

# INFINITY AND THE PAST\*

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Several contemporary philosophers, like G. J. Whitrow, argue that it is logically impossible for the past to be infinite, and offer several arguments in support of this thesis. I believe their arguments are unsuccessful and aim to refute six of them in the six sections of the paper. One of my main criticisms concerns their supposition that an infinite series of past events must contain some events separated from the present event by an infinite number of intermediate events, and consequently that from one of these infinitely distant past events the present could never have been reached. I introduce several considerations to show that an infinite series of past events need not contain any events separated from the present event by an infinite number of intermediate events.

Recently there has been a growing tendency to argue that the past is necessarily finite. Writers who argue this, G. J. Whitrow, W. L. Craig, P. M. Huby, D. A. Conway, and others, acknowledge the formal correctness of contemporary set theory derived from Cantor's writings but deny that such a theory is applicable to the past. I believe their arguments are based on several errors, which I shall expose in the following.

## I

In "Time and the Universe" Whitrow (1966) argues that the series of past events if infinite must be an actual rather than a potential infinity, and that an actual infinity of elapsed events is an impossibility. He begins by defining the phrases "actual infinity" and "potential infinity" in terms of future events (p. 567). The future is potentially infinite in that (1) for any event in the future of  $E$  there will occur future events, and (2) any event in the future of  $E$  is separated from  $E$  by a finite number of intermediate events. Condition (1) asserts that the future is infinite, and condition (2) that this infinity is merely potential. An actual infinity would obtain if there were events separated from  $E$  by an infinite number of intermediate events.

Whitrow then argues that an infinite past is an actual infinity, and consequently is impossible:

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If all the events in a temporal chain culminating in the present are infinite in number, then, because these events actually occurred, the infinity concerned must be an actual, not merely a potential, infinity.

Consequently, if the chain of events forming the past of  $E$  is infinite, there must have occurred events that are separated from  $E$  by an infinite number of intermediate events. For, if not, then any event in the past of  $E$  would be separated from  $E$  by only a finite number of intermediate events. This would mean that the set of past events would, like the set of future events, constitute only a potential infinity, whereas it must constitute an actual infinity. It thus follows that, if the past of  $E$  contains an infinite number of events in a temporal chain culminating in  $E$ , there must have occurred events  $O$  in the past of  $E$  that are separated from  $E$  by an infinity of intermediate events. But this conflicts with our condition that an infinite future with respect to any event, in this case  $O$ , is a potential infinity, for  $E$  is an event that occurs and  $O$  has already occurred. Even if, in this context, we are prepared to forgo the Law of Contradiction, we are still confronted with the same insoluble problem that arose earlier in our discussion: when, in the temporal chain from  $O$  to  $E$ , does the total number of events that have occurred since  $O$  become infinite? . . . We conclude that the idea of an elapsed infinity of events presents an insoluble problem to the mind. (1966, pp. 567–568)

This argument is based on a fallacy of equivocation with respect to the phrases “actual infinity” and “potential infinity.” Whitrow’s proof that the past is infinite is “actually infinite” is based on a different sense of “actually infinite” than that belonging to his proof that an “actually infinite past” is impossible. The former proof utilizes “actually infinite” to mean an infinity of events that *have really occurred*. Whitrow writes, “because these events actually occurred, the infinity concerned must be an actual, not merely a potential, infinity.” In this sense of “actuality,” actuality is opposed to “potentiality” in the sense of *able to occur but not yet having occurred*. However, in his proof that an actually infinite past is impossible, Whitrow uses “actually infinite” in the sense of a *series of events some of which are separated from  $E$  by an infinite number of intermediate events*. Once this equivocation is recognized, Whitrow’s argument loses any sense of plausibility it might have had. For if the past is an “actual infinity” in the sense of being an infinity of events that *have really occurred*, it does not follow that it is also an “actual infinity” in the sense that *some past events are separated from the present event by an infinite number of intermediate events*. It is quite possible for there to be an infinite number of events that have really occurred such that each of these events is separated from the present event by a *finite* number of intermediate events.

