Safety and Pluralism in Mathematics

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Abstract: A belief one has is safe if either (i) it could not easily be false or (ii) in any nearby world in which it is false, it is not formed using the method one uses to form one's actual belief. It seems our mathematical beliefs are safe if mathematical pluralism is true: if, loosely put, almost any consistent mathematical theory is true. It seems, after all, that in any nearby world where one's mathematical beliefs differ from one's actual beliefs, one would believe some other true, consistent theory. Focusing on Justin Clarke-Doane's recent discussion, I argue the thesis that mathematical beliefs are safe given pluralism faces some obstacles. I argue (i) is true of mathematical belief given pluralism only if we deny plausible claims about the interpretation of non-pluralists who many of us could easily be. Unless strong metasemantic theses are true, it is plausible many of us could easily deny or refuse to believe a consistent and true mathematical theory we actually believe. Since philosophical arguments and controversies permeate the methodology of foundational mathematics, I argue we cannot confidently distinguish the methods we use in mathematics between worlds, thus raising doubts about (ii).

1. Introduction

Mathematical pluralism, put roughly for now, is the thesis that almost any consistent mathematical theory is true. Consistent mathematical theories that appear to conflict do not: they are to be interpreted as using similar but ultimately different mathematical concepts. Mathematical pluralism has been suggested as a means to show our beliefs in mathematics are reliably formed.[1] Justin Clarke-Doane has recently argued that the concept of safety is a good explication of the

[1] Such as (Balaguer 1995), (Field 2005, p. 78), and (Beall 1999).

reliability mathematical pluralism intuitively provides us. It is intuitively plausible, he has pointed out, that our beliefs are safe given pluralism is true. A belief one has is safe if and only if either (i) or (ii) is true of it:

(i) There is no nearby world in which it is false—it is *stable*, as I will put it.

(ii) In any nearby worlds in which it is false, it is not formed using the method one uses to form one's actual belief.

It is intuitive that our beliefs are safe given pluralism because it is intuitive our beliefs given pluralism are stable. It seems, after all, that in any nearby world where one's mathematical beliefs differ from one's actual beliefs, one would believe some other true, consistent theory.

Aside from its promise in explicating the reliability of belief formation, safety is sometimes thought of as necessary for the justification of belief. If this is so, mathematical pluralism avoids the undermining of our mathematical justification. Perhaps safety does not always undermine justification. Clarke-Doane himself is explicitly neutral on the question of whether unsafety undermines. Even so, safety seems like a good-making feature of belief to keep track of in our epistemological theorizing. Analyzing the connection between it and mathematical pluralism—an increasingly popular thesis—is worthwhile.

Despite its intuitive pull, I argue the thesis that mathematical belief is safe given pluralism faces some obstacles. I argue (i) is true of mathematical belief given pluralism only if we deny some plausible claims about interpretation. Unless these claims hold, it is plausible many of us could easily deny or refuse to believe a consistent and true mathematical theory we actually believe. Overriding these plausible claims requires strong principles of interpretation (sometimes called principles of charity), or overly strong views about the publicity of linguistic meaning, which at best require argument and at worst are implausible.

Could we establish the safety of mathematical belief given (ii) instead? This is also difficult. I suggest any method most of us actually use to believe mathematical theories involves a weighing of plausibility judgments characteristic of philosophical argumentation. Therefore, even assuming we are pluralists who believe almost all consistent theories, we follow some method or another for weighing up plausibility judgments, leaving us with the difficult task of individuating such methods between worlds. If we decide there is just one such method, then the plausible claims about interpretation I use to raise doubts about (i) suggest our beliefs are *unsafe*, as one would use that single method in nearby worlds where we believe falsely. One upshot I draw from the worries I raise about safety given pluralism is that expertise in the foundation of mathematics threatens the stability of our beliefs, and perhaps their safety as well. Another upshot is that the generality problem for reliabilism, which Clarke-Doane cites as raising trouble for defending the thesis that unsafety in mathematics undermines, raises trouble for the thesis that mathematical belief given pluralism is safe in the first place.

