Relevance, Relatedness and Restricted Set Theory

Barry Smith

From: G. Schurz and G. J. W. Dorn, eds., *Advances in Scientific Philosophy. Essays in Honour of Paul Weingartner*, Amsterdam/Atlanta: Rodopi, 1991, 45–56.

1. Introduction

Relevance logic has become ontologically fertile. No longer is the idea of relevance restricted in its application to purely logical relations among propositions, for as Dunn has shown in his (1987), it is possible to extend the idea in such a way that we can distinguish also between relevant and irrelevant *predications*, as for example between "Reagan is tall" and "Reagan is such that Socrates is wise". Dunn shows that we can exploit certain special properties of identity within the context of standard relevance logic in a way which allows us to discriminate further between relevant and irrelevant properties, as also between relevant and irrelevant relations. The idea yields a family of ontologically interesting results concerning the different ways in which attributes and objects may hang together. Because of certain notorious peculiarities of relevance logic, however,[1] Dunn's idea breaks down where the attempt is made to have it bear fruit in application to relations among entities which are of homogeneous type.

Let us suppose that *is red* and *is prime* are relevant properties in the sense of Dunn. Then the formal machinery underlying his approach dictates that so also are: *is either red or prime, is both red and prime, is if red then prime,* and so on.[2] This consequence is surely both counterintuitive and in conflict with Dunn's stated aims. Still worse, however, are the implications in relation to what Dunn calls "relevant properties of pairs". A pair $<a, b>$ has such a relevant property, Dunn tells us,[3] iff there are relevant monadic properties F and G—relevant monadic properties of any old sort—such that $Fa \vee Gb$. But this seems to be a good first candidate definition of what one might better call an *irrelevant* property of a pair.

Can we, then, provide a more adequate account—a means of sifting out the relevant pairs (Gilbert and Sullivan, Anderson and Belnap, Xanthippe and Socrates) from the irrelevant pairs (Napoleon and the moon, the number 2 and the Eiffel Tower, Quine's left foot and Chisholm's right ear)? Can we, more generally, find a means of sifting out the *relevant sets* from those irrelevant sets which, while constituting a formal unity, lack all material connectedness among their members. Examples of relevant sets would be: The Berlin Philharmonic Orchestra, the set of molecules in this nut, the *natio hungarica*. Examples of irrelevant sets would be: {Quine,

[1] Above all because of the continued acceptance of the validity of $p \rightarrow p \vee q$.

[2] *Op. cit.*, MS p. 362.

[3] *Op. cit.*, MS p. 359.

Gandhi, the number three}; {Napoleon, the moon, redness}, and so on. Intuitively, manifolds or pluralities in the first group ("organic" or "integral" wholes) seem to be admissible as entities in their own right, i.e. in addition to the objects which are their members. This is first of all because there are specially intimate relations among these members which serve to unify them into a whole. But it is also, and no less importantly, because each of the given wholes would seem to enjoy a certain completeness or separateness in relation to the surrounding environment (it would enjoy a special place within the family of those sets in which it is included). The reflections which follow constitute one first step towards making this somewhat metaphorical idea more precise. They have been inspired on the one hand by Dunn's *Relevant Predication* (though any possible formal connection to relevant logic as such, or indeed to relevant set theory as standardly understood, will raise its head only at the very end). On the other hand they owe much to unpublished work of Kit Fine on the formal ontology of dependence.[4] There is moreover an interesting relationship to the work of Orlowska and Weingartner (1986), where relevance relations between predicates are introduced.

