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Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers

Florentin Smarandache

University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA
E-mail: smarand@unm.edu

Abstract. In this paper, we introduce for the first time the neutrosophic quadruple numbers (of the form $\boldsymbol{a + bT + cI + dF}$) and the refined neutrosophic quadruple numbers.
Then we define an absorbance law, based on a prevalence order, both of them in order to multiply the neutrosophic components $\boldsymbol{T, I, F}$ or their sub-components $\boldsymbol{T_j, I_k, F_l}$ and thus to construct the multiplication of neutrosophic quadruple numbers.

Keywords: neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, multiplication of neutrosophic quadruple numbers, multiplication of refined neutrosophic quadruple numbers.

1 Neutrosophic Quadruple Numbers

Let's consider an entity (i.e. a number, an idea, an object, etc.) which is represented by a known part (a) and an unknown part $(bT + cI + dF)$.

Numbers of the form:

$$NQ = a + bT + cI + dF, \quad (1)$$

where a, b, c, d are real (or complex) numbers (or intervals or in general subsets), and

T = truth / membership / probability,
I = indeterminacy,
F = false / membership / improbability,

are called Neutrosophic Quadruple (Real respectively Complex) Numbers (or Intervals, or in general Subsets).

"a" is called the known part of NQ, while "$bT + cI + dF$" is called the unknown part of NQ.

2 Operations

Let

$$NQ_1 = a_1 + b_1T + c_1I + d_1F, \quad (2)$$

$$NQ_2 = a_2 + b_2T + c_2I + d_2F \quad (3)$$

and $\alpha \in \mathbb{R}$ (or $\alpha \in \mathbb{C}$) a real (or complex) scalar.

Then:

2.1 Addition

$$NQ_1 + NQ_2 = (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F. \quad (4)$$

2.2 Substraction

$$NQ_1 - NQ_2 = (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F. \quad (5)$$

2.3 Scalar Multiplication

$$\alpha \cdot NQ = NQ \cdot \alpha = \alpha a + \alpha bT + \alpha cI + \alpha dF. \quad (6)$$

One has:

$$0 \cdot T = 0 \cdot I = 0 \cdot F = 0, \quad (7)$$

and

$$mT + nT = (m + n)T, \quad (8)$$

$$mI + nI = (m + n)I, \quad (9)$$

$$mF + nF = (m + n)F. \quad (10)$$

3 Refined Neutrosophic Quadruple Numbers

Let us consider that Refined Neutrosophic Quadruple Numbers are numbers of the form:

$$RNQ = a + \sum_{i=1}^{p} b_i T_i + \sum_{j=1}^{r} c_j I_j + \sum_{k=1}^{s} d_k F_k, \quad (11)$$

where a, all b_i, all c_j, and all d_k are real (or complex) numbers, intervals, or, in general, subsets,

while $T_1, T_2, \dots, T_p$ are refinements of T;
$I_1, I_2, \dots, I_r$ are refinements of I;
and $F_1, F_2, \dots, F_s$ are refinements of F.

There are cases when the known part *(a)* can be refined as well as $a_1, a_2, \dots$.

The operations are defined similarly.

Let

$$RNQ^{(u)} = a^{(u)} + \sum_{i=1}^{p} b_i^{(u)} T_i + \sum_{j=1}^{r} c_j^{(u)} I_j + \sum_{k=1}^{s} d_k^{(u)} F_k, \quad (12)$$

for $u = 1$ or 2.
Then:

3.1 Addition

$$RNQ^{(1)} + RNQ^{(2)} = \left[a^{(1)} + a^{(2)}\right] + \sum_{i=1}^{p} \left[b_i^{(1)} + b_i^{(2)}\right] T_i + \sum_{j=1}^{r} \left[c_j^{(1)} + c_j^{(2)}\right] I_j + \sum_{k=1}^{s} \left[d_k^{(1)} + d_k^{(2)}\right] F_k. \quad (13)$$

3.2 Substraction

$$RNQ^{(1)} - RNQ^{(2)} = \left[a^{(1)} - a^{(2)}\right] + \sum_{i=1}^{p} \left[b_i^{(1)} - b_i^{(2)}\right] T_i + \sum_{j=1}^{r} \left[c_j^{(1)} - c_j^{(2)}\right] I_j + \sum_{k=1}^{s} \left[d_k^{(1)} - d_k^{(2)}\right] F_k. \quad (14)$$

3.3 Scalar Multiplication

For $\alpha \in \mathbb{R}$ (or $\alpha \in \mathbb{C}$) one has:

$$\alpha \cdot RNQ^{(1)} = \alpha \cdot a^{(1)} + \alpha \cdot \sum_{i=1}^{p} b_i^{(1)} T_i + \alpha \cdot \sum_{j=1}^{r} c_j^{(1)} I_j + \alpha \cdot \sum_{k=1}^{s} d_k^{(1)} F_k. \quad (15)$$

4 Absorbance Law

Let S be a set, endowed with a total order $x \prec y$, named "*x* prevailed by *y*" or "*x* less stronger than *y*" or "*x* less preferred than *y*". We consider $x \preccurlyeq y$ as "*x* prevailed by or equal to *y*" "*x* less stronger than or equal to *y*", or "*x* less preferred than or equal to *y*".