2. Objectivism and Pluralism

I first explain Clarke-Doane's definitions of *pluralism* in mathematics and its opposite, which he calls *objectivism*. It is these definitions which I will use.

To motivate his definitions, Clarke-Doane starts with what he takes to be our attitude toward different geometries. In Euclidean geometry, for every line L and a point not on L, there is exactly one line passing through that point that does not intersect L. Non-Euclidean geometries may be loosely expressed as contradicting it: it is not the case that there is exactly one such line.

But our attitude, Clarke-Doane says, is that we view non-Euclidean geometries as equally true as Euclidean geometry, using different concepts than Euclidean geometry: of point, of line, or both or more. So, a non-Euclidean geometry strictly speaking implies something we may express as follows: for every line* *L* and a point* not on *L*, it is false that there is exactly one line* passing through that point* that intersects with *L.* In Clarke-Doane's terminology, our attitude toward geometry is that geometric truth is not *objective*. We would think geometry is objective if, by contrast, we thought of non-Euclidean geometries as false using Euclidean geometry's concepts rather than true using different concepts: if we thought there was a uniquely true set of axioms of the unique geometric subject matter.

More generally, objectivism about some area of inquiry F is the thesis that there "are not…a plurality of F-like concepts, all satisfied, giving intuitively opposite verdicts to typical F-questions" (Clarke-Doane 2020a, p. 32). Pluralism about F, by contrast, is the thesis that there are such concepts giving intuitively opposite verdicts. In (2020b), Clarke-Doane calls objectivism relative to set theory *set-theoretic universalism.* Universalism implies there is a uniquely true set of axioms of set theory true of a universe of objects satisfying a single concept of set. A pluralist in set theory, by contrast, believes multiple concepts of set are satisfied. For example, she may believe in Zermelo-Fraenkel set theory with the axiom of choice (ZFC) and that it provides a universe that satisfies its concept of set, and therefore believes AC is true: that every set is well-ordered. But she also may believe intuitively opposite theses are true using different concepts of set true of different universes. She may believe, for example, that ~AC* is true: that it is false that every set* is well-ordered. Whether she does have these beliefs depends upon the nature and degree of her pluralism in set theory.

Now we come to Clarke-Doane's non-relative definitions of objectivism and pluralism in mathematics: objectivism and pluralism *simpliciter*. In what follows, 'objectivism' and 'pluralism' will express these notions *simpliciter* unless otherwise noted. Objectivism in mathematics is the thesis that "foundational" areas of mathematics such as set theory, arithmetic, and analysis are objective.[2] (He says that "[b]y "foundational" theories I mean, roughly, theories in which one can carry out metatheoretic reasoning" (2020a, p. 29).) Pluralism in mathematics is "at first pass" the thesis "that every consistent mathematical theory is true of its intended subject, independent of human minds and languages." (p. 152) Clarke-Doane restricts pluralism in mathematics to the claim that every *arithmetically sound* mathematical theory is true. A theory (in first-order logic) is arithmetically sound when no sentence that it implies is false in the standard model of arithmetic. Examples of arithmetically unsound theories are ZFC and (first-order) Peano Arithmetic conjoined with the negations of their respective consistency statements (2020a, pp. 160–61), as those negated consistency statements are false in the standard model of arithmetic. Clarke-Doane excludes theories such as these from his definition of pluralism because he suggests that the plurality of concepts of consistency, finitude, and proof that result from taking arithmetically unsound theories to be true is "too much to swallow" (p. 161).[3]

3. Stability

Clarke-Doane argues that, if objectivism is true, then our beliefs could have easily been false; in short, our beliefs are *unstable*. He accepts what I call *Contingency*, which I will grant in this paper:

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[2] See Clarke-Doane (2020a, pp. 29-30). He says of the objectivist "that she does not believe that typical axioms of our foundational theories, like set theory, are analogous to the Parallel Postulate." (2020a, p. 152)
[3] (2020a, pp. 159-62).

> *Contingency:* What mathematical axioms we find true, false, plausible, or implausible could easily be different.