2. Relevance Relations

We shall write '*xRy*' for '*x* is related to *y*'. This may mean: there is a relevant relation (in something like Dunn's sense) connecting *x* to *y*. Or it may mean any or all of: *x* is in spatial contact with *y*, *x* is consanguineous with *y*, *x* is similar to *y*, *x* has business dealings with *y*, *x* lives in the same place as *y*, *x* looks a bit like *y* from a distance, *x* is causally bound up with *y*, and so on.[5] (In general it seems that what relevant sets there are will be determined by a number of heterogeneous relations of this sort, perhaps acting in consort. The investigation of the consequences of such collaboration amongst different relevance relations will not, however, be attempted here.) All that matters here is that *R* be some symmetric and reflexive relation defined across a space of objects upon which a set-theoretical structure is then built up in the usual way.[6]

Consider, now, the function B defined on this space of objects in such a way that:

$B^0(x) := \{x\}$
$B^{\alpha+1}(x) := B^\alpha(x) \cup \{y: \exists z \in B^\alpha(x) \wedge yRz)\}$ for α a successor ordinal
$B^\alpha(x) := \cup_{\beta<\alpha} B\beta(x)$ for α a limit ordinal

B is an expanding function; thus it has a fixed point α_0, for which

$B^{\alpha_0}(x) = B^{\alpha_0+1}(x)$. (In fact we can prove that $\alpha_0 = \omega$.)

---

[4]The latter is summarized in Simons (1987), pp. 311ff.

[5]We may wish to allow also, for each (relevant?) property *F*, the relevant relation: *x* and *y* are both such as to have *F*. In this way the set of *F*'s, too, may turn out to be a relevant set.

[6]Even the requirement of reflexivity is inessential, since we can of course for any symmetric *R* define a reflexive relation $R^\sim$ by $xR^\sim y := x=y \vee xRy$.

If we now define

$$x\mathrm{R}^{*}y := y \in \mathrm{B}^{\alpha}{}_{0}(x),$$

then *R** is an equivalence relation which partitions the space of *Urelemente* into maximal classes of what we might call (direct and indirect) *relatives*. B defines a basis for a topology on this space whose non-empty open sets are all the unions of the $\mathrm{B}^{\alpha}(x)$.

The $\mathrm{B}^{\alpha}(x)$ are, intuitively, families around x generated by relatednesses of degree $\leq \alpha$. If we define $d(x, y)$ as the least α such that $y \in \mathrm{B}^{\alpha}(x)$, then d provides a measure of the degree of the relatedness of x and y. The function thus defined will be seen to satisfy the usual requirements for a distance function, namely

$d(x, y) = 0$ iff $x = y$,
$d(x, y) = d(y, x)$
$d(x, z) \leq d(x, y) + d(y, z)$.

However d is not, in general, defined. If x and y belong to different *R**-equivalence classes, then there is no relation at all between them, however indirect. Hence, also, there is no sense to the idea that we might measure their relatedness. A fully metric space is, however, obtained if we add the (independently not unattractive) assumption:

$$\forall x, y\ (x\mathrm{R}^{*}y)$$

to the effect that the space of objects is unified (connected) in the sense that it does not collapse into separate regions mutually inaccessible via *R*.

3. Branches, Families, Cores

What, now, are we to allow as the relevant sets? One solution would be to admit precisely the B^{α}, perhaps limiting ourselves (according to scruples) to finite α. In light of the non-transitivity of *R*, the very scrupulous will insist on a restriction to those cases where $\alpha \leq 1$. It turns out, however, that there are certain subsets of the $\mathrm{B}^{1}(x)$ which enjoy just that special sort of rounded-offness from the outside world, and whose members enjoy just that specially intimate sort of relatedness, which we said ought to be characteristic of the relevant sets.

Let us first of all define a *branch* to be any set X all of whose members are directly related by *R*. That is:

$$Br(X) := \forall x, y \in X\ (x\mathrm{R}^{*}y).$$ [7]

[7] This definition enables us to generalize the notion of relevance to apply not merely within but also between sets, for example by setting:

Trivially, by reflexivity of *R*, all singletons are branches. Moreover, and no less trivially, we have

$(X \subseteq Y \wedge Br(Y)) \rightarrow Br(X)$.

A *family* may be defined as a union of branches whose intersection is non-empty.[8] Every non-singleton branch is a subset of some B^1-set, and every B^1-set (and every branch) is a family, but there seem to be no entailments in the opposite direction.

