

Inflation and the Origins of Structure

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Abstract

Guth (1981) provided a persuasive rationale for inflationary cosmology based on its ability to solve fine-tuning problems of big bang cosmology. Yet one of the most important consequences of inflation was only widely recognized a few years later: inflation provides a mechanism for generating small departures from uniformity, needed to seed formation of subsequent structures, by “freezing out” vacuum fluctuations to form classical density perturbations. This paper recounts the historical development of this aspect of inflation, and puts it in context of the development of ideas on structure formation in relativistic cosmology, before turning to the comparison between inflation and a competing account of structure formation based on topological defects. One aim is to assess in what sense inflation is empirically tested through its account of the formation of structure, in light of persistent debates among cosmologists regarding whether inflation is “falsifiable.”

1 Introduction

The initial motivations for a physical theory are sometimes rendered dubious or superfluous by later work. The epistemic load borne by motivating ideas in the first stage of theoretical construction is shifted onto other ideas as work proceeds, leaving the original arguments with a largely ornamental rather than structural role. For example, Einstein described one of the three foundational ideas of general relativity (GR) as Mach’s principle, roughly speaking the claim that spacetime geometry should be fully determined by the distribution of matter without appeal to “background structures.” This principle was one of Einstein’s guiding ideas in the discovery of GR, but few modern relativists grant it the same pride of place in understanding the foundations of the theory he created.¹ Contemporary arguments in favor of accepting GR as the best available classical theory of gravitation barely mention Mach’s principle. The contemporary path to justification of our scientific theories often does not recapitulate the path to discovery.

The central idea of inflationary cosmology is that the early universe passed through a phase of exponential expansion driven by a scalar field displaced from the true minimum of its potential energy. Guth (1981) provided a rationale for this idea that proved to

¹The meaning of Mach’s principle and its status has been a focal point for foundational discussions since Einstein’s day; see Barbour and Pfister (1995) for an entry point to the recent literature.

be quite persuasive: inflation nearly eliminates the need for special initial conditions required by the standard model of cosmology. It was soon discovered that inflation also suggested a solution to a long-standing problem in relativistic cosmology: what is the origin of the seeds for the formation of structure in the universe? A recent textbook draws a distinction between the original rationale for inflation, as a “theory of initial conditions,” and a rationale based on predictions for the seeds of structure, as a “theory of the origins of structure”:

... [T]hese problems [related to initial conditions] can no longer be regarded as the strongest motivation for inflationary cosmology because it is not at all clear that they could ever be used to falsify inflation. [...] By contrast to inflation as a theory of initial conditions, the model of inflation as a possible origin of structure in the Universe is a powerfully predictive one. Different inflation models typically lead to different predictions for observed structures, and observations can discriminate strongly between them. ... *Inflation as the origin of structure is therefore very much a proper science of prediction and observation.* (Liddle and Lyth 2000, 5; my emphasis)

Liddle and Lyth clearly regard the early motivations for inflation as not sufficiently empirical, unlike the case for inflation that can be made given its connection with structure formation. Below I will argue, in agreement with Liddle and Lyth, that there is an important contrast between the historical motivations for inflationary cosmology and the strongest case that can now be made in its favor. But I will suggest a different way of characterizing the contrast, based on how informative different bodies of data are regarding inflation.

The main strategy pursued by inflationary cosmologists is to treat various properties of the early universe as the consequences of the dynamical evolution of a scalar field (or fields) in the early universe. In his persuasive case for inflation, Guth (1981) emphasized that such evolution could lead to a uniform, flat universe for a large range of initial conditions. Shortly after Guth’s paper appeared, several groups of cosmologists formulated accounts of the creation of seeds for structure formation during inflation. The mechanism for generating density perturbations is the most fruitful consequence of inflation, in two different senses. First, the problems Guth emphasized in presenting the theory were regarded as “enigmas” of the standard model of cosmology, when they were discussed at all. By way of contrast, the status of initial “seed” fluctuations was a major problem facing an appealing account of the origin of structure. Given that gravity should be the dominant force at large length scales, it is natural to suppose that structures such as galaxies evolved by the growth of small perturbations to an almost uniform initial distribution of matter. As described below in § 2, while this gravitational instability picture was appealing it seemed to require an extremely implausible initial distribution of matter. Inflation countered this objection and provided theorists with a way of calculating the density perturbations as a consequence of a stage of exponential expansion. § 3 recounts the historical route by which cosmologists developed this account, and contrasts the case of structure formation with the initial motivations for inflation.

§ 4 turns to the second sense in which this aspect of inflation has been particularly

fruitful, namely in providing the grounds for a detailed comparison with an alternative approach to understanding the seeds for structure formation. Structure formation via topological defects was studied extensively as an alternative to inflation throughout the 80s and 90s. Observations of temperature fluctuations in the cosmic microwave background radiation (CMBR) were able to discriminate between these two approaches, and clearly favored inflation. To what extent does this success reflect that inflation has correctly identified the physics of the early universe, as opposed to exhibiting sufficient flexibility to accommodate the observations? The final concluding section attempts to move beyond the way in which questions regarding the empirical status of inflation have been couched in the physics literature, in terms of Popperian “falsifiability.”

2 Structure Formation in The Standard Model

By the early 70s two aspects of what Weinberg (1972) dubbed the Standard Model of Cosmology were well understood theoretically, supported by observational evidence, and accepted as a starting point for further research by most cosmologists.² First, the expanding universe models of general relativity, the Friedmann-Lemaître-Robertson-Walker (FLRW) models, were taken to provide an approximate description of the overall structure and evolution of the universe at some suitably large length scale. In the early days of the field, cosmologists focused on these models for pragmatic reasons. Due to the symmetry assumed to hold in these models, the dynamics of general relativity is reduced to simple equations relating the scale factor $R(t)$ to the matter - energy distribution. Evidence accumulated that this symmetry was not just a useful simplification. In particular, the uniformity of the cosmic microwave background radiation (CMBR), first observed in 1965, supported the applicability of these models even in the early universe. By 1970 almost all cosmologists had accepted the FLRW models as a useful approximation and had turned to the more specific task of measuring the expansion with sufficient accuracy to choose the best model (see, e.g., Sandage 1970).

Second, the theory accounted for two striking features of the universe as relics of the “primeval fireball.” Nuclear reactions in the early universe governed by the rate of expansion leave a telling trace — a helium abundance of about 26 – 28% according to a calculation by Peebles (1967), in agreement with observations. Further development of the theory of big bang nucleosynthesis clarified the dependence of the primordial element abundances on various cosmological parameters. The CMBR was a second natural consequence of a hot big bang. In the early universe radiation and matter are coupled due to interactions, but as the temperature drops low enough for the existence of stable nuclei the universe becomes effectively transparent to photons.³ The photons then cool adiabatically with the expansion of the universe while maintaining a black-

²The steady state theory was no longer a serious rival to the standard “big bang” model by this time, although a small group of proponents (including Hoyle, Narlikar, and others) continued to explore the idea and to challenge the empirical underpinnings of the big bang model (see Kragh 1996).

³Recombination refers to the process by which nuclei capture free electrons and form neutral hydrogen and helium (although “re-” is misleading, as there was no earlier time, in the Standard Model, at which stable nuclei existed). During recombination, photons decouple from matter as the cross section for Thomson scattering drops to zero.

body spectrum, and they carry a tremendous amount of information regarding the universe at the time of recombination. Since 1965 a series of increasingly sophisticated observational missions have succeeded in extracting more and more of this information. Although subsequent research has enriched both ideas considerably, the fundamentals were in place by the early 70s and are presented in the influential texts by Weinberg (1972) and (Peebles 1971).

By contrast with these successes, the Standard Model lacked a compelling account of structure formation. Weinberg prefaced his discussion with the caveat that:

...[w]e still do not have even a tentative quantitative theory of the formation of galaxies, anywhere near so complete and plausible as our theories of the origin of the cosmic abundance of helium or the microwave background.
(Weinberg 1972, 562)

Unlike the successful aspects of the Standard Model, in the case of structure formation it has been much more difficult to link tractable pieces of theory to observations. This reflects the intrinsic difficulty of the subject, which requires integrating a broader array of physical ideas than required for the study of nucleosynthesis or the FLRW models. This section will give a brief overview of the development of the field up to 1980, focusing on the status of initial conditions for structure formation.

The ideas Weinberg (1972) described as a speculative part of the Standard Model were first explored by Lemaître, Georges (1933). Newtonian gravity enhances clumping of a nearly uniform distribution of matter. In the early stages of clumping, small fluctuations in density can be treated as first-order perturbations to a background cosmological model. This will be the case if the density contrast $\Delta =: \frac{\delta\rho}{\rho}$ is less than 1, where $\delta\rho$ is the density enhancement over the background density ρ . A theory of the evolution of small fluctuations must be supplemented on both ends, so to speak. The theory assumes as given an initial spectrum of small fluctuations that are then enhanced via dynamical evolution. An appealing possibility is that the dynamics is unstable, leading to exponential growth of small fluctuations. Then, like the onset of turbulence in fluid mechanics, details regarding the initial state would be relatively unimportant. On the other end, the theory extends up to the point when the fluctuations “freeze out” from the cosmological expansion, and begin to collapse into structures with much higher density contrasts (such as $\Delta \approx 10^6$ for a typical galaxy). Developing a theory governing this later stage of structure formation poses enormous challenges: perturbation theory does not apply, and the non-gravitational interactions of the constituents of the collapsing region can no longer be ignored.⁴ Despite these limitations, the theory of structure formation via gravitational enhancement of non-uniformities covers a large dynamical range. If successful, it would provide a link between the physical processes in the very early universe responsible for the initial fluctuations and the observationally accessible imprints of perturbations at later times.

