BARRY SMITH

Boundaries: A Brentanian Theory

Zeno's Paradox for Colours

Imagine that we proceed along a line through the middle of a disk that is divided into two precisely symmetrical segments, one of which is red, the other green, and that we pass continuously from the red to the green segment. What happens as we pass the boundary between the two? Do we pass through a last point p_1 that is red and a first point p_2 that is green? Clearly not, given the density of every continuum; for then we should have to admit an indefinite number of further points between p_1 and p_2 which would somehow have no colour. To acknowledge one of p_1 and p_2 but not the other, however, as is dictated by the Dedekindian treatment of the continuum, would be to countenance a peculiar privileging of one of the two segments over the other, and an unmotivated asymmetry of this sort we can surely reject as a contravention of the principle of sufficient reason. Perhaps, then, the line becomes colourless at the point where it crosses the segmentary divide, so that the red and green segments would be analogous, topologically, to open regions. One might seek support for this idea by reflecting that extensionless points are not in any case the sorts of things that can be coloured, since colour properly applies only to what is spatially extended. Imagine, however, a perfectly homogeneous red surface. Are the points and lines within the interior of this surface not then also red?

To firm up our intuitions here let us consider two parallel thought-experiments relating to motion and bodily contact. Since one seemingly exceptionally secure result of modern physics is that the four dimensions of space and time are continuous orders of a single type, we might safely assume that any solution to our colour problem should carry over to these other cases also.

Imagine a body which is for a certain period at rest and then begins to move. Is there a last point in time p_1 when the body is at rest and a first point p_2 when it is in motion? Clearly not, given the density of every continuum; for then we should have to admit an indefinite number of further points between p_1 and p_2 at which the body would somehow be

Brentano Studien 8 (1998/99), 107-114

neither at rest nor in motion. To acknowledge one of p_1 and p_2 but not the other, however, would again be to countenance a peculiar privileging of one or other temporal segment. Perhaps, then, the body is neither in motion nor at rest at the point in time where it crosses the segmentary divide. But is it even coherent to suppose that a body might be neither in motion nor at rest at a certain point in time?

Imagine two perfect spheres at rest and in contact with each other. What happens at the point where they touch? Is there a last point p_1 that belongs to the first sphere and a first point p_2 that belongs to the second? Clearly not; for then we should have to admit an indefinite number of further points between p_1 and p_2 and this would imply that the two spheres were not in contact after all. To acknowledge one of p_1 and p_2 but not the other, however, would be to countenance what is here an asymmetry of a quite peculiarly unmotivated sort. And our third alternative seems to be ruled out also. For to admit that the point where the two spheres touch belongs to neither of the two spheres seems to amount to the thesis that the two spheres do not touch after all.

There is, however, an alternative (non-Dedekindian) account of what obtains as far as colour is concerned at the point on the line where the red and green segments meet, an account which can be smoothly extended also to the other cases mentioned. This affirms that there is but one (albeit complex) point of the line which lies precisely on the border between the two segments. This point is in a certain sense *both red and green.* More precisely, it is at one and the same time *a ceasing to be red* and a *beginning to be green*. More precisely still, it is a point where a red point and a green point *coincide.* Similarly in the case of the particle that begins to move: here too there is a single point in time at which the body is both *at rest* and *moving* (or more precisely: it is at one and the same time *ceasing to be at rest* and *beginning to move).* The terminal boundary of the initial interval *coincides* with the initial boundary of the subsequent interval. And the same account can be given also in relation to what occurs when two spheres touch: a point on the boundary of the one sphere *coincides* with a point on the boundary of the other. All bodies and all temporal intervals are on this account analogous, topologically, to closed regions (or perhaps we should more properly say that there is no analogue in the world of spatial and temporal continua of the standard opposition between open and closed).

