Granular Spatio-Temporal Ontologies

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Abstract

We propose an ontological theory that is powerful enough to describe both complex spatio-temporal processes (occurrents) and the enduring entities (continuants) that participate therein. The theory is divided into two major categories of sub-theories: (sub-) theories of type SPAN and (sub-)theories of type SNAP. These theories represent two complementary perspectives on reality and result in distinct though compatible systems of categories. In SNAP we have enduring entities such as substances, qualities, roles, functions; in SPAN we have perduring entities such as processes and their parts and aggregates. We argue that both kinds of ontological theory are required in order to give a non-reductionism account of complex domains of reality.

Introduction

We propose a realist ontological theory that is powerful enough to contain the resources to describe both complex spatio-temporal processes and the enduring entities which participate therein. The theory we have in mind is formal in the sense that it is designed to serve as a re-usable module that can be applied in a variety of material domains. It comprehends two major categories of sub-theories: (sub-)theories of type SNAP and (sub-)theories of type SPAN. As we shall see, these theories represent orthogonal inventories of reality, comparable to the division familiar in the discipline of accounting between *stocks* and *flows*.

SNAP and SPAN reflect two distinct perspectives on reality and result in distinct though compatible systems of categories (Figures 1 and 2). In SNAP we have enduring entities such as substances, qualities, roles, functions; in SPAN we have perduring entities such as processes and their parts and aggregates.

We consider three example domains and we seek to show that the ontological structure manifested in these domains can be understood only in a framework that has the resources of both SNAP and SPAN. Our example domains are medicine, military actions, and natural language understanding. We claim that critical to any adequate ontology of these and related classes of applications is the understanding of the interplay of processes, enduring entities, and changes in enduring entities which are related to their participation in

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processes. In the course of this paper we will provide an ontological theory which will allow us to understand this kind of interplay.

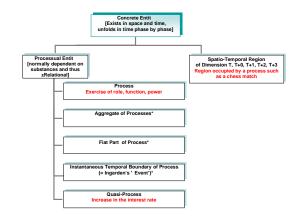


Figure 1: Upper level categories in SPAN.

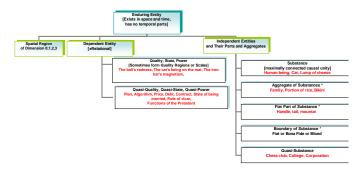


Figure 2: Upper level categories in SNAP.

The limits of four-dimensionalism

One popular position in contemporary analytic metaphysics is that of four-dimensionalism. This holds that all entities in reality (including clinical trials, military actions, the entities of our common-sense ontology) are four-dimensional worms extended in space and time ala Minkowski. (See (Sid01) for further discussion.) The four-dimensionalist

takes a view of the world as consisting exclusively of variously demarcated and variously qualitatively filled spatiotemporal worms. Prominent four-dimensionalists are: Amstrong (Ams80), Carnap (Car67), Cartwright (Car75), and Lewis (Lew).

An important aspect of four-dimensionalism is the thesis that time is just another dimension, in addition to and analogous to the three spatial dimensions. We can think of the four-dimensionalist ontology as what results when reality is described from the perspective of a god-like observer spanning the whole of reality from beginning to end and from one spatial extremity to the other. Human beings take this stance, for example, when they view the world through the lenses of the theory of relativity. We call the resulting ontological theories SPAN ontologies. SPAN ontologies span the entire four-dimensional plenum; hence reality in such ontologies is described atemporally.

The acceptance of this atemporal description of reality does not mean that the existence of the temporal dimension of spatio-temporal entities is denied. Rather, it means that certain aspects of this temporal dimension – above all its subdivision into past, present, and future – are *traced over*. Also traced over in a view of the world as consisting exclusively of spatio-temporal worms is the existence of *enduring* entities, such as people whose identity survives changes such as the gain or loss of molecules and cells, or plans whose identity survives through the different stages of their fulfillment.

