R. SCHMIT, **Husserls Philosophie der Mathematik. Platonistische und konstruktivistische Momente in Husserls Mathematikbegriff.** Bonn: Bouvier Verlag Herbert Grundmann, 1981. 159 pp. No price stated.

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1. *Introduction*. A number of authors, among them Oskar Becker, Marvin Farber, Suzanne Bachelard and Rosado Haddock, have sought to demonstrate the importance of mathematics in the development of Husserl's philosophy. Bachelard, in particular, has demonstrated the relevance to Husserl's thinking of Riemannian manifold theory and (less adequately) of Brouwer's intuitionism.[1] Rosado Haddock has attempted to set Husserl's philosophy of mathematics and logic within the perspective of contemporary research in foundations.[2] The present monograph consists in a detailed exposition of the philosophical relevance of Husserl's own immediate mathematical background, concentrating upon the influence of Weierstrass, Kronecker, Cantor and Frege.

Husserl served for a time as the assistant of Weierstrass in Berlin, where he also attended lectures by Kronecker. He later came into contact with Cantor in Halle, and the principal thesis of Schmit's monograph is that much of Husserl's work in the philosophy of mathematics can be understood as a working out of the tension between what Schmit conceives as the platonism of Cantor and Weierstrass and the constructivism of Kronecker.

For Kronecker, as is well known, only the positive whole numbers have an autonomous existence; all other (legitimate) mathematical objects are built up on the basis of these via mathematical operations. For Weierstrass, in contrast, 'all numbers have identical civil rights' in the domain of mathematical objects. Yet philosophical reflections involving the notion of a mathematical operation were by no means alien to Weierstrass, as notes of his lectures taken by Husserl reveal. Weierstrass was indeed prepared to talk of the operation of *Zusammenfassung* (bringing together) in mathematics as a *psychological act*—and a similar notion is of course present also in the writings of Cantor. One of Husserl's principal tasks in his early writings was to determine the extent to which such talk of operations (whether psychological or not) can do justice to those of our intuitions on the status of mathematical truths which derive from a broadly platonistic standpoint. Mere reference to operations cannot, it is clear, serve to characterise a philosophy of mathematics as constructivist. But there

1. S. Bachelard, *La logique de Husserl* (Paris: P.U.F., 1957). English translation by L.E. Emree; *A Study of Husserl's Logic* (Evanston: Northwestern University Press, 1968).
2. G.E. Rosado Haddock, 'Edmund Husserls Philosophie der Logik und Mathematik im Lichte der gegenwärtigen Logik und Grundlagenforschung' (Dissertation, Bonn University, 1973).

are two further characteristic moments of constructivism: nominalism and finitism, both neatly represented in Kronecker's work. Whilst nominalistic traits were always alien to Husserl, Kronecker's finitism, as Schmit shows, played a significant role throughout Husserl's philosophy of mathematics.

Schmit moves chronologically through the various stages in the development of Husserl's thought on mathematics. Chapter One is devoted to the *Philosophie der Arithmetik* (*PdA*) of 1891, concentrating upon the interplay of constructivism and psychologism in this work. The discussion of constructivism is admirable: Schmit provides a clear account of the role of symbolisation in the construction of number (and of the way in which symbols may, at one and the same time, both construct and signify higher mathematical entities), and of the doctrines underlying Husserl's rejection of the actual infinite. Schmit's treatment of Husser's purported psychologism is however less satisfactory, as we shall see below.

The discussion of the actual infinite is carried further in Chapter Two of the work, devoted to Husserl's encounter with Cantor. Chapter Three moves on to deal with Husserl's masterpiece, the *Logische Untrsuchungen* (*LU*) of 1900/01, and specifically the 'Prolegomena to pure logic' (Vol. I of the German edition), whose extreme, anti-psychologistic platonism Schmit describes—correctly—as a mere 'transitory phase' in Husserl's thinking. With Chapter Four, on Husserl's theory of the definite manifold (complete model), we return to the influence of Cantor, whose discovery of the Burali–Forti paradox in about 1895 had awakened in Husserl a first awareness of the problems nowadays associated with the completeness and consistency of a formal theory. Schmit sets forth here also the influence of Hilbert, and the implications for Husserl's theory of Gödel's results. Chapter Five on 'The idea of pure logic', dealing with Husserl's notion of a formal theory, completes the discussion of the *LU*, and Chapter Six is devoted to the conception of mathematics presupposed in the *Formale und transzendentale Logik*.