Br embraces a tighter restriction than $B^1(x)$. The latter requires only that there be some one (mother) object (x) to which all members of the given $B^1(x)$ are directly related. *Br*, in contrast, requires that all objects be directly related to *each other*. It will not do, however, to take branches as the relevant sets, for a branch x may be formed by selecting some objects at random from some other branch *Y*. Branches may accordingly lack that rounded-offness which is to be part of what justifies our treatment of relevant sets as entities in their own right.

Let us, therefore, introduce the notion of a maximal branch or *Net*, defined by:

$Net\ (X) := \forall Y(Br(X \cup Y) \leftrightarrow Y \subseteq X)$.

It will still not do to restrict ourselves to maximal branches, however, for it seems clear that some relevant sets will be included in others (as for example the wind ensemble is included in the entire orchestra), and this is clearly ruled out if relevant sets are always maximal.

Yet still, we are not too far from the notion we require. A non-maximal branch X is typically contained within the intersection of a larger family of maximal branches which emanate from X is typically contained within the intersection of a larger family of maximal branches which emanate from X in different directions. In each such case there will be what one might call the core of this family, consisting of the largest set of family-members included in all the maximal branches in which X is included. Let us therefore define the *core* generated by X as the set consisting of all those members of this larger family circumcluding X who are members of all the maximal branches extending X.

We shall say that *Y* is a *maximal extension of X*, or

$XRY := Br(X \cup Y)$.

And then, if the definition of branch is itself similarly generalized, it will follow that the property of branchhood is preserved under the operation of taking power sets. Here, however, we shall concentrate exclusively on set of *Urelemente*— which will also mean that the ideas here offered will be capable of being transferred relatively easily to the field of mereology.

[8]This definition starts out from the idea of a family as a set of objects existing simultaneously. We might, however, define a *lineage* (or family-in-the-Wittgensteinian-sense) as a union of branches $X_1, \ldots, X_n$ for which it is required only that each of the $X_i \cap X_{i+1}$ be non-empty.

$X << Y := X \subseteq Y \wedge Net(Y)$.

We now need to adopt the following non-trivial statute of limitations:

$\forall X\,(Br(X) \rightarrow \exists Y\,(X << Y))$.

to the effect that every branch is included in some maximal branch, a condition which can be verified only by appeal to some form of the Axiom of Choice. Suppose $\underline{X}$ is a non-empty set of sets which satisfies the condition that if C is a chain in $< \underline{X}, \subseteq >$, then $\cup_{X \in C} X \in \underline{X}$. Zorn's Lemma tells us that $\underline{X}$ has a maximal member. If, now, we let $\underline{X}$ be the set of all branches including the branch X, and if we suppose $Y_1 \subseteq \ldots Y_i \ldots$is a chain in $\underline{X}$, then the union of the Y_i is also a branch; thus it, too, is a member of $\underline{X}$. Hence we can infer that $\underline{X}$ has a maximal branch. If we now define, for each branch X, its core

$C(X) := \cap_{X<<Y} Y$

then because every branch has at least one maximal extension we know that $C(X)$ is defined for all branches X.

4. The Structure of Restricted Set Theory

The core generated by X has the nice property that it both preserves the inter-relatedness enjoyed by the members of X and is at the same time a natural completion or rounding-off of X. Moreover, it is possible for one core to be included within another.

The operation of taking cores is similar to, though not identical with, a topological closure operation. For while we have, for all branches X and Y,

I.	$X \subseteq C(X)$	(Expansiveness)
II.	$C(C(X)) = C(X)$	(Idempotence)
III.	$X \subseteq Y \rightarrow C(X) \subseteq C(Y)$	(Isotonicity)[9]
IV.	$C(X \cup Y) \rightarrow C(X) \cup C(Y)$	(Additivity)

which is what would be required if C were to yield a full closure algebra. Hence also—despite the presence of

V. $C(\varnothing) = \varnothing$

[9] I. follows trivially from the definitions. To prove II. It suffices to show that $C(C(X))$ $C(X)$, i.e. that $X << Y \rightarrow C(X) << Y$, which follows trivially from the definition of *Net*. To prove III., suppose $x \in C(X)$, then $\forall Z((X \subseteq Z \wedge Net(Z)) \rightarrow x \in Z)$. Whence also x belongs to any Z such that $Y \subseteq Z \wedge Net(Z)$.

