

florentin smarandache

# NIDUS IDEARUM

## annotations on neutrosophy

Did you think about complex neutrosophic graphs where not only the edges but also the vertices are complex neutrosophic numbers (of the form  $a + bi + ci + diI$ , where  $a, b, c, d$  are real numbers,  $i = \sqrt{-1}$ , and  $I =$  literal indeterminacy with  $I^2 = I$ )?

What applications or connections with our everyday life should we find?

We can design Complex Neutrosophic Cognitive Maps or Complex Neutrosophic Cognitive relations and apply them to social sciences for example.

Let's say we have a complex neutrosophic graph. How to interpret an edge:

$$2 - 3i + 4I + 6II$$

A-----B

Also, how to interpret a vertex:  
 $V(1 + 2i - 5I + 7II)$ ?

With respect to our real world.

For example,

since

$$\text{sq}(4-4I) = 2-2I, -2+2I$$

$$(2-2I)^2 = 4-8I+4I^2 = 4-8I+4I = 4-4I.$$

Similarly,

$$(-2+2I)^2 = 4-4I.$$

If  $I =$  indeterminate, then  $I^n =$  undefined for  $n \leq 0$ .  
 Because:

$$I^{(-1)} = 1/I$$

and

$$I^{(-1)} = I^{(1-2)} = I^1 I^{(-2)} = Ix(I)^{(-2)} = Ix(I^{(-1)}) = I^{(1-1)} = I^0 = I.$$

From these two equalities, we get:

$$1/I = I^0 \text{ or } 1 = IxI^0 = I^{(1-0)} = I^1 = I.$$

Therefore we got  $I = 1$  which is absurd.  
 Therefore  $I^n =$  undefined for  $n \leq 0$ .

- Neutrosophic entropy;
- Neutrosophic data mining;
- Neutrosophic network denoising;
- Neutrosophic training set;
- Neutrosophic hyperplane;
- Neutrosophic decision function;
- Neutrosophic optimal solution;
- Neutrosophic constraint;
- Neutrosophic error;
- Neutrosophic performance;
- Neutrosophic specificity;

Florentin Smarandache

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**NIDUS IDEARUM.**

**scilogs, VI: *annotations on neutrosophy***

Brussels, 2019

Exchanging ideas with A.A.A. Agboola,  
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# **Nidus idearum**

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*annotations on neutrosophy*

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## FOREWORD

Welcome into my scientific lab!

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin). I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks (in English, French, and Romanian).

\*

In this *sixth book of scilogs* collected from my nest of ideas, one may find new and old questions and solutions, referring to topics on NEUTROSOPHY – email messages to research colleagues, or replies, notes about authors, articles, or books, and so on. Feel free to budge in or just use the *scilogs* as open source for your own ideas!

*Special thanks to all my peer colleagues for  
exciting and pertinent instances of discussing.*



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Florentin Smarandache: *Nidus idearum*



## **Neutrosophic Science**

We may say, in general, *Neutrosophic Science* (since neutrosophy can be used in so many knowledge fields).

## **Extending/Adjusting Fuzzy Theory**

We always check new things in *fuzzy theory* in order to adjust/extend them to *neutrosophic theory*.

## **Ideas/Proposals for possible future research**

*To W. B. Vasantha Kandasamy*

I collect in a *word* file a list of ideas/proposal for our possible future research. I keep adding more. Some of them might not be worthy, but others may be. You always tell me your opinion. Also, they are not in hurry. Many of them are nontraditional and they might look eccentric at the beginning, but at least to give them a try (I mean a few minutes to think at them in the future)... To you I dare to tell all kind of vanguard scientific ideas...

Such things happened in the history of science (see the Non-Euclidean Geometry when it started with Lobachevski, etc.).

\*

We should vary our research and try if possible to adventure in physics, best would be in creating New Foundations of Physics.

For example, we can use the *neutrosophic real numbers* in the 3D  $(x, y, x)$  coordinate system or even in the 4D  $(x, y, x, t = \text{time})$  coordinate system.

Then checking using the *neutrosophic complex numbers* in these systems of coordinates and how the physical laws and equations should be adjusted (generalized) to this new system of *neutrosophic numbers*.

This is because we have a lot of indeterminacy in our world and the mathematical models of it are just approximations.

\*

We can also develop a *mathematical philosophy*, *mathematical psychology*.

There is also a domain called *mathematical linguistics* that we may approach.

We need to extend our investigation into many fields.

\*

1) *Multispace with a multistructure* = combination of many spaces with different structures into a single one; for example a space  $S_1$  can be a *noncommutative ring*,  $S_2$  a *commutative ring* (and not a field),  $S_3$  a *field*; then we put all three of them together. We need to get applications and connections with other knowledge, and

motivation for studying such structures. This is because our world is a hybridization of many spaces that overlap.

Or as in *Smarandache chain structures*:  $S_1$  included in  $S_2$ , which is included in  $S_3$ , which is included in  $S_4$ , and so on, which  $S_{(n-1)}$  is included in  $S_n$ , such that the structure  $S_1$  is stronger than the structure of  $S_2$ , and so on the structure  $S_{(n-1)}$  is stronger than  $S_n$ .

Similarly if we replace "stronger" with "weaker".

We did chains of only two sets,  $S_1$  and  $S_2$ .

\*

Another idea about the future books would be to consider *tensors of higher dimensions*.

For example, a *matrix* is a *rectangle of numbers*. What about a *prism of numbers*?

I mean by a prism of numbers: a rectangle of numbers over another rectangle of numbers, over another rectangle of numbers, etc.

Then there should be a method to multiply such prisms of numbers.

A matrix is for example in 2D (dimension 2):

$$\begin{array}{ccc} 1 & 2 & 5 \\ 0 & 6 & -1 \end{array}$$

Then we take a such matrix and above it we put another matrix, let's say

-2 9 10

5 7 -27

and we form a 3D (three dimensional) prism of numbers  
(where we have twelve numbers).

More odd, it would be to consider for example a *pyramid of numbers*, and how to multiply, add, etc. them?

\*

Other possible *algebraic structures* to study/develop:

- A set  $S$  closed under union and difference of sets.
- A set  $S$  closed under intersection and difference of sets.
- *Neutrosophic Boolean Algebra (NBA)*.

First one defines the *neutrosophic interval*

$(a_1+b_1I, a_2+b_2I) = \{x+yI, \text{ where } a_1 \leq x \leq a_2 \text{ and } b_1 \leq y \leq b_2\}$ ,

which is actually a rectangle if we consider a rectangular system of coordinates, with  $x$  and  $y$  (where  $y$  is the coordinate of indeterminate  $I$ ) as horizontal and respectively vertical axes.

*Intersection* = common geometrical part of two rectangles.

*Union* = the two geometrical rectangles put together.

*Complement* (of a rectangle) = what's outside of that rectangle.

NBA is isomorphic with  $R^2$ , by the function

$$f(x+yI) = (x, y).$$

- What is the strongest possible algebraic structure?

If we consider a set  $S$  such that:

$(S, \vee, \wedge, \neg) = \text{Boolean Algebra,}$

and  $(S, +, \times, K) = \text{Vector Space over the scalar field } K,$

and  $(S, +, \times) = \text{Field,}$

then  $S$  is a triple algebraic structure [or  $S$  has three different structures: Boolean Algebra, Vector Space, and Field].

Can this very strong structure have a particular interest or application?

\*

Let's have a vector space  $V$  over a field  $F$ .

Then a "double subvector space"  $S \subset V$  defined over the field  $G \subset F$  has the following definition:

- a) if  $s \in S$  and  $g \in G$ , then  $sg \in S$  and  $gs \in S$ ;
- b) if  $s_1 \in S$  and  $s_2 \in S$ , then  $s_1 * s_2 \in S$  (here  $*$  is the composition law in  $S$ ).

\*

Let's have a vector space  $V$  over a field  $F$ . Let  $S \subset V$  and  $G \subset F$ . If  $s \in S$  and  $g \in G$ , then  $sg$  and  $gs \in S$ . So  $S$  is a 'special' vector space [ we have released the above condition b) ].

Now the question: what will  $S$  generate? I.e. if  $s_1$  and  $s_2 \in S$ , then  $s_1 * s_2 \in T$ , such that  $T \subseteq V$ .

Are there any particular cases of interest (especially for applications) when  $S$  generates itself ( $S$ ), or  $S$  generates  $V$ , or  $S$  generates another special vector space  $T$  (in the last case, what  $T$  is equal to?).

\*

A topological space  $S$  is a space formed by  $S$ , the empty set ( $\emptyset$ ), and a family of open subsets of  $S$ , such that  $S$  is closed under union and finite intersection.

What about considering a space  $S$ , formed by  $S$ , the empty set ( $\emptyset$ ), and a family of (open) subsets of  $S$ , such that  $S$  is closed under difference of sets. Any applications of it?

*W. B. Vasantha Kandasamy*

We can give *neutrosophic dimension* to many structures.

*Florentin Smarandache*

Maybe using math in social science for helping in counseling psychology, industrial and organizational psychology and related fields?

Also, Graph Theory and *fuzzy/neutrosophic sets* used in psychology.

\*

In *neutrosophic logic/set/probability* we have three components,  $T =$  truth,  $I =$  indeterminacy,  $F =$  falsehood.

What about a *neutrosophic matrix* where each element has three components, for example:

$$(0.2, 0.5, 0) \quad (0.4, 0.6, 0.1)$$

$$(0.1, 0, 0.8) \quad (0.3, 0.3, 0.4)$$

and we can try doing all kind of operations on such matrices.

We can go even more, since the indeterminacy  $I$  can be split in many subcomponents (such as: uncertainty, contradiction, unknown, etc.), and we consider matrices whose each element has  $n$ -components,  $n \geq 1$ .

We can use the *neutrosophic logic/set product*  $a \wedge_N b$  (as for examples  $\min\{a, b\}$ , or  $ab$ , or  $\max\{0, a+b-1\}$ , etc.), we can use the *neutrosophic logic/set sum*  $a \vee_N b$  (as for examples  $\max\{a, b\}$ , or  $a+b-ab$ , or  $\min\{1, a+b\}$ , etc.).

We can define the *neutrosophic logic/set subtraction* of  $a-b$  as  $a \wedge_N C(b)$ , where  $C(b)$  is the *neutrosophic complement* of  $b$ , and  $\wedge_N$  is the above *neutrosophic/fuzzy product*.

The complement of  $(t, i, f)$  can be defined as  $(f, i, t)$ , or as  $(f, 1-i, t)$ , or as  $(1-t, 1-i, 1-f)$ , or as  $(1-t, i, 1-f)$  etc.

\*

The polynomials with set coefficients can be extended to polynomials with set variables:  $x \in P(X)$ , where  $P(X)$  is the power set of  $X = \{x_1, x_2, \dots, x_n\}$ .

This way, we may propose polynomials whose both coefficients and variables are sets:

$$\{2, 3\}\{x_2, x_3, x_4\}^2 + \{1.2, 4, 5\}\{x_1, x_2, x_3\} - \{4, 5, 6, 9\}.$$

How to interpret for example

$$\{x_2, x_3, x_4\}^2 ?$$

Maybe

$$\{x_2^2, x_3^2, x_4^2\} \text{ or } \{x_1x_2, x_1x_3, x_2x_3\} \text{ or both?}$$

Similarly for other powers of the variables.

For all these structures we need to get applications in order to justify their foundations.

Also, unsolved problems, conjectures, and new directions of research related to all these structures in each book.

\*

We will do work on *neutrosophic probability* and on *refined neutrosophic probability*, that also develops *subset probability* and *subset neutrosophic probability*.

It was possible to find in neutrosophic probability analogues of classical results like *Markov chain*, *Tchebyshev theorem*, etc. For example a *Neutrosophic Markov Chain*.

\*

If  $(G,*)$  is a group, then

$$N_2(G) = \{a+bI_1+cI_2, \text{ where } a, b, c \text{ in } G\}$$

and

$$N_3(G) = \{a+bI_1+cI_2+dI_3, \text{ where } a, b, c, d, \text{ in } G\}.$$

Similarly for other  $n$ -indeterminate algebraic structures (*ring, linear algebra*, etc.).

Can we get any practical applications of them?

*W. B. Vasantha Kandasamy*

Such study can be meaningful in applications.



For if we say  $n$  distinct indeterminates, there can be only like  $n$  variables, for otherwise the basic concept will become shaky.

Like complex number  $I$ , only one, not many.

*Florentin Smarandache*

What about defining an algebraic structure which has two unit elements:  $e$  and  $I$ ? Or more general: an algebraic structure with  $n$  unitary elements, elements which are incompatible with each other. {Like  $e$  and  $I$ , for example.} I have considered the refinement of  $I$  as  $I_1, I_2, \dots$  {for example:  $I_1 =$  uncertainty (truth or false),  $I_2 =$  contradiction (true and false),  $I_3 =$  unknown; etc.}, and so on.

This might generate new structures.

If we can get such applications, I mean when the indeterminacy can be split into more sub-indeterminacies...

For example: using them in social science, where there might be various types of indeterminacies...

*W. B. Vasantha Kandasamy*

We can use them as  $n$  indeterminate variables; we will not weaken this nice structure.

*Florentin Smarandache*

We may have, with respect to a population: an indeterminacy  $I_1$  related to a disease  $D_1$  (indeterminacy  $I_1$  could mean the number of people that we don't

know if they are infected or not with that disease), an indeterminacy  $I_2$  related to a disease  $D_2$ , and so on. I wonder how we may construct graphs to reflect this.

\*

Using *neutrosophic numbers* we study the *mean, medium, standard deviation*, etc. of them.

How to compute *square root of a neutrosophic number*  $(a + bI)$  ?

*W. B. Vasantha Kandasamy*

Actually it is possible to compute *square root* (and any root) of a *neutrosophic number*:

$$\sqrt{a + bI} = x + yI; \text{ solve for } x \text{ and } y.$$

Raise to the second power this equality:

$$a + bI = x^2 + (2xy + y^2)I,$$

whence

$$x = \pm\sqrt{a}$$

and from

$$b = 2(\pm\sqrt{a})y + y^2$$

we can compute  $y$ .

We normally get two *complex neutrosophic numbers*.

We don't get a *field of neutrosophic numbers* (since there is not always an inverse with respect to the multiplication), but a *ring of neutrosophic numbers*. For example  $bI$ , with  $b$  not zero, has no inverse with respect to the multiplication.

For example,

$$\sqrt{4-4I} = 2 - 2I, -2 + 2I$$

since

$$(2 - 2I)^2 = 4 - 8I + 4I^2 = 4 - 8I + 4I = 4 - 4I.$$

Similarly,

$$(-2 + 2I)^2 = 4 - 4I.$$

If  $I =$  indeterminate, then  $I^n =$  undefined for  $n \leq 0$ .

Because:

$$I^{(-1)} = 1/I$$

and

$$I^{(-1)} = I^{(1-2)} = I^1 \times I^{(-2)} = I \times (I^2)^{(-1)} = I \times I^{(-1)} = I^{(1-1)} = I^0.$$

From these two equalities, we get:

$$1/I = I^0 \text{ or } 1 = I \times I^0 = I^{(1+0)} = I^1 = I.$$

Therefore we got  $I = 1$  which is absurd.

Therefore  $I^n =$  undefined for  $n \leq 0$ .

\*

### *Florentin Smarandache*

*Neutrosophic Sampling*: the classical probability distribution function extended to *neutrosophic probability distribution function* (a function of three parameters:  $T, I, F$ ).

\*

For low versions (*vector, matrix*) there are *neutrosophic vectors, neutrosophic matrices*. It would be interesting to consider the higher versions of *neutrosophic tensors*.

\*

*W. B. Vasantha Kandasamy*

Further the degree of partial negation of an inverse element can be adopted once I understand the concept given in your research.

*Florentin Smarandache*

The idea is easy: if we take an axiom (or *law*, or *theorem*, or *theory*, or *property* of a mathematical object, etc.) in a given space, there are points for which this axiom works, and other points for which this axiom does not work.

An easy example in geometry: the fifth postulate by Euclid can behave in many different ways: through a point exterior to a line there is a parallel to it; if we take another point and/or line there is no parallel, and then for other points and lines there are many parallels.

This is because in our reality we have all types of different spaces which connect together, i.e. a space where there is only one parallel, another space where there is no parallel, and so on.

If we put together these spaces we get a *multispace*.

In our case (with inverses of *neutrosophic numbers*) we have only two possibilities: the *neutrosophic numbers* which have an inverse (i.e.  $a+bI$  with  $a \neq 0$ , and  $a \neq -b$ ), and the *neutrosophic numbers* which do not have an inverse (i.e.  $bI$  and  $a-aI$ ). We can study what each such set of numbers generates.

\*

*Neutrosophic Tree* is a tree which has at least one indeterminate vertex or at least one indeterminate edge.

*(t,i,f)-Neutrosophic Tree* is also a tree which has (t,i,f)-vertices or/and (t,i,f)-edges.

Or a combination of the above.

\*

I think we can extend the notion of *Neutrosophic Graph* in a similar way: a graph which has at least one indeterminate edge or at least one indeterminate node.

We can have a graph with, for example, the vertices (nodes)  $V_1, V_2, V_3$ , and we know that nodes  $V_1$  and  $V_2$  are directly connected with another node but we don't know it. So the nodes could be:  $V_1, V_2, V_I$ , where  $I =$  indeterminacy.

\*

We got so far that  $I^n = I$  for  $n > 0$ , and  $I^n =$  undefined for  $n \leq 0$ .

I am not sure how to define and justify, when  $I =$  indeterminacy, the following:

$I^i$ , for  $i = \sqrt{-1}$ , also  $i^i, 2^i$  (and in general  $a^i$ )?

\*

Also, we can talk about *degree of indeterminacy* in  $[0, 1]$ . Something may be more indeterminate than other thing.

\*

We may say that a neutrosophic number  $a+bI$  is formed by a determinate part ( $a$ ) and indeterminate part ( $bI$ ).

\*

We might be able to use the *neutrosophic numbers* in approximations; what do you think?

For example,  $1/3 = 0.3333\dots$ , we can say  $1/3 = 0.333 + 0.0003I$ , or  $1/3 = 0.333 + I$ , or in other way.

The more operations we do, the more indeterminacy increases.

If we take  $1/3 = 0.333 + 0.0003I$  and multiply it by 3 we get  $1 = 0.999 + 0.0009I$ .

How should we better employ the *neutrosophic numbers* in approximation calculations (or even in calculus)?

We may employ the neutrosophic numbers in nonstandard analysis (infinitesimals, monads, binads).

\*

I have used non-standard numbers (hyper-reals) in the first book on *neutrosophics*: <http://fs.unm.edu/eBook-Neutrosophics6.pdf>, in order to make a distinction between relative truth (truth in at least one world) and absolute truth (truth in all possible worlds).

I said that  $NL(\text{relative truth}) = 1$ , while  $NL(\text{absolute truth}) = 1^+$ , where  $NL$  means neutrosophic logic, and  $1^+ = 1 + \varepsilon$ .

In general we can have  $a - \varepsilon$  or  $a + \varepsilon$ , where  $a$  is any real number.

So, we can extend to *nonstandard neutrosophic numbers*:  $c+dI$ , where at least one of  $a$  or  $b$  are non-standard numbers.

The problem would be where to apply them?

\*

Three things are improved in the *neutrosophic codes*, they are: *code rate increase, redundancy decreased, number of codewords is also increased* so it becomes better code than ordinary code; the *minimum distance* remains the same.

\*

*Bilattice* was introduced by M. L. Ginsberg, *Comput. Intell.* 4, No. 3, 265--316 (1988).

It's possible to generalize it to  $n$ -lattice, and to generalize the lattice to *neutrosophic lattice, neutrosophic bilattice*, and maybe to *neutrosophic  $n$ -lattice*.

