NEUTROSOPHIC SETS AND SYSTEMS

{Special Issue: Proceedings of the International Conference called NeutroGeometry, NeutroAlgebra, and Their Applications, Universidad de Habana, Cuban Academy of Sciences et al. Havana, Cuba, 12-14 August 2024}

VOL. 71, 2024

NSS

FLORENTIN SMARANDACHE
MOHAMED ABDEL-BASSET
MAIKEL LEYVA VAZQUEZ
SAID BROUMI

EDITORS-IN-CHIEF

https://fs.unm.edu/NSS/

ISSN 2331-6055(PRINT)
ISSN 2331-608X(ONLINE)

ISSN 2331-6055 (print) ISSN 2331 – 608X (online)

Neutrosophic Science International Association(NSIA)

Neutrosophic Sets and Systems

second version

An International Journal in Information Science and Engineering

University of New Mexico
United States

University of New Mexico

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering

Copyright Notice

Information for Authors and Suscribers

"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their inter actions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).
According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of]-0, 1+[.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.
Neutrosophic Statistics is a generalization of the classical statistics.
What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, igno- rance, imprecision, etc.

All submissions should be designed in MS Word format using our template file:
http://fs.unm.edu/NSS/NSS-paper-template.doc.

A variety of scientific books in many languages can be downloaded freely from the Digital Library of Science: http://fs.unm.edu/ScienceLibrary.htm.

To submit a paper, mail the file to the Editor-in-Chief. To order printed issues, contact the Editor-in-Chief. This journal is non-commercial, academic edition. It is printed from private donations.
Information about the neutrosophics you get from the UNM website:
http://fs.unm.edu/neutrosophy.htm. The home page of the journal is accessed on http://fs.unm.edu/NSS.

University of New Mexico

ISSN 2331-6055 (print) NSS Editorial Board ISSN 2331-608X (online)

University of New Mexico

Preface: Neutrosophic Sets and Systems {Proceedings of the International Conference called NeutroGeometry, NeutroAlgebra, and Their Applications, Universidad de Habana, Cuban Academy of Sciences et al., Havana, Cuba, 12-14 August 2024}.

It is an honor to present this special issue of our journal, dedicated to the Conference on NeutroGeometry, NeutroAlgebra, and Their Applications, organized by the Latin American Association of Neutrosophic Sciences. This event, which took place on August 12-14, 2024, in Havana, Cuba, was made possible by the valuable collaboration of the University of Havana, the University of Physical Culture and Sports Sciences "Manuel Fajardo," the José Antonio Echeverría University of Technology, University of Informatics Sciences and the Cuban Academy of Sciences among other institutions.

In 2019 Smarandache generalized the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) {whose operations and axioms are totally false} and on 2020 he continued to develop them (https://fs.unm.edu/NA/).

The NeutroAlgebras & AntiAlgebras are a new field of research, which is inspired from our real world. In classical algebraic structures, all operations are 100% well-defined, and all axioms are 100% true, but in real life, in many cases these restrictions are too harsh, since in our world we have things that only partially verify some operations or some laws.

Similarly, a classical Geometry structure has all axioms totally (100%) true. A NeutroGeometry structure has some axioms that are only partially true, and no axiom is totally (100%) false. Whereas an AntiGeometry structure has at least one axiom that is totally (100%) false (https://fs.unm.edu/NG/).

And in general, in any field of knowledge one has: Structure, NeutroStructure, and AntiStructure (https://fs.unm.edu/NA/NeutroStructure.pdf), which were inspired from our real world where the laws (axioms) do not equally apply to all people and in the same degree.

This special issue aims to highlight the most recent advances and applications in the fields of NeutroGeometry and NeutroAlgebra, two areas that are at the forefront of contemporary mathematical and scientific thought. During the conference, the mathematical foundations and practical applications of these disciplines were explored, as well as their relevance in the MultiAlism system and other interdisciplinary areas. The following topics, among others, were covered during the conference:

- Mathematical foundations of NeutroGeometry.
- Mathematical foundations of NeutroAlgebra.
- Applications of NeutroGeometry.
- Applications of NeutroAlgebra.
- Interdisciplinary applications.
- Neutrosophic sets and their generalizations.
- MultiAlism System of Thought.
- Neutrosophic Set-Theoretic Methods of Research.
- NeutroStructure and AntiStructure.

The content of this special issue has been carefully selected to reflect the diversity and depth of the topics discussed at the conference. This event and the subsequent publication of these works underline the growing importance of neutrosophic theories in the current scientific landscape. We are confident that the ideas and discoveries shared in these pages will be of great value to researchers, academics, and professionals interested in these innovative areas of knowledge.

We would like to express our gratitude to all the participants of the conference, as well as to the authors who have contributed their research to this special issue. We hope that readers will find in these pages not only valuable knowledge but also inspiration for future research and applications in the field of neutrosophic sciences.

Conference Organizers:

1. Prof. Dr. Florentin Smarandache, PhD, Postdoc, Mathematics Department, University of New Mexico, Gallup, NM 87301, USA
2. Maikel Leyva Vazquez, PhD Universidad de Guayaquil, Guayas, Ecuador
3. Pedro Piñero Pérez, PhD, IADES, Cuba
4. Jesús Estupiñán Ricardo,PhD, Instituto Superior Tecnológico de Investigación Científica e Innovación, Ecuador
5. Iliana Pérez Pupo, PhD, IADES, Cuba
6. Erick González Caballero, PhD, José Antonio Echeverría University of Technology
7. Omar Mar Cornelio, PhD, University of Informatics Sciences, Cuba
8. Salah Hasan Saleh, Qatar Embassy Havana, Cuba
9. Bárbara Bron Fonseca, PhD, University of Informatics Sciences, Cuba
10. Darvin Manuel Ramírez Guerra PhD, Universidad César Vallejo, Lima, Perú
11. PhD, Maura de la Caridad Salabarría Roig, PhD, Center for Studies on Educational Quality and Scientific Research, Toluca, Mexico
12. José Sergio Puig Espinosa PhD, Center for Studies on Educational Quality and Scientific Research, Toluca, Mexico
13. Oscar José Alejo Machado, PhD, Instituto Superior Tecnológico de Investigación Científica e Innovación, Ecuador
14. Karina Pérez-Teruel, PhD, Barna Management School, Santo Domingo, Dominican Republic
15. Ricardo Sánchez Casanova, PhD, University of Havana, Cuba
16. Ariel Romero Fernández, PhD, Universidad Regional Autónoma de los Andes, Ecuador
17. Ángel Martínez Vásquez, MSc, Center for Training and Knowledge Management. Ecuador.
18. Jean Pierre Ramos-Carpio, MSc, Center for Studies on Educational Quality and Scientific Research, Toluca, Mexico.

University of New Mexico

ISSN 2331-6055 (print) NSS Editorial Board ISSN 2331-608X (online)

University of New Mexico

INDEX

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

NeutroGeometry and Fractal Geometry

Erick González-Caballero[1], Maikel Y. Leyva-Vázquez[2], Noel Batista-Hernández[3], and Florentin Smarandache[4]

[1] Member, Latin American Association of Neutrosophic Sciences, Havana, Cuba. E-mail: erickgc@yandex.com
[2] Guayaquil University (UG), Ecuador. E-mail: maikel.leyvav@ug.edu.ec
[3] Bolivarian University of Ecuador (UBE), Durán, Ecuador. E-mail: nbatistah@ube.edu.ec
[4] University of New Mexico, Mathematics, Physics, and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA, E-mail: smarand@unm.edu

Abstract. Geometries are structures with certain elements like points, lines, planes, and spaces, among others, that satisfy certain definitions, axioms, properties, and theorems for the total of the elements. NeutroGeometries are geometric structures that meet at least one of these concepts only partially, never for 100% or 0% of the elements. Until now, NeutroGeometries have been developed from the ideas of classical geometries such as Euclidean, Hyperbolic, Elliptic, Mixed (Smarandache) geometries, among others where axiomatization is the basis of their construction. This paper aims to discuss some ideas about the relationship between NeutroGeometries and fractal geometry. This relation is not necessarily obvious; it is mainly established because fractals are structures used to model deterministic chaotic phenomena. The fractal dimension is a numerical value used to measure the complexity of the figure and the maps that represent chaotic phenomena. The more complex the phenomenon, the more unpredictable it becomes and therefore the more uncertain and indeterminate. This indeterminacy is essentially ontological since it deals mostly with natural phenomena. This relationship is proposed in this article for associating the concepts of NeutroGeometry that present degrees of uncertainty or indeterminacy and fractal geometries that model phenomena where unpredictability exists. This idea is reinforced in some works where a direct relationship between entropy and the fractal dimension is demonstrated.

Keywords: NeutroConcept, NeutroGeometry, indeterminacy, unpredictability, fractal geometry, entropy, chaos, fractal dimension.

1 Introduction

Neutrosophy is the branch of philosophy that studies neutrality, understanding in this concept the neutral, the indeterminate, the erroneous, the contradictory, the imprecise, the paradoxical, the unknown, and the inconsistent, among other concepts. This philosophical theory introduced by Professor F. Smarandache has expanded to various parts of knowledge. Especially mathematics has been enriched with concepts such as Neutrosophic Sets in the form of Single-Valued Neutrosophic Sets or Interval-Valued Neutrosophic Sets. These generalize fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and Pythagorean fuzzy sets, among many others.

One of the fields in which Neutrosophy has been presented is Geometry. Several decades ago Smarandache introduced a movement called Paradoxist, where mixed geometric structures were studied. Specifically, the following definition was used [1]:

"An axiom is said to be Smarandachely denied if the axiom behaves in at least two different ways within the same space, i.e., validated and invalidated, or only invalidated but in multiple distinct ways. A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom."

In this type of geometry, emphasis is placed on the non-satisfaction of some axiom. Above all, the controversial Euclid's fifth postulate about the existence of a single parallel that passes through a point outside a straight line. However, within the framework of Neutrosophy, Smarandache defined NeutroGeometry much more recently as part of a series of concepts coming from neutrosophic logic. He defined what he called a NeutroConcept, compared to Concept and AntiConcept. Each of these categories was associated with a triple of values (T, I, F) that represents the triple of Truthfulness, Indeterminacy, and Falseness, respectively, of satisfaction of the concept [2].

A Geometry satisfies each axiom, definition, property, theorem, and proposition, among others, for all elements, that is, with a true degree of satisfaction $(1,0,0)$. The AntiGeometry satisfies it for none in any Concept that is $(0,0,1)$. The elements of the NeutroGeometry satisfy it for the rest of the intermediate values of (T, I, F). The idea of the existence of indeterminacy within the geometric structure is important, although it is enough that a certain degree of truth exists in NeutroGeometry when $I = 0$ [2-4].

The Mixed Geometries or Smarandache Geometries are included within the concept of NeutroGeometry. However, NeutroGeometries have greater logical implications, they are logic applied to Geometry rather than pure geometry modified in its classical axioms as occurs within Mixed Geometries [5].

There is a relatively large amount of research within the field of Mixed Geometries, e.g. the works of H. Iseri who contributed to the Smarandache Manifolds, where geometric structures are composed of "disks" in the shape of equilateral triangles, such that the length of their edges is taken as the unit of measurement within the structure. In these manifolds, all parallelism properties belonging to elliptic, Euclidean, or hyperbolic geometries appear within the same structure [6]. We also owe the creation of a simple geometric model in [7], although it is mathematically poorly founded and developed. More recently, other authors have worked in this line of research such as Carlos Granados [8]. Other works within the field of Smarandache Geometries can be consulted related to finite geometry [9-11] and some real-life cases where Mixed Geometries are satisfied [12]. This last article also outlines some examples of NeutroGeometry that is not Mixed Geometry. We can also consult the development from Euclidean geometry to Smarandache geometry in [13].

However, a step taken where results are presented in the field of NeutroGeometries beyond the Smarandache Geometries appears in the article [14]. In this article, the author proposes a distance formula within the framework of NeutroGeometry. The mathematical simplicity and at the same time the unreality of the use of distances in classical geometries, whether Euclidean or classical non-Euclidean, becomes evident. This unreality is because the world that geometry tries to model is not homogeneous, and therefore it is not possible to travel from one point to another by the shortest classical distance, since the path may be crossed by obstacles or indeterminacies.

The aforementioned article deals with paths represented in a coordinate system on a two-dimensional surface. The proposed distance formula is valid for any geometry, viz., Euclidean, hyperbolic, elliptic, or mixed. However, the paths proposed to connect one point to another must be rectifiable, that is, there must be a way to measure them. A measure of uncertainty or indeterminacy based on the opinion or experience of a group of people is also proposed and a decision-making algorithm is recommended to solve the problem of moving from one point to another in a practical way.

That is why the idea of distance in this case is associated with a situation of uncertainty or indetermination with a high epistemological component. So, moving from one place to another will depend on people's knowledge of the best path and their cognitive uncertainty or indeterminacy. Another important idea that does not appear, at least explicitly, in the aforementioned works is the relationship of geometry with the dynamic changes of a system. The latter is because these are classical geometries; more emphasis is placed on static realities that do not change over time.

Taking into account all explained so far, it remains to ask if the concept of NeutroGeometry has any relationship with geometry as atypical as fractal since all authors have focused on the idea of associating it with classical geometries. Fractal geometry focuses on the study of the geometric object called fractal which was coined in this way by Benoit Mandelbrot from the Latin word *fractus*, meaning broken or fractured [15]. These objects have been widely studied. Among their characteristics are that they are usually not rectifiable and self-similar. This means that they usually have infinite length even though they occupy a limited space and also have shapes that repeat in the same or similar way at different scales. The visualization of these objects is reminiscent of shapes within nature that cannot be described with the help of classical geometries; however, fractals are usually constructed with simple recurring equations. This is supported by computational development that allows the plotting of these shapes with an adequate resolution.

Associated with the concept of a fractal is the fractal dimension. This is a numerical value of dimension which in fractals are non-integer values. In classical curves or geometric figures, these values are integers, for example, points have dimension 0, rectifiable lines and curves have dimension 1, surfaces have dimension 2, and so on. The fractal dimension has application to study curves that are natural, for example, the crack in a concrete surface, the coastline of a country, and the blood vessel networks of the human body, among many others.

The relationship between fractal forms and the representation of chaotic dynamic systems is known and has been well-studied. These are deterministic phenomena that produce completely different results from small changes in the initial conditions. They are non-random phenomena, since when the same initial value is entered, the results are the same, but on the other hand, a very small change in the initial value gives completely different results. Despite this, they are phenomena that are described with the help of curves included within a limited space.

This paper aims to reflect on and discuss the relationship that may exist between NeutroGeometries and Fractal Geometry. We maintain the thesis that there is a relationship although this has its subtleties. Chaotic phenomena are provided with a high degree of complexity, which is manifested in the fractal dimension of the so-called maps of their graphic representation. In some cases, a direct correlation occurs between the fractal dimension and entropy, for example in the study of the spatial growth of cities. Therefore, it is not difficult to realize that the complexity of natural phenomena implies greater unpredictability and therefore greater uncertainty and indeterminacy. This type of indeterminacy has a primarily ontological nature.

The article is divided into a preliminary section where the most important concepts of NeutroGeometry and fractal geometry are explained. In the following section, we develop the ideas that allow us to link both concepts.

The last section contains the conclusions.

2 Preliminaries

This section contains the basic notions of NeutroGeometry, AntiGeometry, fractal geometry, and fractal dimension.

2.1 NeutroGeometry and AntiGeometry

According to neutrosophic logic, a proposition has a valuation (T, I, F), where T means the degree of truthfulness, I is the degree of indeterminacy and F is the degree of falseness. A Concept is satisfied with valuation according to the triple $(T, I, F) = (1,0,0)$, an AntiConcept is satisfied with the valuation $(T, I, F) = (0,0,1)$, and a NeutroConcept is satisfied for the rest of the possible values of the triple.

Geometry is defined as a geometric structure composed only of Concepts, NeutroGeometry is a geometric structure with at least one NeutroConcept and no AntiConcept, and AntiGeometry is a geometric structure that contains at least one AntiConcept [2, 4].

Specifically, NeutroGeometry is a geometric structure composed of elements such as points, lines, planes, spaces, hyperspaces, and so on, where some definition, axiom, theorem, and property, among others, is a NeutroConcept. That is, it fulfills that there is a NeutroAxiom or a NeutroProperty or a NeutroDefinition, and so on. The intuitive idea to define NeutroGeometry is that the physical world in which we live is not homogeneous and contains indeterminations [2, 4]. For example, although the straight line is the shortest one between two points in a city, it is not always possible to follow the straight path because there may be construction elements such as buildings that can prevent our passage. In this way, the shortest path would be a broken line following the traffic.

It is also clear that Mixed Geometries or Smarandache Geometries are NeutroGeometries. In them, the Smarandachely denied axioms are partially fulfilled. An example that can be consulted is the model shown in Figure 1.

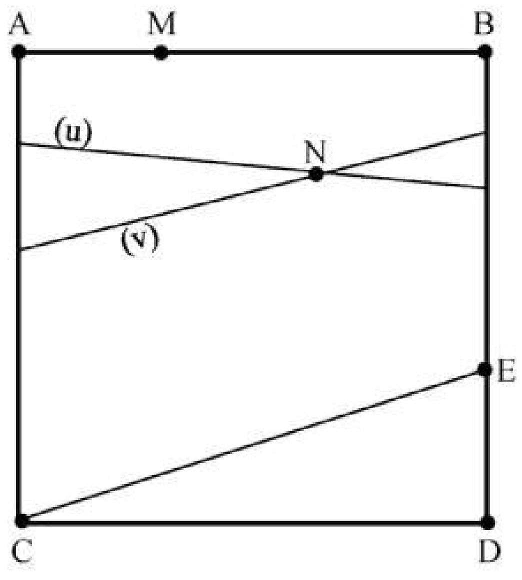

Figure 1: Bhattacharya's model of Smarandache Geometry. See [7].

Figure 1 shows a model made up of a rectangle where the points are the Euclidean points contained in the interior of the rectangle and those on the border. The lines go from one point on the edge AC to another point on the edge BD. If the line CE is taken, it does not have any parallel that passes through any point inside the segment DE. It has at least two parallel lines (u and v) that pass through point N and a single parallel that passes through point M. This model is one of the first to appear in a mixed geometry.

Figure 2 shows another model where a NeutroGeometry is not a Smarandache Geometry.

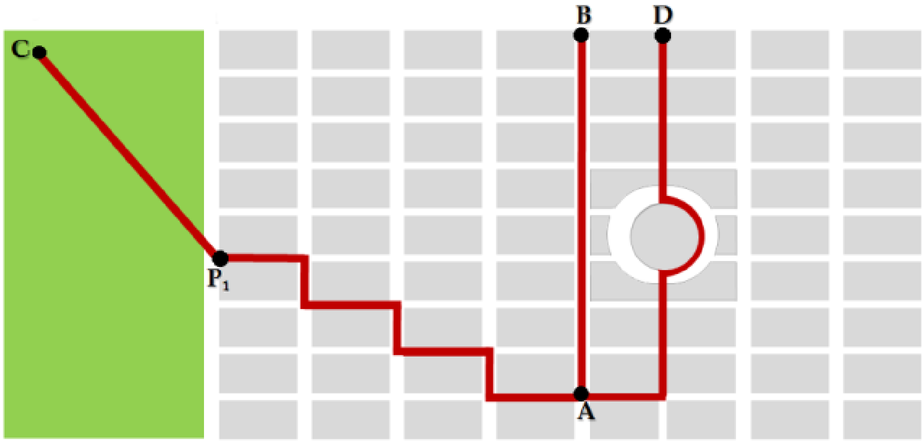

Figure 2: Euclidean model of an urban area showing a park (green area), blocks (gray rectangles), roundabout (gray circle), and paths (red lines). Source: [12].

Formally, the elements in Figure 2 correspond to Euclidean geometry; among the proposed paths, only AB and P_1C can be joined in straight Euclidean lines. There is no geometry defined in the literature where AD is a line in the sense of geodesic. The line AC presents a valid path for Taxicab Geometry in the section AP_1 and the Euclidean geometry in the section P_1C.

Specifically, to travel on the path AC shown in the figure, the shortest line (geodesic) is to follow the path marked in red. However, this path is not valid in both Euclidean geometry and Taxicab Geometry. This is why the property being a geodesic is a NeutroProperty in this example. This is the shortest path in real life, but it is not a line in Euclidean and Taxicab geometries, only partially.

These drawbacks show the rigidity of geometries to provide us with solutions to real-life problems because they are based on axioms that prioritize cognitive simplicity over applicability to daily life problems. However, the search for a more general solution is obtained by sacrificing the simplicity of the classical models. An example of this is the distance proposal for NeutroGeometry as explained below [14].

Definition 1 ([14])**.** Given (G, d_G) is a metric space in a geometric structure. The *NeutroGeometric distance* between x_1 and x_2 for $x_1, x_2 \in X$ is defined as:

$$d_{NG}(x_1, x_2) = inf\left\{\frac{L(p)}{\varepsilon(p)}: p \text{ is a rectifiable path from } x_1 \text{ to } x_2\right\} \qquad (1)$$

Where $\varepsilon(p)$ is the passability function of p.

This formula explains that if we want to calculate the neutrosophic distance that joins the points x_1 and x_2 in a plane corresponding to a certain geometry (Euclidean, hyperbolic, elliptic, mixed, taxicab, another one) having a coordinate system and a metric space associated with the base geometry, then the length of all the rectifiable paths that join both points are divided by a function called passability that measures the uncertainty or indeterminacy that joins both points along the chosen path.

This formula is a decision-making method where the distance is sought taking into account what is the shortest path in the classical sense and also the real possibility of passing through this path taking into account the knowledge of the locals or those people who know the place. It is therefore an uncertainty and indeterminacy that are epistemological.

The example that appears in the article is the following:

Example 1 ([14])**.** Suppose we are in a boat and we want to navigate a river whose bed has irregularities so that the left bank has calm waters and the right bank has turbulent waters in a certain section. However, we wish to go from point A to point B on the right side of the river. Furthermore, the path of the river itself is sinuous, see Figure 3.

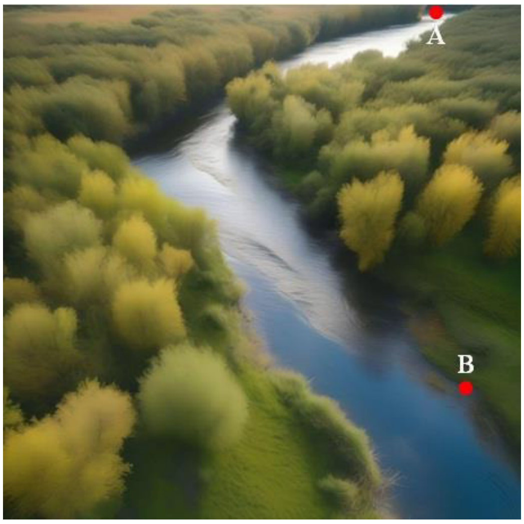

Figure 3: Picture of the river of the example. We need to go from point A to point B. This image was generated by an AI tool. Source: [14].

That is why the shortest path that passes on the right is impassable and hence $\varepsilon(p_1) = 0$. The path that goes through the center has some areas with a percentage of danger, let us say $\varepsilon(p_2) = 0.66667$. While the path that goes left and then turns right is the longest, and it is completely safe, therefore $\varepsilon(p_3) = 1$, see Figure 4.

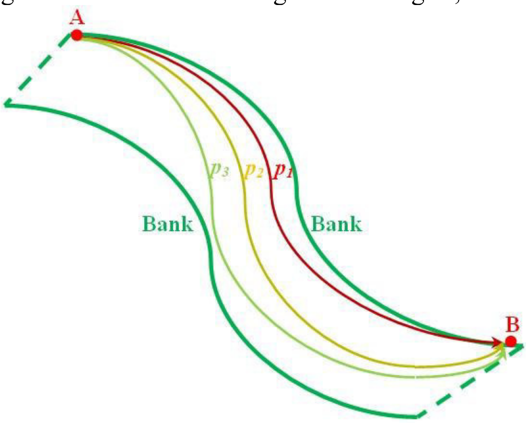

Figure 4: Map of the river showing the three paths, p_1, p_2, and p_3. Source: [14].

Let us suppose that the Euclidean length of each path from A to B is viz., $l(p_1) = 1.5\ km$, $l(p_2) = 1.9\ km$ and $l(p_3) = 2.1\ km$. To go on the shortest path in the Euclidean distance, i.e. a straight line we would have to navigate sections of the river, then go overland in a swampy area, then navigate another section, and so on, which is not practical.

The NeutroGeometric lengths of the three paths in the example are, $l_{NG}(p_1) = \frac{1.5}{0}\ km = \infty\ km$, $l_{NG}(p_2) = \frac{1.9}{0.66667}\ km = 2.85\ km$, while $l_{NG}(p_3) = \frac{2.1}{1}\ km = 2.1 km$, therefore we prefer the path p_3 to make the crossing, even though it is the longest according to Euclidean geometry.

2.2 Fractal Geometry

Fractal Geometry is due to the mathematician Benoît Mandelbrot who introduced it in the mid-1970s [15]. Fractals are geometric objects that have irregularity as a characteristic. They are usually curves or surfaces that are not differentiable at any point and also have infinite lengths (they are not rectifiable). Another characteristic is that they are self-similar, that is, when the scale of a portion of the object is increased, a figure equal or similar to the entire object is obtained, and this occurs at any scale. This self-similarity can be exact as in classical fractals, approximate as in nature where the parts are approximately equal to the whole, and statistical self-similarity where some self-similarity characteristics are preserved.

A classic example of a fractal is the Koch curve; Figure 5 shows the construction of the so-called Koch snowflake.

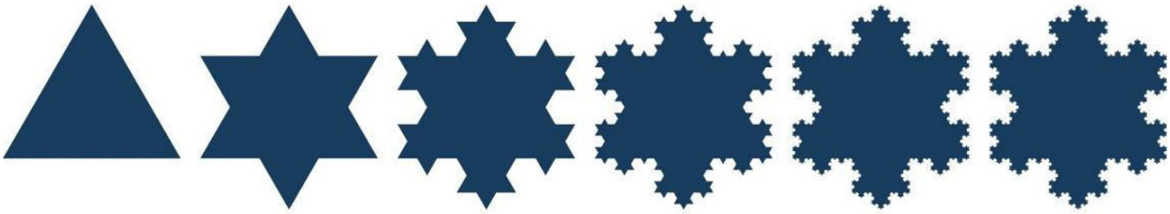

Figure 5: Koch snowflakes fractal. Source: http://www.pixabay.com.

To construct this fractal, an equilateral triangle is taken as seen in the figure. From each of its edges, the segment that is in the middle third is eliminated and replaced by two edges of an equilateral triangle. For the new edges, the process is repeated infinitely. From this process, a bounded object is obtained, but one that has infinite length and is also self-similar.

These objects cannot be described with elements of classical geometry and are reminiscent of elements of nature such as broccoli, cauliflower, and trees, among others. For example, see Figure 6 with the details of a broccoli and Figure 7 contains an artificially generated tree using code in the Python language.

Figure 6: Broccoli showing self-similarity properties. Source: http://www.pixabay.com.

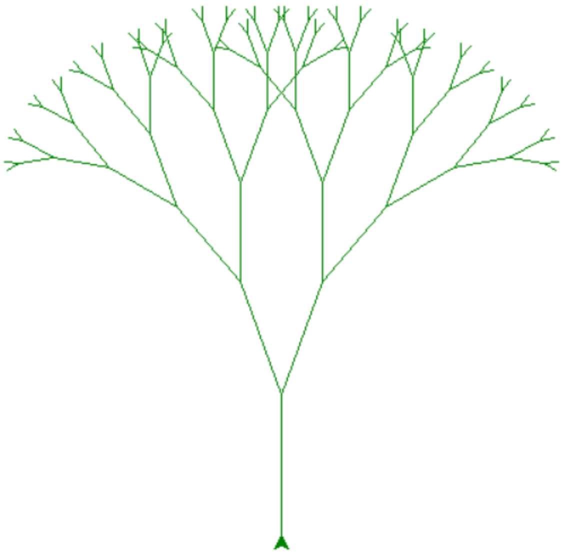

Figure 7: Tree generated using Python code.

An important concept within this theory is the fractal dimension. While classical objects have an integer topological dimension, the fractal dimension can be real, not integer. For example, the Koch curve has a fractal dimension of $d_f = \frac{\ln(4)}{\ln(3)} \approx 1.2619$. This number means that the curve has more complexity than a smooth curve in the two-dimensional space such as a straight line in Euclidean geometry, however, it is not so complex as to occupy a complete two-dimensional space, for example, a square that has topological dimension 2.

The number above is obtained from the general formula:

$$d_f = \frac{\ln m}{\ln r} \qquad (2)$$

Where r is the scale factor, while m is the minimum number of copies of this square needed to cover the object. For example, in the Koch snowflake fractal, 4 squares with a scale factor of 3 are needed to cover the entire figure [16, 17]. This is used when the figure satisfies the self-similarity property.

There is more than one definition for the fractal dimension. A simple one is the Box Dimension, which is calculated from Equation 3 [17].

$$d_B = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)} \qquad (3)$$

If the limit exists, where $N(\varepsilon)$ is the minimum D-dimensional ε side cubes needed to cover the D-dimensional figure.

This dimension is used not only in fractals but also in real objects, a classic example is the box dimension of the coasts of Great Britain [18]. There are phenomena where its fractal dimension is used as a measurement, since classical measurements such as length or area, among others, do not make sense for these objects. Among these cases is the spatial growth of cities over time or the crack in the concrete or wall, see Figures 8 and 9 [19].

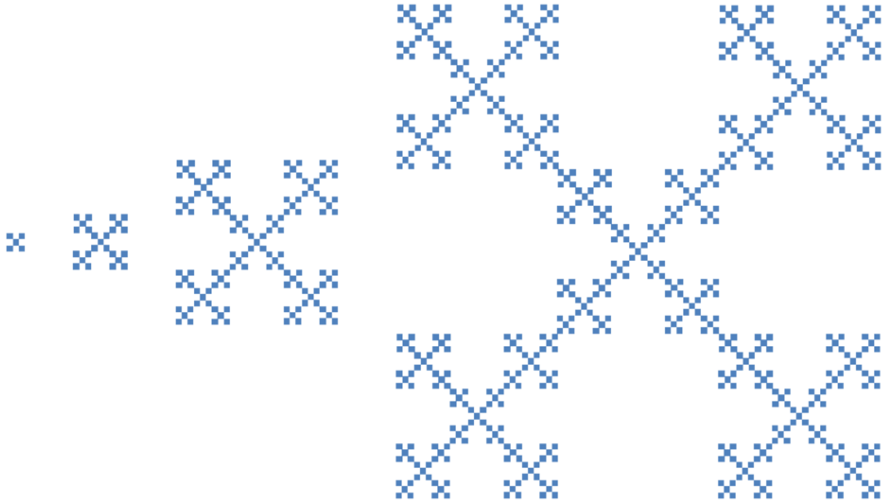

Figure 8: Self-similar fractal used to model the spatial growth of a city. Source: The authors.

Figure 9: Crack in the wall. It is a figure that cannot be represented by classical geometries. Source: http://www.pixabay.com.

3 NeutroGeometry and Fractal Geometry

Perhaps when the idea of creating a theory of NeutroGeometry arose, the possibility of associating this concept with fractal geometry was not thought of. However, there are common points between these two geometries. The NeutroGeometries arose from the Paradoxist movement where only two concepts opposed to each other that co-existed in the same structure were taken into account, mainly the concept of axiom, and within the axioms the fifth postulate of Euclid and its variants of non-Euclidean geometries. However, fractal geometry is not represented axiomatically; therefore it does not make sense to talk about Mixed or Smarandache fractal geometries in this context.

From a formal point of view, both theories can be combined with the self-similarity property. There are curves in the plane that are not self-similar, we have as an example any known smooth curve, a very simple one is the circumference. At the opposite extreme, we have curves that have exact self-similarity such as classic fractals like the one seen in Figure 5. It is also known that there are intermediate curves, where the self-similarity is partial as occurs in several fractals, or even on surfaces like mountains, self-similarity is said to be statistical, since only some parts are somewhat similar to the whole. That is why self-similarity is a Concept that has an AntiConcept (not self-similarity) and a NeutroConcept (approximate or statistical self-similarity).

As for the measure of the complexity of the fractal figure, which is the fractal dimension, which is taken due to the lack of others such as length or area, there is no single measure either. The Box Dimension is just one of them and does not always coincide with the other definitions of fractal dimension.

There are:

$d_T, d_{HB}, d_{MB}, d_E, d_C$,
Where:
d_T: is the topological dimension that is always an integer,
d_{HB}: is the Haussdorff-Besicovitch dimension,
d_{MB}: is the Minkowski-Bouligand or box-counting dimension that had been explained previously,
d_P: is the packaging dimension,
d_C: is the dimension of the Euclidean space that contains the figure and is an integer.

Therefore, if d_f is the fractal dimension of the object, then $d_f \in [d_T, d_C]$ or more specifically for fractal objects we have $d_f \in [min\{d_{HB}, d_{MB}, d_P\}, max\{d_{HB}, d_{MB}, d_P\}]$. That is, if there is a measurement called "fractal dimension" of a fractal object, it must be contained in this last interval. However, in general, it is fulfilled $d_f \in [d_T, d_C]$. Also, any fractal dimension is always calculated approximately.

This is reinforced by the definition of multifractal dimension, because some objects have a self-similarity that does not hold equally for all scales and it is necessary to define a series of fractal dimensions instead of a single numerical value, to describe the object [17].

However, the relationship between NeutroGeometry and fractality is especially noticeable in the representation of deterministic chaos through fractal theory. A system is chaotic when it exhibits aperiodic behavior and is sensitive to initial conditions; therefore it is not predictable [17]. That is, despite it is not random, for small changes in the initial conditions we obtain large changes in the results. We are going to illustrate this idea with the help of the logistic equation shown in (4).

$$x_{n+1} = rx_n(1 - x_n) \qquad (4)$$

This equation represents the growth of a population dynamically over time, note that it is a recurring equation.

The behavior of this equation depends on the parameter r. It has been studied that:

For $r < 1$ it is fulfilled $x_n \to 0$ when $n \to \infty$,
For $1 < r < 3$ it is fulfilled $x_n \to a \neq 0$ when $n \to \infty$, see Figure 10.
For $r > 3$ the graph oscillates between two values (Figure 11), four values (Figure 12), and so on until reaching a chaotic state (Figure 13).

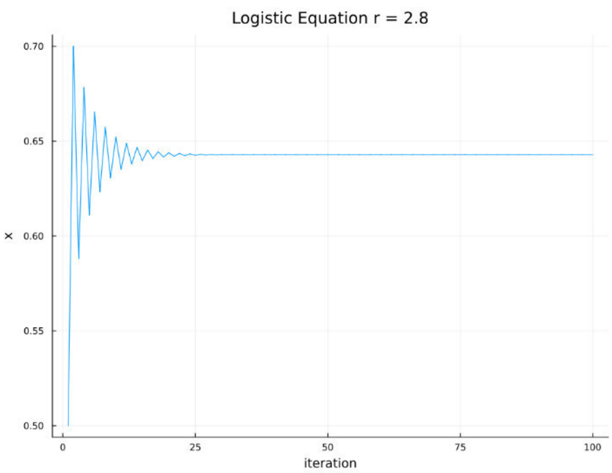

Figure 10: Convergence of the logistic equation for $r = 2.8$. See [16, 17].

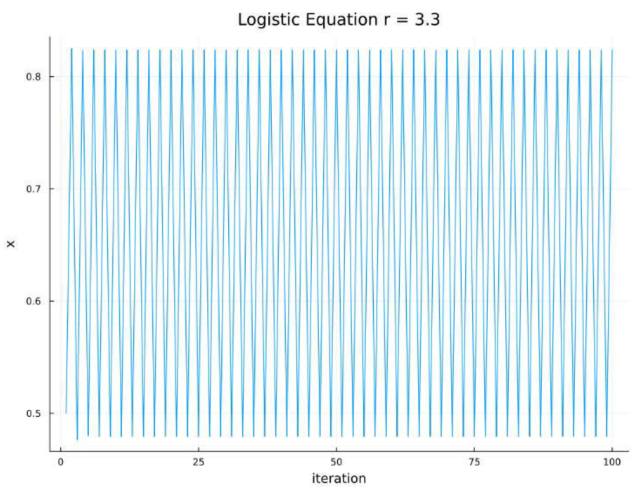

Figure 11: Convergence of the logistic equation for $r = 3.3$. See [16, 17].

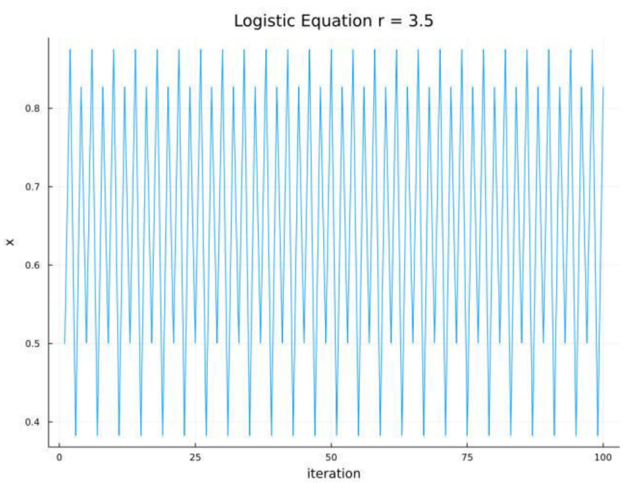

Figure 12: Convergence of the logistic equation for $r = 3.5$. See [16, 17].

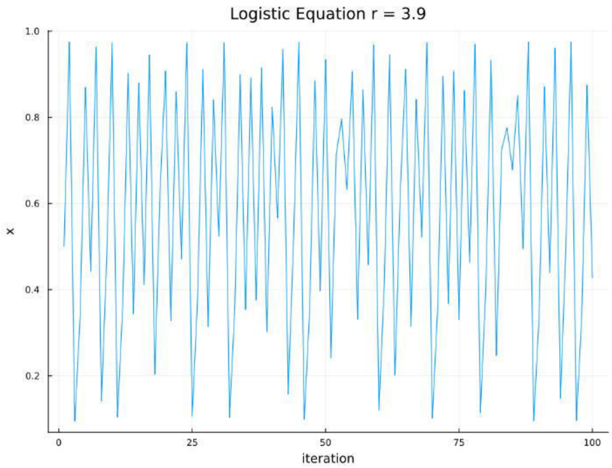

Figure 13: Convergence of the logistic equation for $r = 3.9$. See [16, 17].

To represent the Logistic Map, each value of the parameter r is plotted against the convergence points of Equation 4 for this parameter. This is represented in Figure 14.

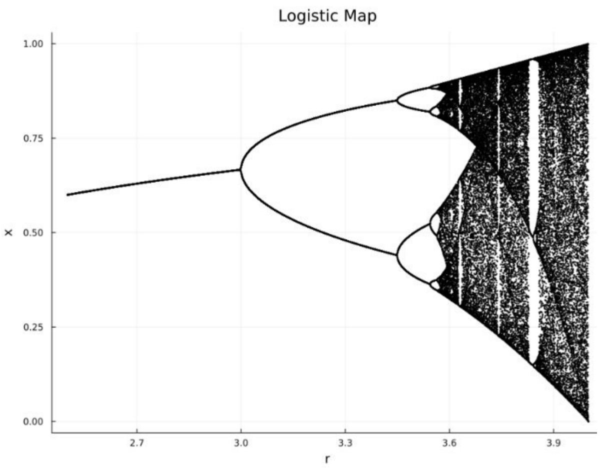

Figure 14: Logistic Map. Plot of parameter vs. point(s) of convergence. See [16, 17].

Figure 14 represents the Logistics Map. It is not difficult to notice that it presents some characteristics of approximate self-similarity. This map requires multifractal dimension theory to calculate its complexity.

We can also notice that for $r = 2.8$ its result is completely predictable, however, for some value $r > 3$ the value begins to bifurcate, obtaining indeterminate prediction results. The larger the r value becomes, the greater the indeterminacy and unpredictability of the system, see Figures 10-12 and Figure 14. This is manifested to a greater degree for certain values close to $r = 4$ which is where the system becomes chaotic, Figures 13 and 14. This is a classic example of the relationship between fractal geometry and the behavior of nonlinear dynamical systems.

This could also be an example where Neutrosophy that studies indeterminacy corresponds totally with the idea of fractal geometry and nonlinear dynamic systems and chaos [20].

This last idea is reinforced by some studies where the fractal dimension is associated with entropy. Entropy is the measure of order of the system [19]. In some phenomena such as the spatial growth of cities or the agglomeration of cities, a positive correlation has been found between the fractal dimension and entropy. This means that in these cases the complexity of urban spatial growth can be measured both with the help of entropy and with the fractal dimension. We can also intuitively link these concepts when it becomes evident that indeterminacy, which is an important component of Neutrosophy, is typical of complex systems. A complex system is understood to be one where very different results are obtained for small variations in the initial conditions. So we can appreciate the relationship between NeutroGeometry[21] and Fractal Geometry[22] when the indeterminacy is intrinsic to the system or what is the same when it is an ontological indetermination [23, 24].

Conclusion

NeutroGeometry is a logical approach to geometric systems based on neutrosophic logic. A NeutroGeometry contains at least one NeutroConcept and never an AntiConcept. However, this idea originally intended for classical geometric systems, such as Euclidean geometry, classical non-Euclidean geometries, or mixed Smarandache geometries, did not take into account other less orthodox geometries such as fractal geometry. Fractal geometry does not respond to any specific axiom, although it is composed of objects called fractals that are immersed in a Euclidean line, plane, space, or hyperspace and are composed of points or segments of points. The objective of this article was to reflect and discuss the relationship that may exist between the ideas of NeutroGeometry, Neutrosophy, and fractal geometry or the phenomena in which it is applied such as nonlinear or chaotic systems. On the one hand, self-similarity properties are NeutroProperties in some cases, when it is approximate or statistical. On the other hand, if it is considered that there is an "intrinsic" fractal dimension of an object, this can only be determined accurately when it is understood as a number within an indeterminate interval since there are several definitions of fractal dimension that do not always coincide each other.

Perhaps the most interesting relation is in the application of fractal geometry to model nonlinear dynamic systems or chaotic systems. These systems can be indeterminate or unpredictable, which falls within the field of study of Neutrosophy. In turn, the maps of some of these systems, such as the logistic map, present self-similarity properties typical of fractal objects. Although this self-similarity may not be exact.

References

[1] Mao, L. (2006). Combinatorial Speculations and the Combinatorial Conjecture for Mathematics. In Smarandache Geometries & Map Theory with Applications (I) (pp. 1-23). Chinese Branch Xiquan House.
[2] Smarandache, F. (2021). NeutroGeometry & AntiGeometry are alternatives and generalizations of the Non-Euclidean Geometries. Neutrosophic Sets and Systems, 46, 456-476.
[3] Singh, P. K. (2021). Anti-geometry and neutrogeometry characterization of non-Euclidean data. Journal of Neutrosophic and Fuzzy Systems, 1, 24-33.
[4] Smarandache, F. (2023). Real Examples of NeutroGeometry & AntiGeometry. Neutrosophic Sets and Systems, 55, 568-575.
[5] Pătrașcu, I. (2023). Smarandache Geometries (or Hybrids). Craiova: Infinite Study.
[6] Iseri, H. (2002). Smarandache manifolds. Craiova: Infinite Study.
[7] Bhattacharya, S. (2004). A model of the Smarandache geometries. Journal of Recreational Mathematics, 33, 166.
[8] Granados, C. (2023). A Review of AntiGeometry and NeutroGeometry and Their Application to Real Life. NeutroGeometry, NeutroAlgebra, and SuperHyperAlgebra in Today's World, 1-16.
[9] González-Caballero, E. (2023). Introduction to the Finite NeutroGeometries: The Mixed Projective-Affine Geometry. In NeutroGeometry, NeutroAlgebra, and SuperHyperAlgebra in Today's World (pp. 52-80). IGI Global.
[10] González-Caballero, E. (2023). “NeutroGeometry Laboratory”: The New Software Dedicated to Finite NeutroGeometries. In NeutroGeometry, NeutroAlgebra, and SuperHyperAlgebra in Today's World (pp. 131-155). IGI Global.
[11] González-Caballero, E. (2023). A Constructive Introduction to Finite Mixed Projective-Affine-Hyperbolic Planes. In NeutroGeometry, NeutroAlgebra, and SuperHyperAlgebra in Today's World (pp. 156-186). IGI Global.
[12] Gonzalez-Caballero, E. (2023). Applications of NeutroGeometry and AntiGeometry in Real World. International Journal of Neutrosophic Science, 21, 14-33.
[13] Gonzalez-Caballero, E. (2024). A Brief Journey from Euclidian to Smarandachean Geometry. London: LAP LAMBERT Academic Publishing. ISBN: 978-620-7-65460-4.
[14] González-Caballero, E. and Smarandache, F. (2024). Theory of Distances in NeutroGeometry. Neutrosophic Sets and Systems, 67, 179-189.
[15] Falconer, K. (2014). Fractal geometry: mathematical foundations and applications. John Wiley & Sons.
[16] Talanquer, V. (1996). Fractus, fracta, fractal. Fractales de Laberintos a Espejos (In Spanish). Mexico D.F.: FCE-Fondo de Cultura Económica.
[17] Strogatz, S.H. (1994) Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering. Cambridge: Westview Press.
[18] Husain, A., Gumma, S., Sajid, M., Reddy, J., and Alresheedi, M. T. (2024). A novel fractal interpolation function algorithm for fractal dimension estimation and coastline geometry reconstruction: a case study of the coastline of Kingdom of Saudi Arabia. The European Physical Journal B: Condensed Matter and Complex Systems, 97, 1-16.
[19] Chen, Y., Wang, J. and Feng, J. (2017). Understanding the Fractal Dimensions of Urban Forms through Spatial Entropy. Entropy, 19, 600.
[20] Al-Kaff, M. O., El-Metwally, H. A., Elsadany, A. E. A., and Elabbasy, E. M. (2024). Exploring chaos and bifurcation in a discrete prey–predator based on coupled logistic map. Scientific Reports, 14, 16118.
[21] Smarandache, F. (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas. Neutrosophic Computing and Machine Learning, 20, 91-104
[22] Sendker, F. L., Lo, Y. K., Heimerl, T., Bohn, S., Persson, L. J., Mais, C. N., ... & Hochberg, G. K. (2024). Emergence of fractal geometries in the evolution of a metabolic enzyme. Nature, 1-7.

[23] Nabeeh, N. (2023). Assessment and contrast the sustainable growth of various road transport systems using intelligent neutrosophic multi-criteria decision-making model. Sustainable Machine Intelligence Journal, 2, 2-1.
[24] Vázquez, M. L., Ricardo, J. E., & Smarandache, F. (2024). Enhancing Set-Theoretic Research Methods with Neutrosophic Sets. HyperSoft Set Methods in Engineering, 2, 96-103.

Received: June 15, 2024. Accepted: August 5, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of M-Learning as a Quechua Learning Instrument

Javier Gamboa-Cruzado[1], María Lima Bendezú[2], Mauro Cruzado Viera[3], Angel Núñez Meza[4] and Edgar Damián Núñez[5]

[1] National University of San Marcos, Lima. Peru. E-mail: jgamboa65@hotmail.com
[2] Micaela Bastidas National University of Apurímac, Apurímac, Peru. E-mail: mlima@unamba.edu.pe
[3] National University of San Marcos, Lima. Peru. E-mail: mcruzadov@unmsm.edu.pe
[4] Daniel Alcides Carrión National University, Pasco. Peru. E-mail: anunezm@undac.edu.pe
[5] National University of San Marcos, Lima. Peru. E-mail: edamiann@unmsm.edu.pe

Abstract. This study focuses on assessing the efficacy of m-learning as a tool for learning Quechua. It uses a unique method of sentiment neutrosophic sentiment analysis, which is based on neutroAlgebra. In the current scenario, where the use of mobile platforms for learning indigenous languages is gaining significance, there is a noticeable absence of methods that can effectively handle the intricate and uncertain nature of users' perceptions towards these systems. Existing literature provides limited solutions that effectively integrate mathematical precision with the ability to recognize and respond to emotions and ambiguous opinions. This article addresses the lack of methodology by utilizing neu-troAlgebra to process and analyze sentiment data. This approach enables a more comprehensive and profound comprehension of the user experience. The study's findings indicate that utilizing neutroAlgebra for neutrosophic sentiment analysis is a highly efficient method for comprehending the intricate and uncertain nature of users' perceptions regarding Quechua m-learning. The findings emphasize that this methodology not only enables a more accurate and thorough evaluation of the tool, but also provides valuable insights to enhance the design and implementation of educational strategies in indigenous languages. The study makes two main contributions. Firstly, it presents a novel theoretical and methodological framework for evaluating m-learning in situations with a high level of uncertainty. Secondly, it proposes practical applications that have the potential to greatly enhance the teaching and learning of Quechua and other minority languages in digital settings and the implemetation in Orange data mining tool.

Keywords: M-Learning, Mobile Platforms, Neutroalgebra, Prospector, Sentiment Analysis, of Indigenous Languages.

1 Introduction

M-learning, also known as mobile learning, has significantly transformed the process of imparting and acquiring knowledge in the 21st century, particularly in educational settings with restricted access to physical resources [1]. Teaching indigenous languages, like Quechua, encounters distinctive obstacles within this context. These challenges arise from the necessity to safeguard the language's integrity while also accommodating advancements in technology [2]. This study aims to assess the efficacy of m-learning as a Quechua teaching and learning tool, employing a novel neutrosophic sentiment analysis approach grounded in neutroalgebra. The significance of this research resides in its capacity to offer a methodology that addresses the intricacy of emotional data and the uncertainty inherent in users' perspectives, thereby providing a more precise and comprehensive assessment.

The preservation and instruction of indigenous languages have historically posed a substantial obstacle, primarily due to limited resources, the dwindling number of native speakers, and the inadequate incorporation of these languages into contemporary educational systems [3]. Nevertheless, due to the proliferation of mobile technology and the growing availability of digital devices, m-learning has emerged as a feasible solution to tackle these difficulties. Although m-learning has been widely used in various contexts, its potential in the teaching of indigenous languages like Quechua has not been fully investigated. The need to assess the efficacy of educational technology in specific linguistic contexts is highlighted by recent advancements in this field and the increasing focus on preserving endangered languages [4]. The main issue that this study aims to solve is the absence of a strong and scientifically rigorous method for assessing the effectiveness of m-learning in the instruction of Quechua, taking into account the intricacies and uncertainties associated with users' perceptions. While there are studies examining the utilization of mobile technologies in education, the majority of them fail to sufficiently consider the diversity of users' emotions and opinions, as well as the uncertainty that arises from interacting with technology [5]. This literature gap prompts the research question: How can a neutrosophic sentiment analysis, utilizing neutroAlgebra [6], be employed to accurately assess the effectiveness of m-learning as a Quechua teaching-learning tool?

Hence, the objective of this study is two-fold. The primary objective is to create and implement a neutrosophic sentiment analysis technique utilizing neutroAlgebra[6]. This method is specifically designed to accurately capture the intricate and uncertain nature of users' perceptions regarding Quechua m-learning. Additionally, it seeks to assess the efficacy of this educational instrument in terms of its capacity to instruct and safeguard the Quechua language, thereby offering a pioneering method that can be duplicated in the instruction of other native languages. The objectives outlined in this article align with the research question and will be thoroughly examined. Their exploration will contribute to the advancement of theory and educational practice in the field of Indigenous languages.

2 Preliminaries

2.1 M-Learning Or Mobile Learning.

M-learning, also known as mobile learning, has become a highly innovative tool in the 21st century education field, providing unparalleled flexibility in accessing knowledge. In a society where technology has infiltrated all facets of everyday existence, m-learning holds the potential to revolutionize not just the timing and location of our learning, but also the manner in which it occurs. Nevertheless, this phenomenon is not devoid of challenges and criticism, which gives rise to a crucial and intricate discourse regarding its actual influence on education. Is m-learning a passing trend or a fundamental shift in education? This question is currently being discussed, and any attempt to answer it must go beyond shallow compliments and explore the profound and complex consequences it entails. M-learning allows for equal access to knowledge by removing geographical and temporal constraints that have historically restricted education [8]. It is presented as an inclusive tool that is particularly advantageous for individuals residing in rural areas or facing barriers that prevent them from accessing traditional educational institutions. Moreover, the capacity to customize learning based on individual speed and requirements is one of the most significant benefits of m-learning. Nevertheless, the process of democratization can be deceptive due to the lack of universal access to technology and the persistent existence of digital disparities, which restrict the availability of this form of communication in numerous global regions. The issue of unequal access to mobile devices and connectivity persists as a significant barrier that must not be overlooked.

M-learning, from a pedagogical perspective, brings forth novel dynamics that have the potential to enhance the educational experience, but also present notable obstacles. M-learning has successfully enhanced interactivity, gamification, and collaborative learning. Nevertheless, the potential for shallowness in education cannot be disregarded when emphasizing the utilization of appealing applications and platforms that lack scholarly rigor. Convenient access to information can result in fragmented learning, where the depth of comprehension is compromised in favor of acquiring a large quantity of knowledge quickly. However, it is crucial to take into account the cognitive effects that mobile learning (m-learning) can have on students. Prolonged exposure to mobile devices and the necessity to allocate attention to multiple tasks can impact concentration and the efficacy of learning [9].

M-learning provides substantial flexibility by allowing learning to take place at any time and in any location. However, the absence of a structured framework can pose difficulties in cultivating efficient study habits and time management skills. This highlights the significance of self-discipline and self-regulation for students, while simultaneously redefining the role of teachers from providers of knowledge to facilitators of learning. Nevertheless, this transition gives rise to apprehensions regarding the preservation of educational excellence in settings with limited in-person interaction, emphasizing the necessity for ongoing teacher education and adjustment to emerging technologies. It is imperative for institutions to allocate resources and provide assistance for this transition, as not all educators are adequately equipped to handle these changes [10].

The influence of mobile learning goes beyond just students and teachers, as it has the potential to reshape traditional educational models and raise concerns about sustainability and fairness within the broader educational system. Integrating mobile technologies into curricula for education must be done carefully to ensure that educational goals are not compromised. Moreover, the growing reliance on technology may impede the acquisition of essential abilities such as analytical reasoning and problem-solving. M-learning should serve as a supplement to, rather than a substitute for, traditional education, guaranteeing that students acquire a comprehensive range of skills. Consistently assessing and adopting a critical mindset towards the utilization of educational technologies are crucial in order to maximize the advantages of mobile learning, while simultaneously tackling its obstacles and guaranteeing equal opportunities for all students[11].

2.2 Neutroalgebra generated by combining function in Prospector

For a given natural number $n > 0$, NeutroGroup is defined from the Prospector combinator function. Prospector is the well-known expert system used to model mining problems [12]. The NeutroGroup set consists of all integers between $-n$ and n plus the symbolic element I to represent indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$and $\oplus_5$is used. This is defined according to the following Cayley table:

Table 1. Cayley table corresponding to $\oplus_5$. Source: [13].

$\oplus_5$	-5	-4	-3	-2	-1	0	Io	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	I
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
I	-5	-4	-3	-2	-1	I	I	I	I	I	I	I
1	-5	-4	-2	-1	0	1	I	2	3	4	4	5
2	-5	-3	-1	0	1	2	I	3	3	4	5	5
3	-5	-2	0	1	2	3	I	4	4	4	5	5
4	-5	0	2	3	4	4	I	4	5	5	5	5
5	Io	5	5	5	5	5	Io	5	5	5	5	5

$\oplus_5$ satisfies the properties of commutativity and associativity and has 0 as a null element. Furthermore, it satisfies each each of the following properties [13]:

- If $x, y < 0$t hen $x \oplus_5 y \leq min(x, y)$,
- If $x, y > 0$ then $x \oplus_5 y \geq max(x, y)$,
- If $x < 0$ and $y > 0$or if $x > 0$and $y < 0$, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

3 Materials and Methods

The following steps outline the process for gathering and processing opinions:

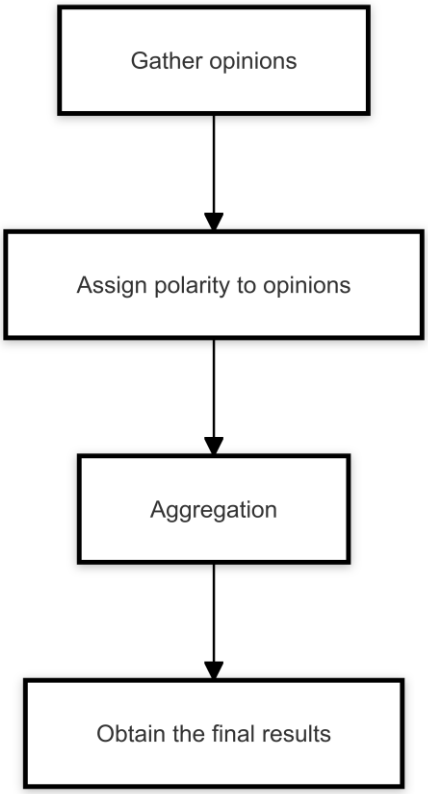

Figure 1. Flowchart of the sentiment analysis method based on neutroAlgebra

Step 1 Gather opinions: Gather opinions from a group of experts $P = \{p_1, p_2, \dots, p_l\}$, in short, informal texts about aspects/variables to evaluate $V = \{v_1, v_2, \dots, v_n\}$.

Step 2 Assign polarity to opinions: Each variable (i) is associated with polarity $v_{ij} \to x_{ij}$, based on a modification on the sentiment analysis algorithm to give values on a discrete scale from -5 to 5. "I" is assigned to indicate the text's unintelligibility

Step 3 Aggregation: For each person and each variable aggregate

$$x_{total,i} = x_{i,1} \oplus_5 x_{i,2} \oplus_5 x_{i,3}, \dots, \oplus_5 x_{i,n} \quad (1)$$

Step 4 **Obtain the final results**:**.** To obtain the final results, we have a group of people whose opinion is studied. Let's call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i}$ it is the total value of the ith variable,

for the i^{th} variable we obtain the final result That is, the arithmetic mean of each of the variables is calculated.

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} \quad (2)$$

Where l is the number of experts

In this process, the examination of opinions and perceptions is conducted using neutrosophic theory [14, 15, 16], which takes into account the degrees of positivity, negativity, and indeterminacy. This method not only captures explicit emotions, such as positive and negative ones, but also takes into account those that are neutral or unclear, thereby achieving a more precise evaluation and enhanced comprehension of how these aspects are perceived in the organizational setting [17, 18].

This technique, which is highly efficient for analyzing brief and casual texts, as explained in the aforementioned method, necessitates the identification of a group of words that are categorized as positive, negative, or neutral. Each word is assigned a strength value ranging from -5 to 5, or labeled as indeterminate. Indeterminacy arises when it is impossible to interpret the person's thoughts regarding the subject at hand, either due to unclear language or incomprehensibility. Moreover, there are instances where a text may contain both highly positive (+5) and highly negative (-5) evaluations for the same variable, resulting in a contradictory and indeterminate classification marked with the letter I [19]. The indeterminacy in this situation can arise from various sources, as demonstrated by the PROSPECTOR expert system [20]. This system assesses the level of evidence provided by an expert on a specific aspect, and it reveals that maximum evidence is found in opposite directions for two different aspects.

This method facilitates the classification of terms associated with the analyzed variables into three categories: Positive, Negative, or Neutral, based on linguistic values. Each term is assigned a value ranging from -5 to 5, or even I, based on the magnitude of its positive or negative charge.

The implementation was developed using the Orange platform, utilizing the "Multilingual Sentiment" [21] widget for lexicon-based sentiment analysis. The processed texts were classified on a discrete scale from -5 to 5, where -5 indicates extremely negative sentiment and 5 indicates extremely positive sentiment. (Figure 2). A Python script was employed to map continuous sentiment values to the desired scale

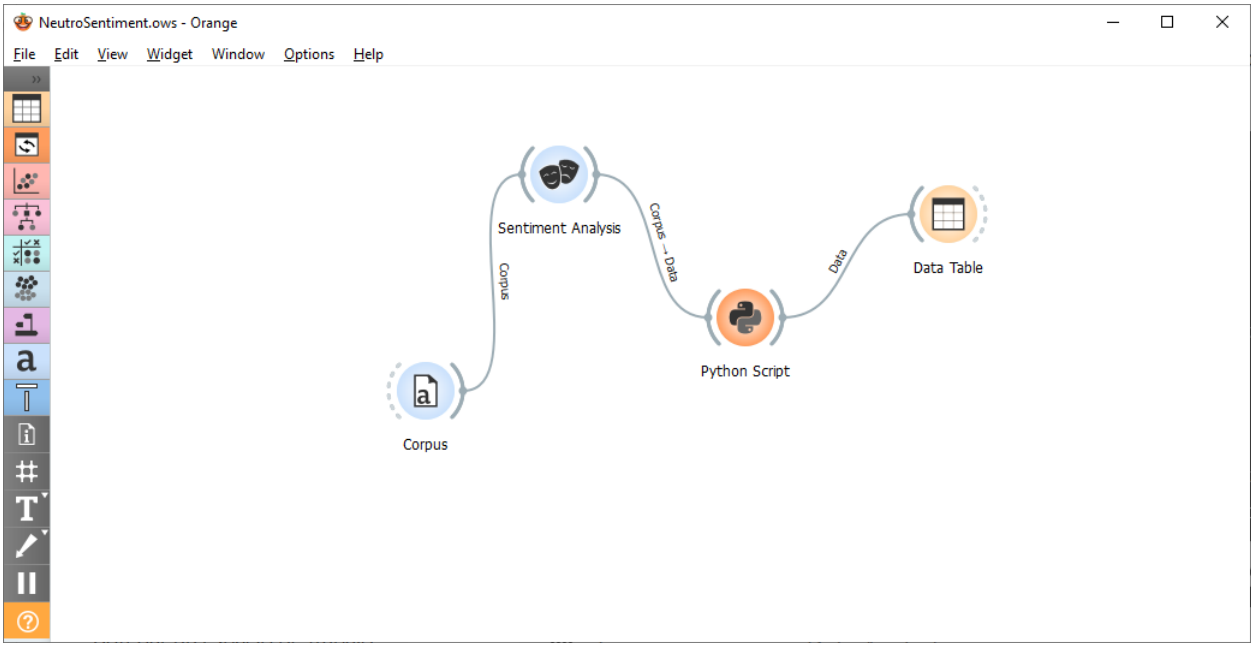

Figure 2. Sentiment Analysis Workflow in Orange.

Additionally, logic was implemented to detect unintelligible texts, defined as those with evident syntactic or semantic issues. In such cases, a special value "I" was assigned to indicate the text's unintelligibility[22]. This approach effectively classified texts, integrating standard sentiment analysis with the handling of exceptional cases within a single platform.

1. Case Study.

To evaluate the effectiveness of m-learning as a tool for teaching and learning Quechua, an approach based on sentiment analysis using neutrosophy is used. The methodology used considers different aspects related to the perception of educators and linguistic anthropologists about m-learning. Below are the steps and calculations carried out in this case.

Step 1: Definition of Variables and Aspects to Evaluate

The aspects to evaluate to measure the effectiveness of m-learning in teaching Quechua are the following:

1. **Accessibility**: Ease of accessing learning materials.
2. **Interactivity**: Degree to which m- learning facilitates the interaction between the content and the user.
3. **Relevance of Content**: Relevance of educational content for learning Quechua.
4. **Ease of Use**: Simplicity of the interface and ease of navigation.
5. **Motivation**: Capacity of m-learning to maintain the interest and motivation of the student.
6. **Learning Efficiency**: Speed with which students acquire new knowledge.
7. **Technical Support**: Quality of support available to resolve technical problems.
8. **Personalization**: Possibility of adapting the content to the individual needs of the students.
9. **Quality of Resources**: Quality and accuracy of the teaching resources available.
10. **Content Update**: Frequency with which materials are updated.
11. **Social Interaction**: Opportunities to interact with other students and educators.
12. **Feedback**: Availability and usefulness of the feedback provided.

Step 2: Data Collection

An open-ended questionnaire was carried out among 15 indigenous language educators and linguistic anthropologists, who evaluated each aspect of m-learning with comments expressed anonymously to ensure sincerity

Answers are processed using Orange pipeline shown in Figure 1. A scale from -5 to 5, where -5 indicates a very negative evaluation, 0 a neutral evaluation, and 5 a very positive. The fragments of survey results are shown in Table 1.

Table 2: Selected Evaluations of M-Learning Aspects by 15 Educators and Linguistic Anthropologists

Aspect	Educator 1	Educator 2	Educator 3	...	Educator 15
Accessibility	4	3	5	...	2
Interactivity	3	2	4	...	3
Content Relevance	5	4	5	...	4
Ease of Use	3	3	2	...	4
Motivation	4	4	5	...	3
Learning Efficiency	3	2	4	...	2

Aspect	Educator 1	Educator 2	Educator 3	...	Educator 15
Technical Support	-1	I	3	...	2
Personalization	4	3	4	...	5
Resource Quality	5	4	4	...	4
Content Update	3	0	3	...	3
Social Interaction	2	3	2	...	3
Feedback	4	3	5	...	4

Step 3: Calculation of Average Values and Evaluation of sentiments.

For each variable evaluated, the total value is calculated using the neutrosophic aggregation operation. This operation is performed by adding the values of each evaluation.

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \dots \oplus_5 x_{i15} \quad (3)$$

It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{15} x_{total,i,j}}{15} \quad (4)$$

the total number of evaluators is 15 in this case.

Table 3: Average Evaluation Values of Aspects of M-Learning.

Aspect	Average Value
Accessibility	3.33
Interactivity	2.67
Content Relevance	4.33
Ease of Use	3.00
Motivation	3.73
Learning Efficiency	2.80
Technical Support	2.53
Personalization	4.00

Aspect	Average Value
Resource Quality	4.27
Content Update	2.87
Social Interaction	2.73
Feedback	4.00

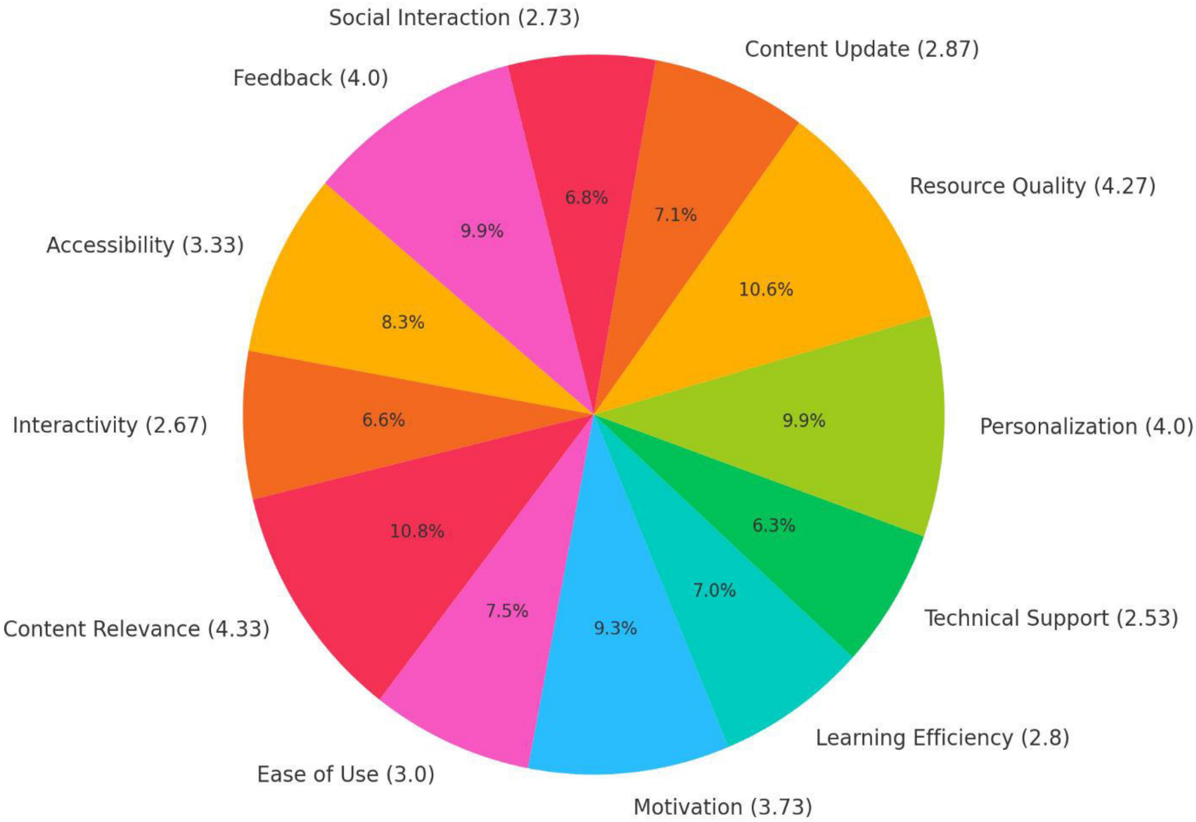

Figure 3. Average Evaluation Values of Aspects of M- Learning.

The average values obtained in the m-learning evaluation reflect diverse perceptions among linguistic educators and anthropologists. Aspects with higher scores, such as **Relevance of Content** and **Quality of Resources** , indicate a positive perception of the ability of m-learning to provide relevant and high-quality materials for teaching Quechua. On the other hand, aspects such as **Technical Support** and **Social Interaction** show lower scores, suggesting areas that could benefit from significant improvements. This analysis provides a detailed vision of the perception of m-learning as an educational tool for Quechua. The data reveals strengths in the relevance and quality of content while highlighting critical areas that require attention, especially in terms of technical support and social interaction opportunities. These findings can guide the future development of m-learning to maximize its effectiveness and better suit the needs of Quechua educators and students. The results obtained in this research highlight the perception of educators and linguistic anthropologists about m-learning as a tool for teaching Quechua. The average values obtained reflect a diverse range of opinions, with highest scores in aspects such as Relevance of Content (4.33) and Quality of Resources (4.27). In contrast, aspects such as Technical Support (2.53) and Social Interaction (2.73) obtain considerably lower evaluations.

The results indicate that m-learning is positively perceived in terms of the pertinence and excellence of educational content for acquiring Quechua language skills, consistent with prior studies that highlight the significance of content quality in m-learning platforms. Nevertheless, the study emphasizes the shortcomings in technical assistance and social engagement, which are essential for user contentment and the overall efficacy of mobile learning. When interpreting the results, it is important to take into account the study's limitations, such as the small sample size and the subjective nature of evaluating certain aspects. To improve the effectiveness of m-learning in Quechua teaching, it is crucial to address these shortcomings. Future research should focus on developing strategies to improve technical support and social interaction. Additionally, it is important to investigate the underlying causes of any anomalies found in the platform's implementation.

Conclusion

The study assessed the effectiveness of m-learning as a pedagogical tool for teaching Quechua, uncovering both advantages and disadvantages of this approach. The content's relevance and the quality of resources were positively perceived, with high average ratings in these aspects. Nevertheless, technical support and social interaction received negative evaluations, highlighting the need for substantial improvement in these areas. The significance of these findings lies in their ability to inform the creation of m-learning platforms for Quechua, specifically in improving technical assistance and fostering social engagement. This has the potential to enhance the overall efficiency and accessibility of m-learning for educators and students alike.

The study has made significant contributions to the field of digital education. It has laid the groundwork for future research and the creation of educational tools that are specifically designed for teaching indigenous languages. It emphasizes the significance of a thorough assessment that takes into account both the favorable and unfavorable aspects of m-learning. Nevertheless, it is important to recognize the limitations of the study, such as the small number of participants and the possibility of subjective assessment in certain areas. Future research should investigate alternative and complementary approaches to mobile learning (m-learning), increase the sample size to ensure a more representative analysis, and examine specific strategies to enhance technical support and social interaction on these platforms. Furthermore, it is worth considering the utilization of alternative sentiment analysis algorithms to enhance and validate the results.

References

[1] S. Iqbal and ZA Bhatti, "A qualitative exploration of teachers' perspective on smartphones usage in higher edu-cation in developing countries," Int. J. Educ. Technol. High. Educ. , vol. 17, no. 1, p. 29, 2020.

[2] C. Montellanos, J. Luis, M. Vasquez, C. Alberto, H. Salazar, and J. Luis, "Augmented reality mobile application and its influence in Quechua language learning," in 2019 IEEE Sciences and Humanities International Research Conference (SHIRCON), vol. 54, no. 8, pp. 1–4, 2019.

[3] J. V. Paragulla, L. Andrade-Arenas, and M. C. Caceres, "Virtual classrooms to improve the teaching-learning process in Systems Engineering courses at a University from Lima North," in 2020 IEEE World Conference on En-gineering Education (EDUNINE), pp. 1–4, 2020.

[4] M. Sundgren, "Blurring time and place in higher education with bring your own device applications: a literature review," Educ. Inf. Technol., vol. 22, no. 6, pp. 3081–3119, 2017.

[5] E. S. Wihidayat, Y. D. Utami, and A. Budianto, "Learn Arabic Language App, Mobile Based Application for Self-Directed Learning," in 2018 4th International Conference on Education and Technology (ICET), pp. 13–17, 2018.