Asymmetrical Boundaries

The picture of the world of continua and boundaries that is dictated by the above is as follows. Boundaries are full-fledged denizens of reality. They serve as objects of perception (and are perhaps the only objects of perception). But boundaries cannot exist in isolation: there are, in reality, no isolated points, lines or surfaces. Boundaries might be compared in this respect to forms or structures (for example the structure of a molecule as this is realized in a given concrete case) in that they are located in space but do not take up space. Further, both boundaries and forms or structures (and holes, and shadows; perhaps also minds or souls) are comparable to universals in that, while they require of necessity hosts which instantiate them, they can in principle be instantiated by a variety of different hosts. (See Casati and Varzi (1994)) Consider, for example, that boundary which is the surface of an apple. The whole apple can here serve as host, but so also can the apple minus core, which might have been eaten away to varying degrees from within.

As Brentano puts it:

> No boundary can exist without being connected with a continuum. ... But there is no specifiable part, however small, of the continuum, and no point, however near it may be to the boundary, which is such that we may say that it is the existence of that part or of that point which conditions the boundary. (Brentano 1981, 56.)

An adequate theory of the continuum must now recognize that boundaries may be boundaries only in certain directions and not in others. Imagine a line that is tangent to a circle and meets the circle at a certain point. The point on the line then *coincides* with a certain point on the circle. To see why the two points are not identical we might think of points (and boundaries in general) as being 'unsaturated' in something like Frege's sense, which is to say: they exist always in consort with certain additional entities of a predetermined sort, or in other words they are required to be completed in certain predetermined directions. Every point must serve as a boundary in at least one direction.

Consider a point within the interior of a solid sphere. This is a boundary in all possible spatial directions and is as it were a boundary of maximal fulness. An external boundary of a body, is of lesser fulness, since it bounds the bodily continuum in only some of the available directions. A boundary is from our present perspective determined in its nature by the continuum which it bounds. This implies, however, that 'the geometer's proposition that only one straight line is conceivable between two points, is strictly speaking false' (Brentano (1988), 12): Lines which enjoy as boundaries less than maximal fulness and which relate to different sides may coincide with one another. The boundary of an area that is red, for example, differs in kind from the boundary of an area that is blue: 'If a red surface and a blue surface are in contact with each other, then a red and a blue line coincide' (Brentano (1988), 41). Or again:

> Imagine the mid-point of a blue circular surface. This appears as the boundary of numberless straight and crooked blue lines and of arbitrarily many blue sectors in which the circular area can be thought of as having been divided. If, however, the surface is made up of four quadrants, of which the first is white, the second blue, the third red, the fourth yellow, then we see the mid-point of the circle split apart in a certain way into a fourness of points. (Brentano (1988), 11)

Points, therefore, may have parts and they coincide with these parts, as the parts, too, coincide among themselves. Each point of a two- or three-dimensional continuum is in fact an infinite (and as it were maximally compressed) collection of distinct but coincident points: punctiform boundaries of lines, of two-dimensional segments of surfaces and of interior regular and irregular cone-shaped portions within three-dimensional continua, etc. Such ontological profligacy has its limits, however: the left punctiform boundary of a one-inch line is identical (and not merely coincident) with the left punctiform boundary of the corresponding initial half-inch segment.

Set Theory

This non-Dedekindian zoology of boundaries and the continuum has its roots in the account sketched by Aristotle in the *Physics*, particularly as this has been interpreted by Franz Brentano and Roderick Chisholm. It proceeds as it were from the top down, taking as its starting point extended and qualitatively filled spatial continua as these are given in perception. Boundaries are then conceived as entities of a certain sort that are capable of being discriminated therein. Standard set-theoretic treatments, in contrast, work upwards, constructing models of the continuum from a starting point consisting of extensionless atoms (or of some abstract equivalent thereof). Such treatments address the concerns of mathematicians and are of great power in applications. Their popularity among philosophers has been sustained further by remnants of older corpularistic ideas to the effect that atomistic physics (or some similar deep-level theory) enjoys a privileged status over against mere perceptual experience. Our contention here, in contrast, is that an alternative, formally coherent and at the same time more realistic theory of the continuum can be arrived at via the top-down approach. — The formal details are set out in Smith (1995) & (1997).