Lifshitz (1946) was the first to treat the evolution of linear perturbations to a background model in general relativity, only to reject gravitational instability as a viable

⁴Modern studies of the non-linear regime employ numerical simulations, although there are a number of analytic techniques that were developed to study non-linear evolution during this time (e.g., Press and Schechter, 1974). See, e.g., Chapter 17 of Peacock (1999) for an introduction.

account of structure formation. He showed that in an FLRW model, the density contrast as a function of time grows very slowly. This result is surprising given the contrast with the account of instability due to Jeans (1902). Jeans derived an equation governing the evolution of small perturbations of a fluid including Newtonian gravity, and showed that the behavior of different modes depends on how their wavelength compares to a critical wavelength, the Jeans length λ_J .⁵ For modes with $\lambda = \lambda_J$ there is a balance between the pressure of the fluid, resisting collapse, and the gravitational force; perturbation modes with $\lambda < \lambda_J$ exhibit oscillatory behavior, whereas those with $\lambda > \lambda_J$ are unstable and grow exponentially. Physically, in the final case the matter density is sufficient to trigger gravitational collapse, leading to exponential growth of the amplitude of the fluctuation. If such rapid growth occurred in an expanding background as well, it would be possible for galaxies to form via gravitational enhancement of thermal fluctuations in the matter density, which Lifshitz (and many others) took to be a reasonable posit for the initial conditions. In this case the fluctuations away from uniformity would be given by the Poisson distribution, $\Delta \propto N^{-1/2}$ for N particles; for a galaxy-scale lump of particles, say 10^{68} particles, thermal fluctuations would give a low density contrast $\Delta_i \propto 10^{-34}$. However, Lifshitz showed that cosmological expansion works against gravitational instability, with the density contrast growing slowly ($\Delta(t) \propto t^{2/3}$) during the matter-dominated era in an expanding model. (Pressure prevents growth of the density contrast during the earlier radiation-dominated era.)⁶ If an initial fluctuation spectrum is imprinted at, say, $t_i = 1$ second (Bonnor 1956), time is too short for the fluctuations to grow into galaxies — with growth on the order of 10^{12} rather than the 10^{40} that is needed. Lifshitz concluded that gravitational instability fails to account for the formation of galaxies. Subsequent work on linear perturbation theory corrected and augmented Lifshitz’s analysis in several significant respects, but with little impact on this line of argument.⁷

In the 50s and early 60s many theorists found this criticism so compelling that they pursued alternative accounts of structure formation. Gamow, for example, turned to developing a theory based on primeval turbulence.⁸ Lifshitz’s line of argument reflects of an assessment of the plausibility of fluctuations at early times. Even a spectrum of thermal fluctuations is not immediately ruled out; Bonnor’s argument shows that thermal fluctuations at $t_i = 1s$ will not undergo sufficient growth, but one can treat t_i as a free variable and simply impose the fluctuation spectrum at an earlier time. Such

⁵See, e.g., Longair (2007), Chapter 11, or Weinberg (2008), Chapter 5, for modern introductions to linear perturbation theory.

⁶Lifshitz (1946) analyzed the behavior of small perturbations for two different equations of state, corresponding to radiation-dominated expansion, i.e. $p = \rho/3$, where p is the pressure and ρ the energy density, and matter-dominated expansion with $p = 0$. See, e.g., Longair (2007) for a modern treatment.

⁷See Peebles (1980, 20–25), and Longair (2006), chapter 15 for historical overviews.

⁸Gamow and Teller (1939) advocated an account of structure formation based on gravitational instability that is undermined by Lifshitz’s results (as Lifshitz explicitly noted). Gamow (1952, 1954) are the original papers on the turbulence theory; see Peebles (1971) for a critical review of Gamow’s proposal and other similar ideas. Two other problems with the gravitational instability account were also important in motivating the search for alternatives. First, there is no preferred length or mass scale in general relativity (with the cosmological constant set to zero), so it is unclear how to introduce scales such as the mass of a typical galaxy (see Harrison (1967a,b) for a detailed discussion of this point). Second, alternative accounts often claimed to give natural explanations of features of galaxies, such as their rotation and spiral structure.

an initial fluctuation spectrum is still mysterious, as we will see in more detail shortly.

The enigmatic nature of the initial conditions was not a sufficient objection to cosmologists who explicitly adopted a more phenomenological approach to galaxy formation (see, e.g., Harrison 1968, Peebles 1968, Zel’dovich 1965). All the available cosmological theories required some specification of the initial conditions, and the gravitational instability account is not obviously more objectionable in this respect. Furthermore, projecting backwards to find the required initial conditions could provide insight into new physics relevant in the early universe. The discovery of the CMBR provided an important new constraint along with the potential to establish observationally the fluctuation spectrum at the time of decoupling. The phenomenological approach focused on giving a more precise characterization of the initial fluctuations that were required for gravitational instability along with a detailed account of their dynamical evolution over time. Throughout the 70s theorists developed competing accounts of structure formation with the common aim of describing the evolution of the different physical degrees of freedom involved — radiation, baryonic matter, and the gravitational field. Solving the complete set of equations capturing all of the details of their interactions and dynamics, the coupled Boltzmann-Einstein equations, would have been computationally intractable. But given the background of an FLRW model, different physical effects are dominant at different stages of evolution. Initial matter and radiation perturbations would in general be a combination of two distinct modes:⁹

- *adiabatic*: Fluctuations in energy density of nonrelativistic matter ρ_m matched by radiation fluctuations (also called “entropy perturbations”), $\frac{4}{3} \frac{\delta\rho_m}{\rho_m} = \frac{\delta\rho_r}{\rho_r}$,
- *isothermal*: Radiation is uniformly distributed, $\frac{\delta\rho_r}{\rho_r} = 0$, although the matter is non-uniformly distributed.

One can then analyze the evolution of these distinct perturbation modes through different stages of the universe’s history. Prior to recombination, radiation ionizes the baryons and the photons and free electrons are coupled via Thomson scattering. As a result, fluctuations in the baryonic matter and radiation move together like a single fluid (Peebles 1965); galactic-scale perturbations undergo acoustic oscillations during this phase. In the later matter-dominated era, radiation and matter decouple and the matter fluctuations can be treated in isolation along the lines of Lifshitz’s analysis, and galactic-scale perturbations grow with $\Delta(t) \propto t^{2/3}$.

There were also debates regarding the appropriate initial spectrum and later stages of structure formation (see Longair 2006). Two different schools of thought dominated the field: Zel’dovich’s school focused on solutions in which large “blinis” (pancakes) formed first from adiabatic perturbations, fragmenting into galaxies and structures much later due to non-gravitational processes. The other school of thought led by Peebles developed a “bottom-up” scenario, in which initial isothermal fluctuations developed into proto-galaxies with larger structures forming later by hierarchical clustering. Despite the

⁹This terminology is due to Zel’dovich and his collaborators. The factor of $\frac{4}{3}$ arises since the energy density of radiation is $\propto T^4$, compared to T^3 for matter (where T is the temperature). These are called “adiabatic” perturbations since the local energy density of the matter relative to the entropy density is fixed. A third mode – tensor perturbations, representing primordial gravitational waves – were not usually included in discussions of structure formation, since they do not couple to energy-density perturbations.

stark differences between the account these theories gave of later stages of structure formation, they had similar implications for the epoch of recombination.

Both schools of thought also needed to address the evolution of density fluctuations, and there was a natural choice for the initial spectrum. Harrison (1970), Peebles and Yu (1970), and Zel’dovich (1972) proposed a scale-invariant (HPZ) spectrum, meaning that $\Delta|_{\lambda} = \text{constant}$ when λ , the perturbations’ wavelength, is equal to the Hubble radius, $\lambda = H^{-1}$.¹⁰ This spectrum lacks any characteristic length scale. For different wavelengths the perturbation amplitude is fixed at different times: in an expanding universe, the wavelength λ increases with the scale factor $R(t)$ whereas the Hubble radius increases at a slower rate as the expansion slows.¹¹ (The Hubble radius is a length scale set by the rate of expansion.)¹² The Hubble radius “crosses” various perturbation wavelengths in an expanding model; a scale-invariant spectrum deserves the name since the perturbations have the same magnitude as the Hubble radius sweeps across different length scales. Estimates of the magnitude of density perturbations when length scales associated with galaxies cross the Hubble radius fall within the range $\Delta \approx 10^{-3} - 10^{-4}$. In addition, the initial perturbations were often also assumed to be “random” in the sense that the mass found within a sphere of fixed radius has a Gaussian distribution (for different locations of the sphere).

Two features of HPZ spectrum are particularly puzzling. The first puzzle arises from the causal structure of the FLRW models. Even though the distance between freely falling particles decreases as $t \rightarrow 0$, the decrease is not rapid enough to insure that sufficiently distant regions of the universe were in causal contact. The FLRW models have particle horizons. Horizons in cosmology measure the maximum distance light travels within a given time period from a time of emission t_e ; the “particle horizon” is defined as the limiting case $t_e \rightarrow 0$.¹³ The existence of particle horizons in the FLRW models indicates that distant regions are not in causal contact (see figure 1). Many discussions mistakenly refer to the Hubble radius H^{-1} as the “horizon.” This is a misnomer because the Hubble radius is not defined in terms of causal structure, but it does indicate the length scale at which expansion has an impact on evolution of

¹⁰In general, for a scale invariant power spectrum the Fourier components of the perturbations obey a power law, $|\delta_k|^2 \propto k^n$; the Harrison-Peebles-Zel’dovich spectrum corresponds to a choice of $n = 1$ (given that the volume element in the inverse Fourier transform is $\frac{dk}{k}$; for the other conventional choice, $k^2 dk$, we then have $n = -3$). The Hubble radius has the appropriate dimension, length: restoring c , it is given by $\frac{c}{H}$, and the Hubble constant H has units of km per second per megaparsec.

¹¹Since the perturbations grow with time, at a “constant time” the shorter wavelength perturbations have greater amplitudes for this spectrum. The difficulty with defining the spectrum of density perturbations in terms of “amplitude at a given time” is that it depends on how one chooses the constant time hypersurfaces.

¹²It is defined as $R_H = H^{-1}$, where H is the Hubble “constant” ($H = \frac{\dot{R}}{R}$); it is also called the speed of light sphere, given that objects moving with the expansion, at a distance R_H , appear to move at speed c .

¹³A horizon is the surface in a time slice t_0 separating particles moving along geodesics that could have been observed from a worldline γ by t_0 from those which could not (Rindler 1956). The distance to this surface, for signals emitted at a time t_e , is given by:

$$d = R(t_0) \int_{t_e}^{t_0} \frac{dt}{R(t)} \quad (1)$$

Different “horizons” correspond to different choices of limits of integration. The integral converges for $R(t) \propto t^n$ with $n < 1$, which holds for matter or radiation-dominated expansion. Thus the integral for the particle horizon ($\lim_{t_e \rightarrow 0}$) converges for the FLRW models (e.g., Ellis and Rothman 1993).

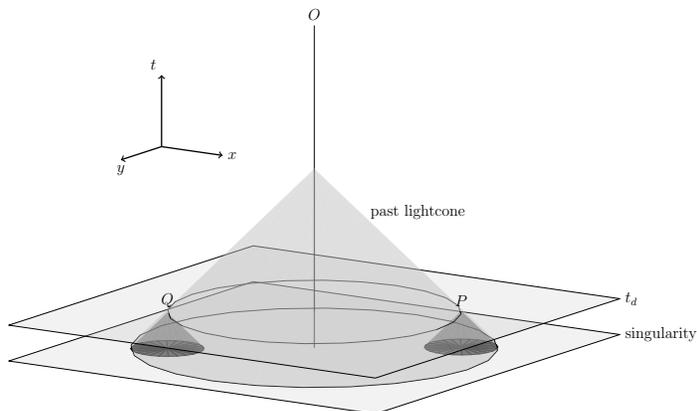


Figure 1: This diagram, with lightcones at 45° , illustrates the causal structure of the FLRW models. Points P, Q on the surface of last scattering t_d , both falling within the past light cone of an observer O , do not have overlapping light cones.

perturbations. A simple scaling argument shows that in standard FLRW expansion perturbation wavelengths cross the “horizon”: the perturbation wavelengths simply scale with the expansion whereas H^{-1} scales as $H^{-1} \propto R^{1/n}$ for $R(t) \propto t^n$. For the length scale associated with a galaxy, horizon crossing occurs at around $t \approx 10^9$ seconds. It is puzzling that the perturbations are coherent prior to this time, at a length scale larger than the Hubble radius.