[6] Smarandache, F. (2022). Introducción a la Super-Hiper-Álgebra y la Super-Hiper-Álgebra Neutrosófica. Neutrosophic Computing and Machine Learning, 20, 1-6. https://doi.org/10.5281/zenodo.6612313

[7] Qazi, A., Qazi, J., Naseer, K., Hasan, N., Hadaker, G., & Bao, D. (2024). M-Learning in Education during COVID-19: A Systematic Review of Sentiment, Challenges, and Opportunities. Heliyon.

[8] I. O. Yahuarcani, L. A. Saravia Llaja, A. M. Nunez Satalaya, E. A. Rojas Alva, K. D. Jeri Lagos, and L. Edith Jeri Lagos, "Effects of Using the WARMA Tool for Learning Quechua in Bilingual Educational Institutions in Huaman-ga City," in 2020 3rd International Conference of Inclusive Technology and Education (CONTIE), pp. 222–227, 2020.

[9] J. E. Zapata-Paulini, M. M. Soto-Cordova, and U. Lapa-Asto, "A Mobile Application with Augmented Reality for the Learning of the Quechua Language in Pre-School Children," in 2019 IEEE 39th Central America and Pana-ma Convention, CONCAPAN 2019, Nov. 2019.

[10] Qazi, A., Hasan, N., Owusu-Ansah, C. M., Hardaker, G., Dey, S. K., & Haruna, K. (2023). SentiTAM: Sentiments centered integrated framework for mobile learning adaptability in higher education. Heliyon, 9(1).

[11] Nadi, A. H., Hossain, S. F. A., Hasan, A. M., Sofin, M. R., Shabab, S., Sohan, M. A. I., & Yuan, C. (2023).

Discovering the role of M-Learning among finance students: the future of online education. In Novel financial applications of machine learning and deep learning: Algorithms, product modeling, and applications (pp. 183-196). Cham: Springer International Publishing.

[12] Caballero, E. G., Leyva, M., Ricardo, J. E., & Hernández, N. B. (2022). NeutroGroups Generated by Uninorms: A Theoretical Approach. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 155-179). IGI Global.

[13] Cruzaty, L. E. V., & Reynoso, A. M. R. (2024). New Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of Ethics in Companies. Neutrosophic Sets and Systems, 70, 264-270.

[14] A. El-Douh, A. (2023) "Assessment the Health Sustainability using Neutrosophic MCDM Methodology: Case Study COVID-19", Sustainable Machine Intelligence Journal, 3, pp. (1):1–

[15] Li, X., Luo, Y., Wang, H., Lin, J., & Deng, B. (2023). Doctor selection based on aspect-based sentiment analysis and neutrosophic TOPSIS method. Engineering Applications of Artificial Intelligence, 124, 106599.

[16] Mohamed, M. (2024) "Partnership of Lean Six Sigma and Digital Twin under Type 2 Neutrosophic Mystery Toward Virtual Manufacturing Environment: Real Scenario Application", Sustainable Machine Intelligence Journal, 8, pp. (6):99–109.

[17] Vázquez, M. Y. L., Cevallos, R. E. H., & Ricardo, J. E. (2021). Análisis de sentimientos: herramienta para estudiar datos cualitativos en la investigación jurídica. Universidad Y Sociedad, 13(S3), 262-266.

[18] Figueroa Cruz, M., Borja Salinas, E., Plúas Salazar, R. M., Castro Castillo, G. J., & Tovar Arcos, G. R. (2021). Studying health and inclusive education: Sentiment analysis using neutrosophy as a research tool. Neutrosophic Sets and Systems, 42, 324-332.

[19] Leyva Vázquez, M., & Smarandache, F. (2021). Resolución de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica. Neutrosophic Computing and Machine Learning, 15, 41-46.

[20] Duda, R., Gaschnig, J., & Hart, P. (1981). Model design in the PROSPECTOR consultant system for mineral exploration. In Readings in Artificial Intelligence (pp. 334-348). Morgan Kaufmann.

[21] Machado Júnior, C., Mantovani, D. M. N., de Sandes-Guimarães, L. V., Romeiro, M. D. C., Furlaneto, C. J., & Bazanini, R. (2023). Volatility of the COVID-19 vaccine hesitancy: sentiment analysis conducted in Brazil. Frontiers in Public Health, 11, 1192155.

[22] Vázquez, M. L., Franco, P. E. D. P., & Palacio, A. J. P. (2022). Neutrosophic dematel in the analysis of the causal factors of youth violence. International Journal of Neutrosophic Science, 18(3), 199-99.

Received: June 16, 2024. Accepted: August 6, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Factors Affecting Educational Quality: A Study Using Neutrosophic Likert Scales and Fuzzy Set Qualitative Comparative Analysis

Carlos Jacinto La Rosa Longobardi[1], Livia Cristina Piñas Rivera[2], Lida Violeta Asencios Trujillo[3], Djamila Gallegos Espinoza[4], Juan Víctor Soras Valdivia[5], Felipe Aguirre Chávez[6], and Elizabeth Gladys Garro Palomino[7]

[1] Enrique Guzmán y Valle National University of Education, Lima. Peru. E-mail: clarosa72@hotmail.com
[2] Enrique Guzmán y Valle National University of Education, Lima. Peru. E-mail: lpinas@une.edu.pe
[3] Enrique Guzmán y Valle National University of Education, Lima. Peru. E-mail: lasencios@une.edu.pe
[4] Enrique Guzmán y Valle National University of Education, Lima. Peru. E-mail: dgallegos@une.edu.pe
[5] Enrique Guzmán y Valle National University of Education, Lima. Peru. E-mail: jsoras@une.edu.pe
[6] Marcellin Champagnat University, Lima. Peru. E-mail: caguirrech@unmsm.edu.pe
[7] Enrique Guzmán y Valle National University of Education, Lima. Peru.

Abstract. This study investigates the educational quality of graduate programs at public universities by employing neutrosophic set theory and fuzzy-set Qualitative Comparative Analysis (fsQCA). The analysis centers on three crucial factors: Teacher Training and Education (TTE), Educational and Technological Resources (ETR), and School Environment and Academic Climate (SEA). The results demonstrate that the combination of ETR (Educational Technology Readiness) and SEA (Student Engagement and Achievement) is the most reliable indicator of educational quality, with the highest overall metric of 0.826292. Although the ETR alone demonstrates substantial coverage, highlighting its wide-ranging significance, the study emphasizes the importance of considering both individual factors and their combinations in comprehending educational outcomes. The research offers crucial insights for enhancing educational quality by implementing focused strategies in resource allocation and improving the environment. Subsequent research should investigate these discoveries in various educational environments and consider supplementary variables such as socio-economic impacts and teaching methods. Another area of future research is to enhance the support of set-theoretic research methods based on neutrosophic sets.

Keywords: Educational Quality, Neutrosophic Likert Scales, Fuzzy Set Qualitative Comparative Analysis, fsQCA.

1 Introduction

Educational quality in graduate programs in public universities is a crucial issue that directly affects the professional and academic development of students and, therefore, the general quality of the educational system [1]. The research focuses on analyzing this quality using methods based on neutrosophic set theory, an innovative approach that allows us to capture the complexity and ambiguity inherent to the educational experience. The importance of the study lies in its potential to improve understanding of the multiple and often subjective dimensions of educational quality, offering a more complete and accurate perspective compared to traditional methods [2]. Historically, the evaluation of educational quality has evolved from approaches based on simple metrics to more sophisticated methods that consider qualitative and quantitative aspects. However, despite advances, many studies still do not adequately address the complexity of students' experiences in the context of graduate programs [3,4]. Neutrosophic set theory, which integrates indeterminacy and uncertainty into analysis, represents a significant advance in this field. This theory has proven to be useful in other research domains, but its application in the analysis of educational quality in graduate programs remains an emerging and underexplored area [5, 6].

Educational quality is a complex concept that involves not only measuring student learning but also evaluating the efficiency of educational processes and ensuring equity in access to education. Traditional methods of assessment, such as standardized tests, often fall short in capturing the richness of educational experiences, as they reduce quality to mere figures and percentages. This has led to a call for more holistic and multifactorial approaches that consider the broader socioeconomic context, curricular structure, and pedagogical practices that influence education. In this context, the application of neutrosophic set theory emerges as a promising alternative, offering a more adaptable evaluation method that can handle the uncertainty, indeterminacy, and inconsistency present in educational environments [7,8,9].

The neutrosophic approach offers a nuanced and inclusive understanding of educational quality by incorporating diverse perspectives and often-overlooked variables. It emphasizes the significance of both academic outcomes and the contexts in which they occur, such as motivation, the school environment, and pedagogical methods. Fuzzy Set Qualitative Comparative Analysis (fsQCA) enhances this by identifying combinations of factors that lead to varying levels of educational quality [10]. This study integrates neutrosophic Likert scales [11] with fsQCA to analyze the complex causality between factors and educational quality, a method recently proposed for more comprehensive evaluation.

2 Preliminaries

2.1 fsQCA and neutrosophic sets.

The relationships between variables are not always simple and often manifest through complex and non-linear patterns, as postulated by complexity theory [12, 13, 14]. This implies that the same cause can trigger different effects depending on the context in which occurs. Three fundamental principles stand out in this theory, conjunction, equifinality and causal asymmetry [15]:

- The concept of conjunction is based on the collaboration between antecedent conditions that function collectively to produce a result, rather than operating individually to account for variation.
- Equifinality refers to the concept that a system can achieve the same result despite starting from various initial conditions and following different paths.
- Causal asymmetry, however, asserts that although certain conditions can cause a result to occur, the absence of those conditions does not necessarily ensure the absence of the result.

Fuzzy Set Qualitative Comparative Analysis (fsQCA) [16, 17] is a modeling technique that incorporates the principles of conjunction, equifinality, and causal asymmetry to capture the complexity of relationships between conditions and outcomes. It identifies the conjunction of antecedent conditions that collectively contribute to the production of an outcome, acknowledging that various initial conditions can lead to the same result. Furthermore, fsQCA demonstrates the notion of causal asymmetry by illustrating that although specific conditions may be required for a particular result, the absence of these conditions does not necessarily hinder the occurrence of that result. This highlights the complex and non-linear nature of causality in intricate systems.

Moreover, the concept of neutrosophy can enhance the comprehension of intricate causality by introducing the inherent qualities of indeterminacy and uncertainty in social phenomena [18, 19]. The application of neutrosophic set theory, which can effectively deal with indeterminacy, provides a more sophisticated perspective for comprehending these intricate relationships [20].

2.2 Neutrosophic Liker Scales

Surveys that utilize neutrosophic Likert scales [21] are efficient in assessing the range of opinions and their impact on public policy and social discourse. These surveys accurately capture areas of agreement, disagreement, and uncertainty.

Here, we provide the essential definitions and concepts pertaining to neutrosophic sets and single-valued neutrosophic sets.

Definition 1 ([22]). Let U be a universe of discourse. $N = \{(x, T(x), I(x), F(x)): x \in U\}$ is a neutrosophic set, denoted by a truth membership function, $TN : U \rightarrow]0^-, 1^+[$; an indeterminacy membership function, $IN : U \rightarrow]0^-, 1^+[$; and a falsity membership function, $FN : U \rightarrow]0^-, 1^+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the universe of discourse U

Definition 2 ([23]). Let U be a universe of discourse. A single-valued neutrosophic set is defined as $N = \{(x, T(x), I(x), F(x)): x \in U\}$, which is identified by a truth membership function, $TN : U \rightarrow [0, 1]$; indeterminacy membership function, $IN : U \rightarrow [0, 1]$; and falsity membership function, $FN : U \rightarrow [0, 1]$, with $0 \leq TN(x) + IN(x) + FN(x) \leq 3$

Using neutrosophic scales with single value neutrosophic sets, responses are classified according to the total of the True, Indeterminate and False components as follows:

- T+I+F=1: Complete
- T+I+F<1: Incomplete
- T+I+F>1: Contradictory

These values are derived from situations where opinions are often incomplete or contradictory. The categorization mentioned here is a notable benefit of employing neutrosophic methods, as it enables a more intricate comprehension of the varying levels of accuracy, uncertainty, and falsehood in responses [24].

For integrating neutrosophic Likert scales into the fsQCA framework is necessary to develop a fuzzification process. Given $AN = \{x, (TA(x), IA(x), FA(x)): x \in X\}$ a NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)): x \in X\}$,where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the similarity equation proposed in [25],

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

The range of the similarity measure function is in the unit interval [0,1], μA (x) ∈ [0,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [0, 1] and satisfies the property of a membership function of a fuzzy set [26].

3 Proposed framework

Each stage of the proposed framework is accompanied by a brief explanation, which makes it easier to understand the process in its entirety.

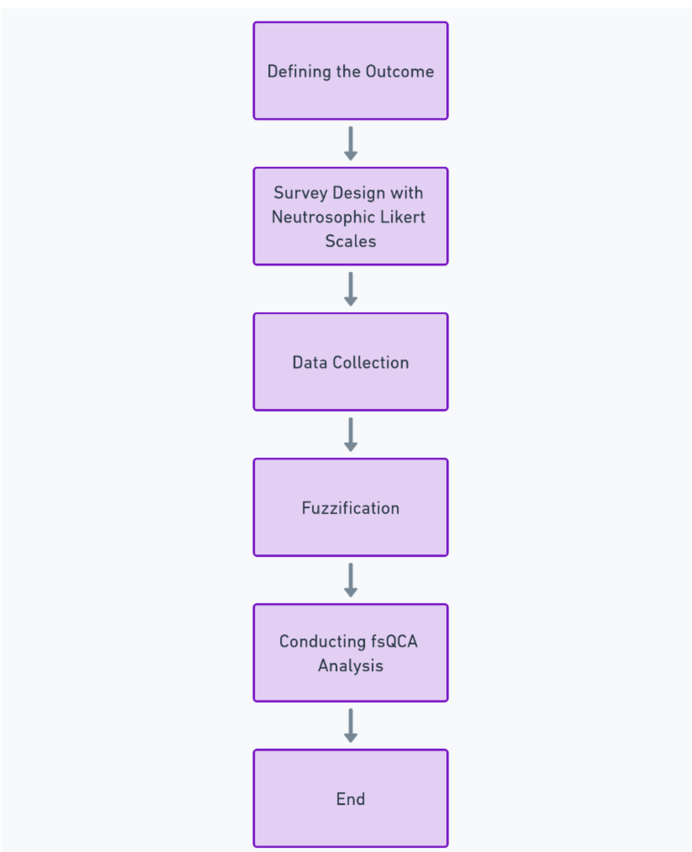

Figure 1. Process Flowchart for Analysis with Neutrosophic Likert Scales and fsQCA

Defining the outcome: Begin by identifying and accurately describing the specific phenomenon, event, or condition you want to investigate. This stage is crucial, as it establishes the focus and frame of reference for the entire analysis.

Survey Design with Neutrosophic Likert Scales: Next, design neutrosophic Likert scales that will be used to measure both the outcome and the associated variables. Unlike conventional Likert scales, which use a fixed scale (such as 1 to 5), neutrosophic scales introduce elements of truth, indeterminacy, and falsehood. Each option on the scale is represented by a triplet (T, I, F), where T indicates the degree of truth, I the degree of indeterminacy, and F the degree of falsehood. This approach allows for a more detailed interpretation of participants' responses and attitudes.

Data Collection: Data collection is conducted through surveys that utilize Neutrosophic Likert scales. These scales allow for the capture of more nuanced and complex opinions and attitudes of respondents, enhancing the richness of the data set. This approach ensures that the collected data is comprehensive and accurately reflects the variables under study, using various indicators or measures related to the defined outcomes. and attitudes.

Fuzzification: the neutrosophic sets obtained are transformed into equivalent fuzzy sets, following the procedure described in Equation 1.

Conducting fsQCA Analysis: Perform fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. The fsQCA program for Windows is used for data processing [18, 19].

The validity of the configuration is evaluated by measuring the consistency and coverage values. Consistency is the measure of the reliability with which the set of pathways produces the desired result. Coverage refers to the degree to which the result is clarified by this arrangement of pathways [20]:

$$Consistency\ (Y_i \leq X_i) = \frac{\sum \min (X_i, Y_i)}{\sum Y_i} \quad (2)$$

$$Coverage\ (Y_i \leq X_i) = \frac{\sum \min (X_i, Y_i)}{\sum X_i} \quad (3)$$

where:
$X_i$is the membership value of case i in the set of causal conditions.
$Y_i$is the membership value of case i in the result set.

Both metrics are used in comparative analysis to evaluate the relationships established between individual conditions, combinations of conditions, path configurations, and the final result. Generally, values greater than 0.8 are considered indicators of a strong relationship [20].

4 Results.

The defined result is the perception of the **Quality of education (QE)** . A Likert scale is developed, represented as single-valued neutrosophic sets. The study also considers the following variables:

Teacher training and education (TTE): The level of preparation and quality of continuous training that teachers receive are essential to guarantee high-quality education.

Educational and technological resources (ETR): The availability and access to teaching materials, educational technologies and learning tools directly impact the quality of the educational process.

School environment and academic climate (SEA): A safe, inclusive and stimulating school environment promotes effective learning and improves the quality of education.

A survey was conducted with a group of 12 university professors from Ecuador (see Table 1).

Table 1. Survey data

No	Teacher training and training	Educational and technological resources	School environment and academic climate	Quality of Education
1	(0.9, 0.8, 0.1)	(0.6, 1, 0.6)	(0.3, 0.7, 0.3)	(0.8, 0.6, 0.7)

No	Teacher training and training	Educational and technological resources	School environment and academic climate	Quality of Education
2	(0.6, 0.6,0.6)	(1,1,1)	(0.6,0.1,0.6)	(0.6, 0.6, 0.7)
3	(0.8, 0.7, 0,4)	(0.7, 0.9, 0.6)	(0.8, 0.6, 0.6)	(0.9, 0.5, 0.3)
4	(1,1,0)	(0.8,0.8,0)	(1,0.9,0.3)	(0.7, 1, 0.9)
5	(1,0.6,0)	(1,0.6,0.4)	(0.8,0.5,0.2)	(0,9, 0,6, 0,1)
6	(0.9, 0.8, 0.7)	(0.8, 0.6, 0.4)	(0.7, 0.6, 0.5)	(0.8, 0.6, 0.6)
7	(0.1, 0.6, 0.8)	(1,0,0)	(0.6, 0.6, 0.6)	(0.8, 0.6, 0.1)
8	(1, 0.9, 0.1)	(0.8, 0.7, 0.1)	(0.8, 0.8, 0.1)	(0,9, 0,9, 0,1)
9	(1,1,0)	(0.8, 0.7, 0.1)	(1,0,0)	(0.9, 0, 0)
10	(0.7, 1, 0.1)	(0.9, 0.4, 0)	(0.6, 0.9, 0.1)	(1,0,0)
11	(0.4, 0.7, 0.1)	(0.3, 0.9, 0.4)	(0.8, 0.4, 0.6)	(0.4, 0.8, 0.3)
12	(0.6, 1, 0.6)	(0.6, 0.6, 0.1)	(0.1, 0.6, 0.7)	(1,0,0.8)

The fuzzification process is developed using Equation 1. (Table 2)

Table 2. Fuzzy values

No	Teacher training and training	Educational and technological resources	School environment and academic climate	Quality of Education
1	0.55	0.30	0.30	0.55
2	0.50	0.50	0.50	0.45
3	0.55	0.40	0.60	0.70
4	0.50	0.50	0.55	0.35
5	0.70	0.70	0.65	0.65
6	0.55	0.60	0.55	0.60
7	0.15	1.00	0.50	0.60
8	0.55	0.55	0.50	0.50
9	0.50	0.55	1.00	0.95
10	0.35	0.75	0.35	1.00
11	0.35	0.20	0.60	0.30
12	0.30	0.50	0.20	0.60

A necessary condition analysis is performed to test consistency and coverage (Table 3).

Table 3. Analysis of necessary conditions

Tested conditions	Consistency	Coverage
Teacher training and education (TTE)	0.717241	0.936937
Educational and technological resources (ETR)	0.806897	0.893130
School environment and academic climate (SEA)	0.786207	0.904762

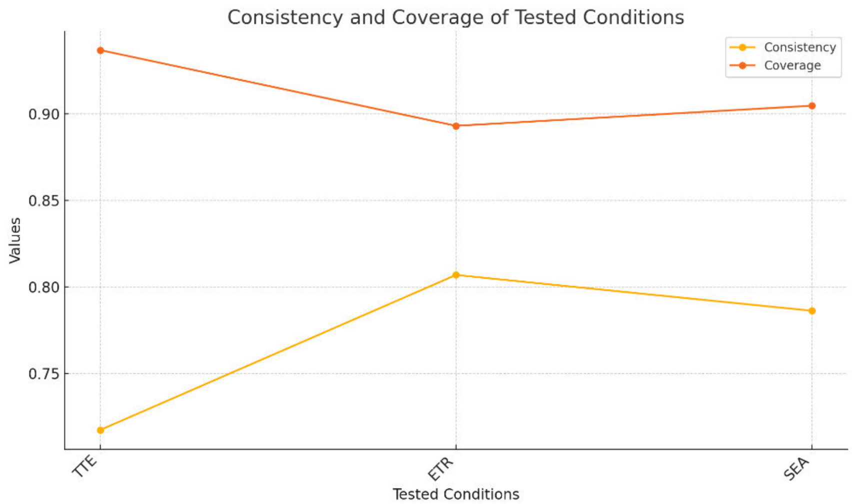

Figure 2. Analysis of necessary conditions (consistency and coverage)

The analysis of necessary conditions using consistency and coverage values for the three evaluated categories (Teacher training and development - FCA, Educational and technological resources - RET, and School environment and academic climate - TCA) reveals key insights within the context of fuzzy-set Qualitative Comparative Analysis (fsQCA). The highest consistency is observed for "Educational and technological resources (RET)," indicating it as the most consistently necessary condition. However, "Teacher training and development (FCA)" exhibits the highest coverage, suggesting it explains a significant proportion of the cases where the outcome occurs. "School environment and academic climate (TCA)" also plays an important role with high values in both consistency and coverage. Overall, while all conditions are crucial, RET stands out as the most necessary condition, and FCA is key in most cases where the outcome is achieved. These findings are essential for understanding the dynamics influencing the outcome and can inform educational policy and strategic planning.

The results of the superset analysis are shown in Table 4.

Table 4. Results of the subset/ superset analysis

Terms	Consistency	Coverage	Combined
TTE*ETR*SEA	0.958333	0.634483	0.792552
ETR*SEA	0.961538	0.689655	0.826292
TTE*ETR	0.940000	0.648276	0.797064
TTE*SEA	0.950980	0.668966	0.809683
ETR	0.893130	0.806897	0.875529
SEA	0.904762	0.786207	0.868768
TTE	0.936937	0.71724	0.838389

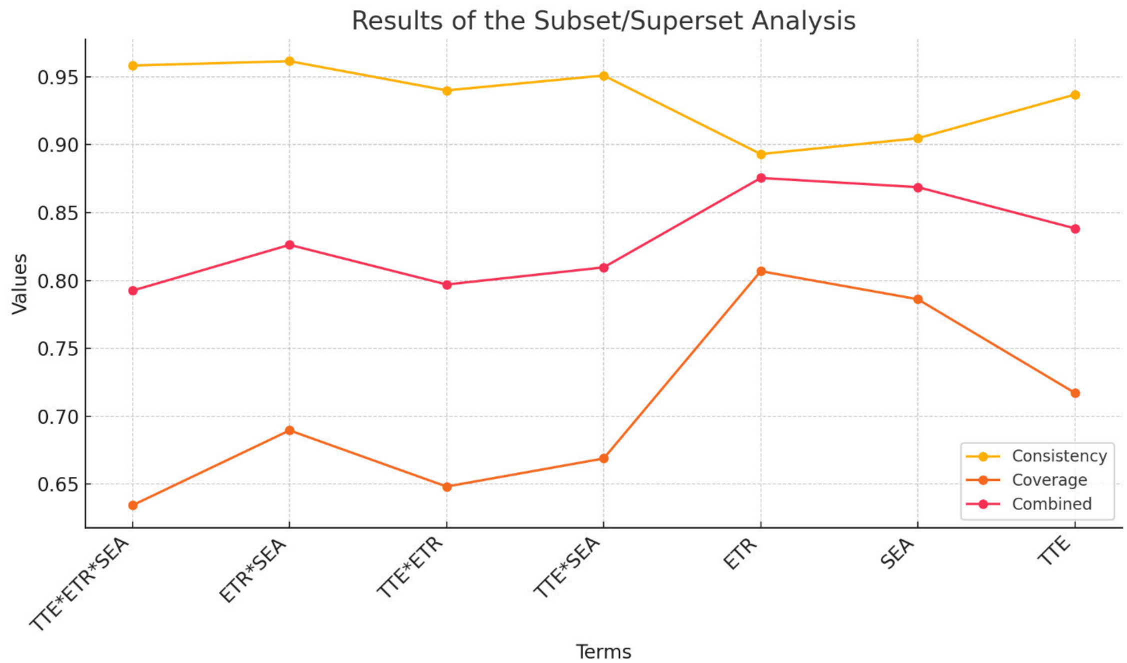

Figure 3. Results of the Subset/Superset Analysis

The subset/superset analysis reveals that the combination "ETR*SEA" (Educational and Technological Resources combined with School Environment and Academic Climate) is the most robust predictor of the outcome, with the highest combined metric of 0.826292, indicating strong consistency and good coverage. While this combination is highly consistent, individual conditions like "ETR" (Educational and Technological Resources) exhibit the highest coverage at 0.806897, suggesting they explain a significant proportion of cases where the outcome occurs. Overall, the analysis highlights the importance of both specific combinations of conditions and individual factors in predicting the desired outcome, providing valuable insights for strategic decision-making and policy development.

Conclusions

The study's conclusions highlight the critical role that Educational and Technological Resources (ETR) and the School Environment and Academic Climate (SEA) play in determining the quality of education in graduate programs. The superset analysis identifies the combination of ETR and SEA as the most robust predictor of educational quality, with the highest combined metric of 0.826292, underscoring the importance of these factors when considered together. While the ETR condition alone also shows significant coverage, indicating its broad applicability, the analysis emphasizes that both individual factors and their combinations are crucial for understanding and improving educational outcomes in public university graduate programs. This nuanced understanding of educational quality is essential for developing targeted strategies that enhance both the resources available to students and the environments in which they learn.

For future research, it is suggested that the application of neutrosophic set theory and fuzzy-set Qualitative Comparative Analysis (fsQCA) be expanded to include a more diverse range of educational settings and larger sample sizes. Further exploration into other combinations of conditions, such as the inclusion of socio-economic factors or variations in pedagogical practices, could provide a deeper understanding of the complexities influencing educational quality. Additionally, longitudinal studies could assess the long-term impact of these identified factors on student outcomes, helping to refine and validate the findings over time. Such research would contribute significantly to the ongoing effort to optimize educational quality across different contexts. Another area of future research is to enhance the support of set-theoretic research methods based on neutrosophic sets.

References

[1] V. Águila-Cabrera, "The concept of quality in university education: Key to achieving institutional competitiveness," Revista Iberoamericana de Educación , vol. 1, pp. 1-8, 2005. [Online]. Available: https://bit.ly/3BBQeus

[2] A. Anastasi, "Evolving concepts of test validation," Annual Review of Psychology , vol. 37, no. 1, pp. 1-16, 1986. [Online]. Available: https://bit.ly/3lz0ttS

[3] A. Alvarado and F. Villegas, "Access and quality in university higher education: The case of low-board professional nursing schools in Metropolitan Lima," MS thesis , Master in Investment Management, Universidad del Pacífico, Lima, Peru, 2016. [Online]. Available: https://bit.ly/2YGCbFj

[4] J. Álvarez, E. Chaparro, D. Reyes, and J. Pérez, "Study of Student Satisfaction with the Educational Services provided by Higher Education Institutions of the Toluca Valley," Ibero-American Journal on Quality, Efficiency and Change in Education , vol. 13, no. 2, pp. 5-26, 2014. [Online]. Available: https://bit.ly/3iSwnQo

[5] Mohamed, Z., M. Ismail, M. and Abd El-Gawad, A. (2023) "Sustainable Supplier Selection using Neutrosophic Multi-Criteria Decision Making Methodology", Sustainable Machine Intelligence Journal, 3, pp. (2):1–9

[6] Leyva Vázquez, M., Estupiñan, J., & Smarandache, F. (2020). Neutrosofía en Latinoamérica, avances y perspectivas. Neutrosophic Computing and Machine Learning, 14, 1-7. https://doi.org/10.5281/zenodo.4384978

[7] Gomez, G. A., Armijos, C. G., & Fernandez, A. R. (2023). Assessment of the Educational Live Action in Uncertainty Environment under Single-Valued Neutrosophic Sets. International Journal of Neutrosophic Science, 23(1), 167-176.

[8] Alvarez, G. Gómez, C. Romero, A. Ahmed, A. (2024). Teaching risk assessment index system using neutrosophic AHP: Data Fusion method. Fusion: Practice and Applications, 14(2), 199-210.

[9] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[10] Vázquez, M. L., Ricardo, J. E., & Smarandache, F. (2024). Enhancing Set-Theoretic Research Methods with Neutrosophic Sets. HyperSoft Set Methods in Engineering, 2, 96-103.

[11] Vázquez, M. Y. L., Ricardo, J. E., Hernández, N. B., Casanova, R. S., & Smarandache, F. (2024). ANÁLISIS NEUTROSÓFICO DE LAS ACTITUDES HACIA LA MÁQUINA DE EXPERIENCIA DE NOZICK. Investigación Operacional, 45(4)

[12] Westbroek, J., Nijhuis, H., & van der Maesen, L. (2020). Evolutionary Thermodynamics and Theory of Social Quality as Links between Physics, Biology, and the Human Sciences. The International Journal of Social Quality, 10(1), 57-86.

[13] Westbroek, J., Nijhuis, H., & van der Maesen, L. (2020). Evolutionary Thermodynamics and Theory of Social Quality as Links between Physics, Biology, and the Human Sciences. The International Journal of Social Quality, 10(1), 57-86.

[14] Leyva-Vázquez, M., Pérez-Teruel, K., Febles-Estrada, A., & Gulín-González, J. (2013). Causal knowledge representation techniques: A case study in medical informatics. Revista Cubana de Información en Ciencias de la Salud (ACIMED), 24(1), 73-83.

[15] Haesebrouck, T., & Thomann, E. (2021). Introduction: Causation, inferences, and solution types in configurational comparative methods. Quality & Quantity, 1-22.

[16] Gligor, D., & Bozkurt, S. (2020). FsQCA versus regression: The context of customer engagement. Journal of Retailing and Consumer Services, 52, 101929.

[17] Pappas, I. O., & Bley, K. (2023). Fuzzy-set qualitative comparative analysis: Introduction to a configurational approach. In Researching and Analysing Business (pp. 362-376). Routledge.

[18] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[19] Pupo, I. P., Pérez, P. Y. P., Vacacela, R. G., Bello, R., Santos, O., & Vázquez, M. Y. L. (2018). Extensions to Linguistic Summaries Indicators based on Neutrosophic Theory, Applications in Project Management Decisions. Neutrosophic Sets and Systems, 87.
[20] Leyva Vázquez, M., & Smarandache, F. (2021). Resolución de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica. Neutrosophic Computing and Machine Learning, 15, 41-46.
[21] Vázquez, M. L., & Smarandache, F. (2024). A Neutrosophic Approach to Study Agnotology: A Case Study on Climate Change Beliefs. HyperSoft Set Methods in Engineering, 2, 1-8.
[22] Peng, X. (2019). New multiparametric similarity measure and distance measure for interval neutrosophic set with IoT industry evaluation. Ieee Access, 7, 28258-28280.
[23] Abdel-Basset, M., & Mohamed, M. (2018). The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. Measurement, 124(10.1016).
[24] Rivieccio, U. (2008). Neutrosophic logics: Prospects and problems. Fuzzy sets and systems, 159(14), 1860-1868.
[25] Mandal, K. On Deneutrosophication. Neutrosophic Sets and Systems, Vol. 38, 2020, 409.
[26] Ricardo, J. E., Fernández, A. J., & Vázquez, M. Y. (2022). Compensatory Fuzzy Logic with Single Valued Neutrosophic Numbers in the Analysis of University Strategic Management. International Journal of Neutrosophic Science (IJNS), 18(4).

Received: June 17, 2024. Accepted: August 7, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Study of the relationship between moderate intermittent exercise and blood pressure in institutionalized older adult patients, using the neutrosophic correlation coefficient

Lisbeth Josefina Reales Chacón[1], Jennifer Pamela Freire Núñez[2], Carmen Analía De la Cruz López[3], Víctor Manuel García Camacho[4], Lizbeth Geovanna Silva Guayasamín[5], and Josselyn Gabriela Bonilla Ayala[6]

[1] National University of Chimborazo, Riobamba. Ecuador. E-mail: lisbeth.reales@unach.edu.ec

[2]Higher Institute of Technology Spain ISTE, Ambato. Ecuador. E-mail: jenny25fn@gmail.com

[3]Higher Institute of Technology Spain ISTE, Ambato. Ecuador. E-mail: analia.delacruz2014@gmail.com

[4]National University of Chimborazo, Riobamba. Ecuador. E-mail: victorgarciacamacho777@gmail.com

[5]National University of Chimborazo, Riobamba. Ecuador. E-mail: lizbethg.silva@unach.edu.ec

[6]Technical University of Ambato, Ambato. Ecuador. E-mail: jg.bonilla@uta.edu.ec

Abstract. This paper aims to analyze the effects of moderate intermittent exercise in institutionalized older adults and its relationship with blood pressure. It is an investigation where the impact of sporadic exercises on blood pressure values over time was evaluated. A total of 29 older adults were analyzed to whom moderate intermittent exercises were applied for 4 weeks, 3 times a week for 30 minutes each session, with 12 interventions. Because blood pressure is an indicator that changes over time during the course of the day and its measurement is not precise, we use single-valued neutrosophic sets as data instead of numerical values. Neutrosophic Statistics techniques were applied to these data. This is a branch of statistics and neutrosophy where the methods of classical statistics are extended to data or parameters in interval form, or in the case of samples or populations whose exact size is not known. Specifically, we use neutrosophic correlation methods applied to neutrosophic data. Although this is not interval data, it could also be considered part of the Neutrosophic Statistics because it is not crisp data.

Keywords: Neutrosophic statistics**,** neutrosophic correlation, blood pressure, older adults, preventive medicine, moderate physical exercises.

1 Introduction

High blood pressure is a condition seen in approximately 20-40% of the adult population in the Americas region, which means around 250 million people experience elevated blood pressure levels. Non-pharmacological treatment, such as physical exercise, has been undervalued by medical personnel, but it can contribute to the control of blood pressure levels in hypertensive patients, reduce their need for medications, improve their quality of life, and reduce cardiovascular mortality, among others [1].

On the other hand, conventional treatment of hypertension, which involves the use of medications, is not only expensive from a financial perspective, but can also lead to various problematic side effects in the case of older adults. Thiazide diuretics, long-acting calcium channel blockers, and angiotensin-converting enzyme inhibitors are some of the drugs considered first-line for the treatment of hypertension in older adults. For this reason, complementary and combined therapeutic approaches are considered, which do not involve the use of medications. These approaches include modifying diet, reducing caloric intake, as well as incorporating regular physical activity, with a minimum of thirty minutes per day [2].

Likewise, physical activity is an effective way to control high blood pressure in older adults. Moderate or intense exercise is especially beneficial for reducing blood pressure; it is one of the most recommended preventive and therapeutic measures to control blood pressure, but some different modalities and protocols can have different effects on this variable.

Therefore, this research aims to contribute to current scientific knowledge by examining the relationship between moderate intermittent exercise and blood pressure in institutionalized older adults. Given the context and data presented above, the need to implement a research project oriented towards the prevention and improvement of conditions such as high blood pressure in institutionalized older adults at the "Hogar Sagrado Corazón de Jesús", in Ambato, Ecuador, becomes evident.

There were 29 older adults undergoing the treatment by performing moderate physical exercises for a period of four weeks. The results of the blood pressure measurements were processed in the form of single-valued neutrosophic numbers instead of precise numerical values. This is because blood pressure is a value subject to inaccuracies due to the nature of the measuring equipment, the differences in blood pressure that occur during the course of the day and that is natural, the difference in blood pressure due to factors external to the person such as stress or performing some physical activity such as walking.

For this study, we carry out an evaluation where the neutrosophic correlation coefficient is used [3-9]. This test generalizes a method from classical statistics to neutrosophic statistics [10-16]. In neutrosophic statistics, results are obtained where the data or parameters are values in the form of an interval. Populations or samples where their exact size is not known are also taken into account. The objective is to gain accuracy by incorporating indeterminate values at the cost of having greater indeterminacy. Usually, neutrosophic methods correlation are not expressed in the form of intervals, however, they can be considered as part of the Neutrosophic Statistics since the data are not numerical values as in classical statistics. Specifically, we have values in the form of single-valued neutrosophic numbers.

This paper consists of the following structure, section 2 is dedicated to offering the basic notions of Neutrosophic Statistics and Neutrosophic Correlation. Section 3 shows the results of the statistical calculations carried out in this research. The last section is dedicated to conclusions.

2 Materials and Methods

This section is dedicated to exposing the basic notions of Neutrosophic Statistics.

2.1 Basic Notions of Neutrosophic Statistics

Definition 1: ([17-19]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 2: ([15-17]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \qquad (1)$$

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indeterminate and falseness of x in A, respectively. For convenience, a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Neutrosophic Statistics extends classical statistics, such that we deal with set values rather than crisp values, ([15-17]). This definition agrees with the type of data used in this paper.

Neutrosophic Descriptive Statistics is comprised of all techniques to summarize and describe the neutrosophic numerical data characteristics.

Neutrosophic Inferential Statistics consists of methods that allow the generalization from a neutrosophic sampling to a population from which the sample was selected.

Neutrosophic Data is the data that contains some indeterminacy. Similarly to classical statistics it can be classified as:

- *Discrete neutrosophic data*, if the values are isolated points.
- *Continuous neutrosophic data*, if the values form one or more intervals.

Another classification is the following:

- *Quantitative (numerical) neutrosophic data*; for example a number in the interval (we do not know exactly), 47, 52, 67, or 69 (we do not know exactly);
- *Qualitative (categorical) neutrosophic data*; for example: blue or red (we don't know exactly), white, black or green or yellow (not knowing exactly).

The *univariate neutrosophic data* is a neutrosophic data that consists of observations on a neutrosophic single attribute [20].

Multivariable neutrosophic data is neutrosophic data that consists of observations on two or more attributes [21, 22].

A *Neutrosophical Statistical Number* N has the form $N = d + I$, where d is called the *determinate part* and I is called the *indeterminate part* [23].

A *Neutrosophic Frequency Distribution* is a table displaying the categories, frequencies, and relative frequencies with some indeterminacy. Most often, indeterminacies occur due to imprecise, incomplete, or unknown data related to frequency. As a consequence, relative frequency becomes imprecise, incomplete, or unknown too.

Neutrosophic Survey Results are survey results that contain some indeterminacy.

A *Neutrosophic Population* is a population not well determined at the level of membership (i.e. not sure if some individuals belong or do not belong to the population).

A *simple random neutrosophic sample* of size n from a classical or neutrosophic population is a sample of n individuals such that at least one of them has some indeterminacy.

A *stratified random neutrosophic sampling* is the pollster groups of the (classical or neutrosophic) population by a strata according to a classification; Then the pollster takes a random sample (of appropriate size according to a criterion) from each group. If there is some indeterminacy, we deal with neutrosophic sampling.

Additionally, we describe some concepts of interval calculus [24, 25].

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ (Addition),

$N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1+b_1 I}{a_2+b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2+b_2)} I$ (Division).

Additionally, given $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ we have the following operations between them:

$I_1 \leq I_2$ if and only if $a_1 \leq a_2$ and $b_1 \leq b_2$.

$I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ (Addition);

$I_1 - I_2 = [a_1 - b_2, b_1 - a_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$\frac{I_1}{I_2} = \left[\frac{a_1}{b_1}, \frac{a_2}{b_2}\right]$, always that $0 \notin I_2$ (Division).

$\sqrt{I} = [\sqrt{a}, \sqrt{b}]$, always that $a \geq 0$ (Square root).

$I^n = \underbrace{I \cdot I \cdot \cdots \cdot I \cdot I}_{n \text{ times}}$.

2.2 Neutrosophic Correlation

Definition 3: ([3, 26]) Let A and B be two single-valued neutrosophic sets in a finite space $X = \{x_1, x_2, \ldots, x_n\}$. The *correlation between the two neutrosophic sets* A and B is defined below:

$$CN(A, B) = \sum_{i=1}^{n}[u_A(x_i)u_B(x_i) + r_A(x_i)r_B(x_i) + v_A(x_i)v_B(x_i)] \quad (2)$$

The *correlation coefficient* between A and B is defined by Equation 3.

$$R(A, B) = \frac{CN(A,B)}{\sqrt{T(A)T(B)}} \quad (3)$$

Where:

$T(A) = \sum_{i=1}^{n}[u_A^2(x_i) + r_A^2(x_i) + v_A^2(x_i)]$ and $T(B) = \sum_{i=1}^{n}[u_B^2(x_i) + r_B^2(x_i) + v_B^2(x_i)]$.

We would like to emphasize that originally this part was called statistics on neutrosophic data. But since it is neutrosophic data, it can also be classified as part of the Neutrosophic Statistics. If we want to be more rigorous in the concepts, each single-valued neutrosophic set can be decomposed into intervals using α-cuts and the proposed statistical methods can be applied to these values in the form of intervals.

3 Results of the statistical study

In the present investigation, numerical blood pressure data is collected from the participating older adults, which will be tabulated and analyzed statistically. Then, in the second experimental stage, an intervention consisting of a program of intermittent physical exercises of moderate intensity is applied.

The initial observational design will allow the sample to be characterized by baseline blood pressure levels. The analytical approach will seek to explore possible associations of this variable with other demographic and clinical factors.

Longitudinal monitoring throughout the exercise program aims to evaluate the effect of this intervention on blood pressure values over time. In this way, the quantitative approach will guide the entire research process from the systematic collection of measurements to the statistical analysis to test the hypothesis.

Selection of area or scope of study

- Field: Health,
- Aspects: Moderate intermittent exercise, blood pressure,
- Province, Canton: Tungurahua, Ambato,
- Place: Hogar Sagrado Corazón de Jesús,
- Time: September 2023-February 2024 Scope of study,
- Research line: Human Health.

For the creation of this degree project, the population will be 29 older adults, and people of both sexes who have blood pressure problems, between 60 and 100 years of age, will be taken into account.

Inclusion criteria

- Patients with controlled blood pressure before exercise,
- Patients who voluntarily sign the informed consent,
- Patients between 60 and 100 years of age.

Exclusion criteria

- Dependent older adults,
- Older adults with diseases such as diabetes mellitus,
- Older adults with a diagnosis of aneurysms,
- Older adults with varicose veins in the lower limbs,
- Older adults with isolated systolic arterial hypertension.

The detailed results of the study are summarized in Table 1.

Table 1. Evaluation of blood pressure in institutionalized older adults.

	Age	Categorization of the Elderly	SBP	DBP	Categorization of the blood pressure
P1	88	Adult elderly fragile	130	80	Prehypertension
P2	98	Adult elderly fragile	126	78	Prehypertension
P3	85	Adult elderly fragile	120	84	Prehypertension
P4	81	Elderly intermediate	130	70	Prehypertension
P5	82	Elderly intermediate	130	66	Prehypertension
P6	90	Adult elderly fragile	120	60	Prehypertension
P7	97	Adult elderly fragile	110	50	Normotension
P8	93	Adult elderly fragile	140	70	Hypertension stage 1
P9	90	Adult elderly fragile	126	86	Prehypertension
P10	71	Adult elderly young	120	80	Prehypertension
P11	88	Adult elderly fragile	130	68	Prehypertension
P12	88	Adult elderly fragile	130	72	Prehypertension
P13	89	Adult elderly fragile	130	70	Prehypertension
P14	87	Adult elderly fragile	130	70	Prehypertension
P15	87	Adult elderly fragile	164	68	Hypertension stage 2
P16	87	Adult elderly fragile	116	66	Normotension
P17	60	Adult elderly young	130	70	Prehypertension
P18	95	Adult elderly fragile	120	70	Prehypertension
P19	81	Elderly intermediate	130	70	Prehypertension
P20	65	Adult elderly young	120	74	Prehypertension
P21	95	Adult elderly fragile	120	70	Prehypertension
P22	83	Intermediate older adult	110	68	Normotension
P23	83	Intermediate older adult	130	70	Prehypertension

	Age	Categorization of the Elderly	SBP	DBP	Categorization of the blood pressure
P24	90	Fragile older adult	120	70	Prehypertension
P25	80	Intermediate older adult	120	84	Prehypertension
P26	94	Fragile older adult	114	68	Normotension
P27	70	Young senior adult	124	70	Prehypertension
P28	78	Intermediate older adult	150	90	Stage 1 hypertension
P29	88	Fragile older adult	130	70	Prehypertension

Data on systolic blood pressure (SBP) and diastolic blood pressure (DBP) were obtained before performing moderate intermittent exercises. The study involved 29 older adults, observing a predominance of frail older adults (≥85 years) in the sample, highlighting the prehypertension category.

Table 2. Variation of SBP and DBP before and after the intermittent exercises.

	SBP			DBP		
Patient	Before	After	SBP variation	Before	After	DBP variation
P1	130	128	-2	80	80	0
P2	126	130	4	78	80	2
P3	120	120	0	84	80	-4
P4	130	128	-2	70	72	2
P5	130	130	0	66	62	-4
P6	120	120	0	60	70	10
P7	110	110	0	50	70	20
P8	140	142	2	70	74	4
P9	126	128	2	86	66	-20
P10	120	118	-2	80	64	-16
P11	130	130	0	68	90	22
P12	130	130	0	72	76	4
P13	130	130	0	70	70	0
P14	130	130	0	70	66	-4
P15	164	160	-4	68	72	4
P16	116	118	2	66	60	-6
P17	130	128	-2	70	70	0
P18	120	120	0	70	70	0
P19	130	128	-2	70	66	-4
P20	120	120	0	74	70	-4
P21	120	120	0	70	72	2
P22	110	110	0	68	70	2
P23	130	130	0	70	70	0
P24	120	120	0	70	70	0
P25	120	120	0	84	80	-4
P26	114	112	-2	68	70	2
P27	124	120	-4	70	70	0
P28	150	150	0	90	88	-2
P29	130	128	-2	70	68	-2

Table 2 shows the variation of the SBP and DBP values before and after the execution of the intermittent exercises. Variation values were calculated using the values after the exercises, subtracted from the values measured before performing the exercises.

Patients who showed negative variations (marked in red) showed lower SBP and DBP values after the exercises performed. On the other hand, patients who present positive variations (marked in green) show that the SBP and DBP values were higher after the exercises performed. Likewise, some patients did not show variation (marked yellow).

Figure 1 represents the membership, indeterminacy, and non-membership functions of the normal DBP. Figure 2 represents the same functions corresponding to SBP.

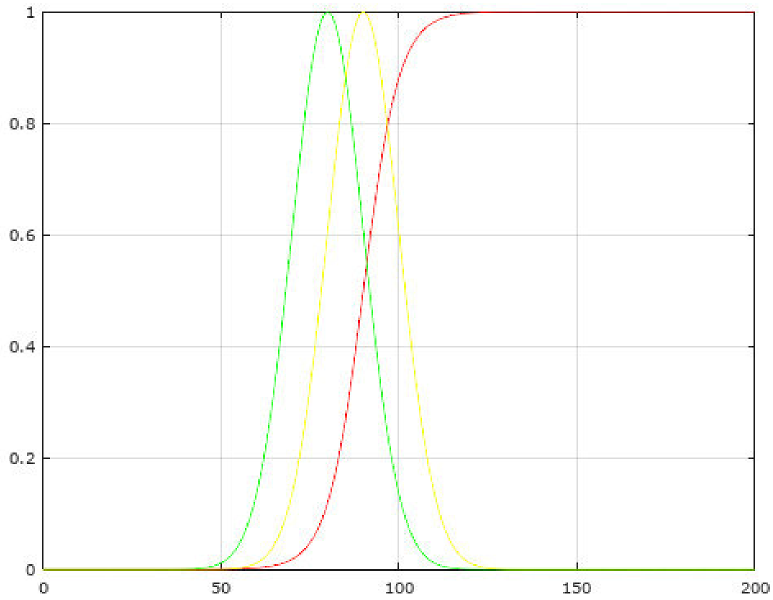

Figure 1: Membership functions $u_A(x)$ (green), indeterminacy $r_A(x)$ (yellow), and non-membership $v_A(x)$ (red) for the DBP.

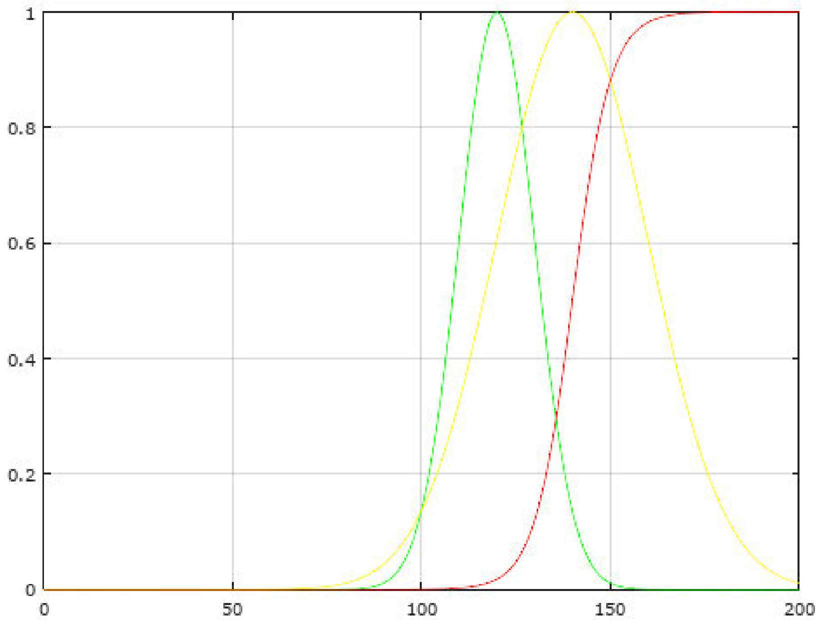

Figure 2: Membership $u_A(x)$ functions (green), indeterminacy $r_A(x)$ (yellow), and non-membership $v_A(x)$ (red) for the SBP.

The calculations carried out with the help of Equation 3 gave the results:

$R(DBP_{before}, DBP_{after}) = 0.87515,$

$R(SBP_{before}, SBP_{after}) = 0.99650.$

This means that there is a great correlation between both values for both DBP and SBP.

On the other hand, when finding the averages of the variations, it was obtained that the average variation of SBP was -0.4137931 and the average variation of DBP was 0.13793103. To find out if this is in line with what normal blood pressure should be, we use Equation 4 shown below, which is the so-called Score Function ([27]):

$N_{PB}(A, x) = \frac{2+u_A(x)-r_A(x)-v_A(x)}{3}$ (4)

To aggregate the results for all values in Equation 4, Equation 5 is used.

$\overline{N}_{PB}(A) = \text{mean}_{x\in X} N_{PB}(A, x)$ (5)

In the case of DBP before, it was obtained $\overline{N}_{PB}(DBP_{before}) = 0.76018$ and for DBP afterward it was obtained $\overline{N}_{PB}(DBP_{after}) = 0.76897$, that is, there was better.

Regarding SBP, we have $\overline{N}_{PB}(SBP_{before}) = 0.61606$ and $\overline{N}_{PB}(SBP_{after}) = 0.62056$, which also indicates an improvement.

Conclusion

Blood pressure is one of the parameters to be followed by current medical treatment. This is an important parameter because an imbalance in it can cause serious consequences for human health. It has been shown that in today's world population, this is a considerable health problem. The treatment of this situation is commonly carried out with medications, which affect the patient's finances and also have side effects on them. A more positive solution is to perform physical exercises as a complementary solution. In the case studied in this article we focus on 29 elderly patients institutionalized in a shelter for older adults in Ecuador. The treatment consisted of moderate exercise for a short period. The results before and after applying the treatment were measured in terms of the patients' DBP and SBP. It was concluded that there was an improvement in the average of the elderly studied in both parameters. For the study, we used the neutrosophic correlation coefficient which demonstrated a high correlation between both parameters. Improvement results were also obtained when DBP and SBP were studied in terms of what is considered normal in a person.

References

[1] Zhou, B., Perel, P., Mensah, G. A., & Ezzati, M. (2021). Global epidemiology, health burden and effective interventions for elevated blood pressure and hypertension. Nature Reviews Cardiology, 18(11), 785-802.

[2] Mancia, G., Cappuccio, F. P., Burnier, M., Coca, A., Persu, A., Borghi, C., ... & Sanner, B. (2023). Perspectives on improving blood pressure control to reduce the clinical and economic burden of hypertension. Journal of Internal Medicine, 294(3), 251-268.

[3] Hanafy, I. M., Salama, A. A., and Mahfouz, K. (2012). Correlation of neutrosophic Data. International Refereed Journal of Engineering and Science (IRJES), 1, 39-43.

[4] Iryna, S., Zhong, Y., Jiang, W., Deng, X., and Geng, J. (2020). Single-valued neutrosophic set correlation coefficient and its application in fault diagnosis. Symmetry, 12, 1371.

[5] Nagarajan, D., Broumi, S., Smarandache, F., and Kavikumar, J. (2021). Analysis of neutrosophic multiple regression. Neutrosophic Sets and Systems. 43, 44-53.

[6] Zulqarnain, R. M., Xin, X. L., Ali, B., Broumi, S., Abdal, S., and Ahamad, M. I. (2021). Decision-making approach based on correlation coefficient with its properties under interval-valued neutrosophic hypersoft set environment. Infinite Study.

[7] Köseoğlu, A., and Şahin, R. (2023). Correlation coefficients of simplified neutrosophic multiplicative sets and their applications in clustering analysis. Journal of Ambient Intelligence and Humanized Computing, 14, 1-22.

[8] Radha, R., Mary, A. S. A., Broumi, S., Jafari, S., and Edalatpanah, S. A. (2023). Improved correlation coefficients in neutrosophic statistics for COVID patients using pentapartitioned neutrosophic sets. In Cognitive intelligence with neutrosophic statistics in bioinformatics (pp. 237-258). Academic Press.

[9] Zhou, X., Lin, M., and Wang, W. (2023). Statistical correlation coefficients for single-valued neutrosophic sets and their applications in medical diagnosis. AIMS Mathematics, 8, 16340-16359.

[10] Silva, P. A. M., Fernández, A. R., and Macías, L. A. G. (2020). Neutrosophic Statistics to Analyze Prevalence of Dental Fluorosis. Neutrosophic Sets and Systems, 37, 395-403

[11] Aslam, M. (2021). Analyzing wind power data using analysis of means under neutrosophic statistics. Soft Computing, 25, 7087-7093.

[12] Duan, W. Q., Khan, Z., Gulistan, M., and Khurshid, A. (2021). Neutrosophic exponential distribution: modeling and applications for complex data analysis. Complexity, 2021, 5970613.

[13] A Afzal, U., Alrweili, H., Ahamd, N., and Aslam, M. (2021). Neutrosophic statistical analysis of resistance depending on the temperature variance of conducting material. Scientific reports, 11, 23939.

[14] lAita, A., and Aslam, M. (2023). Analysis of covariance under neutrosophic statistics. Journal of Statistical Computation and Simulation, 93, 397-415.

[15] AlAita, A., Talebi, H., Aslam, M., and Al Sultan, K. (2023). Neutrosophic statistical analysis of split-plot designs. Soft Computing, 27, 7801-7811.

[16] Aslam, M., and Albassam, M. (2020). Presenting post hoc multiple comparison tests under neutrosophic statistics. Journal of King Saud University-Science, 32, 2728-2732.

[17] Smarandache, F. (2016). Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics. Infinite Study.
[18] Smarandache, F. (2022). Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version) (Vol. 2). Infinite Study.
[19] Woodall, W. H., Driscoll, A. R., and Montgomery, D. C. (2022). A review and perspective on neutrosophic statistical process monitoring methods. IEEE Access, 10, 100456-100462.
[20] Pupo, I. P., Pérez, P. Y. P., Vacacela, R. G., Bello, R., Santos, O., & Vázquez, M. Y. L. (2018). Extensions to Linguistic Summaries Indicators based on Neutrosophic Theory, Applications in Project Management Decisions. Neutrosophic Sets and Systems, 87..
[21] Smarandache, F. (2021). Introducción a la Lógica Plitogénica. Neutrosophic Computing and Machine Learning, 18, 1-6.
[22] Mohamed, M. (2024) "Partnership of Lean Six Sigma and Digital Twin under Type 2 Neutrosophic Mystery Toward Virtual Manufacturing Environment: Real Scenario Application", Sustainable Machine Intelligence Journal, 8, pp.(6):99–109.
[23] Vázquez, M. L., & Torres, L. C. (2024). Integrating Neutrosophic Numbers in Regression Analysis for Enhancing Predictive Modelling through Uncertainty Representation. Plithogenic Logic and Computation, 1, 120-126.
[24] Salman, N. K., Vazquez, M. L., & Noel, B. H. (2024). On The Classification of 3-Cyclic/4-Cyclic Refined Neutrosophic Real and Rational Von Shtawzen's Group. International Journal of Neutrosophic Science, 23(2), 26-32.
[25] Lucero Salcedo, V. H., Cadena Morillo, J. R., & Puetate Paucar, J. M. (2024). Análisis estadístico neutrosófico sobre la sucesión por causa de muerte y sus efectos en la familia pluriparental. Neutrosophic Computing and Machine Learning, 31, 303-311.
[26] Salama, A., Khaled, O. M.; and Mahfouz, K. M. (2019) Neutrosophic Correlation and Simple Linear Regression, Neutrosophic Sets and Systems, 5, 3-8.
[27] Smarandache, F. (2020) The Score, Accuracy, and Certainty Functions determine a Total Order on the Set of Neutrosophic Triplets (T,I,F). Neutrosophic Sets and Systems, 38, 1-14.

Received: June 17, 2024. Accepted: August 7, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Plithogenic Hypothesis in Social Innovation: An Analysis of Its Evolution and Its Relationship with the Social Economy

Mercedes Anabel Monar Verdezoto[1], Hernán Arturo Rojas Sánchez[2], Javier Arturo Rojas Urbano[3], and Jorge Briceño[4]

[1]State University of Bolivar, Guaranda. Ecuador. E-mail: memonar@ueb.edu.ec
[2]State University of Bolivar, Guaranda. Ecuador. E-mail: arojas@ueb.edu.ec
[3]Higher Polytechnic School of Chimborazo, Riobamba. Ecuador. E-mail: javier.rojas@espoch.edu.ec
[4]State University of Bolivar, Guaranda. Ecuador. E-mail: jbriceno@ueb.edu.ec

Abstract. The paper examines the connection between social innovation, social enterprises, and the social economy using the plithogenic hypothesis as a framework. The text highlights a significant deficiency in the existing body of research, specifically pointing out the lack of a unified approach that combines many views to thoroughly evaluate the influence of social innovation on economic growth. The study utilizes sophisticated analytical tools to investigate the interplay between these variables and their impact on the wider social economy. The results demonstrate that although there is a growing acknowledgment of the significance of social innovation and companies, the absence of precise definitions and techniques presents substantial obstacles. The paper provides a novel theoretical framework that integrates the plithogenic hypothesis into the analysis of the social economy. Additionally, it offers practical suggestions for improving policy formulation and strategy creation. The objective of this strategy is to maximize the beneficial effects of social innovation on economic and social development, by enhancing both theoretical understanding and practical implementation in the field.

Keywords: Social Innovation, Social Enterprises, Social Economy, Falsifiability of a Hypothesis, Multivalued Logics, Plitogenic Statistics.

1 Introduction

Social innovation and social enterprises have emerged as key components in contemporary economic and social development. These practices, which focus on solving social problems through innovative and sustainable approaches, are increasingly gaining recognition for their potential to generate positive impact in diverse communities. The relevance of studying the relationship between social innovation and the social economy lies in the need to understand how these initiatives can contribute to inclusive and equitable economic development [1]. The present research focuses on analyzing this relationship through the conceptual framework of the plithogenic hypothesis, a theory that promises to offer a novel and valuable perspective on these issues. Historically, social innovation and social enterprises have played an important role in shaping public policies and implementing economic development strategies. From the rise of cooperatives and solidarity economy movements in the 20th century, to the proliferation of social startups and B Corps today, these forms of organization have evolved significantly [2]. Despite this progress, the integration and impact of these initiatives in the social economy still present significant challenges and areas of opportunity for further research [3].

The central problem that this study addresses is the lack of a clear and systematic understanding of how social innovation and social enterprises interact with the social economy. Despite the growing interest in these issues, a fundamental question remains: how do these initiatives contribute to economic and social development sustainably? The research seeks to answer this question by exploring the relationship between these concepts through plithogenic analysis, which offers a comprehensive and dynamic perspective [4]. This study's main objective is to evaluate the influence of social innovation and social enterprises on the social economy using the plithogenic hypothesis approach. In particular, it aims to unravel how these elements contribute to inclusive economic development and what their impact is on the creation of social value [5]. In addition, it seeks to provide recommendations based on the findings to improve the integration of these practices in economic and social development strategies.

The methodological approach of the study includes the application of the plithogenic hypothesis to analyze empirical and theoretical data related to social innovation and social enterprises. Advanced analytical tools will be used to examine how these initiatives influence the social economy and to identify meaningful patterns and relationships in the collected data [6, 7, 8]. This study not only seeks to advance the theoretical understanding of the relationship between social innovation, social enterprises, and the social economy but also aims to offer practical contributions for the formulation of policies and development strategies [9, 10]. In summary, the article addresses a topic of great relevance in the current context, with the aim of filling a significant gap in the existing literature. Through detailed analysis of the relationship between social innovation, social enterprises and the social economy, it is hoped to provide a more complete and nuanced understanding of how these practices can contribute to sustainable and equitable economic development.

2 Preliminary

2.1. Plithogenic probability

Neutrosophic (or indeterminate) data are characterized by inherent vagueness, lack of clarity, incompleteness, partial unknowns, and contradictory information [11, 12]. Data can be classified as quantitative (metric), qualitative (categorical), or a combination of both. Plithogenic variable data [13, 14] can describe the connections between neutrosophic variables or other multivalued logic variables [15, 16]. Complex problems often demand multiple measurements and observations due to their multidimensional nature, such as measurements needed in scientific investigations. Neutrosophic variables can exhibit dependence, independence, partial dependence, partial independence or partial indeterminacy [17].

A Plithogenic Set [18] is a non-empty set P whose elements within the discourse domain $U(P \subseteq U)$ are characterized by one or more attributes $A_1, A_2, \cdots, A_m$, where m is at least 1. where each attribute can have a set of possible values within the spectrum $S$of values or states, such that Sit can be a finite, infinite, discrete, continuous, open or closed set.

Each element $x \in P$ is characterized by all possible values of the attributes found within the set $V = \{v_1, v_2, \cdots, v_n\}$. The value of an attribute has a degree of membership $d(x, v)$ to an element $x$of the set.P, based on specific criteria. The degree of membership can be fuzzy, fuzzy intuitionistic or neutrosophic, among others [19].

That means,

$$\forall x \in P, d: P \times V \rightarrow \mathcal{P}([0,1]^z) \quad (1)$$

Where$d(x, v) \subseteq [0,1]^z$ and $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$.$z = 1$ (fuzzy degree of membership), $z = 2$(intuitionistic degree of membership belonging) or $z = 3$ (the neutrosophic degree of membership).

Plithogenic [20], derived from the analysis of plithogenic variables, represents a multidimensional probability (" plitho " meaning "many" and synonymous with "multi"). A probability composed of subprobabilities can be considered, where each subprobability describes the behavior of a specific variable. The event under study is assumed to be influenced by one or more variables, each represented by a probability (density) distribution function (PDF).

Consider an event E in a given probability space, whether classical or neutrosophic, determined by $n \geq 2$ variables. $v_1, v_2, \ldots, v_n$, denoted as $E(v_1, v_2, \ldots, v_n)$. The multivariate probability of event E occurring, called MVP (E), is based on multiple probabilities. Specifically, it depends on the probability of event E occurring concerning each variable: $P1(E(v_1))$for variable v_1, $P2(E(v_2))$for variable v_2, etc. Therefore, $MVP(E(v_1, v_2, \ldots, v_n))$it is represented as $\left(P1(E(v_1)), P2(E(v_2)), \ldots, Pn(E(v_n))\right)$. The variables $v_1, v_2, \ldots, v_n$, and the probabilities $P_1, P_2, \ldots, P_n$, can be classical or have some degree of indeterminacy [24].

To make the transition from plithogenic neutrosophic probability (PNP) to univariate neutrosophic probability UNP, we use the conjunction operator [21]:

$$UNP(v_1, v_2, \ldots, v_n) = v_1 \wedge_{i=1}^{n} v_n \quad (2)$$

$\wedge$ In this context, this is a neutrosophic conjunction (t-norm). if we take$\wedge_p$ as the plithogenic conjunction between probabilities of the PNP type, where $(T_A, I_A, F_A) \wedge_p (T_B, I_B, F_B) = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$, such that $\wedge$is the minimum t-norm of fuzzy logic and $\vee$the maximum t-norm [20].

a. Multivalued Hypothesis

Smarandache [22] recently introduced the concept of Partial Falsifiability as an adaptation of Popper's Falsifiability from classical logic to multi-valued logic. This expanded the notion of falsifiability to include hypotheses that involve multiple values in logic. Falsifiability, a concept coined by the Austrian philosopher of science Karl Popper [23], states that for a theory to be considered scientific, it must be capable of being proven false.

Multi-valued logical hypotheses, such as those found in neutrosophic logic, accommodate partial degrees of truth, indeterminacy, and falsehood. In classical logic, the concept of falsifiability pertains to hypotheses that can be unambiguously categorized as either entirely true or entirely false.

Smarandache extension introduces the concept of Partial Falsifiability to multi-valued logic, enabling hypotheses to have partial truth values, uncertainties, and false values [22]. In the field of neutrosophic logic, a hypothesis $NLH(T,I,F)$ is deemed falsifiable if specific conditions are present that enable the hypothesis to be contradicted, leading to the expression $\neg NLH(F,1-I,T)$. Here, T, I, and F represent degrees of truth, indeterminacy, and falsity, respectively, all of which fall within the range of [0,1].

The probabilistic interpretation defines a neutrosophic probabilistic hypothesis [17], NPH (T,I,T), with values for t, i, and f ranging from 0 to 1. These values represent probability, indeterminacy, and falsity, respectively. This approach provides a more thorough and detailed evaluation of the ability to prove false and the likelihood of hypotheses in complex scientific fields such as medical education and social science contexts.

The concept of multivalued logic was recently employed in conjunction with the Consensus artificial intelligence tool to analyze hypotheses [17]. This paper utilizes this framework to examine social innovation and social enterprises.

3 Material and Methods

This framework offers an organized method for testing hypotheses, starting with the explicit articulation of a testable hypothesis that explains the important variables' cause-and-effect relationship. Through the identification of these important variables, the research is focused on investigating certain relationships, such as the independent variables (causes) and dependent variables (effects). Next, the hypothesis is decomposed into specific research questions that explore the suggested association in detail. Consensus Meter techniques are utilized for sentiment categorization and quantification to improve the analysis even more. These algorithms classify scientific assertions into three categories: affirmative, indeterminate, and negative attitudes. Research findings synthesis and interpretation depend on this sentiment analysis designed for scientific material.

The framework then includes the development of probabilistic neutrosophic hypotheses, giving each category a value representing truth, indeterminacy, or untruth. The plithogenic neutrosophic probability (PNP), which assesses the hypothesis's robustness, is then computed by adding these values. Ultimately, these neutrosophic probabilities are analyzed to determine if the hypothesis is generally valid, ambiguous, or false based on the body of scientific research.

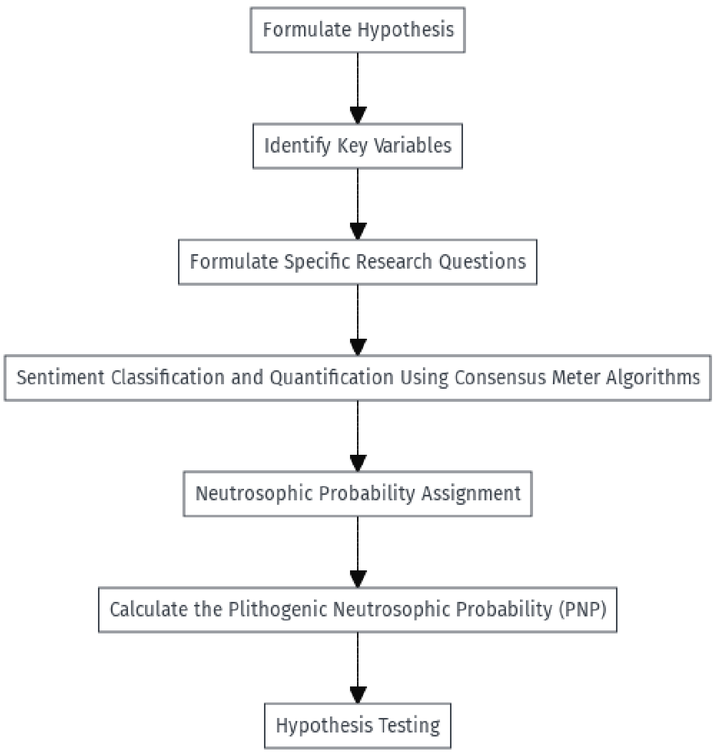

Figure 1. Hypothesis Formulation and Evaluation Process with Neutrosophic Analysis

a. Formulate hypothesis

Begin by explicitly stating the hypothesis intended to be examined. It has to indicate a cause-and-effect relationship between the variables. The hypothesis should be testable through empirical research. It should be possible to collect data that can support or refute the hypothesis.

b. Identify key variables

Identify the crucial variables associated with the hypothesis. These elements usually consist of independent variables (factors that can be manipulated or observed), dependent variables (outcomes that are measured), and any control or intervening variables.

Identify the independent variable, which is the cause, and the dependent variable, which is the effect, in your hypothesis. This helps direct your research queries toward the exact relationship to investigate.

c. Formulate specific research questions

Turn each component or relationship into a research question formulated as "Does X cause Y?" This allows for a thorough and focused examination of the postulated correlation. Ensure that the research questions comprehensively address the entirety of the hypothesis. Collectively, they should enable a thorough examination of the hypothesis.

d. Sentiment Classification and Quantification Using Consensus Meter Algorithms

To perform sentiment analysis on a research paper and quantify the occurrences of "Yes," "Possibility/Indeterminacy," and "No," you need a sentiment analysis tool for scientific statements. In this case, we use Consensus Meter algorithms to categorize statements into three distinct groups: Positive (affirmative), Indeterminate (possibility or indeterminacy), and Negative (negative) [23].

The Consensus Meter differs from traditional sentiment analysis methods used in social media or product reviews. It is a specialized form of sentiment analysis designed specifically for scientific and factual content. The function of categorizing the sentiment or stance of scientific statements regarding a particular hypothesis or question is crucial in the process of synthesizing and interpreting research findings.

e. Neutrosophic probability assignment

In this process, a neutrosophic probability is assigned to each question based on the classification of papers into three stances: Positive, Indeterminate, and Negative, regarding a specific question

f. Calculate the plithogenic neutrosophic probability (PNP)

Using the neutrosophic probabilities assigned to each question, the univariate neutrosophic probability (UNP) is calculated to evaluate the robustness of the overall hypothesis. This process involves combining the separate probabilities to provide a comprehensive evaluation of the overall hypothesis.

$$UNP(v_1,\ v_2, \dots,\ v_n) = (Min(t_1, t_n, \dots, t_n), Max(i_1, i_n, \dots, i_n), Max(f_1, f_n, \dots, f_n)) \tag{3}$$

Where:

$T_1, T_2, \dots, T_n$: are the truth probability values of each question.

$I_1, I_2, \dots, I_n$: are the values of the indeterminacy probabilities of each question.

$F_1, F_2, \dots, F_n$: are the falsehood probability values for each question

g. Hypothesis Testing

Analyze how the responses to these inquiries will collectively enable you to either validate, invalidate, or modify your initial hypothesis. In this case, the negation of NPH is represented as [17]:

$$(T, I, F) = (F, I, T) \tag{4}$$

This step involves analyzing the negated neutrosophic probabilities to evaluate the overall strength and reliability of the general hypothesis. By evaluating the levels of falsehood, uncertainty, and truthfulness, one can determine the degree to which the hypothesis is valid, ambiguous, or incorrect based on the scientific literature.

3.1. Case study.

Social innovation and social enterprises are fundamental components of the social economy. However, despite their growing relevance, they lack a universally accepted definition and a coherent methodological framework to evaluate their impact. This lack of clarity makes it difficult to measure their contribution to economic growth and questions whether these fields are well-researched academically. This study explores these questions by applying the proposed, addressing the uncertainty inherent in the relationship between social innovation, social economy, and corporate social responsibility (CSR).