—the underlying structure is not that of a full topological space. To see that C is not additive, consider the family defined by *Figure 1*:

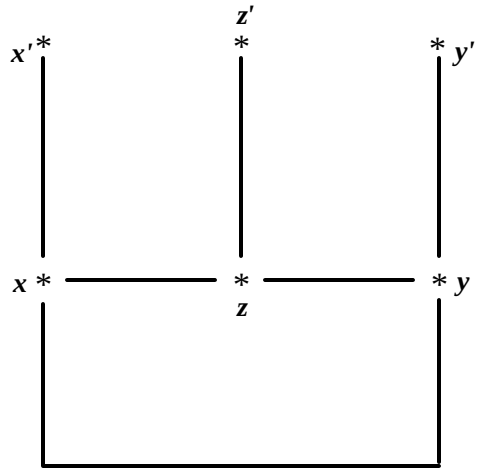


Figure 1

where the links connecting nodes signify that these nodes are related by R. Now set $X=\{x\}$, $Y=\{y\}$, then $C(X)=\{x\}$, $C(Y)=\{y\}$, but $C(X \cup Y)=\{x, y, z\}$.

The failure of additivity tells us that cores cannot in general be split apart, as it were, into constituent cores. Or, from the opposite perspective, it tells us that the forming of unions among relevant sets is no inconsequential matter. Hammer has described the additivity axiom as the axiom of sterility: it requires that two sets cannot produce anything by union that one of them cannot produce alone (Hammer 1962, p. 65). Relevant sets are not sterile in this sense, since if they are unifiable at all, then their relevant union may incorporate something new. The notion of relevant set may in this sense capture part of what is involved in the idea of an emergent whole. In any case it seems to give a formally precise rendering of the idea that there may exist unities at higher levels which are not merely the result of a summing together of unities existing lower down.[10]

If, now, the space of relevant sets does not have the structure of a full closure algebra, then the question arises as to what is structure is. Axioms I.-III. In fact determine a *pre-closure algebra*, a structure first studied extensively by Ore in 1943. Hammer, in particular, has shown in a series of papers that even within this very modest structure it is possible to develop

[10]The notion of relevant set may thereby also throw some philosophically interesting light on the idea that, through an appropriate act of collection or "colligation", any definite objects of thought may be brought together into a whole. Suppose that the relevance of x and y is established solely by such an act of collection z. Then it seems reasonable to suppose that the smallest relevant set containing x and y will contain z also.

analogues of a number of fundamental topological notions.[11] And according to Fine it is a preclosure algebra that "is the mathematical structure underlying the theory of dependence relations put forward by Husserl in his 3rd *Logical Investigation* 'On the Theory of Wholes and parts'".[12]

5. Principles of Restricted Set Theory

If we are right in supposing that the relevant sets are (for some appropriate R) just the core sets as defined above, then the question arises as to how restricted set theory would differ from set theories of the more usual sort. If $\boldsymbol{R}$ is the set of relevant sets, then we have $X \in \boldsymbol{R} \rightarrow X = C(X)$. From this we can easily prove that $<R ; \cap>$, is a complete non-distributive meet-semilattice. This follows from:

$$X, Y \in \boldsymbol{R} \rightarrow X \cap Y = C(X \cap Y),$$

which in turn follows easily from the definitions.