For any elements $x, y \in S$, with $x \preccurlyeq y$, one has the absorbance law:

$$x \cdot y = y \cdot x = \text{absorb}\,(x, y) = \max\{x, y\} = y, \quad (16)$$

which means that the bigger element absorbs the smaller element (the big fish eats the small fish!).

Clearly,

$$x \cdot x = x^2 = \text{absorb}\,(x, x) = \max\{x, x\} = x, \quad (17)$$

and

$$x_1 \cdot x_2 \cdot \ldots \cdot x_n = \text{absorb}(\ldots \text{absorb}(\text{absorb}(x_1, x_2), x_3) \ldots, x_n) = \max\{\ldots \max\{\max\{x_1, x_2\}, x_3\} \ldots, x_n\} = \max\{x_1, x_2, \ldots, x_n\}. \quad (18)$$

Analougously, we say that "$x \succ y$" and we read: "*x* prevails to *y*" or "*x* is stronger than *y*" or "*x* is preferred to *y*".

Also, $x \succcurlyeq y$, and we read: "*x* prevails or is equal to *y*" "*x* is stronger than or equal to *y*", or "*x* is preferred or equal to *y*".

5 Multiplication of Neutrosophic Quadruple Numbers

It depends on the prevalence order defined on $\{T, I, F\}$.

Suppose in an optimistic way the neutrosophic expert considers the prevalence order $T \succ I \succ F$. Then:

$$NQ_1 \cdot NQ_2 = (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) = a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2 + b_1c_2 + c_1b_2 + b_1d_2 + d_1b_2)T + (a_1c_2 + a_2c_1 + c_1d_2 + c_2d_1)I + (a_1d_2 + a_2d_1 + d_1d_2)F, \quad (19)$$

since $TI = IT = T, TF = FT = T, IF = FI = I$,
while $T^2 = T, I^2 = I, F^2 = F$.

Suppose in an pessimistic way the neutrosophic expert considers the prevalence order $F \succ I \succ T$. Then:

$$NQ_1 \cdot NQ_2 = (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) = a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2)T + (a_1c_2 + a_2c_1 + b_1c_2 + b_2c_1 + c_1c_2)I + (a_1d_2 + a_2d_1 + b_1d_2 + b_2d_1 + c_1d_2 + c_2d_1 + d_1d_2)F, \quad (20)$$

since

$$F \cdot I = I \cdot F = F, F \cdot T = T \cdot F = F, I \cdot T = T \cdot I = I$$

while similarly $F^2 = F, I^2 = I, T^2 = T$.

5.1 Remark

Other prevalence orders on $\{T, I, F\}$ can be proposed, depending on the application/problem to solve, and on other conditions.

6 Multiplication of Refined Neutrosophic Quadruple Numbers

Besides a neutrosophic prevalence order defined on $\{T, I, F\}$, we also need a sub-prevalence order on $\{T_1, T_2, \dots, T_p\}$, a sub-prevalence order on $\{I_1, I_2, \dots, I_r\}$, and another sub-prevalence order on $\{F_1, F_2, \dots, F_s\}$.

We assume that, for example, if $T \succ I \succ F$, then $T_j \succ I_k \succ F_l$ for any $j \in \{1, 2, \dots, p\}$, $k \in \{1, 2, \dots, r\}$, and $l \in \{1, 2, \dots, s\}$. Therefore, any prevalence order on $\{T, I, F\}$ imposes a prevalence suborder on their corresponding refined components.

Without loss of generality, we may assume that

$$T_1 \succ T_2 \succ \dots \succ T_p$$

(if this was not the case, we re-number the subcomponents in a decreasing order).

Similarly, we assume without loss of generality that:

$I_1 \succ I_2 \succ \dots \succ I_r$, and

$F_1 \succ F_2 \succ \dots \succ F_s$.

6.1 Exercise for the Reader

Let's have the neutrosophic refined space

$$NS = \{T_1, T_2, T_3, I, F_1, F_2\},$$

with the prevalence order $T_1 \succ T_2 \succ T_3 \succ I \succ F_1 \succ F_2$.

Let's consider the refined neutrosophic quadruples

$NA = 2 - 3T_1 + 2T_2 + T_3 - I + 5F_1 - 3F_2$, and

$NB = 0 + T_1 - T_2 + 0 \cdot T_3 + 5I - 8F_1 + 5F_2$.

By multiplication of sub-components, the bigger absorbs the smaller. For example:

$T_2 \cdot T_3 = T_2$,

$T_1 \cdot F_1 = T_1$,

$I \cdot F_2 = I$,

$T_2 \cdot F_1 = T_2$, etc.

Multiply NA with NB.

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