Formulation of the Hypothesis

The central hypothesis of this study maintains that although social innovation and social enterprises are important components of the social economy, the lack of a coherent methodological framework and universally accepted definition hinders the accurate measurement of their impact on economic growth. However, when aligned with CSR strategies, they can improve the competitive advantage of the organizations involved.

Identification of Key Variables

- **Independent Variable**: Social innovation and social enterprises.
- **Dependent Variable**: Contribution to economic growth and competitive advantage.

Research Questions

1. **Q1**: Is social enterprise an important component of the social economy?
2. **Q2**: Is social innovation a well-researched academic field?
3. **Q3**: Can the impact of social innovation on economic growth be clearly measured in the studies reviewed?
4. **Q4**: Is there a universally accepted definition of social innovation?
5. **Q5**: Does corporate social responsibility improve competitive advantage?

The sentiment analysis applied to the scientific literature on the questions asked used a consensus tool that classifies positions into three categories: Positive, Indeterminate and Negative. The results are presented in Table 1 below.

Table 1. Sentiment Analysis

Questions \ Example of positions in articles	Positive	Indeterminacy	Negative	Neutrosophic probability
Q1	[27, 28, 29]			(1,0,0)
Q2	[30, 31]		[32]	(0.88, 0, 0.12)
Q3	[33, 34]		[35]	(0.83, 0, 0.17)
Q4	[36, 37]		[38, 39, 40]	(0.58, 0, 0.42)
Q5	[41, 42]	[43]		(0.94, 0.06, 0)

Univariate Neutrosophic Probability (UNP)

univariate neutrosophic probability (UNP) is calculated using the neutrosophic conjunction operator on the neutrosophic probabilities of each question.

$$UNP\ ((1,0,0),(0.88,0,0.12),(0.88,0,0.12),(0.83,0,0.17),(0.58,0,0.42),(0.94,0.06,0))$$
$$=\ (min\ (1{,}0.88, 0.83, 0.58, 0.94)\ , max\ (0,0,0,0,0.06)\ , max\ (0{,}0.12, 0.17, 0.42, 0)\ )$$

The univariate neutrosophic probability (UNP) for the set of questions is:

$$UNP = (0.58, 0.06, 0.42)$$

Figure 1: The univariate neutrosophic probability (UNP).

The set of questions has a higher likelihood of being true (58%) compared to being false (42%).

There is a small degree of indeterminacy (6%), indicating some uncertainty or ambiguity in the evaluation of the questions.

This type of probability allows for a more flexible and comprehensive representation of uncertainty, especially in complex or vague situations where traditional binary logic might not be adequate.

Conclusion

The analysis has established that the univariate neutrosophic probability (UNP) for the research questions is (0.58, 0.06, 0.42). The result suggests a strong likelihood that the hypothesis is true, with a confidence level of 67% in its validity. In contrast, the values for indeterminacy and falsehood are 33% and 17%, respectively. These findings emphasize a strong understanding of the relationship between social innovation and the social economy, but the existence of significant uncertainty requires additional investigation.

The conclusions offer practical direction for developing policies and programs that utilize social innovation in the economic sphere. The high probability of truth indicates that tactics based on these discoveries are built on a solid foundation, while the observed uncertainty emphasizes the need for more conceptual refinement and methodological improvement in certain areas. Practitioners and politicians can apply these findings to create projects that maximize the advantages of social innovation while addressing the stated limits.

This study makes a substantial contribution to the research field of social innovation and the social economy. This study introduces the use of the plithogenic method to uncertainty management, providing a comprehensive analysis that can be used as a basis for future research. However, the study does have certain drawbacks, including the uncertainty in the responses, which could affect the applicability of the findings. To improve the accuracy of assessing the influence of social innovation on economic growth, it is essential to create stronger and more comprehensive frameworks and clarify concepts.

References

[1] SD Allen, Q. Zhu, and J. Sarkis , "Expanding the conceptual boundaries of the nexus between sustainable supply chain management and circular economy," Clean Logistics and Supply Chains , vol. 2, p. 100011, 2021.

[2] A. Alonso Puelles and J. Echeverría Ezponda , "What is social innovation? The paradigm shift and its relationship with Social Work," Cuadernos de Trabajo Social , vol. 29, no. 2, pp. 163-171, 2016.

[3] D. Alonso-Martínez, N. González-Álvarez, and M. Nieto, "Social entrepreneurship versus social innovation ," Cuadernos Aragoneses de Economía , vol. 24, no. 1-2, Art. 1-2, 2014.

[4] D. Altamirano Analuisa , P. Bautista Toapanta , and J. Tisalema Borja, "Cooperative government and social innovation in the popular and solidarity economy sector," UNIANDES Episteme Magazine , vol. 6, no. 1, pp. 24-48, 2019.

[5] SE Anglada , "From social innovation to the solidarity economy. Practical keys for the development of public policies," CIRIEC-Spain, Journal of Public, Social and Cooperative Economy , vol. 88, Art. 88, 2017.

[6] R. Arocena and J. Sutz , "Universities and social innovation for global sustainable development from the South," Technological Forecasting and Social Change , vol. 162, p. 120399, 2021.

[7] F. Avelino , J.M. Wittmayer , B. Pel , P. Weaver, A. Dumitru , A. Haxeltine , R. Kemp, MS Jørgensen , T. Bauler , S. Ruijsink , and T. O'Riordan , “Transformative social innovation and (de)empowerment," Technological Forecasting and Social Change , vol. 145, pp. 195-206, 2019.

[8] Ricardo, J. E., Vázquez, M. Y. L., Palacios, A. J. P., & Ojeda, Y. E. A. (2021). Inteligencia artificial y propiedad intelectual. Universidad y sociedad, 13(S3), 362-368.

[9] S. Bhattacharya, K. Govindan , S. Ghosh Dastidar , and P. Sharma, "Applications of artificial intelligence in closed-loop supply chains: A systematic literature review and future research agenda," Transportation Research Part E: Logistics and Transportation Review , vol. 184, p. 103455, 2024.

[10] SE Bibri , J. Huang, SK Jagatheesaperumal , and J. Krogstie , "The synergistic interaction of artificial intelligence and digital twins in sustainable smart city planning: A comprehensive systematic review," Environmental Sciences and Ecotechnology , vol. 20, p. 100433, 2024. [Online]. Available : https://doi.org/10.1016/j.ese.2024.100433 .

[11] M.Ali, A. and Muthuswamy, M. (2023) “Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources”, Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[12] Smarandache, F. (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas. Neutrosophic Computing and Machine Learning, 20, 91-104.

[13] Smarandache, F. (2021). Introducción a la Lógica Plitogénica. Neutrosophic Computing and Machine Learning, 18, 1-6.

[14] Troya-Sarzosa, J. F., & Oña, C. (2024). Using plithogenic statistics to determine optimal guinea pig manure dosing in prickly pear cultivation. Neutrosophic Sets and Systems, 69, 58-66.

[15] Mohamed, M. (2024) “Partnership of Lean Six Sigma and Digital Twin under Type 2 Neutrosophic Mystery Toward Virtual Manufacturing Environment: Real Scenario Application”, Sustainable Machine Intelligence Journal, 8, pp. (6):99–109.

[16] Gafar, M. G., Elhoseny, M., & Gunasekaran, M. (2020). Modeling neutrosophic variables based on particle swarm optimization and information theory measures for forest fires. The Journal of Supercomputing, 76(4), 2339-2356.

[17] De la Cantera, D. H., Quiroz, R. C., Queija, M. L., Gonzalez, J. R., & Vazquez, M. Y. L. (2024). Analyzing Interdisciplinary Education in General Medicine Using Smarandache's Multivalued Logic Hypothesis Theory and Plithogenic Probability. Neutrosophic Sets and Systems, 70, 369-377.

[18] Smarandache, F. (2022). Plithogeny, plithogenic set, logic, probability and statistics: a short review. Journal of Computational and Cognitive Engineering, 1(2), 47-50.

[19] García, T. C. S., Álvarez, L. E. E., & Arias, M. A. I. (2024). Exploring the Impact of Educational Reforms and Their Dimensions on Teacher Performance Through the Analysis of Plithogenic Statistics. Neutrosophic Sets and Systems, 69, 161-170.

[20] Moscoso-Paucarchuco, K. M., Beraún-Espíritu, M. M., Gutiérrez-Gómez, E., Moreno-Menéndez, F. M., Vásquez-Ramírez, M. R., Fernández-Jaime, R. J., ... & Calderon-Fernandez, P. C. (2023). Plithogenic Statistical Study of Environmental Audit and Corporate Social Responsibility in the Junín Region, Peru. Neutrosophic Sets and Systems, 60, 538-547.

[21] Smarandache, F. (2010). Neutrosophic Logic-A Generalization of the Intuitionistic Fuzzy Logic. MULTISPACE & MULTISTRUCTURE. NEUTROSOPHIC TRANSDISCIPLINARITY, 396.

[22] Smarandache, F. (2024). Note on Partial Falsifiability of Fuzzy and Fuzzy-Extension Hypotheses. Plithogenic Logic and Computation, 1, 93-95.

[23] Popper, K. R. (1963). Science as falsification. Conjectures and refutations, 1(1963), 33-39.
[24] Torres Miño, C., Lozano, C., Jácome, E., Carrera Molina, D., & Gins, M. (2024). Neutrosophic statistical analysis of the efficacy of biostimulants in amaranth varieties. Neutrosophic Sets and Systems, 69, 94-101
[26] Consensus. (2023, October 18). Consensus Meter: Guardrails and Limitations - Consensus: AI Search Engine for Research. Consensus: AI Search Engine for Research. https://consensus.app/home/blog/consensus-meter/
[27] Kim, D., & Lim, U. (2017). Social Enterprise as a Catalyst for Sustainable Local and Regional Development. Sustainability, 9, 1427.
[28] Bridge, S., Murtagh, B., & O'Neill, K. (2009). Understanding the social economy and the third sector.
[29] McCabe, A., & Hahn, S. (2006). Promoting Social Enterprise in Korea and the UK: Community Economic Development, Alternative Welfare Provision or a Means to Welfare to Work?. Social Policy and Society, 5, 387 - 398.
[30] Have, R., & Rubalcaba, L. (2016). Social innovation research: An emerging area of innovation studies?. Research Policy, 45, 1923-1935.
[31] Bataglin, J., & Kruglianskas, I. (2022). Social Innovation: Field Analysis and Gaps for Future Research. Sustainability.
[32] Donati, L., & Wigren-Kristoferson, C. (2022). A legitimacy approach to social innovation initiatives at universities. Science and Public Policy.
[33] Wang, W. (2022). Toward Economic Growth and Value Creation Through Social Entrepreneurship: Modelling the Mediating Role of Innovation. Frontiers in Psychology, 13. https://doi.org/10.3389/fpsyg.2022.914700.
[34] Neumann, T. (2020). The impact of entrepreneurship on economic, social and environmental welfare and its determinants: a systematic review. Management Review Quarterly, 71, 553 - 584. https://doi.org/10.1007/s11301-020-00193-7.
[35] Cáceres-Carrasco, F., Santos, F., & Guzmán, C. (2020). Social capital, personal values and economic development: effect on innovation. An international analysis. Innovation: The European Journal of Social Science Research, 33, 70 - 95.
[36] Cáceres-Carrasco, F., Santos, F., & Guzmán, C. (2020). Social capital, personal values and economic development: effect on innovation. An international analysis. Innovation: The European Journal of Social Science Research, 33, 70 - 95.
[37] Edwards-Schachter, M., & Wallace, M. (2017). 'Shaken, but not stirred': Sixty years of defining social innovation. Technological Forecasting and Social Change, 119, 64-79.
[38] Marques, P., Morgan, K., & Richardson, R. (2018). Social innovation in question: The theoretical and practical implications of a contested concept. Environment and Planning C: Politics and Space, 36, 496 - 512.
[39] Mıhçı, H. (2018). Social innovation: a conceptual survey and implications for development studies. Innovation and Development, 9, 105 - 123.
[40] Pue, K., Vandergeest, C., & Breznitz, D. (2015). Toward a Theory of Social Innovation. Development of Innovation eJournal.
[41] Mai, N., Nguyen, A., & Nguyen, T. (2021). Implementation of Corporate Social Responsibility Strategy to Enhance Firm Reputation and Competitive Advantage. Journal of Competitiveness.
[42] Saeidi, S., Sofian, S., Saeidi, P., Saeidi, S., & Saaeidi, S. (2015). How does corporate social responsibility contribute to firm financial performance? The mediating role of competitive advantage, reputation, and customer satisfaction. Journal of Business Research, 68, 341-350.
[43] Vlastelica-Bakić, T., Krstovic, J., & Cicvarić-Kostić, S. (2012). The business case for corporate social responsibility. Marketing Science, 43, 191-198.
[44] Smarandache, F. (2024). Note on the partial falsifiability of the fuzzy and fuzzy extension hypotheses. Logic and Computing Plitogenic, 1, 93-95.
[45] Nabeeh, N. (2023) "Assessing and contrasting sustainable growth of various road transportation systems using a multi-criteria intelligent neutrosophic decision-making model", Sustainable Machine Intelligence Journal, 2, pp. (2): 1–12. doi:10.61185/SMIJ.2023.22102.
[46] Lathamaheswari , M., Sudha , S., B roumi , S., Smarandache, F., & Othman , C. (2022). Neutrosophic perspective of neutrosophic probability distributions and its application. Collected articles. Volume X: On neutrosophic, plitogenic, hypersoft set , hypergraphs and other topics, 267.
[47] Otio . (North Dakota). Consensus AI: an innovative tool to measure group agreement. Otio AI. Retrieved July 28, 2024 from https://otio.ai/blog/consensus-aisi
[48] Consensus. (2023, January 31). Consensus meter: barriers and limitations. Consensus . https://consensus.app/home/blog/consensus-meter/
[49] Potter, T.S., Zalewski, Z., Miao, M., Allsup, C., Thompson, K.M., Hayden, D. , ... and Lankau , E.W. (2024). Apply causal reasoning to investigate multicausality in microbial systems. Ecosphere , 15(5), e4782.

[50] Smarandache, F. (2022). Plitogeny , plitogenic set, logic, probability and statistics: a brief review. Engineering Magazine Computational and Cognitive, 1(2), 47-50.
[51] Mahmood, L., Mohammed, C., & Gilbert, J. (2021). Interprofessional Simulation Education to Improve Teamwork and Communication Skills Among Undergraduate Medical and Nursing Students Using the TeamSTEPPS Framework ® . Magazine medical , Armed Forces India, 77 Suppl 1, S42-S 48. https://doi.org/10.1016/j.mjafi.2020.10.026.
[52] Scherer, Y., Myers, J., O'Connor, T., & Haskins, M. (2013). Interprofessional simulation to foster collaboration between nursing and medical students. Simulation clinic in nursing , 9. https://doi.org/10.1016/J.ECNS.2013.03.001.
[53] Homeyer, S., Hoffmann, W., Hingst, P., Oppermann, R., and Dreier-Wolfgramm, A. (2018). Effects of interprofessional education for medical and nursing students: Facilitators, barriers, and expectations for optimizing future interprofessional collaboration: a qualitative study. BMC Nursing , 17. https://doi.org/10.1186/s12912-018-0279-x.
[54 Kent, F. and Keating, J. (2015). Interprofessional education in primary health care for entry-level students: a systematic review of the literature. Nursing education today, 35 12, 1221-31 . https://doi.org/10.1016/j.nedt.2015.05.005.
[55] Veri, F. (2023). Transform family resemblance concepts into fuzzy sets. Research and methods Sociologicals , 52 (1), 356-388.
[56] Rashno , E., Minaei-Bidgoli , B., and Guo , Y. (2020). An efficient clustering method based on data indeterminacy in the domain of neutrosophic sets. Engineering Applications of Artificial Intelligence, 89, 103411 .
[57] M. Ali , A. and Muthuswamy , M. (2023) "Multi-criteria neutrosophic decision-making framework for sustainable evaluation of energy production systems in renewable energy sources", Sustainable Machine Intelligence Journal , 4, pp. (3):1 –10. doi:10.61185/SMIJ.2023.44103.
[58] Smarandache, F. (2021). Plitogenic probability and statistics are generalizations of multivariate probability and statistics. Neutrosophic sets and systems , 43.
[59] Gafar, M.G., Elhoseny , M., and Gunasekaran , M. (2020). Modeling of neutrosophic variables based on particle swarm optimization and information theory measurements for forest fires. The Journal of Supercomputing , 76(4), 2339-2356.
[60] Reyes Espinoza, LK, Mafla Herrería, C., & Sánchez Sandoval, PA (2024). Hierarchical Neutrosophic analytical process for the evaluation of patients with Deep Vein Thrombosis treated at the IESS Ibarra hospital. Neutrosophic Computing and Machine Learning , 31, 198-209.
[61] Nabeeh , N. (2023) "Assessing and contrasting sustainable growth of various road transportation systems using a multi-criteria intelligent neutrosophic decision-making model", Sustainable Machine Intelligence Journal , 2, pp. (2): 1–12.
[62] Smarandache, F. (2018). Plitogenic set , an extension of the sharp, fuzzy, fuzzy intuitionistic and neutrosophic sets: revised. Neutrosophic Sets and Systems , 21, 153-167.
[63] Smarandache, F. (2021). Introduction to Plitogenic Logic. Neutrosophic Computing and Machine Learning, 18, 1-6.
[64] Rezaei , A. Oner , T. Katican , T. Smarandache, F. Gandotra , N. (2022). A brief history of fuzzy, intuitionistic, neutrosophic and plitogenic sets. Magazine Magazine Neutrosophic Sciences International , 18 (1) , 99-116.
[65] Troya- Sarzosa , JF, & Oña, C. (2024). Use of plitogenic statistics to determine the optimal dosage of guinea pig manure in prickly pear cultivation. Neutrosophic Sets and Systems , 69, 58-66.
[66] Smarandache, F. (2021). Introduction to Plitogenic Logic as a generalization of Multivariate Logic . Neutrosophic sets and systems , 1.
[67] Smarandache, F., Abdel-Basset , M., Vázquez, ML and Broumi , S. (2023). Neutrosophic sets and systems {Special issue: Neutrosophic advances and their impact on research in Latin America}, vol. 62, 2023. Neutrosophic Sets and Systems , 62(1), 40.
[68] Mohamed, Z., M. Ismail , M. and Abd El- Gawad , A. (2023) "Selection of sustainable suppliers using neutrosophic multi-criteria decision-making methodology " , Sustainable Machine Intelligence Journal , 3, pp. (2)
[69] Martin, N., Karuppiah , SJ, Smarandache, F., Rojas, R. and Vázquez, MYL (2023). Post-pandemic impact on occupational change of the rural population: a case study using the neutrosophic comparison t-test. In Cognitive intelligence with neutrosophic statistics in bioinformatics (pp. 259-265). Academic Press.
[70] C. Ashbacher, Introduction to Neutrosophic Logic, American Research Press, Rehoboth, NM, 2002
[71] Petty , G., Dolansky , M., & Luebbers , E. (2016). Practicing interprofessional teamwork between nursing and medical students. The Journal of Nursing Care , 5, 1-3. https://doi.org/10.4172/2167-1168.1000330.

[72] Maclean , S., Kelly, M., Geddes , F., & Della , P. (2017). Using simulated patients to develop communication skills in nursing education: an integrative review. Nursing education today, 48, 90-98 . https://doi.org/10.1016/j.nedt.2016.09.018.
[73] Horsley , T., Reed, T., Muccino , K., Quiñones, D., Siddall , V., and McCarthy, J. (2016). Developing a foundation for interprofessional education within medical and nursing curricula. Nurse educator , 41, 234–238. https://doi.org/10.1097/NNE.0000000000000255.
[74] O'Leary, K., Wayne, D., Haviley , C., Slade, M., Lee, J., & Williams, M. (2010). Improving teamwork: impact of structured interdisciplinary rounds in a medical teaching unit. Medicine Magazine Internal General, 25, 826-832. https://doi.org/10.1007/s11606-010-1345-6.
[75] Henderson, A. and Alexander, H. (2011). Maximizing the integration of medical and nursing students into clinical learning environments: an Australian perspective. , 131-148. https://doi.org/10.1007/978-90-481-3937-8_8.
[76] Berduzco -Torres, N., Choquenaira- Callañaupa , B., Medina, P., Chihuantito - Abal , L., Caballero, S., Gallegos, E., San-Martín, M., Bolton, R., & Vivanco , L. (2020). Factors related to the differential development of interprofessional collaboration skills in Medicine and Nursing students. Borders in Psychology , 11. https://doi.org/10.3389/fpsyg.2020.00432.
[77] Stychno , E. and Kulczycka , K. (2023). Communication in medical teams: a review of the literature. Teka Komisji Prawniczej PAN Oddział w Lublinie . https://doi.org/10.32084/tkp.6863.
[78] Azad , A., Min, J., Syed , S., & Anderson, S. (2020). Continuing nursing education in low- and middle-income countries: a narrative synthesis. BMJ Global Health, 5. https://doi.org/10.1136/bmjgh-2019-001981.
[79] Coster, S., Watkins, M., and Norman, I. (2017). What is the impact of professional nursing on patient outcomes globally? An overview of the research evidence. International Journal of Nursing Studies, 78, 76-83 . https://doi.org/10.1016/j.ijnurstu.2017.10.009.
[80] Bluestone, J., Johnson, P., Fullerton, J., Carr, C., Alderman, J., and BonTempo, J. (2013). Effective design and delivery of in-service training: Evidence from an integrative literature review. Resources Humans for Health, 11, 51 - 51. https://doi.org/10.1186/1478-4491-11-51.
[81] Collins, C., Doran, G., Patton, P., Fitzgerald, R., & Rochfort , A. (2021). Does education of primary care professionals promote patient self-care and improve outcomes in chronic diseases? An updated systematic review. BJGP Open, 5. https://doi.org/10.3399/BJGPO.2020.0186.
[82] Ton, E., Butteri , M., Alba, I., Duong , D., Wang, M., Suekawa , V., and Amin , A. (2017). Closing the clinical knowledge gap: achieving collaborative nursing education between hospitalists and nurses. Current Reports in Hospital and Emergency Medicine, 5, 114-118. https://doi.org/10.1007/s40138-017-0138-1.
[83] Rachwal , C., Langer, T., Trainor, B., Bell, M., Browning, D., and Meyer, E. (2018). Navigating communication challenges in clinical practice: A new approach to team education. Care nurse critics , 38, 15-22. https://doi.org/10.4037/ccn2018748.
[84] Hayat , K. (2023). Transforming medical education and training. Magazine biomedical of Pakistan . https://doi.org/10.54393/pbmj.v6i05.877.
[85] Goldman, M. (2016). Medical Education: Pushing boundaries to foster interdisciplinarity. Borders in Medicine , 3. https://doi.org/10.3389/fmed.2016.00015.
[86] Grift , T., and Kroeze , R. (2016). Design thinking as a tool for interdisciplinary healthcare education. Medicine academic , 91, 1234–1238. https://doi.org/10.1097/ACM.0000000000001195.
[87] Caverzagie , K., Nousiainen, M., Ferguson, P., Cate, O., Ross, S., Harris, K., Busari, J., Bould, M., Bouchard, J., Iobst, W., Carraccio , C. and Frank, J. (2017). General challenges for the implementation of competency-based medical education. Professor of Medicine , 39, 588 – 593. https://doi.org/10.1080/0142159X.2017.1315075.
[88] Walrath, J., Muganlinskaya , N., Shepherd, M., Awad , M., Reuland, C., Makary, M., & Kravet, S. (2006). Interdisciplinary medical, nursing, and administrative education in practice: The Johns Hopkins experience. Academic Medicine : Journal of the Association of American Medical Colleges, 81 8, 744-8. https://doi.org/10.1097/00001888-200608000-00013.
[89] Shepitko , V., Sherstyuk , O., Boruta , N., Tykhonova , O., Tarasenko , Y., Skotarenko , T., and Levchenko , O. (2023). Interdisciplinary integration as a main factor in the development of clinical and professional thinking of students of medical higher educational institutions of Ukraine. Ukrainian žurnal medicine , biology ta sportu . https://doi.org/10.26693/jmbs08.01.268.
[90] Bondarenko , S., Pylypyshyn , O., and Sopiha, M. (2022). TEACHING METHODS AND TECHNOLOGIES FOR MEDICAL STUDENTS ON THE BASIS OF INTERDISCIPLINARY INTEGRATION. doctor освіта . https://doi.org/10.11603/m.2414-5998.2021.4.12694.
[91] Ef , B., Rf , A., Me, B. and On, S. (2011). Interdisciplinary integration as an instrument for the development of a medical education standard. Morfologii ̆a ̆(Saint Petersburg , Russia), 139, 85.
[92] Scicluna , H., Grimm, M., O'Sullivan , A., Harris, P., Pilotto , L., Jones, P., & McNeil, H. (2012). Clinical

capabilities of graduates of an integrated outcomes-based medical program. BMC Education Medical , 12, 23 - 23. https://doi.org/10.1186/1472-6920-12-23.

[93] Scicluna , H., Grimm, M., O'Sullivan , A., Harris, P., Pilotto , L., Jones, P., & McNeil, H. (2012). Clinical capabilities of graduates of an integrated outcomes-based medical program. BMC Education Medical , 12, 23 - 23. https://doi.org/10.1186/1472-6920-12-23.

[94] Sobral, D. (1990). [Interdisciplinarity in medical education: 3 aspects of analysis] . Education medical and health , 24 4, 379-88.

[95] Bandiera , G., Boucher, A., Neville, A., Kuper , A., & Hodges , B. (2013). Integration and timing of education in basic and clinical sciences. Professor of Medicine, 35, 381 – 387. https://doi.org/10.3109/0142159X.2013.769674.

[96] Abouhawwash , M. and Jameel , M. (2023) "Evaluation factors of solar power plants to reduce costs under the multi-criteria neutrosophic decision-making model", Sustainable Machine Intelligence Journal , 2, pp. (1): 1–11. doi:10.61185/SMIJ.2023.22101.

[97] Abdelhafeez , A., Aziz , A., & Khalil , N. (2022). Building a sustainable social feedback loop: An artificial intelligence approach to opinion mining on Twitter. Journal of Sustainable Machine Intelligence , 1, 6-1.

Received: June 18, 2024. Accepted: August 8, 2024

Neutrosophic Sets and Systems {Special Issu: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Assessing Higher Education's Role in Personality Formation Using NeutroAlgebra

Jean Pierre Ramos-Carpio[1], Oscar José Alejo Machado[2], Jesús Estupiñán Ricardo[3], and Angel Braulio Martinez Vasquez[4]

[1] Study Center for Educational Quality and Scientific Research. Mexico. E-mail: jeanpierrer88@gmail.com
[2] Higher Technological Institute for Scientific Research and Innovation (ISTICI). Ecuador. E-mail: oalejo82@hotmail.com
[3] Higher Technological Institute for Scientific Research and Innovation (ISTICI). Ecuador. E-mail: jestupinan2728@gmail.com
[4] CCGECON-Training and Knowledge Management Center. Ecuador. E-mail: vasmarti10@gmail.com

Abstract. This study examines the role of higher education in personality formation, highlighting its influence on the development of critical thinking, autonomy in decision-making, social skills, and creativity. While there is a favorable influence in these areas, the data also highlight limitations in developing problem-solving skills and cementing ethical and moral ideals. Emotional management has a neutral influence, whereas social awareness and citizenship reveal modest improvement, suggesting a need for increased attention on building civic involvement. The study proposes the use of neutroalgebra and neutrosophic theory to more accurately assess these abstract dimensions. Future work could explore the use of other uninorms to generate neutroalgebras, potentially leading to more robust models and precise evaluation tools in higher education.

Keywords: Higher education, personality formation, critical thinking, Neutroalgebra, Prospector.

1 Introduction

The function of higher education in the creation of personality is a subject of rising interest in the academic and social sectors, because education not only strives to convey technical information, but also to shape people with firm values and ideals. The power of educational institutions to impact the personality of students, molding their attitudes, beliefs and actions, is a vital factor in the entire creation of the individual. Over the previous decades, several studies have underlined the necessity of an education that fosters not only academic brilliance, but also the personal and social development of students [1]. However, analyzing this influence remains a difficulty, particularly when it comes to measuring aspects as abstract and complex as personality. Historically, higher education has progressed from an elitist approach, meant for a tiny social elite, towards a more inclusive and democratic model, accessible to a greater section of the public [2]. This transition has brought with it new obligations for educational institutions, which must now ensure not just the transfer of information, but also the training of persons capable of tackling the ethical and social difficulties of modern society. In this context, higher education has been recognized as a critical tool for the holistic development of the human person, but the method in which this effect is exerted and quantified has been a source of continual discussion.

The key challenge that this research tackles consists in the difficulties of precisely and objectively measuring how higher education contributes to the construction of students' personalities. Although there are numerous techniques to evaluate educational effect in academic and professional terms, the assessment of its influence on personality remains an underexplored subject and full of methodological obstacles [3]. The complexity and multidimensional character of human personality implies that any effort at measuring needs sophisticated instruments and methodologies that can manage the ambiguity and indeterminacy inherent in this process. Despite developments in educational psychology and personality theory, there is a considerable vacuum in the research regarding approaches that effectively capture the interplay between higher education and personality formation [4]. Traditional evaluation techniques, which mainly concentrate on cognitive and behavioral components, fail to capture the complete educational process or the subtle impacts that the educational environment imposes on pupils. Furthermore, most research concentrate on practical outcomes, leaving aside the influence on more abstract and subjective characteristics, such as values, attitudes and beliefs [5]. This research presents a solution to this difficulty by employing neutroalgebra, a method that permits a more comprehensive and accurate examination of the effect of higher education on personality development [6]. Neutroalgebra, with its capacity to manage uncertainty and indeterminacy [7,8, 9], is proposed as a unique technique to address the difficulty inherent in the assessment of these abstract elements of human development [10]. Using this method, we strive to not only measure the effect of higher education, but also to better understand how students' personalities grow and change in an educational setting.

The formation of personality is a dynamic and ongoing process shaped by a complex interplay of genetic, environmental, and social factors. From the earliest stages of life, personality develops via distinct and sometimes surprising routes, impacted by experiences, relationships, and social environments such as family, education, and media. This process is distinguished by a conflict between society conformity and individual authenticity, needing a balance that is vital for personal progress. As individuals navigate life's stages, their personalities continue to evolve, demonstrating the capacity for adaptation and self-determination in the face of life's challenges. Ultimately, personality formation is a lifetime process of self-discovery and growth, reflecting both the individual's inner world and their interactions with the larger environment[11].

The application of this method has the potential to offer new perspectives on how educational institutions can improve their training programs to encourage more balanced and robust personal development. By better understanding the relationship between higher education and personality formation, educators and administrators will be able to design more effective strategies to support the holistic growth of their students. This study will therefore not only contribute to the existing theory on education and personality, but will also offer practical tools to improve the quality of higher education as a whole [12].

2 Preliminaries

2.1 NeutroAlgebra and PROSPECTOR function

The idea of NeutroAlgebra proposes a new algebraic structure that generalizes Partial Algebra by including the notion of indeterminacy. In NeutroAlgebra, a given set X is divided into three regions: <A> (true), <antiA> (false), and <neutA> (indeterminate), which may overlap depending on the context. This framework allows for the definition of NeutroFunctions, which are partially well-defined, indeterminate, or outer-defined within their domain, as well as NeutroAxioms, which are true, false, or indeterminate for different elements. These notions expand standard algebraic structures to cover a greater range of possibilities, accommodating uncertainty and partial truth, and are further classed alongside Classical and AntiAlgebras. Additionally, the PROSPECTOR function, applied in the MYCIN expert system [13] , is specified as an uninorm that conforms to particular mathematical criteria, including commutativity, associativity, and monotonicity.

Definition 1 ([14]): Let X be a given nonempty space (or simply set) integrated into a universe of discourse U. Let <A> be an item (concept, attribute, idea, statement, theory, etc.) specified on the set X. Through the process of neutrosophication, we split the set X into three regions [two opposite ones <A> and <antiA>, and one neutral (indeterminate) <neutA> between them], regions which may or may not be disjoint – depending on the application, but they are exhaustive (their union equals the whole space).

A NeutroAlgebra is an algebraic structure characterized by the existence of at least one NeutroOperation or NeutroAxiom. A NeutroAxiom is a unique sort of axiom that is true for certain items, is ambiguous for others, and is untrue for the remaining elements. This allows NeutroAlgebra to incorporate a degree of flexibility and uncertainty, distinguishing it from traditional algebraic systems where axioms are either universally true or false.The NeutroAlgebra is a generalization of Partial Algebra, which is an algebra that has at least one Partial Operation, while all its Axioms are totally true (classical axioms).

Definition 2 ([14]): A function f: X→Y is termed a Partial Function if it is well-defined for certain items in X, and undefined for all the other elements in X. Therefore, there exist certain elements a ∊ X such that f(a) ∊ Y (well-defined), whereas for every other element b ∊ X we obtain f(b) is undefined.

Definition 3 ([14]): A function f: X→ Y is called a NeutroFunction if it has elements in X for which the function is well-defined (degree of truth (T)), elements in X for which the function is indeterminate {degree of indeterminacy (I)}, and elements in X for which the function is outer-defined {degree of falsehood (F)}, where T, I, F ∊ [0, 1], with (T, I, F) ≠ (1, 0, 0) that represents the (Total) Function, and (T, I, F) ≠ (0, 0, 1) that represents the AntiFunction.

Classification of Functions

i. (Classical) Function, which is a function well-defined for all the elements in its domain of definition.
ii. NeutroFunction, which is a function partially well-defined, partially indeterminate, and partially outer-defined on its domain of definition.
iii. AntiFunction, which is a function outer-defined for all the elements in its domain of definition.

Definition 4 ([14]): A (classical) *Algebraic Structure* (or Algebra) is a nonempty set *A* endowed with some (totally well-defined) operations (functions) on *A*, and satisfying some (classical) axioms (totally true) - according to the Universal Algebra.

Definition 5 ([14]): A (classical) *Partial Algebra* is an algebra defined on a nonempty set *PA* that is endowed with some partial operations (or partial functions: partially well-defined, and partially undefined). While the axioms (laws) defined on a Partial Algebra are all totally (100%) true.

Definition 6 ([14]): A *NeutroAxiom* (or *Neutrosophic Axiom*) defined on a nonempty set is an axiom that is true for some set of elements {degree of truth (T)}, indeterminate for other set of elements {degree of indeterminacy (I)}, or false for the other set of elements {degree of falsehood (F)}, where T, I, F ∈ [0, 1], with (T, I, F) ≠ (1, 0, 0) that represents the (classical) Axiom, and (T, I, F) ≠ (0, 0, 1) that represents the AntiAxiom.

Classification of Algebras

i) A (classical) *Algebra* is a nonempty set *CA* that is endowed with total operations (or total functions, i.e. true for all set elements) and (classical) Axioms (also true for all set elements).
ii) A *NeutroAlgebra* (or *NeutroAlgebraic Structure*) is a nonempty set *NA* that is endowed with: at least one *NeutroOperation* (or *NeutroFunction*), or one *NeutroAxiom* that is referred to the set (partial-, neutro-, or total-) operations.
iii) An *AntiAlgebra* (or *AntiAlgebraic Structure*) is a nonempty set *AA* that is endowed with at least one *AntiOperation* (or *AntiFunction*) or at least one *AntiAxiom*.

Additionally, the PROSPECTOR function is defined in the MYCIN expert system in the following way; it is a mapping from $[-1,1]^2$ into $[-1,1]$ with formula, [15, 16]:

$$P(x,y)=\frac{x+y}{1+xy} \quad (1)$$

This function is a uninorm, [17, 18, 19], with neutral element 0, thus it fulfils commutativity, associativity, and monotonicity. Here we respect the condition that $P(-1,1)$ and $P(1,-1)$ are undefined.

3 Material and Methods

This section contains the results of this investigation, for this we explain some characteristics of the method . The set NeutroGroup consists of all integers between $-n$and n plus the symbolic element *I* to represent indeterminacy [20]. This is $NG_7=\{-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,9,10,I\}$and $\odot_{10}$is used

First of all, for convenience $P(x,y)$ is extended to $\bar{P}(x,y)$ such that:

$\bar{P}(x,y)=P(x,y)$ for all $(x,y)\in[-10,10]^2\setminus\{(-10,10),(10,-10)\}$,

$\bar{P}(-10,10)=\bar{P}(10,-10)=I$,

$\bar{P}(I,I)=I$.

$$\bar{P}(I,x)=\bar{P}(x,I)=\begin{cases}\text{I, if x} > 0\\ \text{x, if x} \le 0\end{cases}. \quad (2)$$

Definition 7 : Let S be a finite set defined as $S=\left\{(x,y): x,y\in\left\{\frac{k}{7},I\right\},\ k\in\mathbb{Z}\cap[-7,7]\right\}$.

The operator $\odot_{10}$ is defined for every $(x,y)\in S$, such that:

If $\bar{P}(x,y)$ is not undefined, then $\odot_{10}\ y=\frac{round(\bar{P}(x,y)*10)}{10}$, where *round* is the function that output the integer nearest to the argument.

If $\bar{P}(x,y)$ is undefined then $x\odot_{10} y=I$.

Then $\odot_{10}$ is a finite NeutroAlgebra. This is because $\odot_{10}$ is commutative and associative for the subset of elements of S without any undefined component, but it is not associative otherwise.

E.g., if $a = -6, b = 5, c = I$, then $a \odot_{10} (b \odot_{10} c) = a$ and $(a \odot_{10} b) \odot_{10} c = -1 \neq a$, therefore associativity is a NeutroAxiom.

Function *round* is used for guarantying $\odot_{10}$ is an inner operator.

For the sake of clarity, we use the elements of Tables 1 and Table 2 of Cayley tables.

Table 1: Cayley table of $\odot_{10}$.

$\odot_{10}$	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**
-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
-9	-10	-10	-10	-10	-10	-10	-10	-9	-9	-9	-9
-8	-10	-10	-10	-10	-9	-9	-9	-9	-9	-8	-8
-7	-10	-10	-10	-9	-9	-9	-9	-8	-8	-7	-7
-6	-10	-10	-9	-9	-9	-8	-8	-8	-7	-7	-6
-5	-10	-10	-9	-9	-8	-8	-8	-7	-6	-6	-5
-4	-10	-10	-9	-9	-8	-8	-7	-6	-6	-5	-4
-3	-10	-9	-9	-8	-8	-7	-6	-6	-5	-4	-3
-2	-10	-9	-9	-8	-7	-6	-6	-5	-4	-3	-2
-1	-10	-9	-8	-7	-7	-6	-5	-4	-3	-2	-1
I	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
0	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	-10	-9	-8	-6	-5	-4	-3	-2	-1	0	1
2	-10	-9	-7	-6	-5	-3	-2	-1	0	1	2
3	-10	-8	-7	-5	-4	-2	-1	0	1	2	3
4	-10	-8	-6	-4	-3	-1	0	1	2	3	4
5	-10	-7	-5	-3	-1	0	1	2	3	4	5
6	-10	-7	-4	-2	0	1	3	4	5	5	6
7	-10	-5	-2	0	2	3	4	5	6	6	7
8	-10	-4	0	2	4	5	6	7	7	8	8
9	-10	0	4	5	7	7	8	8	9	9	9
10	I	10	10	10	10	10	10	10	10	10	10

Table 2: Cayley table of $\odot_{10}$ (Continuation).

$\odot_{10}$	**I**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	I.
-9	-9	-9	-9	-8	-8	-7	-7	-5	-4	0	10
-8	-8	-8	-7	-7	-6	-5	-4	-2	0	4	10
-7	-7	-6	-6	-5	-4	-3	-2	0	2	5	10
-6	-6	-5	-5	-4	-3	-1	0	2	4	7	10
-5	-5	-4	-3	-2	-1	0	1	3	5	7	10
-4	-4	-3	-2	-1	0	1	3	4	6	8	10
-3	-3	-2	-1	0	1	2	4	5	7	8	10
-2	-2	-1	0	1	2	3	5	6	7	9	10
-1	-1	0	1	2	3	4	5	6	8	9	10
I	I	I.	I	I	I.	I	I	I.	I	I	I.
0	0	1	2	3	4	5	6	7	8	9	10
1	I	2	3	4	5	6	7	7	8	9	10
2	I	3	4	5	6	6	7	8	9	9	10
3	I	4	5	6	6	7	8	8	9	9	10
4	I	5	6	6	7	8	8	9	9	10	10

$\odot_{10}$	I	1	2	3	4	5	6	7	8	9	10
5	I	6	6	7	8	8	8	9	9	10	10
6	I	7	7	8	8	8	9	9	9	10	10
7	I	7	8	8	9	9	9	9	10	10	10
8	I	8	9	9	9	9	9	10	10	10	10
9	I	9	9	9	10	10	10	10	10	10	10
10	I	10	10	10	10	10	10	10	10	10	10

$\odot_{10}$ It satisfies the properties of commutativity and associativity and has 0 as a null element. Furthermore , it satisfies each one of the following properties [20]:

- If $x, y < 0$ then $x \odot_{10} y \leq min(x, y)$,
- If $x, y > 0$then $x \odot_{10} y \geq max(x, y)$,
- If $x < 0$and $y > 0$ or if $x > 0$and $y < 0$, then we have $\min(x, y) \leq x \odot_7 y \leq \max(x, y)$.
- $\forall x \in G, x \odot_{10} 0 = x$.
- $(-10) \odot_{10} 10 = 10 \odot_{10} (-10) = I$.

The methodology employed in this study to evaluate the effectiveness of higher education in personality formation involves the following steps:

Step 1: Definition of Variables to Evaluate

The first step involves defining the key variables that will be used to evaluate the effectiveness of higher education as a critical instrument in personality formation. These variables, denoted as $V = \{v_1, v_2, \dots, v_n\}$, , represent the specific aspects of personality development that will be assessed throughout the study. Each variable corresponds to a distinct dimension of personality, reflecting how higher education influences the overall formation and growth of an individual's character and abilities.

Step 1 Data Collection: Gather opinions from a group of experts $P = \{p_1, p_2, \dots, p_l\}$, in short, informal texts about aspects/variables to evaluate $V = \{v_1, v_2, \dots, v_n\}$ on a discrete scale from -10 to 10. "I" is assigned to indicate indeterminacy.

To evaluate using scale, where the values range from -10 to 10, where -10 is strongly disagree or extreme disapproval

In this scale, negative values represent levels of disagreement or disapproval, 0 represents a neutral position, and positive values indicate levels of agreement or approval.

Step 3 Aggregation: For each variable each person opinión is aggragated to obtain the final results, we have a group of people whose opinion is studied. Let's call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i}$ it is the total value of the ith variable,

$$x_{total,i} = x_{i,1} \odot_{10} x_{i,2} \odot_{10} x_{i,3} , \dots, \odot_{10} x_{i,n} \quad (1)$$

This aggregated value, $x_{total,i}$ represents the collective assessment for the iii-th variable, taking into account the diverse perspectives of all individuals in the group $P = \{p_1, p_2, \dots, p_l\}$, The operation $\odot_{10}$ used in the aggregation allows for a nuanced combination of opinions, accommodating both consensus and the presence of indeterminacy in the responses[21, 22]. The final result for each variable reflects the overall effectiveness of higher education in influencing that particular aspect of personality formation, providing a comprehensive view of how different educational experiences impact student development [23, 24].

4 Case Study

To evaluate the effectiveness of higher education as a key instrument in personality formation, an innovative approach is adopted based on neutrosophic theory. This approach allows us to more accurately capture and analyze the complex and often contradictory perceptions that students have about their role in personality formation. The applied technique encompasses numerous essential factors, concentrating on how students view and value their effect on the personal development of students. In the next sections, the processes and computations done in this investigation are explained in detail. Neutrosophic techniques were utilized to analyze the positivity, negativity and indeterminacy of the views obtained, which enabled us to acquire a more nuanced and realistic perspective of the usefulness of higher education in the creation of personality.

Step 1: Definition of Variables to Evaluate

The factors to assess The efficiency of higher education as a crucial tool in the creation of personality are the following:

Development of Critical Thinking: To what degree does higher education enhance your capacity to examine, question, and reflect critically on numerous topics?

Problem Solving Ability: How effectively do you believe higher education has enhanced your ability to address and solve complex problems, both in the academic field and in real-life situations?

Autonomy and Decision Making: To what degree does higher education encourage independence in thinking and the capacity to make informed and ethical decisions?

Development of Social abilities: How effectively does higher education help to the growth of your interpersonal abilities, such as effective communication, empathy, and collaboration?

Strengthening Ethical and Moral beliefs: How much has higher education affected the consolidation of your ethical and moral beliefs, and their application in everyday life?

Capacity for Adaptation and Flexibility: How equipped do you believe higher education has enabled you to adjust to changes and tackle problems in diverse personal and professional contexts?

Personal Identity and Self-Concept: To what degree has higher education helped you create a clear sense of personal identity and a positive, realistic self-concept?

Social Awareness and Citizenship: How efficiently has higher education contributed to your creation of social awareness, including your commitment to the community and civic responsibility?

Emotional Management: How well do you feel higher education has qualified you to handle your emotions successfully, especially stress management and resilience in the face of difficulties?

Development of Creativity and invention: How much has higher education spurred your creativity and potential for invention, both in the academic area and in your professional and personal life?

Step 2: Data Collection

A survey was carried out among 10 studnets from different universities in Ecuador, who evaluated each aspect of personality formation on a scale from -10 to 10

Table 3: Results of Data Collection and Aggregation.

Variable	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8	Student 9	Student 10	$\odot_{10}$
v1	6	5	7	4	6	8	3	6	5	6	5
v2	-3	-2	-3	-1	-2	-4	-3	-2	-1	-3	-2
v3	3	4	5	3	3	5	4	5	3	4	4
v4	4	4	5	5	4	5	3	4	5	4	4
v5	-2	-3	-2	-3	-3	-2	-2	-3	-2	-2	-2
v6	0	0	4	0	5	0	0	3	0	2	2
v7	0	0	0	3	0	0	-2	0	0	0	0
v8	1	0	3	4	0	0	4	4	3	9	4
v9	0	0	I	3	0	0	2	0	3	3	3
v10	-3	-2	-3	-1	-2	-3	-4	-3	-2	-3	-3

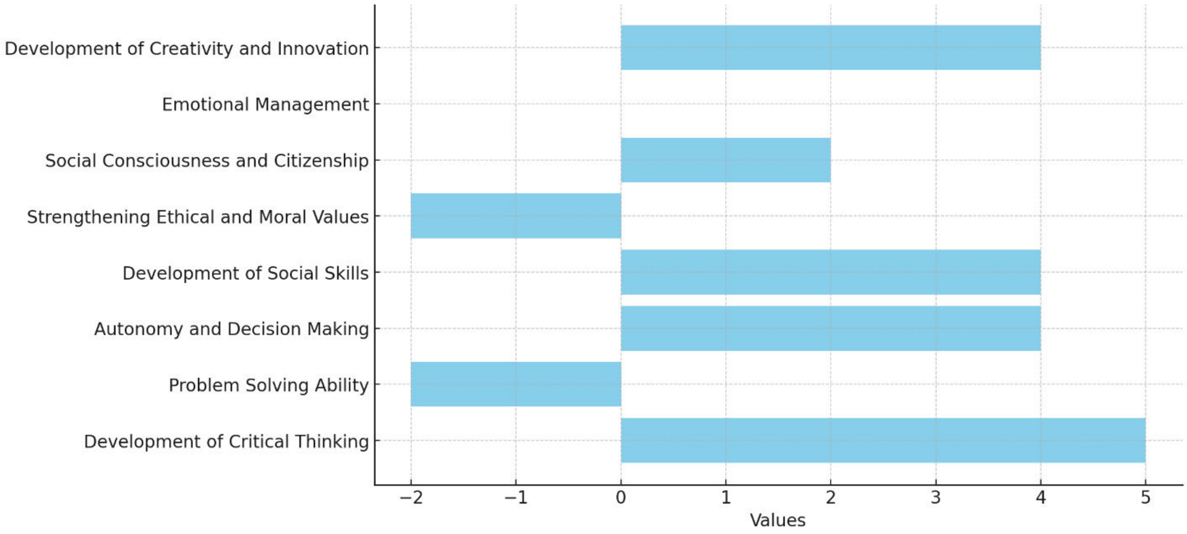

Figure 1: Aggragated alues usisng $\odot_{10}$

The findings reveal a generally beneficial influence of higher education on students' critical thinking (5), autonomy in decision-making (4), social skills (4), and creativity (4). However, there are obstacles in areas such as problem-solving (-2) and the consolidation of ethical and moral ideals (-2). Emotional management is neutral (0), showing that although students gain in numerous important areas, there are major gaps in problem-solving and ethical growth that may require more attention. Social awareness and citizenship obtained a fairly good score (2), showing space for growth in boosting civic involvement.

Conclusion

This research underlines the crucial impact of higher education in forming numerous parts of students' personalities, including critical thinking, autonomy in decision-making, social skills, and creativity. The results show that higher education institutions significantly affect these qualities, contributing to the holistic development of students. However, the research also emphasizes areas where higher education may fall short, notably in strengthening problem-solving ability and the consolidation of ethical and moral beliefs. The neutral impact on emotional regulation offers a possible area for development, while the modest positive effect on social awareness and citizenship refers to the need for greater efforts to encourage civic involvement among students. Future study might examine the use of various uninorms within the framework of neutroalgebra to better represent the complexity and subtleties of personality development in higher education. By experimenting with diverse mathematical frameworks, such as variable uninorms, researchers may construct more robust models that give deeper insights into how educational contexts impact student growth. Additionally, these techniques might assist enhance the assessment instruments utilized in this research, leading to more accurate assessments of the wider implications of higher education on personality development.

References

[1] AEP Roa and PFP Bernal, "Foundation of the epistemological stance of the university teacher-researcher," Revista Entornos , no. 24, pp. 287-294, 2011.
[2] JP Rozo, JPO Quevedo, JN Uribe, and INP Rojas, "Social competencies in university teachers in the city of Bogotá," Global Knowledge , vol. 4, no. 2, pp. 57-68, 2019.
[3] C. Ruiz Bueno, Ó . Mas Torelló , J. Tejada Fernández, and A. Navío Gámez, "Functions and performance scenarios of the university professor: Notes for the definition of the profile based on competencies," Revista de la Educación Superior , vol. 37, no. 146, pp. 115-132, 2008.
[4] JA Ruiz, "What does John Dewey contribute about the role of the teacher in moral education?," Education Forum , vol. 16, no. 24, pp. 125-140, 2018.

[5] REM Salas and PRR Pavón, "Challenges and challenges in Higher Education: a look from the perception of teachers," Education in the Knowledge Society , vol. 23, article e264020, 2022.

[6] Smarandache, F. (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas. Neutrosophic Computing and Machine Learning, 20, 91-104.

[7] Nabeeh, N. (2023) "Assessment and Contrast the Sustainable Growth of Various Road Transport Systems using Intelligent Neutrosophic Multi-Criteria Decision-Making Model", Sustainable Machine Intelligence Journal, 2, pp. (2):1–12.

[8] Vázquez, M. L., Franco, P. E. D. P., & Palacio, A. J. P. (2022). Neutrosophic dematel in the analysis of the causal factors of youth violence. International Journal of Neutrosophic Science, 18(3), 199-99.

[9] Mohamed, Z., M. Ismail, M. and Abd El-Gawad, A. (2023) "Sustainable Supplier Selection using Neutrosophic Multi-Criteria Decision Making Methodology", Sustainable Machine Intelligence Journal, 3, pp. (2):1–9.

[10] PDP Sánchez Cabezas, MM López Rodríguez del Rey, and Y. Alfonso Moreira, "Educational orientation in the professional pedagogical activity of the university teacher," Conrado , vol. 14, pp. 50-57, 2018.

[11] C. Silva, "From natural philosophy to the psychology of morality in John Locke's 'Essay on Human Understanding'," Universidad Autónoma de la Ciudad de México-UACM, 2022.

[12] Falcón, V. V., Martínez, B. S., Ricardo, J. E., & Vázquez, M. Y. L. (2021). Análisis del Ranking 2021 de universidades ecuatorianas del Times Higher Education con el Método Topsis. Revista Conrado, 17(S3), 70-78.

[13] Van Melle, W. (1978). MYCIN: a knowledge-based consultation program for infectious disease diagnosis. International journal of man-machine studies, 10(3), 313-322.

[14] Caballero, E. G., Leyva, M., Ricardo, J. E., & Hernández, N. B. (2022). NeutroGroups Generated by Uninorms: A Theoretical Approach. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 155-179). IGI Global.

[15] A. K. Tsadiras and K. G. Margaritis, "The MYCIN certainty factor handling function as uninorm operator and its use as a threshold function in artificial neurons," Fuzzy Sets and Systems, , vol. 93 pp. 263-274, 1998.

[16] M. Al-Tahan, B. Davvaz, F. Smarandache, and A. Osman, "Properties of Production Neutro Ordered Semigroups," Neutrosophic Sets and Systems, vol. 42, pp. 178-190, 2021.

[17] E. H. Shortliffe, Computer-Based Medical Consultations: MYCIN. New York: Elsevier, 1976.

[18] R. R. Yager and A. Rybalov, "Bipolar aggregation using the Uninorms," Fuzzy Optimization and Decision Making,, vol. 10, pp. 59–70, 2011.

[19] Valdivieso, W. V., Viteri Rodríguez, J. A., Viteri Villa, M. F., Rodríguez Reyes, E. R., & Niño Montero, J. S. (2021). Assessment of Barriers to Access Public Services for Immigrants in Ecuador using a NeutroAlgebra-based Model. Neutrosophic Sets & Systems, 44.

[20] Cruzaty, L. E. V., & Reynoso, A. M. R. (2024). New Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of Ethics in Companies. Neutrosophic Sets and Systems, 70, 264-270.

[21] Smarandache, F. (2022). Introducción a la Super-Hiper-Álgebra y la Super-Hiper-Álgebra Neutrosófica. Neutrosophic Computing and Machine Learning, 20, 1-6.

[22] Martínez, C. R., Hidalgo, G. A., Matos, M. A., & Smarandache, F. (2020). Neutrosophy for Survey Analysis in Social Sciences. Neutrosophic Sets and Systems, 37(1), 49.

[23] Moya, J. R. V., Gómez, G. A. Á., Sánchez, C. V., & Vázquez, M. Y. L. (2021). El liderazgo sincrónico en la transformación de la educación superior. Universidad Y Sociedad, 13(S2), 139-146.

[24] Evans, N. J., Forney, D. S., Guido, F. M., Patton, L. D., & Renn, K. A. (2009). Student development in college: Theory, research, and practice. John Wiley & Sons.

Received: June 19, 2024. Accepted: August 9, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Evaluative Analysis of the Incorporation of ICT in the Solving of Mathematical Problems in Primary Education Students with the Help of Neutrosophic Statistics on Fuzzy Data

Gladys Martha Flores-Choque[1], Beker Maraza-Vilcanqui[2], Nain Maraza-Vilcanqui[3], Queke Maraza-Vilcanqui[4], Jesús Ttito-Quispe[5], Elizabeth Norma Calixto-Arias[6], Liliana Huaranga-Rivera[7], and Aniceto Elías Aguilar-Polo[8]

[1] Institute for Research on Andean and Amazonian Cultures, Puno. Peru. E-mail: gladysmarthaf@gmail.com
[2] National Intercultural University of the Amazon, Pucallpa. Peru. E-mail: bmarazav@unia.edu.pe
[3] National University of the Altiplano, Puno. Peru. E-mail: marazanain@unap.edu.pe
[4] Institute for Research on Andean and Amazonian Cultures, Puno. Peru. E-mail: milqueades@gmail.com
[5] National Intercultural University of the Amazon, Pucallpa. Peru. E-mail: jttitoq@unia.edu.pe
[6] National Intercultural University of the Amazon, Pucallpa. Peru. E-mail: ecalixtoa@unia.edu.pe
[7] National Intercultural University of the Amazon, Pucallpa. Peru. E-mail: ehuarangar@unia.edu.pe
[8] National Intercultural University of the Amazon, Pucallpa. Peru. E-mail: aaguilarp@unia.edu.pe

Abstract. This paper aims to determine the influence of incorporating Information and Communication Technologies (ICT) in solving mathematical problems in students of the Primary Educational Institution N° 70656 Ricardo Palma Puno, Peru. To collect the information, a pre-test and a post-test were applied in the variable "resolution of mathematical problems" to an experimental group that received classes with the help of the use of ICT. The test was applied to a control group of the same age that followed the traditional method of learning mathematics. Teachers were asked to evaluate children using a linguistic scale because it is simpler and more reliable. These values are associated with a triple of fuzzy numbers, to which t-tests are applied to compare the results of the pre-test with the post-test, and also between the experimental group and the control group. The triple represents the truthfulness, indeterminacy, and falseness. For processing, the theory of statistical methods for fuzzy data was generalized to the neutrosophic framework. The results show that the incorporation of ICT in the experimental group has significantly influenced the learning of mathematical problem-solving in the case of the skills of subtracting and dividing. In general, it can be stated that the incorporation of ICT in the resolution of mathematical problems of addition, subtraction, multiplication, and division in primary education has positive effects.

Keywords: Primary education, mathematics teaching, fuzzy data statistics, fuzzy numbers, random fuzzy numbers, t-test, Neutrosophic Statistics, Neutrosophic Probability, Interval-Valued Neutrosophic Sets.

1 Introduction

The application of technology is part of our existence and is a natural phenomenon in the development of humanity. The advancement of science and technology allowed teachers to give pedagogical use to technological instruments in the teaching-learning process.

Currently, extraordinary use is made of tools and devices that improve the interrelation of information and communication between people, the need for their use goes from lesser to greater. This phenomenon has meant a metamorphosis in the way people relate today.

Information and Communications Technologies (ICT) have necessarily become a novel communication option due to the form of changes in the processing and access to information. Therefore, the priorities of using the inclusion of ICT in education as policies at a global level have gained much importance since the seventies of the 20th century and with predominance today, definitely more frequently in recent years in education.

The use of technology appears in various fields and for different purposes. Teachers must know how ICT works, they must be able to face the challenges in the management of technologies, and they must update themselves, investigate, recreate, and be ready to interact in an information society. Immersion in the field of ICT is part of the job of an education professional. It is also important for the social insertion of the child, as well as of young people and adults in the circumstances faced by the information society. Youth are massively involved in the incorporation of ICT as a great opportunity for change in the process of communication transformation. The use of the latest generation mobile devices is a powerful part of the youth population. Educational agents who use technologies open new, more active, and useful spaces compared to those who do not use them or who are passive users.

The use of the Internet to communicate digitally is common today; it is a successful and fast way to communicate asynchronously using personal smartphones. Especially in the time of the COVID-19 pandemic, classes between teachers and students at all educational levels were possible mostly with the use of mobile phones.

There are many advantages of using ICT in education today. Its incorporation into the educational field has generated a more appropriate way to work and learn, above all, applications have been created and many forms of access and use of technological tools for learning have been expanded, which has been positive for the massification of the knowledge. ICT allows the freedom to work from anywhere. As in the field of health, ICT has a high potential to support educational interventions in the promotion of healthy habits and lifestyles, as well as in the prevention of health risk factors. These are more interactive and collaborative tools, they allow students to learn more entertainingly, be better trained, and communicate with each other. Furthermore, according to research, the contribution of ICT to learning is evident when they are used in a collaborative and participatory way.

However, there are also challenges and disadvantages of using ICT. There is inequality in its use by members of society; those individuals with a higher economic level take advantage of it more. There is also the limitation of knowledge of how to use it, especially for those people born in the analog era. The use of ICT can become a mental problem for those people who consume it compulsively; the individual can even isolate himself from direct human contact, although he (she) has the feeling of connection with the rest of the world.

This article aims to determine the influence of the incorporation of Information and Communication Technologies in the learning of mathematical problem-solving in students of the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru. The results obtained from the study can suggest what happens in other schools in the province and the country. For this purpose, the evaluation of basic mathematical skills was collected from 70 boys and girls from the institution for subsequent statistical study as an experimental group and from 71 boys and girls from a control group.

This study was carried out by comparing the results between the group of students who were allowed to use ICT in their learning and the control group who traditionally received classes. Members of each group were randomly selected with simple random sampling. The results between the two groups were compared using an unpaired Student's t-test [1, 2]. To complement, comparisons were also made between the changes obtained by the experimental group before and after using ICT in classes.

It was determined not to use numerical data in the evaluations as is done traditionally, but rather values according to a linguistic scale. Each value of this scale is associated with a triple of fuzzy numbers that are processed [3], one is used to determine truthfulness, another to determine indeterminacy and a third represents falseness. This procedure is justified because natural language is a form more similar to human beings than the numerical scale. Therefore, the theory of what is known as fuzzy data statistics [4-7] is generalized to the neutrosophic field. The idea is to extend the definition of Fuzzy Random Number to the neutrosophic field, where the statistical tests that are carried out for triangular fuzzy numbers are repeated for two more triangular fuzzy numbers. With this procedure, accuracy is gained when indeterminacy is incorporated.

This generalization takes us to the field of Neutrosophic Sets. Since these are numerical intervals, we would have particular cases of Interval-Valued Neutrosophic Sets [8]. In this case, we would have intervals of classic probabilities, another interval where there are undetermined probabilities and a third interval of probabilities that do not hold.

The article is divided into a Materials and Methods section, where the basic notions of fuzzy data statistics are explained, which includes the theory of fuzzy numbers, some notions of neutrosophic sets, and also the Student's t-test. The following section contains the results of the study carried out. The article finishes with a section dedicated to conclusions.

2 Materials and Methods

2.1 Basic Notions of Fuzzy Random Numbers

Fuzzy random numbers are the basis of the theory presented in this article. Below are the formal definitions.

Definition 1. ([7, 9]) A *fuzzy number* is a map $\overline{\mathrm{U}}: \mathbb{R} \rightarrow [0, 1]$ that is normal, convex, and has compact cuts. This means that the α-cuts are given by $\overline{\mathrm{U}}_{\alpha} = \{x \in \mathbb{R}: \overline{U}(x) \geq \alpha\}$ if $\alpha \in (0, 1)$, $\overline{\mathrm{U}}_0 = cl\{x \in \mathbb{R}: \overline{U}(x) > 0\}$, where cl denotes the closure of the corresponding set; these are bounded and closed intervals. $\overline{U}(x)$ is interpreted as the degree of compatibility of x with the property that defines a $\overline{\mathrm{U}}$.

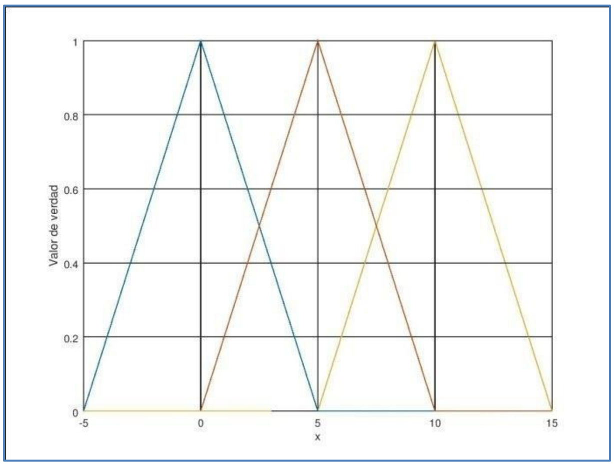

Figure 1. Three triangular fuzzy numbers. Source: [9].

A *triangular fuzzy number* is a particular case of a fuzzy number [10-15]. This is denoted by $A = Tri(a, b, c)$, and, as its name indicates, they have the shape of a triangle such that it has a null image outside the interval $[a, c]$ and reaches a maximum equal to 1 for $x = b$. Figure 1 contains three particular cases of these fuzzy numbers.

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, if\ a \le x \le b \\ \frac{c-x}{c-b}, if\ b \le x \le c \\ 0,\ otherwise \end{cases} \quad (1)$$

Some of the properties of triangular fuzzy numbers are:

- $\overline{\mathrm{U}}_\alpha = [\alpha \mathrm{b} + (1-\alpha)\mathrm{a},\ \alpha \mathrm{b} + (1-\alpha)\mathrm{c}],\ \alpha \in [0, 1]$,
- $\overline{\mathrm{U}}_1 = \{\mathrm{b}\}$ which is considered the value that completely satisfies the property $\overline{\mathrm{U}}$,
- $\overline{\mathrm{U}}_0 = [\mathrm{a}, \mathrm{c}]$ which are all values compatible with $\overline{\mathrm{U}}$ some truth value.

Definition 2. ([7, 9]) Let $\overline{\mathrm{U}}$ and $\overline{\mathrm{V}}$ be two fuzzy numbers. The *sum* of both is denoted by the fuzzy number $\overline{\mathrm{U}} + \overline{\mathrm{V}}$ and is defined from their α-cut as $(\overline{\mathrm{U}} + \overline{\mathrm{V}})_\alpha = [inf\overline{\mathrm{U}}_\alpha + inf\overline{\mathrm{V}}_\alpha, sup\overline{\mathrm{U}}_\alpha + sup\overline{\mathrm{V}}_\alpha]\ \forall\alpha \in [0, 1]$.

Definition 3. ([7, 9]) Let $\overline{\mathrm{U}}$ be a fuzzy number and γ a real number. The *product* $\overline{\mathrm{U}}$ by γ, which is denoted by $\gamma\overline{\mathrm{U}}$ is a fuzzy number that is defined from its α-level as: $(\gamma\overline{\mathrm{U}})_\alpha = \begin{cases} [\gamma inf\overline{\mathrm{U}}_\alpha, \gamma sup\overline{\mathrm{U}}_\alpha], \text{if } \gamma \ge 0 \\ [\gamma sup\overline{\mathrm{U}}_\alpha, \gamma inf\overline{\mathrm{U}}_\alpha], \text{ if } \gamma < 0 \end{cases} \forall\alpha \in [0, 1]$.

Operations on triangular fuzzy numbers are shown below. Let $A_1 = Tri(a_1, b_1, c_1)$ and $A_2 = Tri(a_2, b_2, c_2)$ ([9, 16]):

- $A_1 + A_2 = Tri(a_1 + a_2, b_1 + b_2, c_1 + c_2)$.
- $A_1 - A_2 = Tri(a_1 - c_3, b_1 - b_2, a_3 - c_1)$
- $\gamma A_1 = Tri(\gamma a_1, \gamma b_1, \gamma c_1)$, if $\gamma \ge 0$; $\gamma A_1 = Tri(\gamma c_1, \gamma b_1, \gamma a_1)$, if $\gamma < 0$.

Definition 4. ([9, 16]) Let $\overline{\mathrm{U}}$ and $\overline{\mathrm{V}}$ be two triangular fuzzy numbers $\overline{\mathrm{U}} = Tri(a_1, b_1, c_1)$ and $\overline{\mathrm{V}} = Tri(a_2, b_2, c_2)$. The *distance* between them is defined by the following equation:

$$D(\overline{\mathrm{U}}, \overline{\mathrm{V}}) = \sqrt{\frac{1}{6}[(a_1 - a_2)^2 + 4(b_1 - b_2)^2 + (c_1 - c_2)^2]} \quad (2)$$

Definition 5. ([7, 9]) Given a random experiment modeled in a probability space $(\Omega, \mathcal{A}, \mathcal{P})$. A *fuzzy random number* (FRN) has an associated application χ defined on Ω a fuzzy number, such that $\forall\alpha \in [0, 1]$ the real maps $\inf\chi_\alpha$ and $\sup\chi_\alpha$ are real random variables. Where, $\inf\chi_\alpha(\omega) = inf(\chi(\omega))_\alpha$ and $sup\ \chi_\alpha(\omega) = sup(\chi(\omega))_\alpha$.

Definition 6. ([7, 9]) Given a random experiment and an χ associated FRN, the *mathematical expectation* of χ (Aumann type) is the fuzzy number $\bar{E}(\chi)$, if it exists, such that $\forall\alpha \in [0, 1]$ we have:

$$(\bar{E}(\chi))_\alpha = [E(inf\chi_\alpha), E(sup\chi_\alpha)] \quad (3)$$

Where E is the mathematical expectation of the associated random variable.

In particular, if $\tilde{x}^{(n)} = (\tilde{x}_1, \dots, \tilde{x}_n)$ is a sample of FRN observations χ, then the *sample mean* is the fuzzy number $\overline{\tilde{x}^{(n)}}$ such that $\forall\alpha \in [0, 1]$:

$$\overline{\tilde{x}^{(n)}} = \left[\frac{inf(\tilde{x}_1)_\alpha + \cdots + inf(\tilde{x}_n)_\alpha}{n}, \frac{sup(\tilde{x}_1)_\alpha + \cdots + sup(\tilde{x}_n)_\alpha}{n}\right] \quad (4)$$

Definition 7. ([7, 9]) Given a random experiment and an associated FRN χ, the ρ_2 *−variance* of χ is the real number $Var_{\rho_2}(\chi)$, if it exists, it is defined by:

$$Var_{\rho_2}(\chi) = E\left(\left[\rho_2\left(\chi, \bar{E}(\chi)\right)\right]^2\right) \quad (5)$$

2.2 T-test

T-tests are based on the distribution called this way [1, 2]. The objective of these is to determine if it can be stated that two populations have the same mean through the study of the samples obtained from both. There are several cases to apply this test. If the samples are paired, it means that elements from the same population are compared before and after being subjected to a certain treatment.

There is also a Student's t-test when the samples are independent or unpaired when we want to compare two different samples to prove that they belong to the same population, as is the case of comparing the means of an experimental group and a control group.

The statistic for unpaired tests is shown in Equation 6.

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} \quad (6)$$

Where: $s_j^2 = \frac{\sum_{i=1}^{n_j}(x_i - \bar{X}_j)^2}{n_j - 1}$ with $j = 1, 2$. In this case, it is considered that the standard deviations of both samples are different.

To perform the hypothesis test, if μ_1 and μ_2 denote the means of the two populations, then:

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$.

To perform a similar test for paired samples, the following formula is used:

$$T = \frac{\bar{X}_D}{S_D / \sqrt{n}} \quad (7)$$

Where: $\bar{X}_D$ is the mean of the differences between both samples and S_D is the standard deviation of the differences. It is taken $n - 1$ as the degree of freedom.

If both samples have different sizes and different standard deviation values, the degree of freedom of the Student's t distribution is calculated by Equation 8:

$$g.l. = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\frac{s_2^2}{n_2}\right)^2 / (n_2 - 1)} - 2 \quad (8)$$

The p-value is calculated by setting α in this case, taken $\alpha = 0.05$. The test consists of calculating $t_{1-\alpha,d.f.}$ and is decided according to the following criteria:

If $t_{1-\alpha,d.f.} > T$, the null hypothesis of equal means is not rejected, otherwise the null hypothesis of equal means is rejected.

The initial hypothesis for applying the Student t-test is the normality of the data. This is convenient when the sample size is small.

2.3 Basic Notions on Neutrosophic Sets

Definition 8: ([17, 18]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow \,]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 9: ([17, 18]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (9)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfy the condition $0 \le u_A(x) + r_A(x) + v_A(x) \le 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indeterminate and falseness of x in A, respectively. For convenience a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \le a + b + c \le 3$.

Definition 10: ([19, 20]) A *triangular single-valued neutrosophic number* (TSVNN) $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, is a neutrosophic set over ℝ, whose membership functions of truthfulness, indeterminacy, and falsity, respectively, are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \le a_3 \\ 0, \text{otherwise} \end{cases} \quad (10)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}} & x = a_2 \\ \frac{(x - a_2 + \beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (11)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{(x - a_2 + \gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (12)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 11: ([19, 20]]) Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$ two triangular single-valued neutrosophic numbers and λ is any non-zero real number. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
3. Investment: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$
6. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, ∧ is a t-norm and ∨ is a t-conorm.

Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ is a TSVNN, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (13)$$

$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}})$ (14)

They are called *the score function* and *accuracy function* of ã, respectively.

Let $\{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_n\}$ be a set of n TSVNN, where $\widetilde{A}_j = \langle (a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j} \rangle$ (j = 1, 2, …, n), then the weighted average of the TSVNN is calculated by the following equation:

$\widetilde{A} = \sum_{j=1}^{n} \lambda_j \widetilde{A}_j$ (15)

where λ_j is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

Definition 12 ([21-24]): suppose X is a space of points (objects) with a generic element in X denoted by x. An *interval-valued neutrosophic set* (IVNS) A in X is characterized by the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$.

$$A_{IVNS} = \{\langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle : x \in X\}$$

With $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$ (16)

3 Results

The study carried out consists of tests on basic arithmetic skills of addition, subtraction, multiplication, and division in 141 children from the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru during the year 2023. The children belong to the third grade of primary education.

The students were divided into two groups, experimental group 1 had 70 students, and control group 2 had 71 students, summing up the total of third-grade children at this institution. Membership in one group or another was done with the help of simple random sampling.