One response was to hope that new physics would lead to a different causal structure of the early universe. Bardeen concludes a study of the evolution of density perturbations as follows (Bardeen 1980, 1993):

The one real hope for a dynamical explanation of the origin of structure in the Universe is the abolition of particle horizons at early times, perhaps through quantum modifications to the energy-momentum tensor and/or the

gravitational field equations which in effect violate the strong energy condition.¹⁴

But Bardeen’s focus on particle horizons as a fundamental obstacle set him apart from others in the field; Peebles (1980), for example, mentions the puzzles associated with horizons, but apparently takes this to be one of many indications that we do not sufficiently understand physics near the big bang.

The second puzzle regards the amplitude of the perturbations as they crossed the Hubble radius. While this could be treated as a parameter to be fixed by observations, many theorists hoped for a physical account of how this amplitude was fixed in the early universe. One can evolve backwards to determine the amplitude of the fluctuation spectrum at a given “initial” time t_i . For t_i on the order of the Planck time, for example, these fluctuations are much *smaller* than thermal fluctuations, which are taken to be physically plausible.¹⁵ It seems inappropriate to treat t_i as a free variable, choosing when to “imprint” a spectrum of thermal fluctuations such that the amplitudes match observations. The Planck time is often singled out on dimensional grounds as the scale at which quantum gravity effects should become important. But in the absence of a successor theory, it is unclear how to delimit the boundary of applicability of classical GR and then choose a plausible “initial” perturbation spectrum.

By the late 70s and early 80s, several cosmologists had greater ambitions than merely giving a phenomenological account of structure formation. They sought to understand the origins of initial perturbations based on new physics applicable to the early universe. Those sharing this ambition could draw ideas from the ample storehouse of speculative physics: Planck scale metric fluctuations, gravitational particle production, primordial black holes, “gravithermal” effects, primordial turbulence, non-equilibrium dynamics, and so on.¹⁶ Sakharov (1966) was the first to propose a detailed quantum description of the initial perturbations — remarkably, before the discovery of the CMBR. But this early paper drew no attention, partially because it was an extension of Zel’dovich’s “cold bang” proposal that fell from favor following the discovery of the CMBR. From the mid-70s onward several theorists explored the implications of early universe phase transitions for structure formation, in particular the production of topological defects (discussed in more detail below). This work, along with studies of other possible impacts of phase transitions, illustrates that giving a physical account of the earliest stages of structure formation came to be regarded as a viable research topic. As of 1980 the field was wide open, with the potential to draw on ideas in general relativity and quantum gravity or the many novel ideas recently introduced in particle physics.

In addition to puzzles regarding the initial perturbations, these approaches to struc-

¹⁴Energy conditions are constraints on what is taken to be a reasonable source for the gravitational field equations. Roughly speaking, the strong energy condition requires that the stresses in matter will not be so large as to produce negative energy densities. Formally, $T_{ab}\xi^a\xi^b \geq \frac{1}{2}\text{Tr}(T_{ab})$ for every unit timelike ξ^a ; for a perfect fluid, this implies that $\rho + 3p \geq 0$, where ρ is the energy density and p is the pressure. As Bardeen notes, if the strong energy condition fails then there are solutions such that the integral in eqn. (1) diverges.

¹⁵For example, Blau and Guth (1987) compare the density contrast imposed at $t_i = 10^{-35}$ seconds to the fluctuations obtained by evolving backwards from the time of recombination implies $\Delta \approx 10^{-49}$ at t_i , nine orders of magnitude *smaller* than thermal fluctuations.

¹⁶See Barrow (1980) for a brief review of some of these ideas and references, and Peebles (1980); Zel’dovich and Novikov (1983) for more comprehensive overviews of the field.

ture formation were threatened by tightening observational constraints based on the isotropy of the temperature of the CMBR. Partridge (1980) reached sensitivities of $\Delta T/T \approx 10^{-4}$ in isotropy measurements, and at this level he should have detected fluctuations according to either of the prevailing accounts of structure formation. This problem, along with other events such as experimental evidence in favor of a massive neutrino, led theorists to add hot and cold dark matter to their models of structure formation starting in the early 80s (see, for example, Peebles, 1982, and Pagels, 1984).¹⁷ The early dark matter models established the compatibility between the observational upper limits on temperature anisotropies in the CMBR and the idea of structure formation via gravitational instability. Adding cold dark matter helps to reconcile the uniformity of the CMBR with later clumpiness of matter because, roughly speaking, the cold dark matter decouples from the baryonic matter and radiation early, leaving a minimal imprint on the CMBR, yet after recombination the cold dark matter perturbations regenerate perturbations in the baryonic matter sufficiently large to seed structure formation.

Many contemporary textbooks on structure formation use the puzzles regarding initial perturbations described above to set the stage for the entrance of inflationary cosmology. Rather than pulling the initial spectrum out of a hat, as one might suspect of the earlier proposals, the inflationary theorist can pull an HPZ spectrum with an appropriate amplitude out of the vacuum fluctuations of a quantum field. The performance is captivating because it displays the possibility of *calculating* the features of the initial spectrum from physical principles. The following section will review the route by which the theorists discovered this appealing consequence of inflation, and assess its importance by contrast with the other features of inflation emphasized by Guth (1981).

3 Inflationary Cosmology

The essential idea of inflation is that the early universe went through a transient phase of de Sitter-like expansion.¹⁸ During this phase the scale factor grows exponentially with time, $R(t) \propto e^{\chi t}$, compared to the more sedate radiation-dominated FLRW expansion with $R(t) \propto t^{1/2}$. The idea of modifying FLRW expansion in this way had been suggested several times prior to 1980 (see Smeenk 2005), and the earlier proposals shared two common problems. First, what is the physical source of the accelerated expansion? I will refer to this as the source problem. The source could not be garden-variety matter or radiation, because to drive a stage of exponential expansion it would have to violate the strong energy condition typically assumed to hold for reasonable

¹⁷“Hot” vs. “cold” refers to the thermal velocities of relic particles for different types of dark matter. Hot dark matter decouples while still “relativistic,” in the sense that the momentum is much greater than the rest mass, and relics at late times would still have large quasi-thermal velocities. Cold dark matter is “non-relativistic” when it decouples, meaning that the momentum is negligible compared to the rest mass, and relics have effectively zero thermal velocities.

¹⁸De Sitter spacetime is a solution to Einstein’s field equations with a stress energy tensor given by $T_{ab} = -\rho_v g_{ab}$, where ρ_v is the vacuum energy density. The scale factor then expands exponentially, with $\chi^2 = \frac{8\pi}{3} \rho_v$. During inflation the stress energy tensor has approximately this form. Given that the vacuum energy density remains constant during the expansion while “ordinary” matter and energy is rapidly diluted, the vacuum energy dominates the expansion and the solution, roughly speaking, approaches de Sitter spacetime.

matter fields.¹⁹ Second, how does the exponential expansion transition into the usual FLRW expansion with appropriate matter and energy densities? Solving this second problem, the transition problem, requires an explanation of how the physical source of the expansion ceased to be dynamically relevant and set the stage for the standard big bang model. Any matter or radiation present at the onset of exponential expansion is rapidly diluted away, leaving only the vacuum energy ρ_v , which remains constant throughout the expansion. One needs an account of how the universe is re-populated with normal matter and radiation after the stage of exponential expansion.

Guth (1981) launched a research program not by solving either of these problems but by making a compelling case in favor of inflation. He recognized that a stage of exponential expansion solves two fine-tuning problems of the standard model, the flatness and horizon problems. On this basis he argued that the idea was worth pursuing despite his failure to give an account of the transition to the standard model. The source of exponential expansion in his original account was the vacuum energy of the Higgs field in a proposed Grand Unified Theory (GUT) trapped in a false minima during a first-order phase transition.²⁰ Even though this solution of the source problem would not survive long, by contrast with earlier proposals Guth had shown how to link the idea of inflation with an active area of research in particle physics. In effect, inflation exchanged various large-scale properties of the universe, previously treated as initial conditions, for features of the dynamical evolution of a scalar field in the early universe. This exchange was soon exploited in giving a solution to the transition problem and in giving an account of the origins of the seeds for structure formation. After reviewing Guth’s case and critical responses to it, we will turn to the discovery of the inflationary account of structure formation at the Nuffield workshop and briefly discuss the account itself in more detail.

3.1 Inflation as a Theory of Initial Conditions

Guth identified two problems that inflation was able to solve:

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem) and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy ... (flatness problem). (Guth 1981, 347)

The first could be more aptly called the “uniformity problem”: there is an apparent

¹⁹The stress-energy tensor stated in the previous footnote does not satisfy the strong energy condition formulated in footnote 14; the fact that the vacuum energy density does not dilute with expansion reflects this. A stress-energy tensor that violates this condition is a necessary condition for exponential expansion within classical GR.

²⁰Guth discovered inflation while focusing on a third problem, the monopole problem. GUTs from the late 70s predicted the existence of magnetic monopoles, and the relic abundance of the monopoles would be many orders of magnitude greater than the observed energy density of the universe. See Guth (1997a) for his account of how he discovered inflation. Unlike the monopole problem, which arises for the combination of cosmology and these GUTs, the flatness and horizon problems are problems for the cosmological standard model.

conflict between the strikingly uniform temperature of the CMBR and the horizon structure of the FLRW models (see figure 1). We have seen above that cosmologists working on structure formation noted puzzles due to horizons, and Misner (1969) formulated the problem in terms similar to Guth’s a decade earlier.²¹ The portion of the universe we can see consists of 10^{83} causally disconnected regions at the Planck time. Due to horizons, the fact that all these regions have the same physical properties cannot be explained via causal interactions.

What Guth called the “flatness problem” had not been widely discussed.²² The dynamics of the FLRW models implies that all models approach the “flat” model at early times. This can be seen in the behavior of the density parameter Ω , which is 1 for the FLRW model with Euclidean spatial sections.²³ Assuming normal matter and radiation as sources, $|\Omega - 1|$ increases with time under the FLRW dynamics.²⁴ It is thus surprising to discover that $\Omega(t_0)$, the current observed value, is quite close to 1. If we imagine choosing a value $\Omega(t_i)$ at some early time, it must be *incredibly* close to 1 to be compatible with observations (see figure 2).

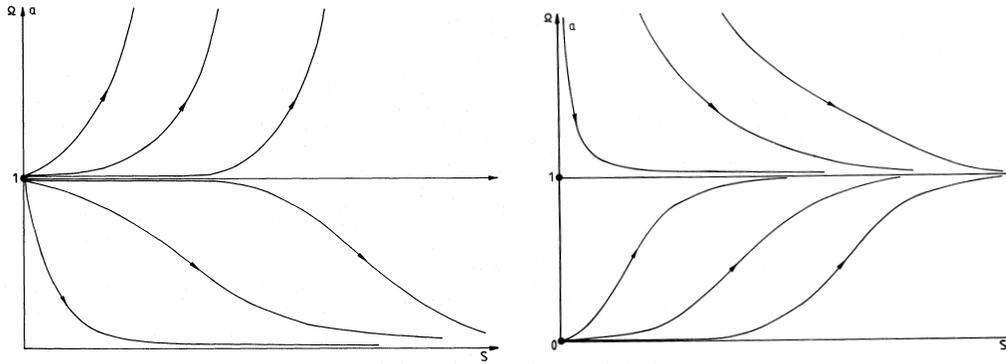


Figure 2: These diagrams from Ellis and Madsen (1988) illustrate the evolution of Ω as a function of S (the scale factor R in the text). In the left diagram, Ω diverges from 1 (for $\gamma > 2/3$), whereas on the right Ω is driven towards one during an inflationary phase (with $\gamma = 0$).

