1]$$

with

$$0 \le T_{11_A}(x) + T_{12_A}(x) + \ldots + T_{1p_A}(x) + I_{11_A}(x) + I_{12_A}(x) + \ldots + I_{1r_A}(x) + F_{11_A}(x) + F_{12_A}(x) + \ldots + F_{1s_A}(x) \le p + r + s,$$

and the single-valued numbers (sub-imaginary parts, or sub-phases)

$$T_{21_A}(x), T_{22_A}(x), \ldots, T_{2p_A}(x), I_{21_A}(x), I_{22_A}(x), \ldots, I_{2r_A}(x), F_{21_A}(x), F_{22_A}(x), \ldots, F_{2s_A}(x) \in [0, 2\pi]$$

with

$$0 \le T_{21_A}(x) + T_{22_A}(x) + \ldots + T_{2p_A}(x) + I_{21_A}(x) + I_{22_A}(x) + \ldots + I_{2r_A}(x) + F_{21_A}(x) + F_{22_A}(x) + \ldots + F_{2s_A}(x) \le 2(p + r + s)\pi,$$

and the integers *p, r, s ≥ 0*, with at least one of *p, r, s* to be *≥ 2*.

Definition 4.2.

In the previous Definition 4.1., if one replaces the single-valued numbers with subset-values i.e., the subset-values

$$T_{11_A}(x), T_{12_A}(x), \ldots, T_{1p_A}(x), I_{11_A}(x), I_{12_A}(x), \ldots, I_{1r_A}(x), F_{11_A}(x), F_{12_A}(x), \ldots, F_{1s_A}(x) \subseteq [0,1]$$

with

$$0 \le \sup(T_{11_A}(x)) + \sup(T_{12_A}(x)) + \ldots + \sup(T_{1p_A}(x)) + \sup(I_{11_A}(x)) + \sup(I_{12_A}(x)) + \ldots + \sup(I_{1r_A}(x)) + \sup(F_{11_A}(x)) + \sup(F_{12_A}(x)) + \ldots + \sup(F_{1s_A}(x)) \le p + r + s,$$

and

$$T_{21_A}(x), T_{22_A}(x), \ldots, T_{2p_A}(x), I_{21_A}(x), I_{22_A}(x), \ldots, I_{2r_A}(x), F_{21_A}(x), F_{22_A}(x), \ldots, F_{2s_A}(x) \subseteq [0, 2\pi]$$

with

$$0 \leq \sup(T_{21_A}(x)) + \sup(T_{22_A}(x)) + \ldots + \sup(T_{2p_A}(x)) + \sup(I_{21_A}(x)) + \sup(I_{22_A}(x)) + \ldots$$
$$\ldots + \sup(I_{2r_A}(x)) + \sup(F_{21_A}(x)) + \sup(F_{22_A}(x)) + \ldots + \sup(F_{2s_A}(x)) \leq 2(p+r+s)\pi,$$

one obtains the *SubSet-Valued Complex Refined Neutrosophic Set (SSVCRNS).*

5 Conclusion

After the introduction of the single-valued and subset-valued complex refined neutrosophic sets as future research is the construction of their aggregation operators, the study of their properties, and their applications in various fields.

6 References

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