To support Contingency, Clarke-Doane notes the variation among mathematicians in their beliefs in truth-values of mathematical axioms and the reasons for them. There is variation over axioms extending ZFC—which is more or less the consensus pick for mathematical foundations, if there is a consensus—and there is variation over belief in ZFC or theories interpretable into it.[4] While mathematicians come to a rough consensus about some axiom sets, it is not hard to find intelligent and knowledgeable mathematicians or mathematically adept philosophers who diverge from these consensuses at just about every point. Clarke-Doane thinks this variability indicates that, if we had gone to a different graduate school or undergraduate program or had read different books, we would have easily found axioms to be true which are, given objectivism, actually false. If AC is objectively true, there is no alternative ~AC* that the pluralist envisages. Thus, were my intellectual history slightly different and I believed ~AC, I would believe a falsehood and not the true ~AC* as the pluralist believes. Thus, if objectivism is true, my beliefs could easily have been false: they are *unstable*.

But on Clarke-Doane's view, if pluralism is true, things are different. He thinks our beliefs in foundational axioms are stable given pluralism is true. Here is why. Suppose, as Clarke-Doane does, that each of us has beliefs in the truth of sets of foundational axioms, and that those beliefs are true. Given Contingency, we could have easily believed different sets of axioms than we actually do. If objectivism is true, some (actually, most) of those beliefs are unstable, since our easily different beliefs would be beliefs in falsehoods. If objectivism is true and we suppose that

[4] See Chapter 2 Clarke-Doane (2020a) and Section 1.4 of Clarke-Doane (2022).

AC is true, I could have believed that ~AC is true. Given objectivism, ~AC is false: ~AC is expressed using the same concept of set as AC does and says something false of the universe of sets which satisfy that concept. By contrast, if pluralism is true, those beliefs we could have easily had are not false, but instead true, unless they are arithmetically unsound. Perhaps I could have easily believed in the truth of the *sentence* 'Not all sets are well-orderable': the sentence '~AC'. But, given pluralism, the proposition I believe in virtue of believing in that sentence's truth is the true proposition that not all sets* are well-orderable: I believe ~AC*. This suggests that, given pluralism, my beliefs could not have easily been false—they are stable.

This is a bit too quick, since we could end up having inconsistent or arithmetically unsound beliefs: if pluralism is true, inconsistent and arithmetically unsound beliefs are false. Clarke-Doane makes two suggestions to fill the gap. First, he cautiously suggests we could not *easily* have had inconsistent or arithmetically unsound beliefs, given pluralism is true. He says the thesis that we could not have easily had inconsistent or arithmetically unsound beliefs—*incoherent* beliefs, for short—is "widely accepted" (p. 160), although it is "not obvious" how to argue for it (*Ibid*.). He continues:

> Probably, any good argument that we could not have easily had incoherent mathematical beliefs will have to be cumulative—noting not just mathematicians' failure to discover an incoherence, but scientists' enormous success in applying mathematical theories to the concrete world, our insight into models of those theories, and so on. (2020a, p. 161).

Second, in endorsing basically all consistent theories, Clarke-Doane in (2020b) suggests that pluralists employ our ability to determine when contradictions do or do not follow from a set of axioms: they are relying on our "mechanism for deductive inference" (2020b, p. 2027). Moreover, "[t]here is a prima facie case to be made that we were selected to have a reliable mechanism for deductive inference…" (2020b, p. 2027). Clarke-Doane admits this is an empirical conjecture. However, on the basis of these two suggestions, Clarke-Doane thinks the stability of our belief in foundational axioms given pluralism is likely.