Thus the standard model requires positing, at the Planck time, the same physical conditions in 10^{83} causally disconnected patches, with a delicately chosen total energy density. Guth argued that inflation is compatible with a much more plausible initial

²¹See Smeenk (2005) for a discussion of how these two features of the FLRW models were treated prior to Guth’s identification of them as problems to be solved by inflation.

²²Guth learned of the problem from lectures given by Robert Dicke (Guth 1997a). See Dicke (1969) and Dicke and Peebles (1979) for Dicke’s formulation of the problem, which he characterized as an “enigma”.

²³ $\Omega =: \frac{\rho}{\rho_c}$, where the critical density ρ_c is the value required for the attraction of gravity due to positive matter-energy density to precisely balance the initial expansion and cosmological constant: $\rho_c = \frac{3}{8\pi} \left(H^2 - \frac{\Lambda}{3} \right)$.

²⁴More precisely,

$$\frac{|\Omega - 1|}{\Omega} \propto R(t)^{3\gamma-2}, \quad (2)$$

where γ is used to classify different types of perfect fluids. The equation of state of a perfect fluid is $p = (\gamma - 1)\rho$, where p is the pressure, ρ the density. For radiation, $\gamma = 4/3$ and for “dust” $\gamma = 1$ (corresponding to zero pressure). For “normal” matter, satisfying the energy conditions defined in footnote 14, $\gamma > 2/3$.

state. Inflation stretches the horizon length; for N “e-foldings” of expansion the horizon length d_h is multiplied by e^N . For $N > 65$ the horizon distance, while still finite, encompasses the observed universe. The observed universe could then have evolved from a single causal patch rather than 10^{83} patches with an astonishing degree of pre-established harmony. In addition, during inflation the density parameter is driven *towards* one.²⁵ An inflationary stage long enough to solve the horizon problem drives a large range of pre-inflationary values of $\Omega(t_i)$ sufficiently close to 1 by the end of inflation, such that $\Omega(t_0) \approx 1$. If inflation occurs, there is no need for a finely-tuned choice of $\Omega(t_i)$. A word of caution is in order, however: it is not the case that inflation *eliminates* dependence on initial conditions entirely. One can choose initial conditions that lead to an arbitrarily non-uniform universe with any value of Ω , despite inflation’s preference for a uniform universe with $\Omega(t_0) = 1$. Inflation *enlarges* the range of initial conditions compatible with observations.²⁶

For those working, like Guth, in particle physics, these problems and the approach to solving them had a familiar ring. Following Wilson and ’t Hooft, many particle theorists sought extensions of the Standard Model of particle physics that would eliminate its “unnatural” features, such as the huge discrepancy between the Higgs boson mass and the scale of the fundamental interactions (the Planck scale). The reception of Guth’s case for inflation in the particle physics community reflects, in part, acceptance of a common strategy: using fine-tuning as a guide to developing new theories. The reception among astrophysicists and cosmologists was more uneven. These communities did not share the methodology implicit in focusing on naturalness or fine-tuning problems. Brawer (1996) argues that Guth’s discovery of a solution to the horizon and flatness problems helped to convince many that they were, in fact, legitimate problems.²⁷ But there have been, since inflation was introduced, vocal critics who have rejected the idea that inflation should be given credence based on solving fine-tuning problems.

One line of criticism grants that an early universe theory should explain how the observed universe arose from “generic” initial conditions, as Guth argued. But does inflation deliver such an explanation? Penrose (1986) argued that the probability of inflation must itself be quite low, on general grounds.²⁸ Suppose that we are given a

²⁵During inflation, the strong energy condition is violated and $\gamma = 0$; it is clear from eqn. 2) that Ω is then driven towards 1.

²⁶This point was first made in response to Misner’s “chaotic cosmology,” which like inflation proposed new dynamics (in Misner’s case, damping of anisotropies due to neutrino viscosity) in order to insure that an isotropic universe emerges from a large range of anisotropic initial conditions. In response to Misner, Collins and Stewart (1971) showed that one can always pick an arbitrarily large anisotropy at a given time t_0 and find a solution of the relevant system of equations as long as there are no processes which could prevent arbitrarily large anisotropies at some $t_i < t_0$. A similar criticism applies to inflation, as Madsen and Ellis (1988) have emphasized. Guth (1997b) has acknowledged this point: “... I emphasize that *NO* theory of evolution is ever intended to work for arbitrary initial conditions. ... In all cases, the most we can hope for is a theory of how the present situation could have evolved from *reasonable* initial conditions” (pp. 240-241, emphasis in the original).

²⁷Brawer’s case is based on published discussions of these problems, as well as the extensive interviews with cosmologists published in Lightman and Brawer (1990).

²⁸Penrose’s original terse statement of this criticism appeared in a book review of the conference proceedings of the Nuffield workshop (discussed below), and he has discussed it further in Penrose (1989, 2004). Although I do not have the space to discuss Penrose’s objection, several more recent papers pursue the issues raised by Penrose, including Unruh (1997); Hollands and Wald (2002).

generic state in a universe that evolves into a “big crunch” singularity in the future. It seems overwhelmingly unlikely that as the universe approaches the final singularity, it will “deflate” by converting all the gravitational energy of the collapsing matter into kinetic energy of a scalar field in just the right way to push it into a false vacuum state. But this is simply the time reverse of the account of inflation, and — on the assumption that the dynamics is time-reversal invariant — the argument concludes that our assessment that the probability of deflation is low should also apply to inflation itself. If Penrose is correct, the features of the field driving inflation, and its pre-inflationary state, are more finely tuned than the initial state required in the standard model without inflation. Inflation shifts fine-tuning from one place to another rather than eliminating it. (There are further specific constraints required on the pre-inflationary state. Vachaspati and Trodden (1999) proved that the field driving inflation must be uniform over a region larger than the Hubble radius in order to trigger inflation; and the scalar field has to be sufficiently uniform to drive exponential expansion during inflation.²⁹)

A second line of argument questions why we should grant that the universe began in a “generic” initial state.³⁰ This is, in effect, a hypothesis regarding the universe at the Planck scale, which is uncertain due to our lack of a theory of quantum gravity. What grounds do we have for accepting such a hypothesis? One problem regards even formulating the hypothesis. Guth and others often write as if the initial state should be regarded as “chosen at random” from among a set of possibilities. It is unclear, however, what theory should be used to define the space of possibilities, since classical GR does not adequately reflect all the laws that would govern this domain. Furthermore, the assessment of an initial state as “generic,” or, on the other hand, “special,” is based on a choice of measure over the allowed initial states of the system. But on what grounds is one measure to be chosen over another? Even if we obtain a clearly delimited space of possibilities, equipped with a measure that allows us to determine the properties of a “generic” choice, what justifies this hypothesis? For a normal experimental system, it is possible to check, at least in principle, whether a large variety of initial states lead to the same final state; if so, there is evidence that the system is governed by dynamics that washes away dependence on the initial state. Obviously, however, such supporting evidence cannot be gathered in cosmology.

A quite different response is that Guth’s motivations for inflation should be disentangled from the physics. In fact, Guth’s precursors in the Soviet Union introduced inflation with essentially the opposite methodology. For example, Starobinsky (1978, 1979) regarded the choice of a specific initial state — de Sitter spacetime — as extremely natural, and it had the advantage of evading the singularity in FLRW models.

Critical responses along these lines did not dim the enthusiasm for Guth’s proposal.

²⁹The stress energy tensor for a scalar field is given by

$$T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (g^{cd} \nabla_c \nabla_d \phi - V(\phi)); \quad (3)$$

inflation requires that the field is “potential-dominated” in the sense that the field is sufficiently uniform that the derivative terms are negligible, $V(\phi) \gg g^{cd} \nabla_c \nabla_d \phi$. If this condition holds, $T_{ab} \approx -V(\phi)g_{ab}$ as required to produce exponential expansion.

³⁰Different versions of this line of argument have been pressed by a number of critics of inflation; see, for example, Earman and Mosterin (1999), Hollands and Wald (2002), and more recently Ijjas et al. (2013).

This is, I will argue, in part because of the answer that was shortly developed to a problem that Guth did see as a clear obstacle to the idea of inflation. Guth noted the advantages of inflation while at the same time admitting that his model failed to solve the transition problem (also called the graceful exit problem). Rather than smoothly joining onto the FLRW expansion, the phase transition Guth considered ended via bubble nucleation, leaving the early universe marred with non-uniformities. The model failed to achieve the delicate balance between overall uniformity and slight perturbations required for the account of structure formation via gravitational instability. As Barrow and Turner (1981) noted, at first blush and provided that bubble nucleation could be avoided, inflation may actually exacerbate the problem by too efficiently smoothing out the universe, leaving it without wrinkles to seed later structures. This worry led to an important success as theorists discovered a mechanism for generating perturbations during inflation.

3.2 New Inflation and the Nuffield Workshop

Guth’s paper and talks based on it introduced many astrophysicists and particle physicists to the very idea of early universe cosmology. By admitting the flaws of his initial model, Guth also left his readers and audiences with a project: to find a working model of inflation. Paul Steinhardt, then a Junior Fellow in the Harvard Society of Fellows, exemplifies this reaction; he described Guth’s talk at Harvard as “the most exciting and depressing talk” he had ever attended (Steinhardt 2002). The excitement stemmed from the promise of connecting the study of phase transitions to fundamental questions in cosmology. But after laying out inflation’s ability to solve the flatness, horizon, and monopole problems, Guth ended by explaining the fatal flaw of his initial model. Steinhardt recalls his reaction (Steinhardt 2002): “Here was this great idea and it just died right there on the table. So I couldn’t let that happen.”

Given Steinhardt’s background in condensed matter physics and familiarity with phase transitions, he was ideally suited to take on the task of reviving Guth’s idea. News of Guth’s paper also led Andrei Linde in Moscow, a pioneer in the study of early universe phase transitions throughout the 70s, to reconsider the possibility of a first-order phase transition. Linde had considered the idea in collaboration with Chibisov, but had dismissed it as unworthy of publication — “there was no reason to publish such garbage” — due to the problem of inhomogeneities.³¹ Steinhardt began studying early universe phase transitions almost immediately, and upon taking a faculty position at the University of Pennsylvania he found a graduate student, Andy Albrecht, eager to join in the project. Linde and Steinhardt and Albrecht independently realized that a symmetry breaking phase transition governed by a different effective potential than that used by Guth could solve the transition problem while providing sufficient inflation to solve the horizon and flatness problems (Albrecht and Steinhardt 1982; Linde 1982). Their proposal is usually called “new inflation.”³²

³¹The collaborative work with Chibisov is mentioned in Linde (1979, 433–434); the quotation is from a 1987 interview (Lightman and Brawer 1990, 486–486).