To make this clear, it can be observed that if past events are infinitely numerous, then they form a set that is open on one end and closed on the other (the present event being the closure). They would correspond to the set of negative numbers:

			Past Events		Present Event
. . .	-4	-3	-2	-1	0

This set has the cardinal number aleph-zero and the order type of regression,  $\omega^*$ . Note that no matter which number one takes in this set, there will be a finite number of numbers intermediate between it and 0. Suppose we take the number  $-128$ , then there are  $-127$  numbers between it and 0. Suppose we take a trillion times  $-128$ , there still is some finite number between it and zero, and so on for any other number in this set. And yet there is not a finite but an *infinite number* of negative numbers in this set; before each negative number, there is another negative number.

Accordingly, we can conceive of past events as forming an infinite set of events that have “actually occurred” without being presented with the “insoluble problem” to which Whitrow refers, namely, that if some past events are separated by an infinite number of events from the present event, when in the chain of past events did the events start being separated from the present event by an infinite rather than finite number of intermediate events?

## II

Apart from Whitrow’s equivocation upon “actual infinity” and “potential infinity,” there is another and deeper fallacy underlying his argument, a fallacy also committed by P. M. Huby<sup>1</sup> and W. L. Craig.<sup>2</sup> The error in question appears in the commonly stated argument that

- (1) Aleph-zero events have occurred before the present event entails
- (2) Events separated from the present event by aleph-zero events have occurred

<sup>1</sup>Pamela Huby argues that past time must be finite for the same reason that the number of objects in space must be finite; namely, because every object in space (or time) must be “a finite distance only from every other object. But between any object and any other object there can then be only a finite number of objects, and therefore, however vast the total number of objects may be, it will still be finite.” See Huby 1971, p. 127. This implies the false assertion that if the total number of objects is infinite, there will be an infinite number of objects between some two of these objects.

<sup>2</sup>Craig reaffirms Whitrow’s argument that “if the chain of events prior to  $E$  is infinite, then there must be an event  $O$  that is separated from  $E$  by an infinite number of intermediate events.” See Craig 1979, p. 200.

which in turn entails

- (3) From one of the events separated from the present event by aleph-zero events the present event could not have been reached.<sup>3</sup>

The fallacy lies in the belief that (1) entails (2). It does not; for aleph-zero events could have occurred before the present event such that no one of these events is separated from the present event by aleph-zero events. It is this state of affairs that I discussed in the last section in connection with the set of negative numbers with the order type  $\omega^*$ . The number of these negative numbers is aleph-zero, but no one of these negative numbers is separated from zero by an aleph-zero number of numbers.

It might be thought that the entailment of (2) by (1), accepted as self-evident by the above authors, could be proven by the following argument (suggested to me by William Vallicella). The set of negative numbers with order type  $\omega^*$  by which the past is represented can be mapped onto the set with order type  $\omega^* + \omega^*$ :

$$\dots -8 -6 -4 -2 + \dots -7 -5 -3 -1 0$$

This set has the same members as the set of negative numbers in their natural order, and consequently, by the axiom of extensionality, is identical with this set. Now the set with the order type  $\omega^* + \omega^*$  contains members separated from other members by an infinite number of intermediate members; for example,  $-4$  is separated from  $-3$  by aleph-zero members. It follows that the set with order type  $\omega^*$  also contains some members infinitely distant from other members. For instance, the number in the set with order type  $\omega^*$  that corresponds (in the one-to-one mapping of the two sets) to  $-4$  in the set with order type  $\omega^* + \omega^*$  is infinitely distant from the number in the set with order type  $\omega^*$  that corresponds to  $-3$  in the set with order type  $\omega^* + \omega^*$ .

I believe this argument can be contested in two areas. First, it assumes that the issue of whether some members in a set are infinitely distant from others is logically independent of the order type of the set. More specifically, it assumes that if two sets have the same cardinality, aleph-zero, and are identical with one another, then irrespective of differences in their order type if one of the sets has infinitely removed members then so must the other. However, it can be proven that whether some members of some set  $S_1$  are infinitely removed from one another is determined by the ordered position of the members in  $S_1$ , and that the ordinal properties of these members are logically independent of the ordinal properties of any set  $S_2$  to which  $S_1$  corresponds or with which it is identical. Take, for example the set  $\omega^* + \omega^*$ ; the number  $-4$  is infinitely removed from  $-3$ .

<sup>3</sup>This error appears in Whitrow's 1978, p. 43; and 1980, pp. 31–32.