Of course, nothing precludes the possibility of constructing models of even a non-Dedekindian theory in set-theoretic terms. That the set-theoretic framework can yield at best a *model* (or family of models) of the continuum, however, and not a theory of the continuum itself as this is given in our experience of spatial bodies, is something which must be insisted upon for at least the following reasons:

1. The latter is a *qualitative* continuum not merely in the sense that it is (standardly) filled by qualities (of colour, temperature, etc.) but also in the sense that it does not sustain the sorts of cardinal number constructions (with the associated talk of 'continuum many', etc.) imposed by the set-theoretic approach. The experienced continuum is not isomorphic to any real-number structure, since the standard mathematical opposition between a dense and a continuous series here finds no application. Nothing like Cantor's continuum problem arises for the experienced continuum, and indeed the very existence of this problem – pointing to a stark absence of relevant intuitions which would decide the issue – may testify to the greater realism of our alternative theory.

2. The set-theoretical construction of the continuum is predicated on the highly questionable thesis that out of unextended building blocks an extended whole can somehow be constructed. The experienced continuum, in contrast, is organized not in such a way that it would be built up out of particles or atoms, but rather in such a way that the wholes, including the medium of space, come before the parts which these wholes might contain and which might be distinguished on various levels within them. The existence of boundaries presupposes as a matter of necessity the existence of continua which they are the boundaries of.

3. The application of set theory to a subject-matter requires the isolation of some basic level of *Urelemente* in such a way as to make possible a simulation of the structures appearing on higher levels by means of sets of successively higher types. If, however, as holds in the case of investigations of the ontology of the experienced world, we are dealing with mesoscopic entities and with their mesoscopic constituents (the latter the products of more or less arbitrary real or imagined division along a variety of distinct axes), then there are no *Urelemente* to serve as our starting-point. Moreover, it seems that we cannot even in principle rely upon physics to supply us with *Urelemente* of the appropriate sort, since even

if ultimate physical particles were capable of being isolated, these would not in themselves constitute a continuum but rather would *presuppose* the continuum of space-time within which they would be located.

4. Set theory sees the continuum as homogeneous, as made up of only one sort of ultimate part (points, atoms, real numbers). From our perspective, in contrast, the continuum is made up of parts of a variety of different sorts: boundaries of different numbers of dimensions, on the one hand, and the extended bodies and regions of space which these boundaries are the boundaries of, on the other. It is this feature of boundaries — that they are as a matter of necessity parts of heterogeneous larger wholes which they bound — which distinguishes our present conception of a boundary most radically from standard mathematical conceptions. The contrast is illustrated by Brentano as follows. We are asked to imagine that space

> would contain at one and the same time a collection of spheres, each moving with a different velocity. For one sphere this would be 0, for another 1 mile per hour, for a third 1/2 a mile per hour, and so on, so that there would be represented by some sphere every intermediate velocity between 0 and 1 mile per hour that is conceivable, whether it manifests a rational or an irrational ratio. If one asks whether one would then have to do with a continuum of velocities, then this question would, according to Dedekind, have to be answered in the affirmative. In truth however it would have to be denied. Where an actual continuum of velocities would be present is in the case of a disc rotating in such a way that the velocity at the circumference is 1 mile per hour while the centre did not change its place. The difference between the two cases is this: in the latter, each of the velocities appears as a boundary which taken in itself is nothing, but when unified with the continuum of velocities is such as to make a contribution thereto; in the case of the collection of spheres, in contrast, *the velocity of each sphere is something for itself*; it is just this which stands in contradiction with its forming a true continuum with the remaining velocities. (Brentano (1988), 41; my underlining)

5. Standard mathematical treatments of the continuum impose a principle of duality according to which every boundary of an entity is also a boundary of the complement of that entity. It is this principle, above all, which is responsible for the Zeno-type problems with which we began. Intuitively, however, it seems that the boundaries given in experience are in many cases *asymmetrical*. This applies, for example, to the external boundaries of bodies and to the beginnings and endings of processes extended in time. Thus it seems not to be the case that the external boundary of a substance is in the same sense a boundary of that entity which is the result of subtracting this substance from the universe as a whole (and even the thesis that there is such an entity is something which from our present perspective has to be taken with a pinch of salt).