\*

The *left* and *right monads* were introduced by Robinson in 1960s, and extended by Smarandache in 2019 to the *left monad closed to the right*, and *right monad closed to the left*.

The *pierced binad* was introduced by Smarandache in 1995, and extended in 2019 to *unpierced binad*; they were needed in order to close the nonstandard

space to arithmetic operations:  
[http://fs.unm.edu/neut/ExtendedNonstandardNeutrosophic Logic.pdf](http://fs.unm.edu/neut/ExtendedNonstandardNeutrosophicLogic.pdf).

I thought that we can introduce the monads and binads into the neutrosophic numbers:  $a + bi + cI + diI$ , where at least one of the real coefficients  $a, b, c, d$  may be either a monad or a binad,  $i = \sqrt{-1}$  and  $I =$  literal indeterminacy.

Monads and binads extended to complex numbers was done.

The hyper-real numbers (monads) are actually used in the general definition of the neutrosophic logic / set / probability.

\*

*Special Pseudo Linear Algebras* using  $[0, n)$  to be extended to *Special Pseudo Linear Neutrosophic Algebras* using  $[0, n) + [0, n)I$ ?

\*

Would it be possible to construct a *vector space* such that we have two vectors which are partially dependent and partially independent? [alike in fuzzy logic].

Or even more: two vectors which are partially dependent, partially independent, partially indeterminate as relation between themselves? [as in *neutrosophic logic*].

\*

In 3D-space a point has three coordinates (lengths, or angles) with respect to a system of three axes.



I thought at a 3D-surface (therefore non-planar surface), whose points might be represented with respect to 2 axes (instead of 3 axes). I mean, in that surface we can designate two intersecting curves that would be a system of coordinates for all its points (I mean, simplify the coordinate system from 3 coordinates to 2 coordinates).

\*

*Neutrosophic Labels* have the form:  $L_a + I \cdot L_b$ , where  $L_a, L_b$  are labels as defined in *Dezert-Smarandache Vector Spaces and Refined Labels* (by W. B. Vasantha Kandasamy & F. Smarandache): <http://fs.unm.edu/DSmVectorSpaces.pdf>.

\*

We should construct some algebraic structure that may MODEL the *Superluminal Physics* (for speeds  $v > c$ , where  $c$  = light speed) and *Instantaneous Physics* ( $v$  equals to, or in a neighborhood of, infinity).

Using  $I$  = numerical indeterminacy (neutrosophic number, or neutrosophic interval) maybe.

Possibly some neutrosophic (algebraic or geometric) structures?

I thought at *neutrosophic physics* a few years ago, and I found *physical neutrosophic triads* ( $\langle A \rangle$ ,  $\langle \text{neut}A \rangle$ ,  $\langle \text{anti}A \rangle$ ), but not yet a neutrosophic triplet algebraic structure.

\*

Thinking at including somehow the indeterminacy  $I$  into the coordinates.

In general, for a Minkovski space-time  $(x,y,z,t)$ , we can define:

$$x=x_1+x_2I, y=y_1+y_2I, z=z_1+z_2I, \text{ and time } t=t_1+t_2I,$$

where  $x,y,z,t$  become now neutrosophic numbers.

\*

It would be interesting to get some applications and to study how well-known equations from math, physics, chemistry, biology, geology etc. behave in such a *neutrosophic system of coordinates*.

For example the simple equation of a line  $ax+by=c$  in 2D would become

$$ax_1+by_1+ (ax_2+by_2)I = c_1+c_2I,$$

or

$$ax_1+by_1=c_1$$

and the indeterminacy part

$$ax_2+by_2=c_2.$$

How should we interpret these? The real part and respectively indeterminacy part of the linear equation?

Any practical examples?

\*

It would be innovatory to use this *neutrosophic system of coordinates* in physics, chemistry, and biology for certain equations and to find a good interpretation.

\*

We maybe can define the approximate inverse, for example for  $a^{-1}$  modulo  $b$ , as

$$\min\{(b+1)/a \text{ modulo } b, \\ (2b+1)/a \text{ modulo } b, \dots, \\ (b^2-b)/a \text{ modulo } b\}.$$

Does it make sense? Any contradiction?

\*

An example of solving a set equation:

$$Ax=B \text{ (modulo } n),$$

where  $A$  and  $B$  are sets; then  $x=A^{-1}B$  modulo  $n$ , where  $A^{-1}=\{a^{-1}, \text{ for all } a \in A\}$ .

We may consider only the  $a$ 's from  $A$  that are invertible modulo  $n$ .

Let the sets  $A=\{1,2,3\}$  and  $B=\{4,5\}$ .

Let's solve the set equation:

$$Ax+B = 0 \text{ modulo } 7.$$

We get:

$$Ax = -B = \{-4,-5\} = \{3, 2\} = \{2, 3\} \text{ modulo } 7.$$

$$\{1,2,3\}x = \{2,3\} \text{ modulo } 7.$$

Let's compute

$$A^{-1} = \{1,2,3\}^{-1} = \{1^{-1}, 2^{-1}, 3^{-1}\} = \{1,4,5\} \text{ modulo } 7.$$

Then

$$\begin{aligned} x &= \{2,3\}\{1,2,3\}^{-1} = \{2,3\}\{1,4,5\} = \\ &= \{2(1), 2(4), 2(5), 3(1), 3(4), 3(5)\} = \\ &= \{2, 8, 10, 3, 12, 15\} = \\ &= \{2, 1, 3, 3, 5, 1\} = \{1, 2, 3, 5\} \text{ modulo } 7. \end{aligned}$$

\*

When the coefficients of an equation are sets, the solution in general of this equation is also a set.

Now, if instead of integers we have reals, we get the real set (interval) equation:

$$[1,3]x+[4,5] = 0 \text{ modulo } 7.$$

Let's try to solve it in the same way as before:

$$[1,3]x = -[4,5] = [-5,-4] = [2,3] \text{ modulo } 7.$$

Whence:

$$x = [1,3]^{-1}[2,3] \text{ modulo } 7.$$

\*

Not all elements in the interval  $[1,3]$  are invertible modulo 7; what do we do in this case?

I think we should get only those invertible elements modulo 7 from the interval  $[1,3]$ .

How do we determine all of them?

\*

We can extend the *neutrosophic real numbers*

$$a+bI,$$

where  $a$  and  $b$  are reals,

to

$$A+BI,$$

where  $A$  and  $B$  are real sets.

Also,

$$A+Bi+CI+DiI,$$

where  $A, B, C, D$  are real sets, and  $i = \sqrt{-1}$ ,  
 $I =$  indeterminacy.

\*

*Nonstandard numbers as neutrosophic numbers.*

We know that approximately  $a^+ = a + \varepsilon$ , and  $a^- = a - \varepsilon$ , where  
 $a$  is a real number, and  $\varepsilon$  a very tiny number close to  
 zero:  $|\varepsilon| < 1/n$  for any integer  $n$ .

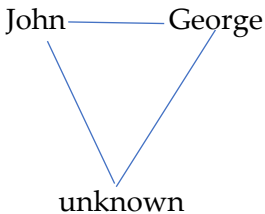
But we can consider  $N = a + \varepsilon$  as a neutrosophic number,  
 where  $a =$  the determinate part of  $N$ , while  $\varepsilon =$   
 indeterminate part of  $N$ .

Similarly,  $M = a - \varepsilon$  or  $M = a + (-\varepsilon)$  can be considered a  
 neutrosophic number, where  $a =$  the determinate part  
 of  $N$ , while  $-\varepsilon =$  indeterminate part of  $N$ .

\*

Besides the graphs with indeterminate edges,  
 we may consider another category of *neutrosophic graphs*,  
 those that have indeterminate nodes.

For example:



\*

The *Neutrosophic Crisp Set* (NCS) was introduced by Salama & Smarandache in 2015, <http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf>.

It is possible to generalize the NCS, and to develop a whole series of *Neutrosophic Crisp Algebraic Structures*.

\*

Always it depends on the way we define the *algebraic law* \*.

For example, let's consider the algebraic neutrosophic set

$$S = \{a+bI, \text{ where } a, b \text{ are non-null real numbers}\}.$$

We define  $(a_1 + b_1I) * (a_2 + b_2I) = a_1a_2 + b_1b_2I$ .

$(S, *)$  is a commutative group, since \* is well-defined, commutative, associative, it has a unit element  $1+I$  because  $(a_1+b_1I) * (1+I) = a_1+b_1I$ , and each element  $a_1+b_1I$  has a unique inverse of the form  $(1/a_1) + (1/b_1)I$  because  $a_1$  and  $b_1$  are non-null, since

$$(a_1+b_1I) * ( (1/a_1) + (1/b_1)I ) = a_1(1/a_1) + b_1(1/b_1)I = 1 + I.$$

\*

We may have an  $m$ -tuple of *neutrosophic extended triplet*:

$$(a; \text{neut}_1(a), \text{neut}_2(a), \dots, \text{neut}_p(a); \\ \text{anti}_1(a), \text{anti}_2(a), \dots, \text{anti}_p(a)),$$

where  $m = 1+2p$ , such that:

- all  $\text{neut}_1(a), \text{neut}_2(a), \dots, \text{neut}_p(a)$  are different from or equal to the unitary element with respect to the composition law \*;
- also

$$a * neut_1(a) = neut_1(a) * a = a$$

$$a * neut_2(a) = neut_2(a) * a = a$$

.....

$$a * neut_p(a) = neut_p(a) * a = a$$

- and

$$a * anti_1(a) = anti_1(a) * a = neut_1(a)$$

$$a * anti_2(a) = anti_2(a) * a = neut_2(a)$$

.....

$$a * anti_p(a) = anti_p(a) * a = neut_p(a).$$

## Neutrosophic Logic

*To Nouran Radwan*

Neutrosophic logic works better than fuzzy logic when dealing with triads.

For example, in voting you can catch all aspects:

*voting Pro, voting Contra, or Neutral voting.*

In games: *winning, losing, or tie game.*

In making a decision: *accepting a decision, rejecting a decision, or pending.*

This middle (neutral, indeterminate) part, i.e. neither true nor false, pending, tie game etc. cannot be caught by fuzzy logic.

That's why I called it (*neutro*)sophic logic, meaning the middle part in between extremes makes the distinction between neutrosophic and fuzzy logic / set / probability.

Neutrosophic logic also permit the refinement of  $T, I, F$  into  $T_1, T_2, \dots, I_1, I_2, \dots, F_1, F_2, \dots$ .

For example, if you want to know what percentage of people didn't vote (denoted by  $I_1$ ) from city  $C_1$ , then what percentage of people didn't vote (denoted by  $I_2$  from city  $C_2$  etc.

See: <http://fs.unm.edu/neutrosophy.htm>.

## **Nedeterminare vs. Terț Inclus**

*Raul Iordăchiță*

Care-i diferența între „nedeterminare” și „terțul inclus”?

*Florentin Smarandache*

“Nedeterminarea” (*Indeterminacy*) din Logica Neutrosofică (*Neutrosophic Logic*) este același lucru cu “Terțul Inclus”, numai că nedeterminarea este mai generală și poate fi rafinată [vezi Logica Neutrosofică Rafinată (*Refined Neutrosophic Logic*):

<http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf>

în multe tipuri de informații vagi, incomplete, confuze, contradictorii etc. [vezi Legea Multiplului-Terț Inclus (*Law of Included Multiple-Middle*): <http://fs.unm.edu/LawIncludedMultiple-Middle.pdf>).

*Raul Iordăchiță*

Adică se revine la *logica terțului exclus*, prin rafinare. Sunt salutare toate interpretările încercărilor de străpungere a metafizicii, însă cred că următorul pas, următoarea



treaptă de relevare majoră a realității, nu a unei noi realități, îl poate face numai un organism colectiv cu conștiință de sine. Pentru conștiința individuală, acesta va fi lumea și transponderul care-i captează unda reverberată de necuprins.

*Florentin Smarandache*

Eu nu mă refer la filozofie, ci la aplicații. Eu nu fac metafizică, ci mă ocup de realitate. Logica (și analog Mulțimea, Probabilitatea și Statistica) Neutrosofică este aceasta: fiecare propoziție logică are un grad de adevăr ( $T$ ), un grad de nedeterminare ( $I$ ) și un grad de falsitate ( $F$ ). Un exemplu din realitate este următorul: Un candidat la președinție are: un procent (grad) de oameni care votează pentru el ( $T$ ), un procent de oameni care nu votează ( $I$ ), și un procent de oameni care votează împotriva lui ( $F$ ). Însa analiștii politici sunt interesați în detalierea rezultatului de votare, adică ei vor să afle cum s-a votat în Ilfov, cum s-a votat în Dolj și în alte județe. Și atunci se face RAFINAREA: procent de oameni din Ilfov care au votat pentru candidat ( $T_1$ ), procent de oameni din Dolj care au votat pentru candidat ( $T_2$ ), etc.; apoi procent de oameni din Ilfov care nu au votat ( $I_1$ ), analog din Dolj ( $I_2$ ) etc.; procent de oameni din Ilfov care au votat împotriva lui ( $F_1$ ), analog din Dolj ( $F_2$ ), etc.

*Raul Iordăchiță*

Am înțeles. Este nevoie și de o apropiere, totuși, matematica este doar o cuantificare a realității, o proteză cu care înțelegem trecutul și provocăm viitorul. Filosofia, prin dragostea ei de armonie, grefează un sens conștienței.

*Florentin Smarandache*

La început, am pornit de la filozofie, neutrosafia ca generalizare a dialecticii:

*In philosophy he introduced in 1995 the 'neutrosophy', as a generalization of Hegel's dialectic, which is the basement of his researches in mathematics and economics, such as 'neutrosophic logic', 'neutrosophic set', 'neutrosophic probability', 'neutrosophic statistics'.*

*Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every notion or idea <A> together with its opposite or negation <antiA> and the spectrum of "neutralities" <neutA> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>. According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium. As a consequence, he generalized*

*the triad thesis-antithesis-synthesis to the tetrad thesis-antithesis-neutrothesis-neutrosynthesis*

[ <http://fs.unm.edu/neutrosophy.htm> ].

Iată un articol în românește despre neutrosofie:

<http://fs.unm.edu/neuro-neutrosafiaNouaRamura.pdf>

*Raul Iordăchiță*

Îl salvez să-l savurez!

## **Neutrosophic Logic of Type 2**

*To Sisalah Bouzina*

Your paper is actually *neutrosophic logic of type 2*, i.e. *neutrosophic logic of neutrosophic logic*. Similarly about the *neutrosophic set of type 2* (*neutrosophic set of neutrosophic set*), and *neutrosophic probability of type 2*.

## **Neutrosophic Logic in Information Fusion**

The *neutrosophic logic* could be used in *information fusion*.

For example, if  $\theta = \{ \theta_1, \theta_2 \}$ , then  $D^0 = \{ \theta_1, \theta_2$  indeterminacy}, where for 'indeterminacy' we consider all other mixed elements. Indeterminacy means neither  $\theta_1$  nor  $\theta_2$ , or  $\theta_1$  and  $\theta_2$  simultaneously, or empty set, or unknown, or incomplete, or paraconsistent. In the decision theory on focus on masses (subjective believes) for  $\theta_1$  and  $\theta_2$  and maybe less interested in others.

If  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , for  $n \geq 2$ , then  $D^\Theta = \{\theta_1, \theta_2, \dots, \theta_n, I\}$  where  $I =$  indeterminacy means neither  $\theta_1$ , nor  $\theta_2$ , ..., nor  $\theta_n$ , or neither any parts (intersections) of some of them, or all of them in the same time, or only a part of them in the same time, or empty set, or unknown.

If so, there are in neutrosophic logic connectors which connect such masses.

### **Three-Way Decision Space as particular case of Neutrosophic Set**

*P. K. Singh*

Three-way decision space provides us a way to classify the given information into the acceptance, rejection, and uncertain regions.

*Florentin Smarandache*

"acceptation, rejection and uncertain" is exactly the *neutrosophy*, or the *neutrosophic set*.

### **Finite Three-Valued Logic & Triple-Infinite Three Valued-Logic**

*Ion Pătrașcu*

Lukasiewicz did a *finite three-valued logic*, while Smarandache did an *infinite three-valued logic* (you have a degree of truth, degree of falsehood, and degree of

indeterminacy - so *neutrosophic logic* is a *triple-infinite logic*).

## **Graph Database**

*To Said Broumi, Vic Christianto*

*Graph Database* is just a particular case of *neutrosophic set/graph*.

We can extend the *Graph Database* to *Neutrosophic Graph Database*.

## **Neutrosophic Psychology / Sociology / Biology**

*Florentin Smarandache*

Neutrosophic theories, based on indeterminacy, gain momentum in more and more fields (now in psychology, sociology, even biology).

I'll always read and wait with interest your new research on neutrosophics.

*Jun Ye*

Your new theory always inspires me for doing more papers and research.

## **Neutrosophic Data Warehouse**

*To A. A. Salama*

Always you try applying the *neutrosophic set* and *logic* in many fields, like your student, Mona Samy Ghareeb Hussin Bondok, did under your supervision, for her

MSc thesis, titled "Neutrosophic Data Warehouse and Dynamic Sets" at Port Said University in Egypt.

## **Neutrosophic Psychology**

*To Ilanthenral Kandasamy*

*Neutrosophic Psychology* means indeterminacy studied in psychology, and connection of opposite theories and their neutral theories together.

If a scale weights are, for example, 1, 2, 3, 4, 5, 6, 7, we can refine in many way, for example:

- pessimistically as T, I<sub>1</sub>, I<sub>2</sub>, I<sub>3</sub>, I<sub>4</sub>, I<sub>5</sub>, F;
- or optimistically as T<sub>1</sub>, T<sub>2</sub>, I<sub>1</sub>, I<sub>2</sub>, I<sub>3</sub>, F<sub>1</sub>, F<sub>2</sub>;
- or more optimistically T<sub>1</sub>, T<sub>2</sub>, T<sub>3</sub>, I, F<sub>1</sub>, F<sub>2</sub>, F<sub>3</sub>;

etc.

Surely, many ideas can be developed on the REFINED neutrosophic set.

## **Neutrosophic Game Theory**

*To Slim Belhaiza*

Yes, I'd be interested in working with you in an extension, if possible, from fuzzy game theory to *neutrosophic game theory*.

## **Picture Fuzzy Set vs. Neutrosophic Set**

The *Picture Fuzzy Set* (PFS) was founded in 2013, while *Neutrosophic Set* (NS) was founded fifteen years ahead - in 1998.

The picture fuzzy set is a particular case of the neutrosophic set.

In PFS the sum of the three degrees (of T, F, refusal) is  $\leq 1$ , while in NS the sum of the three degrees is  $\leq 3$ .

The components in PFS are dependent; in NS they are independent, partially dependent/independent, or dependent.

In PFS the components are in  $[0, 1]$ , in neutrosophic offset they may be outside of  $[0, 1]$ .

*Neutrosophic Refined Set* can describe better the PFS:

T, F, and indeterminacy "I" is refined into "refusal" ( $I_1$ ), and "hesitancy" ( $I_2$ ) - or whatever they are named in PFS.

It would be good if future neutrosophic papers will approach dependence/independence and the neutrosophic over-/under-/off-set - not much done before.

*Theory of Possibility* is equivalent to the *Interval-Valued Probability* (lower probability = necessity, upper probability = possibility), which is a particular case of Neutrosophic Probability (that is a *triplet set probability*), where the chance that an event E occurs is a truth-set  $T \subseteq [0, 1]$ , the chance that the event E does not occur is also a falsehood-set  $F \subseteq [0, 1]$ , and the indeterminate-chance that the event E occurs or not is a set too,  $I \subseteq [0, 1]$ .