Then, all the children were given a test that measured the skills that they were supposed to have sufficiently mastered. Later, they were given eight weeks, two for each arithmetic operation. Group 1 was given a method where they could use some technological means in the first 20 minutes of each class and then the class continued with the traditional teaching method. The traditional teaching method was applied to the members of group 2 the entire class time and they were not allowed to perform calculations with computing means. In addition, the members of group 1 were given tasks where part of that asked them to perform calculations with some computing means or search on the Internet. For this, we had the support of the parents. After the total class sessions had passed, the test was repeated.

The evaluators of both tests were instructed to evaluate the students based on a scale of "Excellent", "Very Good", "Good", "Regular" and "Deficient". The following algorithm adapted from [9] was applied to this linguistic scale.

1. The "Excellent" evaluations were identified with the triple of fuzzy numbers $\langle Tri(15,20,25), 0, Tri(-5,0,5)\rangle$, $\langle Tri(12.5, 17.5, 22.5), Tri(-1,0,1), Tri(0,2.5,5)\rangle$ was identified with "Very Good", $\langle Tri(10,15,20), Tri(0,1,2), Tri(2.5,5,7.5)\rangle$ was identified with "Good", $\langle Tri(6,11,16), Tri(3,4,5), Tri(4,9,14)\rangle$ was identified as "Regular" and $\langle Tri(4,9,14), Tri(3,4,5), Tri(6,11,16)\rangle$ with "Deficient".
2. The T statistic of Equation 6 (or Equation 7) is calculated from the fuzzy numbers of the evaluations, for each element of the triple. For this, Equations 3 and 4 were applied.
3. The triple of fuzzy numbers obtained from the previous step is defuzzified by $\hat{x} = \overline{U}_1$ or the value where the maximum of the fuzzy number is reached. It is a particular case of the defuzzification schemes that appear in [25]. This value coincides with the three schemes that are: the minimum of $\overline{U}_1$, the maximum of $\overline{U}_1$ and the average value of $\overline{U}_1$.
4. The obtained triple of fuzzy numbers is used to perform the t-test hypothesis. The value of the degree of freedom with Equation 8 is carried out with the algebraic operations between fuzzy numbers indicated by the formula and the way of defuzzification indicated in the previous step.

The results did not show behavior according to the normal distribution, however, as indicated in [9, 26], as the samples have a size greater than 25, this test can be considered robust enough. The final results are summarized in Tables 1, 2, 3 and 4.

Tables 1 and 3 contain the results of the evaluations of the control group, before and after traditional classes, respectively. Tables 2 and 4 contain the results of the experimental group before and after, respectively.

Table 1. Level of mathematical problem solving by the students in the control group before the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	9	9
Regular	9	9	0	0
Good	40	40	40	40
Very good	18	18	19	20
Excellent	4	4	3	2

Table 2. Level of mathematical problem solving by the students in the experimental group before the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	10	8	17
Regular	17	36	9	18
Good	32	20	33	34
Very good	18	0	18	0
Excellent	3	4	2	1

Table 3. Level of mathematical problem solving by the students in the control group after the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	0	18
Regular	13	13	13	22
Good	14	22	31	31
Very good	40	36	27	0
Excellent	4	0	0	0

Table 4. Level of mathematical problem solving by the students in the experimental group after the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	0	0
Regular	0	34	21	8
Good	53	32	20	62
Very good	13	4	25	0
Excellent	4	0	4	0

Table 5 contains the results of the algorithm described above of applying the t-test for each of the four skills.

Table 5. Results of the Student t-test for each of the skills comparing the experimental group with the control group in the post-test. According to the results of Tables 3 and 4.

Ability	Triple of Fuzzy Numbers T	$t_{0.95 g.l.}$	Three T defuzzified	Decision
Addition	$\langle [-27.36, 0.58, 14.55], [-8.66, 0.92, 6.08], [-14.01, -0.58, 8.42] \rangle$	$\langle 1.66, 1.66, 1.66 \rangle$	$\langle 0.58, 0.92, -0.58 \rangle$	H_0 is not rejected
Subtraction	$\langle [-19.60, 5.97, 18.76], [13.07, -5.28, -1.39], [-15.82, -5.97, 2.71] \rangle$	$\langle 1.66, 1.66, 1.66 \rangle$	$\langle 5.97, -5.28, -5.97 \rangle$	H_0 is rejected
Multiplication	$\langle [-22.05, 0.54, 11.84], [-8.37, -1.20, 2.62], [-11.14, -0.54, 8.20] \rangle$	$\langle 1.66, 1.66, 1.66 \rangle$	$\langle 0.54, -1.20, -0.54 \rangle$	H_0 is not rejected
Division	$\langle [-31.69, 6.75, 25.97], [-21.10, -6.38, 4.43], [-41.97, -6.75, 3.71] \rangle$	$\langle 1.66, 1.66, 1.66 \rangle$	$\langle 6.75, -6.38, -6.75 \rangle$	H_0 is rejected

The results of Table 5 indicate that there was significant improvement in the experimental group concerning subtraction and division. The latter is the most difficult operation. In the addition and multiplication, the effectiveness was the same in both groups.

To compare the results before and after in the experimental group, the triple of fuzzy numbers was obtained $T = \langle [3.9282, 9.9361, 22.5908], [0.36124, 0.58894, 0.77630], [-0.20553, 0.33689, 1.04023] \rangle$ as a result of applying Equation 7. By defuzzifying, it is $\overline{T} = \langle 9.9361, 0.58894, 0.33689 \rangle$ greater than $t_{0.95,n-1} = t_{0.95,69} = \langle 1.66, 1.66, 1.66 \rangle$ for the truthfulness values, and is not true for indeterminacy or falsity. Therefore, the null hypothesis of equality is rejected, which means that there was an improvement in the learning of the students in this group.

Conclusions

The use of Information and Communications Technologies (ICT) today is no longer foreign to most daily activities within contemporary societies, including teaching. This article proposed to analyze the impact that ICT can have on the learning of a group of children belonging to the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru, in the year 2023. There was a control group and an experimental group. The control group received a completely traditional educational program, while the experimental group was introduced to some extra-class activities and tasks with the support of technological means. Both groups were compared to each other and it was concluded that although in the simplest skills such as addition and multiplication, no significant improvements were obtained, in the case of subtraction and division there was improvement. Furthermore, it was found that the experimental group improved significantly when the skills before and after applying the proposed teaching were compared. To reach these results, the theory of fuzzy statistical tests was used. Specifically, the Student's t-test for paired and unpaired samples on the triangular fuzzy numbers was used. To gain accuracy, the tests were applied to a triple of fuzzy numbers, one indicating truthfulness, the other indicating indeterminacy, and the third one indicating falsehood. The use of a triple of fuzzy numbers, allowed raters to use a linguistic rating scale instead of a numerical scale. This made the evaluations easier for teachers to carry out and also made it possible to obtain a greater degree of interpretability of the results. The use of neutrosophy allowed us to obtain greater accuracy.

References

[1] Adhikari, A., Gorain, S.C., Gayen, P., Pal, I., and Sen, S. (2023). Studying the Differences: A Review on t-Test. International Research Journal of Education and Technology, 5, 338-349.

[2] Liu, Q., and Wang, L. (2021). t-Test and ANOVA for data with ceiling and/or floor effects. Behavior Research Methods, 53, 264-277.

[3] Kumar, R., and Dhiman, G. (2021). A comparative study of fuzzy optimization through fuzzy number. International Journal of Modern Research, 1, 1-14.

[4] Chukhrova, N., and Johannssen, A. (2021). Fuzzy hypothesis testing: Systematic review and bibliography. Applied soft computing, 106, 107331.

[5] Grzegorzewski, P., and Romaniuk, M. (2022). Bootstrap methods for epistemic fuzzy data. International Journal of Applied Mathematics and Computer Science, 32, 285-297.

[6] Watanabe, N. (2021). Fuzzy theories and statistics—fuzzy data analysis. Communications in Statistics: Case Studies, Data Analysis and Applications, 7, 561-572.

[7] Gil, M.A., Lubiano, M.A., De La Rosa De Sáa, S., and Sinova, B. (2015) Analyzing data from a fuzzy rating scale-based questionnaire. A case study. Psychothema. 27, 182-191.

[8] Yazdani, M., Torkayesh, A. E., Stević, Ž., Chatterjee, P., Ahari, S. A., and Hernández, V. D. (2021). An interval valued neutrosophic decision-making structure for sustainable supplier selection. Expert Systems with Applications, 183, 115354.

[9] Huailla, R. Á. M., Paucarchuco, K. M. M., Jaime, R. J. F., and Ramírez, M. R. V. (2023). Análisis Estadístico de Datos Difusos Comparativo de la Rentabilidad de la Producción de Maíz Morado en el Distrito de Huanta, Perú. Investigación Operacional, 44, 571-581.

[10] Dong, J., Wan, S., and Chen, S.M. (2021). Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making. Information Sciences, 547, 1080-1104.

[11] Tešić, D., Božanić, D., and Khalilzadeh, M. (2024). Enhancing multi-criteria decision-making with fuzzy logic: An advanced defining interrelationships between ranked II method incorporating triangular fuzzy numbers. Journal of Intelligent Management Decision, 3, 56-67.

[12] Wang, F. (2021). Preference degree of triangular fuzzy numbers and its application to multi-attribute group decision making. Expert Systems with Applications, 178, 114982.

[13] Xie, Z., and Su, Z. (2021). Evaluation of college English classroom teaching quality dependent on triangular fuzzy number. The International Journal of Electrical Engineering & Education, 00207209211002075.

[14] Zhan, J., Deng, J., Xu, Z., and Martínez, L. (2023). A three-way decision methodology with regret theory via triangular fuzzy numbers in incomplete multiscale decision information systems. IEEE Transactions on Fuzzy Systems, 31, 2773-2787.

[15] Zhang, J. (2021). A study on mental health assessments of college students based on triangular fuzzy function and entropy

weight method. Mathematical Problems in Engineering, 2021, 6659990.
[16] Maese Núñez, J.D.D., Alvarado Iniesta, A., Valles Rosales, D.J., and Báez López, Y.A. (2016) Coeficiente alfa de Cronbach para medir la fiabilidad de un cuestionario difuso. Culcyt//Measuring instruments, 13, 146-156.
[17] Luo, X., Wang, Z., Yang, L., Lu, L., and Hu, S. (2023). Sustainable supplier selection based on VIKOR with single-valued neutrosophic sets. Plos one, 18, e0290093.
[18] Pupo, I. P., Pérez, P. Y. P., Vacacela, R. G., Bello, R., Santos, O., & Vázquez, M. Y. L. (2018). Extensions to Linguistic Summaries Indicators based on Neutrosophic Theory, Applications in Project Management Decisions. Neutrosophic Sets and Systems, 87
[19] Yang, W., Cai, L., Edalatpanah, S. A., and Smarandache, F. (2020). Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement. Symmetry, 12, 588.
[20] Dhouib, S. (2021). Optimization of travelling salesman problem on single valued triangular neutrosophic number using dhouib-matrix-TSP1 heuristic. International Journal of Engineering, 34, 2642-2647.
[21] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.
[22] Torkayesh, A. E., Tavana, M., and Santos-Arteaga, F. J. (2022). A multi-distance interval-valued neutrosophic approach for social failure detection in sustainable municipal waste management. Journal of Cleaner Production, 336, 130409.
[23] Karaşan, A., and Kahraman, C. (2020). Selection of the most appropriate renewable energy alternatives by using a novel interval-valued neutrosophic ELECTRE I method. Informatica, 31, 225-248.
[24] Awang, A., Aizam, N. A. H., Ab Ghani, A. T., Othman, M., and Abdullah, L. (2020). A normalized weighted bonferroni mean aggregation operator considering shapley fuzzy measure under interval-valued neutrosophic environment for decision-making. International Journal of Fuzzy Systems, 22, 321-336.
[25] Jyh-Shing, Rj, Sun, C.-T. and Mizutani, E. (1997) Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence . Upper Saddle River: Prentice Hall.
[26] Sawilowsky, S.S. and Blair, R.C. (1992) A More Realistic Look at the Robustness and Type II Error Properties of the t Test to Departures From Population Normality. Quantitative Methods in Psychology, 3, 352-360.

Received: June 19, 2024. Accepted: August 10, 2024

University of New Mexico

Integration of NeutroAlgebra and Neutrosophic 2-Tuple Linguistic Likert Scales for Feasibility Assessment in Industrial Projects

Pablo Sebastián Volz Oporto[1], Nataly Eny Marcia Paredes Morán[2], Sergio Mysael Mamani Hancco[3] and Augusto Li Chan[4]

[1] National University of San Marcos, Lima Peru. E-mail: paulvolz@hotmail.com
[2] National University of San Marcos, Lima Peru. E-mail: nataly.paredesm@pucp.pe
[3] National University of San Marcos, Lima Peru. E-mail: sergio.mamani@tecsup.edu.pe
[4] National University of San Marcos, Lima Peru. E-mail: alic@unmsm.edu.pe

Abstract. The study addresses the problem of evaluating the viability of complex industrial projects, specifically in the design of a chipboard plant. In the context where decision-making is loaded with uncertainty and multiple interdependent variables, it is critical to have methodologies that can capture this complexity. Despite advances in the feasibility assessment literature, a lack of approaches that effectively integrate indeterminacy and expert opinion in a quantifiable manner has been identified. This article seeks to fill that gap by proposing a method based on neutroAlgebra, combining neutrosophic 2-tuple linguistic models and the Prospector method. The methodology developed in the study allows for a robust evaluation of the viability of an agglomerate plant, considering factors such as sustainability, environmental impact and economic viability. The results obtained show that the proposed method is not only effective in managing uncertainty for strategic decisions. This represents a contribution to the field, offering a practical tool that can be applied in various industrial contexts, thus improving the planning and execution of complex projects.

Keywords: Viability Factors, Critical Elements, PROSPECTOR Function, Neutrofunction, NeutroAlgebra

1. Introduction

The evaluation of the viability of industrial projects is a challenge that has gained relevance in the field of engineering and business management, especially in contexts of high uncertainty[1]. This study focuses on the design of a method for measuring viability factors for an agglomerate plant, using neutroAlgebra [2] generated by the combination function in neutrosophic 2-tuple linguistic models [3] and the Prospector method[4]. The importance of this research lies in the need to have more precise and adapted tools for decision-making in complex projects, where uncertainty and ambiguity are critical factors. Previous research has pointed out the difficulty of integrating subjectivity and uncertainty into feasibility assessment models, something that underlines the need for innovative approaches such as the one proposed in this study.

Historically, project feasibility assessment has evolved from simple methods based on financial analysis to models. more attractive that integrate multiple criteria and consider qualitative factors. With the increasing complexity of industrial projects and the integration of advanced technologies, the need for models that can handle the uncertainty inherent in these processes has become evident. The development of neutrosophic theory and its application in various areas of science and engineering has opened new possibilities. to address problems involving indeterminacy and ambiguity, providing a robust theoretical framework for this type of analysis [5,6].

In this context, measuring the viability of a chipboard plant presents a special challenge due to the diversity of factors that must be considered, including environmental sustainability, economic viability and social acceptance, among others. The fundamental problem addressed by this study is the lack of integrated methods that can effectively evaluate these factors under conditions of high uncertainty [7].

The present study proposes a solution to this problem by combining neutroAlgebra and the Prospector method, intending to develop an evaluation model that is capable of capturing both the technical complexity and the subjectivity inherent in feasibility evaluation. This approach will not only allow for a more holistic assessment but will also offer a more robust decision-making framework adaptable to different industrial contexts. The conceptual background of this research is based on neutrosophic theory, which is effective in managing uncertainty and indeterminacy in various applications [8,9,10]. Furthermore, the use of the Prospector method, widely recognized in the evaluation of natural resources and other industrial projects, offers a solid basis for the development of the proposed model. The combination of these two tools makes it possible to innovatively address the challenges associated with feasibility assessment in complex projects.

The fundamental objective of this study is to develop and validate a method for measuring the viability factors of an agglomerate plant that integrates neutroAlgebra and the Prospector method. This objective is broken down into specific sub-objectives that include the identification of key feasibility factors, the adaptation of neutrosophic 2-tuple linguistic models given these factors, and the validation of the model through proposed studies. of proposed cases and simulations. By achieving these objectives, this study will not only contribute to theoretical advancement in the field of feasibility assessment but will also offer practical tools for application in real industrial projects. In summary, this article seeks to fill a significant gap in the current literature by proposing a new innovative method for the evaluation of the feasibility of industrial projects, using advanced mathematical tools that effectively integrate uncertainty and subjectivity. The results of this study have the potential to significantly improve the quality of decisions in planning and execution. of industrial projects, providing a more complete framework adapted to the needs of the real world.

2. Preliminaries.

This section provides an overview of the Neutrosophic 2-tuple Linguistic Model and NeutroAlgebras necessary for comprehending this page.

2.1 tuple neutrosophic linguistic model

Definition 1 . ([11, 12, 13]) Let there be $S = \{s_0, s_1, \ldots, s_g\}$a set of linguistic terms and $\beta\epsilon[0, g]$a value that represents the result of a symbolic operation, then the linguistic 2-tuple that expresses the information equivalent to β is obtained using the following function:

$\Delta: [0, g] \to S \times [-0.5, 0.5)$

$$\Delta(\beta) = (s_i, \alpha) \quad (1)$$

Where $s_i$is such that $i = \text{round}(\beta)$and $\alpha = \beta - i, \alpha \in [-0.5, 0.5)$"round" is the common rounding operator, $s_i$is the index label closest to β and α is the value of the symbolic translation.

It should be noted that$\Delta^{-1}: \langle S\rangle \to [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. Thus, a linguistic 2-tuple $\langle S\rangle$ is identified by its numerical value in $[0, g]$.

Suppose $S = \{s_0, \ldots, s_g\}$ this is a *2-tuple linguistic set* (2TLS) with odd cardinality g+1. It is defined for $(s_T, a), (s_I, b), (s_F, c) \in$ Lya, b, c ∈[0, g], where $(s_T, a), (s_I, b), (s_F, c) \in$ L they independently express the degree of truthfulness, indeterminacy, and falsity through 2TLS. The 2-tuple linguistic neutrosophic number (2TLNN) is defined as follows [14]:

$$l_j = \left\{(s_{T_j}, a), (s_{I_j}, b), (s_{F_j}, c)\right\} \quad (2)$$

Where $0 \le \Delta^{-1}(s_{T_j}, a) \le g$, $0 \le \Delta^{-1}(s_{I_j}, b) \le g$, $0 \le \Delta^{-1}(s_{F_j}, c) \le g$and $0 \le \Delta^{-1}\left(s_{T_j}, a\right) + \Delta^{-1}\left(s_{I_j}, b\right) + \Delta^{-1}(s_{F_j}, c) \le 3g$.

The score and precision functions allow us to classify 2TLNN.

2.3 Linguistic Neutrosophic Likert Scales

Neutrosophic Likert scales [15] offer an expanded framework that includes indeterminacy as a fundamental element. Fuzzy sets, on the other hand, enable the representation and management of data that do not be precisely specified. Neutrosophic Likert scales are especially suitable for survey responses where participants' opinions not only range from a set of values, as allowed by fuzzy sets but also may include a level of uncertainty or neutrality that is challenging to measure using conventional fuzzy logic or clear Likert scales. Integrating the 2-tuples linguistic mode enables the development of a words-based computer method.

A score function based on neutrosophic theory efficiently converts replies from neutrosophic Likert scales into a unified single value. By combining the neutrosophic Likert scale with a 2TLNN linguist Moldes, we can compute [16]

Let be $l_1 = \{(s_{T_1}, a), (s_{I_1}, b), (s_{F_1}, c)\}$a 2TLNN in L, the score and accuracy functions $l_1$are defined as follows, respectively:

$$S(l_1)=\Delta\left\{\frac{2g+\Delta^{-1}(s_{T_1},a)-\Delta^{-1}(s_{I_1},b)-\Delta^{-1}(s_{F_1},c)}{3}\right\}, \Delta^{-1}(S(l_1)) \in [0, g] \quad (3)$$

$$H(l_1)=\Delta\left\{\frac{g+\Delta^{-1}(s_{T_1},a)-\Delta^{-1}(s_{F_1},c)}{2}\right\}, \Delta^{-1}(H(l_1)) \in [0, g] \quad (4)$$

Utilizing neutrosophic Likert scales accurately quantifies the range of opinions and their impact in analyzing the simultaneous presence of various truths and the spectrum of falsehoods [17].

2.4 NeutralGroups generated by OffUninorms

The theory of NeutroAlgebras generalizes the classical theory of Algebra and partial Algebras within the framework of Neutrosophic theory [18]. NeutroAlgebras continue studying algebraic structures based on ordered pairs formed by a set of elements and an operation. A fundamental distinction between NeutroAlgebras and other types of algebras is the presence of at least one NeutroAxiom. This axiom is characterized by the existence of two sorts of elements: those that adhere to the axiom and those that do not.

Building upon the fundamental concept of Neutrosophy, whenever an Algebra (axiom) \A> is true, there exists a triad (\A>, \ NeutA >, \ AntiA >) where the algebra (axiom) \A> is either 100% true or true. All elements in the set support NeutroAlgebras, which are axioms under NeutA that are satisfied by just a portion of the elements. Conversely, AntiAlgebras, which are axioms under AntiA, are not satisfied by any of the elements in the set. This novel methodology to one of the most traditional fields of mathematics presents a complexity for comprehending these novel concepts. It should be noted that classical algebra operates on mathematical logic, which permits only axioms that are 100% true [19].

A uninorm is a mapping that generalizes the definitions of t-norm and t-conorm . Where there is a neutral element, it is commutative, associative, and non-decreasing with respect to each of the components. In it is generalized to the field of neutrosophic and is further generalized to the field of OffSets , which are sets defined outside the interval $[0, 1]$o $[-1, 1]$and in general, are defined for intervals $[m, n]$where $m, n \in \mathbb{R}$, in particular for $[-n, n]$where the neutral is $e = 0$ [20].

By setting $n > 0$, a NeutroGroup can be defined from the Prospector join function, which is the function used to aggregate elements of a known expert system obtained to model mining problems. This NeutralGroup contains within its structure the symbolic element I which means indetermination. the PROSPECTOR function is defined in the MYCIN expert system as follows; is a mapping from $[-1, 1]^2$within $[-1, 1]$with the formula, [21] :

$$P(x, y) = \frac{x+y}{1+xy} \quad (5)$$

This function is a uninorm with neutral element 0, so it complies with commutativity, associativity and monotonicity. $P(-1,1)$and $P(1, -1)$they are not defined [22].

To obtain a more simplified model in this paper, we will use this five-point Likert scale. Specifically, we will use NeutroGroup with the operation $\oplus_5$ for elements $G = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$. This operation is commutative, associative and the null element is 0. Furthermore, the following properties derived from the properties of the OffUninorm generator are satisfied, taking only the truthfulness component [23]:

- If x, y < 0then $x \oplus_5 y \leq min(x, y)$,
- If x, y > 0then $x \oplus_5 y \geq max(x, y)$,
- If x < 0and y > 0or if x > 0and y < 0, we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

In [24] it is summarized in the following Cayley table :

Table 1. Cayley table corresponding to $\oplus_5$. Source: [16].

$\oplus_5$	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**I**	**1**	**2**	**3**	**4**	**5**
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	I
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
I	-5	-4	-3	-2	-1	I	I	I	I	I	I	I
1	-5	-4	-2	-1	0	1	I	2	3	4	4	5
2	-5	-3	-1	0	1	2	I	3	3	4	5	5
3	-5	-2	0	1	2	3	I	4	4	4	5	5
4	-5	0	2	3	4	4	I	4	5	5	5	5
5	I	5	5	5	5	5	I	5	5	5	5	5

3. Proposed measurement method

To design a method based on [16] for measuring the viability factors of an agglomerate plant in a neutrosophic context, it is important to identify predictors that capture both the particularities of the industrial process and the uncertainties inherent to the feasibility analysis. The predictors considered in this method are the following:

F1. Resource sustainability: This predictor will evaluate the availability and sustainability of the natural resources necessary for the production of chipboard, such as wood and other plant materials. Consideration will be given to the efficiency in the use of these resources and the plant's ability to maintain a constant supply without affecting the surrounding ecosystem.

F2. Environmental impact: This factor will measure the impact that the chipboard plant could have on the natural environment, including waste management, emissions, and energy consumption. The plant's ability to implement clean technologies and minimize its ecological footprint will be important.

F3. Economic viability: The plant's capacity to generate sustainable income will be evaluated, considering production costs, profit margins, and market demand. This predictor will reflect the economic robustness of the plant in different scenarios.

F4. Technological adaptation: This factor will analyze the plant's ability to integrate advanced technologies into its production processes, such as energy efficiency or automation techniques, which influences competitiveness and innovation capacity.

F5. Social acceptance: Finally, this predictor will evaluate the degree of acceptance of the plant by the local community and other social actors, including the impact on local employment, the perception of community benefits, and the management of possible social conflicts.

The expert must evaluate each of these aspects using a linguistic scale $S = \{s_{-5}, s_{-4}, s_{-3}, s_{-2}, s_{-1}, s_0, s_1, s_2, s_3, s_4, s_5\}$, where each element represents a qualitative assessment, as shown in Table 2.

Table 2. Linguistic meaning of each element of the S scale.

Scale element	Linguistic Meaning
s_{-5}	Extremely low
s_{-4}	Very low
s_{-3}	Low
s_{-2}	something low
s_{-1}	Lower than high
s_0	As low as high
s_1	Higher than low
s_2	something high
s_3	High
s_4	very high
s_5	Extremely high

In addition, an importance scale will be used for the expert's opinion, $W = \{w_{-5}, w_{-4}, w_{-3}, w_{-2}, w_{-1}, w_0, w_1, w_2, w_3, w_4, w_5\}$, as described in Table 3.

Table 3. Linguistic meaning of each element of the W scale.

Scale element	Linguistic Meaning
w_{-5}	extremely insignificant
w_{-4}	Very insignificant
w_{-3}	Insignificant
w_{-2}	a little insignificant
w_{-1}	More insignificant than important
w_0	As insignificant as it is important
w_1	More important than insignificant
w_2	a little important
w_3	Important
w_4	Very important
w_5	Extremely important

The elements of the measurement method are defined as follows:

The group $E = \{e_1, e_2, \cdots, e_p\}$ denotes the $p > 0$ experts who measure the sustainability of the company, the town, the region, etc.

$F = \{F_1, F_2, F_3, F_4, F_5\}$ are the five factors to measure.

Steps to measure feasibility

1.Weight assignment: To each expert and i It is assigned a weight on the scale W, based on its knowledge about each factor F_j. Let us denote this weight by w_{ij}.

2.Factor evaluation: Each expert e_i evaluates each factor using three values of Likert scales of elements from Table 2, we have a_{ij} $(i = 1,2,...,p; j = 1,2,...,5)$ is the evaluation of the ith expert on the jth factor; where $a_{ij} = \left(s_{T_{ij}}, s_{I_{ij}}, s_{F_{ij}}\right)$ and $s_{T_{ij}}, s_{I_{ij}}, s_{F_{ij}} \in S$.

3.Calculation of indices: Two indexes ind_i and ind_{wij} associated with (s_T, s_I, s_F) are calculated:

$$\boldsymbol{ind_{ij} = T \oplus_5 I \oplus_5 (-F)} \tag{6}$$
$$\boldsymbol{ind_{w_{ij}}} = \Delta^{-1}(\mathbf{w}_{ih'}) \tag{7}$$

4.Evaluation calculation: The evaluation of each expert for each factor is averaged by:

$$\boldsymbol{\bar{a}_{ij} = \frac{ind_{ij} + ind_{w_{ij}}}{2}} \tag{8}$$

5.Global evaluation of the factor: The global evaluation of each factor is obtained as:

$$\boldsymbol{\tilde{a}_j = \frac{\sum_{i=1}^{p} \bar{a}_{ij}}{p}} \tag{9}$$

Also, to convert it to an integer so $\tilde{a}_j \in [-5, 5]$, formula is used:

$$\boldsymbol{\tilde{\alpha}_j = round(\tilde{a}_j)} \tag{10}$$

6.Global viability calculation: Finally, the global viability of the plant is calculated as:

$$\mathbf{A} = \boldsymbol{\tilde{\alpha}_1 \oplus_5 \tilde{\alpha}_2 \oplus_5 \tilde{\alpha}_3 \oplus_5 \tilde{\alpha}_4 \oplus_5 \tilde{\alpha}_5} \tag{11}$$

If A=I, it indicates disagreement between the experts and the process must be repeated using additional methods such as neutrosophic Delphi.

7.Linguistic Value Determination Finally a linguistic Value is obtaining A is a result of the set $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$.

$$\boldsymbol{V} = \Delta^{-1}(\mathbf{A}) \in [\mathbf{0}, \mathbf{g}] \tag{12}$$

Giving a linguistic value and developing a computing with words process [25].

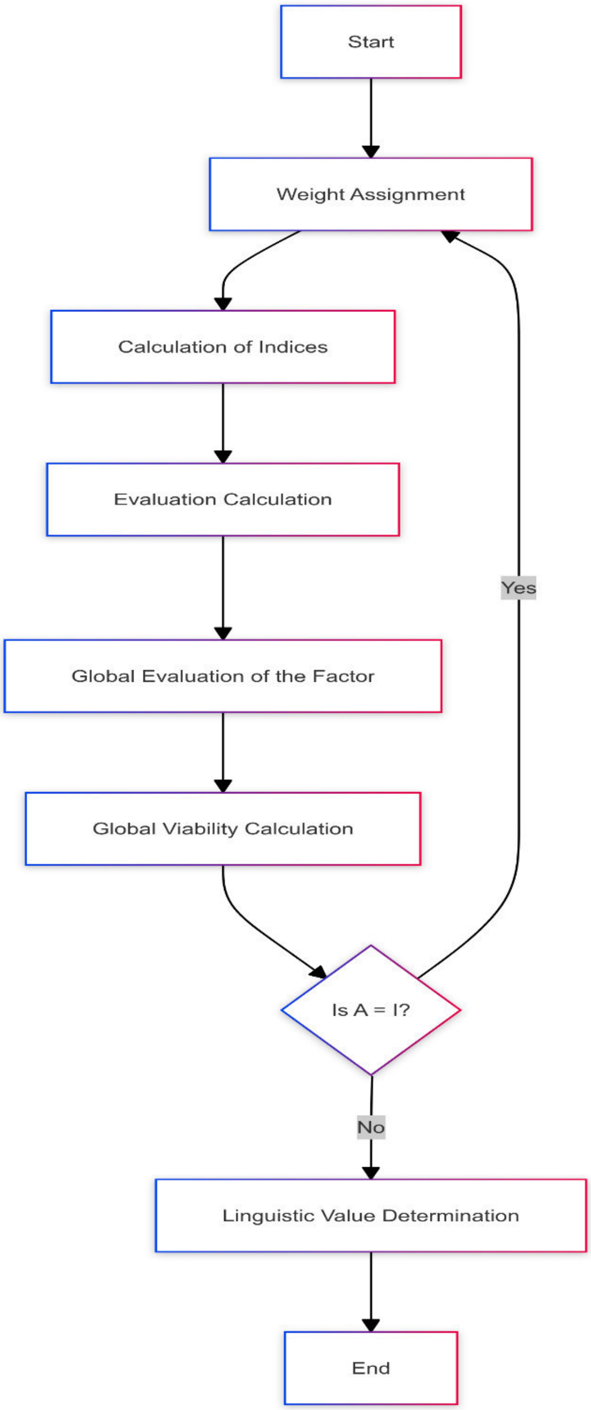

Figure 1: Flowchart of the viability measurement method of a chipboard plant.

Illustrative example

Example 1: Suppose you have 3 experts in the viability of a chipboard plant. Table 4 presents the weights associated with each expert and the evaluations for each factor.

Table 4. Evaluation of each factor and the weights associated with each expert.

Expert/Factor	F1	F2	F3	F4	F5
e_1	$(s_2, s_0, s_1)(w_3)$	$(s_2, s_1, s_{-2})(w_1)$	$(s_{-1}, s_2, s_{-1})(w_2)$	$(s_1, s_2, s_{-2})(w_{-1})$	$(s_3, s_1, s_{-4})(w_2)$
e_2	$(s_{-2}, s_2, s_{-3})(w_{-2})$	$(s_3, s_3, s_{-5})(w_1)$	$(s_{-2}, s_0, s_2)(w_{-1})$	$(s_2, s_1, s_{-3})(w_2)$	$(s_4, s_3, s_{-3})(w_0)$
e_3	$(s_{-1}, s_1, s_0)(w_{-1})$	$(s_0, s_1, s_{-1})(w_{-1})$	$(s_{-1}, s_2, s_2)(w_{-2})$	$(s_0, s_1, s_{-1})(w_2)$	$(s_4, s_4, s_{-5})(w_{-2})$

Table 5. Numerical evaluation of each factor by the experts.

Expert/Factor	F1	F2	F3	F4	F5
e_1	2	2.5	2	1.5	3.5
e_2	-1	3	-1.5	3	2,5
e_3	-1	0.5	-2	2	1.5

Below are the averaged results for each factor, calculated from the experts' evaluations.

- **F1. Resource sustainability:** $\tilde{\alpha}_1$=round((2-1-1) /3)=0
- **F2. Environmental impact:** $\tilde{\alpha}_2$= round((2.5+3+0.5) /3)=3
- **F3. Economic viability:** $\tilde{\alpha}_3$ = round (2-1.5-2) /3) =-1
- **F4. Technological adaptation:** $\tilde{\alpha}_4$= round(1.5+2+3)=2
- **F5. Social acceptance:** $\tilde{\alpha}_5$ = round((3+4.5+4.5) /3)=3

The next step is to combine the global evaluations to obtain the global viability of the chipboard plant:

$$A = 0 \oplus_5 3 \oplus_5 -1 \oplus_5 2 \oplus_5 3$$

V= $\Delta^{-1}(A)$ = very high,

which indicates a -high viability for the installation of the chipboard plant, according to experts.

The results obtained suggest that the proposed chipboard plant has high viability, with specific areas requiring additional analysis to ensure its long-term success. This methodological approach could be applied in future research to evaluate other types of industrial projects or in different organizational contexts.

Conclusion

The combination of neutroAlgebra with neutrosophic 2-tuple linguistic models has proven effective in integrating indeterminacy and expert opinions, offering a detailed evaluation adaptable to various scenarios. This methodology has great potential to enhance decision-making in complex industrial projects by considering both quantitative and qualitative criteria, allowing managers to identify critical areas that need specific interventions, thereby increasing the likelihood of long-term success.

However, the study has limitations, primarily due to its reliance on expert opinions, which introduces subjectivity and could affect replicability in other contexts. Additionally, the focus was on the feasibility of a chipboard plant, potentially limiting the applicability to other types of industrial projects. For future research, it is recommended to integrate additional methodological approaches, such as fuzzy analysis or artificial intelligence techniques, and to apply the method to different industrial projects to validate and expand the findings.

References

[1] Leyva-Vázquez, M., Quiroz-Martínez, M. A., Portilla-Castell, Y., Hechavarría-Hernández, J. R., & González-Caballero, E. (2020). A New Model for the selection of Information Technology Project in a Neutrosophic Environment. Neutrosophic Sets and Systems, 32(1), 22.

[2] Smarandache, F. (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas. Neutrosophic Computing and Machine Learning, 20, 91-104.

[3] Flores-Ledesma, K. N., Rios, M. E. N., Myriam, C. O. V. O., Loza, G. L., Gómez-Zuluaga, C. M., Meneses-La Riva, M. E., ... & Flores-Sotelo, W. S. (2023). A hybrid of ARAS and neutrosophic 2-tuple linguistic

model to evaluate gender equitable policies from the perspective of Latin American professor's. Neutrosophic Sets and Systems, 20.

[4] Gaschnig, J. (1982). Prospector: an expert system for mineral exploration. D. Michie, ed, 1982, 47-64.

[5] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[6] Matute, G., Yusta, J. M., & Naval, N. (2023). Techno-economic model and feasibility assessment of green hydrogen projects based on electrolysis supplied by photovoltaic PPAs. International Journal of Hydrogen Energy, 48(13), 5053-5068.

[7] Halmschlager, V., & Hofmann, R. (2021). Assessing the potential of combined production and energy management in Industrial Energy Hubs–Analysis of a chipboard production plant. Energy, 226, 120415.

[8] Smarandache, F. (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas. Neutrosophic Computing and Machine Learning, 20, 91-104.

[9] Pupo, I. P., Pérez, P. Y. P., Vacacela, R. G., Bello, R., Santos, O., & Vázquez, M. Y. L. (2018). Extensions to Linguistic Summaries Indicators based on Neutrosophic Theory, Applications in Project Management Decisions. Neutrosophic Sets and Systems, 87..

[10] A. El-Douh, A. (2023) "Assessment the Health Sustainability using Neutrosophic MCDM Methodology: Case Study COVID-19", Sustainable Machine Intelligence Journal, 3, pp. (1)

[11] Muhuri, P. K., and Gupta, P. K. (2020). A novel solution approach for multiobjective linguistic optimization problems based on the 2-tuple fuzzy linguistic representation model. Applied Soft Computing, 95, 106395.

[12] Wei, G., Wu, J., Guo, Y., Wang, J., and Wei, C. (2021). An extended COPRAS model for multiple attribute group decision making based on single-valued neutrosophic 2-tuple linguistic environment. Technological and Economic Development of Economy, 27, 353-368.

[13] Chen, H. (2023). A multi-attribute decision-making framework for enterprise competitive intelligence system evaluation with 2-tuple linguistic neutrosophic information. Journal of Intelligent & Fuzzy Systems, 45, 5955-5970.

[14] Mao, C. (2024). Enhanced group decision-making through an intelligent algorithmic approach for multiple-attribute credit evaluation with 2-tuple linguistic neutrosophic sets. International Journal of Knowledge-based and Intelligent Engineering Systems, (Preprint), 1-15.

[15] Vázquez, M. L., Ricardo, J. E., & Smarandache, F. (2024). Enhancing Set-Theoretic Research Methods with Neutrosophic Sets. HyperSoft Set Methods in Engineering, 2, 96-103.

[16] Marilú, K. Michael, M. Rigoberto, U. Marina, S. Angel, M. Hilmer, E. Ramirez-Salas, W. Gamarra-Moreno, A. (2024). Design of a Business Sustainability Measurement Method for Based on NeutroAlgebras Generated by the Combining Function in Prospector and Neutrosophic 2-tuple Linguistic Models. International Journal of Neutrosophic Science, (), 359-366.

[17] Bodur, S., Topal, S., Gürkan, H., & Edalatpanah, S. A. (2024). A Novel Neutrosophic Likert Scale Analysis of Perceptions of Organizational Distributive Justice via a Score Function: A Complete Statistical Study and Symmetry Evidence Using Real-Life Survey Data. Symmetry, 16(5), 598.

[18] Smarandache, F. (2020). NeutroAlgebra is a Generalization of Partial Algebra. International Journal of Neutrosophic Science, 2(1), 08-17.

[19] Smarandache, F. (2022). Introducción a la Super-Hiper-Álgebra y la Super-Hiper-Álgebra Neutrosófica. Neutrosophic Computing and Machine Learning, 20, 1-6.

[20] González-Caballero, E., Leyva-Vázque, M., & Smarandache, F. (2021). On neutrosophic uninorms. Neutrosophic sets and systems, 45(1), 22.

[21] Jiménez, D. S., Mayorga, J. A. V., Ubilla, M. E. R., & Hernández, N. B. (2021). NeutroAlgebra for the evaluation of barriers to migrants' access in Primary Health Care in Chile based on PROSPECTOR function. Neutrosophic Sets and Systems, 39(1), 1.

[22] Homenda, W., & Pedrycz, W. (2005). Balanced fuzzy computing unit. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 13(02), 117-138.

[23] Cruzaty, L. E. V., & Reynoso, A. M. R. (2024). New Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of Ethics in Companies. Neutrosophic Sets and Systems, 70, 264-270.

[24] Batista-Hernández, N., González-Caballero, E., Cruzaty, L. E. V., Ortega-Chávez, W., Huarac, C. F. P., and Chamorro, S. L. C. (2022). Theoretical study of the NeutroAlgebra generated by the combining function in Prospector and some pedagogical notes. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 116-140). IGI Global.

[25] Pérez-Teruel, K., Leyva-Vázquez, M., & Estrada-Sentí, V. (2015). Mental models consensus process using fuzzy cognitive maps and computing with words. Ingeniería y Universidad, 19(1), 173-188.

Received: June 20, 2024. Accepted: August 11, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Plithogenic Analysis in the Optimization of Educational Technologies

Javier Gamboa-Cruzado[1], Augusto Hidalgo-Sánchez[2], Alfonso Tesén Arroyo[3], Angel Núñez Meza[4], Edwin Collazos Paucar[5], and Dante Manuel Macazana Fernández[6]

[1] National University of San Marcos, Lima. Peru. E-mail: jgamboac@unmsm.edu.pe

[2] National University of San Marcos, Lima. Peru. E-mail: ahidalgos@unmsm.edu.pe

[3] National University Pedro Ruiz Gallo, Chiclayo. Peru. E-mail: atesen@unprg.edu.pe

[4] National University Daniel Alcides Carrión, Pasco. Peru. E-mail: anunezm@undac.edu.pe

[5] National University of San Marcos, Lima. Peru. E-mail: ecollazosp@unmsm.edu.pe

[6] National University of San Marcos, Lima. Peru. E-mail: dmacazanaf_af@unmsm.edu.pe

Abstract. This study has investigated the impact of machine learning techniques on the prediction of academic performance, with the objective of analyzing their integration into teaching and their effects on educational quality, costs, and effectiveness of teaching strategies. To this end, neutrosophic and plithogenic statistical analyses were applied to evaluate the relationship between the independent and dependent variables. The results have shown that the combination of machine learning techniques with active teaching participation not only improves academic performance but also optimizes costs and raises educational quality. Consequently, it is concluded that the integration of technology and teaching participation is essential for improving academic results. This evidence provides a path for future research and applications in the educational field, by highlighting the need to balance technology and humanization in educational strategies.

Keywords: Educational technology, Plithogenic statistics, Teacher participation, Educational quality.

1 Introduction

The use of machine learning techniques to predict students' academic performance has become an essential tool in educational management, facilitating informed decision-making that promotes continuous academic progress. This tool not only aims to anticipate students' future performance but also to identify the most effective methodologies that highlight key variables influencing teaching and motivation, thereby contributing to the reduction of school dropout rates [1]. Since academic performance is a critical indicator of educational quality, the implementation of machine learning is positioned as a crucial component for the continuous improvement of academic management [2].

However, despite advancements in using machine learning for predicting academic performance in this field, numerous challenges remain unaddressed [3]. Specifically, understanding learning patterns in contexts with low literacy levels is fundamental to improving academic performance.

Furthermore, although there has been a growing interest in developing predictive models of academic performance, challenges such as low performance and high dropout rates in higher education persist [4, 5]. In this context, machine learning techniques have been intensively adopted, facilitating the processing and analysis of large volumes of data to support educational decision-making [6].

Therefore, the present study aims to analyze the impact of machine learning techniques on predicting students' academic performance and how their integration into teaching practices influences educational quality[7]. Additionally, this study seeks to determine the effect of these tools on academic outcomes, the operational costs of educational institutions, and the effectiveness of teaching intervention strategies.

2 Plithogenic statistics

Plithogenic statistics are applied to analyze complex data in education, particularly in the evaluation of machine learning techniques. This approach allows for the investigation of how these techniques influence various educational factors, such as academic performance, teacher participation, and the quality of teaching. By employing plithogenic statistics, a detailed understanding of the interactions between these factors and how they simultaneously affect the educational environment is obtained. To implement this method, the plithogenic dynamic must be defined in the context of academic performance prediction [8].

Plithogeny is the dynamic of different types of opposites, and/or their neutrals, and/or non-opposites, and their organic fusion. Plithogeny is a generalization of dialectics (the dynamic of one type of opposites: <A> and <antiA>), neutrosophy (the dynamic of one type of opposites and their neutrals: <A> and <antiA> and <neutA>), as Plithogeny studies the dynamic of many types of opposites and their neutrals and non-opposites (<A> and <antiA> and <neutA>, <B> and <antiB> and <neutB>, etc.), and many non-opposites (<C>, <D>, etc.) all together. As an application and particular case derived from Plithogeny, the plithogenic set is an extension of the classical set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set [9], and it has multiple scientific applications [10].

Then, it is called a plithogenic set (P,a,V,d,c):

Where "P" is a set, "a" is an attribute (generally multidimensional), "V" is the range of values of the attribute, "d" is the degree of membership of the attribute value of each element x to the set P for some given criteria (x ∈ P), and "d" means "d_F" or "d_{IF}" or "d_N", when it is a fuzzy membership degree, an intuitionistic fuzzy membership, or a neutrosophic membership degree, respectively, of an element x to the plithogenic set P;

"c" means "c_F" or "c_{IF}" or "c_N", when it is a fuzzy attribute value contradiction degree function, an intuitionistic fuzzy attribute value contradiction degree function, or a neutrosophic attribute value contradiction degree function, respectively [11].

Functions are defined according to the applications that experts need to address. d(·,·) and c(·,·) then use the following notation: x(d(x,V)), where d(x,V) = {d(x,v), for all v ∈ V}, ∀ x ∈ P. Thus, plithogenic statistical analysis allows for addressing the complexity of the perceptions of the analyzed sample [12, 13].

3 Material and Methods

The systematic literature review was conducted using an approach that allowed for an analytical response to the research questions related to the use of machine learning in academic performance [7]. The initial search in academic databases such as Science Direct and Scopus yielded 717,389 articles. These were filtered through exclusion criteria, resulting in the selection of 88 relevant studies (study sample). The selected articles were evaluated based on criteria, including the clarity of objectives and the organization of the document. The data extracted from these sources formed the basis for the modeling of plithogenic statistics.

A linguistic evaluation system was adapted to the plithogenic model to accurately capture the experts' opinions (see Table 1).

Table 1: Plithogenic scales to evaluate the educational impact of machine learning.

Scale	Plithogenic scale	S([T,I,F])	Using ML machine learning techniques (VI1)	Teacher participation (VI2)	Academic performance (VD1)	Operational costs (VD2)	Quality of education (VD3)
7	(0.95, 0.05, 0.05)	0.90	Extremely High (EH)	Very High (VH)	Extremely High (EH)	Very High (VH)	Extremely High (EH)
6	(0.80, 0.15, 0.10)	0.75	Very High (VH)	High (H)	Very High (VH)	High (H)	Very High (VH)
5	(0.65, 0.25, 0.20)	0.65	High (H)	Moderately High (MH)	High (H)	Moderately High (MH)	High (H)
4	(0.50, 0.35, 0.30)	0.50	Moderately High (MH)	Medium (M)	Moderately High (MH)	Medium (M)	Moderately High (MH)
3	(0.35, 0.45, 0.40)	0.45	Medium (M)	Moderately Low (ML)	Medium (M)	Moderately Low (ML)	Medium (M)
2	(0.20, 0.60, 0.50)	0.35	Moderately Low (ML)	Low (L)	Moderately Low (ML)	Low (L)	Moderately Low (ML)
1	(0.10, 0.75, 0.65)	0.25	Low (L)	Very Low (VL)	Low (L)	Very Low (VL)	Low (L)

Consequently, the dataset is evaluated, which is formed totally or partially by data with some degree of indeterminacy and contradiction. The plithogenic statistical method is used to interpret and organize plithogenic data to reveal underlying patterns [14, 15].

For the plithogenic statistical modeling in this study, a random variable P is referenced, representing the lower and upper levels that the studied plithogenic variable can reach, within an indeterminate and contradictory interval. Thus, it follows the plithogenic mean of the variable ($\overline{P}$) when formulating [16]:

$$\overline{P} = \frac{1}{n_P} \sum_{i=1}^{n_P} P_i \tag{1}$$

Where n_P is a plithogenic random sample from the studied population. Once the mean is defined, the next step is to calculate the variance of the plithogenic sample. To do this, it is necessary to convert a plithogenic number to a scalar number according to the methodology analyzed in the study materials. Subsequently, the following equation is used to calculate S_P^2 [16]:

$$S_P^2 = \frac{\sum_{i=1}^{n_P}(P_i - \bar{P}_i)^2}{n_P} \tag{2}$$

Subsequently, the plithogenic coefficient (CV_P) is calculated, which measures the consistency of the variable. The lower the value of CV_P, the more consistent the performance of the analyzed element is compared to the others studied. The following equation is proposed [16]:

$$CV_P = \sqrt{S_P^2} \times 100 \tag{3}$$

4 Results

The following presents an analysis of the implications of applying machine learning techniques in education. The research has addressed how these techniques can influence various aspects of the educational system, from the overall quality of education to the impact on institutional costs. In this context, several hypotheses about the effectiveness and benefits of machine learning in different educational areas are explored (see Table 2).

Table 2: Neutrosophic hypothesis of the study.

Question	Neutrosophic probabilistic hypothesis	Interpretation of studies
1. Does the reliance on machine learning predictions increase the overall quality of education provided to students?	(0.91, 0.09, 0)	Confidence in machine learning predictions improves the overall quality of education by accurately forecasting student performance and assisting in academic decision-making.
2. Does machine learning improve student outcomes in education?	(0.89, 0.11, 0)	Machine learning can enhance student outcomes by precisely predicting academic performance, thus improving learning experiences and aiding in educational decision-making.
3. Does the use of machine learning techniques to predict academic performance result in significant cost savings for educational institutions?	(0.82, 0.16, 0.2)	The use of machine learning techniques can generate significant savings for institutions by providing more accurate predictions, better intervention strategies, and resource optimization.
4. Does the involvement of teachers with predictive learning analytics lead to an improvement in student academic performance?	(0.60, 0.35, 0.15)	Active teacher involvement in predictive analytics can improve student performance, but the mere provision of analytics does not guarantee changes in teaching practices.

To explore how machine learning techniques influence the educational system using a plithogenic approach, it is essential to clearly define the variables to be modeled. The following includes the independent and dependent variables for statistical modeling, as well as the evaluation of each variable in the presented sample (see Tables 3 and 4), and the representation of determining and indeterminate elements (see Figure 1).

Table 3: Variables to measure machine learning.

Variable	Type	Description	Abbreviation
Use of machine learning techniques	Independent variable	Level of application of machine learning techniques	VI1
Teacher involvement in predictive analytics	Independent variable	Degree of teacher involvement in predictive analytics	VI2
Academic performance	Dependent variable	Measured by the average student grades	VD1
Operational costs	Dependent variable	Total expenses at the educational institution	VD2
Quality of education	Dependent variable	Overall satisfaction of students and teachers	VD3

Table 4: Sample analyzed for variables VI1, VI2, VD1, VD2 and VD3.

Source	VI1	VI2	VD1	VD2	VD3
1	(0.78,0.91)	(0.9,0.99)	(0.78,0.89)	(0.76,0.93)	(0.9,0.99)
2	(0.7,0.88)	(0.85,0.92)	(0.79,0.92)	(0.72,0.88)	(0.86,0.88)
3	(0.64,0.75)	(0.84,0.89)	(0.85,0.95)	(0.73,0.89)	(0.87,0.89)
4	(0.77,0.94)	(0.86,0.93)	(0.83,0.92)	(0.78,0.97)	(0.8,0.9)
5	(0.61,0.71)	(0.86,0.94)	(0.76,0.86)	(0.71,0.87)	(0.69,0.71)
6	(0.75,0.91)	(0.9,1)	(0.77,0.99)	(0.76,0.91)	(0.83,0.84)
7	(0.66,0.82)	(0.87,0.97)	(0.7,0.96)	(0.75,0.9)	(0.8,0.82)
8	(0.73,0.86)	(0.88,0.95)	(0.7,0.81)	(0.7,0.88)	(0.86,0.88)
9	(0.62,0.81)	(0.84,0.93)	(0.77,0.94)	(0.8,0.99)	(0.88,0.98)
10	(0.79,0.99)	(0.88,0.96)	(0.76,0.91)	(0.75,0.9)	(0.85,0.95)
11	(0.67,0.82)	(0.89,0.94)	(0.78,0.88)	(0.79,0.98)	(0.9,0.94)
12	(0.69,0.83)	(0.89,0.95)	(0.71,0.96)	(0.73,0.88)	(0.86,0.91)
13	(0.61,0.73)	(0.89,0.99)	(0.85.1)	(0.74,0.91)	(0.82,0.85)
14	(0.67,0.81)	(0.86,0.95)	(0.71,0.86)	(0.79,0.97)	(0.83,0.87)
15	(0.73,0.88)	(0.81,0.87)	(0.79,0.92)	(0.7,0.88)	(0.8,0.81)
16	(0.78,0.88)	(0.84,0.9)	(0.81,0.91)	(0.78,0.95)	(0.85,0.92)
17	(0.75,0.91)	(0.87,0.95)	(0.71,0.82)	(0.72,0.91)	(0.84,0.93)
18	(0.76,0.88)	(0.81,0.89)	(0.7,0.81)	(0.78,0.97)	(0.8,0.89)
19	(0.73,0.92)	(0.85,0.95)	(0.79,0.9)	(0.77,0.92)	(0.84,0.89)
20	(0.69,0.8)	(0.85,0.91)	(0.72,0.83)	(0.78,0.98)	(0.81,0.82)
21	(0.79,0.96)	(0.9,0.97)	(0.85,0.97)	(0.75,0.94)	(0.86,0.96)
22	(0.8,0.97)	(0.82,0.89)	(0.81,0.93)	(0.73,0.89)	(0.85,0.86)
23	(0.69,0.8)	(0.82,0.92)	(0.83,0.97)	(0.79,0.96)	(0.83,0.83)
24	(0.73,0.9)	(0.8,0.89)	(0.79,0.89)	(0.8,0.96)	(0.82,0.89)
25	(0.77,0.89)	(0.9,0.97)	(0.84,0.97)	(0.72,0.9)	(0.82,0.89)
26	(0.74,0.84)	(0.83,0.9)	(0.77,0.9)	(0.76,0.94)	(0.8,0.84)
27	(0.69,0.83)	(0.85,0.9)	(0.79,0.94)	(0.77,0.96)	(0.84,0.85)
28	(0.69,0.79)	(0.81,0.86)	(0.7,0.84)	(0.76,0.93)	(0.87,0.94)
29	(0.69,0.79)	(0.87,0.94)	(0.78,0.88)	(0.78,0.94)	(0.79,0.8)
1-88	(0.64,0.76)	(0.83,0.93)	(0.75,0.89)	(0.76,0.91)	(0.85,0.89)
0-88	**(62.07,75.19)**	**(75.15,81.81)**	**(68.05,79.08)**	**(66.6,81.67)**	**(74.19,78.61)**

Interpretation of Results:

Use of machine learning techniques (VI1): Values for this variable range from 0.61 to 0.99, covering a spectrum from low to extremely high levels. This indicates that educational institutions apply machine learning techniques to varying degrees. Institutions using advanced techniques achieve greater accuracy in predicting academic performance and better resource optimization, contributing to higher educational quality.

Teacher participation in predictive analytics (VI2): Values range from 0.81 to 1, showing high to extremely high levels of teacher involvement in educational data analysis. Intense participation allows teachers to better adapt pedagogical strategies, leading to significant improvements in student academic performance.

Academic performance (VD1): The variable shows values between 0.7 and 1, reflecting generally high academic performance. This suggests that machine learning techniques and effective teaching practices are associated with good academic results. Consistency in high performance validates the effectiveness of the applied educational strategies.

Operational costs (VD2): Operational costs range from 0.71 to 0.99, from medium to extremely high levels. This variability reflects how different institutions manage their expenses related to the implementation of machine learning techniques. Institutions with high costs invest in advanced technologies, while others optimize resources and reduce expenses in the long term.

Quality of education (VD3): Values for this variable range from 0.69 to 0.99, indicating high educational quality. This shows that effective implementation of predictive techniques and active teacher participation contribute to a positive educational experience. High educational quality signifies that machine learning techniques are well-applied and adapted to student needs.

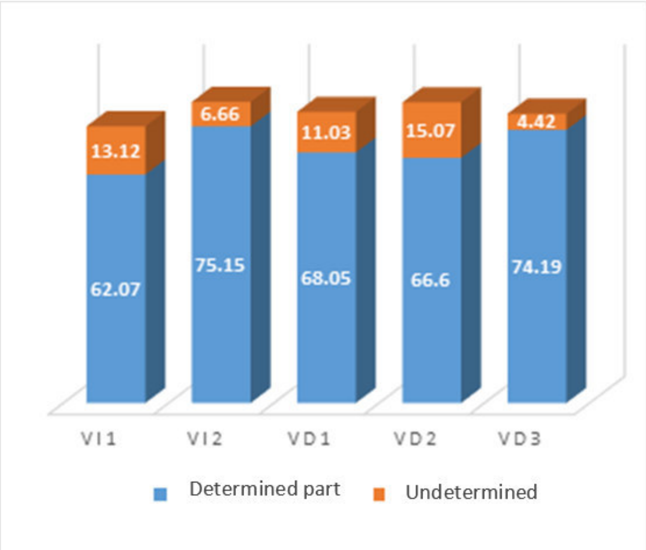

Figure 1: Indeterminacy of the analyzed sample.Source: Own elaboration.

In Figure 1, the levels of indeterminacy for each variable are observed. Among them:

Usage of Machine Learning Techniques (VI1): This section represents the level of application of machine learning techniques, which is clearly defined and measured. The determined part shows that, generally, the use of these techniques is quite high and well understood. However, the undetermined part reflects challenges such as the lack of standardization and variability in the data, introducing uncertainty into the results and the effectiveness of these techniques.

Teacher Participation in Predictive Analysis (VI2): Teacher participation is high and well established, suggesting a good degree of involvement in predictive analysis. Nonetheless, the undetermined part represents difficulties in accurately measuring this participation, including differences in teacher training and motivation.

Academic Performance (VD1): Academic performance is relatively stable and measurable, indicating that student grades are clearly recorded. However, the undetermined part reflects issues such as variability in assessment and uncontrolled external factors, introducing uncertainty into the interpretation of performance data.

Operational Costs (VD2): Operational costs are relatively clear and well documented, facilitating financial management. Despite this, the undetermined part suggests challenges in accurately forecasting costs, such as fluctuations in expenses and changes in budgets, which affect planning and the evaluation of economic efficiency.

Quality of Education (VD3): The quality of education is evaluated quite stably and consistently, reflecting a high level of satisfaction among students and teachers. The undetermined part is low, indicating that uncertainties in this variable are minor. However, variations in the perception of quality or contextual factors not fully reflected in the evaluation should be considered.

Therefore, the impact of the variables in the research is evaluated based on the analysis of the weighted average value, the range of standard deviation, and the weighted coefficient of variation for each variable. These data detail the behavior and consistency of the analyzed variables (see Table 5) [17, 18].

Table 5: Obtaining $\bar{x}_P$, S_P and CV_P of the variables VI1, VI2, VD1, VD2 and VD3.

Variable	$\bar{x}_P$	S_P	CV_P
VI1	0.71 + 0.85 I	0.059 + 0.072 I	0.083 + 0.085 I
VI2	0.85 + 0.93 I	0.033 + 0.037 I	0.039 + 0.04 I
VD1	0.77 + 0.9 I	0.05 + 0.052 I	0.065 + 0.058 I
VD2	0.76 + 0.93 I	0.03 + 0.035 I	0.039 + 0.038 I
VD3	0.84 + 0.89 I	0.04 + 0.055 I	0.048 + 0.062 I

Usage of Machine Learning Techniques (VI1): This variable shows a high average in the use of machine learning techniques but presents moderate uncertainty. Compared to other variables, its high variability makes predictions of academic performance less consistent. However, intensive use of techniques enhances the prediction and reliability of predictive models.

Teacher Participation in Predictive Analysis (VI2): Teacher participation is high and its uncertainty is low, indicating greater consistency in implementing predictive analysis. Compared to VI1, this variable offers greater stability, facilitating the accuracy of predictions. High teacher participation helps make machine learning models more effective and reliable by contributing to more stable predictions of academic performance.

Academic Performance (VD1): Academic performance shows a high average but with considerable uncertainty. Its variability is moderate, affecting the stability of predictive models. Compared to VI2, which has lower variability, VD1 may present additional challenges in prediction due to its lack of consistency. Variability in academic performance complicates the task of modeling and predicting student success with precision.

Operational Costs (VD2): Operational costs show a high mean with low uncertainty, indicating that cost data is stable. Compared to VD1, operational costs are more predictable, which facilitates resource optimization without significantly affecting the accuracy of academic performance predictions. Stability in costs allows machine learning models to adjust more efficiently, contributing to better resource management and intervention strategies.

Quality of Education (VD3): The quality of education has a high average with moderate uncertainty. Its variability, although lower than that of VD1, can still impact the stability of predictions. Compared to VI2 and VD2, the quality of education is less consistent, which may influence the accuracy of predictive models. Improving the stability of this variable is crucial for enhancing the accuracy of predictions related to academic performance.

After performing the comparative plithogenic analysis [18, 19, 20] , the correlation between independent and dependent variables is evaluated to identify the strongest relationships and determine which hypothesis is most viable (see Table 6). The following relationships are proposed:

- H1: Correlation between VI1 and VD3.
- H2: Correlation between VI1 and VD1.
- H3: Correlation between VI1 and VD2.
- H4: Correlation between VI2 and VD1.

Table 6: Pearson correlation coefficients.

	VI1	VI2	VD1	VD2	VD3
VI1	-	-	0.0973	0.0272	0.3695
VI2	-	-	0.491	-	-
VD1	0.0973	0.491	-	-	-
VD2	0.0272	-	-	-	-
VD3	0.3695	-	-	-	-

The table shows the Pearson correlation coefficients, which indicate the strength and direction of the linear relationships between the variables. This analysis focuses on how the independent variables (VI1 and VI2) relate to the dependent variables (VD1, VD2, and VD3). The analysis reveals that teacher participation in predictive analysis (VI2) has a significant influence on academic performance (VD1), suggesting that the active involvement of teachers in using machine learning tools is crucial for improving student outcomes. On the other hand, the use of machine learning techniques (VI1) has a moderate impact on the quality of education (VD3) and a limited effect on operational costs (VD2) and academic performance (VD1). This indicates that while technology is a significant facilitator, the commitment and participation of teachers in the predictive analysis process are essential for achieving better academic performance results.

The acceptance of hypothesis H4 confirms that the active participation of teachers in predictive analysis positively affects student academic performance. This result highlights several key aspects that must be considered in the research:

- Importance of Teacher Intervention: The analysis explores how the integration of machine learning techniques, combined with teacher involvement in predictive analysis, affects critical variables such as academic performance, operational costs, and educational quality. Hypothesis H4 emphasizes the significant impact of teacher participation on improving academic performance, highlighting the crucial role within the analyzed neutrosophic context.
- Direct Connection with Dependent Variables: H4 establishes a clear relationship between teacher participation in predictive analysis and improvement in academic performance (VD1), one of the most relevant dependent variables. Academic performance, as a key indicator of educational process

effectiveness, is directly influenced by this participation, reinforcing the relevance of the hypothesis within the research framework.

- ❖ Applicability and Relevance: The analysis of the data and the involved variables suggests that H4 aligns with the idea that active teacher participation in the interpretation and application of predictive analysis has a significant impact on student academic outcomes.

The correlation between the variables allows for the identification of areas where different variables coincide or overlap, helping to mitigate specific challenges in predicting academic performance. Among those identified are:

- ❖ Correlation between VI1 (Use of Machine Learning Techniques) and VD1 (Academic Performance): The Pearson correlation coefficient is 0.0973, indicating a moderate intersection between these variables. This suggests that while appropriate use of machine learning techniques positively impacts academic performance, it is not a decisive factor. The intersection highlights an area where optimizing these techniques can help overcome challenges in accurately predicting academic performance.
- ❖ Correlation between VI2 (Teacher Participation in Predictive Analysis) and VD1 (Academic Performance): With a coefficient of 0.491, this intersection is significant. The strong correlation indicates that active teacher participation is crucial for improving academic outcomes. Here, the intersection reveals that involving teachers in the predictive process mitigates challenges associated with applying machine learning techniques without human supervision and adjustment.
- ❖ Correlation between VI1 (Use of Machine Learning Techniques) and VD3 (Quality of Education): The high intersection (0.3695) shows that the use of machine learning techniques also affects perceptions of educational quality. This suggests that optimizing these techniques not only improves performance but also enhances overall satisfaction among students and teachers. Addressing challenges in perceiving educational effectiveness accurately is crucial for improving quality.

Conclusion

The validation of hypothesis H4 confirms that active teacher participation in predictive analysis significantly enhances students' academic performance. Additionally, the use of plithogenic statistics has been crucial for identifying and understanding the complex relationships between independent and dependent variables. This approach has allowed for the detection of areas of indeterminacy and contradiction, providing a more comprehensive view of how interactions between technology and teacher participation impact educational outcomes.

The plithogenic analysis has demonstrated that combining machine learning techniques with strong teacher involvement is essential for improving academic performance. The areas of intersection and union indicated that the most effective educational strategies are those that balance technology with human involvement, while the levels of contradiction highlight the need for balanced implementation. These solid and applicable relationships suggest a clear path for designing educational policies and practices that optimize academic performance through machine learning.

References

[1] Pallathadka, H., Wenda, A., Ramirez-Asís, E., Asís-López, M., Flores-Albornoz, J., & Phasinam, K. (2023). Classification and prediction of student performance data using various machine learning algorithms. Materials Today: Proceedings, 80(Part 3), 3782-3785.

[2] Dabhade, P., Agarwal, R., Alameen, K., Fathima, A., Sridharan, R., & Gopakumar, G. (2021). Educational data mining for predicting students' academic performance using machine learning algorithms. Materials Today: Proceedings, 47(Part 15), 5260-5267.

[3] Yağcı, M. (2022). Educational data mining: Prediction of students' academic performance using machine learning algorithms. Smart Learning Environments, 9(1), 11.

[4] Kuadey, N. A., Ankora, C., Tahiru, F., Bensah, L., Agbesi, C. C. M., & Bolatimi, S. O. (2024). Using machine learning algorithms to examine the impact of technostress creators on student learning burnout and perceived academic performance. International Journal of Information Technology, 16(4), 2467-2482.

[5] Rastrollo-Guerrero, J. L., Gómez-Pulido, J. A., & Durán-Domínguez, A. (2020). Analyzing and predicting students' performance by means of machine learning: A review. Applied Sciences, 10(3), 1042.

[6] Maehara, R., Benites, L., Talavera, A., Aybar-Flores, A., & Muñoz, M. (2024). Predicting financial inclusion in Peru: Application of machine learning algorithms. Journal of Risk and Financial Management, 17(1), 34.

[7] Albreiki, B., Zaki, N., & Alashwal, H. (2021). A systematic literature review of student performance prediction using machine learning techniques. Education Sciences, 11(9), 552.

[8] Gutiérrez, P., et al. (2023). Neutrosophic Sets and Systems Special Issue: Neutrosophic advancements Plithogenic Iadov model to study university teaching practices in the complexity of the educational process of comprehensive training by competencies. Neutrosophic Sets and Systems, 62(1), 79-83.

[9] Cristina, E., et al. (2022). Neutrosophic statistical analysis of self-assessment of rehabilitation knowledge in university students. Neutrosophic Sets and Systems, 52(October), 2-5.

[10] Smarandache, F. (2022). Plithogeny, plithogenic set, logic, probability and statistics: A short review. Journal of Computational and Cognitive Engineering, 1(2), 47-50.
[11] González Caballero, E., Leyva Vázquez, M., & Smarandache, F. (2021). On neutrosophic uninorms. Neutrosophic Sets and Systems, 45(2021), 340-348.
[12] La Cantera, D., Hernandez, D., Quiroz, R. C., Queija, M. L., Gonzalez, J. R., & Vazquez, M. Y. L. (2024). Analyzing interdisciplinary education in general medicine using Smarandache's multivalued logic hypothesis theory and plithogenic probability. Neutrosophic Sets and Systems, 70(1), 23.
[13] Ali, M. A., & Muthuswamy, M. (2023). Neutrosophic multi-criteria decision-making framework for sustainable evaluation of power production systems in renewable energy sources. Sustainable Machine Intelligence Journal, 4(3), 1-10.
[14] Sánchez, F. C., Almeida Blacio, J. H., Flores Bracho, M. G., Andrade Santamaria, D. R., & Casanova, R. S. (2021). Neutrosophic and plithogenic statistical analysis in educational development. Neutrosophic Sets & Systems, 44.
[15] Smarandache, F. (2021). Introducción a la lógica plitogénica. Neutrosophic Computing and Machine Learning, 18, 1-6.
[16] Troya-Sarzosa, J. F., & Oña, C. (2024). Using plithogenic statistics to determine optimal guinea pig manure dosing in prickly pear cultivation. Neutrosophic Sets and Systems, 69(1), 8.
[17] Leyva Vázquez, M., & Smarandache, F. (2021). Resolución de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica. Neutrosophic Computing and Machine Learning, 15, 41-46.
[18] Vázquez, M. L., & Torres, L. C. (2024). Integrating neutrosophic numbers in regression analysis for enhancing predictive modelling through uncertainty representation. Plithogenic Logic and Computation, 1, 120-126.
[19] Marín-Eléspuru, C. U., Melgarejo-Mariño, D. M., Solsol-Hidalgo, E. A., Chiroque-Sernaqué, D., & Balbuena Hernández, J. R. (2024). Exploring the impact of performance audits on the management of public organizations through the analysis of plithogenic statistics. Neutrosophic Sets and Systems, 69(1), 21.
[20] El-Douh, A. A., Lu, S., Abdelhafeez, A., & Aziz, A. S. (2023). Assessment of health sustainability using neutrosophic MCDM methodology: Case study COVID-19. Sustainable Machine Intelligence Journal, 3, 1-1.

Received: June 21, 2024. Accepted: August 12, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Application of neutrosophic cognitive maps in the jurisprudence of the National Court of Justice of Ecuador

Alipio Cadena Posso[1], Diana Carolina Quiroz Gordillo[2], Diego Xavier Chamorro Valencia[3], Luis Andrés Crespo Berti[4], and Vladimir Vega Falcón[5]

[1] Autonomous Regional University of Los Andes, Ibarra. Ecuador. E-mail: ui.alipiocadena@uniandes.edu.ec
[2] Autonomous Regional University of Los Andes, Ibarra, Ecuador. E-mail: di.dianacqg10@uniandes.edu.ec
[3] Autonomous Regional University of Los Andes, Ibarra, Ecuador. E-mail: ui.diegochamorro@uniandes.edu.ec
[4] Autonomous Regional University of Los Andes, Ibarra, Ecuador. E-mail: ui.luiscrespo@uniandes.edu.ec
[5] International Center for Entrepreneurs in Barcelona (ICEB), Spain. E-mail: vega.vladimir@gmail.com

Abstract. This research analyzes the use of neutrosophic cognitive maps in the jurisprudential examination of divorce in the National Court of Justice of Ecuador. The suggested technique intends to simulate the links between legal reasons and the psychological and emotional ramifications of divorce, therefore enhancing judicial decision-making. Resolution No. 0071 - 2018 - 38420 is evaluated to determine its influence on the protection of individual rights, presenting an original viewpoint on family law. The results gathered from the study underline the necessity to examine both the legal reasons and the psychological and emotional ramifications of divorce when making court judgments in this area. The findings reveal the important importance of variables such as legal and psychological assistance, family dynamics, and domestic violence in the divorce process. Furthermore, the efficiency of the neutrosophic cognitive mapping approach was proved to model complex and diverse interactions in this context, thus giving a helpful tool to increase the quality and depth of research in legal studies connected to divorce.

Keywords: neutrosophic cognitive map, legal analysis, divorce, neutrosophic model of legal relationships.

1 Introduction

The jurisprudential examination of divorce in the Ecuadorian legal system is an important field of research in modern legal practice. In recent years, divorce has acquired increased importance in Ecuadorian culture, indicating substantial changes in family dynamics and cultural beliefs toward marriage and the breakup of marital partnerships. In this regard, research on information fusion approaches is becoming more relevant in a deep understanding of court cases connected to divorce. The capacity to combine diverse sources of data and information, such as legislation, judicial precedents and socioeconomic elements, becomes a vital ingredient to successfully handle the problems and complexity unique to jurisprudential analysis in this subject. Analyzing jurisprudence connected to divorce has become a difficult and comprehensive undertaking that demands novel and sophisticated analytical techniques. Information fusion approaches provide significant tools to handle this complexity by allowing the integration and coherent analysis of a broad variety of data and elements essential to judicial decision-making. Jurisprudential analysis entails reviewing and assessing a broad variety of legal material and evidence to achieve educated judicial judgments and decisions. However, the information supplied might be varied, fragmented, and frequently inconsistent, making it difficult for judges and attorneys to comprehend and administer the law.

In this sense, information fusion approaches give strong tools and strategies to efficiently combine and evaluate diverse sources of data and knowledge relevant to a specific legal issue. Information fusion enables data from multiple kinds and sources, such as legislation, legal precedents, expert testimony, and forensic evidence, to be merged to generate a fuller and more accurate picture of the issue at hand. By combining data logically and systematically, information fusion approaches may assist in eliminating ambiguity and uncertainty in jurisprudential analysis, so providing a firmer foundation for judicial decision-making [1].

Furthermore, information fusion approaches may also enable the detection of hidden patterns, trends, and linkages in data [2], which can help judges and attorneys better comprehend the nature and context of a certain legal issue. This may be particularly valuable in complicated, multiple instances, where understanding the interconnections between numerous aspects and variables might be vital to obtaining a fair and equitable verdict [3].