Zeno's Paradox for Countries

It might be argued that the account of spatial boundaries presented above is of phenomenological interest at best, and gains no purchase as far as actual reality is concerned, since spatial boundaries of the sorts discussed — which enjoy a sort of geometrical perfection – are not at home in the world of unkempt nature that is described by physics. There is at least one domain, however, where we can find examples of boundaries in spatial reality to which our theory directly applies.

A boundary, for Brentano, bounds only its associated continuum. It is not also, in the same strict sense, a boundary of its surroundings. It is as if the external boundaries of a continuous thing point only inwards. Compare, in this respect, the boundary between the old German Democratic and Federal Republics with the boundary between, for example, Germany and France. The latter is a boundary facing in two directions, or more precisely it is a pair of boundaries facing in opposite directions which are coincident for a certain stretch of their respective total extensions. The former, in contrast, for as long as it existed, was a boundary in one direction only: it faced inwards, to the East, since the Federal Republic did not recognize a boundary in that location at all.

All political and legal boundaries must, it seems, enjoy the sort of geometrical perfection that was presupposed above. Thus they must be infinitely thin (must take up no space), for otherwise disputes would constantly arise in relation to the no-mans-land which the boundaries themselves would then occupy. If a wall or river separates two distinct portions of land, then either the wall or the river must be split equally down the middle, or it must be assigned as a whole to one or other of the two parties, or it must be declared common property (and then there will exist *two* infinitely thin boundaries separating each of the two distinct parcels of land from the commonly owned region which divides them). Note, too, that, as is shown by the case of the United States or of New Zealand, boundaries (like the things they bound) can be *scattered*; they can be built up mereologically out of separate and disconnected Irish-style bits. (See Cartwright (1975) and Smith (1995a)) And as the case of

New York State and the U.S.A. makes clear, distinct geopolitical boundaries may also coincide from within. That is, they may coincide for a part of their length along which they serve as boundaries *on the same side*.

Note, finally, that even though (from the perspective here advanced) political boundaries exist as full-fledged denizens of reality, and even though such boundaries exist always as parts of the things they bound, here, too, coincidence falls short of identity. Thus France and Germany share no common parts. The border of France is, after all, *French*.

References:

Brentano, F. (1981). *The Theory of Categories.* The Hague: Martinus Nijhoff.

Brentano, F. (1988). *Philosophical Investigations on Space, Time and the Continuum.* London/New York/Sydney: Croom Helm.

Cartwright, R. (1975). Scattered Objects. In K. Lehrer (ed.), *Analysis and Metaphysics.* Dordrecht: Reidel, 153-171; repr. in Cartwright, R. (1987), 171-186.

Cartwright, R. (1987). *Philosophical Essays.* Cambridge, Mass.: MIT Press.

Casati, R. & Varzi, A. (1994). *Holes and Other Superficialities.* Cambridge, Mass.: MIT Press.

Chisholm, R.M. (1992/93). Spatial Continuity and the Theory of Part and Whole. A Brentano Study. *Brentano Studien 4*, 11-23.

Smith, B. (1995). Zur Kognition räumlicher Grenzen: Eine mereotopologische Untersuchung. *Kognitionswissenschaft 4*, 177-184.

Smith, B. (1995a). On Drawing Lines on a Map. In A.U. Frank, W. Kuhn & D.M. Mark (eds.), *Spatial Information Theory. Proceedings of COSIT '95.* Berlin/Heidelberg/Vienna/New York/London/Tokyo: Springer Verlag.

Smith, B. (1997). Boundaries: An Essay in Mereotopology. In L. Hahn (ed.), *The Philosophy of Roderick Chisholm* (Library of Living Philosophers XXV). LaSalle: Open Court, 533-561.