Then in the *Theory of Neutrosophic Possibility* the  
truth-necessity (or lower truth-probability) =  $\inf(T)$ ,  
truth-possibility (or upper truth-probability) =  $\sup(T)$ ;  
indeterminate-necessity (or upper indeterminate-  
probability) =  $\sup(I)$ ,  
indeterminate-possibility (or lower indeterminate-  
probability) =  $\inf(I)$ ;  
falsehood-necessity (or upper falsehood-probability) =  
 $\sup(F)$ ,  
falsehood-possibility (or lower falsehood-probability) =  
 $\inf(F)$ .

Thus, *Theory of Neutrosophic Possibility* determines the truth necessity and possibility, the falsehood necessity and possibility, and the indeterminate (neither truth, nor falsehood) necessity and possibility.

## Neutrosophic Triplets

*To A.A.A. Agboola, Young Bae Jun, Arsham Borumand Saeid, Chang Su Kim, Hur Kul, B. Davvaz, Jun Ye, Peide Liu, Pabitra Kumar Maji, Mumtaz Ali, Surapati Pramanik*

The *neutrosophic triplets* are triplets of the form:

$$\langle a, \text{neut}(a), \text{anti}(a) \rangle,$$

where  $\text{neut}(a)$  is the neutral of " $a$ " with respect to a given algebraic law  $*$ , and the neutral is different from the classical algebraic unit element, and  $\text{anti}(a)$  is the



inverse of " $a$ " with respect to the  $neut(a)$  and the same algebraic law  $*$ : i.e. one has the following:

$$a * neut(a) = neut(a) * a = a$$

and

$$a * anti(a) = anti(a) * a = neut(a).$$

An element " $a$ " may have in general more different neutrals  $neut(a)$ , and more different opposite  $anti(a)$ .

These structures are inspired from our everyday world (from triads).

I developed them together with Mumtaz Ali since 2014, but got published barely in 2016.

## Neutrosophic Triplet Function

*Hur Kul*

Recently, we must select only one president from many candidatures. Then at present about 30 % of the total electors is *movable electors*, i.e., *neutrals*.

Thus it is very important for them to select whom.

But we think that  $\langle A \rangle$ ,  $\langle neutA \rangle$ ,  $\langle antiA \rangle$  can select partially another candidate, respectively at voting date. So the final selection is dependent on  $\langle A \rangle$ ,  $\langle neutA \rangle$ ,  $\langle antiA \rangle$ . Of course, it is strongly dependent to  $\langle neutA \rangle$ . Hence we would like to consider  $(\langle A \rangle, \langle neutA \rangle, \langle antiA \rangle)$ ,  $\langle f(\langle A \rangle, \langle neutA \rangle, \langle antiA \rangle) \rangle$  in order to analyze the real world.

Your opinion?

*Florentin Smarandache*

We can simply define a neutrosophic triplet function,

$$f( \langle A \rangle, \langle \text{neut}A \rangle, \langle \text{anti}A \rangle ) = ( f_1(\langle A \rangle), f_2(\langle \text{neut}A \rangle), f_3(\langle \text{anti}A \rangle) ),$$

alike a classical vector function of three variables.

*Hur Kul*

For a group  $(G, *)$ , can consider the following set

$$\langle G \cup \{T, I, F\} \rangle = \{a+bT+cI+dF: a, b, c, d \text{ belong to } G\} ?$$

*Florentin Smarandache*

This is another type of *neutrosophic algebraic structures*, based on *neutrosophic quadruple numbers* (numbers of the form  $a+bT+cI+dF$ , where  $a, b, c, d$  are real or complex numbers), and called *Neutrosophic Quadruple Algebraic Structures*, that I introduced in 2015.

Then Dr. Adesina Agboola started to work on this structure too.

\*

I have defined an ABSORBANCE law, or PREVALENCE order on  $\{T, I, F\}$ . If we consider the pessimistic (prudent) order:  $T < I < F$  which is the most indicated, then:

$$TI = IT = I$$

{ because "I" is bigger, and thus "I" absorbs T;  
or the bigger fish eats the smaller fish }.

$$TF = FT = F$$

{ similarly, the bigger fish eats the smaller fish }.

$$IF = FI = F$$

{ similarly, the bigger fish eats the smaller fish }.

Always ( for any absorbance law, or for any order on {T, I, F} ) one has:

$$T^2 = T, I^2 = I, F^2 = F.$$

*Florentin Smarandache*

We may consider other order: for example  $T < F < I$ , or another one; but it is important to get a justification or an application or an example to support the new order you introduce.

## **Neutrosophic Triplet Group**

*To Kul Hur*

Only two classical axioms on a set  $(N, *)$  are required in order for  $(N, *)$  to be a *Neutrosophic Triplet Group* (NTG), i.e. the law  $*$  has to be well-defined and associative.

But the condition should be that the set  $N$ , embedded with the law  $*$ , be already a set of neutrosophic triplets, that means that for any element  $x \in N$ , there is a neutrosophic triplet  $(a, b, c)$  in  $N$  {i.e. all  $a, b, c \in N$ }, with respect to the law  $*$ , that contains  $a$ , i.e. that

$$x = a \text{ or } x = b \text{ or } x = c.$$

Let  $(N, *)$  be a neutrosophic triplet set. Then  $(N, *)$  is called a *neutrosophic triplet group*, if the following classical axioms are satisfied.

1)  $(\mathbb{N}, *)$  is well-defined, i.e. for any  $a, b \in \mathbb{N}$  one has

$$a*b \in \mathbb{N}.$$

2)  $(\mathbb{N}, *)$  is associative, i.e. for any  $a, b, c \in \mathbb{N}$  one has

$$a*(b*c) = (a*b)*c.$$

NTG, in general, is not a group in the classical way, because it may not have a classical unitary element, nor classical inverse elements.

We consider as the neutrosophic neutrals replacing the classical unitary element, and the neutrosophic opposites as replacing the classical inverse elements.

*To Mumtaz Ali*

This new field of *neutrosophic triplet structures* will become important, because people will realize that they reflect our everyday life [they are not simple imagination!], they are based on real triads: (*friend, neutral, enemy*), (*positive particle, neutral particle, negative particle*), (*positive number, zero, negative number*), (*yes, undecided, no*), (*pro, neutral, against*), and in general ( $\langle A \rangle, \langle \text{neut}A \rangle, \langle \text{anti}A \rangle$ ) as in neutrosophy.

Similarly when I started in 1995 the *neutrosophy* (and consequently *neutrosophic set, logic, probability, statistics, measure* etc.) people were reluctant, attacking and insulting me, especially for the fact that:

$$t + i + f \leq 3, \text{ not } \leq 1.$$

These *neutrosophic triplet structures* will be more practical (therefore more important) than the classical algebraic

structures - the last ones are getting more and more too abstract and too idealistic.

*To Kul Hur*

$a^*anti(a) = anti(a)^*a = neut(a)$  {this is correct axiom as in the published paper} which is an analogous of the classical inverse element.

$a^*neut(a) = neut(a)^*a = a$  (correct axiom) which is analogous of the classical unit element.

$N$  is a set of *neutrosophic triplets*, means that for any element  $a$  in  $N$ , there exists at least one  $neut(a)$  in  $N$  and at least one  $anti(a)$  in  $N$ .

The groupoid may have or may not have a unity element in the classical sense (it does not matter for the neutrosophic triplet structure).  $neut(a)$  has to be different from the classical unit element when we select the neutrals, which means that among the  $neut(a)$ 's you take all except the classical unit element.

*Example 1*

*	1	2
	1	2
	2	2

$(\{1,2\}, *)$  is a groupoid with unit element "1".

Then  $(2,2,2)$  is the only neutrosophic triplet herein.

We do not take  $(1,1,1)$  as a neutrosophic triplet, since "1" is the groupoid unit.

The set of neutrosophic triplets is  $N = \{2\}$ .

*Example 2*

$$\begin{array}{ccc} * & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{array}$$

$(\{1,2\}, *)$  is a groupoid without unit element.

Then  $(1,2,1)$  is a neutrosophic triplet.

The neutrosophic weak triplet set is  $WN = \{1,2\}$ . The definition of a neutrosophic weak triplet set is: for any  $x \in WN$ , there is neutrosophic triplet  $(a, b, c)$  in  $WN$ , such that  $x = a$ , or  $x = b$ , or  $x = c$ .

\*

If  $N$  is the set of the neutrosophic triplets, then for each  $a$  in  $N$ , the corresponding  $neut(a)$  and  $anti(a)$  have to belong to  $N$  too.

This is correct into a neutrosophic triplet group:

$$\begin{aligned} a*anti(a)*neut(a) &= a*anti(a) \text{ or} \\ [a*anti(a)]*neut(a) &= neut(a) \text{ or} \\ neut(a)*neut(a) &= neut(a) \text{ which is true.} \end{aligned}$$

## **Definitions of Neutrosophic Triplet Ring and Neutrosophic Triplet Field**

*To Kul Hur*

*Neutrosophic Triplet Ring* (NTR) is a set endowed with two binary laws  $(M, *, \#)$ , such that:

- a)  $(M, *)$  is a *commutative neutrosophic triplet group*; which means that:
- $M$  is a set of neutrosophic triplets with respect to the law  $*$  (i.e. if  $x$  belongs to  $M$ , then  $neut(x)$  and  $anti(x)$ , defined with respect to the law  $*$ , also belong to  $M$ );
  - the law  $*$  is well-defined, associative, and commutative on  $M$  (as in the classical sense);
- b)  $(M, \#)$  is a set such that the law  $\#$  on  $M$  is well-defined and associative (as in the classical sense);
- c) the law  $\#$  is distributive with respect to the law  $*$  (as in the classical sense).

### **Remarks on Neutrosophic Triplet Ring**

The *Neutrosophic Triplet Ring* is defined on the steps of the classical ring, the only two distinctions are that:

- the classical unit element with respect to the law  $*$  is replaced by  $neut(x)$  with respect to the law  $*$  for each  $x$  in  $M$  into the NTR;
- in the same way, the classical inverse element of an element  $x$  in  $M$ , with respect to the law  $*$ , is replaced by  $anti(x)$  with respect to the law  $*$  in  $M$ .

A *Neutrosophic Triplet Ring*, in general, is different from a classical ring.

### **Neutrosophic Triplet Field**

A *Neutrosophic Triplet Field* (NTF) is a set endowed with two binary laws  $(M, *, \#)$ , such that:

- a)  $(M, *)$  is a *commutative neutrosophic triplet group*; which means that:
- $M$  is a set of *neutrosophic triplets* with respect to the law  $*$  (i.e. if  $x$  belongs to  $M$ , then  $neut(x)$  and  $anti(x)$ , defined with respect to the law  $*$ , also both belong to  $M$ );
  - the law  $*$  is well-defined, associative, and commutative on  $M$  (as in the classical sense);
- b)  $(M, \#)$  is a *neutrosophic triplet group*; which means that:
- $M$  is a set of *neutrosophic triplets* with respect to the law  $\#$  (i.e. if  $x$  belongs to  $M$ , then  $neut(x)$  and  $anti(x)$ , defined with respect to the law  $\#$ , also both belong to  $M$ );
  - the law  $\#$  is well-defined and associative on  $M$  (as in the classical sense);
- c) the law  $\#$  is distributive with respect to the law  $*$  (as in the classical sense).

## Remarks on Neutrosophic Triplet Field

- 1) The *Neutrosophic Triplet Field* is defined on the steps of the classical field, the only four distinctions are that:
- the classical unit element with respect to the law  $*$  is replaced by  $neut(x)$  with respect to the law  $*$  for each  $x$  in  $M$  into the NTF;
  - in the same way, the classical inverse element of an element  $x$  in  $M$ , with respect to the law  $*$ , is replaced by  $anti(x)$  with respect to the law  $*$  in  $M$ ;



- and the classical unit element with respect to the law # is replaced by  $neut(x)$  with respect to the law # for each  $x$  in  $M$  into the NTF;
  - in the same way, the classical inverse element of an element  $x$  in  $M$ , with respect to the law #, is replaced by  $anti(x)$  with respect to the law # in  $M$ ;
- 2) A *Neutrosophic Triplet Field*, in general, is different from a classical field.

*Kul Hur*

If we can find such  $R$ , then we can deal with a *neutrosophic triplet quotient group*?

We try to obtain the relation between  $neut(a)$  and  $anti(a)$ , for example, for any elements  $a$  and  $b$  of semigroup  $(S, *)$ , if there exist  $neut(a)$  and  $neut(b)$ , then there exists an equivalence relation  $R$  on  $S$  such that  $neut(a)*anti(b)$  in  $R$  implies  $neut(b)*anti(a)$  in  $R$  ?

If we can find such  $R$ , then we can deal with a *neutrosophic triplet quotient group*?

*Florentin Smarandache*

$$neut(a)*anti(a) = anti(a).$$

It is not clear to me if  $(S, *)$  is a classical semigroup, or  $S$  is a set of neutrosophic triplets whose law  $*$  is well-defined and associative?

Please specify.

I assume  $(S, *)$  is a set of neutrosophic triplets, and  $*$  is well-defined and associative, so  $(S, *)$  is a neutrosophic triplet group. Otherwise your question does not work...

If  $(a, \text{neut}(a), \text{anti}(a))$  and  $(b, \text{neut}(b), \text{anti}(b))$  are in  $S$ , it is true that  $\text{neut}(a)*\text{anti}(b)$  belongs to  $S$ , and it is also true that  $\text{neut}(b)*\text{anti}(a)$  also belongs to  $S$ .

*Proofs:*

If  $(a, \text{neut}(a), \text{anti}(a))$  and  $(b, \text{neut}(b), \text{anti}(b))$  are in  $S$ , then also if  $a$  is cancellable then  $(\text{anti}(a), \text{neut}(a), a)$ ,  $(\text{neut}(a), \text{neut}(a), \text{neut}(a))$  and if  $b$  is cancellable then  $(\text{anti}(b), \text{neut}(b), b)$ ,  $(\text{neut}(b), \text{neut}(b), \text{neut}(b))$  are also in  $S$ .

These mean that:  $a, \text{neut}(a), \text{anti}(a), b, \text{neut}(b), \text{anti}(b)$  are all in  $S$ .

Since the law  $*$  is well-defined, one has that the composition with the law  $*$  of any two elements are also in  $S$ , i.e. for our cases:  $\text{neut}(a)*\text{anti}(b)$  belongs to  $S$ , and  $\text{neut}(b)*\text{anti}(a)$  belongs to  $S$  as well.

\*

You may define equivalence relationships  $R$  on a neutrosophic triplet group  $G$ , as follows.

For  $a, b$  in  $G$ , one has:

$$aRb \text{ iff } \text{neut}(a) = \text{neut}(b).$$

It is *reflexive*, since  $aRa$  iff  $\text{neut}(a) = \text{neut}(a)$  {which is true};  
*symmetric*: if  $aRb$  then  $bRa$ , which means  $\text{neut}(a) = \text{neut}(b)$  involves  $\text{neut}(b) = \text{neut}(a)$  {which is also true};

and *transitive*, since  $aRb$  and  $bRc$  involves  $aRc$ , which is true:

if  $neut(a) = neut(b)$  and  $neut(b) = neut(c)$  then obviously  $neut(a) = neut(c)$ .

This equivalence relationship is nice, since the class of " $a$ " contains:  $a$ ,  $neut(a)$ ,  $anti(a)$  and maybe other elements.

\*

So, we can get a *neutrosophic triplet quotient group*.

*Example of neutrosophic extended triplet quotient group*  
(extended triplets means the classical unitary element is allowed to be  $neut(x)$ ).

Let  $M = \{0, 2, 4, 6, 8\}$ , with  $*$  (multiplication modulo 10).

Then the neutrosophic triplets are:

$$(0,0,0), (0,0,2), (0,0,4), (0,0,6), (0,0,8), \\ (2,6,8), (4,6,4), (6,6,6), (8,6,2).$$

So,  $M$  is a set of neutrosophic triplets, under  $*$ , and the law is well-defined, associative, and commutative.

Therefore  $(M,*)$  is a commutative neutrosophic triplet group.

Now, let's consider the equivalence relationship:

$$aRb \text{ if and only if } neut(a) = neut(b).$$

Therefore:

$$0R0, \\ \text{but } 2R4, 4R6, 6R8.$$

We get two classes:

class of  $\hat{0} = \{0\} = \{(0,0,0), (0,0,2), (0,0,4), (0,0,6), (0,0,8)\}$ ,

and class of  $\hat{2} = \{2, 4, 6, 8\} =$   
 $\{(2,6,8), (4,6,4), (6,6,6), (8,6,2)\}$ .

Further, the classes  $\{\hat{0}, \hat{2}\}$  form a commutative neutrosophic triplet quotient group, with respect to  $*$  (multiplication modulo 10), since:

$$\hat{0} * \hat{0} = \hat{0} * \hat{2} = \hat{2} * \hat{0} = \hat{0},$$

$$\text{while } \hat{2} * \hat{2} = \hat{2}.$$

The law  $*$  on  $\{\hat{0}, \hat{2}\}$  is well-defined, associative, and commutative.

And the neutrosophic extended triplets of classes are:

$$(\hat{0}, \hat{0}, \hat{0}), (\hat{2}, \hat{2}, \hat{2}),$$

so we have neutrals and anti's of  $\hat{0}$  and  $\hat{2}$  respectively.

Therefore it is proved now that  $(\{\hat{0}, \hat{2}\}, *)$  is a commutative neutrosophic quotient group.

\*

If 0 is the zero element of  $(N, *)$ , then  $(0,0,0)$  is not necessarily a *neutrosophic triplet*. See a *counterexample*:

Let  $N = \{0, 4\}$ , embedded with the law  $*$ , defined as:

$$\begin{array}{ccc} * & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 4 \end{array}$$

Then, the neutrosophic triplets are:  $(0,4,0)$  and  $(4,4,4)$ .

Whence,  $(0,0,0)$  is not a neutrosophic triplet.

## Neutrosophic Fields and Rings

*To Mumtaz Ali*

1) In classical ring, a ring  $R(*, \#)$  has the properties:

$R(*)$  = commutative group;

$R(\#)$  = well-defined, associative law;

and distributivity of  $\#$  with respect to  $*$ .

Then a neutrosophic triplet ring  $NTR(*, \#)$  should be:

$NTR(*)$  = commutative neutrosophic triplet group;

$NTR(\#)$  = well-defined, associative law;

distributivity of  $\#$  with respect to  $*$ .

Then it will result that a neutrosophic triplet commutative group  $NTG(*)$ , endowed with a second law  $\#$  that is well-defined and associative (no need for neutrosophic triplets with respect to  $\#$ ) and distributive with respect to  $*$ , will become a NTR.

2) Going further, a classical ring with a unitary unit with respect to  $\#$  is a unitary ring.

This extends to a Unitary Neutrosophic Triplet Ring (UNTR), i.e. a neutrosophic ring such that each element " $a$ " has a  $neut(a)$  with respect to  $\#$ .

3) Then a Commutative Unitary Neutrosophic Triplet Ring is a UNTR such that  $\#$  is commutative.

4) A Neutrosophic Triplet Field  $NTF(*, \#)$  is a set such that:

NTF(\*) = neutrosophic triplet commutative group;

NTF(#) = neutrosophic triplet group;

# is distributive with respect to \*.

5) If # is commutative, then NTF(\*, #) is a Neutrosophic Triplet Commutative Field.

6) The relationship:

$$neut(a)\#neut(b) = neut(a\#b)$$

is true indeed if  $a$  and  $b$  are cancellable.

*Proof:*

Apply " $a$ " to the left, and " $b$ " to the right above:

$$a\#neut(a)\#neut(b)\#b = a\#neut(a\#b)\#b,$$

and using the associativity one has:

$$(a\#neut(a))\#(neut(b)\#b) = (a\#b)\#neut(a\#b)$$

$a\#b = a\#b$  which is true.