However, if I reorder this set, so that it possesses the order type  $\omega^*$ , then  $-4$  is no longer infinitely removed from  $-3$  but is immediately adjacent to it. This shows that the property of being infinitely or finitely removed from another member of the set is an ordinal property of  $-4$ .

Next, note that if some infinite set  $S_1$  is mapped onto set  $S_2$ , the ordinal property of a member  $x_1$  of  $S_1$  is logically independent of the ordinal property of the member  $y_1$  of  $S_2$  to which  $x_1$  corresponds. This can be proven by a number of instances; thus, the set  $N$  of positive whole numbers can be mapped onto the set  $R$  of rational numbers; in this case, the number 1 in set  $N$  has the ordinal property of being the first member of  $N$ . However, the member in the set  $R$  to which 1 corresponds does not have the ordinal property of being the first member of  $R$ , for set  $R$  by definition has no first member.

Furthermore, each number in set  $N$  but the first member has the ordinal property of being the immediate successor of some other member of  $N$ ; however, none of the numbers in  $R$  to which the numbers in  $N$  correspond have such an ordinal property; for a rational number by definition has no immediate successor.

The above considerations show that the ordinal properties of  $-4$  in the set with the order type  $\omega^* + \omega^*$  are logically independent of the ordinal properties of the members of the set with order type  $\omega^*$  to which  $-4$  corresponds. Accordingly, the fact that  $-4$  has the ordinal property of being infinitely removed from some other member of the set with order type  $\omega^* + \omega^*$  does not entail that the member of the set with order type  $\omega^*$  to which  $-4$  corresponds has the ordinal property of being infinitely removed from some members of the set with order type  $\omega^*$ .

### III

Another thesis common to several of the contemporary “anti-infinetists” is the belief that an actual or completed infinity cannot be instantiated in the past since the set of past events is never completed but is always being added to. David Conway (1974) expresses his difficulties about this:

The notion of a completed infinite series [of past events], i.e. an infinite series all of whose members are already given, a series which is now infinite, infinite without the potential addition of further members, is indeed a strange notion. (1974, pp. 206–207)

This argument is developed at greatest length by William Lane Craig (1979), who claims that “it would be impossible to add to a really existent actual infinite, but the series of past events is being increased daily” (p. 97). Craig sets forth his argument in terms of an example of an actually

infinite collection of material objects, such as library books. Suppose that there is a library with an actually infinite collection of books on its shelves.

Suppose further that each book in the library has a number printed on its spine so as to create a one-to-one correspondence with the natural numbers. Because the collection is actually infinite, this means that *every possible* natural number is printed on some book. Therefore, it would be impossible to add another book to this library. For what would be the number of the new book? Clearly there is no number available to assign to it. Every possible number already has a counterpart in reality, for corresponding to every natural number is an already existent book. Therefore, there would be no number for the new book. (Craig 1979, p. 83)

Craig does not spell out how this argument would apply to past events, but it is implicitly clear what he has in mind. If the collection of past events is actually infinite and has the cardinal number aleph-zero and the order type  $\omega^*$ , then corresponding to each negative number there is a past event. Since *every possible* negative number is assigned to some past event in this collection, it is impossible to add a new past event to it. For what could be the negative number of this event? It could not have a negative number, for *all* the negative numbers have been exhausted. But as a matter of fact the collection “of past events is being increased daily,” new past events *are* being added to the collection; so it follows that the collection of past events cannot have an infinite number. It must have some finite number.

Let us divide the above considerations into two separate arguments, one being that an actually infinite collection cannot be added to, the other being that if all possible negative numbers have been assigned to past events, no new event can be added to this collection. That the first argument is fallacious is apparent if we take any infinite collection of existing items, say, of books, and match them one-to-one with all positive whole numbers greater than or equal to 10. Such a collection is indeed a “really existent actual infinite,” for the books really exist and there is an actually infinite number of numbers in the series 10, 11, 12, . . . , that corresponds to the collection of books. Now add 9 books to the collection, matching them with the first 9 integers; what has occurred is that a really existent actual infinite has been added to.