In Chapter Seven Schmit returns to the general issue of constructivism versus platonism. Husserl, Schmit concludes, has managed to free himself of the platonism which he had embraced in the 'Prolegomena' ot the *LU*, by conceiving mathematics as a 'realm of universal construction'. Iterative construction, which generates ever new formations is—in contrast to platonistic concept-formations—directed always to the effective givenness of the objects of mathematic (p.136).

The volume closes with a brief epilogue on Husserl's 'transcendental phenomenology'. It contains an excellent bibliography, but lacks an index, a serious inadequacy in a volume in which so much disparate material, both mathematical and philosophical and from both the phenomenological and the analytic literature, is discussed.

Schmit's monograph is undoubtedly the best and most well-researched treatment of Husserl's philosophy of mathematics that has appeared to date, and there are many aspects of his account which would merit detailed consideration. Here, however, I shall have space to deal only with the much vexed question of Husserl's purported psychologism in the *PdA*, and with his notion of a formal theory as developed in the *LU*.

2. *On psychologism.* A number, according to Husserl in the *PdA*, is the answer to a *how many* question posed in relation to a given plurality. A number is a property of a plurality. The philosophy of number must begin, therefore, with an understanding of the concept of plurality, and this presupposes in turn that we have an understanding of what it is which binds together the items in a plurality in virtue of which that plurality can be said to exist at all.

Because of the unrestricted generality of the concept of number (of what can be subjected to a count), the relation which these items bear to each other cannot be any real, physical or material relation, for example of proximity or of similarity. And nor can it be a relation which the items bear to each other in virtue of falling under a single concept—at least if the term 'concept' is understood in the traditional manner as signifying that which is connoted by a general term such as 'mammal' or 'human being'. For the objects in a plurality must not merely be picked out by means of some concept word (count noun), they must also be delimited in some way, for example as falling within a certain geographical area, and then the relevant delimitation would seem to be a component of the actually executed *act* of counting. Husserl concludes, in fact, that it is a mental or psychical relation, a relation constituted in or with the act of counting, which binds together the items in a pluality. Such a relation is entirely spurious or, more precisely, entirely extraneous, having no foundation in the material make-up of its relata. We have, according to Husserl, a 'spontaneous power of colligation', and the relation between objects thus colligated 'resides exclusively in the unifying act itself' (*PdA*: *Husserliana*, vol. 12, 43). It is this which explains the complete generality of the process of counting, and thereby also the complete generality of the concept of number.

But how can this appeal to spontaneous operations of consciousness be made consistent with those propositions about number which have their origins in a broadly platonist standpoint? How, above all, is this approach in terms of actually executed acts and operations to be made consistent with the fact that the propositions of arithmetic are necessary truths? Husserl, it must be admitted, gives no entirely satisfactory answers to these questions in the *PdA*, and Frege was able to point to certain passages in the work which suggested that he had indeed adopted a psychologistic standpoint on the issues involved—i.e. that he had sacrificed the necessity of propositions about number in order to uphold his theory of numbers as properties of psychologically generated pluralities.

A careful reading of the text would, however, have revealed to Frege that the work avoids such crude psychologism by distinguishing two quite different sorts of question: those relating to the origins and role of the concept of number in our mental experience, and those relating to the *content* (or ideal significance) of this concept itself. It is only in relation to the former that the properties of actually executed, empirically existing mental acts are of relevance. The necessity of arithmetical truths is guaranteed entirely by the latter. Husserl can justifiably be accused of a variant form of psychologism, but only in the attenuated sense that he assumed that the elucidation of the content of the concept of number must presuppose or at least involve some elucidation of the origin and role of this concept in our mental life. Completely to renounce psychologism of this residual variety is however to run the risk of abandoning *any* attempt to understand how the necessary truths of arithmetic (or, equally, of logic) can come to play a role in our actual thinking. But this, Husserl would argue, is precisely one of the most significant problems in the philosophy of mathematics.