In this context, the implementation of methods such as Cognitive Maps appears as a viable way to enhance the comprehension and analysis of legal opinions linked to divorce. However, the use of cognitive maps to describe and simulate complex systems may lead to the introduction of ambiguity and indeterminacy [4]. This ambiguity may develop owing to the subjective character of the data and the difficulty in correctly defining causal links between various aspects of the system. [5]

Likewise, the intrinsic complexity of many phenomena examined in scientific and technical sectors may lead to uncertainty owing to the necessity to make conclusions based on partial or incomplete evidence [6]. In this setting, obtaining accuracy and perfect certainty may be problematic, leading to the need to create ways that can effectively manage and express ambiguity [7]. The complex interplay between various components that influence divorce situations, encompasses legal and jurisprudential concerns as well as emotional and social considerations [8, 9].

A potential method to remedy this dilemma is the introduction of fuzzy cognitive mapping (FCM). FCMs are an extension of classic cognitive maps that enable modeling the ambiguity and vagueness present in many real-world issues [10]. Unlike traditional cognitive maps, which describe causal links as binary (i.e., there is a connection between two items or there is not), FCMs enable the depiction of the strength and direction of interactions in a more flexible and graded [11]. This is performed by giving numerical values in the range [- 1, 1] to represent the strength of the causal relationship between the nodes in the map. Additionally, FCMs may manage uncertainty by enabling connection values to be imprecise or fuzzy, thereby representing the subjective or ambiguous nature of causal linkages in many complex systems. [12]

On the other hand, the introduction of Neutrosophic in the area of representation of complex systems gives an extra layer of depth and flexibility in the management of uncertainty and indeterminacy. Neutrosophic, a philosophical theory created by Romanian philosopher Florentin Smarandache in the 1990s, presents a more complete way of handling uncertainty that goes beyond typical fuzzy logic [12] . In the context of the representation of complex systems, Neutrosophic enables us to analyze not only the truth or falsity of a statement but also indeterminacy, that is, the inability to identify whether a proposition is true or untrue. [13, 14]. Neutrosophic presents a manner of describing the degree of indeterminacy or neutrosophic of a statement. This permits the underlying ambiguity of many real-world issues to be recorded more exactly and thoroughly than fuzzy logic. While fuzzy logic focuses on describing uncertainty via fuzzy sets and membership values, neutrosophic logic goes a step further by acknowledging the possibility that a statement might be true, false, or indeterminate [15].

In fuzzy cognitive maps, connections are essentially numerical, which restricts the depiction of non-linear interactions between events. However, Neutrosophic emerges as an alternative capable of managing ambiguous and inconsistent information, features that are not well handled by fuzzy sets and intuitive fuzzy sets [16]. Neutrosophic cognitive maps form an extension of fuzzy cognitive maps that contain indeterminacy in their conceptualization, providing a more exact and full depiction of reality. [17]. His study explores and analyzes the potential of Neutrosophic Cognitive Maps in the specific context of divorce jurisprudence in Ecuador. The research aims not only to examine the applicability and effectiveness of NCM to understand judicial decisions related to divorce but also to analyze the legal, social and human impact of these techniques on the interpretation and application of the law. By integrating quantitative and qualitative approaches, as well as jurisprudential and conceptual data, a comprehensive and enriching perspective is provided on the jurisprudential analysis of divorce and its implications for the protection of fundamental rights and the evolution of family law in Ecuador.

1. Related work

Definition 1 [18]. Let N = {(T, I, F): T, I, F ∈[0 ,1]}C be a *neutrosophic evaluation set* . v: $\boldsymbol{P} \to N$ is a mapping of a group of propositional formulas in N, that is, each sentence $p \in \boldsymbol{P}$ is associated with a value in N, as stated in Equation 1, that is, p is T% true, I% indeterminate and F% false.

$$\mathrm{v}\,(\mathrm{p}) = (\mathrm{T}, \mathrm{I}, \mathrm{F}) \tag{1}$$

Therefore, neutrosophic logic is an extension of fuzzy logic, rooted in the principles of neutrosophic according to.

Definition 2 . Let K be the ring of real numbers. The ring generated by $K \cup I$ is called a neutrosophic ring whenever it encompasses the element of indeterminacy within it, where I satisfies $I^2 = I$, $I + I = 2I$, and generally $I + I + \ldots + I = nI$,. If $k \in K$., then $k * I = kI$, and $0I = 0$. The neutrosophic ring is symbolized by K(I), which is generated by $K \cup I$, i.e. $K(I) = \langle K \cup I \rangle$, where $\langle K \cup I \rangle$ denotes the ring generated by K and I.

Definition 3 . A *neutrosophic matrix* is a matrix $A = \left[a_{ij}\right]_{ij}$ $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; $m, n \geq 1$, so that each $a_{ij} \in K(I)$, where K(I) is a neutrosophic ring, see [19].

It is worth noting that an element of the matrix can take the form $a + bI$, where "a" and "b" are real numbers, while (I) represents the indeterminacy factor. The standard operations of neutrosophic matrices can be extended from classical matrix operations. This representation allows us to capture the uncertain nature of some elements of the matrix, which can be crucial for modeling and understanding complex phenomena in various fields of study.

Manipulation and analysis of data that includes both certainty and uncertainty is allowed. This facilitates the application of matrix tools in situations where uncertainty plays an important role, thus expanding the scope and usefulness of matrices in various scientific and technological fields. For example.

$$\begin{pmatrix} -1 & I & 5I \\ I & 4 & 7 \end{pmatrix}\begin{pmatrix} I & 9I & 6 \\ 0 & I & 0 \\ -4 & 7 & 5 \end{pmatrix} = \begin{pmatrix} -21I & 27I & -6+25I \\ -28+I & 49+13I & 35+6I \end{pmatrix}$$

Furthermore, a neutrosophic graph is defined as a graph containing at least one edge or node characterized by indeterminacy, as shown in Figure 1. The neutrosophic adjacency matrix, an extension of the traditional adjacency matrix in graph theory, serves to represent the relationships between nodes. In this matrix, $a_{ij} = 0$it indicates that nodes i and j are not connected, $a_{ij} = 1$ means a connection between these nodes, and $a_{ij} = I$denotes an indeterminate connection state, where it is unknown whether a connection exists or not. In particular, fuzzy set theory lacks such distinctions and representations.

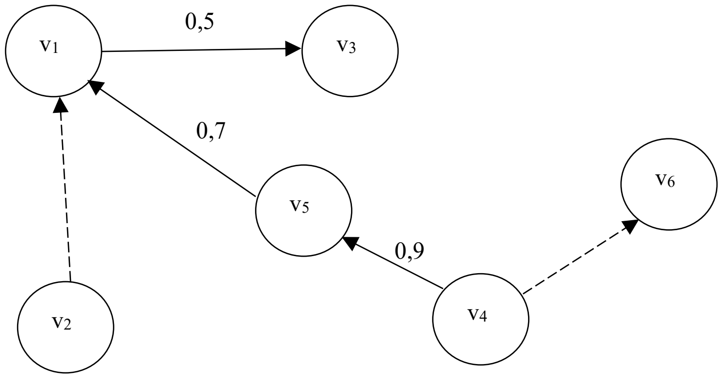

Figure 1. Neutrosophic graph

In contrast, when indeterminacy is incorporated into a cognitive map, the resulting map is called a neutrosophic cognitive map, which is particularly advantageous for representing causal knowledge. Its formal definition is provided in Definition 4.

Definition 4 [20]. A *Neutrosophic Cognitive Map* (NCM) is a directed neutrosophic graph with concepts such as policies and events, among others, as nodes and causalities or indeterminate as edges. It represents the causal relationship between concepts.

2. Material end Methods

As a first step, a Neutrosophic Cognitive Map (NCM) is created by obtaining expert information, which is then used to calculate centrality measures for static analysis.

The measures described below are used in the proposed model, they are based on the absolute values of the adjacency matrix[21].

- The degree is the sum of the row elements in the neutrosophic adjacency matrix. It reflects the strength of the salient relations of the variable.

  $$od(v_i) = \sum_{i=1}^{n} c_{ij} \quad (2)$$

- In degree is the sum of the elements of the column. It reflects the strength of the relationships that come out of the variable.

  $$id(v_i) = \sum_{i=1}^{n} c_{ji} \quad (3)$$

- The total centrality (total degree td (i)), is the sum of the entry degree and the exit degree of the variable.

  $$td(v_i) = od(v_i) + id(v_i) \quad (4)$$

The application of static analysis involves using the adjacency matrix considering the absolute values of the weights. In neutrosophic cognitive maps, as described in [22], static analysis is initially characterized by neutrosophic numbers in the form of $a + bI$, where I represent indeterminacy. This requires a deneutrosofization process, as suggested in, in which $I \in [0, 1]$,and is replaced by its maximum and minimum values.

Is used, calculated using Equation 5 as described in [23]. This calculation is beneficial to obtain a singular value, which helps identify the characteristics to prioritize based on the factors obtained in our case study.

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (5)$$

So,

$$A \succ B \Leftrightarrow \frac{a_1 + a_2}{2} > \frac{b_1 + b_2}{2} \quad (6)$$

3. Results and discussion

Resolution No. 0071 - 2018 - 38420 plays a crucial role in the Ecuadorian legal system by establishing a regulatory framework for divorce procedures, impacting both legal and emotional aspects of marital breakdowns. This research highlights the importance of this resolution in addressing various reasons for divorce, such as infidelity and unjustifiable desertion, while also considering the significant psychological effects on the spouses, including sadness, anxiety, and post-traumatic stress disorder. To thoroughly analyze these complexities, a Neutrosophic Cognitive Map (NCM) was developed, incorporating ten key variables that represent the legal and emotional dimensions of divorce. Expert input was used to assign neutrosophic values to the relationships between these variables, resulting in an adjacency matrix that provides a detailed mathematical foundation for understanding the intricate interrelationships involved in divorce. This approach offers a comprehensive perspective on the legal, emotional, familial, and societal factors at play, contributing to a more nuanced analysis of the divorce process.

Table 1. Selected critical factors to analyze during the study.

Coding	Factor	Description
F1	Legal causes of divorce	It is essential to understand the legal causes of divorce according to the current legal framework, as these establish the legal basis for the divorce process and can influence judicial decisions.
F2	Psychological and emotional impact	Assess the psychological and emotional impact of divorce on spouses and children, as these repercussions can affect the ability to make informed decisions and the well-being of the parties involved.
F3	Family dynamics	Assess the psychological and emotional impact of divorce on spouses and children, as these repercussions can affect the ability to make informed decisions and the well-being of the parties involved.
F4	Legal and Psychological Support	Consider the availability of legal and psychological support for the parties involved in the divorce process, as this may affect their ability to defend their rights and make informed decisions.
F5	Domestic violence	Identify and address cases of domestic violence, as these can have a significant impact on the divorce process and court decisions related to the safety and well-being of the parties involved.

Coding	Factor	Description
F6	Economic Aspects	Evaluate the financial aspects of divorce, including the division of marital assets and debts, as well as the determination of alimony and child support, as these aspects may influence court decisions related to the distribution of financial resources.
F7	Agreements and commitments	Consider the ability of the parties to reach mutual agreements and compromises regarding the divorce, as this can facilitate the judicial decision-making process and promote an amicable and equitable resolution of the conflict.
F8	Professional evaluations	Assess the need for professional evaluations, such as psychological evaluations and expert reports, to inform court decisions in divorce cases, as these evaluations can provide relevant information about the well-being and needs of the parties involved.
F9	Regulations and legal precedents	Consider relevant legal rules and precedents in divorce cases, as they can provide guidance on the interpretation of the law and the application of legal principles in specific situations.
F10	Minors' interests	Prioritize the interests and well-being of the children in the divorce process, as they must be considered a primary consideration in judicial decisions related to the custody, care, and support of children.

The NCM is developed through the capture and representation of relevant knowledge. The resulting adjacency matrix, based on the neutrosophic values assigned by the experts, is detailed in Table 2 as a key tool for the analysis and understanding of causal relationships within the context of the study.

Table 2. Adjacency matrix

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	0	1	0.5	0	0	0.4	I	0.3	0	0
F2	I	0	I	0.2	0	0	0	0	0	0
F3	0.1	0	0	1	0	0	0	0	0	0
F4	0	0.8	0.7	0	0	0	0	0	0	I
F5	1	0.7	1	I	0	0	I	0	0	0
F6	0.6	0.1	1	I	0	0	0	0	0.2	0.8
F7	0	0.5	0	0.9	I	0	0	0.1	0	0
F8	0	0.5	0	0.9	I	0	0	0.1	0	0
F9	1	0	0	1	0	0	I	0.5	0	I
F10	0	0	1	I	0.5	0	0	0.2	1	0

Based on this approach, the centrality measures that have been calculated are presented below. These metrics provide a quantitative assessment of the relative importance of nodes within the context of the network, which is essential to understanding the dynamics and influence of different elements in the system studied.

Table 3: Calculated centrality measures

Factors	$od(v_i)$	$id(v_i)$	$td(v_i)$
F1	2.2+I	2.7+I	4.9+I
F2	0.2+2I	3.6	3.8+2I
F3	1.1	4.2+I	5.3+I
F4	1.5+I	4+3I	5.5+3I
F5	2.7+2I	0.5+2I	3.2+4I
F6	2.7+I	0.4	3.1+I
F7	1.5+I	0+3I	1.5+3I
F8	1.5+I	1.2	2.7+i
F9	2.5+2I	1.2	3.7+2I
F10	2.7+i	0.8+2I	3.5+2I

The static analysis in NCM initially returns neutrosophic values that include the indeterminacy variable. Therefore, it is necessary to apply a deneutrosification process, as suggested by Salmerón and Smarandache, where the parameter I belonging to the interval [0,1] is replaced by its corresponding maximum and minimum values. This procedure is essential to obtain clearer and more precise results that facilitate the interpretation of the relationships within the context of the analysis. See Table 4.

Table 4: Deneutrosophization

Factors	$td(v_i)$
F1	4.9+5.9I
F2	3.8+5.8I
F3	5.3+6.3I
F4	5.5+8.5I
F5	3.2+7.2I
F6	3.1+4.1I
F7	1.5+4.5I
F8	2.7+3.7I
F9	3.7+5.7I
F10	3.5+5.5I

Finally, equation (5) is used to calculate the corresponding values for comparison. This approach allows obtaining a quantitative measure that synthesizes the relevant information, thus providing a more complete and precise perspective of the aspects considered in the analysis.

Table 5: Median of extreme values

Factors	$td(v_i)$
Legal causes of divorce	5.25
Psychological and emotional impact	4.8
Family dynamics	5.8
Legal and Psychological Support	7
Domestic violence	5.2
Economic Aspects	3.6
Agreements and commitments	3
Professional evaluations	3.2
Regulations and legal precedents	4.7
Minors' interests	4.5

The research highlights the importance of "Legal and Psychological Support" as the most influential factor in the legal study of divorce, with the highest centrality value in the neutrosophic map, followed by "Family Dynamics" and "Domestic Violence." These findings emphasize the need to prioritize legal and psychological assistance, family dynamics, and domestic violence prevention in divorce proceedings. While factors like "Economic Aspects" and "Agreements and Commitments" are significant, they are less central. The study demonstrates the effectiveness of neutrosophic cognitive maps in capturing the complexity and ambiguity of divorce-related legal and emotional issues, providing a comprehensive framework for analysis. Resolution No. 0071 - 2018 - 38420 serves as a crucial reference in addressing these elements within Ecuador's legal framework, offering a solid foundation for informed decision-making and policy development that supports justice and well-being in family law.

Conclusion

The current research offered a full investigation of the legal grounds and the psychological, emotional, and social ramifications of divorce, utilizing the approach of neutrosophic cognitive maps as the major instrument. Through this strategy, it was feasible to simulate the complicated interactions between aspects associated with divorce, giving a useful tool to enhance judicial decision-making in this sector. The insights collected from the research underline the need to address both the legal reasons and the psychological and emotional ramifications of divorce when making judicial judgments in this area.

The findings reveal the important importance of variables such as legal and psychological assistance, family dynamics, and domestic violence in the divorce process. Likewise, the efficacy of the neutrosophic cognitive mapping approach was proven to model complicated and multidimensional interactions in this context, giving a beneficial tool to increase the quality and depth of research in legal studies connected to divorce.

References

[1] Barragán-Pazmiño, B., Echeverría, A. O., Guadalupe, M. E., & Toulkeridis, T. (2024). Integrating machine learning models for enhanced soil organic carbon estimation: A multi-model fusion approach. Fusion: Practice and Applications, 2, 22-2.

[2] Li, L., Wang, J., Cao, Z., & Zhong, E. (2008). An information fusion method to identify spatial heterogeneity patterns to improve estimation accuracy. Stochastic Reign Research Risk Assessment, 22, 689-704.

[3] Peñaherrera, J. B. R., Diaz, K. M. T., & Marks, A. (2024). Fusion of forensic analysis of mobile devices: Integrating multi-criteria decision methods and case study insights. Full Length Article, 16(2), 32-2.

[4] Papageorgiou, E. I., & Iakovidis, D. K. (2012). Intuitionistic fuzzy cognitive maps. IEEE Transactions on Fuzzy Systems, 21(2), 342-354.

[5] Best, J. (2008). Ambiguity, uncertainty and risk: Rethinking indeterminacy. International Political Sociology, 2(4), 355-374.

[6] (2023). A neutrosophic multi-criteria model for evaluating sustainable soil enhancement methods and their cost implications in construction. Sustainable Machine Intelligence Journal, 5, 1-11.

[7] Rosli, S. N. I., & Zulkifly, M. I. E. (2023). B-spline bicubic neutrosophic surface interpolation model for uncertainty data. Neutrosophic Systems with Applications, 10, 25-34.

[8] Papageorgiou, E. I. (2011). Learning algorithms for fuzzy cognitive maps: A review study. IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews), 42(2), 150-163.

[9] Papageorgiou, E. I., Hatwágner, M. F., Buruzs, A., & Kóczy, L. T. (2017). A concept reduction approach for fuzzy cognitive map models in decision making and management. Neurocomputing, 232, 16-33.

[10] Leyva-Vázquez, M., Pérez-Teruel, K., Febles-Estrada, A., & Gulín-González, J. (2013). Causal knowledge representation techniques: A case study in medical informatics. Revista Cubana de Información en Ciencias de la Salud (ACIMED), 24(1), 73-83.

[11] Shen, F., Liu, J., & Wu, K. (2020). Evolutionary learning of multi-task fuzzy cognitive maps. Knowledge-Based Systems, 192, 105294.

[12] Quiroz, M., Mora, J., Medina, J., & Leyva, M. (2020). Causal models as an aid to understanding complex systems: Analysis of critical success factors in the development of chatbots. Universidad y Sociedad, 12(4), 64-72.

[13] Dat, L. Q., Thong, N. T., Ali, M., Smarandache, F., Abdel-Basset, M., & Long, H. V. (2019). Linguistic approaches to complex neutrosophic sets of intervals in decision making. IEEE Access, 7, 38902-38917.

[14] Ali, M., Hussain, Z., & Yang, M.-S. (2022). Hausdorff distance and similarity measures for single-valued neutrosophic sets with application in multi-criteria decision making. Electronics, 12(1), 201.

[15] Christianto, V., & Smarandache, F. (2024). Revealing quantum communication: From Bell's inequality to neutrosophic logic. Sustainable Machine Intelligence Journal, 6, 1-6. https://doi.org/10.61356/SMIJ.2024.66106

[16] Henríquez, E. J., Arízaga, J. E., Campoverde, M. R., & Peña, M. E. (2017). Analysis of interdependencies of competencies based on neutrosophic cognitive mapping. Neutrosophic Sets and Systems, 16, 89-92.

[17] Smarandache, F. (2024). Super hiper función y super hiper estructura y sus correspondientes super hiper función neutrosófica y super hiper estructura neutrosófica. Neutrosophic Computing and Machine Learning, 31, 353-359.

[18] Kandasamy, W. B. V., Kandasamy, I., & Smarandache, F. (2019). Semi-idempotents in neutrosophic rings. Mathematics, 7(6), 507.

[19] Abobala, M. (2020). Classical homomorphisms between refined neutrosophic rings and neutrosophic rings. International Journal of Neutrosophic Science, 5, 72-75.

[20] Mariscal Rosado, Z. M., Maldonado Manzano, R. L., & González Alvarado, R. J. (2023). Impacto de la deuda externa en los derechos de Babahoyo: Análisis con mapas cognitivos neutrosóficos. Neutrosophic Computing and Machine Learning, 27, 10-17.

[21] Saavedra Robles, L., Leyva Vázquez, M., & Hechavarría Hernández, J. R. (2021). Application of fuzzy cognitive maps in critical success factors. Case study: Resettlement of the population of the Tres Cerritos Enclosure, Ecuador. In Advances in Artificial Intelligence, Software and Systems Engineering: Proceedings of the AHFE 2020 Virtual Conferences on Software and Systems Engineering, and Artificial Intelligence and Social Computing, July 16-20, 2020, USA (pp. 400-406). Springer International Publishing.

[22] Moscoso-Paucarchuco, K. M., Beraún-Espíritu, M. M., Quispe-Quezada, U. R., Alvarez-Bernuy, S. M., Solano, M. A. Q., Julca-Marcelo, E. H., & Ramirez-Salas, W. (2024). Evaluation of the economic viability of circular models in agriculture based on neutrosophic cognitive maps. International Journal of Neutrosophic Science, 24(4), 352-352.

[23] Salmeron, J. L., & Smarandache, F. (2008). Redesigning decision matrix method with an indeterminacy-based inference process. International Journal of Applied Mathematics & Statistics, 13(M08), 4-11.

Received: June 22, 2024. Accepted: August 13, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Study of knowledge and preservation of traditional stories of the Shipibo-Konibo culture in primary school students based on neutrosophic logic

Hulda Cahuaza Ochavano[1], Beker Maraza Vilcanqui[2], Elizabeth Norma Calixto Arias[3], Aniceto Elías Aguilar Polo[4], Iris Katheryne Mori Cairuna[5], Walter Arturo Quispe Cutipa[6], Maritza Soria Ramírez[7] and Roció del Águila Sandoval[8]

[1] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: huldacahuaza@gmail.com
[2] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: bmarazav@unia.edu.pe
[3] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: ecalixtoa@unia.edu.pe
[4] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: aaguilarp@unia.edu.pe
[5] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: imoric@unia.edu.pe
[6] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: wquispec@unia.edu.pe
[7] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: rociodelaguilas@gmail.com
[8] National Intercultural University of Amazonia, Pucallpa. Peru. E-mail: soriaramirez08@gmail.com

Abstract. This paper aims to determine the knowledge and preservation of traditional stories of the Shipibo-Konibo culture in students of the Primary Educational Institution No. 64161-Bilingue Nuevo Nazareth de Iparia, Peru in 2023. A population of 119 students from the educational institution was considered, whom we interviewed personally after filling out a questionnaire to collect information regarding their knowledge of traditional stories of the Shipibo-Konibo culture. The opinions of the children surveyed are given in the form of linguistic values associated with Single-Valued Neutrosophic Numbers. The data is processed with the help of some propositions according to the neutrosophic logic. The advantage of this logic is that it allows us to deal with indeterminacy explicitly. The results show that children tell stories, fables, myths, and legends belonging to the Shipibo-Konibo culture and that they are transmitted from generation to generation so that their knowledge, beliefs, and teachings are still preserved through morals learned from the traditional stories of this culture.

Keywords: Traditional stories, record literary, Shipibo-Konibo culture, neutrosophy, neutrosophic logic, single-valued neutrosophic number.

1 Introduction

The ancient times of Peru were characterized by traditions, customs, knowledge, and other ways of expressing their culture through oral folklore. The most relevant events of each era were narrated about various actions of the Andean and Amazonian populations. Tradition is the soul of the people, of a society, of a country, a nation. It is their way of thinking, feeling, and acting; oral tradition is a dynamic concept, it renews itself without losing its essence. So we can say that it is born and develops within the community as a natural expression that seeks to preserve and endure identities beyond the oblivion and disappearance of successive generations.

For a clear understanding of this topic, it is necessary to distinguish three concepts: history, understood as narrative content; story, as a narrative signifier or the text itself; narration, as the productive instance of the story. Three categories are also used: time, mode, and voice.

The practices of traditions in a community are diverse, starting with their knowledge, their history, their creators, the events that served the group and consolidate, and strengthen their best customs and ways of being, which are part of the sociocultural characteristics that allow to identify each community, region, and locality.

Like all myths and tales, the stories of ancient Peru have the function of providing listeners (nowadays readers) with an ethical and aesthetic message. They offer a comprehensive worldview, for example, of what was considered socially legitimate, what behaviors were morally reprehensible, or how competitions between men and divinities were recognized.

The essence of traditional stories is the transmission of knowledge from generation to generation. We are not unaware of the complexity of the concept of tradition and its richness when approaching marginalized literature such as national literature. We have proof of this whole world of symbolism and inner reality in the existence of myths, initiation rites, religions, philosophical thoughts, or artistic creations that humanity shares.

In ancient cultures, narrative was the first way of expressing various thoughts, knowledge, facts, stories, and a whole range of understanding of a people when they did not have writing. Narrative thought, on the other hand, produces good stories, dramatic works, and credible chronicles although not necessarily true. As we mentioned before, it deals with human intentions and actions and the vicissitudes and consequences that mark their course. There is an essential element in this modality that modulates human actions: emotions. Narrative thought weaves the basic experience of living, and gives meaning to that experience, to the world in general, and to us as individuals.

Consequently, within cultures, it is possible to develop a living expression through words, which are related to education to achieve communicative skills through the use of oral traditions in their mother tongue, as in the Shipibo-Konibo culture of Peru.

Shipibo-Konibo culture is an indigenous people located in the Peruvian Amazon around the banks of the Ucayali River. In the past, it was made up of three different ethnic groups that were united into one through marriage between their members. They all belong to the same linguistic family called *Pano*. Their main economic activities are fishing, agriculture, crafts, and hunting.

Significant pedagogical actions involving the use of oral communication in the language that the child masters most at school are developed extra-curricular and help in the research of oral traditions and this contributes to the preservation of our culture.

This article interviews and applies a questionnaire to 119 children who study at the Primary Educational Institution No. 64161-Bilingue Nuevo Nazareth de Iparia in Peru. The aim is to determine whether they know traditional stories, tales, and fables and whether they are capable of transmitting them to their peers or to the generations that follow them.

For this purpose, some predicates or logical propositions are used to determine whether this knowledge exists in the students surveyed. To facilitate the completion of the surveys, they were asked to respond based on a linguistic scale where each element has a Single-Valued Neutrosophic Number associated with it [1]. The advantage of dealing with the neutrosophic logic is that it allows dealing with indeterminacy obtained explicitly from different origins that can be due to neutrality, ignorance, contradiction, inconsistency, paradox, and so on. We rely on neutrosophic logic to carry out the study [2-8].

This article is divided into the following sections; the following section is dedicated to explaining the basic notions of neutrosophic logic and neutrosophy in general. The Results section follows. At the end, there is a Conclusions section.

2 Preliminaries

Definition 1: ([1]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow \,]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy, and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 2: ([1]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indeterminate, and falseness of x in A, respectively. For convenience, a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Regarding the neutrosophic logic, if p is a proposition, then a *neutrosophic valuation* of p is defined as:

$$v_N(p) = (t, i, f) \quad (2)$$

Such that $(t, i, f) \subseteq [0, 1]^3$ and in particular $(t, i, f) \in [0, 1]^3$, where t means the degree of truthfulness, i is the degree of indeterminacy, while f is the degree of falseness.

Given the universe of discourse X and $x(T_x, I_x, F_x)$, $y(T_y, I_y, F_y)$ two SVNN, we say that $x \leq_N y$ if and only if $T_x \leq T_y$, $I_x \geq I_y$ and $F_x \geq F_y$, $(X, \leq_N)$ is a *poset*. Whereas, $(L, \wedge, \vee)$ is a lattice, because it is a triple direct product of lattices, see [9, 10]. $x \wedge y = (\min\{T_x, T_y\}, \max\{I_x, I_y\}, \max\{F_x, F_y\})$ and $x \vee y = (\max\{T_x, T_y\}, \min\{I_x, I_y\}, \min\{F_x, F_y\})$. Moreover, it is easy to prove that it is complete.

Let us remark that this definition is valid for interval-valued neutrosophic sets when we substitute their operators with interval-valued operators.

See also that there exist two special elements, viz., $0_N = (0, 1, 1)$ and $1_N = (1, 0, 0)$, which are the infimum and the supremum, respectively, of every SVNS concerning $\leq_N$.

Given two neutrosophic sets, A and B, three basic operations over them are the following, [10, 11]:

1. $A \cap B = A \wedge B$ (Conjunction),
2. $A \cup B = A \vee B$ (Disjunction),
3. $\bar{A} = (F_A, 1 - I_A, T_A)$ (Complement).

Definition 3. A *neutrosophic norm* or *n-norm* N_n [9, 10], is a mapping $N_n: (]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[)^2 \to]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$, such that $N_n\left(x(T_x, I_x, F_x), y(T_y, I_y, F_y)\right) = (N_nT(x,y), N_nI(x,y), N_nF(x,y))$, where N_nT means the degree of membership, N_nI the degree of indeterminacy, and N_nF the degree of non-membership of the conjunction of both, x and y.

For every x, y, and z belonging to $]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$, N_n must satisfy the following axioms:

1. $N_n(x, 0_N) = 0_N$ and $N_n(x, 1_N) = x$ (Boundary conditions),
2. $N_n(x,y) = N_n(y,x)$ (Commutativity),
3. If $x \leq_N y$, then $N_n(x,z) \leq_N N_n(y,z)$ (Monotonicity),
4. $N_n(N_n(x,y),z) = N_n(x, N_n(y,z))$ (Associativity).

Definition 4. A *neutrosophic conorm* or *n-conorm* N_c [9, 10], is a mapping $N_c: (]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[)^2 \to]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$, such that $N_c\left(x(T_x, I_x, F_x), y(T_y, I_y, F_y)\right) = (N_cT(x,y), N_cI(x,y), N_cF(x,y))$, where N_cT means the degree of membership, N_cI the degree of indeterminacy and N_cF the degree of non-membership of the disjunction of x with y.

For every x, y, and z belonging to $]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$, N_c must satisfy the following axioms:

1. $N_c(x, 0_N) = x$ and $N_c(x, 1_N) = 1_N$ (Boundary conditions).
2. $N_c(x,y) = N_c(y,x)$ (Commutativity).
3. If $x \leq_N y$, then $N_c(x,z) \leq_N N_c(y,z)$ (Monotonicity).
4. $N_c(N_c(x,y),z) = N_c(x, N_c(y,z))$ (Associativity).

According to [10] a Singled-Valued Neutrosophic Negator is defined as follows:

Definition 5. A *Single-Valued Neutrosophic Negator* is a decreasing unary neutrosophic operator $N_N: [0,1]^3 \longrightarrow [0,1]^3$, satisfying the following boundary conditions:
1. $N_N(0_N) = 1_N$.
2. $N_N(1_N) = 0_N$.

It is called *involutive* if and only if $N_N(N_N(x)) = x$ for every $x \in [0,1]^3$.

In the following, we show the neutrosophic negators that we have extracted from the literature, see [9, 10]. Given an SVNS A (T_A, I_A, F_A), we have:

1. $N_N((T_A, I_A, F_A)) = (1 - T_A, 1 - I_A, 1 - F_A)$, $N_N((T_A, I_A, F_A)) = (1 - T_A, I_A, 1 - F_A)$, $N_N((T_A, I_A, F_A)) = (F_A, I_A, T_A)$ and $N_N((T_A, I_A, F_A)) = (F_A, 1 - I_A, T_A)$ (Involutive negators).
2. $N_N((T_A, I_A, F_A)) = \left(F_A, \frac{F_A + I_A + T_A}{3}, T_A\right)$ and $N_N((T_A, I_A, F_A)) = \left(1 - T_A, \frac{F_A + I_A + T_A}{3}, 1 - F_A\right)$ (Non-involutive negators).

In the literature, we found neutrosophic implicators, which extend only the notion of *S*-implications [10, 12]. Furthermore, we did not find a general definition of neutrosophic implicators except in [10, 13]. In the following, we conclude this section with definitions and properties.

Definition 6. A *single-valued neutrosophic implicator* is an operator $I_N: [0,1]^3 \times [0,1]^3 \to [0,1]^3$ which satisfies the following conditions, for all $x, x', y, y' \in [0,1]^3$:

1. If $x' \leq_N x$, then $I_N(x,y) \leq_N I_N(x',y)$,
2. If $y \leq_N y'$, then $I_N(x,y) \leq_N I_N(x,y')$,
3. $I_N(0_N, 0_N) = I_N(0_N, 1_N) = I_N(1_N, 1_N) = 1_N$,
4. $I_N(1_N, 0_N) = 0_N$.

Neutrosophic R-implicators can be read in [14-16].

Herein we use the term *neutrosophic implicator* or *n-implicator* to mean single-valued neutrosophic implicator.

It can satisfy the following properties for every $x, y, z \in [0,1]^3$:

1. $I_N(1_N, x) = x$ (Neutrality principle),
2. $I_N(x, y) = I_N(N_{IN}(y), N_{IN}(x))$, where $N_{IN}(x) = I_N(x, 0_N)$ is an n-negator (Contrapositivity),
3. $I_N(x, I_N(y, z)) = I_N(y, I_N(x, z))$ (Interchangeability principle),
4. $x \leq_N y$ if and only if $I_N(x, y) = 1_N$ (Confinement principle),
5. I_N is a continuous mapping (Continuity).

3 The study

The study included 119 students from the Primary Educational Institution No. 64161-Bilingue Nuevo Nazareth de Iparia during the year 2023.

Each student was assessed on their knowledge of the traditional narratives shown in Table 1.

Table 1: Traditional stories of the Shipibo-Konibo Culture. Source: Literary registration form.

No.	Konibo culture (in their native language)	Konibo culture (in English)
1	*Joni jonobaon boní*	The man who was taken by the peccaries
2	*Awajonin westiora ainbo jenemea nokoni*	The Sachava man who found the mermaid.
3	*Xawan kiss manxaman bachi bii kax xono jiwi bochiki baneni*	Xawan benxo who stayed high up in the lupuna tree when he went to collect heron eggs
4	*Atapa betan nonon iní*	The hen and the duck
5	*Chosko joni yapan tetaibo iní*	The four fishermen
6	*Mananxawenin wiso ino paraanan sapenman puppets ninimani*	The fight between the black tiger and the sea cow mama, under the lie of the turtle
7	*Shipibo winoni*	The occurrence of the little lion monkeys
8	*Rononkoniax kené pikoni*	The origin of design from the viper
9	*Joni wanoax mishki kani*	The married man who went fishing with a hook
10	*Joni kabori bachi benai kax wetsa jonin mapon tatí iní*	The man who went looking for taricaya eggs tripped over another man's head
11	*Jonin, boka jawen awin bona kokoai merani*	The man who found the one-handed man giving honey to his wife
12	*Joxo ainbo iní*	The story of the white woman
13	*You are my friend*	Grandfather Korin's story
14	*Rabé ranon iní*	The story of two young people
15	*Ishtonbires bakebo anipaoni*	Child development in the historical past
16	*Jawen chain yoiresabi joni reokooni*	The man who was the victim of his brother-in-law's saying

According to Table 1, it is observed that 16 traditional stories are recorded in primary level students that are the subject of research, which belong to traditional stories of their community in the Shipibo-Konibo language and are tales, fables, and myths.

All literary records are events that occurred in the Amazonian setting, which means that these teachings come from generation to generation, most of the characters are people and animals from the jungle, also, the teachings are consistent with the life of the population of the Shipibo-Konibo culture. The source of the record of stories, myths, and fables was in the mother tongue (Shipibo-Konibo) for research purposes it is translated into English.

Each of these narratives was classified from the point of view of the literary genre as follows as shown in Table 2:

Table 2: Classification of traditional short and medium-length narrative stories. Source: Literary registration form.

No.	Traditional stories	Classification		
		Tale	Fables	Myths or legends
1	*Joni jonobaon boní*	X		
2	*Awajonin westiora ainbo jenemea nokoni*			X
3	*Xawan kiss manxaman bachi bii kax xono jiwi bochiki baneni*	X		
4	*Atapa betan nonon iní*		X	
5	*Chosko joni yapan tetaibo iní*	X		

No.	Traditional stories	Classification		
		Tale	Fables	Myths or legends
6	*Mananxawenin wiso ino paraanan sapenman puppets ni-nimani*		X	
7	*Shipibo winoni*		X	
8	*Rononkoniax kené pikoni*			X
9	*Joni wanoax mishki kani*	X		
10	*Joni kabori bachi benai kax wetsa jonin mapon tatí iní*	X		
11	*Jonin, boka jawen awin bona kokoai merani*	X		
12	*Joxo ainbo iní*	X		
13	*You are my friend*	X		
14	*Rabé ranon iní*	X		
15	*Ishtonbires bakebo anipaoni*			X
16	*Jawen chain yoiresabi joni reokooni*	X		

Table 2 shows the classification of traditional stories of the Shipibo-Konibo culture, which are narratives of the experiences and knowledge that the population has throughout their lives. This table shows that boys and girls record 10 stories, 3 fables, and 3 myths or legends, which means that the highest weighting in traditional stories in the Shipibo-Konibo culture is the story with the greatest conservation.

Of the 16 different records of traditional stories, all belong to the narrative genre, including tales, fables, and legends. All the stories present a moral, where the story teaches knowledge that will be useful for daily practice. Through the stories, values can be taught to the population of the Shipibo-Konibo culture.

Because the children in the study were of different ages, each of them was given an evaluation form, although the tests were different depending on their age. In the end, we obtained evaluations for each child on the same variables regardless of age, although the results are relative to the age of the child. For example, children who do not yet know how to read and write or do not know how to read and write well enough were given oral tests. Older children took written tests. In general, the following variables were evaluated:

1. The child has a good command of oral expression in his (her) indigenous mother tongue.
2. The child knows the stories that are being proposed to him (her).
3. The child can recognize the moral of the stories proposed in Table 1.
4. The child can effectively and motivatingly repeat the stories offered in Table 1.
5. The child knows other stories besides those proposed.
6. The child enjoys telling stories that he (or she) knows to other people.
7. The child recognizes the stories and the essence of what they want to express as part of his (or her) cultural identity.
8. The child can recognize other aspects of his (or her) culture and shows identification with them, such as pottery, and traditional farming methods, among others.
9. The child can explain these stories in Spanish.

Let us denote as V_{ij} the result obtained by the ith student ($i \in \{1,2,\cdots,119\}$) for each of the jth variables, following the order that appears above $j \in \{1,2,\cdots,9\}$.

The values of V_{ij} are obtained from Table 3 that are shown below:

Table 3: Scale of linguistic terms associated with neutrosophic values. See [17].

Linguistic expressions	$(\mathbf{T}, \mathbf{I}, \mathbf{F})$
Very Bad (VB)	$(0.10, 0.75, 0.85)$
Bad (B)	$(0.25, 0.60, 0.80)$
Medium Bad (MB)	$(0.40, 0.70, 0.50)$
Medium (M)	$(0.50, 0.40, 0.60)$
Medium Good (MG)	$(0.65, 0.30, 0.45)$
Good (G)	$(0.80, 0.10, 0.30)$
Very Good (VG)	$(0.95, 0.05, 0.05)$

The linguistic elements appearing in Table 3 are those used by the evaluators for each of the V_{ij}. On the other hand, the SVNN associated with them are those used in the processing of the results.

The algorithm used to carry out the evaluations is the following, see Table 4:

Table 4: Algorithm for obtaining and processing the data from the study carried out.

Proposed algorithm for studying the acceptance of traditions
1. The values of the variables are obtained V_{ij} as a result of the questionnaire and the interviews with the children studied. 2. The following logical predicates are evaluated: $p_{1i} := V_{i1} \wedge_N V_{i2} \wedge_N V_{i3} \wedge_N V_{i4} \wedge_N V_{i9}$ (3) $p_{2i} := V_{i5} \wedge_N V_{i6}$ (4) $p_{3i} := V_{i7} \wedge_N V_{i8}$ (5) $p_{4i} := p_{1i} \wedge_N (p_{1i} \rightarrow_N p_{3i})$ (6) $p_{5i} := p_{2i} \wedge_N (p_{2i} \rightarrow_N p_{3i})$ (7) The predicate in Equation 3 is used to assess students' speaking skills. The predicate in Equation 4 is used to assess children's communication skills and willingness to learn about their culture. It is the attitude. The predicate of Equation 5 indicates the child's identification with his (or her) culture. The predicate in Equation 6 is a Modus Ponens to measure how cultural identity is inferred from children's speaking skills. The predicate of Equation 7 is also a Modus Ponens where the premise is changed by the children's ability to communicate. For the neutrosophic conjunction $\wedge_N$ the minimum n-norm is used: $N_n\left((T_A, I_A, F_A), (T_B, I_B, F_B)\right) = (\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B))$, and together with the maximum n-conorm $N_c\left((T_A, I_A, F_A), (T_B, I_B, F_B)\right) = (\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B))$, and the neutrosophic negation $N_N\left((T_A, I_A, F_A)\right) = (F_A, I_A, T_A)$, the Neutrosophic *S*-implication $SI_N\left((T_A, I_A, F_A), (T_B, I_B, F_B)\right) = N_c\left(N_N\left((T_A, I_A, F_A)\right), (T_B, I_B, F_B)\right)$ is used in $\rightarrow_N$. 3. The results of each predicate are aggregated for all children, using the arithmetic mean operator. So, we get: $\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4, \bar{p}_5$ such that the components T, I, and F are obtained from the means of these components for all the components of the students. 4. The results are interpreted according to the one that has the smallest distance from one of the SVNN values in Table 3. Where: $\text{distance}\left((T_A, I_A, F_A), (T_B, I_B, F_B)\right) = \sqrt{(T_A - T_B)^2 + (I_A - I_B)^2 + (F_A - F_B)^2}$ (8)

The final results are summarized in Table 5.

Table 5: Final results of the evaluation of the predicates and their linguistic interpretation.

Predicate	**SVNN obtained from the algorithm**	**Linguistic interpretation (distance)**
$\bar{p}_1$	$(0.82, 0.13, 0.23)$	Good ($distance = 0.079$)
$\bar{p}_2$	$(0.70, 0.22, 0.38)$	Medium Good ($distance = 0.117$)
$\bar{p}_3$	$(0.76, 0.19, 0.30)$	Good ($distance = 0.098$)
$\bar{p}_4$	$(0.76, 0.13, 0.30)$	Good ($distance = 0.098$)
$\bar{p}_5$	$(0.70, 0.22, 0.38)$	Medium Good ($distance = 0.117$)

From the results shown in Table 5, it can be inferred that all the predicates had values between “Medium Good” and “Good” and that there is also a logical relationship between the children's ability to communicate and their cultural identity, in addition to the children's attitude to communicate with their identification with their culture, with valuations “Medium High” and “High”.

Conclusion

The preservation of the cultural heritage of each ethnic group or society is as important as the preservation of the individual identity of each person. This prevents the uprooting of people and communities. In this paper, we set out to study the degree of identification of a group of 119 children descended from the indigenous Shipibo-Konibo ethnic group of the Peruvian Amazon, all of whom are students at the Primary Educational Institution No. 64161-Bilingue Nuevo Nazareth de Iparia during the year 2023. Specifically, knowledge of the traditional stories of this culture was measured. A linguistic scale was used with elements associated with Single-Valued Neutrosophic Numbers. The use of SVNN allows for inclusion in the evaluation of the uncertainty that exists in decision-making. Five predicates on nine variables were used to evaluate the oral skills of the students in their mother tongue, the ability of the students to express their ideas and knowledge about their culture, and the identification of the children with their culture. In addition, two predicates were added to indicate the deductions corresponding to "if there are oral communication skills then there is identification with their culture" and "if there is an attitude to communicate then there is identification with their culture." It was concluded that all these predicates have values between Medium High (or Medium Good) and High (Good). Therefore, the students of this institution still preserve their cultural identity, which is an encouraging result.

References

[1] Wang, H., Smarandache, F., Zhang, Y.-Q. and Sunderraman, R. (2005) Interval Neutrosophic Sets and Logic: Theory and Applications in Computing, Hexis, Arizona.
[2] Rivieccio, U. (2008). Neutrosophic logics: Prospects and problems. Fuzzy sets and systems, 159, 1860-1868.
[3] Christianto, V., Boyd, R. N., and Smarandache, F. (2020). Three possible applications of Neutrosophic Logic in Fundamental and Applied Sciences. International Journal of Neutrosophic Science, 1, 90-95.
[4] Smarandache, F., Quiroz-Martínez, M. A., Estupiñán-Ricardo, J., Batista-Hernández, N., and Leyva-Vázquez, M. Y. (2020). Application of neutrosophic offsets for digital image processing. Investigación Operacional, 41, 603-612.
[5] Jdid, M., Smarandache, F., and Broumi, S. (2023). Inspection Assignment Form for Product Quality Control Using Neutrosophic Logic. Neutrosophic Systems with Applications, 1, 4-13.
[6] Mandour, S. (2023). An Exhaustive Review of Neutrosophic Logic in Addressing Image Processing Issues. Neutrosophic Systems with Applications, 12, 36-55.
[7] Paraskevas, A. (2024). Extended Event Calculus using Neutrosophic Logic: Method, Implementation, Analysis, Recent Progress and Future Directions. Neutrosophic Systems with Applications, 14, 38-49.
[8] Edalatpanah, S. A., Hassani, F. S., Smarandache, F., Sorourkhah, A., Pamucar, D., and Cui, B. (2024). A hybrid time series forecasting method based on neutrosophic logic with applications in financial issues. Engineering applications of artificial intelligence, 129, 107531.
[9] Smarandache, F. (2017) Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications, Pons Publishing House, Brussels.
[10] González-Caballero, E., Leyva-Vázque, M., & Smarandache, F. (2021). On neutrosophic uninorms. Neutrosophic sets and systems, 45, 340-348.
[11] Ramos, E. L. H., Almache, F. A. G., Kairuz, A. R. P., and Riofrio, C. A. M. (2021). A New Model based on Subjective Logic and Neutrosophic Measure for Legal Reasoning. Neutrosophic Sets and Systems, 44, 456-463.
[12] Broumi, S. and Smarandache, F. (2014) On Neutrosophic Implications. Neutrosophic Sets and Systems, 2, 9-17.
[13] Bao, Y.-L., Yang, H.-L., and Li, S.-G. (2018) On characterization of (I, N)-single valued neutrosophic rough approximation operators. Soft Computing, 23, 6065-6084.
[14] Luo, M., Xu, D., and Wu, L. (2022, March). Fuzzy Inference Full Implication Method Based on Single Valued Neutrosophic t-Representable t-Norm. In Proceedings (Vol. 81, No. 1, p. 24). MDPI.
[15] Luo, M., Sun, Z., and Wu, L. (2024). Fuzzy Inference Quintuple Implication Method Based on Single Valued Neutrosophic t-representable t-norm. Neutrosophic Optimization and Intelligent Systems, 3, 8-22.
[16] Bu, H., Hu, Q., and Zhang, X. (2023). Neutrosophic Pseudo-t-norm and its derived neutrosophic residual implication. Neutrosophic Sets and Systems, 57, 18-32.
[17] Angelica, V. B. G., Favio, V. B. E., Manuel, V. M. V., Amandiz, C. V. D., and del Carmen, P. A. I. (2024). Model of Tax Culture Impact on the Financial Sustainability of Small and Medium Enterprises in Ecuador Based on Neutrosophic HyperSoft Sets. Neutrosophic Sets and Systems, 69, 86-93.

Received: June 23, 2024. Accepted: August 14, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Uncertainty Analysis in Prediction Intervals Using Neutrosophic Numbers

Héctor Félix Cerna Maguiña[1], Domingo Guzmán Chumpitaz Ramos[2], Ada Lucía Gallegos Ruiz Conejo[3], Nancy Isabel Pool Painted[4], and Jesús Isabel Baltazar Ángeles [5]

[1] National University of San Marcos, Lima. Peru. E-mail: hcernam@unmsm.edu.pe
[2] Ricardo Palma University, Lima. Peru. E-mail: dchumpitazr@unmsm.edu.pe
[3] Ricardo Palma University, Lima. Peru. E-mail: agallegosr@unmsm.edu.pe
[4] Ricardo Palma University, Lima. Peru. E-mail: nalbercapintado@gmail.com
[5] Ricardo Palma University, Lima. Peru. E-mail: eb4060@gmail.com

Abstract. This study analyzes uncertainty in prediction intervals by applying neutrosophic numbers to quality of life forecasts based on characteristics such as compensation, job hours, years of experience, and academic level. Three examples were studied, with indeterminacy values ranging from 0.589 to 0.628. These statistics show that between 58.9% and 62.8% of the prediction intervals are affected by uncertainty. Higher indeterminacy levels reflect a stronger effect of external influences or unpredictability within the data. The analysis demonstrates that as variables like compensation and labor hours grow, so does the level of uncertainty in the projections. Neutrosophic numbers provide a valuable framework for quantifying this uncertainty, providing for a better understanding of the model's limitations and the unpredictable nature of the dataset.

Keywords: Neutrosophic numbers, uncertainty analysis, prediction intervals, indeterminacy, remuneration, quality of life predictions.

1 Introduction

The quality of life of teachers is an important factor in the academic sector, which directly influences their performance and professional happiness. This research focuses on examining how high salary and another factor effects the quality of life of professors at the Faculty of Accounting Sciences of the Universidad Nacional Mayor de San Marcos, using neutrosophic regression models to handle the uncertainty inherent to this issue. Historically, the influence of pay on quality of life has been the topic of various research. Early methods concentrated on direct relationships between wage and satisfaction, but over time, it has been recognized that the link is more complicated and multidimensional [1].

Currently, with the growth of sophisticated analytical models, such as neutrosophic models, new perspectives have been established to analyze the connection between economic conditions and well-being [2]. This change in emphasis illustrates the growth in the knowledge of how financial incentives impact teachers and academic workers more broadly.

Neutrosophic machine learning [3] is an emerging field that combines neutrosophic theory, developed by Florentin Smarandache [4, 5], with modern machine learning algorithms to address uncertainty, imprecision, and incompleteness in data. Traditional machine learning assumes data is accurate and complete, but data often contains ambiguities and gaps that can affect model performance. Neutrosophic methods offer a new approach by incorporating uncertainty directly into the learning process, enhancing model accuracy, flexibility, and interpretability. These models are particularly useful in fields like image analysis, natural language processing, medicine, and engineering, where data is often imperfect. However, implementing neutrosophic techniques can be complex, requiring significant customization of existing algorithms, and ongoing research is needed to fully validate their advantages over traditional methods. Despite these challenges, neutrosophic machine learning has the potential to significantly improve outcomes in various applications by better managing uncertainty and incomplete data. In essence, neutrosophic machine learning provides a novel progression in the way models manage uncertainty and imprecision. Although it currently faces hurdles and needs more validation, its ability to give a richer and more flexible representation of data shows tremendous promise to increase the accuracy and application of machine learning models in diverse scenarios.

The purpose of this research is, first, to utilize neutrosophic regression models factor that improves the quality of life of teachers, taking into consideration the uncertainty and variety in individual views. Secondly, it tries to make suggestions based on the results to enhance compensation policies in the academic setting. These aims are related with the study topic and will be elaborated throughout the paper, presenting a complete perspective on how remuneration effects the well-being of teachers and how this connection might be better handled in the academic sector [6].

2 Preliminaries

2.2. Neutrosophic Machine Learning

Machine learning (ML) [7] utilizes mathematical formulations to develop models that can learn from data to make predictions or judgments without being explicitly programmed to accomplish such tasks. Interval prediction in machine learning refers to the approach of predicting a range of probable outcomes for a given input rather than a single point estimate. By offering intervals, these approaches give not just forecasts but also insight into the dependability and uncertainty of the predictions, which is vital for decision making in uncertain contexts [8].

For a data set with independent variables $X = [x_1, x_2, \dots, x_n]$ and a dependent variable, the goal of regression analysis is to model the relationship between X and y accurately. This relationship is expressed mathematically as [9]:

$$y \approx f(X;\, \theta) \quad (1)$$

Where:

$y,$ is the dependent variable or the objective to be predicted.

$X,$ rrepresents the independent or explanatory variables that are used to predict.

f is the regression function, which can vary in shape depending on the type of regression model used (linear, polynomial, logistic, etc.).

θ are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, typically the mean square error (MSE) in regression [10].

In regression analysis, representing predictions as prediction intervals provides a more complete view of the uncertainty associated with the predictions. A prediction interval provides a range within which we expect the true value of the dependent variable to fall with a certain probability, typically 95% or 99%. This is particularly useful because it takes into account variability in the data that might not be captured by prediction alone [2].

To calculate a prediction interval, both the uncertainty in the regression model estimate and the inherent variability of the data must be considered. The interval is built around the predicted value and is usually symmetrical, extending a certain amount above and below the predicted value. This range is determined based on the standard error of the prediction and the residual standard deviation, which reflects the dispersion of the residuals or errors of the model [11, 12].

For example, in a simple linear regression, the prediction interval for a new observation is given by [12]:

$$\hat{y}_0 \pm t_{\propto\,/2,n-2} \cdot SE \quad (2)$$

Where $\hat{y}_0$ is the predicted value of y from the t distribution for a specific confidence level $\propto$and $n-2$ degrees of freedom, and?

Using prediction intervals in regression analysis is beneficial because they offer a realistic spectrum of possible outcomes, which aids in the decision-making process. This recognizes that a single predicted value is not absolute but rather a likely scenario within a range of potential outcomes. This forecasting method effectively incorporates the inherent uncertainties associated with future predictions, providing a more accurate description of what to expect. To further refine this model, neutrosophic statistics can be applied, which excel at managing ambiguity and indeterminacy in data. By converting the interval to a neutrosophic number, the traditional interval is enhanced to include a component of indeterminacy. This addition captures the uncertainty and imprecision that is typically present in real-world data, offering a more nuanced understanding of data variability. The neutrosophic treatment of the interval is as follows [13, 14]:

$$\hat{y}_0 - t_{\propto/\,2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto\,/2,n-2} \cdot SE)I \quad (3)$$

Here, $I_N$represents the indeterminacy factor associated with the prediction, where $I_N \in [I_l, I_u]$, this notation introduces the limits of indeterminacy. I_l (lower indeterminacy) and I_u (upper indeterminacy), which define the range of possible deviations due to uncertain elements that affect the forecast [15].

Neutrosophic techniques are particularly beneficial for fusing diverse predictive models because they allow for the inclusion of ambiguity, indeterminacy, and even contradicting information that may originate from numerous data sources or model outputs. This technique adds to increasing the robustness and reliability of prediction models by presenting a more complete framework that takes into account many types of uncertainty. The neutrosophic mean denoted as X_n, is calculated considering the neutrosophic inclusion $I_N$that belongs to the interval. $[I_l, I_u]$.This mean consists of two main elements: X_l, which is the mean of the bottom part of the neutrosophic samples , and X_u, which is the mean of the top part. The respective definitions are [16]:

$$X_l = \frac{\sum_{i=1}^{n_l} X_{il}1}{n_l} \quad (4)$$

$$X_u = \frac{\sum_{i=1}^{n_u} X_{iu}}{n_u} \quad (5)$$

where $n_l$and $n_u$represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean $X_n$is expressed as the sum of $X_l$and X_u, adjusted by the interval of indeterminacy I_n [17]:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (6)$$

$I_{l,}$=0, andI_u

$$I_u = \frac{X_u - X_l}{X_u} \quad (7)$$

3 Methods.

To better understand the process, it can be broken down into four essential steps, as illustrated in Figure 1.

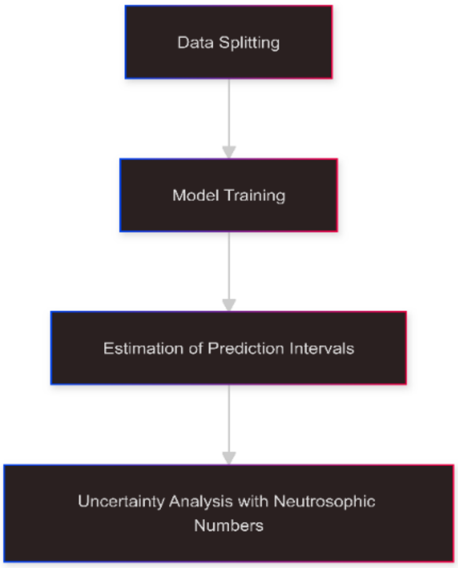

Figure 1. Modeling Process with Uncertainty Analysis using Neutrosophic Numbers

1. Splitting the data: The first step is to split the data into two sets: one for training and one for testing. This separation is critical since it enables you to evaluate the model with data you have not seen before, ensuring that its performance is not simply due to overfitting the training set. Generally, experts opt for a 70-30 or 80-20 split, with most of it utilized for training, and the remainder for testing.
2. Training the models: After separating the data, the next step is to train the models with the training set. This method entails modifying the model parameters so that they correspond as well as feasible with the data. Common regression models such as linear regression, ridge regression, and random forests are employed here. During training, we strive to optimize a loss function to properly capture the underlying pattern in the data.
3. Estimation of prediction intervals: Once the models are trained, the next step is to determine the prediction intervals for the data. Methods for estimating these intervals may vary based on the properties of the model and the nature of the data.
4. Uncertainty analysis using neutrosophic numbers: The final stage is to undertake a complete uncertainty analysis using neutrosophic numbers. This entails studying how the indeterminate component of these numbers varies based on the model, which might reveal insights about the complexity or unpredictability of the data set. For example, higher indeterminacy might suggest the effect of external influences or an unpredictable aspect to the data.

4 Results

This study aims to analyze the impact of high remuneration on the quality of life of teachers at the Faculty of Accounting Sciences of the Universidad Nacional Mayor de San Marcos by applying a neutrosophic logistic regression model. The main motivation for this analysis lies in the need to understand how salary factors influence essential aspects of quality of life, considering the uncertainty and indeterminacy in the data and results.

For this analysis, dataset of 55 events was used that includes information on teacher compensation and various metrics related to their quality of life. The dependent variable is quality of life is a continuous variable with values ranging from 0 to 10.

- **Independent Variables (X):**
  - **Monthly Remuneration (in soles)**
  - **Weekly Work Hours**
  - **Years of Experience**
  - **Academic Level**
- **Dependent Variable (Y):**
  - ✓ Quality of Life High (Values ranging from 0 to 10)

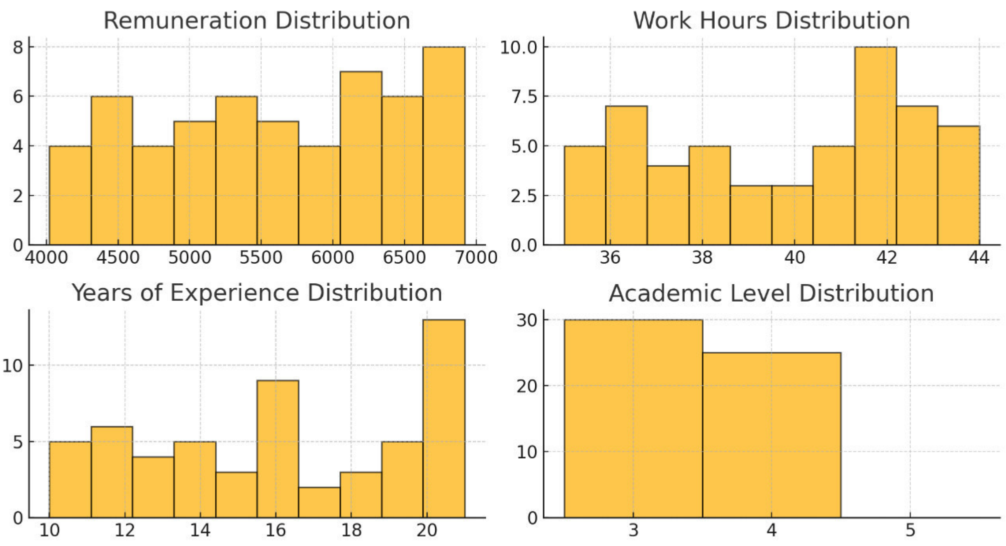

Figure 2. Distribution of Remuneration, Work Hours, Experience, and Academic Level in a Workforce Dataset

The data was divided into 70% for training and 30% for testing.
A linear regression model used is expressed as:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$$

Where

Y is the dependent variable (Quality of Life)
X_1, X_2, X_3, X_4 , are the independent variables (predictors), which could represent Remuneration, Work Hours, Years of Experience, and Academic Level.

β_0 is the intercept of the model
$\beta_1, \beta_2, \beta_3, \beta_4$ are the coefficients associated with each independent variable.

Table 1 shows the predicted quality of life for three different people with varying combinations of pay, hours worked, years of experience, and educational attainment.

Table 1. Prediction intervals and neutrosophic forms

Remuneration	Work Hours	Years of Experience	Academic Level	Prediction Interval [Lower Bound, Upper Bound]	Neutrosophic form
4500	**36**	**10**	**3**	**[3.7, 9]**	**3.7+9**I; $I \in [0, 0.589]$
5500	**40**	**16**	**4**	**[3.4, 8.7]**	**3.4+8.7**I; $I \in [0, 0.609]$
6500	**44**	**20**	**5**	**[3.2, 8.6]**	**3.2+8.6**I; $I \in [0, 0.0.628]$

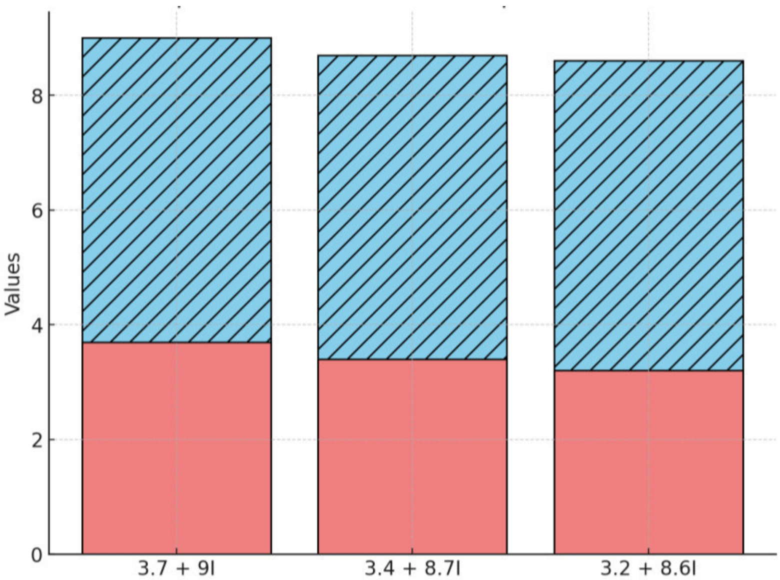

Figure 3. Representation of Neutrosophic Numbers

The analysis reveals varying levels of indeterminacy across the three cases. As the remuneration, work hours, and academic level increase, the indeterminacy slightly increases, indicating more unpredictability. This might suggest the impact of external factors or complexity in the data that the model cannot fully capture. The use of neutrosophic numbers helps quantify and visualize this uncertainty, making it clearer how much of the prediction interval is influenced by indeterminate factors.

The findings point to the possibility of enhancing academic compensation schemes by taking into account the effect that pay has on teachers' general well-being. According to the data, there is more uncertainty in predicting quality of life at higher compensation levels, even if these factors are similarly connected with longer work hours and more experience. This implies that improving well-being might need more than just paying more. Rather, policies ought to strive for a well-rounded strategy that takes into account variables including professional assistance, career advancement, and workload. Academic institutions should better promote the entire well-being of their workers and create a more content and productive workforce by addressing both the financial and non-financial parts of remuneration. This viewpoint will be broadened throughout the study to offer a thorough grasp of the relationship between compensation and well-being in the academic sector.

Conclusion

The findings of this study highlight the importance of understanding the influence of remuneration on the quality of life of instructors, particularly in the academic sector. By using neutrosophic numbers, the analysis captures the uncertainty and indeterminacy present in the relationship between remuneration and well-being. The results suggest that while remuneration plays a significant role in influencing quality of life, its impact is not uniform and varies depending on other factors like work hours, years of experience, and academic level. The inclusion of neutrosophic analysis offers a clearer understanding of this complexity, showing that financial incentives alone are not sufficient. Academic institutions can benefit from this approach by designing compensation policies that consider both financial and non-financial factors, ensuring a more balanced and supportive work environment for their staff.

For future research, it is recommended to explore the impact of various non-economic fac-tors on teachers' quality of life, such as work-life balance, job security, and institutional support. Additionally, extending the sample size and applying neutrosophic models to other academic sectors or areas could provide new insights into how multiple variables combine to promote well-being. Future research should also focus on strengthening neu-trosophic models to better handle multidimensional and dynamic datasets, enabling for even more detailed examination of the factors affecting teachers' quality of life in varied educational environments.

References

[1] Szromek, A. R., & Wolniak, R. (2020). Job satisfaction and problems among academic staff in higher education. Sustainability, 12(12), 4865.
[2] Vázquez, M. L., & Torres, L. C. (2024). Integrating Neutrosophic Numbers in Regression Analysis for Enhancing Predictive Modelling through Uncertainty Representation. Plithogenic Logic and Computation, 1, 120-126.
[3] Liu, J. (2021). Exploring teacher attrition in urban China through interplay of wages and well-being. Education and Urban Society, 53(7), 807-830.
[4] KERVANCI, I. S. (2023). A Review Hybrid Structure of Neutrosophy and Machine Learning Algorithms for Different Types of Problems. 2023 Neutrosophic SuperHyperAlgebra And New Types of Topologies, 133.
[5] [Smarandache, F. (1999). A unifying field in Logics: Neutrosophic Logic. In Philosophy (pp. 1-141). American Research Press.] KERVANCI, I. S. (2023). A Review Hybrid Structure of Neutrosophy and Machine Learning Algorithms for Different Types of Problems. 2023 Neutrosophic SuperHyperAlgebra And New Types of Topologies, 133.
[6] Leyva Vázquez, M., Estupiñan, J., & Smarandache, F. (2020). Neutrosofía en Latinoamérica, avances y perspectivas. Neutrosophic Computing and Machine Learning, 14, 1-7.
[7] Shinde, P. P., & Shah, S. (2018, August). A review of machine learning and deep learning applications. In 2018 Fourth international conference on computing communication control and automation (ICCUBEA) (pp. 1-6). IEEE.
[8] Alanazi, B. and Alrashdi, I. (2023) “Anomaly Detection in Smart Agriculture Systems on Network Edge Using Deep Learning Technique”, Sustainable Machine Intelligence Journal, 3, pp. (4):1–31.
[9] Bilal, M., Ali, M. A., Nichol, J. E., Qiu, Z., Mhawish, A., & Khedher, K. M. (2023). Reduced major axis

regression. In Encyclopedia of Mathematical Geosciences (pp. 1199-1203). Cham: Springer International Publishing.
[10] Anwar, M. B., Hanif, M., Shahzad, U., Emam, W., Anas, M. M., Ali, N., & Shahzadi, S. (2024). Incorporating the neutrosophic framework into kernel regression for predictive mean estimation. Heliyon, 10(3).
[11] Shrestha, D. L., & Solomatine, D. P. (2006). Machine learning approaches for estimation of prediction interval for the model output. Neural networks, 19(2), 225-235.
[12] Doksum, K., & Koo, J. Y. (2000). On spline estimators and prediction intervals in nonparametric regression. Computational statistics & data analysis, 35(1), 67-82.
[13] Smarandache, F., & Aslam, M. (Eds.). (2023). Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics. Elsevier
[14] Leyva Vázquez, M., & Smarandache, F. (2021). Resolución de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica. Neutrosophic Computing and Machine Learning, 15, 41-46.
[15] Delgado Estrada, S. M., Rocio Ruiz Molina, K. D., Ponce Orella-na, F. E., & Chabusa Vargas, J. L. (2024). Neutrosophic Statistics for Enhanced Time Series Analysis of Unemployment Trends in Ecuador. Neutrosophic Sets and Systems, 67(1), 14.
[16] Castro Sánchez, F., Almeida Blacio, J. H., Flores Bracho, M. G., Andrade Santamaria, D. R., & Sánchez Casanova, R. (2021).
[17] Neutrosophic and Plithogenic Statistical Analysis in Educational Development. Neutrosophic Sets and Systems, 44(1), 26.

Received: June 23, 2024. Accepted: August 15, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Plithogenic Statistical Analysis of Teaching Performance in Student Autonomous Learning

Felipe Aguirre Chávez[1], Rolando Oscco Solorzano[2], Freddy Alejandro Soto Zedano[3], María Eva Oscco Solorzano[4], Richard Santiago Quivio Cuno[5], and Delio Merma Saico[6]

[1]Marcelino Champagnat University, Peru. E-mail: faguirre@umch.edu.pe
[2]Enrique Guzmán y Valle National University of Education, Peru. E-mail: rsolorzano@une.gob.pe
[3]National University of San Marcos, Peru. E-mail: fsotoz@unmsm.edu.pe
[4]Enrique Guzmán y Valle National University of Education, Peru. E-mail: maria.oscco.s@upch.pe
[5]Enrique Guzmán y Valle National University of Education, Peru. E-mail: rquivio@une.edu.pe
[6]National University San Antonio Abad of Cusco. Peru. E-mail: delio.merma@unsaac.edu.pe

Abstract. In the current educational context, the effectiveness of teaching performance in blended environments has acquired significant relevance. The present research focused on evaluating the relationship between professional teaching competencies and the achievement of student competencies in the blended context. To this end, perceptions of teacher performance and instrumental, interpersonal, and systemic competencies were evaluated using plithogenic statistics. Among the results, they indicated that the student's perception of teaching performance in relation to classroom management and professional development was positive. However, areas of improvement were identified in interpersonal and technical skills. Observations suggest that teacher performance significantly influences the achievement of student competencies. Consequently, it has been concluded that continuing training programs must be developed for future pedagogical interventions and curricular adjustments in blended environments.

Keywords: teaching performance, blended learning, student competencies, plithogenic statistics.

1 Introduction

In recent decades, alternative basic education has faced a significant mismatch with the growing demands of a globalized world, driven by the rapid advancement of science and technology [1] [2]. This context has highlighted serious deficiencies in the educational system, particularly in basic education, where a pedagogical approach focused on memorization and repetition of content still prevails. This traditional, rigid, and inflexible methodology has limited the development of fundamental competencies in students, such as creativity, innovation, and the ability to solve complex problems [3]. Indeed, this is reflected in low academic performance and widespread dissatisfaction among students and families [4].

Similarly, teaching performance in this context has been insufficient to meet social expectations, leading to a climate of frustration and demotivation [5] [6]. Consequently, this affects the quality of education in blended learning environments, as it significantly depends on the competencies of the students [7]. The area of Mathematical Logic has been impacted by methodologies that do not promote the development of instrumental, interpersonal, and systemic competencies, which are essential for success in an increasingly interconnected and technologically advanced world.

Therefore, this study focuses on evaluating the relationship between teaching performance in the blended learning modality and the development of instrumental, interpersonal, and systemic competencies in students. For this purpose, a plithogenic statistical analysis is proposed to optimize potential solutions in teaching performance in the blended learning modality at the Mother Teresa of Calcutta Pilot Center in San Juan de Lurigancho, Lima.

2 Materials and methods

2.1 Neutrosophic Statistics

Neutrosophic probabilities and statistics are a generalization of classical and imprecise probabilities and statistics [8]. The neutrosophic probability of event E represents the probability that event E occurs, the probability that event E does not occur, and the probability of indeterminacy (not knowing whether event E occurs or not). In classical probability, nsup≤1, whereas in neutrosophic probability, nsup≤3+. The function that models the neutrosophic probability of a random variable x is called the neutrosophic distribution:

$$NP(x) = (T(x), I(x), F(x))$$

Where T(x) represents the probability that the value x occurs, F(x) represents the probability that the value x does not occur, and I(x) represents the indeterminate or unknown probability of the value x. Neutrosophic statistics involve the analysis of neutrosophic events and deal with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, and neutrosophic regression.

It refers to a data set, which is composed, wholly or partially, of data with some degree of indeterminacy and the methods to analyze them. Neutrosophic statistical methods allow for the interpretation and organization of neutrosophic data to reveal underlying patterns.

In summary, neutrosophic logic, neutrosophic sets, and neutrosophic probabilities and statistics have wide applications in various research fields and constitute a novel and developing area of study [9] [10].