In regard to past events, match those that have occurred at some time  $t_1$  with all the negative numbers greater than or equal to  $-10$ ; this is an actually infinite collection of past events. Then match events that newly become past from  $t_2$  to  $t_{11}$  with the negative numbers less than  $-10$ ; the result is that an actually infinite collection of past events has been added to from  $t_2$  to  $t_{11}$ .

The second argument is the one upon which Craig relies most heavily: if all possible negative numbers have been matched with past events, no new past events can be assigned to this collection. However, new assignments can be made if with the arrival of each new event in the past, each negative number is reassigned by being matched with the event immediately earlier than the event to which it had been assigned; such that,  $-3$  is reassigned to the event to which  $-2$  formerly had been assigned, and  $-2$  to the event to which  $-1$  had been assigned, and so on for all the negative numbers greater than  $-3$ . This leaves  $-1$  free to be matched with the event that has newly become past.

To the objection that this leaves some previously past event without a negative number assigned to it there is the following response: Let us call the time before some instance of the above-described reassignment  $t_1$ , and the time of the reassignment  $t_2$ . At  $t_2$  there is a past event belonging to the collection of past events that had not belonged to this collection at  $t_1$ . However, at  $t_2$  there is not a greater number of events belonging to this collection than at  $t_1$ , for the addition of the one event at  $t_2$  to the infinite collection that had existed at  $t_1$  results in a collection with the same number of members as the collection that existed at  $t_1$ , this number being aleph-zero. This is true because aleph-zero plus 1 equals aleph-zero. Consequently, since there are aleph-zero past events at both times, and since there are aleph-zero negative numbers, there is no past event at either time that is unmatched with a negative number.

The collection of past events at  $t_1$  is a proper subset of the collection of past events at  $t_2$ . Craig feels that the equivalence between an infinite set and a proper subset of that set as applied to real things and events is “just not believable” (p. 86). It is only unbelievable, however, if one presupposes erroneously that the definition of an infinite set of real things or events is the same as the definition of a finite set of real things or events; namely, that a set necessarily has more things or events belonging to it than any proper subset of itself. If one does not make this false presupposition, then the equivalence in question is perfectly believable.

#### IV

Craig and Whitrow among others believe that the “Tristram Shandy paradox” is sufficient to demonstrate the impossibility of an infinite past. This paradox, earlier discussed by Russell in reference to the future (1938, #340, pp. 358–359), is based on Sterne’s novel in which a character named Tristram Shandy is writing his autobiography so slowly that it takes him a year to record the events of a single day. Craig applies this story to the past and, relying in part on an argument developed by David Conway (pp. 201–208), he purports to uncover a contradiction in the idea that the past is actually infinite:

. . . suppose Tristram Shandy has been writing from eternity past at the rate of one day per year. Would he now be penning his final page? Here we discern the bankruptcy of the principle of correspondence in the world of the real. For according to that principle, Russell's conclusion would be correct: a one-to-one correspondence between days and years could be established so that given an actual infinite number of years, the book will be completed. But such a conclusion is clearly ridiculous, for Tristram Shandy could not yet have written *today's* events down. In reality he could never finish, for every day of writing generates another year of work. But if the principle of correspondence were descriptive of the real world, he should have finished—which is impossible.

. . . But now a deeper absurdity bursts into view. For if the series of past events is an actual infinite, then we may ask, why did Tristram Shandy not finish his autobiography yesterday or the day before, since by then an infinite series of events had already elapsed? No matter how far along the series of past events one regresses, Tristram Shandy would have already completed his autobiography. Therefore, at no point in the infinite series of past events could he be finishing the book. We could never look over Tristram Shandy's shoulder to see if he were now writing the last page. For at any point an actual infinite sequence of events would have transpired and the book would have already been completed. Thus, at no time in eternity will we find Tristram Shandy writing, which is absurd, since we supposed him to be writing from eternity. And at no point will he finish the book, which is equally absurd, because for the book to be completed he must at some point have finished. What the Tristram Shandy story really tells us is that an actually infinite temporal regress is absurd. (Craig 1979, pp. 98–99).

It is not clear at first glance why Craig believes the Tristram Shandy story to result in this “absurdity,” so it is best to reconstruct the logic of this story and try to pinpoint where the “absurdity” is supposed to arise.

(1) Tristram Shandy has been writing his autobiography at every moment in the past, and it takes him one year to write about one day.