Variables $X, Y, Z, \ldots$ shall henceforth range over relevant sets. Since relevant intersection as defined on $\boldsymbol{R}$ differs not at all from normal intersection, we shall represent it by means of '$\cap$' as usual. Membership and set inclusion, too, like the null-set, will turn out to coincide with the membership, inclusion and null-set of normal set theory. Only with the operation of relevant union, defined by

$$X \cup^R Y := C(X \cup Y)$$

do problems arise, since $C(X \cup Y)$ is not, in general, defined. Where it is defined, however (for which a necessary and sufficient condition is that X and Y together form a branch), then it is unique. Moreover, $\cup^R$ is commutative. It is associative, in the sense that, if $(X \cup^R Y) \cup^R Z)$ is defined, then so also is $X \cup^R (Y \cup^R Z)$, and the two are equal. And it satisfies the usual

$$X \cup^R X = X$$
$$X \cup^R (X \cap Y) = X$$
$$X \cap (X \cup^R Y) = X$$

Further, the equivalence of $X \subseteq Y$ and $X \cup^R Y = Y$ is preserved, and we have also the reassuring

$$\forall X\, (X \cup^R Y = X) \rightarrow Y = \varnothing$$

---

[11] See Hammer 1960, etc. and compare Netzer 1978.

[12] Cf. Fine (unpublished). Husserl's discussion of separation and of connectedness via chains in §§ 8 and 20 of this work may also be of relevance to the issues under discussion here.

That restrictive set theory is non-distributive can be seen if we consider again the family defined in Fig. 1 above. (To see the failure of $X \cap (Y \cup^R Z) = (X \cap Y) \cup^R (X \cap Z)$, set $X = \{z\}$, $Y = \{y\}$, $Z = \{z\}$.) However, we do have the distributive inequalities (where all the relevant unions are assumed to be defined):

$(X \cap Y) \cup^R (X \cap Z) \subseteq X \cap (Y \cup^R Z)$,
$(X \cup^R (Y \cap Z) \subseteq (X \cup^R Y) \cap (X \cup^R Z)$,
$(X \cap Y) \cup^R (Y \cap Z) \cup^R (Z \cap X) \subseteq (X \cup^R Y) \cap (Y \cup^R Z) \cap (Z \cup^R X)$,

and we also have the modular inequality:

$(X \cap Y) \cup^R (X \cap Z) \subseteq X \cap (Y \cup^R (X \cap Z))$,

Perhaps one of the most interesting features of restricted set theory is that the operation of taking singletons is not in general defined, something which nicely captures our intuition that there are objects—for example smiles—which in and of themselves do not form self-existent wholes but rather stand in need of other objects in order to exist. x is dependent or non-self-existent in this sens iff there is some y such that all objects to which x is related are related also to y. The smallest relevant set including x in such circumstances will include also y. The idea of dependence is similar, in some respects, to Frege's idea of unsaturatedness. Unlike Frege, however, we can distinguish in our present framework not only unilateral but also mutual unsaturatedness, for is x depends for its existence on y in the sense indicated, then it may or may not be the case that y also depends for its existence on x. In a system like Wittgenstein's *Tractatus*, where the *Sachverhalte* might be said to serve as the relevant sets (integral wholes) in our present sense, it seems that all simple objects are mutually unsaturated in this sense.

We may now define the notion of a *relevant atom*, as follows:

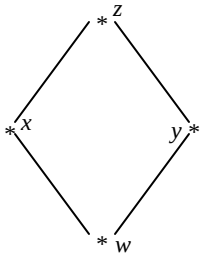
$Atom(X) := \forall Y(\varnothing \subseteq Y \subseteq X \rightarrow Y = \varnothing \vee Y = X)$.

Clearly atoms are mutually exclusive, and every atom X is join-irreducible, in the sense that if $X = Y \cup^R Z$, then $X = Y$ or $X = Z$. As Figure 1 makes clear, however, not all join-irreducible elements are atoms.

Finally, it goes without saying that the operation of taking complements, too, is not in general defined within the theory of relevant sets. It seems, indeed, that (for natural interpretations of the theory) it is never defined.[13] Unity, as Aristotle might have said, is not to be arrived at through privation.