For the processing of neutrosophic numbers, they are defined as numbers of the form a + bI, where a and b are real or complex numbers, while "I" represents the indeterminate part of the neutrosophic number N. The study of neutrosophic statistics refers to a neutrosophic random variable where X_l and $X_u I_N$ represent the lower and upper levels, respectively, that the studied variable can reach within an indeterminate interval $[I_l, I_u]$. Thus, the neutrosophic mean of the variable $(\bar{x}_N)$ is formulated as:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \tag{1}$$

Where, $\bar{x}_a = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{il},\ \ \bar{x}_b = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{iu},\ \ n_N \in [n_l, n_u],$ (2)

is a neutrosophic random sample. However, neutral frames (NNS) can be calculated as follows

$$\sum_{i=1}^{n} N(X_i - \bar{X}_{iN})^2 = \sum_{i=1}^{n} N \begin{bmatrix} \min\begin{pmatrix} (a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U) \end{pmatrix} \\ \max\begin{pmatrix} (a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U) \end{pmatrix} \end{bmatrix}, I \in [I_L, I_U] \tag{3}$$

Where $a_i = X_l b_i = X_u$. The variance of the neutrosophic sample can be calculated by

$$S_N^2 = \frac{\sum_{i=1}^{n_N}(X_i - \bar{X}_{iN})^2}{n_N};\ S_N^2 \in [S_L^2, S_U^2] \tag{4}$$

The neutrosophic coefficient (NCV) measures the consistency of the variable. The lower the NCV value, the more consistent the performance of the factor compared to other factors. The NCV can be calculated as follows.

$$CV_N = \frac{\sqrt{S_N^2}}{\bar{X}_N} \times 100;\ CV_N \in [CV_L, CV_U] \tag{5}$$

The neutrosophic scale is applied to evaluate the importance and frequency of the dimensions of teaching performance and the achievement of student competencies. The following criteria are used as seen in the following tables (see Table 1 and 2).

Table 1: Plithogenic scale to evaluate the importance of teaching performance. Source: Own elaboration.

Linguistic expression	Scale	Plithogenic number (T, I, F)	$\mathcal{S}([\mathbf{T}, \mathbf{I}, \mathbf{F}])$	Description
Extremely High (EH)	10	(1.00, 0.00, 0.00)	0.95	Extremely High Achievement of Student Competencies.
Very Very High (VVH)	9	(0.90, 0.05, 0.10)	0.85	Very High Achievement of Student Competencies.
Very High (VH)	8	(0.80, 0.15, 0.20)	0.75	High Achievement of Student Competencies.
High (H)	7	(0.70, 0.25, 0.30)	0.65	Significant Achievement of Student Competencies.
Moderately High (MH)	6	(0.60, 0.35, 0.40)	0.55	Moderately High Achievement of Student Competencies.
Medium (M)	5	(0.50, 0.45, 0.50)	0.50	Moderate Achievement of Student Competencies.
Moderately Low (ML)	4	(0.40, 0.55, 0.65)	0.45	Moderately Low Achievement of Student Competencies.
Low (L)	3	(0.30, 0.65, 0.70)	0.35	Low Achievement of Student Competencies.
Very Low (VL)	2	(0.20, 0.75, 0.80)	0.25	Very Low Achievement of Student Competencies.
Very Very Low (VVL)	1	(0.10, 0.85, 0.90)	0.15	Very Very Low Achievement of Student Competencies.
Extremely Low (EL)	0	(0.00, 0.95, 1.00)	0.05	Insignificant Achievement of Student Competencies.

Table 2: Plithogenic scale to evaluate the frequency of achievement of student competencies. Source: Own elaboration.

Linguistic expression	Scale	Plithogenic number (T, I, F)	$S([T, I, F])$	Description
Extremely High (EH)	10	(1.00, 0.00, 0.00)	0.95	Extremely High Achievement of Student Competencies.
Very Very High (VVH)	9	(0.90, 0.05, 0.10)	0.85	Very High Achievement of Student Competencies.
Very High (VH)	8	(0.80, 0.15, 0.20)	0.75	High Achievement of Student Competencies.
High (H)	7	(0.70, 0.25, 0.30)	0.65	Significant Achievement of Student Competencies.
Moderately High (MH)	6	(0.60, 0.35, 0.40)	0.55	Moderately High Achievement of Student Competencies.
Medium (M)	5	(0.50, 0.45, 0.50)	0.50	Moderate Achievement of Student Competencies.
Moderately Low (ML)	4	(0.40, 0.55, 0.65)	0.45	Moderately Low Achievement of Student Competencies.
Low (L)	3	(0.30, 0.65, 0.70)	0.35	Low Achievement of Student Competencies.
Very Low (VL)	2	(0.20, 0.75, 0.80)	0.25	Very Low Achievement of Student Competencies.
Very Very Low (VVL)	1	(0.10, 0.85, 0.90)	0.15	Very Very Low Achievement of Student Competencies.
Extremely Low (EL)	0	(0.00, 0.95, 1.00)	0.05	Insignificant Achievement of Student Competencies.

2.2 Plithogenic statistics.

Plithogenic statistics is a methodology that focuses on the inclusion of indeterminacy, contradiction, and the interrelation of variables within a plithogenic set and its dimensions [11] [12]. Plithogenic logic presents the following characteristics according to the methodology analyzed in the study materials [13] [14].

3 Results

3.1 Case study. Variable characteristics.

Case study: Teaching performance in student competencies in a blended learning environment.

The case study focuses on the Mother Teresa of Calcutta Pilot Center for Alternative Basic Education in San Juan de Lurigancho. The aim is to analyze how teaching performance in a blended learning environment influences the achievement of student competencies. The analysis seeks a clear understanding of how teaching performance in the blended modality affects the achievement of competencies in students. It is expected to identify significant relationships between the evaluated variables and dimensions (see Table 3).

Table 3: Characteristics of the plithogenic set, performance, and teacher-student competencies. Source: Own elaboration.

COD	Variable	COD	Dimension	Definition
V1	Teaching performance in blended education.	D1	Student knowledge and understanding	Reflects the teacher's knowledge and understanding of the characteristics and needs of the students.
		D2	Classroom relationships and management	Assesses the teacher's ability to build positive relationships and manage the classroom effectively.
		D3	Collaboration and participation in the educational community	Measures the teacher's collaboration with colleagues and participation in institutional management and improvement.
		D4	Reflection and professional development	Values reflection on practice and commitment to professional and ethical development.
V2	Achievement of student competencies.	D5	Instrumental competencies	Evaluates the basic skills and knowledge acquired by students and their ability to apply them.
		D6	Interpersonal competencies	Measures students' interpersonal skills, such as teamwork and effective communication.
		D7	Systemic competencies	Reflects students' ability to apply knowledge and skills in broader and more complex contexts.

The plithogenic set is defined for three variables, V1 and V2. Therefore, a plithogenic set is defined as consisting of 7 attributes, each of which contains values with their respective plithogenic characteristics. It is focused on determining the level of importance of the variable within the plithogenic set. The multi-attribute set of dimension 3 has a cardinality of 3x4 = 12.

3.2 Teaching performance in blended education

When modeling the variable using plithogenic statistics, frequencies are obtained to determine the level of importance of each dimension in the variable of teaching performance in blended learning. For this purpose, a sample of 127 students was used to measure each dimension according to the plithogenic scale proposed in Table 1. Below, the neutrosophic frequency and neutrosophic statistical analysis of each evaluated dimension are detailed (see Table 4 and Table 5).

Table 4: Plithogenic frequency for the variable teaching performance in blended care. Source: Own elaboration.

Students	Plithogenic frequencies			
	D1	**D2**	**D3**	**D4**
1	(0.73,0.87)	(0.85,0.94)	(0.64,0.8)	(0.59,0.84)
2	(0.77,0.88)	(0.85,0.86)	(0.53,0.83)	(0.59,0.95)
3	(0.78,0.97)	(0.85,0.94)	(0.62,0.87)	(0.6,0.86)
4	(0.66,0.74)	(0.83,0.83)	(0.59,0.76)	(0.54,0.88)
5	(0.7,0.76)	(0.87,0.93)	(0.64,0.76)	(0.51,0.91)
6	(0.6,0.79)	(0.89,0.93)	(0.58,0.83)	(0.52,0.86)
7	(0.66,0.72)	(0.89,0.92)	(0.52,0.8)	(0.56,0.9)
8	(0.7,0.89)	(0.89,0.98)	(0.64,0.88)	(0.51,0.89)
9	(0.8,0.85)	(0.82,0.92)	(0.66,0.78)	(0.53,0.75)
10	(0.68,0.74)	(0.89,0.99)	(0.55,0.72)	(0.59,0.85)
11	(0.77,0.93)	(0.86,0.86)	(0.57,0.77)	(0.58,0.89)
12	(0.69,0.72)	(0.9,0.99)	(0.61,0.87)	(0.54,0.76)
13	(0.5,0.62)	(0.9,0.92)	(0.52,0.78)	(0.6,0.87)
14	(0.58,0.62)	(0.83,0.93)	(0.56,0.71)	(0.53,0.91)
15	(0.64,0.78)	(0.81,0.82)	(0.7,0.85)	(0.54,0.79)
16	(0.52,0.57)	(0.88,0.91)	(0.52,0.71)	(0.5,0.81)
17	(0.76,0.88)	(0.87,0.92)	(0.58,0.69)	(0.5,0.75)
18	(0.79,0.94)	(0.84,0.9)	(0.52,0.69)	(0.6,0.9)
19	(0.57,0.63)	(0.88,0.98)	(0.7,0.83)	(0.54,0.76)
20	(0.76,0.8)	(0.8,0.8)	(0.63,0.84)	(0.56,0.76)
21	(0.63,0.67)	(0.9,0.9)	(0.58,0.83)	(0.6,0.85)
22	(0.61,0.61)	(0.89,0.93)	(0.52,0.66)	(0.5,0.82)
23	(0.57,0.71)	(0.82,0.88)	(0.63,0.91)	(0.57,0.78)
24	(0.67,0.73)	(0.83,0.84)	(0.68,0.97)	(0.54,0.83)
25	(0.63,0.72)	(0.84,0.86)	(0.61,0.88)	(0.52,0.89)
26	(0.55,0.73)	(0.87,0.91)	(0.62,0.76)	(0.57,0.94)
27	(0.65,0.79)	(0.82,0.9)	(0.63,0.92)	(0.51,0.74)
28	(0.53,0.61)	(0.87,0.93)	(0.7,0.99)	(0.56,0.83)
29	(0.63,0.66)	(0.85,0.87)	(0.53,0.65)	(0.58,0.89)
30	(0.73,0.83)	(0.89,0.93)	(0.61,0.88)	(0.54,0.85)
0-127	**(82.96,94.73)**	**(108.43,114.79)**	**(76.17,101.68)**	**(69.92,107.63)**

Analysis of neutrosophic frequencies:

Table 4 shows the neutrosophic frequency for each dimension of teaching performance in blended learning. Each interval represents the range of student perception regarding performance in the various dimensions.

- In Dimension D1: Student Knowledge and Understanding, the neutrosophic frequency ranges between "Moderately High (MH)" and "Extremely High (EH)," indicating generally high performance in this area, though with notable variability. This suggests that while some students demonstrate excellent understanding, others are at moderately high levels.
- Dimension D2: Classroom Relationships and Management shows values between "Very High (VH)" and "Extremely High (EH)," reflecting consistently high performance in classroom management and relationships. The high neutrosophic frequency in this dimension indicates strong positive consensus, with students perceiving excellent management of the learning environment.
- Dimension D3: Collaboration and Participation in the Educational Community ranges between "Moderately High (MH)" and "Extremely High (EH)." Although collaboration and participation are evaluated positively, there is variability in perceptions, with some evaluations reaching extremely high

levels, showing a disparity in participation within the educational community.
- Finally, Dimension D4: Reflection and Professional Development falls between "Moderately High (MH)" and "Very High (VH)." This indicates a generally positive perception of reflection and professional development, although with greater variability compared to classroom management. It suggests areas where efforts could be made to achieve greater consistency in professional development.

Table 5: Plithogenic statistical analysis of the teaching performance variable in blended care. Source: Own elaboration.

Dimension	$\bar{x}_N$	S_N	CV_N	
D1	0.65 + 0.75 I	0.092 + 0.107 I	0.142 + 0.143 I	0.65 + 0.75 I
D2	0.85 + 0.9 I	0.031 + 0.043 I	0.036 + 0.048 I	0.85 + 0.9 I
D3	0.6 + 0.8 I	0.062 + 0.083 I	0.103 + 0.104 I	0.6 + 0.8 I
D4	0.55 + 0.85 I	0.031 + 0.066 I	0.056 + 0.078 I	0.55 + 0.85 I

Table 5 provides the neutrosophic statistical analysis for each dimension, showing the neutrosophic mean, the neutrosophic standard deviation, and the neutrosophic coefficient of variation. These data allow us to evaluate the consistency of perceptions. Among them, it is observed:

- Student Knowledge and Understanding: The neutrosophic average for D1, ranging between "Moderately High (MH)" and "High (H)," indicates overall performance in the moderately high range. The relatively low standard deviation and coefficient of variation suggest reasonable consistency in perceptions.
- Classroom Relationships and Management: D2 has a neutrosophic average in the range of "Very High (VH)" to "Extremely High (EH)," with very low standard deviations and coefficients of variation. This reflects high consistency and precision in the perception of classroom management and relationships.
- Collaboration and Participation in the Educational Community: The neutrosophic average for D3 falls between "Moderately High (MH)" and "Very High (VH)." Moderate standard deviation and coefficient of variation indicate some variability in perceptions, though a generally acceptable level of consistency is observed.
- Reflection and Professional Development: The neutrosophic average for D4 is between "Moderately High (MH)" and "Very High (VH)," suggesting generally high performance but with greater variability compared to D2. The standard deviation and coefficient of variation indicate reasonable consistency with some differences in evaluations.

3.3 Student Competencies.

For modeling the plithogenic statistics of the variable student competencies achievement using plithogenic statistics, a sample of 17 teachers was analyzed (see Table 6). Consequently, a neutrosophic statistical analysis of each evaluated dimension of the variable is carried out (see Table 7). Each teacher used Table 1 to conduct their evaluations of the analyzed dimensions.

Table 6: Plithogenic frequency for the student competency achievement variable. Source: Own elaboration.

Teachers	Plithogenic frequencies		
	D5	**D6**	**D7**
1	(0.51,0.53)	(0.72,0.77)	(0.66,0.91)
2	(0.69,0.97)	(0.76,0.89)	(0.68,0.93)
3	(0.7,0.75)	(0.56,0.71)	(0.69,0.97)
4	(0.63,0.64)	(0.51,0.63)	(0.66,0.96)
5	(0.6,0.86)	(0.66,0.78)	(0.67,0.93)
6	(0.46,0.64)	(0.68,0.81)	(0.69,0.97)
7	(0.56,0.67)	(0.5,0.62)	(0.68,0.98)
8	(0.46,0.71)	(0.52,0.62)	(0.69,0.97)
9	(0.56,0.67)	(0.56,0.67)	(0.66,0.95)
10	(0.42,0.67)	(0.54,0.64)	(0.65,0.94)
11	(0.66,0.9)	(0.6,0.69)	(0.65,0.94)
12	(0.62,0.74)	(0.74,0.78)	(0.66,0.91)
13	(0.68,0.9)	(0.72,0.89)	(0.66,0.94)
14	(0.5,0.58)	(0.71,0.77)	(0.65,0.95)
15	(0.66,0.67)	(0.64,0.74)	(0.66,0.91)
16	(0.64,0.79)	(0.53,0.55)	(0.65,0.9)
17	(0.7,0.84)	(0.5,0.57)	(0.67,0.93)
1-17	**(10.05,12.53)**	**(10.45,12.13)**	**(11.33,15.99)**

Analysis of plithogenic frequencies:

- The results in Dimension D5 show that the neutrosophic frequencies for teachers range between values indicating achievement from "Medium (M)" to "High (H)" in terms of instrumental competencies. However, this variability suggests that not all students achieve a high level, indicating a need to strengthen certain areas. Critical points to enhance in D5 include improving students' technical and methodological skills to ensure that all reach at least a "High" level.
- In Dimension D6, the neutrosophic frequencies also range between "Medium (M)" and "High (H)," similar to Dimension D5. However, some students achieve higher levels, close to "Very High (VH)," indicating significant variability in interpersonal competencies. Critical areas to address in D6 include promoting communication and teamwork skills to standardize competency levels among all students and raise the overall standard.
- Dimension D7 shows a greater concentration of results at the levels of "High (H)" and "Very High (VH)," indicating generally superior performance in systemic competencies. Although the average level is high, there is an opportunity to further enhance advanced systemic skills, such as solving complex problems and integrating interdisciplinary knowledge, to ensure that students consistently achieve the highest levels of performance.

Table 7: Plithogenic statistical analysis of the student competency achievement variable. Source: Own elaboration.

Dimension	$\bar{x}_N$	S_N	CV_N
D5	0.59 + 0.74 I	0.092 + 0.123 I	0.156 + 0.166 I
D6	0.61 + 0.71 I	0.094 + 0.102 I	0.154 + 0.144 I
D7	0.67 + 0.94 I	0.015 + 0.024 I	0.022 + 0.026 I

Analysis of the consistency of the dimensions and neutrosophic evaluation:

- The statistical analysis of D5 indicates that the average achievement of students in instrumental competencies falls between "Medium (M)" and "High (H)," with moderate dispersion in results. This suggests that while some students reach a good level of instrumental competencies, others do not exceed a moderate level. It is crucial to enhance consistency in this aspect by focusing on the uniform development of practical and methodological skills.
- In Dimension D6, the average achievement also ranges between "Medium (M)" and "High (H)," with moderate dispersion. This reflects that, similar to D5, there is significant variability in interpersonal competencies. Interventions should focus on developing greater coherence in these competencies by promoting collaborative learning experiences and leadership skills among all students.
- Dimension D7 stands out with an average achievement approaching "Very High (VH)," with significantly less dispersion than in the other two dimensions. This suggests greater consistency in students' performance in systemic competencies. However, it is essential to maintain and enhance this high level of performance by ensuring that students continue to develop complex skills, such as the ability to integrate and apply knowledge in multivariate contexts.

3.4 Improvement proposals to enhance each dimension

To improve teaching performance and address the areas identified in the neutrosophic evaluation dimensions, it is essential to develop specific actions based on the plithogenic categories assigned to each dimension. The following concrete actions are proposed for each dimension to enhance overall performance and address critical areas:

- To improve Dimension D1: Student Knowledge and Understanding, it is proposed to implement a continuous training program for teachers that includes advanced teaching techniques and content updates. Periodic diagnostic assessments are also recommended, along with promoting the use of interactive educational technologies to adapt teaching methods to student needs.
- For Dimension D2: Classroom Relationships and Management, which shows very high performance, the focus should be on training in interpersonal skills and advanced management techniques. Additionally, implementing constructive feedback between teachers and students will improve classroom dynamics and strengthen the learning environment.
- In Dimension D3: Collaboration and Participation in the Educational Community, it is recommended to encourage more joint projects and school events that integrate teachers, students, and parents. Developing satisfaction surveys would also help adjust collaboration strategies to strengthen community engagement.
- Finally, for Dimension D4: Reflection and Professional Development, personalized professional de-

velopment plans for teachers should be established, with an emphasis on pedagogical reflection. Mentorship and coaching programs, along with periodic performance evaluations, would improve teaching practice and facilitate continuous improvement.

To improve the achievement of students' competencies and address the areas identified in the dimensions of neutrosophic evaluation, the following strategies are proposed:

- To improve the achievement of instrumental competencies (D5), it is essential to review and update the curriculum to ensure a solid foundation in technical skills. Additional workshops should be implemented for students facing difficulties, and a mentoring program should be established where outstanding students support their peers. This approach will enhance overall performance and foster collaborative learning.
- In the dimension of interpersonal competencies (D6), it is crucial to improve communication and teamwork skills. Integrating activities such as group presentations and debates into the curriculum, along with group dynamics and team projects, is recommended. Additionally, offering leadership training programs will prepare students for coordination roles in academic and professional contexts.
- To enhance systemic competencies (D7), promoting interdisciplinary projects and fostering creativity through methodologies like design thinking is advised. These strategies enable students to develop a more focused perspective by applying innovative solutions to complex problems. Implementing a continuous evaluation system with regular feedback will maintain and improve the high level of systemic competencies.

3.5 Plithogenic intersection integrated into the dominant dimensions

To enhance the results within the plithogenic set of teaching and student competencies performance, the neutrosophic intersection of each variable is conducted based on the dominant dimensions. Therefore, the plithogenic intersection matrix is presented (see Table 8).

Table 8: Plithogenic intersection worksheet. Source: Own elaboration.

Aspect	Details
Variable 1:	Teaching performance in blended care.
▪ Selected dimension	**D2:** Relationship and management in the classroom.
▪ Neutrosophic evaluation	Extremely high (EH).
Variable 2:	Achievement of student competencies.
▪ Selected dimension	**D7:** Systemic competencies.
▪ Neutrosophic evaluation	Very high (VH).
	Plithogenic intersection of dimensions
Dimension D2	▪ The teacher's ability to build effective relationships and manage the classroom efficiently is essential in blended contexts. So it directly impacts the quality of interaction and support provided to students.
Dimension D7	▪ Systemic competencies reflect students' ability to apply knowledge in broader contexts. Ensures that the skills acquired are useful and transferable to various situations.
	Synergy between D2 and D7
Intersection and empowerment	▪ Strengthening D2: Improving classroom relationships and management (D2) allows teachers to provide a more inclusive and adaptive learning environment. ▪ D7 Facilitation: Better classroom management facilitates the development of systemic competencies (D7) in students. The more effective application of knowledge in diverse contexts is enhanced. ▪ General impact: The improvement in the teacher-student relationship and classroom management enhances contextualized learning and the ability of students to integrate and apply their skills in more complex situations.

The degree of contradiction between the values for each dominant attribute is defined below:

- $c_N(D_2, D_7) = 0.20$

As can be seen, the dominant values for each attribute are D2 and D7. When dimensions D2 and D7 are activated, other related aspects are also activated, suggesting that classroom relationships and management, along with the development of systemic competencies, directly influence student academic performance and the quality of blended teaching. To propose effective solutions, it is crucial to identify the intersection of these key attributes within the plithogenic set. The relative importance of Dimension D2 (Classroom Relationships and Management) and Dimension D7 (Systemic Competencies), as well as their impact on educational effectiveness, should be evaluated. Therefore, the intersection between these dimensions is assessed using the following formula:

- (a1, a2, a3)∧p(b1, b2, b3) = (0.84, 0.08, 0.04)

An evaluation for this intersection shows a level just above Very High (MA) and closer to Very Very High (VVH). This result reflects a strong connection between classroom management and the development of systemic competencies, surpassing the influence of other dimensions on teaching performance. Consequently, it is suggested that solutions focus on addressing the plithogenic areas that converge in dimensions D2 and D7, as they significantly affect academic performance and educational effectiveness in blended learning environments.

To maximize the impact on teaching performance and student competency achievement, it is recommended to implement training programs that enhance classroom management skills and improve relationships with students in blended learning environments. Additionally, it is essential to promote practical activities that allow students to apply their competencies in real or simulated situations, reinforcing systemic skills. Finally, continuous evaluations should be conducted to adjust teaching strategies and ensure the effective development and application of student competencies. This integration significantly enhances both teaching performance and competency achievement by creating a positive feedback loop.

Conclusion

The student perception of teaching performance in blended learning environments has shown a predominantly positive evaluation regarding classroom management and reflective practice. Results indicated that students highly value the ability of teachers to manage the classroom effectively and foster a reflective learning environment, highlighting the importance of these competencies for educational success. This positive assessment underscores the effectiveness of the pedagogical approaches used but also suggests potential areas for improvement, particularly in collaboration and participation within the educational community.

The achievement of student competencies has revealed that interpersonal and technical skills need strengthening. Despite a generally positive appreciation of systemic competencies, instrumental and interpersonal skills are perceived in a range suggesting significant opportunities for improvement. This points to the need to review and adjust teaching strategies and educational resources to ensure a comprehensive development of students' technical and communicative skills.

The intersection between classroom management (D2) and the development of systemic competencies (D7) demonstrated notable synergy. The plithogenic analysis revealed that this intersection achieved a neutrosophic level close to Very Very High (VVH), indicating a strong relationship between the quality of classroom management and student's ability to apply knowledge across various contexts. This result emphasizes the need to optimize both dimensions to improve academic performance and practical application of knowledge. Additionally, it is recommended to develop continuous training programs for teachers, including specific modules on the use of educational technologies and fostering interpersonal skills. Updating the curriculum to more effectively address students' needs in blended learning environments is also suggested.

References

[1] Ramírez-Montoya, M. S., Loaiza-Aguirre, M. I., Zúñiga-Ojeda, A., & Portuguez-Castro, M. (2021). Characterization of the teaching profile within the framework of education 4.0. *Future Internet*, 13(4), 91.

[2] Alieto, E., Abequibel Encarnacion, B., Estigoy, E., Balasa, K., Eijansantos, A., & Torres Toukoumidis, A. (2024). Teaching inside a digital classroom: A quantitative analysis of attitude, technological competence and access among teachers across subject disciplines. *Heliyon*, 10(2), e24282.

[3] Prasetya, F., et al. (2024). The impact of augmented reality learning experiences based on the motivational design model: A meta-analysis. *Social Sciences & Humanities Open*, 10(October), 1-10.

[4] Park, D., & Ramirez, G. (2022). Frustration in the classroom: Causes and strategies to help teachers cope productively. *Educational Psychology Review*, 34(4), 1955-1983.

[5] Moè, A., Consiglio, P., & Katz, I. (2022). Exploring the circumplex model of motivating and demotivating teaching styles: The role of teacher need satisfaction and need frustration. *Teaching and Teacher Education*, 118(October), 2-5.

[6] López-García, G. D., Granero-Gallegos, A., Carrasco-Poyatos, M., & Burgueño, R. (2023). Detrimental effects of disempowering climates on teaching intention in (physical education) initial teacher education. *International Journal of Environmental Research and Public Health*, 20(1), 878.

[7] Lauriola, I., Lavelli, A., & Aiolli, F. (2021). An introduction to deep learning in natural language processing: Models, techniques, and tools. *Neurocomputing*, 470(January), 443-456.

[8] Mayorga, C. A., Padilla, P. E. A., Tello, G. B., & del Sol, A. B. (2021). Statistical methods and management of indeterminacy in medical sciences. Surgical site infection study after vascular surgery procedures. *Neutrosophic Sets and Systems*, 44(1), 133-139.
[9] Jaramillo, K., Quilambaqui, J., & Yanez, J. (2022). Blockchain in healthcare from a neutrosophic analysis. *International Journal of Neutrosophic Science*, 18(3), 177-188.
[10] González Caballero, E., Leyva Vázquez, M., & Smarandache, F. (2021). On neutrosophic uninorms. *Neutrosophic Sets and Systems*, 45, 340-348.
[11] Cristina, E., et al. (2022). Neutrosophic statistical analysis of self-assessment of rehabilitation knowledge in university students. *Neutrosophic Sets and Systems*, 52(October), 2-5.
[12] Gutiérrez, P., et al. (2023). Neutrosophic sets and systems special issue: Neutrosophic advancements Plithogenic Iadov model to study university teaching practices in the complexity of the educational process of comprehensive training by competencies. *Neutrosophic Sets and Systems*, 62(1), 79-83.
[13] Martin, N., Smarandache, F., & Sudha, S. (2023). A novel method of decision making based on plithogenic contradictions. *Neutrosophic Systems with Applications*, 10(October), 12-24.
[14] Smarandache, F. (2022). Plithogeny, plithogenic set, logic, probability and statistics: A short review. *Journal of Computational and Cognitive Engineering*, 1(2), 47-50.

Received: June 24, 2024. Accepted: August 16, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Feasibility Study of the Application of Proposals for the Implementation of Compliance in the Low-Quantity Process in Public Procurement in Ecuador Using Plithogenic SWOT Analysis

Víctor-Gustavo Gómez-Rodríguez[1], Noel Batista-Hernández[2], Willian-Paul Avilés-Quiñonez[3], Johanna-Irene Escobar-Jara[4], Roberto-Eduardo Vargas-Zambrano[5], Ronald-Marcelo Sánchez-Rovalino[6], Alejandro Reigosa-Lara[7], and Duniesky Alfonso-Caveda[8]

[1] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: vgomez@ube.edu.ec
[2] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: nbatistah@ube.edu.ec
[3] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: paviles45@gmail.com
[4] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: jiescobarj@ube.edu.ec
[5] Southern Manabi State University, Jipijapa. Ecuador. E-mail: rovarza2006@gmail.com
[6] University of the Pacific, Guayaquil, Ecuador. E-mail: ronald.sanchez@upacifico.edu.ec
[7] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: areigosal@ube.edu.ec
[8] Bolivarian University of Ecuador (UBE), Durán. Ecuador. E-mail: dalfonsoc@ube.edu.ec

Abstract. Compliance is a concept that means that the organization conforms to the legal regulations in force in the country where it is located. This avoids additional expenses on the part of the company and economic losses due to non-compliance with the regulatory legal framework. In the case of the low-value process in public purchases in Ecuador, it is necessary to take this aspect into account, especially in the anti-corruption fight that is currently being carried out worldwide. To study the positive and negative factors that influence the application of compliance in Ecuadorian public purchases, we use the plithogenic SWOT analysis. The SWOT analysis studies the Strengths, Weaknesses, Opportunities, and Threats to carry out a project. The hybridization with the plithogenic theory allows us to analyze the data taking into account the complexity and dynamics of the scenarios studied.

Keywords: Compliance, low-value, public procurement, plithogerny, SWOT analysis, plithogenic number.

1 Introduction

Public procurement in Ecuador represents on average 4% of the Gross Domestic Product (GDP) and 15.2% of the General State Budget, which is a significant part of public spending and crucial for the economic and social development of the country. Through the acquisition of goods, services, and works, the State can execute public policies, build infrastructure, and provide essential services to citizens. In this context, ensuring transparency, integrity, and efficiency in the use of public resources is of vital importance. However, public procurement is also exposed to risks of corruption and embezzlement, which can undermine citizens' trust in public institutions and negatively affect economic development.

In Ecuador, the technical and regulatory body for public procurement is the National Public Procurement Service. This body is responsible for articulating, controlling, and supervising all actions related to public procurement under criteria of effectiveness, efficiency, transparency, quality, and concurrence. Its mission is to ensure that public institutions and suppliers comply with established regulations and that procurement processes are carried out fairly and transparently.

Public procurement procedures in Ecuador are determined by the amount and purpose of the contract, which may include standardized and non-standardized goods and services, works, and consultancies. Each type of purchase is subject to monetary limits that vary annually according to the Initial State Budget. This allows contracting thresholds to be adjusted to the economic and financial realities of the country, guaranteeing flexible management adapted to the needs of the State.

The regulatory framework governing public procurement in Ecuador is broad and detailed, providing a clear structure for all actors involved. Below are the main types and procedures of public procurement, which are reflected in the following table:

Table 1: Regime, Types, and Procedures of Public Procurement in Ecuador.

Contracting Regime	**Type of Contract**	**Procedure**
Common Regime	Standardized Goods and Services	Electronic Reverse Auction, Public Competition
	Non-Standard Goods and Services	Quotation, Public Tender
	Works	Direct Contracting, Tendering
	Consultancies	Merit and Opposition Competition, Direct Hiring
Special Regime	Urgent Acquisitions	Direct Contracting
	International Acquisitions	Simplified Procedure, Based on International Organization Standards
	Artistic Works	Special Procedure, Direct Selection

The term SWOT (Strength, Weakness, Opportunity, and Threat) is a widely used method within companies and in other areas as well, to determine the development prospects of the entity and the impediments to achieving this development [1-4]. It studies the balance between the positive and negative for the advancement of the entity, both internally and externally within the organization. The internal can be improved within the organization itself, while the external must be dealt with.

It is known that companies and their environment exchange in a dynamic way. That is why when studying the four aspects of the SWOT this exchange must be analyzed concerning time, since it is not static. In addition, it consists of multiple variables, some of which are unknown, or in other cases the relationship between variables is unknown. That is why the crisp treatment of the SWOT tool is not necessarily realistic since some uncertainty tools such as fuzzy or neutrosophic have been incorporated into it [5-11].

In this article, we apply the theory of plithogeny [12]. This term was coined by Professor Florentin Smarandache and is used to indicate the dynamic exchange between concepts where not only the concept <A> and its complementary opposite <AntiA> intervene as in classical dialectics, but also the interaction between <A> and <AntiA> and also that which is neither one nor the other denoted by <NeutroA>. It also includes the interaction with other concepts and their neuters or opposites such as <B> or <AntiB> or <NeutroB>, as well as <C> or <AntiC> or <NeutroC> and so on. Etymologically this word comes from Greek as a combination of the words *plitho* (many) and *geniá* (generation).

Plithogenic SWOT analysis is used here to study the implementation of compliance in the process of low-value public procurement in Ecuador. With this combination of tools, we will obtain more realistic results because we base ourselves on the changing dynamics of companies.

The paper is divided into a Materials and Methods section, where SWOT analysis and plithogenic theory are explained. In the next section, we explain the procedures and results applied to the problem we propose to study. The last section is devoted to giving the conclusions.

2 Materials and Methods

2.1 SWOT Analysis

A SWOT analysis serves as a tool for assessing a company or a specific project. It evaluates its internal strengths and weaknesses alongside external opportunities and threats, utilizing a square matrix.

The analysis must be conducted by four stages:

- Examining external influences,
- Exploring internal elements within the organization,
- Developing a SWOT analysis matrix,
- Determining the appropriate strategy to implement.

An organization cannot exist without its surroundings. Looking at what is happening around it helps us understand the chances and challenges that might affect the organization. These elements involve examining external factors. Opportunities are favorable circumstances in our environment that we can capitalize on once we become aware of them. Threats refer to challenging external situations related to a program or project that may lead to difficulties. When these threats occur, it is important to create a good plan to deal with them.

During a SWOT analysis, we need to look at the internal parts of a business, specifically its strengths and weaknesses. This encompasses factors such as the financial resources and assets available, the expertise of its workforce, the caliber of its offerings, the structure of the organization, and customer perceptions of the company.

Internal analysis helps the organization identify its strengths and weaknesses by studying the amount and quality of its resources and processes. Strengths refer to the positive aspects of a program or project that set it apart from comparable initiatives. Weaknesses refer to existing skills, resources, and attitudes within the organization that hinder its overall performance. Weaknesses are issues inside a person or a group. When we find them and come up with a good plan, we can and should fix them.

The four parts of the analysis are arranged in a matrix and checked by experts. These results are combined based on the percentages from their ratings. We can find more information about this method in sources [1, 2].

When we put together our strengths and opportunities, it shows us possibilities that can help the organization succeed best. Weaknesses and threats create limitations that are a serious warning. At the same time, the risks (which come from both strengths and threats) and challenges (which come from both weaknesses and opportunities) will need to be thought about carefully. These considerations will direct the organization's journey toward its envisioned future.

2.2 Basics of Plithogeny Theory

Let U be a universe of discourse, and P a non-empty set of elements, $P \subseteq U$. Let A be a non-empty set of *one-dimensional* attributes $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$, $m \geq 1$; and let $\alpha \in A$ be a given attribute whose spectrum of all possible values (or states) is the nonempty set S, where S can be a discrete finite set $S = \{s_1, s_2, \dots, s_l\}$, $1 \leq l < \infty$, or an infinitely countable set $S = \{s_1, s_2, \dots, s_\infty\}$, or an infinitely uncountable (continuous) set $S =]a, b[$, $a < b$, where] ... [is any open, half-open, or closed interval of the set of real numbers or another general set.

Let V be a nonempty subset of S, where V is the range of all attribute values needed by experts for their application. Each element $x \in P$ is characterized by the values of all attributes in $V = \{v_1, v_2, \dots, v_n\}$, for $n \geq 1$.

In the set of attribute values V, generally, there is a *dominant attribute value*, which is determined by experts in their application. The dominant attribute value means the most important attribute value that experts are interested in.

Each attribute value $v \in V$ has a corresponding *degree of appurtenance* $d(x, v)$ of the element x, to the set P, for some given criteria.

The degree of membership can be a *fuzzy degree of appurtenance*, an *intuitionistic fuzzy degree of appurtenance*, or a *neutrosophic degree of appurtenance* to the plithogenic set [12].

Therefore, the *attribute value appurtenance degree function is:*

$\forall x \in P, d: P \times V \rightarrow P([0,1]^z)$ (1)

Thus $d(x, v)$ is a subset of $[0,1]^z$, where $\mathcal{P}([0,1]^z)$ is the power set of $[0,1]^z$, where z = 1 (*fuzzy degree of appurtenance*), z = 2 (for the *intuitionistic fuzzy degree of appurtenance*), or z = 3 (for the *neutrosophic degree of appurtenance*).

Let $|V| \geq 1$ be the cardinal. Let $c: V \times V \rightarrow [0,1]$ be the *attribute value contradiction degree function* between any two attribute values v_1 and v_2, denoted by $c(v_1, v_2)$, and satisfying the following axioms:

1. $c(v_1, v_1) = 0$, the degree of contradiction between the same attribute values is zero;
2. $c(v_1, v_2) = c(v_2, v_1)$, commutativity.

We can define a *fuzzy attribute value contradiction degree function* (c as before, which we can denote by c_F to distinguish it from the next two), an *intuitionistic fuzzy attribute value contradiction function* $c_{IF}(\cdot)$, or more generally, a $c_{IF} : V \times V \rightarrow [0,1]^2$. *Neutrosophic attribute value contradiction degree function* ($c_N : V \times V \rightarrow [0,1]^3$) can be used, increasing the complexity of the calculation, but also increasing the precision.

We mostly figure out how much one-dimensional attribute values disagree with each other. For multi-dimensional attribute values, we break them down into one-dimensional attribute values.

The attribute value contradiction degree function helps improve the results of certain grouping methods and the related order system.

The way we measure how much different values disagree with each other is set up for each area where a special type of group is used, based on the specific problem that needs solving. If we disregard the details, the computations will still be performed, but they may not yield reliable results.

So (P, a, V, d, c) is called a *plithogenic set*, [12-16]:

1. Where "P" is a set, "a" is an attribute (multidimensional in general), "V" is the rank of the attribute values, "d" is the degree of appurtenance of the attribute value of each element x to the set P concerning some given criteria ($x \in P$), and "d" means "d_F" or "d_{IF}" or "d_N", when it is a fuzzy degree of appurtenance, an intuitionistic fuzzy degree of appurtenance, or a neutrosophic degree of appurtenance respectively of an element x to the plithogenic set P;

2. "c" stands for "c_F" or "c_{IF}" or "c_N", when it comes to the fuzzy degree of contradiction, fuzzy intuitionistic degree of contradiction, or neutrosophic degree of contradiction between attribute values respectively.

Functions $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are defined according to the applications that experts need to solve.

The notation used is:

$x(d(x,V))$, where $d(x,V) = \{d(x,v), \text{ for all } v \in V\}, \forall x \in P$.

The degree of attribute value contradiction is calculated between each attribute value concerning the dominant attribute value (denoted by v_D) in particular and to other attribute values as well.

The attribute value contradiction degree function c between the attribute values is used in the definition of *plithogenic aggregation operators* (intersection (AND), union (OR), implication (⟹), equivalence (⟺), inclusion relation (partial order), and other plithogenic aggregation operators combining two or more attribute value degrees acting on the t-norm and t-conorm.

Most plithogenic aggregation operators are linear combinations of the fuzzy t-norm (denoted by $\wedge_F$), and the fuzzy t-conorm (denoted by $\vee_F$), but nonlinear combinations can also be constructed.

If the t-norm is applied on the dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then on the attribute value v_2 the following applies:

$[1 - c(v_D, v_2)] \cdot t_{norm}(v_D, v_2) + c(v_D, v_2) \cdot t_{conorm}(v_D, v_2)$ (2),

Or, by using symbols:

$[1 - c(v_D, v_2)] \cdot (v_D \wedge_F v_2) + c(v_D, v_2) \cdot (v_D \vee_F v_2)$ (3).

Similarly, if the t- conorm is applied on the dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then on the attribute value v_2 the following applies:

$[1 - c(v_D, v_2)] \cdot t_{conorm}(v_D, v_2) + c(v_D, v_2) \cdot t_{norm}(v_D, v_2)$ (4).

Or, by using symbols:

$[1 - c(v_D, v_2)] \cdot (v_D \vee_F v_2) + c(v_D, v_2) \cdot (v_D \wedge_F v_2)$ (5).

The *Plithogenic Neutrosophic Intersection* is defined as:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right) \quad (6),$$

The *Plithogenic Neutrosophic Union* is defined as:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right) \quad (7),$$

In other words, concerning what applies to belonging, the opposite applies to non-belonging, while in indeterminacy the average between them applies.

The *Plithogenic Neutrosophic Inclusion* is defined as follows:

Since the degrees of contradiction are $c(a_1, a_2) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5$, it applies: $a_2 \geq [1 - c(a_1, a_2)]b_2$ or $a_2 \geq (1 - 0.5)b_2$ or $a_2 \geq 0.5b_2$ while $c(a_1, a_3) = c(b_1, b_3) = 1$.

Having $a_1 \leq b_1$ the opposite is done for $a_3 \geq b_3$, where $(a_1, a_2, a_3) \leq_P (b_1, b_2, b_3)$ if and only if $a_1 \leq b_1$ and $a_2 \geq 0.5b_2$, $a_3 \geq b_3$.

3 The Study

Regardless of the type and procedure of contracting, all public contracting in Ecuador is carried out in three fundamental phases, by the provisions of the National Public Procurement Service Resolution No. 72, of January 2018. These phases are:

1. **Preparatory Phase**: The contracting entity defines its needs and specifies the technical and administrative requirements. In addition, a market study is carried out to determine the reference budget. This stage ends with the signing of the resolution to start and publish the process authorized by the highest authority of the entity or its delegate.
2. **Pre-Contractual Phase**: This stage begins with the publication of the process on the public procurement portal and includes the call for tenders, the reception and evaluation of offers, and the award to the supplier with the best offer, and ends with the signing of the contract.
3. **Contractual Phase**: Once the contract has been awarded, it is administered and monitored to supervise its compliance. This is the most important stage in a public procurement process because it includes the management of resources, verification of compliance with the contractual conditions, and evaluation of the results obtained.

The legal framework that regulates public procurement in Ecuador is made up of a series of laws, regulations, and standards that establish the principles and procedures that must be followed for the acquisition of goods, services, and works by the State. Among the main regulations are:

- Organic Law of the National Public Procurement System (*LOSNCP* in Spanish): This law establishes the general framework for public procurement in Ecuador, defining the principles of transparency, equality, free competition, efficiency, and publicity that must govern all procurement processes. The *LOSNCP* seeks to ensure that procurement processes are carried out fairly and equitably, promoting the participation of a greater number of suppliers and avoiding monopolistic practices.
- General Regulations of the Organic Law of the National Public Procurement System: This regulation develops and specifies the provisions of the *LOSNCP*, establishing the specific procedures for public procurement. It details the stages of the procurement process, from planning and calls for tenders to the award and execution of the contract, ensuring that the principles established in the law are met.
- Comprehensive Organic Criminal Code (*COIP* in Spanish): Includes specific provisions on crimes related to corruption in the field of public procurement, highlighting the importance of compliance as a tool to prevent and punish these crimes. The *COIP* establishes severe sanctions for those who commit acts of corruption and promotes transparency and accountability in public institutions.
- Resolutions and Guidelines Issued by the National Public Procurement Service (*SERCOP* in Spanish): These resolutions and guidelines complement the main regulations, providing guidelines and criteria for the implementation of public procurement policies. *SERCOP* is the body in charge of supervising and regulating public procurement processes in Ecuador, and its guidelines are essential to ensure the correct application of the *LOSNCP* and the corresponding regulations.

The variables related to the implementation of compliance in Ecuador in public procurement can be classified as follows:

Internal variables:

Strengths (Positive)

S1: There are monitoring and internal audit mechanisms that are facilitated by the adoption of technological tools, such as anonymous reporting systems and data analysis platforms, which allow effective monitoring of suspicious transactions and operations.

S2: The Ecuadorian government has taken important measures to promote the implementation of compliance systems, such as the creation of the Organic Law on the Fight against Corruption, which establishes clear guidelines for the prevention and punishment of corruption.

S3: Collaboration between the public and private sectors is also essential to create an enabling environment for compliance, promoting a culture of transparency and accountability.

Weaknesses (Negative)

W1: A scheme must be manifested in several dimensions through the implementation of codes of conduct, anti-corruption policies, and continuous training programs for employees with the main functions performed within compliance in this area.

External variables

Opportunities (Positive)

O1: There are legal mechanisms at the state level that help implement compliance, these are:

- **Prevention**: It is a fundamental pillar of compliance that is achieved through the implementation of clear policies and procedures that establish the expected behaviors and the consequences of inappropriate actions.
- **Detection**: Crucial to minimizing the impact of corrupt acts is the use of data analysis software that can help detect unusual patterns in financial transactions that could indicate corrupt practices.
- **Investigation:** Once a possible act of corruption has been detected, it is essential to carry out a thorough investigation impartially and rigorously.
- **Sanctioning:** The imposition of appropriate sanctions is essential to deter future acts of corruption. In addition, organizations must cooperate with judicial authorities to ensure that corrupt acts are punished according to the law.
- **Remediation**: Remediation refers to corrective actions that are implemented to prevent the recurrence of corrupt acts.

O2: Compliance in Ecuador can be improved by adopting advanced technologies, such as blockchain, which is a mechanism that can revolutionize the way corrupt acts are monitored and detected.

O3: There is international cooperation in the exchange of best practices to strengthen local capacities to implement robust compliance programs.

Threats (Negative)

T1: There are corruption problems in the country, which has been involved in several international scandals for this reason.

T2: The main threat is resistance to change within public institutions, especially in sectors where corruption has been deeply rooted, such as justice and health.

T3: Lack of resources and adequate training can also limit the effectiveness of compliance programs.

As in [17], in this article, we summarize in Tables 2 and 3 the linguistic values associated with plithogenic numbers:

Table 2: Linguistic values associated with plithogenic numbers for the assessment. Source: [17, 18].

Linguistic Expression	Plitogenic number (T, I, F)
Very poor (VP)	(0.10, 0.75, 0.85)
Poor (P)	(0.25, 0.60, 0.80)
Moderately poor (MP)	(0.40, 0.70, 0.50)
Medium (M)	(0.50, 0.40, 0.60)
Moderately good (MG)	(0.65, 0.30, 0.45)
Good (G)	(0.80, 0.10, 0.30)
Very good (VG)	(0.95, 0.05, 0.05)

Table 3: Linguistic values associated with plithogenic numbers for the evaluation of the weight of the criteria. Source: [17, 18].

Linguistic Expression	Plitogenic number (T, I, F)
Low significance (LS)	(0.10, 0.70, 0.80)
Equal significance (ES)	(0.30, 0.40, 0.80)
Robust significance (RS)	(0.50, 0.40, 0.60)
Very robust significance (VRS)	(0.70, 0.30, 0.10)
Absolute significance (AS)	(0.90, 0.10, 0.10)

A group of five experts was hired, who evaluated the Strengths, Weaknesses, Opportunities, and Threats according to the scale that appears in Tables 2 and 3. The results of the associated plithogenic numbers are processed to obtain the balance in this method.

Each of the four aspects to be evaluated is located in a matrix, where the evaluations of the internal variables appear in rows and the evaluations of the external variables in columns.

To calculate the results of the aggregation by the 4 quadrants Strength-Opportunity (SO), Strength-Threat (ST), Weakness-Opportunity (WO), and Weakness-Threat (WT), equations 8, 9, 10, and 11 are used, respectively. In this article, no dominant aspect is differentiated for these formulas.

$$\text{SO} = \wedge_{p\,i=1}^{\;3} \wedge_{p\,j=1}^{\;3} \left(\left(\omega_{o_i} \wedge_p \omega_{s_j} \right) \wedge_p E_{o_i s_j} \right) \quad (8),$$

Where $\wedge_p$ is the plithogenic conjunction based on the minimum t-norm and the maximum t- conorm, Equation 6, ω_{o_i} is the weight given to the i-th opportunity, ω_{s_j} is the weight given to the j-th strength, $E_{o_i s_j}$ is the evaluation assigned to the intersection of the i-th opportunity with the j-th strength.

$$\text{ST} = \wedge_{p\,i=1}^{\;3} \wedge_{p\,j=1}^{\;3} \left(\left(\omega_{T_i} \wedge_p \omega_{s_j} \right) \wedge_p E_{T_i s_j} \right) \quad (9),$$

ω_{T_i} is the weight given to the i-th threat, $E_{T_i s_j}$ is the evaluation assigned to the intersection of the i-th threat with the j-th strength.

$$\text{WO} = \wedge_p \wedge_{p\,j=1}^{\;3} \left(\left(\omega_w \wedge_p \omega_{o_j} \right) \wedge_p E_{wo_j} \right) \quad (10),$$

ω_w is the weight given to the weakness, E_{wo_j} is the evaluation assigned to the intersection of the weakness with the j-th opportunity.

$$\text{WT} = \wedge_p \wedge_{p\,j=1}^{\;3} \left(\left(\omega_w \wedge_p \omega_{T_j} \right) \wedge_p E_{wT_j} \right) \quad (11),$$

E_{WT_j} is the evaluation assigned to the intersection of the weakness with the j-th threat.

Table 3 contains the aggregated evaluation given by the five experts of the combination of the results of each internal variable value with each external variable value. The median was used for the aggregation. Note that we used the algorithm proposed in [17] to perform the evaluation.

Table 4: SWOT matrix of the aspects measured in plithogenic numbers. Source: The authors.

		Opportunities			**Threats**		
	.	**O1**	**O2**	**O3**	**T1**	**T2**	**T3**
Strengths	**S1**	(0.95, 0.05, 0.05)	(0.95, 0.05, 0.05)	(0.95, 0.05, 0.05)	(0.65, 0.30, 0.45)	(0.65, 0.30, 0.45)	(0.50, 0.40, 0.60)
	S2	(0.95, 0.05, 0.05)	(0.65, 0.30, 0.45)	(0.95, 0.05, 0.05)	(0.50, 0.40, 0.60)	(0.50, 0.40, 0.60)	(0.65, 0.30, 0.45)
	S3	(0.65, 0.30, 0.45)	(0.80, 0.10, 0.30)	(0.65, 0.30, 0.45)	(0.65, 0.30, 0.45)	(0.65, 0.30, 0.45)	(0.95, 0.05, 0.05)
Weaknesses	**W**	(0.95, 0.05, 0.05)	(0.95, 0.05, 0.05)	(0.95, 0.05, 0.05)	(0.65, 0.30, 0.45)	(0.50, 0.40, 0.60)	(0.50, 0.40, 0.60)

All variables were assigned the value of absolute significance weight or $(0.90, 0.10, 0.10)$, therefore the following result was obtained for the variable combinations:

$\mathrm{SO} = (0.65, 0.16328, 0.45)$, $\mathrm{ST} = (0.50, 0.14023, 0.60)$, $\mathrm{WO} = (0.90, 0.075, 0.10)$, and $\mathrm{WT} = (0.50, 0.23750, 0.60)$.

From these results, compared with the values in Table 2 and their linguistic values, it is inferred that the potentialities (SO) are “Moderately good”, the limitations (WT) are “Medium”, the risks (ST) are “Medium” and finally the challenges (WO) are between “Good” or “High” and “Very Good” or “Very High”.

Conclusion

The implementation of compliance in the small-value procurement process not only strengthens transparency and integrity in public procurement but also promotes a culture of accountability and responsibility. By following the proposed steps and actions, public institutions can minimize corruption risks and ensure a more efficient and ethical use of public resources. The effective implementation of compliance programs not only helps combat corruption but also contributes to improving public trust in institutions, promoting a more ethical and sustainable business environment. Ultimately, compliance is an ethical concept that has become an indispensable tool for achieving a more efficient and responsible public administration, essential for the development and well-being of Ecuador.

This paper proposed a feasibility study to implement a compliance program in the current situation of Ecuador. To do so, we utilized SWOT analysis using plithogenic numbers, following the method applied in [17]. A group of five experts gave their opinion to carry out the required evaluation. A linguistic scale was used that facilitated the evaluation by the experts. The plithogenic theory allowed capturing the complexity of the dynamics of what is being studied. The final result was that the limitations and risks are “Medium”, the potentialities are “Moderately Good” and the challenges are “High”.

Acknowledgment.

The authors thank the collaboration of the Universidad Bolivariana del Ecuador (UBE) and the Research Network for the Study of Intercultural Education and the Internationalization of Teaching in Higher Education (RIIIES) for their support in carrying out the research project: Compliance as a preventive strategy for risk management and the adoption of best practices in public administration, of which this publication is a product.

References

[1] Al-Hashimy, H. N. H. (2022). The Impact of Corona virus Pandemic on the International and Domestic Economy: Analysis the Strengths and Weaknesses Based on SWOT Analysis. International Journal of Business and Management Invention, 11, 90-96.

[2] Longsheng, C., Shah, S. A. A., Solangi, Y. A., Ahmad, M., and Ali, S. (2022). An integrated SWOT-multi-criteria analysis of implementing sustainable waste-to-energy in Pakistan. Renewable Energy, 195, 1438-1453.

[3] Yusuf, M., Saiyed, R., and Sahala, J. (2022, December). Swot Analysis in Making Relationship Marketing Program. In

Proceeding of The International Conference on Economics and Business (Vol. 1, No. 2, pp. 573-588).
[4] Benzaghta, M. A., Elwalda, A., Mousa, M. M., Erkan, I., and Rahman, M. (2021). SWOT analysis applications: An integrative literature review. Journal of Global Business Insights, 6, 54-72.
[5] Đalić, I., Ateljević, J., Stević, Ž., and Terzić, S. (2020). An integrated swot–fuzzy piprecia model for analysis of competitiveness in order to improve logistics performances. Facta Universitatis, Series: Mechanical Engineering, 18, 439-451.
[6] Meskauskas, Z., Jasinevicius, R., Kazanavicius, E., and Petrauskas, V. (2020, July). XAI-based fuzzy SWOT maps for analysis of complex systems. In 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) (pp. 1-8). IEEE.
[7] Villanueva, L. K. B., Intriago, D. A. V., Gómez, L. K. Á., and Morán, A. M. I. (2020). Business plan for entrepreneurs, actors and organizations of social and solidarity economy based on neutrosophic AHP-SWOT (Vol. 37). Infinite Study.
[8] Villalba, M. F. G., Pután, R. A., Romero, J. M., and Oviedo, J. E. E. (2020). Analysis of an strategic plan to increase the sales level of the company" TIENS" of Babahoyo using neutrosophic methods. Neutrosophic Sets and Systems, 34, 183-192.
[9] Büyüközkan, G., Mukul, E., and Kongar, E. (2021). Health tourism strategy selection via SWOT analysis and integrated hesitant fuzzy linguistic AHP-MABAC approach. Socio-Economic Planning Sciences, 74, 100929.
[10] Wu, P., Zhao, G., and Li, Y. (2022). Green mining strategy selection via an integrated SWOT-PEST analysis and Fuzzy AHP-MARCOS approach. Sustainability, 14, 7577.
[11] Obbineni, J., Kandasamy, I., Vasantha, W. B., and Smarandache, F. (2023). Combining SWOT analysis and neutrosophic cognitive maps for multi-criteria decision making: a case study of organic agriculture in India. Soft Computing, 27, 18311-18332.
[12] Smarandache, F. (2022). Plithogeny, plithogenic set, logic, probability and statistics: a short review. Journal of Computational and Cognitive Engineering, 1, 47-50.
[13] Abdel-Basset, M., Mohamed, R., Smarandache, F., and Elhoseny, M. (2021). A new decision-making model based on plithogenic set for supplier selection. Computers, Materials and Continua, 66, 2751-2769.
[14] Martin, N., Smarandache, F., and Sudha, S. (2023). A novel method of decision making based on plithogenic contradictions. Neutrosophic Systems with Applications, 10, 12-24.
[15] Tayal, D. K., Yadav, S. K., and Arora, D. (2023). Personalized ranking of products using aspect-based sentiment analysis and Plithogenic sets. Multimedia Tools and Applications, 82, 1261-1287.
[16] Yon-Delgado, J. C., Yon-Delgado, M. R., Aguirre-Baique, N., Gamarra-Salinas, R., Ponce-Bardales, Z. E., and Gianinna Yon-Delgado, G. (2023). Neutrosophic Plithogenic AHP Model for Inclusive Higher Education Program Selection. Full Length Article, 21, 50-63.
[17] Batista-Hernández, N., Leyva-Vázquez, M. Y., González-Caballero, E., Valencia-Cruzaty, L. E., Ortega-Chávez, W. and Smarandache, F. (2021). A new method to assess entrepreneurship competence in university students using based on plithogenic numbers and SWOT analysis. International Journal of Fuzzy Logic and Intelligent Systems, 21, 280-292.
[18] Abdel-Basset, M., El-Hoseny, M., Gamal, A., and Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. Artificial intelligence in medicine, 100, 101710.

Received: June 24, 2024. Accepted: August 17, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Assessing the need for a feminist foreign policy in Ecuador through a sentiment analysis based on neutroAlgebra

Daniela Cecilia Márquez Carriel[1], Luis Oña Garces[2], Arnaldo Vergara Romero[3], and Fidel Márquez Sanchez[4]

[1] ECOTEC University, Guayaquil. Samborondon. Ecuador. E-mail: danielacmarquezc@outlook.com
[2] ECOTEC University, Guayaquil. Samborondon. Ecuador. E-mail: luis.onagarces@gmail.com
[3] Bolivarian University of Ecuador, Duran. Samborondon. Ecuador. E-mail: avergarar@ube.edu.ec
[4] Espiritu Santo University, Samborondón. Ecuador. E-mail: fmarquez@uees.edu.ec

Abstract. This paper examines the potential for a feminist foreign policy in Ecuador, a concept gaining global attention but still new in local debates. While Ecuador's foreign policy has historically focused on economic and strategic interests, the rising influence of feminist movements is prompting discussions about integrating gender perspectives into diplomacy, trade, and international aid. Using neutrosophic sentiment analysis, the study finds divided opinions: while "Gender Equality" and "Human Rights" receive stable support, the "Economic Impact" variable shows significant uncertainty. Some view a feminist foreign policy as a way to enhance Ecuador's global standing and gender equity, while others are skeptical of its feasibility in the current political and social climate. The study calls for more research to understand public and political perceptions, particularly on the economic impacts and how such a policy could be adapted to Ecuador's specific needs.

Keywords: Foreign Policy, Feminist, Neutroalgebra, Prospector, Sentiment Analysis.

1 Introduction

The topic of feminist foreign policy [1] has been gaining strength in global discussions and, little by little, has crept into local debates. Ecuador is no exception. This research delves into an area that may seem novel but touches deep fibers about how the country relates to the world and how international policies reflect or do not, gender equity values. Some authors have pointed out that feminist foreign policies seek to integrate the gender perspective in matters such as diplomacy, trade, and international aid [2, 3]. However, in Ecuador, the question is not only whether we need a feminist foreign policy, but how it could adapt to our realities.

Historically, Ecuador's foreign policy has been marked by economic interests, strategic alliances, and, to a lesser extent, human rights [4]. Few imagined, just a few years ago, that one could talk about diplomacy from a gender perspective. But today, with the growing visibility of feminist movements and demands for greater equality of rights, the possibility of foreign policy also reflecting these struggles has begun to be discussed. Cases such as Sweden, which became the first country to officially adopt a feminist foreign policy in 2014, have served as inspiration, although not without criticism [3]. But of course, replicating this model in a country with such different realities as Ecuador is not so simple.

Now, the main question driving this research is: Is a feminist foreign policy viable and necessary in Ecuador? It seems like a simple question, but when exploring it, layers of complexity emerge. It is not just about adding the term “feminist” to foreign policy; it is about understanding what that means in a context where gender inequality remains a latent problem and where national interests do not always align with human rights agendas [4]. In addition, it must be considered that any change in foreign policy could have implications both for the country’s international image and its strategic positioning. Could, then, a feminist policy contribute something more than just a discourse? The gap in current literature on the subject is notable. While there are studies on feminist foreign policies in Europe and North America, in Latin America, and particularly in Ecuador, the lack of research is evident [5].

This could be due, in part, to the fact that foreign policy has traditionally been a field dominated by more conservative visions, where gender issues are not a priority. Furthermore, how citizen perception of this idea influences its possible implementation has not been sufficiently explored. This is where our neutrosophic approach comes into play: capturing those perceptions that are often contradictory or indefinite. To address this complexity, this study employs sentiment analysis based on neutroAlgebra [6], a tool that allows for dealing with the indeterminacy and ambiguity inherent in human opinions [7, 8]. This approach not only examines the arguments for and against a feminist foreign policy but also unravels those grey areas, those opinions that do not align neatly into a "yes" or "no". The methodology includes qualitative and quantitative data collection, combining sentiment analysis techniques with the mathematical structure of neutroAlgebra to assess the need for a feminist foreign policy.

Preliminary results suggest that perceptions about the need for a feminist foreign policy in Ecuador are divided, with a strong presence of indeterminate opinions reflecting both skepticism and cautious support [9, 10]. While some see the possibility of improving Ecuador's representation in international forums through a gender lens, others doubt its viability, especially in a political and social context that is not always receptive to changes in traditional norms. This finding is key, as it highlights the need for a deeper analysis that goes beyond the obvious. In conclusion, the objectives of this study are clear: to assess the viability of a feminist foreign policy in Ecuador through a sentiment analysis based on neutral algebra and to provide a balanced view of its potential benefits and limitations. This approach seeks not only to fill an academic gap but also to offer a practical tool for policymakers who wish to better understand how the population perceives these issues. In doing so, it is hoped to contribute to a more informed and less polarized debate on the future direction of Ecuadorian foreign policy.

2 Preliminaries

2.2 Sentiment analysis

Sentiment analysis, at its core, relies on the use of natural language processing tools, combined with text analysis and computational linguistics techniques, to unravel the subjective part hidden in various sources of information [8]. In the field of text mining, this approach stands out for its ability to mass classify the polarity of data, that is, to determine whether something is positive, negative, or neutral.

Given a corpus T containing a set of texts, the goal of sentiment analysis is to assign a **polarity score** to each text element $t_i \in T$ This score reflects whether the sentiment of the text is **positive**, **neutral**, or **negative [11]**

The sentiment scores are typically derived from [12] :

- Lexicon-based models: Predefined lists of words or phrases with associated positive, negative, or neutral scores.
- Machine learning models: Trained classifiers that output probabilities for each sentiment category.

Using neutroAlgebra [13] we can extend the sentiment classification by introducing the concept of $I(T_i)$ This accounts for uncertainty or ambiguity in the sentiment of the text using the symbol I
The sentiment analysis function becomes:

$$S_{neut}(t_i) = (P(t_i), N(t_i), U(t_i), I(t_i)) \quad (1)$$

Where:

$P(t_i)$, represents the **positive** sentiment score in the text
$N(t_i)$, represents the **negative** sentiment score in the text
$U(t_i)$, represents the **neutral** sentiment score in the text
$I(t_i)$ represents the **indeterminate** sentiment component, reflecting uncertainty or mixed sentiment in the text.

When measuring these sentiments, experts insist that it is essential to include a neutral option, since an emotional state cannot always be labeled as simply positive or negative. Sometimes, people are somewhere in the middle, with their emotions not leaning towards one side or the other. This is where Neutrosophy comes into play, since this theory not only covers the positive and the negative but also recognizes the existence of neutral. This is especially useful when analyzing texts, because not all words have a clear connotation and many times the meaning can be ambiguous, adding extra complexity to the analysis.

a. **Method**

For a given natural number $n > 0$, NeutroGroup is defined from the combinator function of Prospector. Prospector is a well-known expert system [14, 15]. The set NeutroGroup consists of all integers between $-n$ and n plus the symbolic element I to represent indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$and $\oplus_5$is used. This is defined according to the following Cayley table:

Table 1. Cayley table corresponding to $\oplus_5$. Source: [16].

$\oplus_5$	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	I
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
I	-5	-4	-3	-2	-1	I	I	I	I	I	I	I
1	-5	-4	-2	-1	0	1	I	2	3	4	4	5
2	-5	-3	-1	0	1	2	I	3	3	4	5	5
3	-5	-2	0	1	2	3	I	4	4	4	5	5
4	-5	0	2	3	4	4	I	4	5	5	5	5
5	I	5	5	5	5	5	I	5	5	5	5	5

$\oplus_5$It satisfies the properties of commutativity and associativity and has 0 as a null element. In addition, it satisfies each of the following properties:

- If $x, y < 0$then $x \oplus_5 y \leq min(x, y)$,
- If $x, y > 0$then $x \oplus_5 y \geq max(x, y)$,
- If $x < 0$and $y > 0$or if $x > 0$and $y < 0$, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

Sentiment analysis, through the neutrosophic method, focuses on assessing integrity, transparency, and accountability within organizations. Using this theory, opinions and perceptions are examined by considering the degrees of positivity, negativity, and indeterminacy. This approach not only captures clear sentiments, such as positive and negative ones but also addresses those that are neutral or ambiguous, thus achieving a more accurate assessment and a better understanding of how these aspects are perceived in the organizational environment [17].

This method, particularly effective in the analysis of short and informal texts, as described in the technique mentioned above, requires the identification of a set of words that are classified as positive, negative or neutral, each with a strength value evaluated in a range from -5 to 5, or that are marked as indeterminate. Indeterminacy occurs when it is not possible to decipher the individual's thoughts on the subject in question, which may occur due to a lack of clarity in the semantics of the text or because the text is unintelligible. Furthermore, in certain cases, it is possible that in the same text extreme evaluations of positivity (+5) and negativity (-5) are presented for the same variable, which generates a contradiction that is classified as indeterminate, marked with the letter I. This indeterminacy can have different origins, which becomes evident when the function used in the PROSPECT expert system, which evaluates the degree of evidence of an expert on a particular aspect, finds maximum evidence but in opposite directions for two different aspects.

This method, which borrows some elements from the SentiStrength sentiment strength detection algorithm [18], allows terms related to the analyzed variables to be classified as Positive, Negative or Neutral in a list using linguistic values. Each of these terms is associated with a value between -5 and 5, or even I, depending on the intensity of its positive or negative charge. For example, the term "I like" increases its positive value if expressed as "I like it a lot", while "I don't like it" becomes more negative by saying "I don't like it a lot". What applies is that for the word “much” or “a lot” that modifies one of the positive or negative classifying words, is used $x \oplus_5 x$, and for “too much” $x \oplus_5 x \oplus_5 x$, where $x$is the value that is associated with the word. For example, $x > 0$it results in “very” with an even more positive value. On the other hand, when $x < 0$, the result is more negative.

Also, the modification of "quite" is converted to $\left[sig(x)\sqrt{|x|}\right]$.

- They take into account words that reverse the meaning of what is said. In this case, the sign is changed. For example, “I like” has a value of $x = 3$, when it comes to “I don't like” it is calculated as $x = -3$, both have the same strength, but with opposite meaning.
- In this algorithm, very complex cases, where there are exclamation or question marks, are ignored, since we want to evaluate what the members of the organization or clients write, if it makes sense, about each of the twelve aspects of ethics mentioned in the previous points.
- Another aspect that is taken into account in the proposed algorithm taken from the previous one is the evaluation of the emoticons.
- Spell checking also applies here.

The next step is the evaluation of a short informal text written by a person. To do this, natural language processing is performed, where words that express feelings or opinions about each of the twelve aspects mentioned above are searched for. Let us denote these aspects as $V = \{v_1, v_2, \cdots, v_{12}\}$:

Then, within the text processing, the words referring to each of these variables are identified. These words are identified with a value from -5 to 5 o I. Let us denote this as follows, for the i- th variable, the set $X_i$of word ratings that appear in the text:

$v_i \rightarrow X_i = \{x_{i1}, x_{i2}, \dots, x_{im_i}\}$, where$x_{ij}$ It is the set of elements between -5 and 5 o I, used to qualify the words that refer to the i- th variable.

Note that even the individual evaluation of each word can be complicated. For example, when modifiers such as "very" appear, the value of the modified word changes. Also when there are spelling errors that make an evaluation illegible, it is necessary to use the value I. The final value associated with eachv_i is:

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \dots \oplus_5 x_{im_i} \quad (2)$$

Let us keep in mind that we do not consider it convenient to obtain an aggregate ethical value for all the variables since the separate value is more useful to have an idea of the individual opinion or feeling.

If we have a set of people whose opinion is being studied. Let us call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i,j}$it is the total value of the i - th ethics variable in the organization, according to the jth person.

It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} \quad (3)$$

That is, the arithmetic mean of each of the variables is calculated.

Finally, if the I symbol appears a de-neutrosophication process is developed replacing $I \in [-5, 5]$ both maximum and minimum values []

3 Case Study.

An analysis was conducted using the neutrosophic sentiment analysis method to assess the perception of the need for a feminist foreign policy in Ecuador. Opinions were collected from 12 specialists on the topic, categorized by their areas of specialization. Each specialist provided brief texts in which they assessed various variables related to the topic.

Variables Evaluated:

1. Gender Equality: his variable assesses how a feminist foreign policy would promote gender equality both domestically and internationally. It looks at whether the policy can enhance the representation of women in diplomatic roles and integrate gender equity into international treaties and political agendas.
2. Human Rights: This variable examines the impact of a feminist foreign policy on the defense and promotion of human rights, especially the rights of women and vulnerable groups. It assesses whether the policy incorporates women's rights in international negotiations and supports global human rights movements.
3. Economic Impact: This variable measures the economic consequences of implementing a feminist foreign policy. It analyzes whether the policy would have positive or negative economic implications for the country, focusing on trade agreements that favor gender equity and the cost-benefit of adopting feminist policies internationally.

Evaluation and Calculations:

The evaluations of each specialist for each variable were processed, applying the neutrosophic method of sentiment analysis. The values of each word were adjusted with modifiers and corrections, and the means for each variable were calculated.

Evaluation Data:

For each specialist, words related to the variables were identified, with assigned values ranging from -5 to 5 or I (indeterminate). Calculations were performed according to the rules of the neutrosophic method.

Table 2. Gender Equality Assessment.

Specialist	Evaluation 1	Evaluation 2	Evaluation 3	Final Result ⊕ 5
Specialist 1	3	4	1	5
Specialist 2	4	2	1	4
Specialist 3	5	4	3	5
Specialist 4	2	3	2	3
Specialist 5	2	5	1	5
Specialist 6	3	2	4	5
Specialist 7	3	2	1	3

Specialist	Evaluation 1	Evaluation 2	Evaluation 3	Final Result ⊕ 5
Specialist 8	3	2	0	4
Specialist 9	2	4	1	4
Specialist 10	5	5	2	5
Specialist 11	0	3	2	3
Specialist 12	4	2	3	5

Table 3. Human Rights **Assessment.**

Specialist	Evaluation 1	Evaluation 2	Evaluation 3	Final Result $\oplus_5$
Specialist 1	4	1	2	5
Specialist 2	3	2	4	4
Specialist 3	5	4	1	5
Specialist 4	2	3	0	4
Specialist 5	4	3	4	5
Specialist 6	3	2	2	4
Specialist 7	3	3	4	5
Specialist 8	2	1	3	4
Specialist 9	1	5	4	5
Specialist 10	4	1	2	5
Specialist 11	3	2	4	4
Specialist 12	2	4	4	5

Table 4. Economic Impact **Assessment.**

Specialist	Evaluation 1	Evaluation 2	Evaluation 3	Final Result $\oplus_5$
Specialist 1	4	3	2	5
Specialist 2	3	2	I	I
Specialist 3	5	4	3	5

Specialist	Evaluation 1	Evaluation 2	Evaluation 3	Final Result ⊕ 5
Specialist 4	2	1	I	I
Specialist 5	4	5	4	5
Specialist 6	3	2	2	4
Specialist 7	I	3	4	I
Specialist 8	2	1	3	4
Specialist 9	1	I	4	I
Specialist 10	4	-1	2	5
Specialist 11	3	2	4	4
Specialist 12	2	4	I	I

Calculating the Mean for Each Variable

For each variable, the arithmetic mean is calculated as follows:

$$\bar{x}_{totai,12} = \frac{\sum_{j=1}^{12} x_{total,i,j}}{12} \quad (4)$$

Table 5: Aggregation of the variables.

Variable	Average
1. Gender Equality	4.25
2. Human Rights	4.58
3. Economic Impact	2.6667+0.46I

In the next step the de-neutrosophication process is developed

Table 6: De-neutrosophication of variables**.**

Variable	Average
1. Gender Equality	4.25
2. Human Rights	4.58
3. Economic Impact	[0.3667, 4.9667]

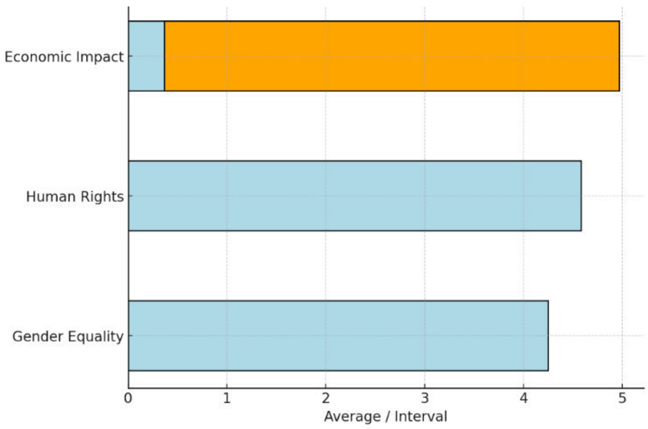

Figure 1: Horizontal Bar Chart of Variables with Interval Representation for Economic Impact.