Taken alone, a four-dimensionalist ontology is too weak for the purposes of giving an account of the ontology of clinical trails, of military actions or of the world as described by the simple sentences of natural language. This is because an ontology adequate to these purposes needs to have the resources to deal not only with processes but also with enduring entities and with the change and preservation of identity over time that is associated therewith. Enduring entities are not spread out in time in the way in which they are spread out in space and in the way processes are spread out in time. Rather they exist *in toto* at every instant at which they exist at all. This is true not only of substances such as you and me, but also of our qualities, roles and functions, of the plans and intentions we form, and of the diseases from which we suffer.

Medical treatment. An ontology of medical treatment must provide a framework for a systematic account of processes inside and outside the human body – the domain of physiology – and of the corresponding effects on the side of patients (enduring entities) – the domain of anatomy. From this perspective it is clear that enduring entities need to be recognized by any adequate ontology of medical treatment. To see that enduring entities are as important as the processes in which they are involved one also needs to consider the fact that doctors may interact with patients (for examination or treatment) over long periods of time and this means that they interact with the *same* patients at *different* times, or in other word with enduring entities. In order to monitor a course of treatment they need to track the processes tak-

ing place within some given human being. Such tracking presupposes the enduring existence not only of doctor and patient but also of the doctor's enduring intention (plan) to carry out the tracking.

The given processes, too, are two-fold: processes of tracking on the one hand, and the tracked processes themselves on the other. Some of the latter can be detected only indirectly. When your family doctor examines you, then she may initiate new processes in order to gauge from your responses the underlying processes taking place in the interior of your body. Also she may detect the occurrence of processes in the interior of your body via the measurement of qualities at its exterior.

The military domain Similar observations can be made in relation to the military domain. Focusing only on processes and thereby tracing over the involvement of substance-like enduring entities such as human beings, platoons, armies, as well as plans, powers, functions and roles, misses important aspects of military reality. The efficient structuring of military units and their co-ordination on the battlefield presupposes relations of authority – persons with specific roles and powers in virtue of which armies become organized in nested hierarchies of larger and smaller units. It presupposes an opposition between friendly and enemy forces, an opposition which obtains even when fighting is at a standstill. All these aspects can be recognized by an ontology only if the corresponding enduring entities are recognized also.

The psychological aspect, too, plays an important role. The evaluation of the morale of the troops is an important aspect of the evaluation of the overall power of military units. Again, processes do not have low of high morale. Morale is a quality of enduring entities.

Ontology for Natural Language Processing. A third class of applications is the ontology of common sense. Consider the sentence

A 'John is kissing Mary'

The entities denoted by 'John' and 'Mary' here are substances or enduring entities. The sentence as a whole, on the other hand, refers to a certain process of kissing and it asserts that this process is occurring over a time interval including the time of utterance. At the same time it presupposes that there is some spatial environment within which the process unfolds itself through time. An ontology describing the entities referred to by this sentence needs to have the resources to describe both complex spatio-temporal processes (the kissing event) and the enduring entities which participate therein (John and Mary).

We claim that critical to an adequate ontology of all three application domains is the understanding of the interplay of processes and enduring entities. The four-dimensionalist ontology fails to do justice to this kind of interplay. We shall thus offer a view which differs from four-dimensionalism in holding that the SPAN ontology does not exhaust the totality of what exists.

The limits of Three-Dimensionalism

In order to take account of the existence of enduring entities we need to develop a second type of ontological view, in addition to the SPAN ontology of the four-dimensionalists. We can think of this second type of view as being analogous to the taking of an instantaneous snapshot of reality, thereby apprehending all enduring entities existing at a given time (Gea66; Zem70). We call the resulting view a SNAP ontology.

The SNAP view gives us access to enduring entities such as John, his house, his car, etc., as well as the relations between them, their qualities, roles, functions, and so forth – but not to processes such as John's life or John's digestion of his lunch or his drive to work. This is because the SNAP view apprehends only those entities which exist in full at a given instant of time and processes exist only in such a way as to unfold themselves through time.

On the other hand, what one gets when slicing four-dimensional entities within a SPAN ontology at a certain point in time are not enduring entities but (three-dimensional) *slices of processes*. Whereas gluing together an arbitrary number of SNAP ontologies does not yield a SPAN ontology. No slicing and gluing operations can allow the translation from SNAP into SPAN or vice versa. In order to establish a relationship between SNAP and SPAN ontologies one needs rather to analyze SNAP and SPAN ontologies on some meta-level.