This entails (2) that the temporal distance between any past day and the later time at which it is recorded increases with passage of time.

And this in turn entails (3) that there is no later day finitely distant from any earlier day at which all prior days have been written about.

Now, (4) the present day is finitely distant from any past day.

Therefore, (5) at the present day all past days will not have been written about. Tristram Shandy's autobiography will *not* have been completed.

Nevertheless, (6) the number of days written about is the same as the

number of years elapsed prior to the present (aleph-zero), for in each year Tristram Shandy had written about one day.

At this point, we can see that Craig is tacitly appealing to this supposed contradiction: If in relation to any present day there are an infinite number of past days and an infinite number of past days written about, then in relation to any present there are no past days unwritten about—which contradicts (5).

However, it is false that the proposition “the number of past days written about is the same as the number of past days” entails “there are no past days unwritten about.” For, the number of past days written about is a proper subset of the infinite set of past days, and a proper subset of an infinite set can be numerically equivalent to the set even though there are members of the set that are not members of the proper subset. Just as the infinite set of natural numbers has the same number of members as its proper subset of even numbers, yet has members that are not members of this proper subset (these members being the odd numbers); so the infinite set of past days has the same number of members as its proper subset of days written about, yet has members that are not members of this proper subset (these members being the days unwritten about).

In conclusion, the fact that the number of past days written about corresponds to the number of past days does *not* entail that at each point in the past Tristram Shandy has completed his autobiography. Rather, at no point in the past, and at no present, will Tristram Shandy’s autobiography be complete. The story of Tristram Shandy is internally consistent and so is the idea of an actually infinite past.

## V

In chapter 6 of *Our Knowledge of the External World*, Russell writes:

. . . classes which are infinite are given all at once by the defining property of their members, so that there is no question of ‘completion’ or of ‘successive synthesis’. (Russell 1960, p. 123)

Some “anti-infinetists” have inferred from this that the past cannot be infinite since events are not given all at once but successively. Craig writes that if the past is infinite then

time and the events in it are like an actual infinite; the whole class of events and moments are given simultaneously, as Russell would say. . . . But, of course, such a picture is a crude caricature of time, For events in time, unlike events in space, exist serially. . . . The collection of all past events . . . is formed by successive addition or, to use Kant’s phrase, successive synthesis. (Craig 1979, p. 203, n. 25)

This argument is based on a confusion of givenness *in thought* with givenness *in reality*. The infinite class of events is given simultaneously *in thought*, but it is given successively *in reality*. In the thought of all events, all events are thought of “all at once,” rather than “one at a time.” But that does not entail that all events *existed* all at once; rather they existed one at a time.

The idea behind the quoted passage from Russell’s work is that classes that are infinite are given *in thought* all at once by the defining property of their members, so that there is no question of completion or of successive synthesis *in thought*. This definition of the manner of givenness *in thought* of infinite classes implies nothing about how these classes are given *in reality*. Whereas *every* infinite class is given simultaneously in our thought, *some* of the classes are also given simultaneously in reality (for example, the class of material objects if it is infinite), and *others* are given successively in reality (for example, the class of events in time if it is infinite).

## VI

By acknowledging that events are given successively *in reality*, are we not admitting that *in reality* they can never add up to an infinite collection? Is it not true that for an infinite class to be given at all, whether in thought or reality, it must be given all at once?

The reply is that the collection of events cannot add up to an infinite collection in a finite amount of time, but they do so add up in an infinite amount of time. And since it is coherent to suppose that in relation to any present an infinite amount of time has elapsed, it is also coherent to suppose that in relation to any present an infinite collection of past events has already been formed by successive addition.

This suffices to disprove Kant’s thesis that an infinite series “can never be completed through successive synthesis” (Kant 1960, A426/B454, p. 413); for, although such a series can never be completely synthesized in a finite time, it can be completely synthesized in an infinite time.