While the values for "Gender Equality" (4.25) and "Human Rights" (4.58) are fixed, "Economic Impact" is represented as an interval ranging from 0.3667 to 4.9667. This indicates greater uncertainty or variability in the economic impact, suggesting it could fluctuate significantly compared to the more stable and predictable values of the other two variables. The interval highlights the wide range of possible outcomes for the economic impact, which could be very low or reach levels comparable to the other variables.

Conclusion

The conclusions of this paper indicate that the viability of a feminist foreign policy in Ecuador is a subject of considerable debate, with perceptions divided across specialists. While variables such as "Gender Equality" and "Human Rights" received stable, positive evaluations, the "Economic Impact" variable presented a notable degree of uncertainty. This is reflected in the range of possible outcomes, with values fluctuating between 0.3667 and 4.9667, highlighting both the potential for positive economic effects and the risk of instability. The use of neutrosophic sentiment analysis reveals that there are mixed feelings, with some individuals expressing support for a feminist foreign policy based on its potential to enhance international representation and gender equity, while others remain skeptical of its feasibility given Ecuador's political and social context.

For future work, it is recommended that more research be conducted to better understand the public and political reception of such policies. Expanding the study to include a broader range of stakeholders and conducting in-depth qualitative interviews could provide a more nuanced understanding of how feminist foreign policy might be adapted to Ecuador's unique realities. Additionally, further exploration of the economic implications is needed, particularly in the areas of trade and international cooperation, to assess the long-term sustainability and potential economic benefits of such a policy.

References

[1] Scheyer, V., & Kumskova, M. (2019). Feminist foreign policy. Journal of International Affairs, 72(2), 57-76.
[2] van Wyk, J. A. (2023). South Africa: A Growing Embrace of Feminist Foreign Policy?. The Thinker, 94(1), 64-76.
[3] von Hlatky, S. (2021). Women, peace, and security: feminist perspectives on international security (Vol. 15). McGill-Queen's Press-MQUP.
[4] Bustamante, F. (1992). Ecuador: Putting an end to ghosts of the past?. Journal of Interamerican Studies and World Affairs, 34(4), 195-224.
[5] Rosén Sundström, M., & Elgström, O. (2020). Praise or critique? Sweden’s feminist foreign policy in the eyes of its fellow EU members. European Politics and Society, 21(4), 418-433.
[6] Smarandache, F. (2024). Super Hiper Función y Super Hiper Estructura y sus correspondientes Super Hiper Función Neutrosófica y Super Hiper Estructura Neutrosófica. Neutrosophic Computing and Machine Learning, 31, 353-359.
[7] Mohamed, Z., M. Ismail, M. and Abd El-Gawad, A. (2023) “Sustainable Supplier Selection using Neutrosophic Multi-Criteria Decision Making Methodology”, Sustainable Machine Intelligence Journal, 3,

pp. (2):1–9.
[8] Vázquez, M. Y. L., Cevallos, R. E. H., & Ricardo, J. E. (2021). Análisis de sentimientos: herramienta para estudiar datos cualitativos en la investigación jurídica. Universidad Y Sociedad, 13(S3), 262-266.
[9] Hinojosa, M., & Kittilson, M. C. (2020). Seeing women, strengthening democracy: How women in politics foster connected citizens. Oxford University Press, USA.
[10] Márquez-Carriel, D. C., Márquez-Sánchez, F., Oña-Garcés, L., & Vergara-Romero, A. (2024). Feminist Foreign Policy's Prospects in The Context of Global Cooperation. Kurdish Studies, 12(1), 3038-3050.
[11] Kandasamy, I., Vasantha, W. B., Obbineni, J. M., & Smarandache, F. (2020). Sentiment analysis of tweets using refined neutrosophic sets. Computers in Industry, 115, 103180.
[12] Kolchyna, O., Souza, T. T., Treleaven, P., & Aste, T. (2015). Twitter sentiment analysis: Lexicon method, machine learning method and their combination. arXiv preprint arXiv:1507.00955.

[13] Cruzaty, L. E. V., & Reynoso, A. M. R. (2024). New Neutrosophic Sentiment Analysis Method Based on NeutroAlgebra for the Evaluation of Ethics in Companies. Neutrosophic Sets and Systems, 70, 264-270.
[14] Zhang, C., & Luo, X. (1999). Isomorphic transformations of uncertainties for incorporating EMYCIN-style and PROSPECTOR-style systems into a distributed expert system. Journal of Computer Science and Technology, 14, 386-392.
[15] Reiter, J. E. (1983). A Review of the PROSPECTOR Project. IEEE Database Eng. Bull., 6(4), 38-45.
[16] Batista-Hernández, N., González-Caballero, E., Valencia-Cruzaty, L. E., Ortega-Chávez, W., Huarac, C. F. P., and Chamorro, S. L. C. (2022). Theoretical study of the NeutroAlgebra generated by the combining function in Prospector and some pedagogical notes. In Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras (pp. 116-140). IGI Global.
[17] Awajan, I., Mohamad, M., & Al-Quran, A. (2021). Sentiment analysis technique and neutrosophic set theory for mining and ranking big data from online reviews. IEEE Access, 9, 47338-47353.
[18] Thelwall, M., Buckley, K., Paltoglou, G., Cai, D., & Kappas, A. (2010). Sentiment strength detection in short informal text. Journal of the American society for information science and technology, 61(12), 2544-2558.
[19] Salmerona, J. L., & Smarandacheb, F. (2010). Redesigning Decision Matrix Method with an indeterminacy-based inference process. Multispace and Multistructure. Neutrosophic Transdisciplinarity (100 Collected Papers of Sciences), 4, 151.

Received: June 25, 2024. Accepted: August 18, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Evaluating the direct effect of an increase in the Value Added Tax on business sales using the Delphi and NAHP+NSC methods

Guido Macas-Acosta[1], Jesús Estupiñán Ricardo[2], Arnaldo Vergara-Romero[3], and Fidel Márquez Sánchez[4]

[1] ECOTEC University. Ecuador. E-mail: gmacas@ecotec.edu.ec
[2] Higher Institute of Technology for Scientific Research and Innovation, Quito. Ecuador. E-mail: canciller@istici.edu.ec
[3] Bolivarian University of Ecuador, Duran. Ecuador. E-mail: avergarar@ube.edu.ec
[4] University of Specialties Holy Spirit. Ecuador. E-mail: fmarquez@uees.edu.ec

Abstract. This article uses the Delphi and neutrosophic analytic hierarchy process (NAHP) and neutrosophic social choice theory (NSC). NAHP+NSC methodology is used to investigate the potential direct effects of a rise in the Value Added Tax (VAT) on company sales. The primary question is how a change in VAT may affect corporate activity; this is a simple enough question despite its weighty ramifications. Despite the large number of economic research, it seems that the literature has not yet gone into great length on how these particular techniques might provide an in-depth understanding of possible company responses to tax increases. It's interesting to note that the study not only closes a significant research gap, but also uses advanced approaches to examine the impact. Findings that would not have been reached by more conventional methods are achieved by combining the Delphi technique for expert viewpoints with NAHP+NSC for a more in-depth study. The results imply that, depending on a number of variables, including industry type and company size, a rise in VAT might have varying impacts on business sales. This research provides helpful tools for firms and politicians looking to adjust to possible changes in the tax environment, in addition to offering a fresh viewpoint on the topic of tax policy. In the end, the study broadens our theoretical knowledge and offers helpful advice for navigating the intricate realm of tax laws and their implications on the economy.

Keywords: Value Added Tax, Neutrosophic Analytic Hierarchy Process, Neutrosophic Social Choice Theory.

1 Introduction

The impact of an increase in Value Added Tax (VAT) on business sales is a topic that, although it seems technical and specific, has profound implications for the economy at large. In the current context, where tax policies are constantly changing and businesses are looking to adapt, understanding how an adjustment in VAT could influence sales becomes crucial. VAT decisions not only affect the finances of businesses but can also have a domino effect on consumer behavior and the economic stability of local and global markets [1, 2]. This study delves into this issue by applying advanced analytical methods, such neutrosophic analytic hierarchy process (NAHP) [3] and neutrosophic social choice theory (NSC) [4], to gain insight into this phenomenon [5].

Historically, VAT has been a fundamental pillar in the tax policies of many countries, and its modification can be a powerful tool to adjust national finances [6]. Since it was implemented in various nations, different responses have been observed from the business and consumer sectors. For example, in the 1990s, several European countries experienced adjustments in their VAT rates, leading to a variety of studies on their economic effects. This background highlights the relevance of the topic and the need to examine how a change in VAT could influence the current business landscape [7].

The central issue explored in this study is how an increase in VAT could directly affect business sales. Despite the existence of studies on the general impact of tax changes, there seems to be a gap in the application of specific methodologies that can capture the complexities of these effects at the business level [8]. How can businesses adjust their sales strategies in response to an increase in VAT, and what specific factors should they consider? This question guides the research, seeking to provide concrete and practical answers. The main objective of this study is to assess the direct impact of an increase in VAT on business sales using a combination of Delphi and NAHP+NSC methods [9]. The Delphi method will allow for expert opinions on the potential impact of the tax change, while NAHP+NSC will provide a more detailed and structured analysis [10]. The idea is to integrate these approaches to provide a comprehensive assessment that not only identifies potential impacts, but also helps businesses adapt to changes in the tax environment.

This study seeks, first, to understand how an increase in VAT could alter sales dynamics in different business sectors. Second, it aims to provide practical recommendations based on the findings obtained, in order to help businesses prepare and adapt to future tax adjustments. These objectives are designed to address the research question comprehensively and offer valuable insights for both academics and sector practitioners. The proposed methodological approach aims to fill a gap in the current literature by combining expert analysis and advanced methods for a more accurate assessment. As the research progresses, the results are expected to offer a new perspective on how tax policies impact business behavior and ultimately the economy at large. This study not only contributes to theoretical knowledge, but also provides practical tools for decision-making in a changing tax context.

2 Related work.

2.1 Delphi and AHP Methods

The Delphi Method [11] It is a forecasting and decision-making technique based on collecting and analyzing expert opinions on a specific topic through a series of iterative questionnaires. After each round of questionnaires, the information collected is compiled and summarized, providing feedback to all participants. They are then asked to reconsider and, if they wish, revise their previous responses based on the feedback received. This process is repeated over several rounds until a consensus is reached or a stabilization and convergence in responses is observed. The goal is to reach a consensus or common understanding on the topic in question, taking advantage of the collective knowledge and experience of experts. Statistical control employs means and standard deviations to summarize expert opinions and observe the convergence of opinions throughout the rounds [12].

On the other hand, the Analytical Hierarchy Process (AHP) [13] is a theory-oriented decision-making tool that uses pairwise comparisons to assign weights to criteria and alternatives. This method employs a structured matrix, often referred to as a comparison matrix $A = [a_{ij}]$, where each element a_{ij} represents the relative importance of criterion

i over criterion j. The principal eigenvector of the matrix is then calculated to derive the priority vector, which provides the weights for each criterion.

The consistency of the judgments is assessed through a consistency ratio (CR), calculated using the eigenvalue method, to ensure reliable results. AHP combines subjective and objective measurements, and when integrated with Delphi, it allows for iterative feedback and refinement of expert opinions. This hybrid approach ensures both consensus-building and rigorous decision analysis, making it applicable across a wide range of fields. Together, these methods offer a powerful decision-making framework that balances expert knowledge with quantitative rigor [14].

2.2. Neutrosophic concepts.

Definition 1 : ([15]) The neutrosophic set N is characterized by three membership functions, which are the truth membership function T_A , the indeterminacy membership function I A , and the falsity membership function F_A , where U is the Universe of Discourse and ∀x ∈U, $T_A(x)$, $I_A(x)$, and $F_A(x) \subseteq]^-0,1^+[$, y $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$. Note that by the definition, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are standard or nonstandard real subsets of $]^-0, 1^+[$ and therefore $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be subintervals of [0, 1].

Definition 2 [16, 17]: The single-valued neutrosophic set (SVNS) N over U is A = {<x; TA(x), IA(x), FA(x)>: x ∈U}, where TA: U →[0, 1], IA: U →[0, 1], and FA: U →[0, 1], 0 ≤TA(x) + IA(x) + FA(x) ≤3. The Single-Value Neutrosophic Number (SVNN) is represented by N = (t, I, f), such that 0 ≤t, I, f ≤1 and 0 ≤t + I + f ≤3.

Definition 3: The single-valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2 . a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$is a neutrosophic set in ℝ, whose truth, indeterminacy and falsity membership functions are defined in [18].

Definition 4: Given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$two $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$single-valued trapezoidal neutrosophic numbers and λany nonzero number on the real line. Then the operations are defined in [19].

Definitions 3 and 4 refer to the *single-valued triangular neutrosophic number* when the condition a2 = a3 . For simplicity, we use the linguistic scale of triangular neutrosophic numbers, see Table 1, and also compare it with the scale defined in.

Table 1: Saaty scale translated into a neutrosophic triangular scale. Source [20]

Definition	Neutrosophic Triangular Scale
Equally influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
Very influential	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
Absolutely influential	$\tilde{9} = \langle(9,9,9); 1.00, 1.00, 1.00\rangle$
Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

For verification of the analytic hierarchy process model in a neutrosophic environment (N-AHP) methodology, see [21].

2.3 Neutrosophic social choice theory

This subsection summarizes the main concepts of the Neutrosophic Social Choice theory developed in [22].

Definition 5: ([22]) Let a = (Ta, Ia , Fa) be a single-valued neutrosophic number with truth value Ta, indeterminacy value Ia and falsity value Fa. *The distributed indeterminacy form* (DIF) of *a* is defined as $_{\text{DIF}}$ = (Ta − TaIa , 0, Fa − FaIa).

The DIF aims to distribute the result of the indeterminacy regarding truth and falsehood, thus, it measures the degree of affectation of truth and falsehood, when the indeterminacy varies.

Definition 6: ([24]) Let a be *a* single-valued neutrosophic number. A *score function* H of *a* is:

$$H(a) = \frac{(1+T_a - I_a\,(1-T_a) - F_a(1-I_a)}{2} \quad (1)$$

Where for all *a* , H $_{(a)}$ Є [0, 1]. H is an order relation representing an information accuracy score of *a* . If H(a_1) = H(a_2), then a_1= a_2, i.e., they have the same information, whereas, if H(a_1)< H(a_2), then $a_2$is greater than a_1.

Let S = {s1, s2, . . . , sn } will be a set of alternatives and m will be a set of individuals. Each individual declares his preferences over S, which are represented by an individual neutrosophic preference relation Rk , where N R_k: S × S → [0,1] × [0,1] × [0,1] and matrix Rk =[r_{ij}^k], j= 1,2,3,…, n; k = 1,2,3, . . . , m, where r_{ij}^k= N R_k(r_i^k, r_j^k).

$$R_k = \begin{matrix} (0.5, 0.5, 0.5) \\ r_{21}^k \\ r_{n1}^k \end{matrix}$$

The function H (called *the neutrosophic index* or *neutrosophic hesitation function)* maps each a_{ij} neutrosophic value to a number in [0, 1]. Thus, the *neutrosophic index* or *neutrosophic hesitation function* is defined as follows [25]:

$$H(a) = \frac{(1+T(a_{ij}) - I\,(a_{ij})\,(1-T(a_{ij})) - F(a_{ij})(1-I(a_{ij}))}{2} \quad (2)$$

The matrix R_k^H= [H(r_{ij}^k)], i, j = 1,2,3, . . , n; k = 1, 2, 3, ..., m. R_k^H is *quasi-reciprocal* if and only if $H(r_{ij}^k) \le 1 - H(r_{ij}^k)$. If R_k^Hit is not quasi-reciprocal, we call ak an *irrational individual* . Other definitions are stated in [26].

Definition 7: ([26]) *:* $S_i \in W$ is called *a consensus winner* if and only if $\forall S_j \neq S_i: r_{ij} > 0.5$,where$r_{ij} \in H_{\pi}$.

Definition 8: ([26]) The *average social aggregation function C* is defined to calculate the order of $S_i$in the group to the extent that individuals are not against the option yes, using the following equation:

$$C(S_i) = \frac{1}{m-1}\sum_{i\neq j} r_{ij} \quad (3)$$

Where i, j = 1, 2, ⋯ , m.

3.0 Material and Methods

In this section, the proposed NAHP+NSC method for the analysis of the object of study in the article is presented. The first element is defined for any neutrosophic triangular number ã as the triangular precision function of ã = ‹(a1, a2, a3); αa, βa, γa ›, which is the TA function defined as follows:

$$TA(\tilde{a}) = A(‹(a1, a2, a3);\ DIF((\alpha a, \beta a, \gamma a))›) \quad (4)$$

This is the degree of accuracy of Equation 6 calculated for the DIF of the neutrosophic number contained in ã. The inclusion of DIF follows the idea of [27], where the accuracy function H calculates the effect of Indeterminacy on truth and falsity.

It can be seen that the reciprocal or quasi-reciprocal properties in NSC theory are similar to the reciprocal property in NAHP, from the perspective of the rationality of the decision maker.

The method analyzed consists of the following steps:

1. The objective of the problem is established and the group of experts is selected accordingly. Attributes, sub-attributes and alternatives are then specified.
2. The expert group is divided into M interest subgroups, denoted by IG = {IG $_1$, IG $_2$, ⋯ , IG $_M$ }. In the analysis, it is assumed that the members of each subgroup form a homogeneous decision group.
3. Each expert evaluates his/her own NAHP. However, for each IG $_i$, the equivalent matrices of the subgroup members are aggregated using formula 5.

Given {Ãi1, Vi2,…, Ã in_i} as a set of n $_i$ SVTNN representing the evaluation of each member of the i- th subgroup, where Ã $_{ij}$ = ‹(a $_{ij}$, a $_{ij}$, a $_{ij}$); α $\tilde{a}_{ij}$, β $\tilde{a}_{ij}$, γ $\tilde{a}_{ij}$› (i = 1, 2, …, M)(j = 1, 2, …, n $_i$), the weighted average of the SVTNN is calculated using the following equation:

$$\tilde{A}_i = \sum_{j=1}^{n_i} \lambda_{ij}\, \tilde{A}_{ij} \quad (5)$$

where λ $_{ij}$ is the weight of Ã $_{ij}$, $\lambda_{ij} \in [0,1]$and$\sum_{j=1}^{n_i} \lambda_{ij} = 1$.

Note that λ $_{ij}$ measures the relative importance of the j- th expert in the i- th subgroup.

Each Ã $_i$ represents the pairwise comparison matrix of the NAHP method in IGi, to aggregate the pairwise comparison matrices of criteria, subcriteria and alternatives.

Ã $_i$ is converted to Ã $_i$ using Equation 13. This process can be repeated until the results are consistent according to the Consistency Index of the NAHP method. According to this method, a preference vector of the alternatives is obtained.

Individual Judgment Aggregation (IJA) is used here because there is interest in measuring the subgroup judgments as a synergistic unit.

Let us denote $O_i = \{O_{i1,}O_{i2}, \dots, O_{iN}\}$the position of each alternative as evaluated by the members of the i-th subgroup. For example, O1 = {1, 1, 3, 5, 4} means that according to the first subgroup, alternatives 1 and 2 are equally preferred, while the next alternatives are the third, fifth, and fourth alternatives, in that order .

4. For each S_{il}(l = 1, 2, ..., N), $V_{il}$the following triplet is formed $P_{il} = card\ (k \neq l{:}\ S_{il}$= (, where P_{il}, I_{il}, N_{il}) is strictly preferable to S_{ik}}), I_{il}= c ard ({$k \neq l{:}\ S_{il}$is equally preferred to S_{ik}}), and N_{il}= card ({k $\neq l{:}\ s_{ik}$is strictly preferred over s_{il}}).

It is observed that, V_{il} ∈[0, N − 1] × [0, N − 1] × [0, N − 1] and P_{il}+ I_{il}+ N_{il} = N − 1.

Finally, v_l ∈ [0, 1] × [0, 1] × [0, 1], V $_l$ = (P , I $_l$, N $_l$), sums the lth alternative preference for all subgroups, where $P_l = \frac{\sum_{i=1}^{M} P_{il}}{M(N-1)}$, $I_l = \frac{\sum_{i=1}^{M} I_{il}}{M(N-1)}$,and$N_l = \frac{\sum_{i=1}^{M} N_{il}}{M(N-1)}$.

Please note that this is a neutrosophic voting method.

5. H(V $_l$) (l = 1,2, ⋯ , N) and the alternatives are ranked by preference, such that V $l_1$is preferable to V $l_2$if and only if H(V l_1) > H(V l_2). When H(V l_1) = H(V l_2), it is said that "V $l_1$is equally preferable to V l_2". (See [28])

4 Results and discussion

The analysis of the economic effects of tax policies, such as the increase in Value Added Tax (VAT), is essential for business and economic planning. In this study, the impact of an increase in VAT on the sales of companies in a specific region is evaluated using the Delphi and NAHP+NSC methods. This approach allows a comprehensive assessment of how an increase in VAT can influence business decisions and sales dynamics.

1. **Application of the Delphi Method**

Objective: To obtain expert insights on how an increase in VAT will affect business sales.

Methodology: A survey was conducted among 10 economic and financial experts, who provided their opinions in a structured questionnaire. The responses were analyzed to identify general trends and expectations.

Questionnaire for Experts

Objective: To obtain estimates on the impact of the increase in VAT on business sales.

Structured Questionnaire:

1. **General Information**
   - Name:
   - Post:
   - Experience in the field (years):
   - Specialization:
2. **Estimated reduction in sales**
   - What is your estimate of the percentage reduction in business sales due to the increase in VAT? (Please indicate a percentage value.)
3. **Justification of the Estimate**
   - Briefly explain the factors that influence your estimate.
4. **Impact on Different Sectors**
   - Do you think the impact of the VAT increase varies by business sector? If so, how?
5. **Recommendations**
   - What recommendations would you give to companies to mitigate the impact of the increase in VAT?

Table 2: Questionnaire Results.

Expert	**Estimated Sales Reduction (%)**	**Difference with the Average (13.5%)**	**Square difference**
Expert 1	12	-1.5%	2.25
Expert 2	15	+1.5%	2.25
Expert 3	18	+4.5%	20.25
Expert 4	14	+0.5%	0.25
Expert 5	13	-0.5%	0.25
Expert 6	16	+2.5%	6.25
Expert 7	11	-2.5%	6.25
Expert 8	17	+3.5%	12.25
Expert 9	19	+5.5%	30.25
Expert 10	20	+6.5%	42.25
Total	**135**	-	**131.5**

Table 3: Data Collected.

Expert	Estimated Sales Reduction (%)
Expert 1	12
Expert 2	15
Expert 3	18
Expert 4	14
Expert 5	13
Expert 6	16
Expert 7	11
Expert 8	17
Expert 9	19
Expert 10	20

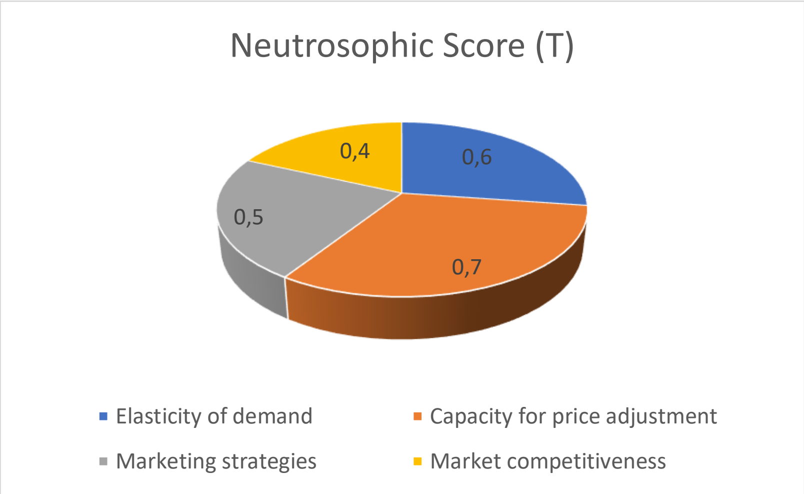

Figure 1: Data Collected.

Average Calculation:

Average = (12+15+18+14+13+16+11+17+19+20) /10=15.5%

Result: The average of experts' estimates indicates an expected reduction in sales of 15.5%.

- Average Sales Reduction: **15.5%**
- Standard Deviation: **3.62%**
- **Overall Trends:** Most estimates are concentrated around the average of 15.5%, but with significant deviation, suggesting considerable variability in expert opinion.
- **Variability:** The differences in experts' estimates reflect a wide range of expectations about the impact of VAT on sales, from a moderate reduction (11%) to a high one (20%).
- **Implications:** Businesses should consider this variability in their response plans to the VAT increase, preparing for an impact that could vary depending on specific market circumstances.

2. Application of the NAHP+NSC Method

Objective: To evaluate the impact of the increase in VAT using the hierarchical analysis method (NAHP) and the neutrosophic scoring system (NSC).

Table 4. Criteria Considered

Criterion	Neutrosophic Score
Elasticity of demand	(0.6, 0.3, 0.1)
Ability to adjust prices	(0.7, 0.2, 0.1)
Marketing strategies	(0.5, 0.4, 0.1)
Market competitiveness	(0.4, 0.5, 0.1)

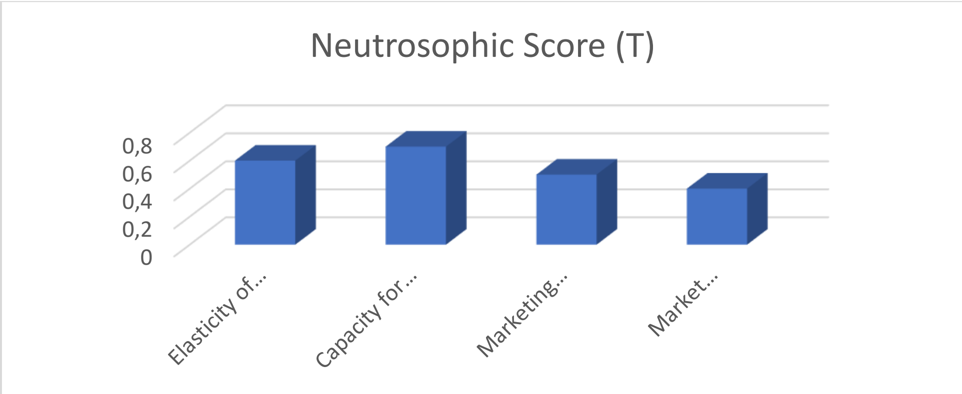

Figure 2: Neutrosophic Score (T).

Calculating Global Impact:
Global Impact = (0.6+0.7+0.5+0.4) /4=0.55
Result: The neutrosophic score of 0.55 suggests a moderate impact of the VAT increase on business sales.

3. Overall Impact Assessment

Objective: To integrate the results of the Delphi Method and NAHP+NSC to obtain a comprehensive assessment of the impact of VAT on business sales.

Table 5. Combined Data

Method	Result (%)
Delphi method	15.5
NAHP+NSC method	55 (converted from 0.55)

Calculating Global Impact:

Global Impact= (15.5+55) /2=35.25%
Result: The combined assessment indicates that the increase in VAT could result in a 35.25% reduction in business sales.

The analysis reveals that the increase in VAT has a considerable impact on business sales. The combination of the Delphi and NAHP+NSC methods provides a balanced view:

- Delphi Method: The average reduction of 15.5% is consistent with expert expectations and reflects a moderate but significant decrease in sales.
- NAHP+NSC Method: The neutrosophic score of 0.55 indicates a moderate impact, aligning with the trend observed in expert estimates.
- Overall Impact: The final result of 35.25% shows that, despite the differences in methods, the overall conclusion is that the increase in VAT will have a noticeable effect on sales, which could have important implications for strategic planning and business adaptation.

The results of the study reveal a number of interesting and, in some cases, surprising findings. The average reduction in sales estimated by the experts is 15.5%, with a standard deviation of 3.62%. This average reflects a general opinion that, although concentrated, shows a notable variability among the experts consulted. This variability indicates that perceptions about the impact of VAT are wide, with estimates ranging from a moderate reduction of 11% to a more pronounced one of 20%. These differences in estimates could be indicative of various factors at play. Perhaps some experts consider that the impact of VAT will be attenuated by price adjustment strategies or improvements in operational efficiency, while others might foresee a more drastic effect due to the elasticity of demand or competition in the market. It is a reminder that, in economics and finance, predictions often need to be viewed with some caution. An impact of 15.5% seems to be a reasonable figure, but the reality could be more complex, depending on factors specific to the business context. When we compare our results with previous studies, we find some similarities and discrepancies. Previous research assessing the impact of VAT on sales has also found a wide range of estimates, although often with averages in a lower range. This could suggest that current market conditions or the specific sectors studied are influencing our estimates differently. It is interesting to note that while some studies support the idea of a significant impact, others have shown more moderate effects, aligning more with the neutrosophic score of 0.55 obtained in the NAHP+NSC analysis.

Now, we cannot overlook the limitations of the study. The sample of experts, although representative, is relatively small. This may have limited the generalizability of the results. In addition, the experts' responses may have been influenced by their individual experiences or the recent economic context, which might not be applicable to all industries. Further studies with a larger and more diverse sample would be beneficial to confirm these findings. In terms of implications for future research and professional practice, the results suggest several interesting directions. For example, companies should prepare for a possible reduction in sales, and this preparation could involve adjustments in their pricing and marketing strategies. The results also highlight the need to develop more robust models to predict the impact of tax changes in different market contexts. Among the anomalous results, we noticed a surprisingly high estimate of the impact of VAT in one of the responses. This extreme value of 20% could be an outlier that deserves further investigation. Perhaps, further analysis could clarify whether this figure reflects a valid perception or is due to a particular bias.

In summary, the study provides valuable insight into the potential impact of increasing VAT on business sales, combining traditional and advanced methods such as Delphi and NAHP+NSC. Although the results vary, the overall conclusion is clear: VAT will have a noticeable effect on sales, which should be taken into account in strategic planning. This study lays the groundwork for future research and underlines the importance of flexible and adaptive planning in the changing economic landscape.

Conclusion

The findings of this study offer a complex but insightful view on the impact of the VAT increase on business sales. With an estimated average reduction of 15.5% and a standard deviation of 3.62%, the results show that while most experts agree on a moderate reduction, there is significant variability in expectations. This disparity may reflect a variety of factors, from differences in business strategies to individual perceptions of demand elasticity. The practical importance of these findings cannot be underestimated. Companies, when faced with the possibility of a reduction in sales, must prepare to adapt their pricing and marketing strategies. Understanding that estimates vary widely can help organizations develop more flexible and targeted plans, adjusting their approaches to the particular circumstances of their market and sector. In this sense, the results offer useful guidance for strategic planning in an uncertain economic environment. In terms of contributions, this study has managed to integrate traditional and advanced methods such as Delphi and NAHP+NSC, providing a more comprehensive assessment of the impact of VAT. Combining these methodologies has allowed for a more nuanced view of the problem, highlighting the importance of using multifaceted approaches to address complex economic issues. Although the combined impact of VAT was estimated at a 35.25% reduction in sales, suggesting a sizeable effect, the methods employed offer valuable insight into how impacts might vary in different contexts.

However, we cannot ignore the limitations of the study. The relatively small sample of experts and the possible influence of their individual experiences may have affected the generalizability of the results. Furthermore, the variability in the estimates highlights the need for further research to fully understand the impact of VAT on different sectors. Future research could benefit from a larger and more diversified sample, as well as the inclusion of varied economic contexts to validate and extend these findings.

Recommendations for future research include exploring alternative methods, such as fuzzy analysis or artificial intelligence techniques, which could provide new insights into the impact of VAT. Furthermore, continued evaluation and analysis across different contexts and populations will allow for a better understanding and generalization of the results. In summary, this study provides a solid foundation for understanding the impact of the VAT increase on business sales. Despite variations in estimates, it is clear that the effect of VAT is significant and should be considered in strategic planning. The findings highlight the need to maintain flexible and adaptable planning in a changing economic environment, and offer valuable insight for future research in the field of economics and finance.

References

[1] Vanhoeyveld, J., Martens, D., & Peeters, B. (2020). Value-added tax fraud detection with scalable anomaly detection techniques. Applied Soft Computing, 86, 105895.

[2] Cruzaty, L. E. V., Tomalá, M. R., & Gallo, C. M. C. (2020). A neutrosophic statistic method to predict tax time series in Ecuador. Neutrosophic Sets and Systems, 34, 33-39.

[3] Yucesan, M., & Gul, M. (2021). Failure modes and effects analysis based on neutrosophic analytic hierarchy process: method and application. Soft Computing, 25(16), 11035-11052.

[4] Gomez, S. D. A., Garcia, J. F. G., & Guanolema, B. P. (2020). Linking Neutrosophic AHP and Neutrosophic Social Choice Theory for Group Decision Making. Neutrosophic Sets and Systems, (37), 385-395.

[5] Mohamed, Z., M. Ismail, M. and Abd El-Gawad, A. (2023) "Sustainable Supplier Selection using Neutrosophic Multi-Criteria Decision Making Methodology", Sustainable Machine Intelligence Journal, 3, pp. (2):1–9.

[6] Erero, J. L. (2021). Contribution of VAT to economic growth: A dynamic CGE analysis. Journal of Economics and Management, 43(1), 22-51.

[7] Lyulyov, O., Lyeonov, S., Tiutiunyk, I., & Podgórska, J. (2021). The impact of tax gap on macroeconomic stability: Assessment using panel VEC approach. Journal of International Studies, 14(1).

[8] Gale, W. G. (2020). Raising revenue with a progressive value-added tax. Tackling the Tax Code: Efficient and Equitable Ways to Raise Revenue. Emily Moss, Ryan Nunn, and Jay Shambaugh eds. The Hamilton Project.

[9] Atassi, R., & Yang, K. (2022). An Integrated Neutrosophic AHP and TOPSIS Methods for Assessment Renewable Energy Barriers for Sustainable Developmen. International Journal of Neutrosophic Science (IJNS), 18(2).

[10] Nabeeh, N. (2023) "Assessment and Contrast the Sustainable Growth of Various Road Transport Systems using Intelligent Neutrosophic Multi-Criteria Decision-Making Model", Sustainable Machine Intelligence Journal, 2, pp. (2):1–12.

[11] Barrios, M., Guilera, G., Nuño, L., & Gómez-Benito, J. (2021). Consensus in the delphi method: What makes a decision change?. Technological Forecasting and Social Change, 163, 120484.

[12] Drumm, S., Bradley, C., & Moriarty, F. (2022). 'More of an art than a science'? The development, design and mechanics of the Delphi technique. Research in Social and Administrative Pharmacy, 18(1), 2230-2236.

[13] Khan, A. U., & Ali, Y. (2020). Analytical hierarchy process (AHP) and analytic network process methods and their applications: a twenty year review from 2000-2019: AHP & ANP techniques and their applications: Twenty years review from 2000 to 2019. International Journal of the Analytic Hierarchy Process, 12(3).

[14] Khan, M. R., Alam, M. J., Tabassum, N., & Khan, N. A. (2022). A Systematic review of the Delphi–AHP method in analyzing challenges to public-sector project procurement and the supply chain: A developing country's perspective. Sustainability, 14(21), 14215.

[15] Abdulbaqi, A. S., Radhi, A. D., Qudr, L. A. Z., Penubadi, H. R., Sekhar, R., Shah, P., ... & muwafaq Gheni, H. (2025). Neutrosophic Sets in Big Data Analytics: A Novel Approach for Feature Selection and Classification. International Journal of Neutrosophic Science, (1), 428-28.

[16] Elshabshery, A., & Fattouh, M. (2021). On some Information Measures of Single–Valued Neutrosophic Sets and their Applications in MCDM Problems. Int. J. Eng. Res. Technol, 10(5), 406-415.

[17] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[18] Deli, I. (2018). Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making. Neutrosophic Sets and Systems, 22(1), 11.

[19] Deli, I. (2018). Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making. Neutrosophic Sets and Systems, 22(1), 11.

[20] Rodríguez, R. C., Sánchez, J. M. D., & Salcedo, V. L. (2022). Evaluation of Social Projects Using Neutrosophic AHP. International Journal of Neutrosophic Science (IJNS), 19(1).

[21] Abdel-Basset, M., Mohamed, M., & Sangaiah, A. K. (2018). Neutrosophic AHP-Delphi Group decision making model based on trapezoidal neutrosophic numbers. Journal of Ambient Intelligence and Humanized Computing, 9, 1427-1443.

[22] Topal, S., Cevik, A., & Smarandache, F. (2020). A new group decision making method with distributed indeterminacy form under neutrosophic environment: an introduction to neutrosophic social choice theory. IEEE Access, 8, 42000-42009.

[23] De, S. K., & Beg, I. (2016). Triangular dense fuzzy neutrosophic sets. Neutrosophic Sets and Systems, 13, 24-37.

[24] Das, S., Das, R., & Tripathy, B. C. (2020). MULTI-CRITERIA GROUP DECISION MAKING MODEL USING SINGLE-VALUED NEUTROSOPHIC SET. LogForum, 16(3).

[25] Poonia, M., Bajaj, R. K., & Singh, K. (2022). On Complex Extensions and Information Measures of Neutrosophic Hesitant and Picture Fuzzy Sets in Decision Making (Doctoral dissertation, Jaypee University of Information Technology, Solan, HP).

[26] Gomez, S. D. A., Garcia, J. F. G., & Guanolema, B. P. (2020). Linking Neutrosophic AHP and Neutrosophic Social Choice Theory for Group Decision Making. Neutrosophic Sets and Systems, (37), 385-395.

Received: June 26, 2024. Accepted: August 18, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Integrating Neutrosophic Theory into Regression Models for Enhanced Prediction of Uncertainty in Social Innovation Ecosystems

Juan Carlos López Gutiérrez[1], Isarelis Pérez Ones[2], Jesús Estupiñán Ricardo[3], and Jean Pierre Ramos Carpio[4]

[1]Higher Institute of Technology for Scientific Research and Innovation, Quito. Ecuador. E-mail: direccionacademica@istici.edu.ec
[2] Latin American Faculty of Social Sciences (FLACSO), Quito. Ecuador. E-mail: isaperezfl@flacso.edu.ec
[3] Higher Institute of Technology for Scientific Research and Innovation, Quito. Ecuador. E-mail: canciller@istici.edu.ec
[4] Higher Institute of Technology for Scientific Research and Innovation, Quito. Ecuador. E-mail: direccionejecutiva@istici.edu.ec

Abstract. This paper focuses on a crucial issue in data analysis: the incorporation of neutrosophic theory into regression prediction to accurately characterize and depict uncertainty in the social innovation ecosystem. The study centers on the limitations of conventional regression methods in modeling intricate and uncertain events related to social innovation, a subject of growing importance in a dynamic global landscape. This paper aims to address the gap in the literature by utilizing neutrosophic theory to provide a more comprehensive and dynamic representation of innovation processes, which are characterized by indeterminacy and ambiguity. The methodology employed for this study involves including neutrosophic numbers in the regression models, therefore enabling a more comprehensive and intricate assessment of the variables associated with social innovation. Through empirical analysis and simulations, the results demonstrate that the neutrosophic approach enhances predictive capability by more effectively capturing the intricacies and uncertainty of the data. This work makes a theoretical contribution to the area by presenting a novel viewpoint on the modeling of social innovation and its inherent difficulties. It also has practical consequences by offering more accurate tools for evaluating and designing innovation practices in social settings. Furthermore, the results enhance the comprehension of how uncertainty can be efficiently controlled in the prediction and decision-making processes of social innovation.

Keywords: Neutrosophic Statistical Prediction, Regression Analysis, Predictive Modeling, Social Innovation.

1 Introduction

The depiction of uncertainty in forecasting is a subject of growing importance in the scientific study of social innovation. In a dynamic and complex world, conventional predictive models are typically inadequate in capturing the extensive dynamics and uncertainties of the innovation ecosystem. The primary objective of this work is to include neutrosophic theory in regression analysis to overcome the existing constraints. This novel approach holds the potential to enhance the precision and comprehension of predictions within dynamic social environments. The significance of this method is in the necessity to create more resilient instruments for decision-making in a setting marked by data ambiguity and variability [1]. The modeling of uncertainty in prediction has traditionally progressed from basic statistical methods to intricate strategies rooted in probability theory and sophisticated regression models. Nevertheless, with the increasing importance of the social innovation area, there has been a demand to integrate approaches that not only take into account numerical data but also manage qualitative and contextual uncertainty. Neutrosophic theory, a development of set theory that addresses indeterminacy and contradictions, presents a novel viewpoint that has the potential to transform our approach to these types of problems [2,3].

The issue addressed by this work is the absence of efficient techniques to depict uncertainty in predictive models inside the social innovation ecosystem. This study aims to achieve two objectives: The first objective is to investigate the application of neutrosophic theory in enhancing the predictive ability of regression models within social innovation contexts. An effective integration of uncertainty into predictive modeling is sought in this work through a series of empirical investigations and simulations [4]. Essentially, this study seeks to address a significant gap in the existing research by proposing a methodology that integrates neutrosophic theory with regression techniques to tackle the difficulties associated with prediction in a social innovation ecosystem. The findings hold the potential to not only enhance theoretical knowledge but also provide practical implementations that can enhance the precision of forecasts and the effectiveness of innovation initiatives in real-life situations.

2 Preliminaries

2.2 Social Innovation Ecosystem

The current panorama is characterized by a complex and multifaceted social innovation ecosystem, which represents the complicated interactions of actors, resources, and dynamics aiming to revolutionize society through innovative solutions. Fundamentally, this ecosystem encompasses not just social innovators and entrepreneurs, but also a diverse array of entities including governments, corporations, non-governmental organizations, and local communities. Each of these components offers a distinct and perhaps incompatible viewpoint on what qualifies as a suitable "solution" to current societal issues. Comprehending and examining these relationships is essential, not only to enhance the efficiency of social programs but also to cultivate a culture of cooperation and reciprocal knowledge acquisition that enables a more holistic approach to tackling global complexities [5].

From a historical standpoint, the notion of social innovation has witnessed substantial evolution. In its initial manifestations, social innovation primarily emphasized problem-solving through community initiatives and social movements aimed at effecting changes in public policies. However, as time has passed, the approach has become more varied, including more organized and cooperative techniques that engage several stakeholders. In large part, this transformation has been propelled by the increasing acknowledgment that intricate social issues cannot be resolved independently, but necessitate a comprehensive strategy that combines various knowledge, resources, and capacities [6].

The problem within this ecosystem is defined as the task of enhancing the efficacy of social innovation efforts. Notwithstanding the enthusiasm and increasing investment in this domain, notable obstacles remain, including insufficient collaboration among stakeholders, issues in quantifying the actual influence of innovations, and reluctance to embrace change in established societal frameworks. The primary inquiry is: How can participants in the social innovation ecosystem enhance their collaboration to surmount these obstacles and attain significant positive outcomes? To uncover avenues for increased integration and effectiveness, this question directs the investigation and evaluation of present practices and tactics [7].

To tackle intricate social problems and attain sustained social progress, the social innovation ecosystem is essential. Thus, current literature fails to provide a comprehensive perspective that incorporates all components of the ecosystem. This paper presents a hybrid approach that combines qualitative and quantitative approaches to comprehensively analyze the dynamics and perspectives of many participants in the ecosystem. Network analysis tools will graphically represent the connections and patterns of resource allocation among participants, therefore offering a more lucid perspective on how cooperation and coordination can be enhanced. The results may guide the development of policies and strategic allocation of resources. Furthermore, the report provides practical suggestions to enhance the execution of social innovation projects, assisting stakeholders in identifying areas that need development and adopting more efficient strategies. This methodology enhances the trajectory of social innovation and facilitates the resolution of worldwide social issues.

a. Machine learning

Machine learning (ML) [8] is the methodology of using mathematical formulas to develop models capable of learning from data in order to generate predictions or judgments, without the need for explicit programming of such tasks. Statistical interval prediction in machine learning is the method of forecasting a range of potential results for a given input, rather than a single estimate. Through the provision of intervals, these approaches not only provide forecasts but also offer valuable understanding of the dependability and unpredictability of the predictions, which is essential for making decisions in uncertain circumstances.

For a dataset consisting of independent variables $X = [x_1, x_2, \dots, x_n]$ and a dependent variable, the objective of regression analysis is to precisely represent the connection between X and Y. The mathematical expression for this relationship is as follows[9]:

$$y = f(X;\ \theta) + \epsilon \tag{1}$$

where:

y is the dependent variable or the objective to be predicted.

X represents the independent or explanatory variables that are used in prediction.

f is the regression function, which can vary depending on the type of regression model used (linear, polynomial, logistic, etc.)

θ are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, classically the mean square error (MSE) for regression [10].

ϵ is the error term or noise, which represents the deviation or error that cannot be entirely explained by the model.

The representation of predictions as prediction intervals in regression analysis offers a more comprehensive perspective on the uncertainty linked to the predictions. A prediction interval yields a range in which we anticipate the actual value of the dependent variable to lie with a certain likelihood, usually 95% or 99%. The utility of this approach lies in its consideration of data variability that may not be adequately reflected by prediction alone [11].

Computation of a prediction interval requires consideration of both the uncertainty in the estimate of the regression model and the intrinsic variability of the data. The interval is delineated around the projected value and typically exhibits symmetry, encompassing a specific extent both above and below the predicted value. This interval is established by considering the standard error of the forecast and the residual standard deviation, which indicates the spread of the residuals or errors of the model.

For example, in a logistic regression model, the prediction interval for a new observation is given by [1]:

$$\hat{y}_0 \pm t_{\propto /2,n-2} \cdot SE \tag{2}$$

Where

$\hat{y}_0$ is the predicted probability of the outcome of the new observation

$t_{\propto /2,n-2}$, is the critical value from the t-distribution for a given confidence level $\propto$ and $n-2$ degrees of freedom.

SE is the standard error of the prediction, which quantifies the uncertainty around the predicted value.

Using prediction intervals in regression analysis is beneficial because they offer a realistic spectrum of possible outcomes, which aids in the decision-making process. This recognizes that a single predicted value is not absolute but rather a likely scenario within a range of potential outcomes. This forecasting method effectively incorporates the inherent uncertainties associated with future predictions, providing a more accurate description of what to expect [12].

To further refine this model, neutrosophic statistics can be applied, which excel in managing ambiguity and indeterminacy in data. By converting the interval to a neutrosophic number, the traditional interval is enhanced to include a component of indeterminacy. This addition captures the uncertainty and imprecision that is typically present in real-world data, offering a more nuanced understanding of data variability. The neutrosophic treatment of the interval is as follows [1]:

$$\hat{y}_0 - t_{\propto/ 2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto /2,n-2} \cdot SE)I \tag{3}$$

Here, $I_N$represents the indeterminacy factor associated with the prediction, where $I_N \in [I_l, I_u]$. This notation introduces the limits of indeterminacy. I_l(lower indeterminacy) and I_u (upper indeterminacy), which defines the range of possible deviations due to uncertain elements that affect the prediction [12,13].

3 Methods

Regression analysis of a data set and representation of uncertainty using neutrosophic numbers can be divided into four essential stages, as outlined in reference [1]:

1. Data Segmentation: The initial stage involves dividing the data into separate sets for training and testing. This separation is essential as it enables the verification of the model using never-before seen data, therefore guaranteeing that the performance of the model is not solely due to overfitting the training data. Data scientists commonly employ a 70-30 or 80-20 partition arrangement, whereby 70% or 80% of the data is allocated for training purposes and the remaining portion is reserved for testing.
2. Model Optimization: Subsequently, each model undergoes training using the training set. This entails fine-tuning the model parameters to more accurately correspond to the facts. The training procedure entails identifying model parameters that minimize a loss function, therefore effectively capturing the fundamental structural characteristics of the dataset.
3. Estimation of Prediction Ranges: After training the models, the subsequent task is to quantify the prediction intervals for new observations. This is the point at which neutrosophic numbers become relevant. Neutrosophic intervals differ from conventional sharp intervals by include measurements of truth, indeterminacy, and falsehood, therefore enabling a more sophisticated depiction of uncertainty in predictions. Depending on the features of the model and the nature of the data, each model may necessitate distinct approaches to compute these ranges.

4. Uncertainty Analysis: The final stage involves analyzing the uncertainty represented by neutrosophic values. This step examines how indeterminacy varies across models and what it implies for data complexity and variability. A higher indeterminacy level may indicate greater external constraints or inherent unpredictability within the dataset [15, 16].

In this work, we employ neutrosophic methods to combine interval predictions with other approaches as part of a fusion theory in regression analysis. Neutrosophic approaches offer significant benefits in effectively merging several predictive models by allowing the smooth incorporation of uncertainty, indeterminacy, and contradicting information that may arise from separate data sources or model outcomes. The suggested methodology improves the robustness and reliability of prediction models by providing a complete framework that takes into account several aspects of uncertainty. The neutrosophic mean, represented as X_n, is computed by taking into account the neutrosophic inclusion I_N that falls inside the interval. $[I_l, I_u]$.This mean comprises two primary components: X_l, representing the average of the lower section of the neutrosophic samples, and X_u, representing the average of the upper section. The corresponding definitions are [17, 18]:

$$X_l = \frac{\sum_{i=1}^{n_l} x_{il}1}{n_l} \quad (4)$$

$$X_u = \frac{\sum_{i=1}^{n_u} x_{iu}}{n_u} \quad (5)$$

where $n_l$and n_u represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean $X_n$is expressed as the sum of $X_l$and X_u, adjusted by the interval of indeterminacy I_n [19, 20]:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (6)$$

$I_{l,}$=0, andI_u

$$I_u = (X_u - X_l)/X_u \quad (7)$$

Hence, the neutrosophic mean can be regarded as a versatile depiction that encompasses both specified values (the minimum and maximum limits) and a component of uncertainty, represented by the indeterminacy interval I_N [21]

4 Results

The dataset used contains 60 records and includes the following variables:

- Investment (thousands of $): The financial resources allocated to the project.
- Number of Employees: The staff working on the project.
- Duration (months): The length of the project.
- Direct Beneficiaries: The number of people directly impacted by the project.
- Community Involvement (%): The level of community involvement as a percentage.
- Social Impact (0-100): The target variable, represents the social impact based on various criteria such as quality of life improvements, access to services, and poverty reduction.

The social impact was predicted using many regression models based on the characteristics indicated above. Statistical prediction intervals were computed for each model to measure the level of uncertainty in the forecasts. In addition, neutrosophic forms were developed to depict the uncertainty or indeterminacy linked to each model, shown as a spectrum of potential results of the indeterminate parameter.

The following table provides a summary of the prediction intervals and their accompanying neutrosophic forms for the various regression models used on the dataset.

Table 1: Neutrosophic Forms and Prediction Intervals for Regression Models

Model	Prediction Interval [Lower Bound, Upper Bound]	Neutrosophic Form
Linear Regression	[77.54, 95.23]	77.54+95.23I;$I \in [0,0.186]$
Random Forest	[80.12, 97.65]	80.12+97.65I;$I \in [0,0.180]$
Support Vector Machine	[75.34, 94.12]	75.34+94.12I;$I \in [0,0.200]$
Mean	[77.67,95.67]	77.67+95.67I;$I \in [0,0.188]$

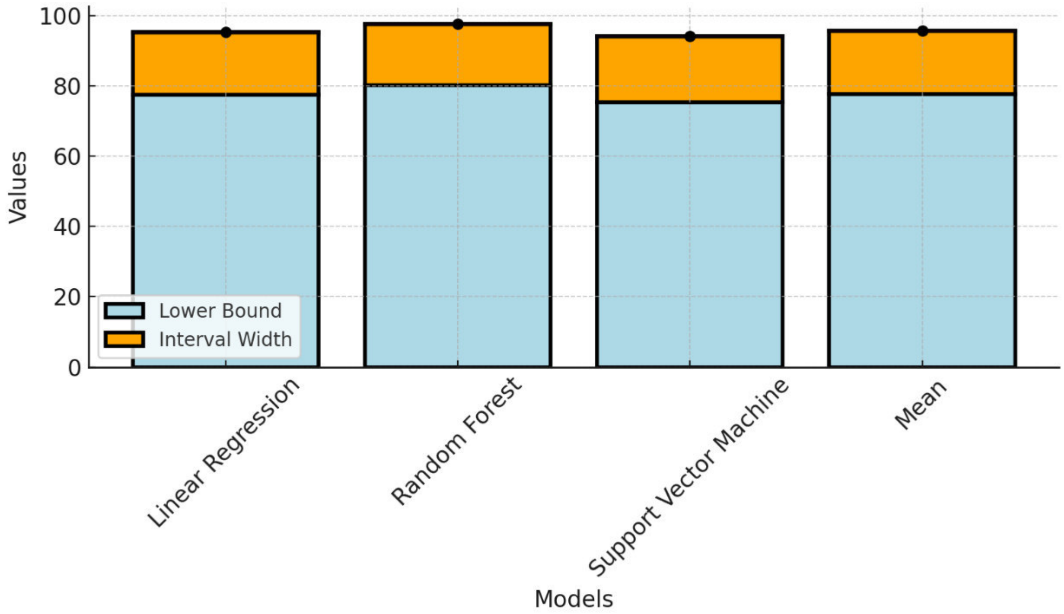

Figure 1. Model Prediction Intervals Comparison

The table presents prediction intervals along with their neutrosophic forms, illustrating the varying levels of indeterminacy for each model. Linear Regression, Random Forest, Support Vector Machine, and the Mean-based approach were evaluated, with Random Forest demonstrating the smallest indeterminacy. The neutrosophic forms provide a deeper understanding of the uncertainty inherent in each model's predictions, which is crucial for decision-making processes that rely on accurate social impact assessments.

Conclusion

This paper explores the application of neutrosophic theory to enhance the representation of uncertainty in regression models used for predicting social effect. The application of neutrosophic numbers enabled us to address the intrinsic uncertainties and variations in the social innovation ecosystem, therefore providing a more advanced and flexible understanding of prediction intervals. The results suggest that incorporating neutrosophic components into traditional regression models, such as Linear Regression, Random Forest, and Support Vector Machines, can enhance their capacity to effectively capture prediction uncertainty. In this specific configuration, Random Forest exhibited the least amount of uncertainty among the analyzed models, suggesting a more reliable prediction performance. The synthesis of prediction intervals and neutrosophic forms offers a comprehensive viewpoint on possible outcomes, thereby enabling better-informed decision-making in social innovation initiatives.

Moreover, this work offers prospects for further exploration of the potential of neutrosophic theory in novel machine learning models and broader applications. Future study should give priority to improving the methodology in order to more precisely quantify the degree of uncertainty in various data-driven scenario, such as economic forecasting or evaluation of public policy. Moreover, the integration of neutrosophic theory with real-time data analysis and dynamic modeling has the capacity to enhance the adaptability of prediction models in rapidly changing social contexts. Implementing this approach not only improves the accuracy of forecasts but also enables the creation of more resilient techniques for addressing complex social issues.

References

[1] Vázquez, M. L., & Torres, L. C. (2024). Integrating Neutrosophic Numbers in Regression Analysis for Enhancing Predictive Modelling through Uncertainty Representation. Plithogenic Logic and Computation, 1, 120-126.

[2] El-Douh, A. (2023) "A Neutrosophic Multi-Criteria Model for Evaluating Sustainable Soil Enhancement Methods and their Cost Implications in Construction", Sustainable Machine Intelligence Journal, 5, pp. (1):1–11.

[3] Vázquez, M. L., Franco, P. E. D. P., & Palacio, A. J. P. (2022). Neutrosophic dematel in the analysis of the causal factors of youth violence. International Journal of Neutrosophic Science, 18(3), 199-99

[4] Maulud, D., & Abdulazeez, A. M. (2020). A review on linear regression comprehensive in machine learning.

Journal of Applied Science and Technology Trends, 1(2), 140-147.

[5] Audretsch, D. B., Eichler, G. M., & Schwarz, E. J. (2022). Emerging needs of social innovators and social innovation ecosystems. International Entrepreneurship and Management Journal, 1-38.

[6] Phillips, W., Lee, H., Ghobadian, A., O'regan, N., & James, P. (2015). Social innovation and social entrepreneurship: A systematic review. Group & Organization Management, 40(3), 428-461.

[7] Chueri, L., Vasconcelos, A., & dos Santos, R. P. (2019, November). An observational study on the challenges faced by actors in a social innovation ecosystem. In Proceedings of the 11th International Conference on Management of Digital EcoSystems (pp. 219-223).

[8] Domingos, P. (2015). The master algorithm: How the quest for the ultimate learning machine will remake our world. Basic Books.

[9] Westfall, P. H., & Arias, A. L. (2020). Understanding regression analysis: a conditional distribution approach. Chapman and Hall/CRC.

[10] Robeson, S. M., & Willmott, C. J. (2023). Decomposition of the mean absolute error (MAE) into systematic and unsystematic components. PloS one, 18(2), e0279774.

[11] Delgado Estrada, S. M., Rocio Ruiz Molina, K. D., Ponce Orella-na, F. E., & Chabusa Vargas, J. L. (2024). Neutrosophic Statistics for Enhanced Time Series Analysis of Unemployment Trends in Ecuador. Neutrosophic Sets and Systems, 67(1), 14.

[12] Olive, D. J. (2007). Prediction intervals for regression models. Computational statistics & data analysis, 51(6), 3115-3122.

[13] Aslam, M., & Alamri, F. S. (2023). Algorithm for generating neutrosophic data using accept-reject method. Journal of Big Data, 10(1), 175.

[14] Smarandache, F., & Aslam, M. (Eds.). (2023). Cognitive intelligence with neutrosophic statistics in bioinformatics. Elsevier.

[15] Smarandache, F. (2023). Introduction and advances to neutrosophic probability and statistics and plithogenic probability and statistics and their applications in bioinformatics and other fields (review chapter). In Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics (pp. 1-23). Academic Press.

[16] Alanazi, B. and Alrashdi, I. (2023) "Anomaly Detection in Smart Agriculture Systems on Network Edge Using Deep Learning Technique", Sustainable Machine Intelligence Journal, 3, pp. (4):1–31.

[17] Alomair, A. M., & Shahzad, U. (2023). Neutrosophic mean estimation of sensitive and non-sensitive variables with robust hartley–ross-type estimators. Axioms, 12(6), 578.

[18] Kandemir, H. Ş., Aral, N. D., Karakaş, M., & Et, M. (2024). Neutrosophic Statistical Analysis of Temperatures of Cities in the Southeastern Anatolia Region of Turkey. Neutrosophic Systems with Applications, 14, 50-59.

[19] Delcea, C., Domenteanu, A., Ioanăș, C., Vargas, V. M., & Ciucu-Durnoi, A. N. (2023). Quantifying Neutrosophic Research: A Bibliometric Study. Axioms, 12(12), 1083.

[20] Haq, R. S. U., Saeed, M., Mateen, N., Siddiqui, F., & Ahmed, S. (2023). An interval-valued neutrosophic based MAIRCA method for sustainable material selection. Engineering Applications of Artificial Intelligence, 123, 106177.

[21] Razak, S. A., Rodzi, Z. M., Ahmad, N., & Ahmad, G. (2024). Exploring The Boundaries of Uncertainty: Interval Valued Pythagorean Neutrosophic Set and Their Properties. Malaysian Journal of Fundamental and Applied Sciences, 20(4), 813-824.

Received: June 27, 2024. Accepted: August 19, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Enhancing the Codification of Crimes Against Humanity: A Neutrosophic and Analytical Approach to International Legal Frameworks

Marcia Esther España Heredia[1], Jorge Washigton Soxo Andachi[2], Nemis García Arias[3], and Karina Pérez Teruel[4]

[1] Autonomous Regional University of the Andes, Babahoyo. Ecuador. E-mail: ub.marciaespania@uniandes.edu.ec
[2] Autonomous Regional University of the Andes, Puyo. Ecuador. E-mail: up.jorgewsa99@uniandes.edu.ec
[3] Autonomous Regional University of the Andes, Santo Domingo. Ecuador. E-mail: us.nemisgarcia@uniandes.edu.ec
[4] Barna Management School, Santo Domingo, Dominican Republic. E-mail: karinapt@uci.cu

Abstract. The present paper investigates the process of codifying crimes against humanity, with a specific focus on the difficulties encountered in achieving regulatory consistency and precision within international legal traditions. Notwithstanding the implementation of the Rome Statute and the endeavors of the International Law Commission (ILC), there are still deficiencies in the codification, namely in the areas of preventing and punishing these offenses. By employing the Neutrosophic Analytic Hierarchy Process (NAHP), Delphi method, and Neutrosophic Cognitive Maps (NCM), this study reveals key elements that impact the codification process. These elements include the necessity for a distinct convention and the sufficiency of current legal norms. Neutrosophic Cognitive Maps provide the visualization of the connections and interdependencies among important elements, therefore providing a deeper comprehension of their influence. An expert consensus highlighted the need for precise definitions, regulatory consistency, and a strong theoretical and legal basis. To enhance accountability and avoid future crimes against humanity, the report asserts that stronger international collaboration and a systematic approach to integrating these elements are essential. Subsequent investigations should prioritize the pragmatic application of suggested standards, comparative examination with other global legal systems, and ongoing expert verification through neutrosophic techniques.

Keywords: Crimes against humanity, codification, international criminal law, International Law Commission, neutrosophic cognitive maps, Neutrosophic Analytic Hierarchy Process, Delphi method.

1. Introduction

The lack of adequate codification for crimes against humanity, in contrast to the codification of genocide and war crimes in the late 1940s, highlights a significant disparity in international regulation. Despite the adoption of the Rome Statute in 1998, which established the permanent International Criminal Court (ICC) and sought to address these crimes, insufficient acceptance and support for the ICC have posed challenges to effective accountability for crimes against humanity[1].

Throughout history, individual accountability for crimes against humanity has evolved and is less controversial today compared to the stance following World War II. However, detailed and complex issues persist that require proper addressing through solid codification [2].

The codification efforts carried out by the International Law Commission (ILC) reflect the importance of addressing the complexities of crimes against humanity and establishing clear definitions and norms for their prevention and punishment. The existence of disjointed regulations also highlights the need for more coherent codification for these crimes.

The adoption of the draft articles on the prevention and punishment of crimes against humanity by the ILC in 2019, on the second reading, represents a significant opportunity for scholarly analysis. Through this process, a valuable opportunity is presented to examine both the content of the document and the role played by the ILC in the development of international criminal law since its establishment.

The scientific analysis of this draft article will allow for an objective and rigorous assessment of the theoretical, conceptual, and legal foundations underpinning the proposals contained in the document. Researchers will be able to study and compare the approach adopted by the ILC in the prevention and punishment of crimes against humanity with existing legal frameworks and international standards [3].

Furthermore, the scientific analysis can shed light on how the ILC has evolved in its role as a subsidiary body of the UN General Assembly and its contribution to the development of international criminal law. Its working methods, decision-making processes, and the degree of participation and cooperation from member states and other international actors in its endeavors can be investigated [4].

The scientific study of this project can also address key issues related to the implementation and effectiveness of the proposed norms in the prevention and punishment of crimes against humanity. Researchers can examine the adequacy and coherence of the document concerning other international legal instruments, like the Rome Statute, and assess its potential impact on the protection of human rights and accountability.

The current debate naturally addresses the need to consider a separate Convention for crimes against humanity. This study aims to provide a critical and well-founded assessment of the need for more robust codification for crimes against humanity and its impact on the effective accountability and punishment of those responsible through the Saaty AHP [5] and neutrosophic Delphi methods [6]. Additionally, it sought to understand the role of the ILC in this context and its contribution to the development of international criminal law. The study also highlighted the implications and challenges related to the prevention and punishment of these crimes and how more coherent codification could more effectively address them.

2. Background

2.1 Analytic Hierarchy Process Method

The Analytic Hierarchy Process (AHP) is a methodology widely used to tackle complex decision-making problems that involve multiple criteria [7]. The AHP approach involves creating a hierarchy that contrasts the decision-making process, placing the main objective at the highest level and the available options at the lowest level, with relevant criteria and attributes outlined at intermediate levels.

This technique is particularly useful for decisions that require the consideration of technical, economic, political, social, and cultural aspects, providing a scientific method to manage elements that are difficult to comprehend.Through its structured approach, which includes element prioritization, pairwise comparisons, weight assignment, and synthesis, the AHP aids in identifying the best alternative based on available resources [8].

Originally proposed by Thomas Saaty in 1980, this method is distinguished by its ability to structure complexity, quantify preferences on a scale, and synthesize the outcomes to aid decision-making [7]. Moreover, the introduction of neutrosophic sets and their application in this context is noted. These sets are characterized by three membership functions: truth, indeterminacy, and falsehood, applicable to both standard and non-standard subsets[9,10].

Single-Valued Neutrosophic Set (SVNS) and Single-Valued Neutrosophic Number (SVNN) are mathematical representations of items that can be stated as values representing truth, indeterminacy, and untruth within a specified range. Additionally, the concept of a single-valued neutrosophic trapezoidal number is mentioned, providing a more detailed structure for these sets[11, 12].

This methodology represents the issue that results in the development of a hierarchical structure that reflects the corresponding decision-making paradigm. The initial and primary step is to formulate the decision-making problem into a hierarchical structure [13]. This stage is the phase in which the decision-maker must deconstruct the problem into its pertinent elements. The hierarchy is designed to ensure that the elements have a consistent magnitude and can correlate with certain levels below. A typical hierarchy is structured such that the top level identifies the challenge of decision-making. The factors that influence the process of making decisions are depicted at the intermediate level, where the criteria are situated. At the lowest level, the choice alternatives are positioned. The relative level of significance or importance of the criteria is determined by conducting paired comparisons between them. The comparison is conducted by applying a scale, as denoted by equation (1).

$$S = \left\{1,3,5,7,9,\frac{1}{9},\frac{1}{7},\frac{1}{5},\frac{1}{3}\right\} \tag{1}$$

Within a neutrosophic framework, the theory of the Analytic Hierarchy Process (AHP) allows for the modeling of decision-making indeterminacy via the application of neutrosophic AHP, otherwise known as NAHP. Equation 1 presents a universal neutrosophic pairwise comparison matrix

To convert neutrosophic triangular numbers into crisp numbers, two indexes are defined: the so-called score and accuracy indexes, as seen in Equations 2 and 3, respectively [14].

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \tag{2}$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \tag{3}$$

A widely accepted method for comparing the relative importance of two alternatives involves the Saaty scale. By employing a comparison model technique, which involves assessing alternatives based on several criteria, precision is improved. Priorities are allocated numerical values ranging from 1 to 9, as shown in Table 1.

Table 1. Saaty's scale translated into a neutrosophic triangular scale. Source: [15].

Numerical	Scale	Neutrosophic Triangular Scale
1	Equally preferred	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
3	Moderately preferred	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly preferred	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
7	Very strongly preferred	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
9	Extremely preferred	$\tilde{9} = \langle(9,9,9); 1.00, 1.00, 1.00\rangle$
2	Equally to moderately preferred	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$
4	Moderately to strongly preferred	$\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$
6	Strongly to very strongly preferred	$\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$
8	Very strongly to extremely preferred	$\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

To apply the AHP methodology among a set of criteria, the following steps are necessary[16]:

1. Determine the specific criteria for comparison.
2. Compute the Comparison Matrix for pairs of components, sub-factors, and strategies by utilizing the linguistic terminology provided in Table 1.
3. To normalize the comparison matrix, divide each integer by the total of the values in its corresponding column.
4. Determine the weight of each criterion by converting the neutrosophic pairwise comparison matrix into a deterministic matrix by applying Equations 9 and 10.
5. Determine the value of the consistency index..

In Step 3, the application of this technique requires the consideration of the calculus of the Consistency Index (CI), which is a function that depends on λmax, the greatest eigenvalue of the matrix. The consistency of the evaluations can be ascertained by the equation proposed by Saaty [17].

$$\mathrm{CI} = \frac{\lambda_{\max} - \mathrm{n}}{\mathrm{n} - 1} \tag{4}$$

where *n* is the order of the matrix. In addition, the *Consistency Ratio* (CR) is defined by equation:

$$CR = \frac{CI}{RI} \tag{5}$$

RI is given in Table 2.

Table 2. RI associated with every order. Source: [18].

Order (n)	1	2	3	4		5	6	7	8	9	10
RI	0	0	0.52	0.89		1.11	1.25	1.35	1.40	1.45	1.49

If CR 0.1, it is considered that the experts' evaluation is sufficiently consistent, and thus, attempting to utilize NAHP is feasible.

2.2 Delphi Neutrosophic Method

The establishment of the group including the expert panel should guarantee the essential diversity and importance in order to effectively tackle the topic of the research. The neutrosophic expert competence coefficient (K_N) is used by the coordination group to determine the final selection of experts [19].

Let X be a universe of discourse. A Single-Valued Neutrosophic Set (SVNS) A over X is an object in the form described in the following equation [20]:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \tag{6}$$

In this coefficient, two factors were averaged, the knowledge coefficient (K_{cn}) and the argumentation coefficient (K_{an}).

$$K_N = \frac{1}{2}(K_{aN} + K_{cN}) \quad (7)$$

Where, by applying equation (7), it is obtained:

$$K_{aN} = \{\langle x, u_{Ka}(x), r_{Ka}(x), v_{Ka}(x)\rangle : x \in X\} \quad (8)$$

$$K_{cN} = \{\langle x, u_{Kc}(x), r_{Kn}(x), v_{Kn}(x)\rangle : x \in X\} \quad (9)$$

The neutrosophic knowledge coefficient is derived from the information provided by the expert on the researched subject. This information is obtained through a self-assessment procedure using a scale to measure the level of knowledge on the examined topic and subject of study (see Table 3).

Table 3. Linguistic terms used to determine $K_{aN,}K$ and evaluate the proposed criteria.

Linguistic term	SVNN
Full knowledge of the subject of study (FK)	(1,0,0)
Very very good in the subject of study (VVGK)	(0.9, 0.1, 0.1)
Very good in the subject of study (VGK)	(0.8,0,15,0.20)
Good in the subject of study (GK)	(0.70,0.25,0.30)
Moderately good in the subject of study (MGK)	(0.60,0.35,0.40)
Know the topic of study (K)	(0.50,0.50,0.50)
Moderately poorly knows the subject of study (MPK)	(0.40,0.65,0.60)
Poorly knows the topic of study (PK)	(0.30,0.75,0.70)
Know the topic of study very poorly (VPK)	(0.20,0.85,0.80)
Very very poor knowledge of the topic of study (VVPK)	(0.10,0.90,0.90)
No knowledge of the study topic (NK)	(0,1,1)

The Neutrosophic Argumentation Coefficient utilizes linguistic terms with Single-Valued Neutrosophic Numbers (SVNN) for the consensus of the expert opinion's substantiation (see Table 3).This is calculated by summing the weighted values derived from a set of influence factors identified by the Coordination Group. These factors include the experience acquired through activity and practice, understanding of the current state of the subject at national and international levels, intuition about the subject, knowledge of technology, and review of writings and publications on the research topic[21, 22]

The evaluation of the experts' responses is established as an objective criterion of the Neutrosophic Expert Competence Coefficient with a critical level required set by the Coordination Group for a value (A) (see Table 4).

Table 4. Linguistic terms used to determine K_{CN}. Source: [23].

Linguistic term	SVNN
Very High (VH)	(0.9, 0.1, 0.1)
High (H)	(0.75,0.25,0.20)
Medium (M)	(0.50;0.5;0.50)
Low (L)	(0.35,0.75,0.80)
Very Low (VL)	(0.10,0.90,0.90)

To determine the consensus among the participants of the Expert Panel, the agreement coefficient determined with the expression:

$$Cc = \left(1 - \frac{V_n}{V_t}\right)100 \quad (10)$$

For binary questions (yes or no), the consensus is considered reached with an agreement of 75%.

In this work, the application of the method was established in four stages:

- Design by the Coordination Group based on the variables identified in the undetermined dimensions in the neutrosophic analysis.

- Selection of the Expert Panel.
- Interpretation of responses and assessment of actions.
- Interpretation of the answers.

3. Materials and Methods

This study employed a neutrosophic cognitive mapping (NCM) [24, 25, 26] approach to analyze the factors influencing the codification and treatment of crimes against humanity. The process involved expert evaluation of key factors identified in the context of international legal frameworks. The methodology consisted of the following steps:

1. Identification of Key Factors.
   Critical factors were classified by experts in the domain of international law, particularly in the codification of crimes against humanity.
2. Neutrosophic Cognitive Map (NCM) Construction.
   The second step is the construction of the neutrosophic cognitive map (NCM) to visualize the relationships and influences among these factors. This map served as the basis for understanding the relative importance of each factor and their interdependencies.
3. Centrality Analysis and Node Elimination.
   Using centrality analysis, the significance of each factor was evaluated. Factors with higher centrality were determined to have a greater influence on the system, while those with lower centrality were considered less impactful.
4. Neutrosophic Analytical Hierarchy Process (NAHP).
   Once the nodes with lower centrality values were eliminated, the remaining factors were further analyzed using the Neutrosophic Analytical Hierarchy Process (NAHP). This method enables pairwise comparisons between the remaining factors to determine their relative importance.
5. Consistency Analysis.
   A consistency analysis was conducted to ensure that the pairwise comparison matrix was logically coherent.
6. Validation and Expert Consensus.