The entities recognized in a SNAP ontology are enduring entities and the different sorts of enduring relations between them such as spatial relations, relations of authority, family relations, and so forth. The entities in a SPAN ontology are processes and the (timeless) relations between them (e.g. the spatio-temporal overlap of the process of the spreading of an infection through your body and the rising of your body temperature).

When describing reality in terms of a SNAP ontology we take a perspective which is modeled on the everyday experiences of reality which we enjoy in successive present moments. We can also call this perspective the *human perspective*, as opposed to the god's eye perspective which we adopt when using the SPAN ontology.

Spatial change occurs whenever enduring entities have different qualities (of color, temperature and so on), parts, locations, roles, etc., at different moments in time. It is important to note that the succession of times is itself outside the scope of each SNAP ontology. From the SNAP perspective time is an index which we assign to the inventories of the world we take at different moments – one index per ontology. This indexing – which we can make explicit by writing $SNAP_{t_1}$, $SNAP_{t_2}$, ... – occurs not within the ontology itself but on a meta-level. For this reason there is no representation of the flow of time in SNAP.

Within each SNAP_{t_i} ontology processes are invisible, just as enduring entities are invisible within the SPAN ontology. Thus, however many SNAP_{t_i} ontologies we have at our disposal, they will always be insufficient to constitute a complete inventory of reality.

Kinds of ontologies and cross categorical relations

The underlying idea in all of the above is that the we can associate ontologies with ways we humans project onto reality, i.e., with the different perspectives we take when describing or perceiving reality. We can indeed define an ontology as an inventory of those entities existing in reality which are visible from a certain perspective. We concentrate here on the SPAN/SPAN perspectives, but we do not rule out the possibility that other ontology-generating perspectives might be distinguished.

Since ontologies reflect certain views or ways of projecting onto reality we consider an ontology not as a collection of terms or sentences but rather as a collection of judgments bound by a certain context. The context hereby may be more or less general in its scope and it can be specified (1) according to the *kind* of ontology at hand, which means according to the view of reality that is adopted; (2) according to the *scope* of the ontology (e.g., the medical domain vs. the domain of warfare); and also (3) according to the level of *granularity* at which an inventory of the domain is taken.

Factors (2) and (3) ensure selectivity. Thus in the medical domain the selectivity feature is responsible for the fact that the target domain is the human body, which is made up of different kinds of enduring entities at different levels of granularity: body parts such as organs, limbs, torso, etc. at a very coarse level of granularity, cells at a finer level of granularity, molecules at a still finer level of granularity, and so on. On the other side there are also processes taking places within the human body at different levels of granularity. At the most granular level there is the process which is John's life (with its constituent processes such as John's youth, John's adulthood, and so on); at lower levels of granularity there are processes of digestion, breathing, and so on. Obviously, there are relationships between SNAP entities at successive levels of granularity, and the same is true for SPAN entities.

Of critical importance is that there are also cross-ontological or *cross-categorical* relationships between SNAP entities and SPAN entities in the sense that every process *depends* on its carrier-substances. For example, the process which is John's live depends on the substance John, a kissing process depends on those who perform the kissing, the process of John's digestion depends on John's digestive organs, and so on. A classification of those cross-categorical dependence relations can be found in (Gre03a; Gre03b).

For the purposes of this paper it is above all the the granularity issue that is involved in such dependence relations that is of interest. For example it is the enduring SNAP-entity John *John as a whole* which is the bearer of that coarsegrained SPAN-entity which is *John's life-process*. On the other hand John's digestive tract is the bearer of his digestion process, rather than John as a whole. The process of oxygen exchange in the lungs depends on tiny blood vessels in the lung and on the blood cells transported through them. Cross-categorical dependence relations thus hold between SNAP and SPAN entities which are of *compatible granu-*

larity. This is the motivation for our definition of granular ontologies below.