7) Similarly if  $a$  and  $b$  are cancellable we can prove that:

$$anti(a)\#anti(b) = anti(a\#b)$$

by applying " $a$ " to the left and " $b$ " to the right, then the associativity:

$$(a\#anti(a))\#(anti(b)\#b) = (a\#b)\#anti(a\#b)$$

$$\text{or } neut(a)\#neut(b) = neut(a\#b)$$

which was proved to be true.

## Special Cases of Neutrosophic Numbers

*To W. B. Vasantha Kandasamy*

In physics one measures various physical objects, but the measuring is not exact. We may consider such inexact measures as *special cases of neutrosophic numbers*.

For a simple example: if we build a real right triangle from wires, such that each leg is 1 unit, then its hypotenuse is  $\sqrt{2}$ , according to the Pythagorean Theorem.

But,  $\sqrt{2}$  is an irrational number, which has infinitely many decimals, so it is impossible in practice to make a wire (or an object) to have the exact size  $\sqrt{2}$ .

We can generalize that to all irrational numbers, transcendental numbers, etc.

I mean: how should we neutrosophically interpret these practically inexact numbers?

\*

A non-standard number  $a^+ = a + \varepsilon$ , where  $\varepsilon$  is a very tiny positive number (close to zero).

We may then consider  $det(a^+) = a$ , and  $indet(a^+) = \varepsilon$ , where  $det(x)$  means the determinate part of  $x$ , while  $indet(x)$  means indeterminate part of  $x$ .

Similarly, for the non-standard number  $\bar{a} = a - \varepsilon$ .

We may then consider  $det(\bar{a}) = a$ , and  $indet(\bar{a}) = \varepsilon$ .

Even more, the non-standard number (called *binad*, introduced by Smarandache in 1995)  $\bar{a}^+ = a \pm \varepsilon$ , where  $\pm$  means + or -.

We may then consider  $\det(-a+)=a$ , and  $\text{indet}(-a+)=\pm \varepsilon$ .

\*

We can define as **neutrosophic law** either a law that is defined on a set of neutrosophic numbers; or a law defined on non-neutrosophic numbers but having some indeterminate values; or both.

\*

Actually, since for any two elements from

$N(P) = \{a+bI; \text{with } a, b \in [0, 1]\}$  there is an infimum 0 and a supremum  $1+I$ , so it is a lattice as *poset* (partially ordered set).

Or simply *min* and *max* are: commutative, associative, and absorbent.

## Derivatives and integrals of neutrosophic number functions

If  $f(z)=u(x)+v(y)I$ , where  $z=x+yI$  is a neutrosophic real or complex number:

- What is the neutrosophic derivative  $f'_N(z)=?$
- What is the neutrosophic integral  $\int_N f(z) dz$  ?

## Definition of Neutrosophic Laws

A *neutrosophic law* is:

- a) either a law defined on a neutrosophic set, or on a set of neutrosophic numbers;



- b) or a law defined on a non-neutrosophic set, but this law is partially defined;
- c) or a law defined on non-neutrosophic set, but which has at least one partially invalid axiom; [this is a particular case from geometry, i.e. from Smarandachely denied axiom, which is an axiom which is in the same space validated and invalidated, or only invalidated but in at least two different ways]
- d) or any combinations of the above a), b), c).

A neutrosophic lattice is a lattice having at least one neutrosophic law.

## Hybrid numbers

*Said Broumi*

This paper is mainly about the decision-making method based on prospect theory in heterogeneous information environment. The heterogeneous information includes *exact number, interval number, linguistic term, intuitionistic fuzzy number, interval intuitionistic fuzzy number, neutrosophic numbers, and trapezoidal fuzzy neutrosophic numbers*. One of the comments of a reviewer is as follows: "In *Introduction*, the authors mentioned that "the Neutrosophic sets is the generalization of the theory of fuzzy sets, interval-valued fuzzy sets and intuitionistic fuzzy sets, etc." However, they still take the interval-valued fuzzy sets

and intuitionistic fuzzy sets as the heterogeneous information of the Neutrosophic sets, why?"

*Florentin Smarandache*

The best for the authors will be to extend all different types of numbers to neutrosophic numbers, which are the most general. Otherwise it will be almost impossible to aggregate (union, intersection, implication, equivalence etc.) them.

How for example we combine an IFN (intuitionistic fuzzy number) and a NN (neutrosophic number):

$$(0.2, 0.5) \wedge (0.6, 0.1, 0.3) ?$$

We transform the first into a NN:

$$(0.2, 0.3, 0.5) \wedge (0.6, 0.1, 0.3)$$

which is *neutrosophic intersection*.

How do you combine a FN (fuzzy number) with a NN ?

$$(0.7) \wedge (0.6, 0.1, 0.3) ?$$

You transform the fuzzy into neutrosophic number:

$$(0.7, 0, 0.3) \wedge (0.6, 0.1, 0.3)$$

which is a *neutrosophic intersection*.

*Another example:*

For a crisp number, say 0.4, we transform it into an interval:  $[0.4, 0.4]$ .

One problem is with the linguistic number, which has to be converted/approximated into an *Interval Neutrosophic Number*.

For my curiosity, how did the authors combined these hybrid numbers?

## **Subtraction and Division of Neutrosophic Numbers**

*To Kajal Chatterjee*

So far there have been defined the  $A + B$ ,  $A \times B$ ,  $\lambda \times A$ , and  $A^\lambda$ , where  $A$  and  $B$  are neutrosophic numbers, while  $\lambda > 0$  is a scalar. See: *A Multi-Criteria Decision-Making Method Using Power Aggregation Operators for Single-valued Neutrosophic Sets*, by Lihua Yang and Baolin Li. Published in *International Journal of Database and Theory and Application*, Vol. 9, No. 2 (2016), pp. 23-32; <http://fs.unm.edu/AMultiCriteriaDecisionMaking.pdf>

The  $+$  was defined as the neutrosophic union,  $\times$  as neutrosophic intersection, while " $\lambda \times A$ " as a generalization of  $+$  [as repeated addition], and similarly " $A^\lambda$ " as a generalization of  $\times$  (as repeated multiplication).

I have done something for  $A - B$  and  $A / B$  [they only partially work].

See this paper which is under press that you can cite at references as: F. Smarandache, *Subtraction and Division of Neutrosophic Numbers*, *Critical Review*, Vol. XIII, 103-110, 2016, <http://fs.unm.edu/CR/SubstractionAndDivision.pdf>.

Please try to use them, and let me know if they work for your paper.

\*

*To Kajal Chatterjee*

For  $t_1, t_2, \dots, t_n$  as neutrosophic truth components of neutrosophic numbers, one has:

$$t_1 \oplus t_2 = t_1 + t_2 - t_1t_2 = \{t_1 + t_2\} - \{t_1t_2\} = S_1 - S_2.$$

$$\begin{aligned} (t_1 \oplus t_2) \oplus t_3 &= (t_1 + t_2 - t_1t_2) \oplus t_3 = \\ &= t_1 + t_2 + t_3 - t_1t_2 - t_1t_3 - t_2t_3 + t_1t_2t_3 \\ &= \{t_1 + t_2 + t_3\} - \{t_1t_2 + t_2t_3 + t_3t_1\} + \{t_1t_2t_3\} = \\ &= S_1 - S_2 + S_3. \end{aligned}$$

$$\begin{aligned} (t_1 \oplus t_2 \oplus t_3) \oplus t_4 &= (t_1 + t_2 + t_3 - t_1t_2 - t_1t_3 - t_2t_3 + t_1t_2t_3) \oplus t_4 \\ &= t_1 + t_2 + t_3 + t_4 - t_1t_2 - t_1t_3 - t_2t_3 - t_1t_4 - t_2t_4 - t_3t_4 + t_1t_2t_3 + \\ &\quad t_1t_2t_4 + t_1t_3t_4 + t_2t_3t_4 - t_1t_2t_3t_4 \\ &= \{t_1 + t_2 + t_3 + t_4\} - \{t_1t_2 - t_1t_3 - t_2t_3 - t_1t_4 - t_2t_4 - t_3t_4\} + \{t_1t_2t_3 + \\ &\quad t_1t_2t_4 + t_1t_3t_4 + t_2t_3t_4\} - \{t_1t_2t_3t_4\} \\ &= S_1 - S_2 + S_3 - S_4. \end{aligned}$$

In general:

$$t_1 \oplus t_2 \oplus \dots \oplus t_n = S_1 - S_2 + \dots + (-1)^{k+1}S_k + \dots + (-1)^{n+1}S_n,$$

and, for  $1 \leq k \leq n$ , one has  $S_k = \sum_{\{j_1, j_2, \dots, j_k\}} t_{j_1} t_{j_2} \dots t_{j_k}$ , where

$\{j_1, j_2, \dots, j_k\}$  are permutations of  $n$  elements  $\{1, 2, \dots, n\}$

taken by groups of  $k$  elements.

\*

*To Jun Ye*

Did you try to consider different definitions for the addition and multiplication of neutrosophic numbers?

For example:

$$(t_1, i_1, f_1) + (t_2, i_2, f_2) = \\ = (\max\{t_1, t_2\}, \min\{i_1, i_2\}, \min\{f_1, f_2\})$$

or

$$(t_1, i_1, f_1) + (t_2, i_2, f_2) = \\ = (\min\{t_1 + t_2, 1\}, \max\{i_1 + i_2 - 1, 0\}, \max\{f_1 + f_2 - 1, 0\})$$

Respectively:

$$(t_1, i_1, f_1) \times (t_2, i_2, f_2) = \\ = (\min\{t_1, t_2\}, \max\{i_1, i_2\}, \max\{f_1, f_2\})$$

or

$$(t_1, i_1, f_1) \times (t_2, i_2, f_2) = \\ = (\max\{t_1 + t_2 - 1, 0\}, \min\{i_1 + i_2, 1\}, \min\{f_1 + f_2, 1\})$$

Also, can you find some applications to the subtraction and division of neutrosophic numbers?

## General Intersection and Union of Neutrosophic Sets

*To Mumtaz Ali*

For the intersection and union of neutrosophic sets, the most general definitions are:

$$(t_1, i_1, f_1) \wedge_N (t_2, i_2, f_2) = (t_1 \wedge_F i_1, f_1 \vee_N f_2, i_1 \vee_N i_2)$$

$$(t_1, i_1, f_1) \vee_N (t_2, i_2, f_2) = (t_1 \vee_N i_1, f_1 \wedge_F f_2, i_1 \wedge_F i_2)$$

where  $\wedge_F$  is the (fuzzy) t-norm, and  $\vee_N$  is the (fuzzy) t-conorm both from fuzzy set and logic.

So we can use for  $\wedge_F / \vee_N$  respectively:

$$\min\{t_1, t_2\} / \max\{t_1, t_2\} \\ t_1 t_2 / t_1 + t_2 - t_1 t_2$$

$$\max\{t_1+t_1-1, 0\} / \min\{t_1+t_2, 1\}$$

and other such functions verifying the axioms of t-norm and t-conorm respectively.

### **Extended union of soft sets in order to form a multi-soft set**

For three soft sets  $(F_1, A_1)$ ,  $(F_2, A_2)$ ,  $(F_3, A_3)$  over the same universe set  $U$ , we have their union as  $(G, B)$ , where  $B = A_1 \cup A_2 \cup A_3$  in the following way:

For any  $b$  in  $B$  we have:

$$\begin{aligned} G(b) = & F_1(b) \text{ if } b \text{ in } A_1 - (A_2 \cup A_3), \\ & \text{or } F_2(b) \text{ if } b \text{ in } A_2 - (A_3 \cup A_1), \\ & \text{or } F_3(b) \text{ if } b \text{ in } A_3 - (A_1 \cup A_2), \\ & \text{or } F_1(b) \cup F_2(b) \text{ if } b \text{ in } A_1 \cap A_2 - A_3, \\ & \text{or } F_2(b) \cup F_3(b) \text{ if } b \text{ in } A_2 \cap A_3 - A_1, \\ & \text{or } F_3(b) \cup F_1(b) \text{ if } b \text{ in } A_3 \cap A_1 - A_2, \\ & \text{or } F_1(b) \cup F_2(b) \cup F_3(b) \text{ if } b \text{ in } A_1 \cap A_2 \cap A_3. \end{aligned}$$

There are  $2^3 - 1 = 7$  possibilities.

\*

In the general case, for  $n$  soft sets:  $(F_1, A_1)$ ,  $(F_2, A_2)$ , ...,  $(F_n, A_n)$  over the same universe set  $U$ , we have their union as  $(G, B)$ , where  $B = A_1 \cup A_2 \cup \dots \cup A_n$  in the following way:

For any  $b$  in  $B$  we have:

$$\begin{aligned} G(b) = & F_{i1}(b) \cup F_{i2}(b) \cup \dots \cup F_{ik}(b) \\ \text{if } b \text{ in } & (A_{i1} \cap A_{i2} \cap \dots \cap A_{ik}) - (A_{ik+1} \cup A_{ik+2} \cup \dots \cup A_{in}), \end{aligned}$$

where  $(i_1, i_2, \dots, i_k, i_{k+1}, i_{k+2}, \dots, i_n)$  are all possible permutations of the indexes  $(1, 2, \dots, n)$ , and  $k = 1, 2, \dots, n$ . There are  $2^n - 1$  possibilities.

## **Trapezoidal Neutrosophic Number vs. Trapezoidal Fuzzy Neutrosophic Number**

*To Said Broumi*

The *Trapezoidal Neutrosophic Number* should be the most general:

$$\langle a_1, a_2, a_3, a_4 \rangle, \langle b_1, b_2, b_3, b_4 \rangle, \langle c_1, c_2, c_3, c_4 \rangle;$$

while *Trapezoidal Fuzzy Neutrosophic Number* should be a particular case, i.e. when all these three vectors are equal:

$$\langle a_1, a_2, a_3, a_4 \rangle = \langle b_1, b_2, b_3, b_4 \rangle = \langle c_1, c_2, c_3, c_4 \rangle.$$

## **Neutrosophic Multiset**

*To W. B. Vasantha Kandasamy*

The neutrosophic membership of an individual John, in our everyday life, may vary from a time to another time with respect to a neutrosophic set A. For example, we may have:

$\langle \text{John}, 0.5, 0.2, 0.4 \rangle$  at time  $t_1$ , then again  $\langle \text{John}, 0.5, 0.2, 0.4 \rangle$  at time  $t_2$ , but later  $\langle \text{John}, 0.6, 0.1, 0.3 \rangle$  at time  $t_3$ , and  $\langle \text{John}, 0.7, 0.0, 0.2 \rangle$  at time  $t_4$  etc.

This is a *neutrosophic multiset*.

## **Neutrosophic Multiset BCI/BCK-Algebras**

*To Young Bae Jun, Seok-Zun Song, Hashem Bordbar*

As future research: what about introducing and studying  
Neutrosophic Multiset BCI/BCK-Algebras?

See: <http://fs.unm.edu/NeutrosophicMultisets.htm>

and a small chapter on *Neutrosophic Multisets* (an extension of classical multisets) into the book *Neutrosophic Perspectives...* (2017), pp. 115-122:

<http://fs.unm.edu/NeutrosophicPerspectives-ed2.pdf>.

Never the Neutrosophic Multisets were studied (except the little work I did), it is almost a virgin territory, so it would be good to extend your BCK/BCI-algebras expertise to various fields, for diversity of research.

## **Neutrosophic Multiset**

*To Xiaohong Zhang*

Can you also check the *neutrosophic multisets* (<http://fs.unm.edu/NeutrosophicMultisets.htm>)?...

They are from our everyday life, since the neutrosophic degree of membership / indeterminacy / nonmembership of an element with respect to a set CHANGES over time.

## **Neutrosophic Multiset Applied in Physical Processes**

Let  $U$  be a universe of discourse and a set  $M \subseteq U$ . The *Neutrosophic Multiset*  $M$  is defined as a neutrosophic set



with the property that one or more elements are repeated either with the same neutrosophic components, or with different neutrosophic components.

For example,

$$Q = \{a(0.6,0.3,0.2), a(0.6,0.3,0.2), a(0.5,0.4,0.1), \\ b(0.7,0.1,0.1)\}$$

is a *neutrosophic multiset*.

The *Neutrosophic Multiplicity Function* is defined as:

$$nm: U \rightarrow N = \{1, 2, 3, \dots\},$$

and for each  $x \in M$  one has

$$nm(x) = \{(k_1, \langle t_1, i_1, f_1 \rangle), (k_2, \langle t_2, i_2, f_2 \rangle), \dots, (k_j, \langle t_j, i_j, f_j \rangle), \dots\},$$

which means that in the set  $M$  the element  $x$  is repeated  $k_1$  times with the neutrosophic components  $\langle t_1, i_1, f_1 \rangle$ , and  $k_2$  times with the neutrosophic components  $\langle t_2, i_2, f_2 \rangle$ , ...,  $k_j$  times with the neutrosophic components  $\langle t_j, i_j, f_j \rangle$ , ... and so on. Of course,  $\langle t_p, i_p, f_p \rangle \neq \langle t_r, i_r, f_r \rangle$  for  $p \neq r$ .

For example, with respect to the above neutrosophic multiset  $Q$ ,  $nm(a) = \{(2, \langle 0.6, 0.3, 0.2 \rangle), (1, \langle 0.5, 0.4, 0.1 \rangle)\}$ .

Neutrosophic multiset is used in time series, and in representing instances of a physical process at different times, since its neutrosophic components vary with time.

## **Complex Neutrosophic Multiset**

*To Mumtaz Ali, Ganeshsree Selvachandran*

I think we can do a paper together on *Complex Neutrosophic Multiset* (CNMS) and Applications.

I attach the definition of neutrosophic multiset, neutrosophic multiplicity function, and examples of neutrosophic multiset.

What was not done before (upon my knowledge), the possibility of having the same element  $x$  with the same and also with various  $t, i, f$  components into the same (multi)set.

We can define a CNMS as a complex neutrosophic set, whose either the amplitude or phase or both as neutrosophic multisets.

And Dr. Ganeshsree will do some application.

## **Applications of Neutrosophic Duplets**

The *neutrosophic duplets* are applicable to the cases of items that do not have opposites, but have only neutrals.

For example, there are animals that have no predators.

Or objects for whom the opposites make no sense. Let's say: what is the opposite of a... table? While the "neutrals" of a "table" are all other objects.

## Neutrosophic Multisets in MCDM

*To Ganeshsree Selvachandran*

We may connect the *neutrosophic set* MCDM with neutrosophic "multisets" MCDM.

## Interval Complex Neutrosophic Sets

*To Luu Dat, Le Hoang Son, Mumtaz Ali*

Let's consider an interval neutrosophic complex item:

$$([t_1, t_2] \times e^{[t_3, t_4]}, [i_1, i_2] \times e^{[i_3, i_4]}, [f_1, f_2] \times e^{[f_3, f_4]}),$$

then one defines:

- *the score function parts:*

$$\text{AMPLITUDE: } (1/6)(4+t_1+t_2-i_1-i_2-f_1-f_2)$$

$$\text{PHASE: } (1/6)(4+t_3+t_4-i_3-i_4-f_3-f_4)$$

- *the accuracy function parts:*

$$\text{AMPLITUDE: } (1/2)(t_1-f_1+t_2-f_2)$$

$$\text{PHASE: } (1/2)(t_3-f_3+t_4-f_4)$$

- *the certainty function parts:*

$$\text{AMPLITUDE: } (1/2)(t_1+t_2)$$

$$\text{PHASE: } (1/2)(t_3+t_4)$$

In this way, for each computed amplitude we can associate a corresponding phase, and then re-interpret the results in terms of phase as well.

## Interval Bipolar Complex Neutrosophic Set

We may extend, if you all are interested, the previous paper to *Interval Bipolar Complex Neutrosophic Set*, never done before upon my knowledge.