The principal inadequacies in the *PdA* lie in fact not in any crude psychologism, but in certain terminological inadequacies on Husserl's part. Of these the most important derives from the fact that Husserl had inherited from his teacher Brentano a dualism which divided all phenomena (everything that is given in experience) into two categories of 'psychical and physical phenomena'. Experience itself is correspondingly divided into 'inner and outer perception', according as to whether what is experienced is a psychical phenomenon (a part or moment of one's own mental experience, in particular a mental act), or a physical phenomenon (something given as external or transcendent to consciousness, for example an experienced colour-datum). Constrained by this dualistic framework Husserl was initially compelled to consign to the realm of the *psychical* everything non-physical, i.e. everything not founded in the material make-up of given objects. Identity, for example, he was constrained to describe as a psychical relation, and so also in the case of the (empty, extraneous) relation which links the items in a plurality. But then 'psychical' comes to signify nothing other than 'not real', 'not founded in any material peculiarities of the things themselves', and it was so used by Husserl—albeit falteringly—until the terminology of 'psychical' and 'physical' was replaced, in his later writings, by the much more straightforward and consistently applicable terminology of 'formal' and 'material'. This in turn reflected Husserl's rapid progress, in the last decade of the 19th century, in the understanding of the peculiarity of formal concepts and of the fundamental importance of the discipline of formal ontology.

Schmit, unfortunately, ignores this line of development in Husserl's thought, and indeed he makes no reference to the influence of Brentano. In this respect Schmit reveals himself to be rooted in the post-War German tradition of Husserl scholarship, where the immediate philosophical influences upon the early Husserl, deriving largely from psychological and ontological writings of the Brentano school, have tended to be ignored in favour of an epistemologised account of his philosophy developed within the perspective of his later 'transcendental-phenomenological' writings.

3. *On the notion of a formal theory*. The predominance of this same, epistemological, perspective may explain also the brevity of Schmit's treatment (pp.87–99) of the theory of formal ontology developed by Husserl in the *LU*. Husserl develops there a two-fold distinction between the formal and the material on the one hand, and between formal (apophantic) logic and formal ontology on the other. The first distinction lies at the root of Husserl's account of the *applicability* of mathematics (and of formal theories in general): application is identified with materialisation, that is to say with the substitution of material terms of a greater or lesser generality for purely empty formal terms—terms ranging over all material regions without

restriction—in the propositions of a formal theory. The second distinction may be explained as follows.

Each and every formal theory, each and every theoretically complete and consistent system of proposition-forms (propositions containing no material terms) determines, or corresponds ideally to, an appropriate domain of object-forms. Formal logic is the discipline which has as its subject-matter the pure forms of propositions ('apophanses') and of meaning-entities in general, and which concerns itself specifically with the logical (deducibility) relations amongst these meaning-forms. Formal ontology is the discipline which has as its subject-matter the pure forms of object-entities in general: forms such as thing, state of affairs, unity, plurality, number, relation, connection, etc. (*LU*, 'Prolegomena', §67). The sub-disciplines of formal ontology would therefore include, for example, number theory, set theory, topology, measure theory, and certain other branches of mathematics. But they would include also the theory of parts and wholes, embracing not only the theory of *extensive* part-whole relations (i.e. Leśniewskian mereology), the theory of the relations amongst wholes and their various detachable 'pieces', but also the theory of *non-extensive* part-whole relations amongst wholes and their non-detachable 'moments' (and amongst the pieces of such moments) developed by Husserl in the 3rd Logical Investigation. (Schmit, unfortunately, seems to confuse the two kinds of theory: see p.94*f*.)

Whilst the extensive fragment of the formal ontological theory of part-whole relations is, mathematically speaking, almost completely trivial, the full theory of part-whole relations has a much more interesting structure (illuminated by Kit Fine, in as yet unpublished writings, from the standpoint of algebraic topology). And the apparent mathematical fruitfulness and wide applicability of this full theory, in association with the other formal ontological disciplines, lends substance to Husserl's enthusiastic claims, advanced not only in the *LU* but also in his later works, to the effect that it would be possible to develop an absolutely general formal *mathesis universalis*, a single theory within which the various branches of mathematics would find their (natural) place. And whilst some of Husserl's own explicit statements on the structure of this *mathesis universalis* have been shown, as a result of Gödel's theorems, to be ill-founded, the fundamental idea of such a theory—and specifically the idea of a rigorous working out of the strictly mathematical implications of the general theory of part and whole—continues to hold out much promise.

But now Schmit's identification of Husserlian philosophy of mathematics with the theory of mathematical *constructions* appears, in this light, to require supplementation: not by any element of platonism—the ever-present but ultimately irrelevant bugbear of Schmit's account—but by a conception of mathematics formulated within the framework of Husserl's idea of a formal *mathesis universalis*.