---

[13]Even the complement, $Y \setminus X$, of X relative to Y, characterised by $X \subseteq Y \wedge X \cap Y \setminus X = \varnothing \wedge X \cup^R Y \setminus X = Y$)), is not always defined, as is clear for example from the family:



6. Models of Restricted Set Theory

(i) Let $x, y, \ldots$ range over points in three-dimensional Euclidean space and set xRy iff $|x–y| \leq 1$. Then the relevant sets are all intersections of closed balls of unit diameter.

(ii) Let $x, y, \ldots$ range over people, and set xRy iff x is directly related to (is a sibling, parent, child or spouse of) y. Then the relevant sets are all maximal family groups of siblings-plus-parents, together with all singletons of non-dependent individuals (where a dependent individual is for example a child with no children of its own).

(iii) Let $x, y, \ldots$ range over ultimate, microphysical particles and set xRy iff there is some (physical, chemical, biological,...) object to which both x and y belong. The relevant sets are then the sets of particles constituting objects at different levels, and inclusion relations between relevant sets will mimic the constituency relations between the corresponding objects.

(iv) Let $x, y, \ldots$ range over *Urelemente* and set xRy iff x and y exist simultaneously. The relevant sets are then those sets of objects which at some time actually exist. There are two different ways of understanding this idea. On the one hand we might regard the relevant sets as constituting a sub-set of the sets classically conceived and as existing no less abstractly and timelessly than these. On the other hand however we could see the set of relevant sets a changing from moment to moment; R would then, as it were, bring temporality into the realm of sets.

(iv) Let $x, y, \ldots$ range over topics in some universe of discourse, and suppose xRy is a relevance-relation defined on $x, y, \ldots$ in some intuitively acceptable way. The relevant sets will then be organized hierarchically, in light of the fact that topics will stand in relations of greater or lesser generality. Define the *content* $T(p)$ of a proposition p as the set of its topics. We now let $p, q, \ldots$ range over those propositions which are themselves relevant, in the sense that their respective contents are each such as to be included in a relevant set. We then define relevance *between* propositions by:

pRq iff $T(q)$ is included in some relevant set.

We can now exploit this idea to cope with our conflicting intuitions as to the validity of entailments like $p \rightarrow p \vee q$. There is on the one hand a long-standing intuition to the effect that a sentence of the form $A \rightarrow B$ can express an analytic truth only if it is the case that $T(B) \subseteq T(A)$,[14] Anderson and Belnap, on the other hand, point out that we are predisposed to accept as analytic also a case like *all brothers are siblings* which (give or take the quantifiers) they would have us regard as an instance of $p \rightarrow p \vee q$ (Anderson and Belnap, p. 155). Hence, they argue (and without pausing for air), that $p \rightarrow p \vee q$ is valid in general. Surely, however, we can find some way to allow the sibling-brother case without opening the floodgates to $p \rightarrow p \vee q$ in general. This is in fact precisely what is achieved by imposing on valid entailments of the form $A \rightarrow B$ the restriction that they be relevance-preserving in the sense that $T(A) \cup T(B)$ should be included in a relevant set.

Previous attempts at using topic- or content-related notions as a means of carving out the relevant entailments have failed, we might argue, because they have employed set-theoretic

[14] See, e.g., Parry 1933.

means which are themselves at bottom still irrelevant in orientation. This is particularly clear for example in the case of Carnap and Bar-Hillel (1952), who would have it that the content of the negation of a proposition is the usual set-theoretic complement of the content of the proposition itself. Even the more sophisticated framework described by Dunn (1976, §6) still admits the formation of contents via unrestricted union, and so he, too, is forced to allow $p \rightarrow p \vee q$ to slip through the net. By exploiting the resources of a restricted set theory of the sort described above, however, it may be possible to do justice to the idea of a relevance logic by allowing relevance among propositions to be determined, in part at least, by those ontological relevance relations which obtain amongst the objects in the world.

7. Relevant Logic

Let *A*, *B*, … range over those propositions which are themselves relevant, in the sense that their respective contents are each such as to be included in a relevant set. We can now define relevance *between* propositions by:

$ARB :\!= T(A) \cup T(B)$ is included in some relevant set.