We have:

$$( [t_{p1}, t_{p2}] \times e^{j([tp3, tp4])}, [i_{p1}, i_{p2}] \times e^{j([ip3, ip4])}, [f_{p1}, f_{p2}] \times e^{j([fp3, fp4])}, \\ [t_{n1}, t_{n2}] \times e^{j([tn3, tn4])}, [i_{n1}, i_{n2}] \times e^{j([in3, in4])}, [f_{n1}, f_{n2}] \times e^{j([fn3, fn4])} )$$

Where the positive neutrosophic components

$$[t_{p1}, t_{p2}], [t_{p3}, t_{p4}], [i_{p1}, i_{p2}], [i_{p3}, i_{p4}], [f_{p1}, f_{p2}], [f_{p3}, f_{p4}],$$

are included in  $[0, 1]$ ; while the negative neutrosophic components:

$$[t_{n1}, t_{n2}], [t_{n3}, t_{n4}], [i_{n1}, i_{n2}], [i_{n3}, i_{n4}], [f_{n1}, f_{n2}], [f_{n3}, f_{n4}],$$

are included in  $[-1, 0]$ .

The advantage is that, besides the positive qualitative values, we get the negative qualitative values too.

Now, when we apply the neutrosophic union or neutrosophic intersection, we do the opposite for the combination of the negative neutrosophic components, in comparison with the operations to the positive neutrosophic components.

*Example:*

if we use  $\vee$  for the positive truth ( $t_p$ ), we need to use  $\wedge$  for the negative truth ( $t_n$ ).

Similarly for positive indeterminacy and respectively negative indeterminacy; as well as for positive falsehood and respectively negative falsehood.

Let's say the neutrosophic union:

$$( \text{for positive components } (\vee, \vee), (\wedge, \wedge), (\wedge, \wedge); \\ \text{for negative components } (\wedge, \wedge), (\vee, \vee), (\vee, \vee) ),$$

where the first  $(\vee, \vee)$  means the first  $\vee$  is for the real part, and second  $\vee$  for the complex part for the positive truth;  
next  $(\wedge, \wedge)$  means  $\wedge$  for the real part, and second  $\wedge$  for the complex part of the positive indeterminacy;  
and so on.

## **Neutrosophic (...) BCI-Algebras**

*To J. Kim, K. Hur, P. K. Lim, J. G. Lee*

I think you can extend your research from *neutrosophic BCI-algebras* to the *neutrosophic triplet BCI-algebras* and to: *neutrosophic duplet BCI-algebras* and *neutrosophic multiset BCI-algebras*.

## **Neutrosophic Duplet Structures & Neutrosophic Multiset Structures**

*To J. Kim, K. Hur, P. K. Lim, J. G. Lee*

In my book *Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications*, 2017, I proposed for the first time two new types of algebraic structures.

These completely new types of algebraic structures are:

- 1) *Neutrosophic Duplet Structures* ( see book pages: 109-114, and <http://fs.unm.edu/NeutrosophicDuplets.htm> );
- 2) and *Neutrosophic Multiset Structures* ( see book pages: 115-123, and <http://fs.unm.edu/NeutrosophicMultisets.htm> ),

where the *neutrosophic multiset* was an extension of the classical multiset, never done before.

\*

So, our theorem should be:

Let  $(M, *)$  be a *neutrosophic triplet group*, where the law  $*$  satisfies the left and right cancellation laws.

If  $\langle a, b, c \rangle$  is a neutrosophic triplet, then  $\langle b, c, a \rangle$  and  $\langle b, b, b \rangle$  are also neutrosophic triplets.

The cancellation laws include the zero-divisors and the zero elements, I mean if there are zero-divisors then the cancellation laws do not hold.

Another think: *zero* is included in the cancellation laws.

I mean: if  $0^*b = 0^*c$  it shouldn't involve that  $b = c$ .

Or is it a restriction that  $a, b, c$  are different from *zero*?

\*

I feel that we need to keep the associativity, that's why I said that  $M$  has to be a group, and not just a set... What do you think?

\*

The following axiom:

- for any  $a, b, c$  in the set  $(M, *)$ , if  $a^*b = a^*c$ , then  $b = c$ ;

and

- for any  $a, b, c$  in the set  $(M, *)$ , if  $b^*a = c^*a$ , then  $b = c$ .

\*

Or how is called a set  $(M, *)$  that satisfies this axiom?

That's the problem for our *Theorem*:

If  $\langle a, b, c \rangle$  is a neutrosophic triplet, then  $\langle c, b, a \rangle$  and  $\langle b, b, b \rangle$  are neutrosophic triplets.

## Internal Laws of Algebraic Structures

*To W. B. Vasantha Kandasamy*

As we know from the classical abstract algebra, we have algebraic structures with one binary internal law (*groupoid, semigroup, loop, group*), and with two binary internal laws (*ring, semiring, field, semifield*).

What about introducing algebraic structures with 3 or more internal binary laws?

And each law being defined on many arguments (*binary, ternary, m-ary* in general for integer  $m \geq 1$ ).

We may consider a non-empty set, on which we define many internal *m-ary* laws:

law  $L_1$  which is  $m_1$ -ary,

law  $L_2$  which is  $m_2$ -ary,

...

law  $L_n$  which  $m_n$ -ary.

Have such things being study in the literature so far?

Can we find any applications of them?

Any connections with other theories as well?

## **External Laws of Algebraic Structures**

*To W. B. Vasantha Kandasamy*

The *vector space* has, behind the addition and multiplication of vectors (i.e. *internal laws*), also an external law (*scalar multiplication of vectors*).

Besides the *internal laws* defined previously, we can all introduce more *external laws* in our algebraic structures.

Therefore, an algebraic structure with  $m$  internal laws, and  $r$  external laws.

What has been done in the literature about these?

## **Neutrosophic n-ary Algebraic Structures**

*To Young Bae Jun*

What about structures where we have  $n$ -ary laws,  $n \geq 3$ , not only binary laws, extended to *neutrosophic environment*?

## **Neutrosophic Quadruple Algebraic Structures**

For a group  $(G, *)$ , we may consider the following set

$$\langle G \cup \{T, I, F\} \rangle = \{a+bT+cI+dF: a, b, c, d \in G\}.$$

This is another type of neutrosophic algebraic structures, based on neutrosophic quadruple numbers (numbers of the form  $a+bT+cI+dF$ , where  $a, b, c, d$  are real or complex numbers), and called *Neutrosophic Quadruple Algebraic Structures*, that I introduced in 2015.



Then Dr. Adesina Agboola started to work on this structure too.

\*

There is another type:  $(t,i,f)$ -neutrosophic algebraic structures, when the elements of a set only partially belong to the set, and we apply laws of composition on both: the elements themselves (as in classical algebraic structures), and on the  $(t,i,f)$ -neutrosophic degrees of appurtenance to the neutrosophic set.

\*

Then I have refined these neutrosophic structures since 2013.

T is refined into sub-truth components  $T_1, T_2, \dots$ ;

Similarly I into  $I_1, I_2, \dots$ ; and F into  $F_1, F_2, \dots$ .

Examples are many from our everyday life about neutrosophic refinements.

The neutrosophic research, including the neutrosophic triplet research, are very related to the real world.

## **Neutrosophic Quadruple BCK/BCI-Algebras**

*To Young Bae Jun*

The *neutrosophic quadruple algebraic structures* can be extended to *Neutrosophic Quadruple BCK/BCI-Algebras*, what do you think?

## Neutrosophic Cube

There is a *Neutrosophic Cube*, whose three axes are  $OT$ ,  $OI$ ,  $OF$ .

$bT + cI + dF$  can be the position of a particle (or point) inside the neutrosophic cube, while " $a$ " ( from  $a + bT + cI + dF$  ) could be time (or another parameter that the particle depends on) =  $a$ .

Then we extend, and  $OT$ ,  $OI$ ,  $OF$  can be the whole *Oxyz* Cartesian real system of coordinates.

For the multiplication, the most common order is be  $T < I < F$  (the pessimistic/prudent way), so the bigger absorbs the smaller when multiplying.

But you can choose your own order depending on the application to solve.

## Refined Neutrosophic BCK/BCI-algebras

*To Young Bae Jun*

I think it is possible to extend the Neutrosophic BCK/BCI-algebras to Refined Neutrosophic BCK/BCI-algebras [see Florentin Smarandache, *n-Valued Refined Neutrosophic Logic and Its Applications in Physics*, Progress in Physics, 143-146, Vol. 4, 2013;

<https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf> ],

i.e.  $T$  is refined into  $T_1, T_2, \dots$  (sub-truths);  $I$  is refined into  $I_1, I_2, \dots$  (sub-indeterminacies); and  $F$  is refined into  $F_1, F_2, \dots$  (sub-falsehoods).

## Support-Neutrosophic Set

*To Nguyễn Xuân Thảo*

It is easy to extend from *Support-Intuitionistic Fuzzy Set* to *Support-Neutrosophic Set*.

## (t,i,f)-Neutrosophic Structures

*To Young Bae Jun*

There was defined a structure such as "(t, i, f)-Neutrosophic Structures" when we consider a law \* on x's and another law # on the neutrosophic components, thus:

$$x_1(t_1, i_1, f_1) + x_2(t_2, i_2, f_2) = [x_1 * x_2] \{ (t_1, i_1, f_1) \# x_2(t_2, i_2, f_2) \}.$$

[See <http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf>.]

## (t,i,f)-Neutrosophic Multiset Group/Loop

*To Tèmitópé Gbóláhàn Jaíyéolá*

The problem and the distinction between classical multiset structures and neutrosophic multiset structures, is that the classical multiset structures have no coordinates, while in neutrosophic set one has the neutrosophic components, for example  $a(0.5, 0.2, 0.6)$ , while in classical multiset structures each element is  $a(1, 0, 0)$ .

I have defined in my book *Symbolic Neutrosophic Theory* (2015) the *(t,i,f)-neutrosophic algebraic structures*, but they are not much studied, since are brand new...

What does it mean and it was never done before?

If we make a neutrosophic law  $*$  defined as:

$$x_1(t_1, i_1, f_1) * x_2(t_2, i_2, f_2)$$

we might need two classical laws  $\#$  and  $\diamond$ ,

one that combines the elements  $x_1\#x_2$ , and the other that combines their neutrosophic coordinates

$$(t_1, i_1, f_1) \diamond (t_2, i_2, f_2).$$

I have constructed this example:

$$x_1(t_1, i_1, f_1) * x_2(t_2, i_2, f_2) = \max\{x_1, x_2\}(\min\{t_1, t_2\}, \max\{i_1, i_2\}, \max\{f_1, f_2\}).$$

Using this example, we can construct a  $(t, i, f)$ -neutrosophic group/loop, and then a  $(t, i, f)$ -neutrosophic multiset group/loop.

This is a case when  $\#$  and  $\diamond$  work separately,

But it might be possible to combine  $x_1, x_2, t_1, i_1, f_1, t_2, i_2, f_2$  all together...

## Seira-Group

Etymologically, (Gr.) *σειρά* [ *seirá* ] means *set, string*.

Therefore, Seira-Group is a group whose unitary element is replaced by a set of two or more distinct elements.

*Definition of Seira-Group*

Let  $(G, E, *)$  be a nonempty set, where the law  $*$  is:

- Well-Defined: if  $x, y \in G$ , then  $x*y \in G$ ;
- Associative: if  $x, y, z \in G$ , then  $(x*y)*z = x*(y*z)$ ;
- The set  $E \subsetneq G$  (i.e.  $E \neq G$ ) has at least two distinct elements:  $E = \{e_1, e_2, \dots, e_n\}$ , for  $n \geq 2$ .

Each element  $x \in G$  has at least one corresponding unit (neutral) element  $e \in E$ ,

$$\text{i.e. } x * e = e * x = x.$$

And for each element  $e_i \in E$  there exist at least one element  $x_i \in G$ , such that

$$x_i * e_i = e_i * x_i = x_i, \text{ with } i \in \{1, 2, \dots, n\}.$$

{Therefore, the classical unit (neutral) element is replaced by a set of two or more distinct unit (neutral) elements.}

- For each element  $x \in G$  there exist at least an inverse element  $x^{-1} \in G$  such that

$$x * x^{-1} = x^{-1} * x \in E.$$

If the law  $*$  is commutative, then  $(G, E, *)$  is a *Commutative Seira-Group*.

*Example 1 of Seira-Group*

Using the Cayley Table:

$$G = \{a, b, c\}, E = \{a, b\}.$$

$*$	$a$	$b$	$c$
$a$	$a$	$a$	$b$
$b$	$b$	$b$	$c$
$c$	$a$	$c$	$b$

The neutrals are:  $neut(a) = a, neut(b) = b, neut(c) = b$ .

The inverse elements are:  $anti(a) = a, anti(b) = b, anti(c) = c$ .

$(G, *)$  is also a *Neutrosophic Triplet Group*, whose neutrosophic triplets are:

$$\langle a, a, a \rangle, \langle b, b, b \rangle, \langle c, b, c \rangle.$$

*Theorem 1.*

A *Seira-Group* is also a *Neutrosophic Triplet Group*.

*Proof:*

Let  $(G, E, *)$  be a *Seira-Group*. The law  $*$  is well-defined, and associative.

For each element  $x \in G$ , there exist at least a neutral element that we denote  $e_x \in E \subset G$  and at least an inverse element that we denote  $x^{-1} \in G$ , therefore a neutrosophic triplet  $(x, e_x, x^{-1})$  included in  $G$ .

Therefore  $(G, E, *)$  is also a *Neutrosophic Triplet Group*.

*Theorem 2.*

If a *Neutrosophic Triplet Group* has at least two distinct neutral elements, then it is also a *Seira-Group*.

*Example 2 of non Seira-Group*

We change the law  $*$  to  $\#$ .

$$G = \{a, b, c\}, E = \{a, b\}.$$

$$\# \quad a \quad b \quad c$$

$$a \quad a \quad a \quad a$$

$$b \quad b \quad b \quad c$$

$$c \quad a \quad c \quad a$$

The unit/ neutrals are:  $neut(a) = a, neut(b) = b, neut(c) = b$ .

The inverse/anti elements are:  $anti(a) = a, anti(b) = b, anti(c) = none$ .

Therefore  $(G, \#)$  is not a *Seira-Group* (since “ $c$ ” has no inverse), nor a *Neutrosophic Triplets Group*, because:

$$\langle a, a, a \rangle, \langle b, b, b \rangle,$$

are neutrosophic triplets, but

$\langle c, b, none \rangle$

is a neutrosophic duplet – not triplet (since “ $c$ ” has no inverse).

## **Multispace of Fuzzy Set + Intuitionistic Fuzzy Set + Neutrosophic Set**

*To Said Broumi*

Yes, we can consider Neutrosophic Set (NS) in one space, Fuzzy Set (FS) in another space [as in *Combined Effect of Neutrosophic set and Fuzzy logic for Enhancing old manuscripts*, by Jaspreet Kaur, Rupinder Kaur, International Research Journal of Engineering and Technology (IRJET), Volume: 03, Issue: 08, Aug. 2016].

We may tackle the case when all three: FS, IFS, NS are used, each of them into a separate space, but forming together a multispace.

## **Neutrosophic Graphs**

*To Muhamed Akram*

1. *Bipolar neutrosophic graphs.*

This is the most general definition of "bipolar neutrosophic graphs", meaning that the values of vertices and edges can be any subsets of the interval  $[0, 1]$  and respectively of  $[-1, 0]$ .

2. *Bipolar single-valued neutrosophic graphs.*

This is a particular definition of the "bipolar neutrosophic graphs", meaning that the values of vertices and edges should be only single numbers from the interval  $[0, 1]$  and respectively of  $[-1, 0]$ .

Similarly,

3. *Intuitionistic neutrosophic graphs.*

Similarly: the values of vertices and edges are any subsets of  $[0, 1]$ , respecting the condition of intuitionistic.

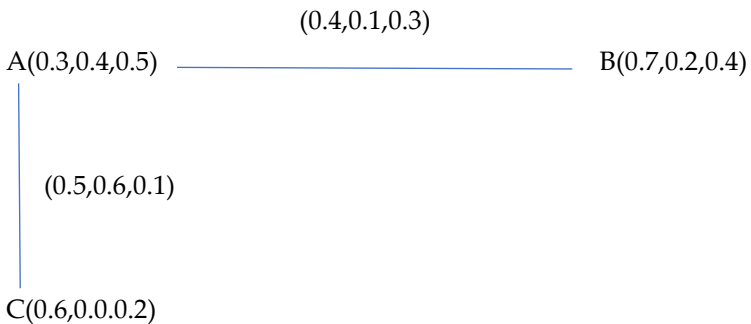
4. *Intuitionistic single-valued neutrosophic graphs.*

Similarly: the values of vertices and edges are single numbers from  $[0, 1]$ , respecting the condition of intuitionistic.

## Neutrosophic Graphs

*To Said Broumi*

Do you also consider the edge's value in computing the path from let's say:





Said, please tell me what is the shortest path: AB or AC?  
And how did you compute this?

## Neutrosophic Isolated Graphs

*To Said Broumi*

We compare the isolated neutrosophic graph with isolated fuzzy graph and with isolated intuitionistic fuzzy graph.

The relationship between any two vertices of isolated graphs is zero?

We may consider the neutrosophic value of the edge, for isolated graphs, as  $(0,0,1)$  or  $(0, 1, 1)$ .

For a bipolar neutrosophic graph we have  $T^+, T^-, I^+, I^-, F^+, F^-$  while for a bipolar intuitionistic graph you have only  $T^+, T^-, F^+, F^-$ .

\*

We may go checking the theorems and properties in a neutrosophic triplet group considering:

$$\langle a, \{neut(a)\}, \{anti(a)\} \rangle$$

I mean to take all neutrals ( using braces in order to consider all of them:  $\{neut(a)\}$  ) and all anti's of an element "a" ( similarly, using braces in order to consider all of them:  $\{anti(a)\}$  )?

So, we could check properties directly on all  $\{neut(a)\}$  and all  $\{anti(a)\}$  not only on individual ones.

A problem herein might be that some anti's correspond to different neutrals.

Also, let's say  $neut(a) \in \{s, t\}$ , as you tried to explain.

Then when we say: " $neut(a)*neut(a)$ " we may have  $s*s$ ,  $s*t$ ,  $t*s$  (in case of non-commutativity), and  $t*t$ . How should we avoid confusions?

\*

Let's see an example of a false proof.

In a neutrosophic triplet group, if  $\langle a, neut(a), anti(a) \rangle$  is a neutrosophic triplet, then  $\langle anti(a), neut(a), a \rangle$  is also a neutrosophic triplet.

*Proof:*

$$anti(a)*a = a*anti(a) = neut(a)$$

since  $\langle a, neut(a), anti(a) \rangle$  is a neutrosophic triplet.

Let's prove that:

$$anti(a)*neut(a) = anti(a)$$

{ and similarly for  $neut(a)*anti(a) = anti(a)$  }.

Multiply by "a" to the left:

$$a*[anti(a)*neut(a)] = a*anti(a).$$

Apply associativity:

$$[a*anti(a)]*neut(a) = a*anti(a),$$

we get:

$$neut(a)*neut(a) = neut(a).$$

Apply again "a" to the left, and then associativity:

$$[a*neut(a)]*neut(a) = a*neut(a)$$

or

$$a^*neut(a) = a^*neut(a)$$

which is true.

But the proof is wrong, because if "a" is non-cancellable, the proof does not work.

*Counter-example:*

In  $Z_{10} = \{0, 1, 2, \dots, 9\}$ , with  $\times$  as classical multiplication modulo 10, one has:  $\langle 2, 6, 3 \rangle$  is a neutrosophic triplet since  $2 \times 6 = 6 \times 2 = 12 = 2 \pmod{10}$ , and  $2 \times 3 = 3 \times 2 = 6 \pmod{10}$ . But  $\langle 3, 6, 2 \rangle$  is not a neutrosophic triplet since  $3 \times 6 = 6 \times 3 = 18 = 8 \pmod{10}$ , that is different from 3 !

