Review of *Geometric Possibility* (2011), by Gordon Belot. Oxford and New York: Oxford University Press. x + 219 pp.

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How are we to understand the truth conditions for claims about spacetime geometry, e.g. that a cyclist's front tire is trailing the rear tire of another cyclist by $10cm$, or that both cyclists are accelerating as they go downhill? A substantivalist regards the truth or falsity of such claims as underwritten by geometrical relations among the regions of spacetime occupied by the tires at different times. Yet do we need to treat these claims as parasitic on structural properties of spacetime? A relationalist argues that we only need the geometrical relations among bodies, but then owes us an account of the truth conditions for geometrical claims to replace the substantivalist's. Belot's aim, partially inspired by Leibniz, is to "retool the substantivalist truth conditions so that they demand not that certain patterns of geometric relations be (un)instantiated by actual material points, but rather that the instantiation of such patterns be geometrically (im)possible" (p. 4). The bulk of the book is devoted to a clever and engaging exploration of different ways to flesh out this idea of "geometric possibility," taken as a distinctive kind of modality.

The relationalist needs geometric possibility because a *conservative* relationalism, formulated solely in terms of *actual* configurations of bodies, fails, according to Belot. The conservative hopes that the details of a particular configuration of matter will suffice to fix all the geometrical properties of spacetime. Yet couldn't our cyclists fit into spacetimes with different geometries? How is one to select the "best" geometry into which a given material configuration can be embedded? Belot argues that the conservative cannot solve this selection problem, yielding a unique geometry for any given material configuration, even in a relatively simple case (Chapter 2). The committed relationalist can respond by characterizing truth conditions for geometrical claims in terms of *possible* material configurations rather than only actual ones. In historical terms, Belot sees this shift taking place between Descartes and Leibniz, in whose writing the position is "all but made explicit" (p. 178). Once the proposal is made explicit, it is obvious that the relationalist needs to clarify the nature of this distinctive modality.

Belot identifies three plausible desiderata for an account of geometric possibility. An account is: *grounded* if agreement between the material configurations in two possible worlds (called geometrical duplicates) implies agreement on facts about geometrical possibility; *ambitious* if the relationalist is able to give an account of geometrical possibility that matches the geometrical claims made by the substantivalist; and *metric* if two material configurations instantiating the same distance relations are geometrical duplicates. Yet, Belot argues, no relationalist can satisfy all three. For consider treating geometric possibility, by analogy with physical possibility, as an accessibility relation $\mathcal{G}(w_1, w_2)$ on a space of possible worlds. For the substantivalist, $\mathcal{G}(w_1, w_2)$ holds iff the points and regions of w_1 and w_2 instantiate the same geometries. An ambitious relationalist would like to follow this analysis, but cannot do so while also giving a grounded and metric account. For consider two worlds including a single particle (pp. 5, 80-81), and suppose that a substantivalist can legitimately regard the particle as moving in Euclidean (w_1) or hyperbolic (w_2) space. These two possibilities are $\mathcal{G}$-inaccessible because they do not instantiate the same geometries. Yet the metric relations considered by the relationalist will be the same (trivial) ones in both cases. For a grounded relationalist, that the two cases are geometric duplicates further implies agreement about geometric possibility. A grounded, ambitious, metric relationalist must therefore classify these two geometries as being $\mathcal{G}$-accessible. The three commitments together lead to this incorrect conclusion. The bulk of the book is devoted to considering three accounts of geometric possibility, each of which saves two of the three mutually incompatible desiderata. These proposals are modeled on accounts of physical possibility: the best-systems approach – grounded, metric but unambitious; primitivism – metric, ambitious, but ungrounded; and necessitarianism – grounded, ambitious, but non-metric. Belot is most critical of the best-systems approach, and his line of argument makes a contribution to assessing this approach well beyond the case at hand. He does not advocate either of the remaining approaches, although he regards the necessitarian line as "more intriguing" (p. 136).

The discussion of primitivism (Chapter IV) illustrates Belot's approach to assessing these positions. First, Belot regards a much wider range of geometries as relevant, moving well beyond Euclidean geometry, the "classical" geometries of constant curvature, and Riemannian geometry. Chapter I surveys different ways of characterizing geometry, leading to the position that any *finite metric space* represents a possible spatial structure.[1] Broadening the conception of geometry in this way undermines many arguments that hold in more restrictive settings. A case in point from Chapter IV: congruent regions of space need not be superposable. (Regions A and B are *congruent* iff there is an isometry $\phi : A \rightarrow B$, and *superposable* if there is an isometry such that $A = \phi(B)$, p. 87. Cf. Appendix E, where Belot discusses what is needed for the two to coincide, which he calls "lability.") Consider, for example, a plane with a hemispherical bulge. Two congruent triangles, one drawn on the plane and one over the bulge, fail to be superposable in this sense. Belot discusses this point in the course of refining the sense in which an ambitious relationalist aspires to "mirror" the substantivalist. For in an inhomogeneous space, there are qualitative geometric facts about the space that are not captured by distance relations alone — e.g., that point p is the apex of a hemispherical bulge. This forces a more precise formulation of ambition: what facts does the relationalist have to "mirror" from the substantivalist account? It further leads to parallel discussions of the accessibility relation $\mathcal{G}(w_1, w_2)$ for a metric (§4) and non-metric (§5) primitivist. This line of argument is typical of the book: Belot refines and criticizes the three positions by considering how they fare with a conception of geometry much broader than that usually considered by philosophers. As with the discussion of necessitarianism, the aim is to formulate a position subtle enough to handle the complexities of metric spaces, rather than to criticize or advocate a particular approach to modality. But I expect that many philosophers will find the discussion illuminating for debates about physical possibility and other forms of modality.