In the remainder of the paper we assume that every ontology is characterized at a meta-level by three independent axes: the underlying view (SNAP vs. SPAN), the target domain (selectivity), and the level of granularity.

Granular partitions and granular ontologies

In our papers (SB02; BS02) we introduced the notion of a granular partition in order to capture formally the ways in which human cognition selectively targets entities in reality above all in light of the granular structure of the latter. Consider for example, the human body, which can be subdivided into head, torso, limbs. Limbs are subdivided into: arms, legs, and so on. Such subdivisions reflect acts of human cognition whereby boundaries are imposed by fiat upon targeted objects at different levels of granularity (Smi01). Examples of fiat boundaries are: the boundary between your hand and your lower arm, between your torso and your neck, etc. (Compare the hierarchies in Figure 3.) Such boundaries do not correspond to any discontinuities in the underlying reality. However, they are in the given example, not imposed arbitrarily but rather in a manner that is consistent with constraints of form and function.

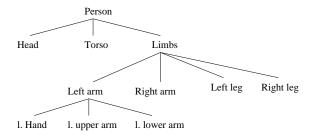


Figure 3: Hierarchical subdivision of the human body.

Other examples of granular partitions are political subdivisions such as the subdivision of Europe into countries, regions, towns; the subdivision of the animal kingdom into genus and species; the subdivision of an army into divisions, battalions, and platoons, and so on.

Definition: A granular partition is a triple,

$$GS = \langle (Z, \subset), (\Delta, <), \pi \rangle$$
.

Here (Z,\subseteq) is a *cell structure* with a partial ordering defined by \subseteq which forms a finite tree. (Δ,\leq) is the *target domain* which is a partial ordering which satisfies the axioms of general extensional mereology (GEM) (Sim87). The projection mapping $\pi:Z\to\Delta$ is an order-homomorphism from Z into Δ such that for all $z_1,z_2\in Z$ and for all $o_1,o_2\in\Delta$ we have:

$$z_1 \subseteq z_2 \Rightarrow (\pi \ z_1) \leq (\pi \ z_2)$$

 $(\pi \ z_1) \leq (\pi \ z_2) \Rightarrow z_1 \subseteq z_2.$

Since we are only interested in those entities in Δ which are targeted by cells in Z it will be sufficient to refer to the cell structure as a proxy for the granular partition as a whole. Consequently we will write (Z,\subseteq) as an abbreviation for $<(Z,\subseteq),(\Delta,\leq),\pi>$. We hereby assume that the cells in Z are labeled with the names of the corresponding targeted portions of reality.

Consider the following examples:

- (E1) A spatial granular partition formed by the cells Hyde Park, Soho, Buckingham Palace, Congestion Zone, London, York, Edinburgh, Glasgow, England, Scotland, Great Britain, Germany, Europe with the corresponding nesting (Figure 4);
- (E2) The partition of a human body into head, torso, limbs, left arm, left upper arm, left lower arm, left hand, etc., again with the corresponding nesting (Figure 3).

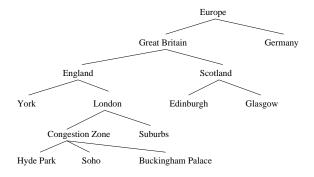


Figure 4: Hierarchical structure of places

Let $GS = (Z_{GS}, \subseteq)$ be a granular partition and let \mathcal{GS} be the corresponding tree representation. A *level of granularity* δ_{GS} in GS is then a *cut* in the tree-structure in the sense of (RS95): (1) Let X be the root of \mathcal{GS} , then $\{X\}$ is a cut; (2) sons(X) is a cut, where sons(a) is the set of immediate descendants of a; (3) Let C be a cut and $v \in C$ such that $sons(v) \neq \emptyset$ then $C' = (C - v) \cup sons(v)$ is a cut. This definition ensures that: (i) the elements forming a level of granularity are pair-wise disjoint, i.e., $\neg \exists v_1, v_2 \in C : v_1 \subset v_2 \lor v_2 \subset v_1$; and (ii) levels of granularity are exhaustive in the sense that $\forall v \in Z_{GS}$: if $v \notin C$ then $\exists v' \in C : v \subseteq v' \lor v' \subseteq v$.