The error is herein:

$$anti(a)^*neut(a) = anti(a)$$

becomes:

$$3 \times 6 = 3 \pmod{10}$$

which is false,

and multiply to the left by "a" = 2 (which is non-cancellable):

$$2 \times (3 \times 6) = 2 \times 3 \text{ or } 36 = 6 \text{ or } 6 = 6 \pmod{10}$$

which is true.

Same problem may arise to all propositions and theorems in our first paper.

My general conclusion is the following:

All propositions and theorems from our paper one are true if the multipliers are cancellable on the left or right (where it is multiplied to at).

If the multipliers are not cancellable, there are cases when the propositions and theorems are true and others when they are false.

Going back to the previous example, one has:  $\langle 2,6,8 \rangle$  is a neutrosophic triplet, and  $\langle 8,6,2 \rangle$  is also a neutrosophic triplet, although 2 is non-cancellable!

## **Subset Graphs and Hypergraphs**

The *subset graph* is referring to *vertices*, while the *hypergraph* to *edges*.

Therefore, we can have two types:

- *subset graph* = *graph* that has vertices as groups of vertices, and normal edges;
- *subset hypergraph* = *subset graph*, with *hyperedges*.

Besides these I have thought at having more than one edge between two vertices  $V_1$  and  $V_2$ .

For example, an edge referring to an attribute value and other edges referring to other attribute values.

Then, we may take the values of vertices and values of edges as neutrosophic.

## **Refined Neutrosophic Hypergraph**

*To A. Hassan, M. A. Malik*

We can extend your *Generalized Neutrosophic Hypergraph* to the *Generalized Refined Neutrosophic Hypergraph*, where both the vertexes and the hyperedges can be refined.

## **Independent Neutrosophic Degrees between Vertices and Edges in a Neutrosophic Graph**

*To Said Broumi*

Let's consider two people:

John, with the neutrosophic degree of membership  $J(t_1, i_1,$

$f_1)$  with respect to an organization  $\alpha$ ,

and George, with the neutrosophic degree of membership

$G(t_2, i_2, f_2)$  with respect to the same organization  $\alpha$ .

Now, the neutrosophic degree of association between the

vertices J and G within this organization (neutrosophic

graph), i.e. the neutrosophic degree of the edge JG,

does not depend on the neutrosophic degrees of

memberships of these two people.

### **Example with the distance**

*To Said Broumi*

1) If it is said: "The distance is 5 km."

Then it is as you did:

Then  $T = 5, I = [4.9, 5.1]$ , considering the threshold 0.1 (but

threshold 0.2, 0.3, etc. can also be considered,

depending on the expert and application),

and F is what's left:  $[0.0, 4.9) \cup (5.1, \infty)$ .

2) If it is said: "The distance is about 5 km."

We can use a *neutrosophic triangular or trapezoidal number*.

Or we may take, for example:  $T = [4.9, 5.1]$ ,  $I = [4.7, 4.9) \cup (5.1, 5.2]$ , considering thresholds 0.1 and 0.2 respectively;

and F what is left:  $F = [0, 4.7) \cup (5.2, \infty)$ .

In each case the threshold may vary depending on expert and application.

## Complex Neutrosophic Graph

*To W. B. Vasantha Kandasamy*

Did you think about *complex neutrosophic graphs* where not only the edges but also the vertices are *complex neutrosophic numbers* (of the form  $a + bi + cI + dIi$ , where:  $a, b, c, d$  are real numbers,  $i = \sqrt{-1}$ , and  $I =$  literal indeterminacy with  $I^2 = I$ )?

What applications or connections with our everyday life should we find?

We can design *Complex Neutrosophic Cognitive Maps* or *Complex Neutrosophic Cognitive relations* and apply them to social sciences for example.

Let's say we have a complex neutrosophic graph.

How to interpret an edge:

$$2 - 3i + 4I + 6iI$$

A ————— B

Also, how to interpret a vertex:

$$V(1 + 2i - 5I + 7iI)$$

with respect to our real world ?

*W. B. Vasantha Kandasamy*

We see if  $A$  and  $B$  are two vertices then edge  $AB$  may or may not exist, this is universally true for all graphs.

In math model we say  $A$  and  $B$ , concept or node, are both complex that is imaginary and indeterminate and if there is some complex and imaginary relational tie existing between them we have the edge regards, we need no comparison they are not lattices, we usually never compare two vertices of a graph in general regards.

See we can say a edge can exist according to the model in hand for an expert may feel the relation between two vertices is imaginary and is an indeterminate.

For instance take the case of the mentally ill patient; he may suffer from imaginary grievances and the true problem in such case it is imaginary, but the true cause is indeterminate regards.

So such situations occur there are concept or more or vertices which can be both imaginary and indeterminate regards.

More in medical diagnostics, also child psychology.

Interpretations of poetry verses in literature regards.

## Neutrosophic Group

*To Tèmitópé Gbóláhàn Jaíyéolá*

I see that the author considered "A" a set where for each  $x$  there is an  $y$  such that  $xy$  and  $yx$  are in  $A$ .

If  $y$  is the neutrosophic neutral element of  $x$ , this condition is verified.

## Neutrosophic Score Function

*To Nirmal Nital*

For ranking, you can use the neutrosophic score function:

$$(t,i,f) \rightarrow 1/3 (t + 2 - i - f).$$

## Neutrosophic Offset

*To Muhammad Gulistan*

Even more general than the *neutrosophic set* is the *neutrosophic offset*, when the degree of membership can be  $< 0$  and also  $> 1$ .

Besides this book of mine, no more work has been done on neutrosophic offset. You may try to do so if interested.

## Interval Complex Neutrosophic Set

*To Luu Quoc Dat, Mumtaz Ali, Le Huang Son*

The advantage of CNS (*Complex Neutrosophic Set*) over the NS (*Neutrosophic Set*) is the fact that, in addition to the membership degree provided by the NS (and represented in the CNS by amplitude), the CNS also



provides the phase (which is an attribute characterizing the amplitude). In many problems we need to be able to also characterize the type of membership.

Yet, in many real applications it is not easy to find a crisp neutrosophic membership degree (as in single-valued neutrosophic set), since we deal with unclear and vague information in our everyday life. In order to overcome this, we have to use an interval neutrosophic membership degree.

Therefore, in the ICNS (*Interval Complex Neutrosophic Set*) both, the amplitude and its phase (attribute), may be represented by intervals, which better catch the unsure values of the membership.

## **Neutrosophic Differential Equations and Neutrosophic Integral Equations**

*To Jun Ye*

I tried a little in my book on *neutrosophic measure* and on *neutrosophic integral*, but you can approach *neutrosophic differential equations* and *neutrosophic integral equations* from a different perspective and in a deeper study and applications.

## **t-norm and t-conorm within Neutrosophic Operators**

*To S. Khalil*

You apply the t-norm and t-conorm on the neutrosophic components T, I, F.

You may refine only T as  $T_1, T_2, \dots$ , or refine only I in the same way, or refine only F;

or you may refine only two of T, I, F;

or you may refine all T, I, F.

You also apply t-norm and t-conorm on the neutrosophic subcomponents  $T_1, T_2, \dots; I_1, I_2, \dots; F_1, F_2, \dots$  as well.

Where:

$$\text{t-norm}(a, b) = ab, \text{ or } \min\{a, b\}, \text{ or } \max\{0, a+b-1\} \text{ etc.}$$

and similarly

$$\text{t-conorm}(a, b) = a+b-ab, \text{ or } \max\{a, b\}, \text{ or } \min\{a+b, 1\}$$

etc.

## **Continuous Neutrosophic Operators**

I do not remember in the fuzzy or intuitionistic fuzzy environments to have been done publications when the components T (or respectively T, F) were continuous functions (of time, or of other variables), but of course there is a vast literature that I could not check all of it.

The neutrosophic operators using  $T(t)$ ,  $I(t)$ ,  $F(t)$  as functions of time ( $t$ ) are the same as in the case when  $T$ ,  $I$ ,  $F$  are constant crisp numbers and we did before.

Myself, I straightforwardly extended from crisp numbers to continuous functions for the neutrosophic operators.

But there might be a different possibility to design (new) 'continuous neutrosophic operators', instead of 'discrete neutrosophic operators' as used previously.

The fact, that the degree of membership of the same element  $x$  with respect to the same set  $S$  varies, is real.

For example, a worker can be hired to work 50% (part-time), then later he can be changed to work 75%, and so on.

We need some good practical possible example, where the neutrosophic degree of the membership of an element  $x$  with respect to a set  $S$  changes continuously upon time.

For example the degree of quality of a piece: very high at the beginning, but little by little it degrades over the time.

## **Refined Indeterminacy in retina Image**

### **Analysis**

*To Abdolreza Rashno*

*Mouchkeram!*

You may try to use the *refined neutrosophic indeterminacy* into retina image analysis, for example:

$I_1$  = indeterminacy in abnormal retina regions;

and

$I_2$  = indeterminacy in normal retina regions.

Or you can refine (split) "I" (Indeterminacy) into other ways, depending on your application.

Refined neutrosophic set gives you a more detailed answer than nonrefined neutrosophic set.

## **Refined Cubic Neutrosophic Set**

*To Young Bae Jun & S. Z. Song*

The *neutrosophic cubic set* can be extended to *refined neutrosophic cubic set* –

i.e. when T is refined into  $T_1, T_2, \dots$ ;

similarly I is refined into  $I_1, I_2, \dots$ ;

and F is refined into  $F_1, F_2, \dots$ .

Therefore, for each neutrosophic subcomponent  $T_j, I_k, F_l$  we can consider an interval from  $[0, 1]$  together with a crisp number from  $[0, 1]$  respectively.

## **Q-Refined Neutrosophic Soft Relation**

*To Majdoleen Abu-Qamar, Nasruddin Hassan*

You can extend the *Q-Neutrosophic Soft Relation* to *Q-Refined Neutrosophic Soft Relation*.

## Neutrosophic Triplet Strong and Weak Sets

*To Tèmitópé Gbóláhàn Jaíyéolá*

We say that a *neutrosophic triplet set*  $N$  is *strong* if: for any  $x$  in  $N$  there is a *neut*( $x$ ) and an *anti*( $x$ ) in  $N$ .

Then we say that a *neutrosophic triplet set*  $M$  is *weak* if: for any  $x$  in  $M$  there is a neutrosophic triplet  $\langle y, \text{neut}(y), \text{anti}(y) \rangle$  in  $M$  such that  $x = y$  or  $x = \text{neut}(y)$  or  $x = \text{anti}(y)$ .

For example,  $Z_3 = \{0, 1, 2\}$ , with respect to the multiplication modulo 3, is a neutrosophic triplet weak set, whose neutrosophic triplets are:

$$\langle 0,0,0 \rangle, \langle 0,0,1 \rangle, \langle 0,0,2 \rangle,$$

but it is not a neutrosophic triplet strong set, since for the element 2, for example, there is no *neut*(2) nor *anti*(2).

Applications are for the triads:

$$\langle A, \text{neut}(A), \text{anti}(A) \rangle$$

as in neutrosophy, where  $A =$  entity, idea, notion, concept etc.:

$$\langle \text{friend}, \text{neutral}, \text{enemy} \rangle,$$

$$\langle \text{winning}, \text{tie game}, \text{loosing} \rangle,$$

$$\langle \text{accept}, \text{pending}, \text{reject} \rangle,$$

$$\langle \text{positive particle}, \text{neutral particle}, \text{negative particle} \rangle,$$

etc.

\*

*To Xiaohong Zhang, Xinliang Liang*

*Theorem.* Let  $(N, *)$  be a *neutrosophic triplet strong set* (i.e. for  $x$  in  $N$ , there also is a *neut*( $x$ ) and an *anti*( $x$ ) in  $N$ ), with a well defined law  $*$ .

If  $y$  in  $N$  is cancellable, then  $y$  has only a neutral and only one opposite.

*Proof:*

Let's suppose there are two neutrals  $n_1$  and  $n_2$  in  $N$  for  $x$ .

Then:  $y*n_1 = y$  and  $y*n_2 = y$ , or  $y*n_1 = y*n_2$ , but since  $y$  is cancellable, we cancel  $y$ , and we get:

$$n_1 = n_2.$$

Now, let suppose that  $y$  has two opposites:  $a_1$  and  $a_2$  in  $N$ .

Then  $y*a_1 = n_1$  and  $y*a_2 = n_1$ , or  $y*a_1 = y*a_2$  (since there is only one neutral for  $y$ ); but again  $y$  is cancellable, so we cancel  $y$  and we get:

$$a_1 = a_2.$$

## **Neutrosophic Triplet and Extended Triplet Relationships**

Also, it will be interesting to check the *relationships of neutrosophic triplet sets* and of *neutrosophic extended triplet sets* as well.

## **Total Indeterminacy**

*To Mohamed Abdel-Basset*

The idea of using

$$T(x) = *$$

(where \* means unknown)

is good.

Surely, this idea of complete indeterminacy (i.e. the value \*) is interesting.

The neutrosophic theories have the capabilities to be extended in many directions.

## **Hesitancy as sub-part of Indeterminacy**

*To Harish Garg*

The *hesitation degree* in *Intuitionistic Fuzzy Set* is just the *Indeterminacy "I" degree* in *Neutrosophic Set*.

Actually, Indeterminacy "I" is more than hesitancy in general.

When one considers the components T, I, F (or sources that provide them) are totally dependent (as in IFS), i.e.  $T + I + F = 1$ , then  $I = 1 - T - F$ , i.e. your "I" = hesitancy from IFS.

If T, I, F are partially dependent and partially independent, i.e.  $T + I + F > 1$ , then indeterminacy "I" is including your hesitancy, and even more, i.e. Indeterminacy is split in many types of subindeterminacies in function of the problem to solve, so hesitancy can be considered a subpart of the indeterminacy.

In neutrosophic logic you can also refine T, I, F and go by sub-components.

For example, you can only refine "I", that you're interested in, for example as:

$I_1$  = contradiction,  $I_2$  = vagueness,  $I_3$  = hesitancy etc.

\*

*To W.B. Vasantha Kandasamy*

If we divide, let's say:

$$(2+2I)/(1+I) = x + yI.$$

We need to find  $x$  and  $y$  as real numbers.

Then:

$$(1+I)(x+yI) \equiv 2+2I \text{ (identical)}$$

$$\text{or } x + (y+x+y)I \equiv 2+2I,$$

whence  $x = 2$  and  $y = 0$ ,

therefore alike in classical algebra:

$$(2+2I)/(1+I) = 2.$$

Yet, if we compute  $I/I = x+yI$ , we get:

$$I \equiv I(x+yI),$$

$$\text{or } I \equiv (x+y)I, \text{ whence } x+y = 1,$$

therefore  $I/I = (-a+1) + aI$ , where " $a$ " is any real number,

therefore we have infinitely many results for  $I/I$ .

It is curious that  $(a+bI)/(dI)$  has no solution if  $a \neq 0$ .

And  $(a+bI)/(c+dI)$  has one solution only if  $ac \neq 0$ .

What do you think?

Can we divide by  $a+bI$  or not ?



## Neutrosophic Dynamic Systems

*Elemer Elad Rosinger*

Now, as far as I am interested, I find of MAJOR interest precisely those possible physical compositions which are MORE complex than tensor products.

Indeed, they could possibly be able to SIMULATE quantum computation!

And HOPEFULLY, they could be EFFECTIVELY implemented, and they would NOT suffer from the CURSE of quanta which is... decoherence, that makes it so immensely DIFFICULT to build quantum computers ...

So that, perhaps, you could give a thought or two to that issue :

- 1) find compositions of physical systems MORE complex than tensor products;
- 2) and such that they can be EFFECTIVELY implemented physically, and WITHOUT decoherence.

By the way, could you, please, tell me where could I learn quickly and easily enough about NEUTROSOPHY ?

*Florentin Smarandache*

About the neutrosophics, please see the UNM website:

*<http://fs.unm.edu/neutrosophy.htm> .*

Regarding complex physical systems, but regarded from a different point of view from tensor products, see the book Symbolic Neutrosophic Theory

[<http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf>] the neutrosophic dynamic systems [which are systems that take into consideration the indeterminacy as well, and goes on true-indeterminate-false degree of elements and relationships inside and/or with respect to the whole system, with attracting and rejecting points in the system, evolution of neutrosophic dynamic systems, and so on.

Is it possible to somehow connect the tensor products to the neutrosophic dynamic systems?

## **Ultra Neutrosophic Crisp Set**

*To A. A. Salama*

When you define something new, try to get applications, justifications for it -- I mean for *Ultra Neutrosophic Crisp Set*, otherwise it will be just a mathematical artifact. I suggest you to read my small article on *n*-valued REFINED neutrosophic set.

I also suggest you to extend the neutrosophic crisp set  $\langle A, B, C \rangle$  in a refined way, i.e. to a *refined neutrosophic crisp set*  $\langle A_1, A_2, \dots; B_1, B_2, \dots; C_1, C_2, \dots \rangle$  and try to see some applications (I mean similarly to *n*-valued refined neutrosophic logic and set attached).

I changed your notations  $\langle A_1, A_2, A_3 \rangle$  to  $\langle A, B, C \rangle$  so we better see the refinement.

## Neutrosophic Notions

*To Mumtaz Ali, John Mordeson*

The adopted *fuzzy notions* to *neutrosophic notions* are correct.

I think we can say in a more general way.

Let's use the notations:

$$NS(x) = \langle T(x), I(x), F(x) \rangle,$$

where  $NS$  means neutrosophic set values,

$$NS(y) = \langle T(y), I(y), F(y) \rangle,$$

$$NS(xy) = \langle T(xy), I(xy), F(xy) \rangle.$$

Then one has a Neutrosophic Fuzzy Subgroupoid  $A$  of a groupoid  $(G, \cdot)$  if for all  $x, y$  in  $G$  one has:

$$NS(xy) \geq NS(x) \wedge NS(y).$$

And the definition of the Neutrosophic Fuzzy Subgroup can be written as:

For  $x, y$  in  $A$  one has:

$$NS(xy) \geq NS(x) \wedge NS(y)$$

$$NS(x) \geq NS(x^{-1}).$$

We may have classes of *neutrosophic fuzzy subgroupoids*, and *neutrosophic fuzzy subgroups*, since there are classes of *neutrosophic conjunctions*, i.e.:

$$NS(x) \wedge NS(y) = \langle \min\{T(x), T(y)\}, \max\{I(x), I(y)\}, \max\{F(x), F(y)\} \rangle$$

or

$$NS(x) \wedge NS(y) = \langle \min\{T(x), T(y)\}, \min\{I(x), I(y)\}, \max\{F(x), F(y)\} \rangle$$

or

$$NS(x) \wedge NS(y) = \langle T(x)T(y), 1-I(x)I(y), 1-F(x)F(y) \rangle$$

or

$$NS(x) \wedge NS(y) = \langle T(x)T(y), I(x)I(y), 1-F(x)F(y) \rangle$$

etc.

And we also have two types of *neutrosophic inequalities* of sets:

$$\langle T_1, I_1, F_1 \rangle \geq \langle T_2, I_2, F_2 \rangle,$$

if  $T_1 \geq T_2$  and  $I_1 \leq I_2$  and  $F_1 \leq F_2$ ,

or

$$\langle T_1, I_1, F_1 \rangle \geq \langle T_2, I_2, F_2 \rangle$$

if  $T_1 \geq T_2$  and  $I_1 \geq I_2$  and  $F_1 \geq F_2$ .