4. Safety

Clarke-Doane makes these assertions about the stability of belief to suggest some theses about the *safety* of belief. Here is the definition Clarke-Doane uses:

> *Safety: S*'s belief that *P* is safe if and only if *S* could not easily have had a false belief as to whether *P*, using the method that *S* actually used to determine whether *P*.[5]

Put alternatively: a belief is safe if I could not have easily believed a false answer to the question "*P*?" using the method I actually used to believe an answer to that question. In terms of worlds: a belief that *P* is safe when we do not have a false belief as to whether *P* in all nearby worlds, using the method we use in the actual world. Restriction to "the method we actually use to believe *P*" is essential to have a useful concept of safety. Suppose I wonder what time it is. I have a working smart phone in my pocket whose battery and internet connectivity I check fastidiously. I pull out my phone and believe the time is what my phone says it to be. It turns out I could have easily asked for the time from a stranger sitting by me who, unbeknownst to me, loves lying to strangers about the time. My belief could have been easily false, but the method I actually used to form the belief

[5] See (2020a, pp. 108-109 and 148). How best to formulate safety is a matter of dispute. Here I am simply taking on Clarke-Doane's preferred formulation. He follows Pritchard (2009) in stating safety a matter of easily having a false belief *as to whether P* rather than easily having a false belief that *P*. Also, safety as Clarke-Doane understands it technically is:

> A belief that *P* is safe if and only if, for *Q* relevantly similar to *P*, we could not have easily had a false belief as to whether *Q*, using the method we actually use to determine whether *Q*.

This is due to a counterexample from Williamson (2000, p. 124). This amendment is not relevant to my purposes.

likely makes it safe, or at least safe according to a concept of safety useful for assessing my beliefs, given I use the excellent method of checking my well-maintained smart phone.

For my purposes, I will break down the definition of safety into two parts. A belief is safe if and only if at least one of (i) and (ii) is true of it:

> (i) My belief is stable. That is: there is no nearby world in which I believe a false answer to the question "*P*?".
>
> (ii) In any nearby world where I believe falsely, I do not use the method I actually used to form my answer to "*P*?".

Therefore, a belief is unsafe if both (i) and (ii) are false of it. It suffices to show my belief is safe if I show it is stable. Clarke-Doane's suggestion that our mathematical beliefs are stable given pluralism would imply that those beliefs are safe given pluralism. It also suffices to show my belief is safe if, in any nearby worlds in which I believe falsely, I do not use the method I used to form my actual belief. If we assume the nearby worlds in which I falsely believe what the lying stranger says about the time in my above example are the only nearby worlds where I do not use my phone to form a belief about the time, my actual belief about the time formed using my smart phone is plausibly safe in virtue of meeting (ii).

5. The Metasemantics of Stability

Here is an argument that, given pluralism is true, the beliefs that some individuals have in some, probably most, foundational axioms are unstable. I will label it *the instability argument*. I argue in this section that its premises are plausible and that it can only be contested with strong metasemantic theses.

Consider someone who has beliefs in the truth of foundational axioms. To fix ideas, suppose that person is me, and that I believe ZFC is true. To whom the argument applies will be discussed later. Given Contingency, there is a nearby world with four features. First, I have the commitment to arithmetically sound foundational axioms in at least one area F which objectivists relative to F have. Given our assumption to fix ideas that actual me believes ZFC, F here equals set theory. Second, I use and affirm at least one negation of the sentences of foundational mathematics I actually endorse: to fix ideas, let us make one of those sentences '~AC'. Third, I am a committed objectivist about area F: I believe that, in F, there are no other F-like concepts satisfied by distinct but similar mathematical universes. Note this does not imply I am an objectivist *simpliciter.* But it does imply I am not a pluralist *simpliciter*. Let us thus say that I am a committed *non-pluralist*. Fourth, I am well-versed in the foundations of mathematics and philosophical and methodological issues surrounding it; in short, I am an *expert* non-pluralist. Perhaps I am not an expert mathematician, but I am adept enough at both mathematical and philosophical topics to have well-formed opinions in these areas. Given these four features, in that world, in virtue of affirming '~AC', I believe ~AC. Given pluralism is true, ~AC is false, since AC is true. (If objectivism *simpliciter* were true, we could not infer ~AC is false unless we also endorse that ZFC is true. But, by the definition of pluralism *simpliciter*, ZFC and hence AC is true if pluralism is true). Thus, given pluralism is true, I could have easily believed a falsehood. Thus, given pluralism is true, my beliefs in foundational axioms are unstable. The argument is perfectly general, applying to any foundational axiom: restrictions will be considered below.