Following (RS95) we define a partial order on cuts C and C' of a given tree as: $C \ll C'$ if and only if $\forall y \in C' : \exists x \in C$ such that $x \subseteq y$.

Consider Figures 3 and 4. Levels of granularity are for example:

 $\{Fred\}$ t_0 t_1 {head, torso, limbs} {head, torso, l. arm, r. arm, l. leg, r. leg} t_2 {head, torso, l. leg, r. leg, r. arm, l. hand, t_3 *l. lower arm, l. upper arm*} (1) g_0 Great Britain, Germany \ g_1 York, London, Scotland, Germany g_2 { York, Hyde Park, Soho, Buckingham Palace, g_3 *Suburbs, Edinburgh, Glasgow, Germany*}

¹The notion of granular partitions introduced in (BS02) is more general. Here we focus on granular partitions which are mereologically monotonic.

We can now define the notion of a *granular ontology*:

(G) A granular ontology is an inventory of entities existing in reality all of which belong to the same level of some granular partition.

It follows from the constraints laid on levels of granularities that the entities recognized by a given granular ontology have the property that they do not have parts recognized by the ontology in question. Such parts, if they exist at all, are not visible in the ontology in question in the same sense in which enduring entities are not visible in SPAN ontologies. The entities recognized by a granular ontology are marked by what we might call relative atomicity. They have this property, however, only relative to whatever is in the underlying level of granularity.

Granular spatio-temporal ontologies

Consider the sentence

(B) 'John was in Hyde Park from 6am to 7am on Monday morning'.

Here we have a process which is recognized by the corresponding SPAN ontology. We can call it Johnbeing-in-Hyde-Park-on-Monday-morning. The level of granularity underlying this particular granular ontology we call SPAN_{coarse}. At a finer level of granularity, e.g., that of SPAN fine, the same entity might be recognized as the mereological sum of John-enteringthe-park, John-walking-to-his-favorite-bench, John-sittingdown-on-his-favorite-bench, John-walking-to-the-exit, and John-exiting-the-park, and so on. The important point is that in SPAN_{coarse} the particular mereological structure visible in SPAN_{fine} is traced over. Of course, in SPAN_{fine} even finer mereological structures such as John's-first-stepin-the-park, John's-second-step-in-the-park, and so on, are traced over. Consequently, there is no single granular SPAN ontology of a given domain, but rather a system of such ontologies, which form a granularity lattice. Each granular ontology within this lattice is an inventory of the same domain but at a different level of granularity.

Corresponding to granular SPAN ontologies there are granular SNAP ontologies. For example, we have on the one hand a granular ontology SNAP_{coarse} with entities like *Hyde Park* and on the other hand a granular ontology SNAP_{fine} with entities like *Entrance-of-Hyde-Park*, the-path-from-the-entrance-to-the-bench, and so on.

Consider once again our sentence (B) and the granular ontologies $SPAN_{coarse}$ and $SPAN_{coarse}$. The whole family of $SNAP_{t_i}$ ontologies for $6am \le i \le 7am$ represents a series of coarse-grained SNAP-shots, each of which involves the SNAP entity John standing in a certain relation to another SNAP entity $Hyde\ Park$, which we might express by $In(John, Hyde\ Park)$. This relation holds no matter whether

John is near the entrance of the park or sitting on his favorite bench in the center of the park. This corresponds to the *John-being-in-Hyde-Park-on-Monday-morning* process in the SPAN_{coarse} ontology. This indicates that there is a rather complex interrelationship between granular SPAN ontologies and granular SNAP ontologies, which goes beyond the already mentioned dependence relationships. We accordingly define the notion of *matching* SPAN–SNAP pairs in order to specify aspects of this interrelationship in a more formal manner.