³²At roughly the same time, Stephen Hawking and Ian Moss proposed an alternative solution to the transition problem. Although Hawking and Moss (1982) is sometimes cited as a third independent discovery

Albrecht and Steinhardt (1982) and Linde (1982) both developed models of the phase transition based on a Coleman-Weinberg effective potential for the Higgs field. (The Lagrangian density for a classical scalar field is given by $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$, where $V(\phi)$ is the potential. The effective potential includes quantum corrections to the classical potential.)³³ This change leads to a dramatically different phase transition. Most importantly, inflation continues after the formation of an initial bubble: rather than tunnelling directly to the global minimum, in this scenario the field ϕ evolves to the minimum over a “long” timescale τ (i.e., much longer than the expansion time scale). Throughout this evolution ϕ is still displaced from the global minimum, and the non-zero $V(\phi)$ continues to drive exponential expansion. Linde (1982); Albrecht and Steinhardt (1982) both argue that for natural values of τ the expansion lasts long enough for the initial bubble to become much, much larger than the observed universe. Finally, as in Guth’s scenario any pre-inflationary matter and energy density are diluted during the extended inflationary stage. In the new scenario, oscillations of the field ϕ near its global minimum would produce other particles via baryon-number nonconserving decay in order to “reheat” the universe to an energy density compatible with standard cosmology.

The initial proposals were quickly developed into a general account of new inflation. The features of the phase transition can be described simply in terms of the evolution of ϕ , which is determined by the form of the potential $V(\phi)$. The classical equations of motion for a scalar field ϕ with a potential $V(\phi)$ in an FLRW model are given by:

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \Gamma_\phi\frac{d\phi}{dt} + \frac{dV(\phi)}{d\phi} = 0, \quad (4)$$

where t is the time coordinate in the FLRW model, and Γ_ϕ is the decay width of ϕ .³⁴ New inflation requires a long “slow roll” followed by reheating. Assume that the field ϕ is initially close to $\phi = 0$. Slow roll occurs if the potential is suitably flat near $\phi = 0$ and the $\ddot{\phi}$ term is negligible; given the further assumption that the Γ_ϕ term is negligible, then the evolution of ϕ can be approximately described by:

$$3H\dot{\phi} \approx -\frac{dV(\phi)}{d\phi}. \quad (5)$$

(The name is due to the similarity between the evolution of ϕ and that of a ball rolling down a hill, slowed by friction.) During slow roll the potential energy $V(\phi)$ dominates over the kinetic energy $\frac{\dot{\phi}^2}{2}$, and $V(\phi)$ drives inflationary expansion. The slow roll ap-

of new inflation, it differs substantially from the other proposals. The aim of the paper is to show that including the effects of curvature and finite horizon size leads to a different description of the phase transition. This phase transition proceeds from a local minimum at $\phi = 0$ to the global minimum ϕ_0 via an intermediate state ϕ_1 ; rather cryptic arguments lead to the conclusion that “the universe will continue in the essentially stationary de Sitter state until it makes a quantum transition everywhere to the $\phi = \phi_1$ solution” (p. 36). They further argue that following this transition to a coherent Hubble scale patch, ϕ will “roll down the hill” (for an appropriate values of parameters in the effective potential), producing an inflationary stage long enough to match Guth’s success.

³³See, e.g., Coleman (1985), Chapter 5 for an introduction to the effective potential, and Kolb and Turner (1990) for a detailed discussion of the differences between old and new inflation.

³⁴One of the main differences between the initial papers on new inflation is that Albrecht and Steinhardt (1982) explicitly include the $3H\dot{\phi}$ term (aka the “Hubble drag” term), whereas Linde (1982) does not.

proximation breaks down as the field approaches the global minimum. The Γ_ϕ term is put in “by hand” to describe the process of reheating: roughly, ϕ oscillates around the minimum and decays into other types of particles. The details depend on the coupling of ϕ to other fields, and are heavily model-dependent. The reheating stage is necessary to “repopulate” the universe, given that any pre-existing matter or radiation is rapidly diluted during the inflationary expansion.

By the spring of 1982 several groups were at work fleshing out the details of the new inflationary scenario: a group at the University of Chicago and Fermilab including Turner and Kolb, Steinhardt and Albrecht at the University of Pennsylvania, Guth at MIT, Linde and various collaborators in Moscow, Laurence Abbott at Brandeis, Hawking and others in Cambridge, and John Barrow in Sussex. With notable exceptions, such as Hawking and Barrow, nearly everyone in this research community came from a background in particle physics. The framework described in the previous paragraph left ample room for innovation and new ideas: the connections with particle physics were poorly understood at best, the various approximations used were generally on shaky footing, and there were numerous hints of interesting new physics. Several of these researchers recognized the most important hint: homogeneity at all scales at the end of inflation would be incompatible with accounts of galaxy formation, which required an initial spectrum of perturbations. There appeared to be several ways to avoid *too much* homogeneity at the end of inflation; Linde (1982), for example, mentions a later phase transition without supercooling or quantum gravity effects as a possible means for generating inhomogeneities.

The first international conference focusing on “very early universe cosmology ($t < 1$ sec)” convened in Cambridge from June 21 - July 9, 1982.³⁵ Nearly half the lectures at the Nuffield workshop were devoted to inflation, and the intense collaborations and discussions during the workshop led to the “death and transfiguration” of inflation (from the title of the conference review in *Nature*, Barrow and Turner 1982). One focus of the conference was the calculation of density perturbations produced during an inflationary stage: Steinhardt, Starobinsky, Hawking, Turner, Lukash and Guth had all realized that this was a “calculable problem” (in Steinhardt’s words), with the answer being an estimate of the magnitude of the density perturbations, measured by the dimensionless density contrast Δ , produced during inflation. Preliminary calculations of this magnitude disagreed by an astounding 12 orders of magnitude: Hawking circulated a preprint (later published as Hawking 1982) that found $\Delta \approx 10^{-4}$, whereas Steinhardt and Turner (1984) initially estimated a magnitude of 10^{-16} . After three weeks of effort, the various groups working on the problem had converged on an answer, but the answer proved to be disastrous for new inflation.

The calculations drew on an idea introduced prior to Guth’s paper. Mukhanov and Chibisov (1981) had argued that a de Sitter phase could generate perturbations by “stretching” zero-point fluctuations of quantum fields to significant scales. This

³⁵The description is taken from the invitation letter to the conference (Guth 1997a, 223). The Nuffield Foundation had previously sponsored conferences in quantum gravity, but shifted the focus to early universe cosmology in response to interest in the inflationary scenario. A 1981 conference in Moscow on quantum gravity also included numerous discussions of early universe cosmology (Markov and West 1984), but Nuffield was the first conference explicitly devoted to the early universe.

idea would become the basis for the generation of seed perturbations in inflationary cosmology. The details were worked out at the Nuffield workshop, which seems to be a rare example of a scientific workshop that fulfilled the goal of bringing together the relevant research groups and successfully forging a consensus on an important problem.

Prior to the workshop, Hawking circulated a preprint which argued that initial inhomogeneities in the ϕ field would imply that inflation begins at slightly different times in different regions; the inhomogeneities reflect the different “departure times” of the scalar field. Hawking’s preprint claimed that this results in a scale-invariant spectrum of adiabatic perturbations with $\Delta \approx 10^{-4}$, exactly what was needed in accounts of structure formation. But others pursuing the problem (Steinhardt and Turner; Guth and his collaborator, So-Young Pi) did not trust Hawking’s method; Steinhardt has commented that he “did not believe it [Hawking’s calculation] for a second” (Steinhardt 2002, cf. Guth 1997a, 222–230). There were two closely-linked concerns with Hawking’s method (beyond the sketchiness of his initial calculations): it is not clear how this approach treats the evolution of the fluctuations in different regimes, and it is also not gauge invariant.

The “gauge problem” in this case reflects the fact that a “perturbed spacetime” cannot be uniquely decomposed into a background spacetime plus perturbations. Slicing the spacetime up along different surfaces of constant time leads to different magnitudes for the density perturbations. The perturbations “disappear,” for example, by slicing along surfaces of constant density. In practice, almost all studies of structure formation used a particular gauge choice (synchronous gauge), but this leads to difficulties in interpreting perturbations with length scales greater than the Hubble radius.³⁶ Press and Vishniac (1980) identify six “tenacious myths” that result from the confusion between spurious gauge modes and physical perturbations for $\lambda > H^{-1}$. This problem is significant for the inflationary account because over the course of an inflationary stage perturbations of fixed length go from $\lambda \ll H^{-1}$ to $\lambda \gg H^{-1}$. Length scales “blow up” during inflation since they scale as $R(t) \propto e^{Ht}$, but the Hubble radius remains fixed since H is approximately constant during the slow roll phase of inflation. For this reason it is especially tricky to calculate the evolution of physical perturbations in inflation using a gauge-dependent formalism. The first problem mentioned in the previous paragraph is related: determining the imprint of initial inhomogeneities requires evolving through several regimes, from the pre-inflationary patch, through the inflationary stage and reheating to standard radiation-dominated evolution.

Hawking and Guth pursued refinements of Hawking’s approach throughout the Nuffield workshop.³⁷ The centerpiece of these calculations is the “time delay” function characterizing the start of the scalar field’s slow roll down the effective potential. This “time delay” function is related to the two-point correlation function characterizing fluctuations in ϕ prior to inflation, and it is also related to the spectrum of density perturbations, since these are assumed to arise as a result of the differences in the time

³⁶Synchronous gauge is also known as “time-orthogonal” gauge: the coordinates are adapted to constant time hypersurfaces orthogonal to the geodesics of comoving observers. All perturbations are confined to spatial components of the metric, i.e., the metric has the form $ds^2 = R^2(t)(dt^2 - h_{ij}dx^i dx^j)$, with $i, j = 1, 2, 3$. The coordinates break down if the geodesics of co-moving observers cross.

³⁷These efforts were later published as (Hawking 1982; Guth and Pi 1982).

at which inflation ends. However, these calculations treat the perturbations as departures from a globally homogenous solution to the equations of motion for ϕ , and do not take gravitational effects into account. How this approach is meant to handle the gauge problem is also not clear.

Starobinsky’s approach leads to a similar conclusion via a different argument: as in the first approach, the time at which the de Sitter stage ends is effectively coordinate dependent (Starobinsky 1982). The source of these differences is traced to the production of “scalarons” during the de Sitter stage rather than a “time delay” function for the scalar field (see, in particular Starobinsky 1983, 303). Finally, Steinhardt and Turner enlisted James Bardeen’s assistance in developing a third approach; he had recently formulated a fully gauge invariant formulation for the study of density perturbations (Bardeen 1980). Using Bardeen’s formalism, the three aimed to give a full account of the behavior of different modes of the field ϕ as these evolved through the inflationary phase and up to recombination. The physical origin of the spectrum was traced to the qualitative change in behavior as perturbation modes expand past the Hubble radius: they “freeze out” as they cross the horizon, and leave an imprint that depends on the details of the model under consideration.