The best way Clarke-Doane could challenge the instability argument is by challenging this premise: in the nearby world, in virtue of affirming '~AC', I believe ~AC. If this is right, we cannot believe our beliefs are safe under pluralism while also taking pluralism only to be a first-order *metaphysical* view about what actually is true or exists. We also have to commit to a *semantic*, or *metasemantic*, theory about how to interpret our sentences and the contents of our beliefs. Here is a tempting way to put this theory:

> *Metasemantic pluralism:* Any arithmetically sound sentential set (in foundational mathematics) anyone sincerely affirms is true, interpreted as expressing a true arithmetically sound theory containing some F-like concept or another for the relevant mathematical F.

An *arithmetically sound sentential set* is a set of sentences (in first-order logic) which is syntactically consistent, and, in addition, is arithmetically sound: implying no falsehoods in the standard model of arithmetic. 'Arithmetically sound theory', as I have been using the phrase, refers to a set of propositions. Clarke-Doane affirms metasemantic pluralism as a consequence of pluralism, saying that the pluralist's "metasemantics" is "cooperative," in that "the consistent mathematical sentences that we accept are automatically about the parts of mathematical (or mathematical-like) reality of which they are true, and there always are such parts" (2020a, p. 159). The thought is that, if I affirm '~AC', it is about a different part of mathematical (or mathematical-like) reality than ZFC. It is instead about a theory with a concept of set*: call it ZF~C*. In virtue of affirming '~AC', I affirm ~AC*, not ~AC. In virtue of affirming 'ZF~C', I affirm ZF~C*, not ZF~C, where ZF~C is a theory using the orthodox concept of set involving the negation of the axiom of choice.

However, adding metasemantic pluralism is insufficient to justify this response to the instability argument. Suppose non-pluralist nearby-me affirms '~AC', and then strenuously denies thereby believing ~AC*. The non-pluralist nearby-me, being a committed and expert non-pluralist, is aware of the distinction between sentences and propositions, the pluralist's views on these things, and the like. Nearby-me insists: "I believe ~AC, not ~AC*." In virtue of what is nearby-me mistaken? We need some kind of principled reason to think that, try as nearby-me might, I cannot believe ~AC instead of ~AC*. After all, the pluralist thinks the *proposition* ~AC exists, either by itself or embedded in a variety of logically complex propositions—what prevents belief in it?

There are two strategies I will consider in this section for providing the principled reason I seek, which would thereby disarm the instability argument and help defend stability given pluralism. The first appeals to a specific principle of interpretation specific to the case of mathematics, principles in turn likely justified by broader, and strong, metasemantic principles. The second appeals to the publicity of linguistic meaning. A third strategy will be mentioned in the conclusion. These are the only three strategies I can think of.

To the first strategy. Let us offer the following principle to do the job of preventing denial of AC. I call it *No Denial:*

> *No Denial:* No one who fully understands an arithmetically sound theory denies it. If a speaker who fully understands an arithmetically sound sentential set denies those sentences (that is, affirmatively utters one of their negations), those sentences are to be interpreted so that the speaker does not deny an arithmetically sound theory.

We can entertain ~AC, or suppose ~AC, or affirm logically complex thoughts with ~AC as a constituent. But according to No Denial, no one could actually sincerely *believe* ~AC, no matter how sophisticated their beliefs about their own beliefs are, unless they fail to understand what they purport to deny. Correlatively, those who understand *sentences* that express some arithmetically sound theory or another do not thereby deny an arithmetically sound theory. (This principle therefore uses the notion of understanding propositions and the notion of understanding sentences).

It is not enough to justify such ideas to argue that, as a matter of fact, the axioms are true in virtue of the F-like concepts in other arithmetically sound axioms, or true in virtue of the conventions governing the concepts' expression in language. The non-pluralist, after all, could deny such metaphysical claims, even if they are true, and then deny those axioms. What is needed is a thesis that someone who truly understands and grasps the relevant concept cannot deny the axioms which utilize that concept. In short, it requires epistemic and not metaphysical analyticity, to use Paul Boghossian's distinction (Boghossian 1996).