\*

An direct edge of a graph, let's say from  $A$  to  $B$  (i.e. how  $A$  influences  $B$ ), may have a neutrosophic value  $(t, i, f)$ , where  $t$  means the positive influence of  $A$  on  $B$ ,  $i$  means the indeterminate influence of  $A$  on  $B$ , and  $f$  means the negative influence of  $A$  on  $B$ .

Then, if we have, let's say:  $A \rightarrow B \rightarrow C$ , such that  $A \rightarrow B$  has the neutrosophic value  $(t_1, i_1, f_1)$ , and  $B \rightarrow C$  has the neutrosophic value  $(t_2, i_2, f_2)$ , then  $A \rightarrow C$  has the neutrosophic value  $(t_1, i_1, f_1) \wedge_N (t_2, i_2, f_2)$ , where  $\wedge_N$  is the AND neutrosophic operator.

What do you think about new interpretation of the *neutrosophic graphs, neutrosophic trees*? Can we apply it to *neutrosophic cognitive maps* as well?

## Neutrosophic Refined Soft Set

*To Mumtaz Ali*

We can refine the *soft set* and write a paper on this new idea.

Let  $U = \{u_1, u_2, u_3, \dots, u_n\}$  the universe of discourse and  $E = \{e_1, e_2, \dots, e_m\}$  the set of parameters.

Then we refine some of the parameters:  $e_1$  refined as  $e_{11}, e_{12}, \dots, e_{1p}$ , similarly for any  $e_j$ .

For example, if  $U = \{c_1, c_2, c_3, c_4\}$  the universal set of houses and  $E = \{e_1=\text{good}, e_2=\text{big}\}$ , we can define "good" as "very good", "medium good", and "little good"; similarly "big" refined as "very big", and "not that big".

Then  $f(\text{good}) = f(\text{very good, medium good, little good}) = (P_1(U), P_2(U), P_3(U))$  where  $P_1(U)$  is a part of the power set of  $U$ , similarly for  $P_2(U)$  and  $P_3(U)$ .

Then we extend them to *neutrosophic refined soft set*.

We may refine later the neutrosophic too.

## Interval Triangular Neutrosophic Set

Let  $U$  be a universe of discourse.

*Triangular Neutrosophic Set* can be generalized to *Interval Triangular Neutrosophic Set*.

And similarly a *Trapezoidal Neutrosophic Set* to an *Interval Trapezoidal Neutrosophic Set*.

We firstly extend a *trapezoidal neutrosophic number*  $(a,b,c,d)$  to a *trapezoidal neutrosophic interval*  $([a_1,a_2], [b_1,b_2], [c_1,c_2], [d_1,d_2])$ .

And similarly for a *triangular neutrosophic number*  $(a,b,c)$  to a *triangular neutrosophic interval*  $([a_1,a_2], [b_1,b_2], [c_1,c_2])$ .

### **Possibility Interval Neutrosophic Soft Sets**

The similarity measure between *possibility neutrosophic soft sets* developed by Faruk Karaaslan, where the author uses only *single valued neutrosophic sets*.

What about generalizing it to POSSIBILITY interval neutrosophic soft sets?

Also, the possibility measure, which is a degree of possibility of belongingness, is also singled valued. But we can extend it too to interval valued belongingness.

### **Neutrosophic Sub-structures**

About the *fuzzy subgroupoid*, *fuzzy subsemigroup*, and *fuzzy subgroup*, I observed the following:

Let  $x(T, F)$  be a generic fuzzy element, where  $T+F=1$ .

If  $T(xy) \geq \min\{T(x), T(y)\}$ , then it results that

$$F(xy) \leq \max\{F(x), F(y)\}.$$

For example:

In  $Z_4 = \{0, 1, 2, 3\}$ , with  $0(0.9, 0.1)$ ,  $1(0.7, 0.3)$ ,  $2(0.6, 0.4)$ ,  $3(0.8, 0.2)$ , with respect to addition modulo 4:

$$T(1+2) = T(3) = 0.8 \geq \min\{T(1), T(2)\} = \min\{0.7, 0.6\} = 0.6.$$

But  $F(1+2) = F(3) = 0.2 \leq \max\{F(1), F(2)\} = \max\{0.3, 0.4\} = 0.4$ .

Therefore  $F(1+2) = 0.2$  is not bigger than or equal to  $\min\{0.3, 0.4\}$ .

Whence, for the *extension from fuzzy to neutrosophic substructures*, we need to take, in my opinion:

$$I(xy) \leq \max\{I(x), I(y)\}$$

$$\text{and } F(xy) \leq \max\{F(x), F(y)\}.$$

### Neutrosophic Subsemigroup

1) The *Neutrosophic Set*  $\mu$  in a Semigroup  $S$  is a function:

$$\mu: S \rightarrow [0,1]^3,$$

for all  $x$  in  $S$ ,  $\mu(x) = (T(x), I(x), F(x))$ .

2) A *Neutrosophic Set*  $\mu$  in a Semigroup  $S$  is a *Neutrosophic Subsemigroup* if:

$$T(xy) \geq \min\{T(x), T(y)\}, I(xy) \leq \max\{I(x), I(y)\},$$

$$\text{and } F(xy) \leq \max\{F(x), F(y)\}.$$

A second variant of this definition would be to replace Axiom 2 by:

$$T(xy) \geq \min\{T(x), T(y)\}, I(xy) \geq \min\{I(x), I(y)\},$$

$$\text{and } F(xy) \leq \max\{F(x), F(y)\}.$$

And a third variant of this definition would be to replace the Axiom 2 by:

$$T(xy) \geq \min\{T(x), T(y)\}, I(xy) = (I(x)+I(y))/2,$$

$$\text{and } F(xy) \leq \max\{F(x), F(y)\}.$$

These distinctions are due to the fact that the intersection of neutrosophic sets can be defined in many ways.

## **Lingvistică neutrosofică**

*To Ștefan Vlăduțescu, Mihaela Colhon*

Logica neutrosofică este o generalizare a logicii fuzzy.

În logica neutrosofică, cum știți, avem: grad de adevăr, grad de fals, și grad de nedeterminare.

Ca la fotbal când joacă două echipe: șansa ca Univ. Craiova să câștige, ori să piardă, ori să aibă meci nul. Deci 3 componente.

În lingvistică nu s-a folosit prea mult, deci este bună ideea doamnei Mihaela Colhon și a Dvs. de a aborda acest domeniu (*lingvistica neutrosofică*).

Am un articol cu un cercetător turc { *A Lattice Theoretic Look: A Negation Approach to Adjectival (Intersective, Neutrosophic and Private) Phrases*, de Selcuk Topal and Florentin Smarandache } de aplicații neutrosofice în lingvistică.

Al doilea articol, *Neutrosophic Actions, Prevalence Order, Refinement of Neutrosophic Entities, and Neutrosophic Literal Logical Operators*, de F. Smarandache, *Neutrosophic Sets and Systems*, Vol. 10, pp. 102-107, tratează tocmai negarea, <neutA> a unei propoziții, și antitetizarea <antiA>, dar și <nonA>.

*Ștefan Vlăduțescu*

Acele valențe care au rolul de a nuanța un enunț care în lingvistică sunt împărțite în 3 categorii:



- *intensificatori*: foarte, extrem, destul de ...
- *diminuatori*: puțin, abia ...
- *negatori*: schimbă polaritatea unui enunț: din pozitiv în face negativ și invers: deloc, ...

*Florentin Smarandache*

Desigur, se poate merge și pe nuanțe/grade de intensificatori ( $T_1, T_2, \dots$ ), nuanțe/grade de diminuatori ( $F_1, F_2, \dots$ ), și nuanțe/grade de negatori (folosind operatorul de negație neutrosofică). Vedeți articolul despre *Refined Neutrosophic Logic*.

Vă rog, înaintați email-ul meu doamnei Colhon, să îmi scrie și dânsa, și cu Dvs. în CC de asemenea.

Să avem o discuție în trei dacă vă interesează.

*Mihaela Colhon*

Am citit azi (parțial) articolele trimise. Concluziile mele sunt:

- pentru a contura *lingvistica neutrosofică* e nevoie de multă documentație, și aș vrea să mă ajutați cu sugestii în această privință, dar clar un astfel de studiu îl văd materializat destul de departe;
- ce aș putea însă încerca, drept un prim exercițiu ca să mă familiarizez cu domeniul, și în care aș putea folosi rezultatele studiilor mele anterioare: analiza sentimentelor (cu asta m-am ocupat în ultimii ani) are la bază termeni care exprimă sentimente pozitive (de exemplu "bun", "reușit", "măreț"), și termeni care

exprimă sentimente negative (ex: "prost", "nereușit" ...).

Asta ar fi nivelul cel mai de jos al analizei.

Apoi aceste două categorii de termeni pot fi însoțite de modificatori de valență (ce vă spuneam în e-mailul anterior): negatori, diminuatori și intensificatori.

Putem să aplicăm operatorii logicii neutrosofice unor termeni lingvistici (care ar putea fi văzuți ca având valori booleene "true" (pentru termenii pozitivi) sau "false" (termenii negativi)?

Prin intermediul acestor operatori am putea determina mai ușor gradul de pozitivitate/negativitate al unui termen lingvistic.

Spuneați dvs în unul din articolele trimise "*This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities*".

Deci acest studiu s-ar adresa strict termenilor lingvistici, fără a lua în calcul modificatorii de valență, care și ei trebuie analizați pentru a face un studiu complet. Dar strict la termenii lingvistici am avea, de exemplu:

<A> = "frumos", atunci <antiA> = "urât" și între ei există mulțimea termenilor a căror semnificație este cuprinsă între aceștia, ca de exemplu: formele diminuate ale acestora "frumușel", "urâțel" ... nu știu dacă între doi termeni de polarități diferite s-ar putea încadra, în afară de formele diminuate ale acestora și alți termeni,

în cazul exemplului considerat ar fi "drăguț" și poate "dezagreabil"

În orice caz, un astfel de studiu ar micșora analiza sentimentelor, deoarece am putea trata doar termenii dintr-o singură categorie, atașând acestora pe cei din categoria opusă.

Mai trebuie să mai studiez, o să mă consult și cu Ștefan Vlăduțescu, văd că aveți termeni <antiA> și <nonA> care din punctul de vedere al analizei sentimentelor nu știu prin ce s-ar diferenția. De asemenea m-ar interesa să aflu dacă pot fi aplicați operatorii logici unor astfel de termeni (având în vedere că aceștia pot fi văzuți ca fiind "true", "false" și nuanțele dintre aceste valori). Am putea defini și/sau ordona aceste nuanțe? Există un suport matematic în acest sens?

Scuzați-mi multitudinea de întrebări. Dacă aveți răgaz să îmi răspundeți v-aș fi recunoscătoare. Oricum vă voi ține la curent (împreună cu Ștefan) cu stadiul cercetărilor noastre.

\*

*Mihaela Colhon*

Cred că se poate modela teoria dvs pe rețeaua *SentiWordNet*, unde fiecare termen are atașat un scor de *positivity*, *negativity* și *objectivity*. Dacă aș putea obține o variantă mai comasată a acestei rețele în care toți termenii să poată fi conectați la termenii

corespunzători de polaritate inversă, sau/și la termenii de aceeași polaritate (caz în care am avea o nuanțare a respectivului sentiment) ... așa, am obține mult mai ușor scorurile pe baza cărora se face analiza sentimentelor și de asemenea, elementele din această resursă ar fi mult mai bine structurate, deci actualizarea rețelei s-ar face mult mai ușor (bineînțeleles, ca și exploatarea ei ar fi înlesnită).

*Florentin Smarandache*

Dați-mi, vă rog, un exemplu simplu - concret, or a problemă simplă – concretă, de rezolvat privind sentimentele (subiectul care vă interesează).

Și voi încerca să văd cum îl/o pot rezolva.

În fond, orice teorie (și a mea și a Dvs.), pleacă de la exemple simple.

*Mihaela Colhon*

Primul meu gând a fost să încerc să mapez relațiile semantice din rețeaua lexicală WordNet în SentiWordNet. Actualmente această din urmă resursă lexicală e o înșiruire de cuvinte, fiecare având atașat un scor de pozitivitate, negativitate, obiectivitate sau neutralitate (adică nu exprimă niciun sentiment).

În schimb ... în rețeaua WordNet avem cuvintele grupate în synseturi (adică au un același înțeleles) iar între aceste synset-uri avem RELAȚII SEMANTICE: *sinonimia* (același înțeleles), *hiperonimia* (leagă un înțeleles particular

de unul general), *hiponimia* (inversă hiperonimiei),  
*meronimia* (parte din) ...

Asta e la WordNet, unde se tratează sensurile cuvintelor.

Iar aceste relații semantice sunt de mare ajutor în  
utilizarea acestei resurse.

La SentiWordNet, așa cum v-am spus, nu avem astfel de  
relații între termeni, pur și simplu se listează cuvintele  
însoțite de scorurile lor dpdv al analizei sentimentelor,  
deci fiecare cuvânt are atașate 3 scoruri:

word(positive\_score, negative\_score, objective\_score)

am putea să avem termeni "pozitivi" ca să zic așa, să  
zicem:

frumos(100, 0, 0)

...

sau termeni "negativi"

urât(0,0,100)

...

(exagerez scorurile acum) și o mulțime de termeni care  
exprimă nuanțe, ca de exemplu

drăguț(90,5,5)

...

După cum vedeți, așa cum am citit în lucrarea dvs "n-  
Valued Neutrosophic Logic", ajungem la valorile  
neutrosofice ( $T_x$ ,  $I_x$ ,  $F_x$ ) și mai departe ... putem să  
aplicăm operatorii definiți de dvs între termenii din  
SentiWordnet.

Încă nu am clar cum putem transpune operatorii logicii neutrosifice pe SentiWordnet, dar avem deja o punte de legătură.

*Florentin Smarandache*

Mă gândesc dacă nu ați putea folosi și grafuri neutrosifice: unde fiecare cuvânt/sentiment este un nod  $A(t_A, i_A, f_A)$ , iar între două noduri  $A(t_A, i_A, f_A)$  și  $B(t_B, i_B, f_B)$  sunt relații de legătură  $AB(t_{AB}, i_{AB}, f_{AB})$ , adică și relațiile între cuvinte/sentimente sunt tot aproximative/neutrosifice (un grad de pozitivitate, unul de negativitate, și altul de neutralitate / obiectivitate) între A și B.

*Mihaela Colhon*

Da, relațiile de orice natură determină structuri de tip graf. Întrebarea mea ar fi, putem avea o metodă de a stabili gradul de "apropiere semantică" (pentru studiul considerat apropierea această semantică este considerată dpdv al sentimentului descris) între doi termeni

$$A(t_A, i_A, f_A)$$

$$B(t_B, i_B, f_B)$$

pe baza scorurilor  $t_A, i_A, f_A, t_B, i_B, f_B$ ?

Avem următoarele proprietăți: pentru orice termen  $X$ , avem  $t_X + i_X + f_X = 100$ , deci dacă am lua un *threshold*  $\theta$ , am considera că termenii A și B sunt apropiați semantic (de aceeași polaritate) dacă

$$|t_A - t_B| < \theta, |i_A - i_B| < \theta, |f_A - f_B| < \theta$$

E corectă o astfel de abordare? E prea simplistă?

Pentru că dacă am avea lucru acesta riguros stabilit, aş putea să implementez pe reţeaua SentiWordNet ... am senzaţia că am putea obţine nişte lucruri interesante (clustere semantice de termeni).

*Florentin Smarandache*

Suntem pe aceeaşi lungime de undă.

Aţi intuit perfect! Este corect ceea ce spuneţi.

Există în literatură, denumirea de "delta-equality of neutrosophic sets", definită exact cum aţi zis Dvs.

Adică, de exemplu, dacă vrem ca două cuvinte  $C_1(t_1, i_1, f_1)$  şi  $C_2(t_2, i_2, f_2)$  să aibă sinonimie de 90%, punem atunci condiţiile ca

$$|t_1 - t_2| \leq 0.1, |i_1 - i_2| \leq 0.1, |f_1 - f_2| \leq 0.1,$$

aşa cum aţi scris Dvs. în email-ul anterior.

Fiindcă  $1 - 0.9 = 0.1$ .

Dacă vrem ca ele să aibă sinonimie de 95%, în mod similar punem:

$$|t_1 - t_2| \leq 0.05, |i_1 - i_2| \leq 0.05, |f_1 - f_2| \leq 0.05.$$

\*

În articolul ataşat se consideră cazul general: când gradele de pozitivitate/neutralitate/negativitate nu sunt simple numere (*crisp numbers*), ci sunt submulţimi ale intervalului  $[0, 1]$ , de aceea s-a pus "sup" (*supreme*).

De pildă cuvântul "frumos"([0.5, 0.7], {0.0, 0.1, 0.2}, [0.3, 0.4]) are gradul de adevăr între 0.5 și 0.7, gradul de neutralitate: fie 0.0 or 0.1 or 0.2, și gradul de negativitate între 0.3 și 0.4.

Sunt cazuri când nu putem estima cu precizie (adică un număr crisp, ci prezicem cu aproximație - adică un interval, ori o submulțime din [0, 1]).

Altceva, privind suma componentelor  $t, i, f$ : aceasta este 1 când aceeași sursă le estimează pe toate trei:  $t+i+f = 1$ . Spunem că  $t, i, f$  sunt dependente.

Dacă, însă, surse independente furnizează respectiv aceste componente  $t, i, f$ , atunci putem avea:

$$0 \leq t+i+f \leq 3.$$

Acest lucru poate părea contra-intuitiv, dar nu este așa. Dacă, de pildă Universitatea Craiova joacă fotbal cu Dinamo, întrebând un chibit al Craiovei ce crede el? Acesta va spune că Universitatea va câștiga, să zicem cu șansa de 70%, dar întrebând un chibit al lui Dinamo, acesta va spune că Dinamo va câștiga, să zicem cu șansa de 60%, iar o persoană nefiind în nicio tabără poate spune că va fi șansa de meci egal de 80%... deci suma  $0.7 + 0.6 + 0.8 > 1$ .

Altceva, care este posibil să vă fie util. Există și noțiunea de "neutrosophic hypergraph" [*hipergraf neutrosofic*], adică un graf cu noduri de cuvinte/sentimente, dar relațiile între noduri nu se fac numai între câte două



noduri, ci și între 3 sau mai multe noduri deodată (relație în grup).

Adică, dacă *A*, *B*, *C* sunt trei noduri (trei frați de pildă) dintr-un graf, aceștia pot avea o relație între ei de cooperare în grup, adică toți trei deodată, nu separat *AB*, sau *AC*, sau *BC*.

Se întâmplă acest lucru, când fiecare frate știe să facă un lucru pe care ceilalți doi frați nu știu să-l facă, iar ca treabă să meargă fiecare frate trebuie să-și facă lucrul sau. Deci, numai doi frați nu vor putea face lucrul celui de-al treilea, de aceea trebuie considerată doar relația în trei (deoarece relația în doi nu merge).

Puteți considera 3 sau mai multe cuvinte cu un anumit grad de sinonimie, și să aveți o "relație de sinonimie de grup" dacă vă interesează, etc.

### *Mihaela Colhon*

Ce am obținut până acum: deoarece SentiWordNet (SWN) urmărește organizarea pe synseturi a WordNet-ului (WN) avem următoarea situație: un același cuvânt poate aparține mai multor synset-uri, în funcție de sensurile pe care le are (este evident cazul cuvintelor polisemantice)

SWN însă atașează un scor de polaritate pozitiv, negativ pe sensuri și putem avea ceva de genul:

```
scor_poz1 scor_neg1  
word1_numarSynset1
```

*word2*\_numarSynset2

*word3*\_numarSynset3

scor\_poz2 scor\_neg2

*word2*\_numarSynset4

unde:

scor\_poz1 si scor\_poz2 notează scoruri pozitive oarecare, scor\_neg1 si scor\_neg2 notează scoruri negative oarecare, *word1*, *word2*, *word3* sunt cuvinte oarecare între care "word2" e cuvânt polisemantic deoarece îl găsim și în "numarSynset2" și în "numarSynset4".