I have explained that this infinite synthesis is to be understood in terms of the collection of negative numbers. But doubts may arise in connection with the idea of a *successive synthesis* corresponding to the collection of negative numbers. It seems, first of all, that a successive synthesis corresponding to the collection of negative numbers must be a potential infinite, not an actual infinite; this is because the collection of negative numbers itself cannot be completely synthesized. David Conway concludes from this that the representation of past events as corresponding to the negative numbers does not show that past events form an actual infinite:

. . . we cannot understand the completeness of the series [of past events] on the analogy to the negative number series, since no one imagines that an infinite number of negative numbers is ‘given’, that, e.g., each number has actually been already written down prior to ‘arriving at’  $-1$ . Rather, the latter series is infinite in the sense that there is an unending number of potential additions to it. (Conway 1974, p. 207)

Craig asserts, in a similar vein, that “we cannot conceive of anyone writing down all the negative numbers from eternity past so that he ends at  $-1$ ” (1979, p. 203, n. 25).

Moreover, there is the further problem that the negative numbers in being counted are counted *in reverse* to the order in which the past events existed. This problem is brought out by Huby and Whitrow, the latter writing:

A potentially infinite sequence of future events can be enumerated as 1, 2, 3, . . . , and so on indefinitely. Similarly, it has been argued that an infinite sequence of past events can be associated with the sequence of negative integers ending with  $-1$  and that this demolishes Kant’s objection to the possibility of an infinite sequence of past events. However, we can only enumerate the events in such a sequence by counting backwards, that is by beginning with  $-1$  instead of ending with it. That is the reverse of the way in which the events would actually occur and yields only a potentially infinite sequence. (Whitrow 1980, p. 31)

These objections can be dealt with in turn. Certainly Conway is right in believing that no one imagines that an infinite series<sup>4</sup> of negative numbers *has been* written down. But he is wrong if he believes, with Craig, that no one can conceive *the possibility* of this series being written down. It may be the case that in a finite period of time this series cannot be written down,<sup>5</sup> but it certainly is the case that it could be written down in an infinite period of time. It is coherent to suppose that in relation to any present event, an infinite series of past events has already elapsed, and that each number in the negative number series has been written down at a time corresponding to each one of these past events.

P. F. Strawson believes that this is impossible “because we think of

<sup>4</sup>Negative numbers form a series as well as a collection (set), a series being a collection of *sequentially ordered* members. Collections that are not series are exemplified by the collection of grains of sand.

<sup>5</sup>Can any infinite series of numbers be written down in a finite time? For a discussion of an issue directly related to this, namely, whether an infinite task can be completed in a finite time, see the articles in Salmon’s edition of *Zeno’s Paradoxes* (Salmon 1970).

the process of counting as having to *start* at some time” (1966, p. 176). But this belief in the sense that it is true is irrelevant, and in the sense that it is relevant is false. The *empirically observable* processes of counting with which we are familiar all start at some time, but that has little bearing upon the issue of whether all *logically possible* processes of counting must start at some time. If we define a logically possible process of counting as a *synthetic series of acts of counting*, then it makes sense to conceive each past event to correspond to one act of counting, such that each earlier past event is correlated with an act of counting a greater negative number and each later past event is correlated with an act of counting a smaller negative number. In relation to any present event, the immediately preceding event can be conceived as correlating to an act of counting the number  $-1$ , such that at the time of this immediately preceding event the series of counting acts *is being* brought to completion, and at the time of the present event the series of counting acts *has already been* brought to completion.

These reflections enable the second objection, Whitrow’s, to be disposed of. It may be true in the empirical sense that ‘we’ can only enumerate the series of past events by counting backwards from  $-1$ , and that such an enumeration yields only a potential infinite. But what we can or cannot do given our empirical limitations is not essentially relevant to the issue of whether it is *logically possible* to enumerate the series of past events in accordance with the negative number series. It may be the case that *we* must start at  $-1$  and can only count some ways backwards, but a *logically possible counter* could have been counting at every moment in the past *in the order* in which the past events occurred. And this logically possible counter in relation to any present would have completely counted the negative numbers.

I conclude that these arguments have given us no reason to believe that an actually infinite past is logically impossible.<sup>6</sup>

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<sup>6</sup>Some theses assumed in or related to this paper are defended elsewhere. The assumption that the past, present, and future are mind-independent elements of time has been defended in Smith 1986, section 38, and in Smith 1985b. The thesis that the past may be infinite, but is not necessarily so, implies that time may have begun, an idea elaborated upon in Smith 1985c. If the universe began, say, with the big bang, it is not necessary that time began also; an infinitude of empty time may have elapsed before the universe. See Smith 1985a.

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