Is No Denial sufficient to justify the above response to the instability argument? Not entirely. It is not enough that nearby-me is unable to believe that ~AC. It also must be the case that nearby-me believes ~AC* in virtue of affirming '~AC'. But we can argue for this conclusion as follows. Given metasemantic pluralism, any arithmetically sound set of sentences anyone sincerely affirms expresses a true arithmetically sound theory. 'ZF~C' is arithmetically sound, and nearby-me sincerely affirms it. By No Denial, nearby-me is unable to believe ZF~C (assuming that nearby-me understands it). The *only plausible available interpretation* of the 'ZF~C' I affirm, given pluralism, is ZF~C*. So, nearby-me believes ~AC* in virtue of affirming '~AC'.

This argument works, but we need another principle about belief and meaning to support the last premise and also avoid a contradiction. It seems that nearby-me *refuses to affirm* ~AC* and thereby ZF~C*. As a committed and expert non-pluralist, I would say: "I refuse to affirm ~AC* and therefore ZF~C*. It utilizes an empty, or contentless, concept satisfied by no universe I recognize to exist. ~AC* is neither true nor false." I can give sophisticated philosophical and mathematical arguments to back up this utterance. Taking my words at face value, I am sincerely committed to refusing to believe ~AC*. I do not purport to *deny* it, since I do not believe that ~~AC*. Taking my words at face value, I refuse to believe ZF~C*. As a committed non-pluralist, I can plausibly be imagined as *uttering* "set-prime" or "set-star" or "set-asterisk" or "shmet" and uttering, for example: "I refuse to believe ZF~C*" or "It is not true that there are any shmets." Thus, if I take my words at face value, and if the argument just given that I believe ZF~C* in virtue of affirming 'ZF~C' is sound, I both believe *and* refuse to believe ZF~C*. My state of belief is impossible. Even though a dialetheist thinks she believes contradictions, she knows she cannot both believe and refuse to believe a proposition.[6]

We can avoid this contradiction, and finally respond to the instability argument, if we endorse No Denial and, in addition, *No Refusal:*

> *No Refusal:* No one who fully understands an arithmetically sound theory refuses to believe it. If a speaker who fully understands an arithmetically sound sentential set refuses to believe it (that is, refuses to utter affirmatively their conjoined truth), those sentences are to be interpreted so that the speaker does not refuse to believe an arithmetically sound theory.

Try as non-pluralist nearby-me might, I cannot refuse to accept an arithmetically sound theory I fully understand. So, here is how we can respond to the instability argument. Given No Denial, non-pluralist nearby-me does not believe ~AC. Given No Refusal, non-pluralist nearby-me also does not refuse to believe ~AC*. Given all of this, given that I affirm 'ZF~C', and given also metasemantic pluralism, the most plausible interpretation of the 'ZF~C' I affirm is ZF~C*. Thus,

[6] See Priest (2006, pp. 96–99), who uses 'rejection' for what I label as 'refusal to believe'.

my belief in ZFC and AC in the actual world is safe given pluralism, because it could not easily be false: in nearby worlds, I believe the arithmetically sound alternatives.

I emphasize that No Denial and No Refusal do not go without saying given the metaphysical commitments of mathematical pluralism. They are additional metasemantic views. To see what is at stake here, let us take a look at philosophers who hold versions of No Denial and No Refusal in similar contexts.

Jared Warren, in his recent conventionalist project in (Warren 2020), would accept a version of No Denial and No Refusal, replacing "arithmetically sound set of sentences" with "consequences of logical inference rules" (and also "consequences of mathematical inference rules," at least in some cases). He wields a strong version of what he calls a principle of charity to do so. On his view, an intuitionist who refuses to accept classical logical truths is to be interpreted not as doing so. He says that, if those of us who speak English interpret other speakers of English at face value if they do not accept the law of excluded middle in its full generality, it makes them "shockingly irrational and unaccountably foolish" (2020, p. 131). These speakers must be re-interpreted as using words with different meanings: the sentences they refuse to believe are true are to be translated as not using the same logical connectives as we believers in classical logic use. Some object to this principle of charity (Field 2023).