E evident că vom avea: *word1*, *word2* și *word3* au sinonimie 100% deoarece au același scor poz și negativ, dar *word2* apare cu alte scoruri atunci când e considerat vis-a-vis de synsetul "numarSynset4".

Trebuie să vedem cum tratăm aceste cazuri, deoarece acestea formează însăși esența WN și implicit a SWN...

\*

*To Mihaela Colhon*

Se pot face distanțe (sau *măsuri de similaritate*) între tipurile de sensuri, deci dacă sunt "n" sensuri,  $s_1, s_2, .. s_n$  ale unor cuvinte, se poate obține sub-distanțe pentru fiecare sens, sau se poate obține o singură distanță totală.

Acum, o distanță mai generală ar fi:

➤ pentru

$w_1(\langle p_{11}, n_{11} \rangle, \langle p_{12}, n_{12} \rangle)$

$$w_2(\langle p_{21}, n_{21} \rangle, \langle p_{22}, n_{22} \rangle)$$

(considerând doar două sensuri pe cuvânt)

$$d(w_1, w_2) =$$

➤ pentru sensul pozitiv:

$$\{|p_{11}-p_{21}|^\lambda + |p_{12}-p_{22}|^\lambda\}^{(1/\lambda)},$$

➤ respectiv pentru sensul negativ:

$$\{|n_{11}-n_{21}|^\lambda + |n_{12}-n_{22}|^\lambda\}^{(1/\lambda)}.$$

Pentru  $\lambda = 2$ , avem distanța clasică euclidiană, ca distanță geometrică dintre două puncte în plan.

Acestea sunt pentru cuvintele cu același număr de sensuri.

Problema ar fi, în cazul când cuvintele au numere de sensuri diferite, cum v-ar plăcea să fie calculată distanța similară?

\*

### *Florentin Smarandache*

Ca aplicație, eu am un program de computer care îmi compară volumele de bancuri anterioare de fiecare dată cu noul volum de bancuri, pentru a sesiza dublurile de bancuri din noul volum și a le elimina.

Problema a devenit spinoasă, deoarece s-au strâns peste 90.000 de bancuri anterioare, iar compararea cu noul volum care are 6.000 bancuri ia câte o noapte întreagă...

Programul îmi face compararea pe un număr de cuvinte consecutive, dacă se află și în volumele anterioare și în cel nou, sau nu...

Însă nu sesizează dacă același banc a fost spus prin cuvinte diferite (sinonime, sau alt stil).

La comparare, problema ar mai fi și de... retorică.

Unii le spun pe direct, alții pe ocolite, sau dau apropouri.

La bancuri, unii zic Bulă, alții Gigel, alții Aghiuță, alții soacră, alții o blondă etc. la același banc... Ar trebui să putem adăuga clase de sinonime pe fiecare domeniu (adică Bulă este sinonim cu Gigel, cu Aghiuță etc. când e vorba de bancuri).

Știu că se fac comparații de plagiate când trimiți o lucrare spre publicare, că până și referințele (pe care nu le poți schimba) se adaugă la procentajul de... plagiat...

## **Teoria Neutrosofică a Evoluției**

*To Gabriela Tonț*

L-am citat pe Wallace cu "Darwinismul" în articolul atasat. De ce spuneți că vă gândiți la Wallace?

Nu știu dacă o să mi-l publice vreo revistă, susțin ceva adevărat, și anume: există evoluție parțială, involuție parțială (vedeți exemplele din articol), și neutralitate parțială (sau indeterminare, adică ceea ce nici nu evoluează, nici nu involuează).

\*

*To Vali Ichim*

Teoria Evoluției Neutrosofice (evoluție, involuție, neutralitate sau indeterminare) n-ar fi greu de înțeles, fiind în consens cu viața.

**Neutrosophic Topics to study**

- Constantele naturii sunt doar  $(t, i, f)$ -constante;
- Simetria este tot o  $(t, i, f)$ -simetrie;
- Proporționalitatea este o  $(t, i, f)$ -proporționalitate;
- An axiomatic system that destroys itself;
- Neutrosophic entropy;
- Neutrosophic data mining;
- Neutrosophic network denoising;
- Neutrosophic training set;
- Neutrosophic hyperplane;
- Neutrosophic decision function;
- Neutrosophic optimal solution;
- Neutrosophic constraint;
- Neutrosophic error;
- Neutrosophic performance;
- Neutrosophic specificity;
- Neutrosophic classifier (used by. V. Jaiganesh and P. Rutravigneshwarai etc.);
- Neutrosophic set used in Genetic Algorithm;
- Neutrosophic decision tree;
- Neutrosophic signature;
- Neutrosophic rule extraction techniques.

## Lucrare neutrosifică și Viața neutrosifică

*Emenia Cera*

Ani neutrosofici, e de bine dacă ți-i urez? Cum ai vrea să-ți fie anii ce vor urma? Îți doresc mulți, frumoși și sănătoși cu împliniri pe toate domeniile! La mulți ani!

*Florentin Smarandache*

La propriu e bine, la figurat nu.

*Emenia Cera*

Care e diferența?

*Florentin Smarandache*

La propriu ar fi: lucrări de logici / mulțimi etc. neutrosofice. La figurat ar fi: combinații de bune, rele și neutre în viață (care cam așa se-ntâmplă în orice viață)...

*Emenia Cera*

Ambele variante sunt ok. Adică la figurat, e normalitatea de zi cu zi...

## n-ary Algebraic Structures

### Definition of *n-ary* Law

Let  $U$  be a universe of discourse, a non-empty set  $M \subseteq U$

, an integer  $n \geq 2$ ,

and an *n-ary law* constructed, as follows:

$$\circ_n : M^n \rightarrow M .$$

The  $n$ -ary law is also called *hyperoperation*, while  $n$  is called *arity* of this hyperoperation.

**$n$ -ary Well-Defined Law**

The law  $\circ_n$  is  *$n$ -ary well-defined* if for any  $x_1, x_2, \dots, x_n \in M$  one has  $\circ_n(x_1, x_2, \dots, x_n) \in M$ .

**$n$ -ary Associativity Law**

The  $n$ -ary law  $\circ_n$  is associative, if for any  $x_1, x_2, \dots, x_{2n-1} \in M$  one has:

$$\begin{aligned} &\circ_n(\circ_n(x_1, x_2, \dots, x_n), x_{n+1}, x_{n+2}, \dots, x_{2n-1}) = \\ &\circ_n(x_1, \circ_n(x_2, x_3, \dots, x_{n+1}), x_{n+2}, x_{n+3}, \dots, x_{2n-1}) = \\ &\circ_n(x_1, x_2, \circ_n(x_3, x_4, \dots, x_{n+2}), x_{n+3}, x_{n+4}, \dots, x_{2n-1}) = \\ &\dots = \circ_n(x_1, x_2, \dots, x_{n-1}, \circ_n(x_n, x_{n+1}, \dots, x_{2n-1})) \end{aligned}$$

The  $n$ -ary associativity is very strong/restrictive in comparison to the binary associativity.

**$n$ -ary Commutativity Law**

The  $n$ -ary law  $\circ_n$  is commutative, if for any  $x_1, x_2, \dots, x_n \in M$  and any distinct permutations of these  $n$  elements,  $\varphi_i(x_1, x_2, \dots, x_n)$  and  $\varphi_j(x_1, x_2, \dots, x_n)$ , one has

$$\circ_n(\varphi_i(x_1, x_2, \dots, x_n)) = \circ_n(\varphi_j(x_1, x_2, \dots, x_n))$$

where  $i, j \in \{1, 2, \dots, n!\}$ ,  $i \neq j$ , while  $n$  factorial,  $n! = 1 \cdot 2 \cdot \dots \cdot n$ , represents the total number of permutations of  $n$  elements.

The  $n$ -ary commutativity is very strong/restrictive with respect to the binary commutativity.

### **$n$ -ary Neutral/Unitary Element**

If there exist some element  $e \in M$ , such that for any  $x \in M$  and any two distinct permutations  $\varphi_i(x, e, \dots, e)$  and

$$\varphi_j(x, e, \dots, e), \text{ of the } n \text{ elements } \{x, e, \dots, e\}, \text{ one has}$$

$$\circ_n^H(\varphi_i(x, e, \dots, e)) = \circ_n^H(\varphi_j(x, e, \dots, e)) = x$$

where  $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ , while  $n$  represents the total number of permutations of these elements, then element  $e$  is the  $n$ -ary neutral/unitary element of the set  $M$ .

### **$n$ -ary Inverse/Opposite Element**

Let  $x \in M$  and  $e \in M$  be the  $n$ -ary neutral/unitary element of the set  $M$ . If there exist some element  $x^{-1} \in M$ , such that for any two distinct permutations  $\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$  and  $\varphi_j(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$ , of the  $n$

elements  $\{x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1}\}$ , one has

$$\circ_n^H(\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) = \circ_n^H(\varphi_j(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) = e$$



where  $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ , while  $n$  represents the total number of permutations of these elements, then element  $x^{-1}$  is the  $n$ -ary inverse/opposite element of  $x$ .

The  $n$ -ary inverse element is not defined as existence of unique solutions to several equations as done in previous  $n$ -ary laws.

### **$n$ -ary Group**

$n$ -ary group is a set  $(M, \circ_n)$ , such that its  $n$ -ary law  $\circ_n$  is  $n$ -ary well-defined,  $n$ -ary associative, has an  $n$ -ary neutral/unitary element  $e$ , and each element,  $x \in M$ , has an  $n$ -ary inverse with respect to the set  $n$ -ary neutral element  $e$ .

If the  $n$ -ary law is, in addition,  $n$ -ary commutative, then one has an  **$n$ -ary Commutative Group**.

If the set  $(M, \circ_n)$ , is such that its  $n$ -ary law  $\circ_n$  is  $n$ -ary well-defined, and  $n$ -ary associative, then the set is called  **$n$ -ary Semigroup**.

## **$n$ -ary Neutrosophic Extended Triplet Structures**

Let  $U$  be a universe of discourse, a non-empty set  $M \subseteq U$   
 an integer  $n \geq 2$ ,

and an  $n$ -ary law constructed, as follows:

$$\circ_n : M^n \rightarrow M .$$

### **n-ary Neutrosophic Extended Neutral/Unitary**

Let  $x \in M$ . If there exist some element  $e_x \in M$ , such that for any two distinct permutations  $\varphi_i(x, \underbrace{e_x, \dots, e_x}_{n-1})$  and

$\varphi_j(x, \underbrace{e_x, \dots, e_x}_{n-1})$ , of the  $n$  elements  $\{x, \underbrace{e_x, \dots, e_x}_{n-1}\}$ , one has

$$\circ_n^H(\varphi_i(x, \underbrace{e_x, \dots, e_x}_{n-1})) = \circ_n^H(\varphi_j(x, \underbrace{e_x, \dots, e_x}_{n-1})) = x$$

where  $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ , while  $n$  represents the total number of permutations of these elements, then the element  $e_x$  is the *n-ary neutrosophic extended neutral/unitary element* of the element  $x$ , denoted by  $e_x = {}^e\text{neut}_n(x)$ .

### **n-ary Neutrosophic Extended Inverse/Opposite Element**

Let  $x \in M$  be an element, and  $e_x \in M$  be the *n-ary neutrosophic extended neutral/unitary element* of  $x$ . If there exist some element  $x^{-1} \in M$ , such that for any two distinct

permutations  $\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$  and  $\varphi_j(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$ , of the  $n$  elements  $\{x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1}\}$ ,

one has

$$\circ_n^H(\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) = \circ_n^H(\varphi_j(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) = e_x$$

where  $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ , while  $n$  represents the total number of permutations of these elements, then

element  $x^{-1}$  is the  $n$ -ary extended inverse/opposite element of  $x$ , and it is denoted by  $x^{-1} = {}^e\text{anti}_n(x)$ .

### **n-ary Neutrosophic Extended Triplet Set**

A set  $M$ , such that for any  $x \in M$  there is an  ${}^e\text{neut}_n(x)$  and an  ${}^e\text{anti}_n(x)$ , that we call **n-ary neutrosophic extended triplet**, denoted as  $(x, {}^e\text{neut}_n(x), {}^e\text{anti}_n(x))$ , is called  $n$ -ary *Neutrosophic Extended Triplet Set*.

### **n-ary Neutrosophic Extended Triplet Group**

An  $n$ -ary neutrosophic extended triplet set whose  $n$ -ary law is associative is called  $n$ -ary *Neutrosophic Extended Triplet Group*.

In addition, if the  $n$ -ary law is commutative, one has an **n-ary Neutrosophic Extended Triplet Commutative Group**.

## **n-ary Algebraic HyperStructures**

### **n-ary HyperLaw**

Let  $U$  be a universe of discourse, a non-empty set  $M \subseteq U$ ,  $P(M)$  the power set of  $M$  (all subsets of  $M$ ), an integer  $n \geq 2$ , then an  $n$ -ary *HyperLaw* is constructed follows:

$$\circ_n^H : M^n \rightarrow P(M).$$

The associativity and commutativity are defined similarly as above for  $n$ -ary Law:

### **n-ary HyperAssociativity**

The  $n$ -ary hyperlaw  $\circ_n^H$  is hyperassociative, if for any

$x_1, x_2, \dots, x_{2n-1} \in M$  one has:

$$\begin{aligned} & \circ_n^H (\circ_n^H (x_1, x_2, \dots, x_n), x_{n+1}, x_{n+2}, \dots, x_{2n-1}) = \\ & \circ_n^H (x_1, \circ_n^H (x_2, x_3, \dots, x_{n+1}), x_{n+2}, x_{n+3}, \dots, x_{2n-1}) = \\ & = \circ_n^H (x_1, x_2, \circ_n^H (x_3, x_4, \dots, x_{n+2}), x_{n+3}, x_{n+4}, \dots, x_{2n-1}) = \\ & \dots = \circ_n^H (x_1, x_2, \dots, x_{n-1}, \circ_n^H (x_n, x_{n+1}, \dots, x_{2n-1})) \end{aligned}$$

### **$n$ -ary HyperCommutativity**

The  $n$ -ary hyperlaw  $\circ_n^H$  is hypercommutative, if for any

$x_1, x_2, \dots, x_n \in M$  and any distinct permutations of these  $n$  elements,  $\varphi_i(x_1, x_2, \dots, x_n)$  and  $\varphi_j(x_1, x_2, \dots, x_n)$ , one has

$$\circ_n^H (\varphi_i(x_1, x_2, \dots, x_n)) = \circ_n^H (\varphi_j(x_1, x_2, \dots, x_n)),$$

where  $i, j \in \{1, 2, \dots, n!\}$ , while  $n$  factorial  $n! = 1 \cdot 2 \cdot \dots \cdot n$  represents the total number of permutations on  $n$  elements.

### **$n$ -ary HyperNeutral/HyperUnitary Element**

If there exist some element  $e \in M$ , such that for any  $x \in M$  and any permutation  $\varphi_i(x, e, \dots, e)$  of the  $n$  elements

$\{x, e, \dots, e\}$ , one has

$$\circ_n^H (\varphi_i(x, e, \dots, e)) \ni x$$

where the symbol “ $\ni$ ” means “contains”, with  $i \in \{1, 2, \dots, n\}$ , while  $n$  represents the total number of permutations of these elements, then element  $e$  is the  $n$ -ary hyperneutral/hyperunitary element of the set  $M$ .

**$n$ -ary HyperInverse/HyperOpposite Element**

Let  $x \in M$  and  $e \in M$  be the  $n$ -ary hyperneutral/hyperunitary element of the set  $M$ . If there exist some element  $x^{-1} \in M$ , such that for any permutations  $\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$  of the  $n$  elements

$\{x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1}\}$ , one has

$$\circ_n^H (\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) \ni e$$

where  $i \in \{1, 2, \dots, n\}$ , while  $n$  represents the total number of permutations of these elements, then element  $x^{-1}$  is the  $n$ -ary hyperinverse/hyperopposite element of  $x$ .

The  $n$ -ary hyperinverse element is not defined as existence of unique solutions to several equations as done in previous  $n$ -ary hyperlaws.

**$n$ -ary Neutrosophic Extended Triplet HyperStructures**

Let  $U$  be a universe of discourse, a non-empty set  $M \subseteq U$ ,  $P(M)$  the power set of  $M$  (all subsets of  $M$ ), an integer  $n \geq 2$ , then an  $n$ -ary *HyperLaw* is constructed follows:

$$\circ_n^H : M^n \rightarrow P(M).$$

### **$n$ -ary Neutrosophic Extended**

#### **HyperNeutral/HyperUnitary Element**

Let  $x \in M$  be an element. If there exist some element  $e_x \in M$ , such that for any permutation  $\varphi_i(x, \underbrace{e_x, \dots, e_x}_{n-1})$  of the

$n$  elements  $\{x, \underbrace{e_x, \dots, e_x}_{n-1}\}$ , one has

$$\circ_n^H(\varphi_i(x, \underbrace{e_x, \dots, e_x}_{n-1})) \ni x$$

where the symbol “ $\ni$ ” means “contains”, with  $i \in \{1, 2, \dots, n\}$ , while  $n$  represents the total number of permutations of these elements, then the element  $e_x$  is the  $n$ -ary *neutrosophic extended hyperneutral/hyperunitary element* of  $x$ , and it is denoted as  $e_x = {}^e neut_n^H(x)$ .

### **$n$ -ary Neutrosophic Extended**

#### **HyperInverse/HyperOpposite Element**

Let  $x \in M$  and  $e_x \in M$  be the  $n$ -ary neutrosophic extended hyperneutral/hyperunitary element of  $x$ . If there exist some element  $x^{-1} \in M$ , such that for any permutations

$\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})$  of the  $n$  elements  $\{x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1}\}$ , one

has

$$\circ_n^H(\varphi_i(x, \underbrace{x^{-1}, \dots, x^{-1}}_{n-1})) \ni e_x$$

where  $i \in \{1, 2, \dots, n\}$ , while  $n$  represents the total number of permutations of these elements, then element  $x^{-1}$  is the  $n$ -ary neutrosophic extended hyperinverse/hyperopposite element of  $x$  with respect to its neutrosophic extended hyperneutral/hyperunitary  $e_x$ , and it is denoted as:

$$x^{-1} = {}^e anti_n^H(x).$$

The  $n$ -ary neutrosophic extended hyperinverse element is not defined as existence of unique solutions to several equations.

### **n-ary Neutrosophic Extended Triplet HyperSet**

A set  $M$ , such that for any  $x \in M$  there exist an  ${}^e neut_n^H(x)$  and an  ${}^e anti_n^H(x)$ , that we call **n-ary Neutrosophic Extended HyperTriplet**, denoted as  $(x, {}^e neut_n^H(x), {}^e anti_n^H(x))$ , is called *n-ary Neutrosophic Extended Triplet HyperSet*.

### **n-ary Neutrosophic Extended Triplet HyperGroup**

An  $n$ -ary neutrosophic extended triplet hyperset whose  $n$ -ary hyperlaw is associative is called *n-ary Neutrosophic Extended Triplet HyperGroup*.

In addition, if the  $n$ -ary hyperlaw is commutative, one has  
an  **$n$ -ary Neutrosophic Extended Triplet  
Commutative HyperGroup.**

**References:**

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My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin). I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος (*lógos*) – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks (in English, French, Spanish, and Romanian).

In this *sixth book of scilogs* collected from my nest of ideas, one may find new and old questions and solutions, referring to topics on NEUTROSOPHY – email messages to research colleagues, or replies, notes about authors, articles, or books, future projects, and so on.

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