Let SPAN^g be a granular SPAN ontology at granularity level g, let \mathbf{SNAP}^f be a set of granular \mathbf{SNAP}_{t_i} ontologies at granularity level f, and let a slice of a SPAN entity be its projection on the spatial domain at a certain point in time at which it exists. We then say that $(\mathbf{SPAN}^g, \mathbf{SNAP}^f)$ is a *matching* \mathbf{SPAN} -SNAP *pair* if and only if for each $\mathbf{SNAP}_t^f \in \mathbf{SNAP}^f$ there is a corresponding slice of \mathbf{SPAN}^g such that:

- 1. the time indexes of the SPAN g -slice and the SNAP f_t ontology are identical;
- 2. for every entity s recognized in SNAP $_t^f$ there exists a slice of a life-process p recognized in the corresponding SPAN g -slice such that s and p occupy the same spatial region;
- 3. for every SNAP $_t^f$ relation there is a corresponding SPAN g -slice-entity with the appropriate time index; and
- 4. for every SPAN^g-entity there is an n-ary SNAP^f_t relation R ranging over SNAP^f_t-entities e_1, \ldots, e_n such that $R(e_1, \ldots, e_n)$ holds in SNAP^f_t; (this is the identity relation for lives of SNAP entities, kissing relations for kissing events, and so on).

Note that this kind of correspondence can only hold if the granular ontologies SPAN^g and $\operatorname{SNAP}^f_t \in \operatorname{SNAP}^f$ inventarize entities which are recognized by granular SNAP and SPAN ontologies with compatible levels of granularity. If the granularity of the one ontology is too fine compared to the granularity of the other, then there are either lives of entities in SPAN without the corresponding entities in SNAP or vice versa; or there are SPAN-entities for which there is no corresponding SNAP relation; or there are SNAP-relations without corresponding SPAN-entities.

Consider, for example the pair (SPAN $_{fine}$, SNAP $_{coarse}$) with SPAN $_{fine}$ recognizing atomic entities like John-entering-the-park and John-walking-to-his-favorite-bench, and SNAP $_{coarse} \in \mathbf{SNAP}_{coarse}$ on the other hand recognizing atomic entities like John, Hyde Park, etc. and relations like $in(John, Hyde\ Park)$. For (SPAN $_{fine}$, SNAP $_{coarse}$) to be a matching SPAN–SNAP pair there would have to be SNAP $_{t_1}$, SNAP $_{t_2}$, etc. in SNAP $_{coarse}$ such that (1) SNAP $_{coarset_1}$ recognizes the atomic entities John, the-entrance-of-the-park, and the relation in(John, the-entrance-of-the-park) and (2) SNAP $_{coarset_2}$ recognizes the atomic entities John, the-path-from-the-entrance-to-the-bench, and the relation in(John, the-path-from-the-entrance-to-the-bench), and so on. Entities like the-entrance-of-the-park are however of a granularity which is different

²Notice that having both John and Hyde Park in our granular SNAP ontology does not violate the no-overlap-principle. This is because John is *located* in Hyde Park but no part of John is also a part of Hyde Park and vise versa. For more about objects that occupy the same space without overlapping each other see (CV99) or (Don03).

from that of *Hyde Park*. Therefore such entities are not recognized by the ontologies in **SNAP**_{coarse}, and that is why (SPAN_{fine},**SNAP**_{coarse}) is not a *matching* SPAN–SNAP *pair*.

Conclusions

We have proposed a realist ontological theory that is powerful enough to contain the resources to describe both complex spatio-temporal processes and the enduring entities which participate therein. This theory is divided into two major categories of sub-theories: (sub-)theories of type SPAN and (sub-)theories of type SNAP. These theories represent orthogonal inventories of reality; they presuppose different perspectives on reality and result in distinct though compatible systems of categories. In SNAP we have enduring entities such as substances, qualities, roles, functions; in SPAN we have perduring entities such as processes and their parts and aggregates.

We argued that ontologies reflect views or ways of projecting onto reality that one needs to consider an ontology not as a collection of terms or sentences but rather as a collection of judgments bound by a certain context. We then argued that ontologies can be classified according to: (a) the underlying view of reality; (b) The scope or domain; and (c) the level of granularity at which an inventory of this domain is compiled.

Acknowledgements

Our thanks go to Jonathan Simon and Pierre Grenon for helpful comments. Support from the the Wolfgang Paul Program of the Alexander von Humboldt Foundation is gratefully acknowledged.

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