Define S_R to be that subset of the set of formulae of the propositional calculus which satisfies the following conditions:

(i) all atomic formulae are in S_R,
(ii) $\neg B$ is in S_R iff B is in S_R,
(iii) $(B \rightarrow C)$, $(B \vee C)$, $(B \wedge C)$, are in S_R iff B and C are in S_R and BRC.

What, now, can be said about the set $S_R \cap PC$, defined as the intersection of the set S_R with the set of *PC*-valid formulae? If *R* is defined so weakly that *ARB* for all *A* and *b*, then $S_R \cap PC$ is just the set of propositional tautologies. If, on the other hand, *R* is defined in such a way that, for atomic formulae *p* and *q*, *pRq* iff $p = q$, then $S_R \cap PC$ consists exclusively of tautologies in a single sentential variable. Between these two extremes there is a range of possibilities, all of which exclude formulae like $p \rightarrow p \vee q$ but at the same time allow in principle certain cases of $A \rightarrow B$ where the content of *B* goes *beyond* the content of *A*. Such implications would express what we might call synthetic truths, the analytic implications being implications identified as satisfying.

(A) $T(B) \subseteq T(A)$.

Our approach, therefore, represents a generalisation of those varieties of relevance logic which rest on taking principle (A) as a constraint on validity. The family of such logics embraces, familiarly, not only the original system of Parry (1933) and its modification by Dunn (1972), but also, and not least, the A_0-system of Weingartner and Schurz (1986).

References

Anderson, A. R. and Belnap, N. D. Jr. (1975), *Entailment – The Logic of Relevance and Necessity*, vol. 1, Princeton/London: Princeton University Press.

Dunn, J. M. (1972), "A Modification of Parry's Analytic Implication", *Notre Dame Journal of Formal Logic* 13, 195-205.

Dunn, J. M. (1976), "Intuitive Semantics for First Degree Entailments and 'Coupled Trees'", *Philosophical Studies* 29, 149-168.

Dunn, J. M. (1987), "Relevant Predication I: The Formal Theory", *Journal of Philosophical Logic* 16, 347-381.

Fine, K. (unpublished), "Husserl, Part I: The Pure Theory".

Hammer, P. C. (1960), "Kuratowski's Closure Theorem", *Nieuw Archief voor Wiskunde* 8, 74-80.

Hammer, P. C. (1962), "Extended Topology: Set-Valued Set-Functions", *Nieuw Archief voor Wiskunde* 10, 55-77.

Hammer, P. C. (1962a), "Extended Topology: Additive and Sub-Additive Sub-functions", *Rend. Circ. Mat. Palermo, Ser 2*, 11, 262-70.

Hammer, P. C. (1964), "Extended Topology: Structure of Isotonic Functions", *Journal für die reine und angewandte Mathematik* 213, 174-186.

Husserl, E. (1970), *Logical Investigations*, transl, by J.N. Findlay, 2 vols., London: Routledge and Paul Kegan.

Netzer, N. (1978), "Verallgeneinerte topologische Strukturen", *Juhrbuch Überblicke Mathematik*, 87-106.

Ore, O. (1943), "Some Studies on Closure Relations", *Duke Mathematical Journal 10*: pp. 761-785.

Orlowska, E. and Weingartner, P. (1986), "Semantic Considerations on Relevance", *Reports of the Institute of Computer Science 582*, Polish Academy of Sciences.

Parry, W. T. (1933), "Ein Axiomensystem für eine neue Art von Implikation (analytische Implikation)", *Ergebnisse eines mathematisehen Kolloquiums*, 4, pp. 5-6.

Simons, P. M. (1987), *Parts. A Study in Ontology*, Oxford: Clarendon Press.

Weingartner, P. and Schurz, G. (1986) "Paradoxes Solved by Simple Relevance Criteria", *Logique et Analyse* 29, 3-40.