Here I will not give a more detailed comparison of these three approaches. Despite the conflicting assumptions and other differences, the participants of the Nuffield workshop apparently lent greater credibility to their conclusions due to the rough agreement between the three different approaches.

During the three weeks of collaboration at Nuffield these different approaches converged on the following results. In the notation of Bardeen et al. (1983), the spectrum of density perturbations is related to the field ϕ by:

$$\Delta|_{\lambda} = AH \frac{\Delta\phi}{\dot{\phi}}, \quad (6)$$

where $\lambda \approx H^{-1}$, and A is a constant depending on whether the universe is radiation ($A = 4$) or matter ($A = 2/5$) dominated when λ “re-enters” the Hubble radius. The other quantities on the RHS are both evaluated when λ “exits” the Hubble radius: $\Delta\phi$ is the initial quantum fluctuation in ϕ , on the order of $\frac{H}{2\pi}$. The value of $\dot{\phi}$ is given by (from 5) $\dot{\phi} \approx \frac{V'(\phi)}{3H}$, and V' depends on the coupling constants appearing in the effective potential. For a Coleman-Weinberg effective potential with “natural” coupling constants, $\dot{\phi} < H^2$; plugging this all back into the initial equation we have:

$$\Delta|_{\lambda} > A \frac{H^2}{2\pi H^2} \approx .1 - 1 \quad (7)$$

Inflation naturally leads to an *almost* HPZ spectrum, which is also Gaussian (see, e.g., Bardeen et al. 1983). But reducing the magnitude of these perturbations to satisfy observational constraints requires an unnatural choice of coupling constants. In particular, the self-coupling for the Higgs field apparently needs to be on the order of 10^{-8} , although a “natural” value would be on the order of 1.³⁸

³⁸See Steinhardt and Turner (1984, 2165–2166) for a clear discussion of this constraint, which is also discussed in detail in Kolb and Turner (1990); Linde (1990).

Calculations of the perturbation spectrum culminated in a Pyrrhic victory: a Coleman-Weinberg potential provided a natural mechanism for producing perturbations, but it could be corrected to give the correct amplitude only by abandoning any pretense that the field driving inflation is a Higgs field in an $SU(5)$ GUT. However, it was clear how to develop a “newer inflation” model, since before the conclusion of the conference Bardeen, Steinhardt, and Turner had suggested that the effective potential for a scalar field in a supersymmetric theory (rather than the Higgs field of a GUT) would have the appropriate properties to drive inflation.

Finding a particular particle physics candidate for the scalar field driving inflation would provide for an important independent line of evidence. The Nuffield workshop marked the start of a different approach, as the focus shifted to implementing inflation successfully rather than starting with a candidate for the field driving inflation derived from particle physics.

The introduction of an “inflaton” field, a scalar field custom-made to produce an inflationary stage, roughly a year later illustrates this methodological shift.³⁹ An inflaton field may resemble the Higgs, but the rules of the game have changed: an inflaton is a new fundamental field distinct from any scalar field appearing in particle physics. The fact that inflation has not been closely tied to $SU(5)$ GUTs has been a boon to the field. Experiments carried out throughout the early to mid 80s failed to detect proton decay on time scales predicted by the minimal $SU(5)$ GUTs (Blewitt et al. 1985). Following the demise of the minimal GUTs, there has been an ongoing effort to implement inflation within new models provided by particle physics.

Following the Nuffield workshop, inflation turned into a “paradigm without a theory,” borrowing Turner’s phrase, as cosmologists developed a wide variety of models bearing a loose family resemblance. The models share the basic idea that the early universe passed through an inflationary phase, but differ on the nature of the “inflaton” field (or fields) and the form of the effective potential $V(\phi)$. Keith Olive’s review of the first decade of inflation ended by bemoaning the ongoing failure of any of these models to renew the strong connection with particle physics achieved in old and new inflation:

A glaring problem, in my opinion, is our lack of being able to fully integrate inflation into a unification scheme or any scheme having to do with our fundamental understanding of particle physics and gravity. . . . An inflaton as an inflaton and nothing else can only be viewed as a toy, not a theory. (Olive 1990, 389)

In a similar vein, Dennis Sciama commented that inflation had entered “a Baroque state” as theorists constructed increasingly ornate toy models (Lightman and Brawer 1990, p. 148). The sheer number of versions of inflation is incredible; Guth (1997a, 278) counts over 50 models of inflation in the nearly 3,000 papers devoted to inflation (from 1981 to 1997), and both numbers have continued to grow. Cosmologists have even complained about the difficulty of christening a new model with an original name, and a partial list of the inflationary menagerie has been used as comic relief in conference

³⁹Several researchers studied scalar fields with the appropriate properties to drive inflation, but the term “inflaton” seems to have appeared first in Nanopoulos et al. (1983); see Shafi and Vilenkin (1984) for a similar model. I thank Keith Olive for bringing the first paper to my attention.

talks.⁴⁰

4 Demise of a Rival Approach: Topological Defects

The development of scientific theories is shaped by competing approaches and the prospects for fruitful engagement with observations or experiments. The CMBR, aptly called the “cosmic Rosetta stone,” has provided a stable target for early universe cosmologists: the physical understanding of the CMBR is well-established, and the quality and variety of observations has improved steadily. These observations have been guided by assessments of what distinguishes among inflation and alternative accounts of the early universe. Throughout the 80s and 90s the most important alternative account of the origins of structure was based on topological defects. These ideas were first studied in the 70s prior to inflation, as a general feature of symmetry-breaking phase transitions in the early universe. Guth invented inflation to avoid an over-abundance of one kind of defect, monopoles. But there are many types of defects that can be produced, and several theorists took up the challenge of understanding whether defects formed in the early universe could produce appropriate seeds for structure formation. This line of work is too diverse to be characterized as a single competing theory; it is, instead, a general approach, characterized by the assumption that topological defects are the primary mechanism of structure formation in the early universe. This brief discussion will focus on contrasting inflation with this approach, with no attempt to give a detailed account of the historical development of these ideas.⁴¹

The formation of topological defects is determined by properties of the vacuum manifold \mathcal{M} . The vacuum manifold consists of the degenerate vacuum states of the system after the phase transition. Suppose the theory initially has a symmetry group G that is then spontaneously broken to a subgroup H .⁴² The symmetry is broken in the sense that the vacuum states of the theory are degenerate: although the vacuum state is not invariant under the action of some $g \in G$, these distinct vacuum states are degenerate in that the Hamiltonian has the same eigenvalue. The subgroup H consists of those elements of G under which the vacuum state remains invariant. The space of degenerate vacuum states is then in one-to-one correspondence with sets of elements of the form gH ; in other words, the vacuum manifold \mathcal{M} is topologically equivalent to the quotient space G/H . Topological features of the vacuum manifold then determine

⁴⁰Rocky Kolb used such a slide in a talk at the Pritzker Symposium (Chicago, 1998); for an example of such a list see Shellard (2003), figure 41.3.

⁴¹I have left aside one important aspect of the comparison between inflation and topological defect theories, namely the role of different types of dark matter in each scenario. The mechanisms for structure formation are part of package deal, including assumptions about the overall matter budget and other factors more significant for later stages of structure formation.

⁴²This means that, roughly speaking, for all $g \in G$ the Hamiltonian of the system is invariant under the action of g , but the vacuum or ground state of the system is not. (This is only a rough gloss; in quantum mechanics the action of a symmetry g is usually represented by a unitary operator on the Hilbert space, but in the case of broken symmetry there is not a well-defined operator mapping between degenerate vacua, as these each define different Hilbert spaces.) The degenerate vacuum states are labeled by different values of the “order parameter” of the transition. The order parameter is the thermodynamic quantity that changes discontinuously through the transition and characterizes different phases, corresponding to degenerate vacua in this case; it is the vacuum expectation value of the relevant field(s).

what kinds of topological defects may form in the course of the phase transition.⁴³

Starting in the early 70s these ideas were applied to cosmology. Extrapolating the FLRW models, the early universe reaches arbitrarily high temperatures at early times. Kirzhnits (1972) suggested that symmetries in particle physics would be restored at sufficiently high temperatures, by analogy with symmetry restoration in condensed matter systems. Further calculations of symmetry restoration in the Standard Model of particle physics supported the idea that as the universe cooled it passed through a series of phase transitions that broke the symmetries between various interactions. Many symmetry breaking phase transitions in condensed matter systems lead to the formation of topological defects, such as vortices in liquid helium, so it is natural to expect that defects to also arise in early universe phase transitions.

In a seminal paper, Kibble (1976) argued that topological defects would be produced due to the horizon structure of the early universe. (His account is sometimes referred to as the “Kibble mechanism.”) Given that the correlation length of the order parameter is bounded by the horizon distance, the phase transition produces domains in which the order parameter takes on different values determined by random fluctuations, assuming that the dynamics is not completely adiabatic. Whether defects form depends on the topology of the vacuum manifold. For example, suppose that there is a curve through \mathcal{M} that cannot be smoothly contracted to a point. Each point within the space \mathcal{M} represents a different degenerate vacuum state, which is labeled by different values of the order parameter for the phase transition.

Suppose that the values of the order parameter around a spatial loop take the same values given along the loop in \mathcal{M} . Since the loop cannot be continuously contracted to a point within \mathcal{M} , it is also not possible to assign values of the order parameter continuously in the region bounded by the spatial loop while remaining in \mathcal{M} . This implies that there must be a “defect,” namely a region of space in which the fields cannot reach the vacuum state and instead remain trapped in a state of higher energy. The nature of these regions of higher energy is fixed by the structure of \mathcal{M} . In the case at hand, with a non-simply connected vacuum manifold, the phase transition leads to two-dimensional defects called “cosmic strings.” There are several other possibilities. A phase transition breaking a *discrete* symmetry leads to regions in which the order parameter takes on discrete values separated by domain walls, which are three-dimensional surfaces in spacetime. If the vacuum manifold has non-contractible two-spheres rather than circles, then the phase transition produces point-like defects (such as magnetic monopoles); for non-contractible three-spheres the corresponding zero-dimensional defects are called “textures,” event-like defects that do not have a stable localized core.⁴⁴

Early studies showed that domain walls and some types of monopoles had disastrous consequences, conflicting with observational constraints by several orders of magnitude (see, e.g., Zel’dovich et al. 1975; Zel’dovich and Khlopov 1978; Guth and Tye 1980).

⁴³The relevant structure is given by the homotopy groups of the space. For further discussion, see, e.g., Vilenkin and Shellard (2000).

⁴⁴Additional types of defects arise due to the distinction between gauge and global symmetries and the possibility of “hybrid” defects. Defects formed in a transition breaking a global symmetry tend to have energy density distributed throughout a region, whereas those formed by gauge symmetry breaking are more localized. Hybrid defects are produced by a series of phase transitions, leaving an interacting network of defects of different kinds. See, e.g., Vilenkin and Shellard (2000), for further discussion.