The expert panel's responses were validated through multiple iterations using the Delphi method, structured around the criteria derived from the NAHP analysis. Experts provided feedback on the validity of the criteria, and the results demonstrated strong consensus across the key areas.

The NAHP method and NCM analysis allowed for the clear identification of the most critical factors influencing the codification and treatment of crimes against humanity. By reducing the number of criteria based on centrality values and using neutrosophic logic, this methodology provided a structured and reliable approach for decision-making, incorporating uncertainty and indeterminacy inherent in complex international legal contexts.

In addition, the Delphi method played a crucial role in validating the findings by facilitating expert consensus on the prioritized factors. The iterative nature of the Delphi process enabled the refinement of opinions and ensured that the final set of criteria was grounded in collective expert knowledge. This helped strengthen the reliability of the decision-making framework, ensuring that the identified factors were not only analytically significant but also practically relevant according to expert judgment. The combined use of the NAHP, NCM, and Delphi method resulted in a comprehensive, consensus-driven approach to addressing the complex issue of codification in international legal frameworks.

3. Results

Experts classified nine factors to draw in the context of the codification and treatment of criminal offenses against humanity.

- Need for a Separate Convention (NCS)
- Adequacy of Codification (AC)
- Clarity of Definitions and Norms (CDN)
- Regulatory Coherence (CR)
- Lack of Coordination (LC)
- Theoretical and Legal Foundations (FTJ)
- Role and Evolution of the ILC (RE)
- Implementation Problems (IP)

- International Participation and Cooperation(PCI)
- Impact on Human Rights Protection (IPDH)
- Effectiveness of Legal Responsibility: (ERL)

The process begins by constructing a neutrosophic cognitive map (NCM) to visualize the relationships and influences between the identified factors. This map serves as the foundation for understanding the impact and relevance of each factor(Figure 1).

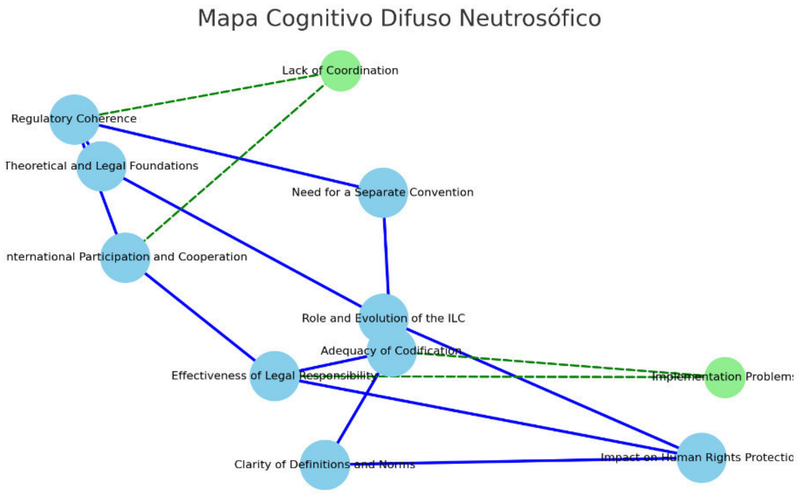

Figure 1. Neutrosophic Cognitive Map with Low-Centrality Nodes Highlighted in Green

The list of all the nodes with their centrality values, along with the two nodes that were eliminated is as follows:
AC (Adequacy of Codification): 0.375
CR (Regulatory Coherence): 0.375
IPDH (Impact on Human Rights Protection): 0.375
ERL (Effectiveness of Legal Responsibility): 0.375
NCS (Need for a Separate Convention): 0.25
CDN (Clarity of Definitions and Norms): 0.25
FTJ (Theoretical and Legal Foundations): 0.25
RE (Role and Evolution of the ILC): 0.25
PCI (International Participation and Cooperation): 0.25
Eliminated nodes (lower centrality):
LC (Lack of Coordination): 0.2
IP (Implementation Problems): 0.2
These two nodes were removed due to having the lowest centrality values.

Once the nodes with lower centrality values (LC - Lack of Coordination and IP - Implementation Problems) have been eliminated, and the number of criteria has been reduced, you can apply the Neutrosophic Analytical Hierarchy Process (NAHP) method to further refine the decision-making process .

The weights of the factors acting in the codification and treatment of crimes against humanity are determined using the Neutrosophic AHP method.

Table 5. Neutrosophic AHP Pairwise Matrix.

Factors	AC	NCS	CDN	CR	FTJ	RE	PCI	IPDH	ERL
NCS	Equally influential	5	7	5	7	5	7	5	7
AC	1/5	Equally influential	3	5	5	7	5	7	5
CDN	1/7	1/3	Equally influential	3	3	5	5	5	5
CR	1/5	1/5	1/3	Equally influential	1	3	3	3	3
FTJ	1/7	1/5	1/3	1/1	Equally influential	1	3	3	3
RE	1/5	1/7	1/5	1/3	1/1	Equally influential	1	1	1

Factors	AC	NCS	CDN	CR	FTJ	RE	PCI	IPDH	ERL
PCI	1/7	1/5	1/5	1/3	1/3	1/1	Equally influential	1	1
IPDH	1/5	1/7	1/5	1/3	1/3	1/1	1/1	Equally influential	5
ERL	1/7	1/5	1/5	1/3	1/3	1/1	1/1	1/5	Equally influential

Table 6. Determination of criteria weights using the Neutrosophic AHP method.

Factors	AC	NCS	CDN	CR	FTJ	RE	PCI	IPDH	ERL	WEIGHT
NCS	0,42	0,67	0,56	0,31	0,37	0,20	0,26	0,19	0,23	0,36
AC	0,08	0,13	0,24	0,31	0,26	0,28	0,19	0,27	0,16	0,21
CDN	0,06	0,04	0,08	0,18	0,16	0,20	0,19	0,19	0,16	0,14
CR	0,08	0,03	0,03	0,06	0,05	0,12	0,11	0,11	0,10	0,08
FTJ	0,06	0,03	0,03	0,06	0,05	0,04	0,11	0,11	0,10	0,07
RE	0,08	0,02	0,02	0,02	0,05	0,04	0,04	0,04	0,03	0,04
PCI	0,06	0,03	0,02	0,02	0,02	0,04	0,04	0,04	0,03	0,03
IPDH	0,08	0,02	0,02	0,02	0,02	0,04	0,04	0,04	0,16	0,05
ERL	0,06	0,03	0,02	0,02	0,02	0,04	0,04	0,01	0,03	0,03

Table 7. Analysis of the consistency of the paired matrix.

Factors	|	Approximate eigenvalues
NCS	4.11	11.53051666
AC	2.33	10.89193966
CDN	1.43	10.14890395
CR	0.74	9.650934083
FTJ	0.65	9.889946348
RE	0.37	9.741562706
PCI	0.32	9.799366333
IPDH	0.44	9.105433668
ERL	0.28	9.682770364
Eigenvalue - 10.049042		

The consistency analysis confirms that the modeling satisfies the desired parameters, with an eigenvalue of 6.40, CI=0.13, and CR=0.09. The values are believed to quantify the relative significance (weights) of several aspects assessed using the Analytic Hierarchy Process (AHP) approach devised by Saaty. The final weights are indicative of their relative relevance in the ultimate choice. Thus, NCS carries the greatest significance with a weight of 0.36, signifying its paramount role in the process of decision-making. The weight of 0.21 assigned to AC indicates that a distinct convention is equally important, albeit to a lower degree than AC.

Weighted at 0.14, the Clarity of Definitions and Norms indicates a modest level of significance. CR and FTJ assign weights of 0.08 and 0.07 correspondingly, suggesting a quite modest level of significance. The implication is that while these elements do influence decision-making, they are of lower importance compared to AC, NCS, and CDN. The minimum weights (0.04, 0.03, 0.05, and 0.03) assigned to RE, PCI, IPDH, and ERL indicate that these parameters are of the least importance in this study. This implies that although these features are taken into account, they exert a relatively smaller direct impact on strategic decision-making in comparison to other elements. Thus, the most pertinent criteria are:

1. Need for a Separate Convention: (NCS)
2. Adequacy of Codification: (AC)
3. Clarity of Definitions and Norms: (CDN)
4. Regulatory Coherence: (CR)
5. Theoretical and Legal Foundations: (FTJ)

To model the Delphi method the following strategies based on the mentioned criteria are contemplated:

- The creation of a clear framework for the convention: it is necessary to identify and reach consensus on specific areas that require detailed attention and distinct codification. This involves determining the boundaries and scope of such a convention, ensuring that it addresses needs identified and articulated by experts (a).
- The establishment of a Multidisciplinary Expert Panel: Given the criterion of "Adequacy of Codification," selecting a diverse group of experts who possess not only legal knowledge but also experience in areas that are relevant to the codification at hand is crucial. This ensures a broad and appropriate vision in the drafting of codifications or conventions that are integrally fair and applicable (b).
- Definition and setting of key terms: In response to "Clarity of Definitions and Standards," the strategy should clarify and reach a consensus on definitions of key terms and regulations. This facilitates a uniform understanding and reduces ambiguities in future interpretations (c).
- Evaluation and harmonization of existing regulations: Due to the importance of "Regulatory Coherence," discrepancies between existing regulations and proposals should be determined.The goal would be to harmonize the new codifications with the existing legal and regulatory framework and promote cohesive integration that strengthens the international legal system without generating normative conflicts (d).
- Validation of theoretical and legal foundations: To ensure that new norms or codifications are based on solid "Theoretical and Legal Foundations," the theoretical and legal principles that apply to the proposals can be validated.This involves critically reviewing the legal and theoretical basis and ensuring that the recommendations are well-founded and defensible from an international legal perspective (e).
- The selected strategies will be implemented and a questionnaire will be sent to the experts, following a structured approach.The Delphi process is characterized by being iterative, anonymous, and controlled, to reach consensus among experts on a specific topic. Below are the steps for implementing this:

Step 1: Expert Selection

- A panel of 9 experts with a wide range of expertise and experiences pertinent to the subject of interest is selected, including jurists, academics specialized in international law, experts with backgrounds in crimes against humanity, specialists in law formulation, and other relevant professionals. To appropriately select these specialists, the Neutrosophic Argumentation Coefficient is used, which is based on the evaluation of the solidity of the experts' opinions through a weighted aggregation of values obtained from various Influence Factors.
- These criteria are established by the Coordination Group and encompass the expert's professional and practical expertise, their understanding of the present national and international situation, their intuitive ability pertaining to the subject being discussed, their familiarity with technology, and their contribution to the relevant literature and studies.

Step 2: Design of the Initial Questionnaire

- The first questionnaire should be prepared to investigate the main areas delineated in the strategy, using open-ended questions to facilitate comprehensive answers and precise recommendations. This comprises:
    1. What are the main strategies you identify in the context of the codification and treatment of crimes against humanity?
    2. Based on your experience, how would you assess the relative frequency of the current strategies implemented?
    3. Which strategy do you consider the most effective?
    4. From your perspective, do you think the strategies are useful for overcoming current challenges?

Step 3: Distribution and Collection of Responses

- Send the questionnaire to the selected experts and ensure anonymity to promote honesty and avoid biases in the responses. Establish a clear deadline for the return of the responses.

Step 4: Analysis of Responses and Preparation for the Second Round

- Determine areas of agreement and disagreement by analyzing the responses. Compile a concise overview of the replies and incorporate statistical data if feasible (such as the degree of agreement with each statement) and prepare a second questionnaire based on the obtained results. This second questionnaire should specifically target areas where agreement was not achieved and request the experts to reassess their answers within the framework of the group's overall recommendations.

Table 8. Validation of the criteria

Expert	a	b	c	d	e
E1	(0.35;0.75;0.80)	(0.35;0.75;0.80)	(0.35;0.75;0.80)	(0.10;0.90;0.90)	I
E2	(0.75;0.25;0.20)	(0.9;0.1;0.1)	(0.10;0.90;0.90)	(0.35;0.75;0.80)	(0.9;0.1;0.1)
E3	(0.9;0.1;0.1)	I	(0.10;0.90;0.90)	(0.35;0.75;0.80)	(0.75;0.25;0.20)
E4	(0.75;0.25;0.20)	(0.9;0.1;0.1)	(0.75;0.25;0.20)	(0.75;0.25;0.20)	(0.75;0.25;0.20)
E5	(0.35;0.75;0.80)	(0.75;0.25;0.20)	(0.10;0.90;0.90)	(0.75;0.25;0.20)	(0.35;0.75;0.80)
E6	(0.35;0.75;0.80)	(0.9;0.1;0.1)	I	(0.9;0.1;0.1)	(0.75;0.25;0.20)
E7	(0.10;0.90;0.90)	(0.10;0.90;0.90)	(0.35;0.75;0.80)	(0.35;0.75;0.80)	(0.10;0.90;0.90)
E8	(0.10;0.90;0.90)	(0.9;0.1;0.1)	(0.9;0.1;0.1)	(0.9;0.1;0.1)	(0.35;0.75;0.80)
E9	(0.9;0.1;0.1)	(0.75;0.25;0.20)	(0.75;0.25;0.20)	(0.75;0.25;0.20)	I

According to the evaluation given by the experts, strategies b, c, and e each show a level of indeterminacy.

Table 9. Relative frequency.

Indicators	Very high	High	Medium	Low	Very Low
a	0.2222	0.4444	0.4444	0.7778	1,0000
b	0.4444	0.6667	0.7778	0.8889	1,0000
c	0.1111	0.3333	0.4444	0.6667	1,0000
d	0.2222	0.5556	0.5556	0.8889	1,0000
e	0.1111	0.4444	0.6667	0.8889	1,0000

Table 10. Calculation of cut points.

Indicators	Very high	High	Medium	Low	Very Low
a	-0.76	-0.14	-0.14	0.76	3.50
b	-0.14	0.43	0.76	1.22	3.50
c	-1.22	-0.43	-0.14	0.43	3.50
d	-0.76	0.14	0.14	1.22	3.50
e	-1.22	-0.14	0.43	1.22	3.50

Table 11. Scale of neutrosophic indicators. Source: own elaboration.

Indicators	Average	N - Avg.	SVNN
a	0.64	**-0.95**	(0.35;0.75;0.80)
b	1.15	**-1.46**	(0.75;0.25;0.20)
c	0.43	**-0.74**	(0.35;0.75;0.80)
d	0.85	**-1.16**	(0.35;0.75;0.80)
e	0.76	**-1.07**	(0.35;0.75;0.80)
	-0.31	=N	
	N =	**-0.31**	

Table 12. Expert validation. Source: own elaboration.

Expert	C1	C2	C3	C4	C5
E1	YES	YES	YES	YES	YES
E2	YES	YES	YES	YES	YES
E3	YES	YES	YES	NO	YES
E4	YES	NO	YES	YES	YES
E5	YES	YES	YES	YES	YES
E6	YES	YES	YES	YES	YES
E7	NO	YES	YES	YES	YES
E8	YES	YES	YES	YES	YES
E9	YES	YES	YES	YES	YES
YES	8	8	9	8	9
NO	1	1	0	1	0
Coefficient	88.89	88.89	100	88.89	100

From the results it is clear that in all cases a consensus was reached; therefore, a second round was not necessary.

The analysis was based on a series of key indicators, evaluated through a consensus of experts. The indicators, identified as a, b, c, d, and e, were examined in terms of their average, average negativity). The results suggest significant differences in experts' perceptions of the importance and impact of these indicators. The details are as follows:

Indicator a: Shows a balance between positive and negative perspectives suggesting a moderately positive opinion.

Indicator b: Stands out for having the most positive evaluation on average.

Indicator c: Similar to indicator *a* in terms of perception, which shows a moderately positive opinion.

Indicators d and e: Both indicators present intermediate values, with a slight inclination towards a more positive than negative perception.

The overall average negativity (N) was calculated at -0.31, indicating a general trend towards more negative opinions in the dataset.

Based on collective knowledge and expert recommendations, the following strategic actions are suggested for each of the key areas represented by the indicators:

Indicators a and c: A cautious approach is recommended to optimize and continuously evaluate current strategies to ensure their effectiveness. The moderate positivity suggests that, although there are benefits, there are significant areas for improvement.

Indicator b: The strategy provides a basis for considering this indicator as strategically advantageous, with significantly positive attributes that experts find valuable.

Indicators d and e: Given their intermediate position with a tendency towards the positive, it is crucial to strengthen aspects that are already perceived favorably, while identifying and addressing the underlying causes of any negative perception.

Conclusion

The analytical findings emphasize the importance of progressing the codification and handling of crimes against humanity, with a specific emphasis on enhancing regulatory consistency and precision in defining relevant terms. Neutrosophic approaches and the Analytic Hierarchy Process (AHP) have enabled the identification of crucial elements that impact this process, including the necessity for a distinct convention and the sufficiency of codification. Nevertheless, various obstacles persist about the execution, global involvement, and synchronization endeavors aimed at attaining a stronger and more efficient legal structure. Although there was agreement on the majority of the assessed criteria, it is still crucial to enhance the consistency of regulations and clarify the theoretical and legal basis.

For further study, it is advisable to examine the realistic application of the norms suggested by the International Law Commission (ILC), with a specific emphasis on how domestic legislation might conform to global benchmarks. Furthermore, it would be beneficial to investigate the influence of these standards on human rights and legal responsibility in particular situations, by employing comparative methodologies with other global frameworks. Furthermore, it is imperative to persist in the use and advancement of the Delphi method and neutrosophic analysis to guarantee continuous expert validation in the creation of worldwide legal guidelines.

References

[1] N. Myroshnychenko, D. Baranenko, A. Murzanovska, Z. Dilna, and S. Suleimanova, "Crimes against humanity in the conditions of military conflicts of the 21st Century: The practice of international courts," Amazonia Investiga, vol. 11, no. 54, pp. 9-17, 2022

[2] R. D. Hare, E. León-Mayer, J. R. Salinas, J. Folino, and C. S. Neumann, "Psychopathy and crimes against humanity: A conceptual and empirical examination of human rights violators," Journal of criminal justice, vol. 81, no. July–August, p. 101901, 2022.

[3] W. I. Pons, J. E. Lord, and M. A. Stein, "Disability, Human Rights Violations, and Crimes Against Humanity," American Journal of International Law, vol. 116, no. 1, pp. 58-95, 2022.

[4] Bassiouni, M. C., & Schabas, W. A. (Eds.). (2021). The Legislative History of the International Criminal Court: Introduction, Analysis, and Integrated Text (3 vols) (Vol. 18). Brill.

[5] Saaty, T. L. (2004). Decision making—the analytic hierarchy and network processes (AHP/ANP). Journal of systems science and systems engineering, 13, 1-35.

[6] Smarandache, F., Estupiñán Ricardo, J., González Caballero, E., Leyva Vázquez, M. Y., & Batista Hernández, N. (2020). Delphi method for evaluating scientific research proposals in a neutrosophic environment. Neutrosophic Sets and Systems, 34(1), 26.

[7] Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems, 33(6), 4055-4066.

[8] Baby, S. (2013). AHP modeling for multicriteria decision-making and to optimise strategies for protecting coastal landscape resources. International Journal of Innovation, Management and Technology, 4(2), 218.

[9] Nabeeh, N. (2023) "Assessment and Contrast the Sustainable Growth of Various Road Transport Systems using Intelligent Neutrosophic Multi-Criteria Decision-Making Model", Sustainable Machine Intelligence Journal, 2, pp. (2):1–12

[10] Rodríguez, R. C., Sánchez, J. M. D., & Salcedo, V. L. (2022). Evaluation of Social Projects Using Neutrosophic AHP. International Journal of Neutrosophic Science (IJNS), 19(1).

[11] Mishra, A. R., Pamucar, D., Rani, P., Shrivastava, R., & Hezam, I. M. (2024). Assessing the sustainable energy storage technologies using single-valued neutrosophic decision-making framework with divergence measure. Expert Systems With Applications, 238, 121791.

[12] M.Ali, A. and Muthuswamy, M. (2023) "Neutrosophic Multi-Criteria Decision-Making Framework for Sustainable Evaluation of Power Production Systems in Renewable Energy Sources", Sustainable Machine Intelligence Journal, 4, pp. (3):1–10.

[13] Albayrak, E., & Erensal, Y. C. (2004). Using analytic hierarchy process (AHP) to improve human performance: An application of multiple criteria decision making problem. Journal of intelligent manufacturing, 15, 491-503.

[14] Guerrero, O. E. A., Bandera, N. D. L. M. H., Bernal, L. I. M., & Espinosa, J. S. P. (2023). Neutrosophic Modeling of Healthcare Resource Allocation. Neutrosophic Sets and Systems, 62, 60-68.

[15] Molina Chalacan, L. J., Checa Cabrera, M. A., Aguirre Paz, L. M., Lalama Flores, R. V., & Martín Rabelo, A. (2021). Selection of criteria for the prioritization of information technology services in a neutrosophic environment. Neutrosophic Sets and Systems, 44(1), 41.

[16] Franek, J., & Kresta, A. (2014). Judgment scales and consistency measure in AHP. Procedia economics and finance, 12, 164-173.

[17] Bulut, E., Duru, O., Keçeci, T., & Yoshida, S. (2012). Use of consistency index, expert prioritization and direct numerical inputs for generic fuzzy-AHP modeling: A process model for shipping asset management. Expert Systems with Applications, 39(2), 1911-1923.

[18] Elvis, B. W. W., Arsene, M., Theophile, N. M., Bruno, K. M. E., & Olivier, O. A. (2022). Integration of shannon entropy (SE), frequency ratio (FR) and analytical hierarchy process (AHP) in GIS for suitable

groundwater potential zones targeting in the Yoyo river basin, Méiganga area, Adamawa Cameroon. Journal of Hydrology: Regional Studies, 39, 100997.

[19] Falcon, V. V., Martinez, B. S., & Sanchez, F. C. (2020). Experts' Selection for Neutrosophic Delphi Method. A Case Study of Hotel Activity. Neutrosophic Sets and Systems, (37), 115-125.

[20] Ricardo, J. E., Fernández, A. J., & Vázquez, M. Y. (2022). Compensatory Fuzzy Logic with Single Valued Neutrosophic Numbers in the Analysis of University Strategic Management. *International Journal of Neutrosophic Science (IJNS)*, *18*(4).

[21] Deveci, M., Özcan, E., John, R., Covrig, C. F., & Pamucar, D. (2020). A study on offshore wind farm siting criteria using a novel interval-valued fuzzy-rough based Delphi method. Journal of Environmental Management, 270, 110916.

[22] Vázquez, M. Y. L., Cevallos, R. E. H., & Ricardo, J. E. (2021). Análisis de sentimientos: herramienta para estudiar datos cualitativos en la investigación jurídica. Universidad Y Sociedad, 13(S3), 262-266.

[23] Elkadry, H., Shamsuzzaman, M., Piya, S., Haridy, S., Bashir, H., & Khadem, M. (2023). A fuzzy Delphi-AHP framework for identifying and prioritizing factors affecting students' satisfaction in public high schools: Insights from the United Arab Emirates. Journal of Engineering Research.

[24] Mariscal Rosado, Z. M., Maldonado Manzano, R. L., & González Alvarado, R. J. (2023). Impacto de la deuda externa en los derechos de Babahoyo: análisis con mapas cognitivos neutrosóficos. Neutrosophic Computing and Machine Learning, 27, 10-17.

[25] Ricardo, J. E., Flores, D. F. C., Díaz, J. A. E., & Teruel, K. P. (2020). An Exploration of Wisdom of Crowds using Neutrosophic Cognitive Maps. Neutrosophic Sets and Systems, 37(1), 2.

[26] Wajid, M. S., Terashima-Marin, H., Paul Rad, P. N., & Wajid, M. A. (2022). Violence detection approach based on cloud data and Neutrosophic cognitive maps. Journal of Cloud Computing, 11(1), 85.

Received: June 28, 2024. Accepted: August 20, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Enhancing Judicial Impartiality in Ecuador: A Fuzzy Cognitive Map Approach Using Neutrosophic Logic and Fuzzification

Robert Alcides Falconi Herrera[1]

[1] Autonomous Regional University of the Andes, Babahoyo. Ecuador. E-mail: ur.robertfalconi@uniandes.edu.ec

Abstract. Fuzzy Cognitive Map (FCM) approach, with neutrosophic logic and fuzzification applied to assign weights to the relationships between key concepts. Through expert consensus, eight critical factors were identified: Judicial Independence, Transparency, Corruption, Judicial Training, Accountability, Access to Justice, Procedural Guarantees, and Supervision and Control. Centrality analysis revealed that Judicial Independence and Transparency are the most influential, while Judicial Training has the lowest impact. The use of neutrosophic logic and fuzzification allowed for a flexible and accurate representation of causal relationships, accounting for uncertainty within the system. The findings highlight the need for reforms focused on enhancing judicial independence and transparency, as well as addressing corruption and access to justice. Future research should incorporate dynamic simulations and machine learning to further improve predictive capabilities and deepen the understanding of policy impacts on judicial impartiality.

Keywords: Judicial impartiality, fuzzy cognitive maps, expert knowledge, Neutrosophic Logic fuzzification.

1. Introduction

The issue of judicial impartiality in Ecuador is a critical and multifaceted challenge, reflecting the ongoing difficulties within the nation's legal system. Despite numerous efforts to reform and strengthen the judiciary over recent decades, doubts persist regarding the genuine autonomy of the courts. A significant barrier to achieving full impartiality lies in the persistent political interference in administrative procedures, particularly in the appointment of judges. Such interventions compromise public confidence in the fairness of judicial decisions, as many citizens perceive rulings to be influenced by external political agendas rather than legal principles [1].

An additional and severe obstacle is the pervasive corruption at various levels of the Ecuadorian government, including the judiciary itself. Corrupt practices within the judicial system severely undermine both the actual and perceived fairness of court rulings, leading to doubts about the integrity of judges and the outcomes of legal processes. Although efforts to combat corruption have been implemented, they have been insufficient in eradicating the issue. Stronger oversight and enforcement mechanisms are urgently needed to restore public trust in the judicial system [2].

Another contributing factor to the erosion of judicial impartiality is the lack of resources and inadequate training for judges. Heavy workloads and limited access to specialized training often result in biases or insufficient decision-making processes. Particularly in complex cases requiring technical expertise, the absence of continuous education and infrastructure support poses significant challenges. Addressing these deficiencies requires a comprehensive commitment to strengthening the judicial framework and providing judges with the resources necessary to carry out their responsibilities impartially [3,4].

One potential solution to these issues is the incorporation of expert information into judicial decision-making processes. In cases involving technical or scientific evidence, such as environmental or intellectual property disputes, expert participation can provide judges with the necessary specialized knowledge to make well-informed and impartial rulings. This approach reduces the likelihood of errors or biases, ensuring that judicial decisions are based on a thorough understanding of the facts [5,6].

Finally, the use of neutrosophic tools, including fuzzy cognitive maps (FCMs), offers a novel method for representing and understanding the complexities of the judicial process. These tools facilitate the depiction of uncertainty and ambiguity, which are inherent in legal decision-making. The application of neutrosophic approaches allows for a deeper understanding of the factors that affect judicial impartiality and enables the development of more effective strategies to improve transparency, fairness, and accountability in the Ecuadorian judiciary [7,8]. This study aims to explore these issues and propose solutions to enhance the efficiency and fairness of the judicial system.

2. Backgound

2.1 Fuzzy Cognitive Maps: Structure, Causality, and Integration with Neutrosophic Logic

Fuzzy Cognitive Maps (FCMs) are conceptual structures that employ feedback to describe causality. These models merge theoretical elements from cognitive maps, fuzzy logic, neural networks, semantic networks, expert systems, and nonlinear dynamic systems. This methodology allows for the representation of the system with feedback using fuzzy degrees of causality within the interval [0,1]. Each node in the diagram represents a fuzzy set or an event that occurs with a certain degree of certainty. These nodes are causal concepts that can represent events, actions, values, goals, or processes. The use of this technique provides additional advantages such as visual modeling, simulation, and predictive capability [9, 10].

In Fuzzy Cognitive Maps (FCMs), different types of relationships are established between the represented concepts. These relationships are characterized by their influence on the change of values among the associated concepts [9]. Next, the three main types of causality relationships are described[11, 12]:

1. Positive Causality: When the weight of the relationship between two concepts, represented by W_{ij}, is greater than zero ($W_{ij} > 0$), it indicates a positive causality between the concepts C_i and C_j. In other words, an increase (or decrease) in the value of concept C_i leads to an increase (or decrease) in the value of concept C_j.
2. Negative Causality: If the weight of the relationship between two concepts, W_{ij}, is less than zero ($W_{ij} < 0$), a negative causality between the concepts C_i and C_j is established. This implies that an increase (or decrease) in the value of C_i leads to a decrease (or increase) in the value of C_j.
3. Absence of Relationships: When the weight of the relationship between two concepts is null ($W_{ij} = 0$), it is concluded that there is no direct causal relationship between C_i and C_j.

A directed graph, in which nodes represent concepts and edges represent causal relationships, can be used to represent an FCM. Fuzzy values show the intensity of the causal relationship. Each step of the simulation involves calculating the values of the concepts. The initial vector will determine whether the FCM will converge to a fixed point, limit cycle, or chaotic attractor[13, 14].

In this article, the calculation will be developed as follows[15]:

1. Selection of the factors of interest
2. Elaboration of the adjacency matrix.
3. Static analysis: calculated for the absolute values of the adjacency matrix:
   - *Outdegree*, denoted by $od(v_i)$, is the sum for each row of the absolute values of a variable from the fuzzy adjacency matrix. It is a measure of the accumulated strength of the existing connections in the variable.
   - *Indegree*, denoted by $id(v_i)$, is the sum for each column of the absolute values of a variable from the fuzzy adjacency matrix. It measures the accumulated strength of input to the variable.
   - The *centrality* or *total degree* of the variable is the sum of $od(v_i)$ with $id(v_i)$, as indicated below:

$$td(v_i) = od(v_i) + id(v_i) \quad (1)$$

Finally, the variables are classified according to the following criteria, see[14]:

a) The transmitting variables are those with $od(v_i) > 0$ and $id(v_i) = 0$.
b) The receiving variables are those with $od(v_i) = 0$ and $id(v_i) > 0$.
c) Ordinary variables satisfy both $od(v_i) \neq 0$ and $id(v_i) \neq 0$.

They are ordered in ascending order according to the degree of centrality.

The adjacency matrix is created using an aggregation operator, such as the arithmetic mean, when a set of experts (k) is involved. The simplest method is to calculate the arithmetic mean of each connection for each expert. The final adjacency matrix of the FCM (E) for k experts is obtained as [16]:

$$E = \frac{(E_1 + E_2 + \cdots + E_k)}{k} \quad (2)$$

The creation of collective mental models is relatively easy thanks to this aggregation ease.

The relationships in fuzzy cognitive maps can be expressed using many-valued logic, such as computing with words, hesitant, and neutrosophic logic [17, 18, 19]. Neutrosophic logic, in particular, allows for greater flexibility by incorporating the notions of truth, indeterminacy, and falsity, making it suitable for dealing with uncertainty and hesitation in decision-making processes. Using neutrosophic logic in fuzzy cognitive maps enhances the ability to model complex, real-world systems with a more nuanced representation of uncertainty and conflicting information[20, 21].

3. Materials and Methods

This study utilized a Fuzzy Cognitive Map (FCM) to analyze the factors influencing judicial impartiality in Ecuador. The methodology was based on the consensus of a panel of experts who identified the key concepts impacting impartiality in the Ecuadorian judicial system.

A group of 12 specialists in law and justice, including judges, academics, and lawyers with experience in the Ecuadorian judicial system, were selected. These experts participated in structured interviews and workshops to identify the most relevant factors related to judicial impartiality.

The experts provided a list of key concepts through a modified Delphi process, involving rounds of discussion and review. The agreed-upon concepts were: judicial independence, corruption, transparency, judicial training, accountability, access to justice, procedural guarantees, and supervision and control. Using these concepts, the FCM was constructed with the Mental Modeler software.

The procedure is as follows:

1. Construction of the Fuzzy Cognitive Map

The Fuzzy Cognitive Map was built using an Adjacency Matrix, which describes the influences between the identified concepts. Each cell in the matrix contains a value representing the magnitude and direction of influence between pairs of concepts, as assessed by the experts. In this case, the weights are expressed using linguistic terms and then represented as singlevalued neutrosophic numbers according to the scale shown in Table 1.

Table 1: Linguistic Terms and Corresponding Neutrosophic Values

Linguistic Term	Neutrosophic Values $\left(TA(x), IA(x), FA(x)\right)$
Strong Negative Influence	(0,0,-1)
Negative Influence (NI)	(0.25, 0,0)
Neutral (N)	(0.5, 0.5, 0.5
Positive Influence (PI)	(0.75, 0,0.25)
Strong Positive Influence (SPI)	(1,0,0)

The function presented in [22] transformed for the fuzzification process. Given $AN = \{x, \left(TA(x), IA(x), FA(x)\right): x \in X\}$ a NS. Its equivalent fuzzy membership set is defined as $AF = \{\left(x, \mu A(x)\right): x \in X\}$,where $\mu A(x) = s\left(\left(TA(x), IA(x), FA(x)\right), (1,0,0)\right)$. So, using the similarity equation proposed in [22],

$$\mu A(x) = T_A(x) - \max\{I_A(x), F_A(x)\} \quad (3)$$

The range of the similarity measure function is in the interval [-1,1], μA (x) ∈ [-1,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [-1, 1] and satisfies the property of a membership function of a fuzzy cognitive map.

2. Centrality Analysis

A centrality analysis was performed for each concept within the FCM, calculating "Indegree" (incoming influences) and "Outdegree" (outgoing influences) values. Centrality was determined as the sum of both values, helping to assess the overall importance of each node in the system.

3. Nodes Calsification

In a fuzzy cognitive map, nodes are classified into three categories:

- Receivers: Nodes that only receive influences, with outdegree = 0.
- Ordinary: Nodes that both receive and emit influences, with indegree and outdegree greater than 0.
- Transmitters: Nodes that primarily emit influences, with low or zero indegree, and high outdegree.

This classification helps to understand the flow of influences within the system.All analyses were conducted using Mental Modeler software, which facilitated both the construction of the cognitive map and the calculations for centrality and weighted sum. This software is particularly suited for modeling complex systems with qualitative and fuzzy variables, as is the case with judicial impartiality.

4. Results

From the consensus among the experts, eight relevant concepts for an individual cognitive map centered on judicial impartiality in Ecuador were determined. Below the reasons for their relevance are explained and listed.

1. Judicial independence: The independence of the judiciary is fundamental to ensure that judges can make decisions free from external influences, contributing to the impartiality of the judicial system.
2. Corruption: Corruption can undermine judicial impartiality by compromising the integrity of judges and the judicial process, negatively affecting public trust in the judicial system.
3. Transparency: Transparency in the judicial system is crucial to maintaining public confidence and ensuring that judicial decisions are made fairly and impartially.
4. Judicial training: Proper training of judges and judicial personnel is essential to ensure they can perform their duties impartially and effectively.
5. Accountability: Holding judges accountable for their actions is important to ensure responsibility and integrity in the exercise of their duties, contributing to the impartiality of the judicial system.
6. Access to justice: Ensuring equitable access to justice for all citizens is essential to maintain the impartiality of the judicial system and prevent unfair discrimination.
7. Procedural guarantees: Procedural guarantees, such as the right to a fair trial and due process of law, are fundamental to protecting individual rights and ensuring impartiality in the judicial system.
8. Supervision and control: Effective supervision and control of judicial activities are important to prevent abuses of power and ensure that the judicial system operates impartially and transparently.

These concepts are fundamental to understanding and addressing the challenges related to judicial impartiality in Ecuador, as well as to promoting a fair, transparent, and effective judicial system (Figure 1) and (Tables 2 and 3).

Table 2: Adjacency Matrix.

	Judicial independence	Transparency	Accountability	Access to justice	Corruption	Procedural guarantees	Supervision and control	Judicial training
Judicial independence	(0.5, 0.5, 0.5) **N**	(0.75, 0, 0.25) PI	(0.75, 0, 0.25) PI	(0.75, 0, 0.25) PI	(0.75, 0, 0.25) PI	(0.75, 0, 0.25) PI	(0.75, 0, 0.25) PI	(0.5, 0.5, 0.5) **N**
Transparency	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.75, 0, 0.25) **PI**	(1, 0, 0) **SPI**	1, 0, 0) **SPI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**
Accountability	(0.25, 0, 0.75) **NI**	(0.75, 0, 0.25) **PI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**
Access to justice	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N** 0	(0.5, 0.5, 0.5) **N** 0	(0.5, 0.5, 0.5) **N**	(0.25, 0, 0.75) **NI**	(0.75, 0, 0.25) **PI**	(0.5, 0.5, 0.5) **N**	(0.75, 0, 0.25) **PI**
Corruption	(0.5, 0.5, 0.5) **N**	(0, 0, 1) **SNI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.25, 0, 0.75) **NI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**
Procedural guarantees	(0.75, 0, 0.25) **PI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.75, 0, 0.25) **PI**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**
Supervision and control	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.75, 0, 0.25) **P**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**
Judicial training	(0.5, 0.5, 0.5) **N** 0	(0.5, 0.5, 0.5) **N** 0	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**	(0.5, 0.5, 0.5) **N**

Fuzzification of the values occur cording to (3) graphically equation 3.

Table 3: Adjacency Matrix associated after de fuzzification process.

	Judicial independenc e	Transparenc y	Accountabilit y	Acces s to justice	Corruptio n	Procedur al guarantee s	Supervisio n and control	Judicia l trainin g
Judicial independenc e	0	0.5	0.5	0.5	0.5	0.5	0.5	0
Transparency	0	0	0.5	1	1	0	0	0
Accountabilit y	-0.5	0.5	0	0	0	0	0	0
Access to justice	0	0	0	0	-0.5	0.5	0	0.5
Corruption	0	-1	(0	0	0	-0.5	0	0
Procedural guarantees	0.5	0	0	0.5	0	0	0	0
Supervision and control	0	0	0.5	0	0	0	0	0
Judicial training	0	0	0	0	0	0	0	0

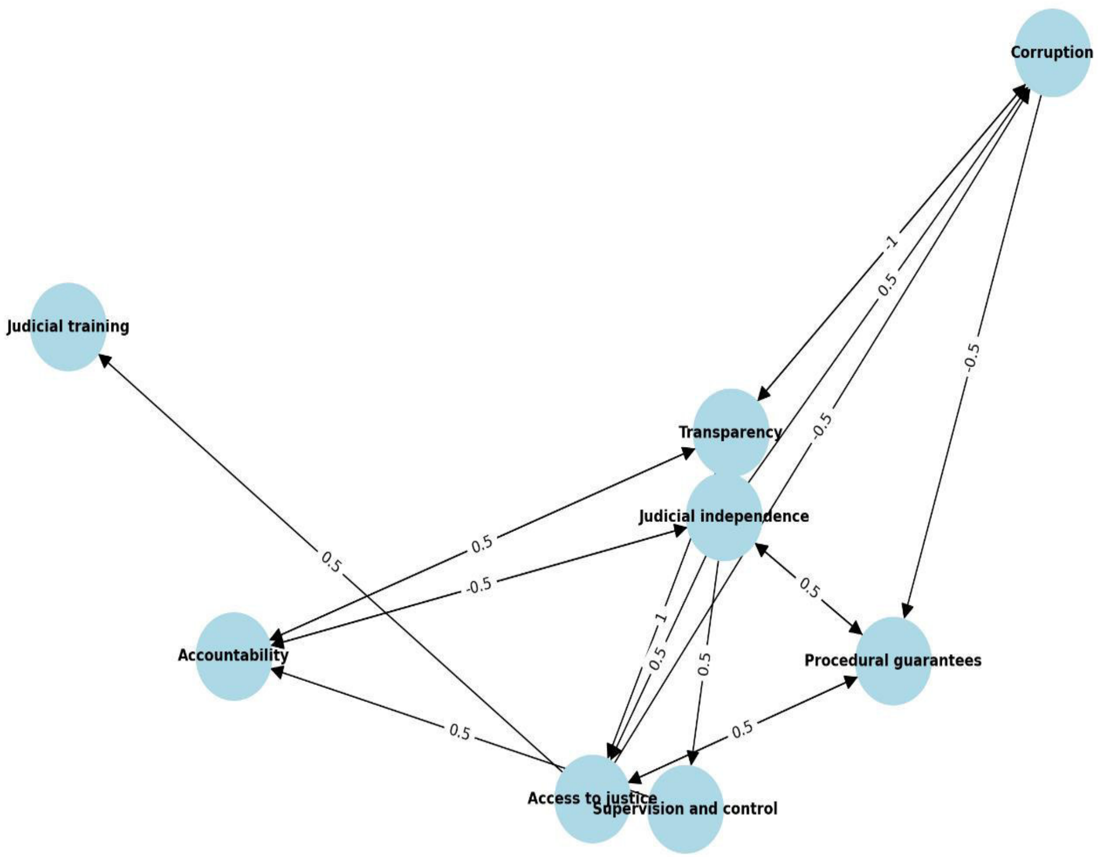

Figure 1. Fuzzy Cognitive Map of the Judicial System

The centrality analysis for each element is presented in the following table:

Table 4: Centralized Analysis. Note: Mental Modeler software was used for method execution. Source: own elaboration.

Components	Indegree	Outdegree	Centrality	Type
Judicial independence	3	1	4	Ordinary
Transparency	2	2.5	5.5	Ordinary
Accountability	1.5	1	2.5	Ordinary
Access to justice	2	1.5	3.5	Ordinary
Corruption	2	1.5	3.5	Ordinary
Procedural guarantees	1.5	1	2.5	Ordinary
Supervision and control	0.5	0.5	1	Ordinary
Judicial training	0.5	0	0.5	Ordinary

The centrality analysis of the judicial system components reveals the following key insights:

- Judicial independence and Transparency emerge as the most influential nodes, with high centrality values (4 and 5.5, respectively). Both nodes exhibit strong incoming and outgoing influences, classifying them as Ordinary nodes.
- Access to justice, Corruption, and Procedural guarantees hold moderate centrality values (between 2.5 and 3.5), indicating their importance within the system. These are also classified as Ordinary.
- Accountability and Supervision and control display lower centrality but remain Ordinary nodes due to their capacity to both influence and be influenced.
- Judicial training has the lowest centrality (0.5) and outdegree = 0, indicating its limited role in the system, although it is classified as Ordinary.

This classification helps to highlight the varying degrees of influence and importance among the different components within the system.

Conclusion

This study provides valuable insights into the factors influencing judicial impartiality in Ecuador, using a Fuzzy Cognitive Map (FCM) to model the relationships between key concepts. The centrality analysis highlights the prominence of Judicial Independence and Transparency as the most influential factors, suggesting that these areas are crucial for enhancing impartiality. Corruption, Access to Justice, and Procedural Guarantees also hold significant importance, while Judicial Training is identified as having a lesser impact. These results emphasize the need for targeted interventions in these areas to improve the judicial system's effectiveness and public trust.

Additionally, this research has applied neutrosophic logic and fuzzification techniques to determine the weights of the arcs in the FCM, allowing for a more flexible and accurate representation of the relationships between concepts. Future research should further refine this approach by incorporating more dynamic simulations and expanding the range of stakeholders involved. Additionally, integrating machine learning and expert systems with FCMs could enhance predictive capabilities and improve decision-making processes within the judiciary.

References

[1] Villanueva Flores, R. (2021). Impartiality, gender stereotypes and judiciary corruption. *Derecho PUCP*, (86), 363–392.

[2] Leino, M., Kulha, K., Setälä, M., & Ylisalo, J. (2022). Expert hearings in mini-publics: How does the field of expertise influence deliberation and its outcomes? *Policy Sciences*, 55(3), 429–450.

[3] Jáuregui Bustamante, K. del R., & Arista Bustamante, L. L. (2023). The incidence of algorithmic bias in the predictive justice of the judicial system. *TZHOECOEN*, 15(2), 79–97.

[4] Fiol Matta, L. (2019). Unconscious prejudices and access to justice. *Revista de Derecho de Puerto Rico*, 59, 173.

[5] Piñas Piñas, L. F., Viteri Naranjo, B. D. C., & Freire Sánchez, N. F. (2023). The solution of conflicts in the model of the constitutional state of rights and justice. *Russian Law Journal*, 11(13S), 234–239.

[6] González Vélez, A. C., & Jaramillo, I. C. (2017). Legal knowledge as a tool for social change: La mesa por la vida y la salud de las mujeres as an expert on Colombian abortion law. *Health and Human Rights*, 19(1), 109–118.

[7] Chamorro Valencia, D., Molina Gutiérrez, T. de J., Burbano, L. H., & Cadena Posso Garcia, A. A. (2019). Case-based reasoning and neutrosophic logic to identify the employment limitations for law school graduates at UNIANDES Ibarra. *Neutrosophic Sets and Systems*, 26(Special Issue), 82–90.

[8] Huarhua Beltran, R. M., & Navarrete Dilas, D. M. (2023). Public ethics and dignity in the judiciary within the judicial system: Proposals for its maximization. *Revista de Climatología*, 23(Special Issue), 524–534.

[9] Calderón Ramírez, M. A., et al. (2019). Pestel based on neutrosophic cognitive maps to characterize the factors that influence the consolidation of the neo constitutionalism in Ecuador. *Neutrosophic Sets and Systems*, 26(Special Issue), 61–68.

[10] Aguilar Berrezueta, R., Mayorga Aldaz, E., Estrella López, B. C., & Arrunátegui Aguirre, J. A. (2023). Neutrosophic assessment of personality traits. *Neutrosophic Sets and Systems*, 62(1), 137–145.

[11] Hasan Al-subhi, S., Román Rubio, P. A., Piñero Pérez, P., García Vacacela, R., & Mahdi, G. S. S. (2020). Neutrosophic clinical decision support system for the treatment of pregnant women with heart diseases. *Investigación Operacional*, 41(5), 773–783.

[12] Castillo Villacrés, H. P., Machado Maliza, M. E., & Tixi Torres, D. F. (2021). Method for recommending guardianship to minors based on parental responsibility using a fuzzy cognitive map. In W. Pedrycz, L. Martínez, R. A. Espin-Andrade, G. Rivera, & J. Marx Gómez (Eds.), *Computational Intelligence for Business Analytics* (pp. 245–258). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-73819-8_14

[13] Teruel, K. P., Vázquez, M. Y. L., & Sentí, V. E. (2014). Consensus process in mental models and its application to agile software development in bioinformatics. *Revista Cubana de Información en Ciencias de la Salud (ACIMED)*, 25(3), 317-331.

[14] Khajeh, F., & Shahbandarzadeh, H. (2023). Modeling factors affecting the interests of the container terminal using fuzzy cognitive map and fuzzy DEMATEL. *Journal of Modelling in Management*, 18(5), 1364-1388.

[15] Khajeh, F., & Shahbandarzadeh, H. (2023). Modeling factors affecting the interests of the container terminal using fuzzy cognitive map and fuzzy DEMATEL. *Journal of Modelling in Management*, 18(5), 1364-1388.

[16] Papageorgiou, E. I., & Salmeron, J. L. (2012). A review of fuzzy cognitive maps research during the last decade. *IEEE Transactions on Fuzzy Systems*, 21(1), 66–79.

[17] Liu, X., Wang, Z., Zhang, S., & Liu, J. (2019). A novel approach to fuzzy cognitive map based on hesitant fuzzy sets for modeling risk impact on electric power system. *International Journal of Computational Intelligence Systems*, 12(2), 842–854.

[18] Pérez-Teruel, K., Leyva-Vázquez, M., & Estrada-Sentí, V. (2015). Mental models consensus process using fuzzy cognitive maps and computing with words. *Ingeniería y Universidad*, 19(1), 173–188.

[19] Ghaderi, H., Gitinavard, H., Mousavi, S. M., & Vahdani, B. (2017). A hesitant fuzzy cognitive mapping approach with risk preferences for student accommodation problems. *International Journal of Applied Management Science*, 9(4), 253–293.
[20] Vázquez, M. L., Franco, P. E. D. P., & Palacio, A. J. P. (2022). Neutrosophic DEMATEL in the analysis of the causal factors of youth violence. *International Journal of Neutrosophic Science*, 18(3), 199-99.
[21] Ali, M. A., & Muthuswamy, M. (2023). Neutrosophic multi-criteria decision-making framework for sustainable evaluation of power production systems in renewable energy sources. *Sustainable Machine Intelligence Journal*, 4(3), 1–10.
[22] Mandal, K. (2020). On deneutrosophication. *Neutrosophic Sets and Systems*, 38, 409–423.

Received: June 28, 2024. Accepted: August 20, 2024

Neutrosophic Sets and Systems {Special Issue: NeutroGeometry, NeutroAlgebra, and Their Applications}, Vol. 71, 2024

University of New Mexico

Determining the Relationship between Interculturality and Bilingualism in Bilingual Teaching in Peru Based on Plithogenic Statistics

Rafael Pinedo Coral[1], Aniceto Elías Aguilar Polo[2], Juan Ernesto Ríos Ángeles[3], Edgar Ricardo Yauri Rivera[4], Janett Karina Vásquez Pérez [5], Liliana Huaranga Rivera[6], Roberth Lozano Tacuri Toribio[7], and Miriam Esther Campos Llana[8]

[1] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: rpinedoc@unia.edu.pe
[2] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: aagularp@unia.edu.pe
[3] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: ernestoriosangeles@gmail.com
[4] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: eyaurir@unia.edu
[5] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: jvasquezp@unia.edu.pe
[6] National Intercultural University of the Amazon, Pucallpa, Perú. E-mail: lhuarangar@unia.edu.pe
[7] Daniel Alcides Carrion National University Pasco, Perú. E-mail: rtacurit@undac.edu.pe
[8] Daniel Alcides Carrion National University Pasco, Perú. E-mail: mcamposl@undac.edu.pe

Abstract. The development of this research was propitiated as its main goal to determine the level of relationship that exists between interculturality and bilingualism in bilingual teachers of regular basic education in the rural area of Callería, Ucayali, Peru, during 2023. The study was developed through simple random sampling, in the population that is made up of 581 bilingual teachers. The sample comprises 174 of these teachers, to whom two questionnaires were applied as data collection instruments prepared from the survey technique; thus concluding that there is a direct and significant relationship between interculturality and bilingualism. We determined that the use of traditional statistical methods is insufficient to carry out a complete study because there is uncertainty in the data collected since we wanted to consider the degree of mastery of the language by each teacher. The study is based on plithogenic statistics because this theory is dedicated to studying the indeterminacy in complex phenomena, where several factors influence the object of study. The plithogenic statistics is an extension of Multivariate Statistics.

Keywords: Interculturality, bilingualism, Plithogeny, Plithogenic Statistics, Single-Valued Neutrosophic Number, Plithogenic Neutrosophic Probability, Spearman Rho coefficient.

1 Introduction

The analysis of educational processes must start from an understanding of the historical and social conditions in which they occur. In this sense, intercultural education is found within the context of the political, social, economic, and cultural projects of globalization. Globalization, with its face of internationalization, has fragmented cultural identities and transformed customs, traditions, and ways of life. Faced with such acculturation processes, we find ourselves today in the 21st century with a "crisis of progress" in which the universe of techniques, markets, and finances is dissociated on the one hand, and the inner universe, that which we call our identity, on the other.

Interculturality in a pluricultural and multilingual society emphasizes its attention on the communication search, also on the recognition of existing cultural differences, and not forgetting the roots that gave rise to our national culture.

Communication is conceived as sharing, interaction, the action of sharing values, and a way of establishing links and relationships between people. Communication, from this perspective, is the basic process for the construction of life in society; it is the activating mechanism of dialogue and coexistence between subjects. In this context, communication can be considered the concrete and objective manifestation of the permanent processes of reconstruction of the different contexts of reality that we build and cultivate in everyday life. Thus, communication is the only way we have to get in touch with others and, even when we do not realize how much we depend on it, it constitutes the center of our existence.

In Latin America, interculturality was born under the auspices of related educational modalities called ethnoeducation, indigenous education, and/or intercultural bilingual education, in response to policies of extermination, assimilation, and forced integration directed at different indigenous peoples, which were developed by the national

States of the region. From there, it initially emerged as a proposal for recognition and reparation of the fundamental rights of these peoples, and throughout the 20th century, it was assumed from different paradigms (compensatory, integrationist, democratizing, resistance) and emphases regarding the way of approaching Indigenous school education.

There are still tendencies or thoughts in the different schools, that teachers tend to associate and approach interculturality more as a content - mainly languages and certain knowledge and practices of the main indigenous cultures officially recognized in the country - than as an educational approach. Therefore, interculturality as part of a study carried out, was associated with the teaching of languages such as *Mapuzugun* or *Aymara*, or certain aspects of the cultures of these or other indigenous peoples present in the country, but not to an approach to the teaching-learning process that implies, for example, putting these worldviews into dialogue with the worldview of the hegemonic culture in the national educational system, on an equal footing in terms of its legitimacy, or problematizing one's own culture based on the different cultural perspectives that are present in the school, to mention just a few ways in which the intercultural educational approach could materialize.

The process of globalization essentially ignores the human aspect, giving significant weight to innovation in educational processes as a necessary element for the formation of people and groups, which is why innovation is required to encompass not only what is traditionally understood, but also advance to intercultural training as a human quality.

All of this implies that the planet is immersed in a maelstrom of diffusion, development, and consolidation of neo-positivist and neo-liberal economic unidimensional philosophical currents; of information and communication technologies (ICT) and computing; of new scenarios, challenges and uncertainties; of mass education; of educational currents based on systemic educational technology; as a context of tendencies and causalities, in which diverse cultures of humanity develop, and on which educational models with tendencies towards cultural homogeneity and at the same time heterogeneity of reality and society, opposed to interculturality, impact.

Our focus is also on bilingualism, which broadly refers to the ability to express oneself in two languages. When a person is fluent in more than one language, he or she is called bilingual. Bilinguals are people who can, to any degree, understand or produce written or spoken expressions in more than one language. We can see that they have a very high command of both languages, both written and spoken. They demonstrate comprehension skills and/or abilities, depending on the experience in which they are called to use both languages.

Bilingualism is therefore a phenomenon that is studied from two major perspectives: psycholinguistics and sociolinguistics. From the psycholinguistic perspective, the interest is in the bilingual subject considered individually and the emphasis is placed on the psychological processes involved in bilingual learning and behavior, that is, all those factors that have an impact on the capacity and mastery of the different languages that the subject acquires. From the sociolinguistic perspective, the interest is focused on the linguistic community, on the social group where the use of the different languages converges, and the determinants that influence the interrelation of these considering their historical evolution and their social, demographic, and cultural context.

In the context of bilingual classes, in Peru, its educational system adopted a coordinated bilingualism, under the criterion of the relationship between language and thought; since it has different cognitive units for the linguistic units depending on whether they are in L1 or L2. In this sense, the academic training process of education professionals does not respond to the challenges of the 21st century, which requires integration to face globalization, especially based on social needs and intercultural processes.

This paper aims to statistically demonstrate that there is a direct and significant relationship between interculturality and bilingualism in bilingual teachers of regular basic education in the rural area of Callería. To meet this objective, Plithogenic Statistics methods are applied [1]. Neutrosophy is the branch of philosophy that studies neutrality, which contains the neutral, the unknown, the contradictory, the erroneous, the inconsistent, and the paradoxical, among others. More recently, F. Smarandache created the Plithogeny theory where the dynamic relationship between different concepts is captured, with what Smarandache called the AntiConcepts and the NeutroConcepts (which are neither Concepts nor Anticoncepts) [2-4]. This replaces the traditional dialectic which contains only Concepts and AntiConcepts.

The Plithogenic Statistics generalizes the Neutrosophic Statistics, where not only data or parameters are taken into account in the form of intervals as in the latter one, but also any set where there is indeterminacy, either in the form of intervals or discrete values, or mixed [5-8]. The Plithogenic Statistics generalizes multivariate statistics.

This article uses Plithogenic techniques Statistics to determine the relationship between interculturality and bilingualism. We considered using plithogenic statistics because it allows us the study complex phenomena, where multiple factors cannot be studied in a simplified way. Interculturality and language phenomena are these types of phenomena since they depend on culture, psychology, sociology, education, and politics, among many other concepts that interact with each other to emerge in complex situations.

The article is divided into a Materials and Methods section, where the main concepts of Plithogenic Probability and Plithogenic Statistics are explained. The next section is devoted to presenting the results of the study. The article concludes with a Conclusions section.

2 Materials and Methods

The Plithogenic Probability of an event is composed of the probabilities of the event occurring for all the random variables or parameters that constitute it [6, 9-15]. The Plithogenic Probability based on the Plithogenic Variation Analysis, is multi-dimensional. It could be said that it is a probability of sub-probabilities, where each sub-probability refers to the behavior of a variable by assuming that the event is produced by one or more variables. Each variable is represented by a Probability Distribution (Density) Function (PDF).

According to the classification of F. Smarandache, the Subclasses of Plithogenic Probability are as follows:

(1) **Classical MultiVariate Probability**: If all PDFs are classical.

(2) **Plithogenic Neutrosophic Probability** is defined when the PDF is expressed in the form of $(\mathrm{T}, \mathrm{I}, \mathrm{F})$, where T is the probability that the event occurs, I is the probability of indeterminacy that the event occurs and F is the probability that the event does not occur. Such that the following is fulfilled: $\mathrm{T}, \mathrm{I}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T}+\mathrm{I}+\mathrm{F} \leq 3$.

(3) **Plithogenic Indeterminate Probability**: When all PDFs have indeterminate data or arguments.

(4) **Plithogenic Intuitionistic Fuzzy Probability**: When PDFs have the form (T, F) where $\mathrm{T}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T}+\mathrm{F} \leq 1$.

(5) **Plithogenic Picture Fuzzy Probability**: When PDFs have the form $(\mathrm{T}, \mathrm{N}, \mathrm{F})$. $\mathrm{T}, \mathrm{N}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T}+\mathrm{N}+\mathrm{F} \leq 1$; where T is the probability that the event occurs, N is the neutral probability that the event occurs or not, and F is the probability that the event does not occur.

(6) **Plithogenic Spherical Fuzzy Probability**: When PDFs have the form $(\mathrm{T}, \mathrm{H}, \mathrm{F})$. $\mathrm{T}, \mathrm{H}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T}^2+\mathrm{H}^2+\mathrm{F}^2 \leq 1$; where T is the probability that the event occurs, H is the neutral probability that it occurs or not, and F is the probability that the event does not occur.

(7) **Plithogenic (fuzzy-extension) Probability**: when we have that all PDFs are in the form of (fuzzy-extension set) style.

(8) **Plithogenic Hybrid Probability**: When some PDFs are in one of the above styles and others are in other styles.

Plithogenic Statistics (PS) comprises the analysis and observations of the events studied by the Plithogenic Probability.

Plithogenic Statistics generalizes the classical Multivariate Statistics, which in turn allows an analysis of many output variables that are neutrosophic or indeterminate. It is also a multi-indeterminate statistic.

Various Subclasses of Plithogenic Statistics are the following:

- Multivariate Statistics,
- Plithogenic Neutrosophic Statistics,
- Plithogenic Indeterminate Statistics,
- Plithogenic Intuitionistic Fuzzy Statistics,
- Plithogenic Picture Fuzzy Statistics,
- Plithogenic Spherical Fuzzy Statistics,
- and in general: Plithogenic (fuzzy-extension) Statistics,
- and Plithogenic Hybrid Statistics.

On the other hand, Plithogenic Refined Statistics are the most general form of statistics that studies the analysis and observations of the events described by the Plithogenic Refined Probability.

In classical inference statistics estimates the population's average of the variable from the sample's average.

When we have a classical random variable, the exact sample size is known and all elements of the sample belong 100% to the population. However, this does not reflect the dynamics of a population such as the student population, which is the example illustrated by F. Smarandache, where there is fluctuation of students within courses, in addition to the fact that the membership of each student varies depending on whether he or she is studying a course full-time, part-time or over-time [1].

In a Neutrosophic Population, each element has a triple probability of membership equal to (T_j, I_j, F_j) such that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume we have the dataset (T_j, I_j, F_j) for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability for all data in the sample is calculated by Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

3 The Plithogenic Study

The concepts (variables) on which this research is based are:

Variable 1: Interculturality

Interculturality is the set of political, social, legal, and educational processes generated by the interaction of cultures in a relationship of reciprocal exchanges caused by the presence in the same territory of human groups with different origins and histories. This variable can be operationalized through the dimensions: of “intercultural skills”, “intercultural knowledge”, and “intercultural attitudes”.

Variable 2: Bilingualism

Bilingualism is not the perfect and identical command of two languages, but the ability to use two or more languages in different contexts and with different modalities. This variable can be operationalized through the dimensions of “language” and “communication”.

Table 1 contains the details of each dimension mentioned above:

Table 1. Variables used in the study, their dimensions, and definitions.

VARIABLE	DIMENSIONS	INDICATORS
Interculturality	Intercultural Knowledge	Identify the local culture and that of students whose parents come from other cultures: stories, beliefs, songs, games, instruments, etc.
		Describe general characteristics of the cultures from which the parents or grandparents of their students come, such as geographic location, physical features, ways of speaking, cultural manifestations, etc.
		Identify how cultures of regional origin are formed throughout history according to thoughts, beliefs, and ideas.
		Identify words from other cultures (names, foods, beliefs, etc.).
	Intercultural Attitudes	Shows openness to other cultural manifestations different from one's own.
		Show empathy towards other cultural realities.
		It shows an appreciation of the implications of cultural diversity in society.
		Shows willingness to resolve conflicts.
	Intercultural Skills	Participate positively in group activities that involve interaction between peers from different cultural manifestations, such as games, making murals, or storytelling.
		Recognizes foreign cultural practices and manifestations.
		Identify the negative aspects of discrimination or prejudice towards different cultural manifestations.
Bilingualism	Language	Recognizes the code systems of his/her language.
		Rules of use.
		Values the use of his/her language in different contexts.
		Uses his/her native language and second language to communicate, taking into account the contexts.
	Communication	Makes use of different types of communication in his/her daily activities.
		Recognizes the different ways of communicating in his/her family and community, also considering the moments.
		Uses assertive and respectful forms of communication to achieve goals and objectives.
		Assess the development of his/her writing using various connectors appropriately, allowing for a clear sentence.
		Communicates ideas coherently and accurately, using both his/her native language and his/her second language.

There is a population of 581 bilingual teachers of Regular Basic Education from the district of Callería. The sample is made up of 174 teachers selected by simple random sampling. Each of them was asked to evaluate how they perceived each of the dimensions corresponding to the two variables studied. For this purpose, a linguistic measurement scale was used, where each linguistic value has an associated Single-Valued Neutrosophic Number (SVNN) in the form of (T, I, F).

Table 2. The linguistic scale and the Single- Valued Neutrosophic Numbers were used in the survey of the study carried out.

Linguistic Value	Associated Single-Valued Neutrosophic Number
Unfavorable	(0.1,0.1,0.8)
Moderately Favorable	(0.55,0.1,0.35)
Favorable	(0.8,0.1,0.1)

Table 3 contains a summary of the results of the surveys regarding interculturality:

Table 3. General results obtained from the survey on interculturality and its dimensions.

	Interculturality		Intercultural knowledge		Intercultural attitudes		Intercultural skills	
	f	%	f	%	f	%	f	%
Unfavorable	0	0	10	5.7	0	0	5	2.9
Moderately favorable	165	94.8	164	94.3	170	97.7	159	91.4
Favorable	9	5.2	0	0	4	2.3	10	5.7
Total	174	100	174	100	174	100	174	100

Table 4 summarizes the results regarding bilingualism:

Table 4. General results obtained from the survey on bilingualism and its dimensions.

	Bilingualism		Language		Communication	
	f	%	f	%	f	%
Unfavorable	15	8.6	15	8.6	15	8.6
Moderately favorable	135	77.6	135	77.6	145	83.3
Favorable	24	13.8	24	13.8	14	8.0
Total	174	100	174	100	174	100

From these tables, the first plithogenic probabilities are obtained:

If x is a teacher of the population which is studied, we have the Plithogenic Neutrosophic Probability as follows:

$PNP_{InterCult}(x) = \big((0.52435, 0.1, 0.37565); (0.55575, 0.1, 0.34425); (0.5512, 0.1, 0.3488)\big)$,

$PNP_{Biling}(x) = \big((0.5458, 0.1, 0.3542); (0.5307, 0.0999, 0.36835)\big)$.

That is, we infer from $PNP_{InterCult}(x)$ that the probability of x having intercultural knowledge is approximately 52%, with 10% of indeterminacy and 37.6% of falsehood. For intercultural attitudes, this is expressed in approximately 56%, 10% of indeterminacy, and 34% of dissatisfaction. For intercultural skills, there is practically 55% probability, 10% of indeterminacy, and around 35% of non-occurrence.

Regarding language skills, there is approximately a 56% probability of occurrence, 10% of indeterminacy, and 35% of non-occurrence, while regarding communication, there is a probability of approximately 53% of occurrence, almost 10% of indeterminacy, and 37% of non-occurrence.

Let us do a logical calculation, where the probabilities of each variable are aggregated using the following n-norm given both, $x = (T_x, I_x, F_x)$ and $= \big(T_y, I_y, F_y\big)$ [16, 17]:

$$N(x, y) = \big(\min(T_x, T_y), \max(I_x, I_y), \max(F_x, F_y)\big) \quad (2)$$

Therefore, the aggregated probability for Interculturality is $(0.52435, 0.1, 0.37565)$, and for Bilingualism is $(0.5307, 0.1, 0.36835)$. These results indicate that the opinion on both variables is around moderately favorable.

Let us now analyze the relationship between both variables. Table 5 contains the summary of applying the Spearman Rho coefficient between Interculturality and Bilingualism. In this case, we reduce the multivariate statistics to the different univariate statistics.

Table 5. Application of correlation tests using Spearman's Rho coefficient between interculturality and bilingualism.

			Interculturality	Bilingualism
Spearman's Rho	**Interculturality**	Value	$(1.000, 1.000, 1.000)$	$(0.502, 1.000, 0.486)$
		Significance (Bilateral)	-	$(0.000, 0.000, 0.000)$
		N	174	174
	Bilingualism	Correlation coefficient	$(0.502, 1.000, 0.486)$	$(1.000, 1.000, 1.000)$
		Significance (Bilateral)	$(0.000, 0.000, 0.000)$	-
		N	174	174

Table 6 shows the results of applying Spearman's Rho coefficient to determine the relationship between interculturality and language.

Table 6. Application of correlation tests using Spearman's Rho coefficient between interculturality and language.

			Interculturality	Language
Spearman's Rho	**Interculturality**	Value	$(1.000, 1.000, 1.000)$	$(0.529, 1.000, 0.467)$
		Significance (Bilateral)	-	$(0.000, 0.000, 0.000)$
		N	174	174
	Language	Correlation coefficient	$(0.529, 1.000, 0.467)$	$(1.000, 1.000, 1.000)$
		Significance (Bilateral)	$(0.000, 0.000, 0.000)$	-
		N	174	174

Table 7 shows the correlation between interculturality and communication:

Table 7. Application of correlation tests using Spearman's Rho coefficient between interculturality and communication.

			Interculturality	Communication
Spearman's Rho	**Interculturality**	Value	$(1.000, 1.000, 1.000)$	$(0.498, 0.999, 0.501)$
		Significance (Bilateral)	-	$(0.000, 0.000, 0.000)$
		N	174	174
	Communication	Correlation coefficient	$(0.498, 0.999, 0.501)$	$(1.000, 1.000, 1.000)$
		Significance (Bilateral)	$(0.000, 0.000, 0.000)$	-
		N	174	174

Applying Equation 2 to the Spearman Rho values that appear in Tables 5-7, the value obtained is $(0.498, 1.000, 0.501)$. The result is a moderate and direct correlation between the two variables studied.

Conclusion

With this article, we set out to study whether there is a relationship between the concepts of interculturality and bilingualism in Peruvian bilingual schools. To do so, we took as an object of study a sample of size 174 from a population of 581 bilingual teachers of Regular Basic Education in the area of the Callería district in Peru. The teachers were asked to fill out questionnaires determining their opinions on interculturality and bilingualism within their educational centers. For the evaluation, we used a linguistic scale with values associated with Single-Valued Neutrosophic numbers. In the processing of the data obtained, we use the Plithogenic Statistics. In general, it was found that the highest probability is that the teacher has an average performance in each of the variables; in addition, the correlation between both variables is moderate and direct with the application of the Spearman Rho coefficient adapted to the Plithogenic Neutrosophic Probabilities.

References

[1] Sarzosa, J. F. T., and Oña, C. (2024). Using Plithogenic Statistics to Determine Optimal Guinea Pig Manure Dosing in Prickly Pear Cultivation. Neutrosophic Sets and Systems, 69, 58-66.
[2] Smarandache, F. (2017). Plithogeny, plithogenic set, logic, probability, and statistics. Infinite Study.
[3] Smarandache, F. (2022). Plithogeny, plithogenic set, logic, probability and statistics: a short review. Journal of Computational and Cognitive Engineering, 1, 47-50.
[4] Smarandache, F. (2023). An overview of Plithogenic set and symbolic Plithogenic algebraic structures. Journal of fuzzy extension and applications, 4, 48-55.
[5] Sánchez, F., Almeida Blacio, J. H., Flores Bracho, M. G., Andrade Santamaria, D. R., and Sánchez Casanova, R. (2021). Neutrosophic and Plithogenic Statistical Analysis in Educational Development. Neutrosophic Sets and Systems, 44, 223-234.
[6] Smarandache, F. (2022). Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (Third version). Bulletin of Pure & Applied Sciences-Mathematics and Statistics, 41, 172-183.
[7] Martin, N., Smarandache, F., and Sudha, S. (2023). A novel method of decision making based on plithogenic contradictions. Neutrosophic Systems with Applications, 10, 12-24.
[8] Smarandache, F. (2023). Introduction and advances to neutrosophic probability and statistics and plithogenic probability and statistics and their applications in bioinformatics and other fields (review chapter). In Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics (pp. 1-23). Academic Press.
[9] De la Cantera, D. H., Quiroz, R. C., Queija, M. L., Gonzalez, J. R., and Leyva Vazquez, M. Y. (2024). Analyzing Interdisciplinary Education in General Medicine Using Smarandache's Multivalued Logic Hypothesis Theory and Plithogenic Probability. Neutrosophic Sets and Systems, 70, 369-377.
[10] Martin, N., Smarandache, F., and Priya, R. (2022). Introduction to Plithogenic Sociogram with preference representations by Plithogenic Number. Journal of fuzzy extension and applications, 3, 96-108.
[11] García, T. C. S., Álvarez, L. E. E., and Arias, M. A. I. (2024). Exploring the Impact of Educational Reforms and Their Dimensions on Teacher Performance Through the Analysis of Plithogenic Statistics. Neutrosophic Sets and Systems, 69, 161-170.
[12] Rezaei, A., Oner, T., Katican, T., Smarandache, F., and Gandotra, N. (2022). A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets. Infinite Study.
[13] Villacres, G. E. F., Naranjo, F. A. V., and Lopez, R. R. L. (2021). Plithogenic Logic for Determination of Strategic Solutions for Ergonomic Occupational Health Risks. Neutrosophic Sets and Systems, 44, 342-357.
[14] Gutiérrez, P. B., Flores-Ledesma, K. N., Gómez, S. L. G., Bulnes, J. L. L., Nakayo, J. L. J., Carranza, C. F. C., and Ayala, A. C. N. (2023). Plithogenic Iadov model to study university teaching practices in the complexity of the educational process of comprehensive training by competencies. Neutrosophic Sets and Systems, 62, 78-85.
[15] Sharma, A., Kumar, V., and Ganaie, I. R. (2023). Some remarks on I (Sθ)-summability via neutrosophic norm. Filomat, 37(20), 6699-6707.
[16] Alkhazaleh, S. (2015). More on Neutrosophic Norms and Conorms. Neutrosophic Sets and Systems, 9, 23-30.
[17] Zeina, M. B., Hemeda, S., Koreny, M., and Abobala, M. (2023). On Symbolic n-Plithogenic Random Variables Using a Generalized Isomorphism. Neutrosophic Sets and Systems, 59, 330-339.

Received: June 28, 2024. Accepted: August 21, 2024

Information about the journal

Neutrosophy provides elements to strengthen Latin American philosophy on solving social problems and on a new rebirth of philosophy, science and art in the continet. This special issue serves as a sample of the advances of neutrosophy in the continent in resolving social problems and the results of the Latin American Association of Neutrosophics Sciences.

Editors-in-Chief of this Neutrosophic Sets and Systems Special Issue: *Proceedings of the International Conference called NeutroGeometry, NeutroAlgebra, and Their Applications* Vol.SI 71, 2024:

Prof. Dr. Florentin Smarandache, Postdoc, Mathematics Department, University of New Mexico, Gallup, NM 87301, USA, Email: smarand@unm.edu.
Dr. Mohamed Abdel-Basset, Faculty of Computers and Informatics, Zagazig University, Egypt, Email: mohamed.abdelbasset@fci.zu.edu.eg.
Prof. Dr. Maikel Leyva Vazquez, Autonomous Regional University of the Andes (UNIANDES Ambato), Ecuador, Email: mleyvaz@gmail.com.
Dr. Said Broumi, Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco, Email: s.broumi@flbenmsik.ma.

ISSN 2331-6055 (print)
ISSN 2331-608X (online)
23316055