Belot's methodology takes advantage of having a clean, well-lighted space of possibilities in assessing these accounts of geometric possibility. Once we accept that all possible spatial geometries are metric spaces, and that all metric spaces represent possible spatial geometries, we can press the best-systems analyst to tell us how to balance simplicity vs. strength in choosing Euclidean, discrete, or elliptical geometries, or the primitivist to provide an account of mirroring that applies to non-labile geometries.

Yet, turning now to critical points, there are problems with basing the analysis on the class of metric spaces in this way. It is common practice within mathematics to isolate and clarify a quite general structure such as that of a metric space, for a variety of reasons. But it is not clear that the resulting generalization aids a metaphysician who aims to characterize the possible structures of space. Why should we take *all* metric spaces as representing possible spatial geometries – can a sensible physical theory even be formulated in all of them? Belot aims to remain neutral on the interplay between geometry and physics, based on the "hope that for any spatial geometry to be considered, we could if pressed cook up a ... story about what sort of physics would go along with that spatial geometry ..." (p. 10). This neutrality presupposes that metrical structure alone is sufficient to ground physics, a contentious point. It is not obvious that all finite metric spaces will support other structures used in physical theories, such as projective and affine structure. Belot has replied in conversation that it is natural to take the metrical structure as basic, and that it will suffice for grounding other structures needed in the cooked-up physical story. This may be correct, especially for the cases pressed into use as counter-examples as the argument unfolds. Yet this issue surely deserves more than a dismissive aside.

[1] A finite metric space, in Belot's terminology, is a metric space defined over a finite set of points (pp. 24-25). Any such space is discrete, in that for any point in the space, there is a non-zero ϵ such that there is an open ball of radius ϵ which includes only that point. This is a quite general conception of geometry that lacks some intuitive features. As Belot emphasizes, in finite metric spaces the distance between two arbitrary points is not, as in Riemannian geometry, the greatest lower bound on the lengths of paths connecting them (since there are no paths in discrete spaces). Belot does not take a position on whether arbitrary metric spaces should count as representing possible spatial geometries (p. 27).

In addition, the advantage of taking the clearly-circumscribed concept of a metric space as essential has to be weighed against a disadvantage Belot eventually acknowledges, in the final paragraph of the main text (p. 138). The assessment of modal relationalism is restricted to *spatial* rather than *spatio-temporal* geometry, because there is no analog of "metric space" for spacetime geometry — i.e., a general concept that can encompass classical and relativistic spacetime geometries. What, then, is the relevance of Belot's discussion to motion and dynamics, a central issue in the debate regarding spacetime from Newton and Liebniz forward? One might hope that the only obstacle here is the lack of mathematical work devoted to elucidating the appropriate general structure. There is, in any case, important unfinished business in understanding how to extend Belot's analysis from *space* to *spacetime*.

It is striking how much Belot's approach runs counter to the historically fruitful interplay between physics and geometry, in the work of luminaries from Helmholtz to Einstein. A related critical point regards the need for a *sui generis* conception of geometric possibility. Logical and metaphysical possibility are not world-relative; is there some other modality that could ground the accessibility relation $\mathcal{G}(w_1, w_2)$? Belot argues (p. 50) that the relationalist should "resist the temptation" to see geometric possibility as an aspect of physical possibility. Maxwell's equations, for example, admit solutions in infinite as well as compact spacetimes. Physical possibility according to Maxwell's equations thus does not suffice to distinguish, for example, between a configuration of matter and fields with respect to which it is possible for a particle to fly off to infinity and one in which it is not. An infinite spacetime will, however, be *geometrically* impossible with respect to a compact spacetime, providing the desired contrast. Yet this brief argument does not entirely dispel the temptation, given the rich tradition of treating physics and geometry as intertwined. There is no reason to expect every physical theory to be equally committal regarding spacetime geometry. Physical possibility according to a particular theory may not suffice for drawing the relevant distinctions, but perhaps we need to invoke a different theory rather than a new modality. One might also question the need to draw the distinctions made using this more fine-grained modality.

In sum, along with exploring modal relationalism, Belot's book raises a more fundamental question regarding what philosophy of geometry should be about. Belot's analysis focuses on metric spaces given that these best capture an intuitive, natural extension of Euclidean geometry. His work amply demonstrates how the philosophical tools of modal metaphysics can be applied and studied within this setting, exploiting the clarity and generality provided by mathematics. Belot's insights regarding the three approaches to modality have broader interest given that they may be transferrable to other debates in metaphysics. Yet, ironically, it is clearer how Belot's methodology and results fit into contemporary work on modality than into the debate he uses to frame his discussion. Leibniz suggested modal relationalism in the midst of a debate with Newton regarding the appropriate spacetime geometry for formulating dynamics. The subsequent debate, historically as well as in recent philosophy of space and time, focuses on "physical geometry," in the sense that geometry is treated in close connection with dynamics rather than purely mathematically. The generality and precision afforded by the mathematical treatment of metric spaces comes at the cost of apparently changing the subject.

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