However, other types of defects — in particular, cosmic strings — were more plausible candidates for the seeds for structure formation. The defects are inherently stable regions of higher energy density, whose scale is set by the energy scale of the phase transition. The defects have an important impact on the dynamical evolution of the system following the phase transition, and in particular it is plausible that they will provide seeds that are subsequently enhanced via gravitational instability as described by linear perturbation theory. For GUT-scale phase transitions the energy density is the appropriate order of magnitude to seed large-scale structure. Some defect theories have “scaling solutions,” in which the network of defects evolves such that there is no preferred length scale imprinted at a particular time. These theories then pass an important initial test, in that they lead to an approximately scale-invariant HPZ spectrum of perturbations.⁴⁵ They are thus compatible with the first generation of CMBR observations and the general picture of structure formation described above. However, there are important general differences between the inflationary account and that provided by topological defects, and these were clarified by a substantial research effort throughout the 80s and 90s.

To determine whether topological defects suffice as the primary mechanism for producing seeds for structure formation, researchers had to tackle two challenging problems. The first was to describe the phase transition itself and determine the nature of the defects produced, with sufficient quantitative detail to determine the consequences for later stages of evolution. In principle these details should be calculable given a particular extension of the Standard Model of particle physics. But the sheer complexity of the models, and the nature of the quantities needed to assess the implications for structure formation, have made it quite difficult in practice to carry out such calculations. Second, one had to describe the subsequent evolution of the network of defects left over following the phase transition over a wide range of dynamical scales. Solving this second problem requires determining the interactions among the defects and their gravitational effects. The problem is exceedingly difficult because the evolution of defects is non-linear, and researchers have relied primarily on numerical simulations. Physically plausible suggestions regarding evolution of defects have often been undercut by numerical work. Throughout the 80s, for example, the general picture of how strings seeded galaxy formation changed considerably in light of numerical simulations establishing details regarding the size of typical closed loops of strings and the behavior of open strings.⁴⁶ These two problems are exacerbated by uncertainty regarding the relevant fundamental physics. The details of the phase transitions depend on specific features of the physics — for example, the vacuum manifold is fixed by the full symmetry group G and its unbroken subgroup H , but these differ among proposed extensions of the Standard Model.

Despite these difficulties, by around 1997 there was a consensus regarding the generic

⁴⁵However, the sense in which the two theories are scale-invariant is different; see, e.g., § 5.1.1 of Martin and Brandenberger (2001). Many defect models are scale-invariant only over a limited dynamical range; for example, in models of defect formation via strings scale invariance is broken at the matter-radiation transition.

⁴⁶See Vilenkin and Shellard (2000, Chapter 11) for an overview; the closing section (p. 342) emphasizes the changes in the account due to numerical simulations of the evolution of string networks.

consequences of structure formation via defects and the contrast with the consequences of inflation.⁴⁷ Structure formation via topological defects is “active” in the sense that the network of defects persists over time and continues to interact gravitationally with the other constituents. More precisely, in the evolution equation for perturbations of the cosmological model there is a source term, representing the stress-energy of the network of defects. Determining the evolution of the perturbations thus requires calculating the evolution of this source term, based on the non-linear dynamics of the network of defects. (For example, in the case of cosmic strings the non-linear dynamics describes the growth of the network of strings with the cosmological expansion and the different kinds of interactions among strings.) Perturbations produced in defect theories “decohere” (as first noted by Albrecht et al. 1996) in the sense that fluctuations at all wave-numbers are not in phase. This is a consequence of the non-linear evolution of the source term, which leads to mixing of perturbations across different modes. The perturbations are also non-Gaussian due to the correlations that this mixing produces between perturbations. Finally, defects generate scalar, vector, and tensor perturbations of roughly equal magnitude.

This account of structure formation contrasts sharply with that based on inflation. Despite debates regarding how inflation related to particle physics, consensus was achieved regarding the consequences of inflation for structure formation.⁴⁸ Consider a massless, minimally coupled scalar field ϕ evolving in a background FLRW model. Due to the symmetry of the FLRW models the Fourier modes ϕ_k of ϕ are uncoupled, and each mode evolves during slow-roll inflation according to the equation of a damped harmonic oscillator.⁴⁹ For modes such that $\frac{k}{R} \ll H$, the damping term is negligible, whereas those with $\frac{k}{R} \gg H$ will evolve like an over-damped oscillator and “freeze in” with a fixed amplitude. The inflationary account runs very roughly as follows. All the modes ϕ_k are assumed to be in their ground state prior to inflation. For $\frac{k}{R} \ll H$ the modes evolve adiabatically, remaining in their ground states, given that eqn. (8) is approximately the equation for a harmonic oscillator. This account is not sensitive to exactly when a given mode is assumed to be “born” in its ground state. During inflation the modes scale with the exponential expansion whereas H is approximately constant. Due to this scaling behavior, modes will reach the horizon scale $\frac{k}{R} \approx H$ — “horizon exit”. The damping term is no longer negligible and the modes “freeze in” as they cross the horizon. Modes then “re-enter” the horizon later given that the Hubble radius grows more rapidly than the modes after the inflationary stage has ended. Finally, these modes are treated as classical density perturbations upon re-entering the horizon.⁵⁰ This leads to a nearly scale invariant spectrum; it is not *exactly* scale invariant

⁴⁷Several groups published calculations at around this time supporting the general picture I summarize here; see, e.g., Magueijo et al. (1996); Pen et al. (1997). See Durrer et al. (2002) for a comprehensive review of this area with further references to the original papers, and Brandenberger (1994) for an earlier review.

⁴⁸Mukhanov et al. (1992) is the canonical review article regarding structure formation.

⁴⁹The equation can be derived from the action for the scalar field minimally coupled to gravity (with various simplifications, such as neglecting metric perturbations):

$$\frac{d^2\phi_k}{dt^2} + 3H\frac{d\phi_k}{dt} + \frac{k^2}{R^2}\phi_k = 0. \quad (8)$$

⁵⁰Although I do not have space to discuss the issue further here, this step involves a quantum to classical

because the Hubble radius is not exactly constant throughout inflation. The amplitude of the perturbations that are frozen in at horizon exit depends upon the details of the particular inflationary model under consideration.

The inflationary account differs in a number of respects from structure formation via defects. Inflation is a “passive” account of structure formation: there is no source term in the evolution equation, and in the linear regime the solution is fixed by the initial conditions. Roughly speaking, in inflation the perturbations evolve “on their own” after being imprinted at early times, whereas in the defect theories the network of defects persists and continues to seed structure formation. The most striking contrast is that inflation leads to phase coherence of the perturbations, because the dynamics described above leads to synchronization of the Fourier modes. Generically inflation predicts an oscillatory pattern in the angular power spectrum of temperature fluctuations in the CMBR, known as Doppler peaks.⁵¹

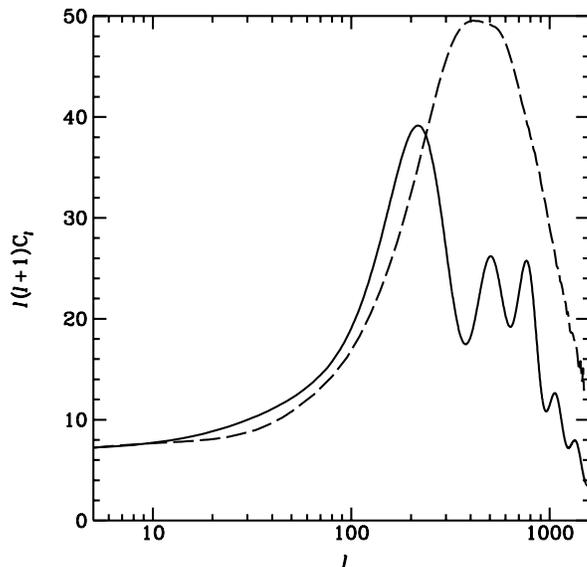


Figure 3: This figure (from Albrecht et al. 1996) shows the predicted angular power spectrum of temperature fluctuations in the CMBR from a particular model of cosmic strings (dashed line), and a generic inflationary model (solid line).

These contrasts between defect formation and inflation lead to quite strikingly different predictions for what should be observed in the CMBR (as illustrated in figure 3). In addition, theorists working on these calculations through the 90s were correct to expect that further observations of the CMBR would be able to discriminate among competing accounts. In particular, defect theories fail to predict strong secondary oscillations evident in subsequent CMBR observations. These features are “washed out”

transition.

⁵¹The angular power spectrum characterizes the variations in temperature of the CMBR, i.e. the amount of temperature variation across different points of the sky versus the angular frequency ℓ . Small values of ℓ correspond to temperature variations with a large angular scale. See, e.g., Liddle and Lyth (2000), § 5.2, for further discussion of the angular power spectrum.

due to decoherence, whereas in inflationary accounts there are coherent standing wave oscillations in the baryon density that lead to strong secondary peaks. The position of the first peak also differs in inflation and topological defect models, with defect models generally predicting a primary peak at a larger multipole moment ($\ell \geq 300$) than inflation ($\ell \approx 200$). Observational results starting in the late 90s and culminating in the WMAP results (3 year results published in 2003) provided decisive support for inflation with respect to both of these features.

In addition to the physical contrast between the mechanisms for structure formation, there are important methodological contrasts between the two approaches. First, despite uncertainty regarding the detailed physics of the phase transitions, the account of structure formation via defects is constrained enough by general theoretical principles to produce specific observational signatures. Physicists working on defects often highlighted this rigidity as a virtue of the theory, characterizing it as “falsifiable” in a Popperian sense. Second, accounts based on topological defects do not address the problems related to initial conditions highlighted by Guth. In effect, the theory starts from the same initial conditions as the standard FLRW models, with the exception that the initial seed perturbations were produced dynamically rather than fixed by hand. This methodological contrast did not play a role in the detailed evaluation of structure formation via topological defects in light of CMBR observations. Those who accepted Guth’s approach to fine-tuning and initial conditions could still use defects, however. Inflation could still be invoked to solve the problems related to initial conditions (see, e.g., Vilenkin and Shellard 2000), as long as inflation set the stage for a subsequent phase transition that would produce appropriate topological defects.

The sharp contrast described above is based on assuming that there is only one primary mechanism for the formation of primordial perturbations. Yet the current theory of early universe phase transitions does not enforce such exclusivity. Ruling out models combining inflation and topological defects makes for a clearer theoretical contrast, amenable to decisive observational tests. But, as far as I am aware, there is little evidence that Nature’s choices are so conveniently circumscribed.

5 Characterizing Empirical Success

The fate of topological defects illustrates the power of contemporary cosmological observations. Within the last 50 years cosmology has gone from being a field with only “2 1/2 facts”⁵² to a field with data that is sufficiently rich to warrant conclusions regarding novel physics far beyond the reach of earthbound accelerators. By the turn of the millenium, structure formation via topological defects was not only falsifiable but apparently falsified by cosmological observations.⁵³ Inflation did not share this fate; it is clearly compatible with the CMBR observations that ruled out defects. But in

⁵²Peter Scheuer made this remark in the course of warning a student, Malcolm Longair, about the current status of cosmology in 1963; the list included (1) that the sky is dark at night, (2) that the galaxies recede, and (2 1/2) that the universe is evolving (qualified as a half fact due to its uncertainty).

⁵³Observations seem to rule out topological defects as the primary mechanism for generating large-scale structure. However, defects might still play a role as part of the full account of the formation of structure or in other aspects of early universe cosmology, such as baryogenesis.

what sense does current observational data support inflationary theory? How should we characterize the empirical success or predictive power of the theory?

Debates in the physics literature have typically framed this question in terms of falsifiability. Has inflation avoided falsification just because it is unfalsifiable? Consider, for example, whether inflation could be falsified by finding that $\Omega_0 \not\approx 1$.⁵⁴ Flatness is often cited as an unambiguous, correct prediction of inflation. Guth (1997a), for example, emphasizes the extraordinary precision of the inflationary prediction – a correct value of Ω at the end of inflation to 15 significant figures! There are two reasons, however, to doubt that this is so straightforward.⁵⁵ First, for *any* particular value of Ω_0 there is a corresponding “initial” value $\Omega(t_p)$, whether inflation occurred or not. Thus the prediction has to be regarded as a probabilistic claim: for “highly probable” or “reasonable” initial conditions inflation yields $\Omega_0 = 1$. But, as discussed in § 3.1 above, it is not clear what to make of these probabilistic claims without a measure over the space of initial values of Ω . The second objection is that the inflationary paradigm is too flexible to yield falsifiable predictions. In the mid 90s theorists constructed “open models” of inflation that yield a lower value of Ω_0 (see, for example, Bucher et al. 1995). At most one might claim that a subset of inflationary models could be ruled out by finding $\Omega_0 \not\approx 1$, with further disagreement over whether this subset includes all of the “natural” or “reasonable” models of inflation. Rather than an unambiguous, falsifiable prediction, we are left with equivocal judgments regarding the probability assigned to initial conditions and the plausibility of different inflationary models.

Discussions of the falsifiability of inflation often draw Liddle and Lyth’s distinction quoted in the introduction between inflation “as a theory of initial conditions” and inflation as a theory of structure formation.⁵⁶ The account of structure formation appears to have definitive, falsifiable consequences. Several observational signatures — gaussianity, near scale invariance — follow directly from the description of the dynamical evolution of the modes of a quantum field through horizon-crossing. This dynamical mechanism for generating perturbations is a direct consequence of the defining feature shared by all inflationary models, given that it depends on the evolution of the Hubble constant during exponential expansion. Thus one might hope to avoid the above objections: the production of density perturbations is independent of assumptions regarding initial conditions, and the account is generic in the sense of being common to all models of inflation. But does the success of inflation simply exploit the malleability of the

⁵⁴The falsifiability of inflation, focusing in part on flatness, is addressed quite directly in a number of papers in Turok (1997), in particular the contributions by Linde, Steinhardt, Guth, and Albrecht. This has been a perennial subject of debate since the early days of inflation.

⁵⁵The question was particularly pressing throughout the 90s, when the evidence seemed to favor open cosmological models with $\Omega_0 \approx 0.2 - 0.3$, although there was not a general consensus. See, e.g., Coles and Ellis (1997) for a detailed argument in favor of an open universe. However, the consensus had begun to shift in favor of a flat universe by 1998. Peebles and David Schramm were invited to convene a “great debate” on the issue in April of 1998. Due to Schramm’s death the debate was rescheduled for October of 1998, with Michael Turner taking Schramm’s place. But given that Peebles and Turner both agreed that the evidence decisively favored a flat universe, they changed the subject of the debate to “Is Cosmology Solved?” (Peebles 1999a; Turner 1999).

⁵⁶The distinction is perhaps too quick, given that there are some predictions related to initial conditions. For example, inflation predicts that the observed universe is topologically simply-connected; inflation is incompatible with compact topology at sub-horizon scales. Evidence that the universe is multiply connected would rule out inflation.

“inflaton” field and its potential? Note, for example, that the amplitude of the density perturbations needed for accounts of structure formation is used to constrain the parameters of the inflaton field. For this reason, Peebles (1999b) classifies the amplitude of the density perturbations as a “diagnostic” rather than a successful prediction.

Hollands and Wald (2002) argued that there is not such a clear contrast in terms of initial conditions. In particular, the inflationary account of the dynamical evolution of the modes of a quantum field through horizon crossing assumes that the modes are initially in their ground state. This is a plausible assumption given that the modes with cosmologically significant length scales will be well inside the Hubble radius prior to the inflationary phase. Since the modes evolve adiabatically before horizon crossing the exact time at which they are taken to be “born” in their ground state is unimportant. Hollands and Wald (2002) construct a simple model that produces a similar spectrum of density perturbations *without an inflationary phase* based on a different *Ansatz* for the initial conditions for these modes. Their model describes quantized sound waves in a perfect fluid, with the same “overdamping” of modes with $\lambda \gg H^{-1}$ as in inflation. By contrast with inflation, there is no horizon crossing, so it is significant precisely when the modes are taken to be in a vacuum state. Hollands and Wald (2002) propose to take the modes to be “born” in a ground state when their proper wavelength is equal to the Planck scale, motivated by considerations of the domain of applicability of semi-classical quantum gravity.⁵⁷ This hypothesis combined with the dynamics governing the evolution of the modes leads to a scale-invariant perturbation spectrum. The significance of this result for present purposes is that it undermines claims that the theory of structure formation does not depend on arguments regarding plausible initial conditions.

Stepping back from the details of inflation for a moment, it should be clear that there are important questions regarding both how to characterize a theory’s empirical success and what a given degree of success establishes. It is unfortunate that these questions are still treated in the physics literature in terms of “falsifiability,” and I will briefly sketch an alternative drawing on recent studies of Newton’s methodology (Harper 2002; Smith 2002). On this approach, empirical success is defined in terms of the ability to determine consistent values of theoretical parameters from multiple, independent bodies of data. Consider, for example, Newton’s argument in favor of a universal force of gravity in the *Principia*. Newton takes the theoretical framework provided by the laws of motion to be exact, and the array of mathematical results applying to forces in general then allows him to infer properties of the gravitational force from the observed motions of the planets, their satellites, and various other bodies (such as pendulums). The famous precession theorem is a particularly beautiful example: Newton shows that for approximately circular orbits, the motion of the apsides measures the exponent of the power law.⁵⁸ Taking the exponent in the power law for gravity as our example of

⁵⁷The modes will be “born” at different times, continually “emerging out of the spacetime foam” (or whatever description the full theory of quantum gravity provides), with the modes relevant to large-scale structure born at times much earlier than the Planck time. By way of contrast, in the usual approach the modes at all length scales are specified to be in a ground state at a particular time, such as the Planck time. But the precise time at which one stipulates the field modes to be in a vacuum state does not matter given that the sub-horizon modes evolve adiabatically.

⁵⁸The apsidal angle θ is the angle through which the radius vector rotates between two consecutive apsides, which are points on the orbit of maximum (aphelion) or minimum (perihelion) distance from the force center.

a theoretical parameter, there are several lines of argument from diverse, independent bodies of data that fix the value as very close to -2 . This account acknowledges that the theory requires some data as “input” to enable further predictions. Other bodies of data that can be used to constrain the same parameter value then provide independent checks. Harper (1990, 2007) argues that Newtonian characterization of empirical success is much more demanding than mere predictive accuracy. A theory that achieves predictive accuracy by “curve-fitting” (exploiting theoretical flexibility) will suffer by comparison with a more rigid theory on the Newtonian account.

The strength of Newton’s empirical argument for universal gravitation bears directly on two potential objections. First, why should one accept gravity as a “real force” given that it apparently involved action-at-a-distance? Although the issue is complicated, Newton clearly held that the empirical case was sufficient to establish the reality of gravitational force despite uncertainty regarding its underlying cause and certainty that it is not a “mechanical” cause (i.e., due to contact action). Second, from a modern perspective, why should Newton’s theory be preserved as a limiting case of general relativity? If we regarded the theory merely as a predictively accurate curve-fit, rather than an accurate systematic treatment of physical relationships within a limited domain, there would be no reason to expect general relativity to recover anything more than the predictions themselves. Speaking more generally, the first kind of objection relates to unresolved problems. In some cases the empirical success of a theory is sufficient to warrant acceptance even in light of open physical questions. The second challenge regards the use of a theory as a step towards further theories. Sufficient empirical success warrants preserving not just the predictions of the theory but the physical relationships it ascribes to systems within its domain.

Returning to the case of inflation, there are two similar challenges. First, there are various open problems regarding the place of an “inflaton” field within particle physics at the appropriate energy scales and the coupling of a scalar field to gravity. The cosmological constant problem is sometimes characterized as the Achilles heel of inflation. Inflation is built on the assumption that the false vacuum energy of the inflaton field couples to gravitation. But if this is so, the vacuum energy density of other quantum fields should contribute to gravity as an effective cosmological constant. A comparison between the vacuum energy density calculated in QFT and observational limits on the cosmological constant in GR reveals an incredible discrepancy of some 120 orders of magnitude! As Frank Wilczek commented in a review of the Nuffield workshop:

It is surely an act of cosmic *chutzpah* to use this dismal theoretical failure [in understanding the cosmological constant] as a base for erecting theoretical superstructures, but of course this is exactly what is done in current inflationary models. (Hawking et al. 1983, 476, original emphasis)

Second, cosmologists have often suggested that the requirement to find an inflaton

Newton establishes (Book I, Proposition 45) that for approximately circular orbits under a centripetal force varying as $f \propto r^{n-3}$, the apsidal angle is given by $n = \left(\frac{\theta}{\pi}\right)^2$. For stable orbits, the radius vector rotates through π between the aphelion and perihelion, such that $n = 1$ and $f \propto r^{-2}$; and for nearly stable orbits, the force is approximately $f \propto r^{-2}$.

field should serve as a constraint on particle physics. This is certainly appealing, as a successful case for inflation would provide a strong constraint at energy scales with few observational constraints from earthbound accelerators.

On this approach, the question to ask regarding inflation is not whether it makes various “falsifiable” predictions, but to what extent do the observational data allow us to infer the details of inflation? On the assumption that inflation is correct, what do the data allow us to infer about the inflaton field, and its effective potential $V(\phi)$? In these terms the account of inflation as a theory of structure formation provides a richer set of constraints on the theory. The solution of the horizon and flatness problems constrains the duration of the inflationary phase: the pre-inflationary patch has to grow larger than the observed universe, at a minimum. The inflationary stage will last sufficiently long if the potential $V(\phi)$ is suitably flat, and satisfies the “slow-roll” conditions described in § 3.2 above. The account of structure formation, by contrast, provides more detailed constraints. The fluctuation modes that seed the formation of structure depend on the properties of the effective potential $V(\phi)$ at the time when they cross the horizon. (There is a limit on the part of the potential that can be constrained in this way, given that only some of the modes will have re-entered the horizon as observable density perturbations.) This opens up the prospect of reconstructing the inflaton potential based on observations of the CMBR. Whether the reconstruction provides sufficient empirical warrant to answer the challenges above is another question.

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