W.V. Quine in (Quine 1980) argues that those who try to deny or refuse believing classical logical truths would refuse to believe what is obviously true or would believe what is obviously false. He thinks charity demands we interpret others as not denying or not refusing to believe classical logical truths, using (what he calls) a principle of charity which re-interprets those who sincerely affirm obvious falsehoods or refuse to accept obvious truths. These speakers must be re-interpreted as using words with different meanings: the sentences they refuse to believe are true are to be translated as not using the same logical connectives as we believers in obvious classical logic use. Some object to this principle of charity (Morton 1973), (Parsons 1974).

Such principles of charity may not be necessary to derive No Denial and No Refusal. However, it seems a paternalistic (if you will) interpretive principle like a strong principle of charity is necessary in order for one not to take the knowledgeable beliefs of experts at face value and attribute beliefs to them other than those that they sincerely take themselves to have or take themselves to refuse to have.[7] Anyone who thinks we have at least a defeasible commitment to take expert logicians, mathematicians, and mathematical and logical philosophers at face value when they refuse to affirm classical or non-classical logical and mathematical theories will reject No Denial and No Refusal, at least in full generality. It is consistent with the defeasibility of this commitment that experts can sometimes be wrong about their beliefs. Thus, we do not have to go as far as Timothy Williamson, who in (Williamson 2007) forcefully articulates the commitment but adheres to a strong version of it, arguing that it is likely we can interpret *any* expert as refusing to believe *any* truth she fully understands. But experts' sometimes being wrong about their beliefs

[7] (Rosen 2023) argues that stability is sometimes false given pluralism is true. Rosen makes a similar point as I do when talking about how the pluralist has to interpret those like finitists who accept what he calls "restrictive theses": "It is uncharitable in the extreme to construe proponents of restrictive views as asserting bland tautologies when they take themselves to be taking a bold stand against the ovine groupthink that surrounds them." (p. 797) Rosen does not develop his view on interpretation as I develop mine (or, at least, the view I put forward as plausible). Moreover it seems he agrees with Clarke-Doane that metasemantic pluralism implies that, at least in the case of set-theoretic axioms such as ~AC, we could not easily believe falsely that ~AC since we would believe ~AC* instead: he argues that the "cooperative metasemantics" of pluralism is "plausible in many cases" (*Ibid*.), but not in cases of "restrictive theses" such as finitism and nominalism. (Clarke-Doane interprets him this way in his reply (Clarke-Doane 2023)). The way Rosen characterizes the cooperative metasemantics seems to be in terms of metasemantic pluralism: "So long as our mathematical beliefs are consistent, they are inevitably about the structures that make them true (if any)" (p. 796). As I have argued, metasemantic pluralism is not sufficient to justify stability of belief given pluralism. Further, Rosen thinks the pluralist would have to interpret finitists as "asserting bland tautologies" such as "Every finite set is finite," something I do not commit to. Moreover, as I suggest below, my points may be consistent with interpreting some unorthodox commitments in the philosophy of mathematics such as ultrafinitism and fictionalism in a "paternalistic" way.

or refusals to believe is not sufficient for the perfectly general No Denial and No Refusal: when it comes to arithmetically sound theories, these principles say self-proclaimed non-pluralists are *always* wrong about their foundational beliefs.

Although we saw that Clarke-Doane affirms metasemantic pluralism, there is some indirect textual evidence he would deny No Denial and No Refusal. For one, as I noted above, one way to support No Denial is with a notion of epistemic analyticity: one could use it to support No Refusal as well. But Clarke-Doane rejects epistemic analyticity in mathematics in (2022, pp. 7-9). Also, consider:

> Mathematical fictionalists do not believe any plausible mathematical propositions, because they do not believe any mathematical propositions at all. (More exactly, fictionalists do not believe any atomic or existentially quantified mathematical propositions, and believe that all universally quantified mathematical propositions are vacuous.) (2020a, p. 47)

He takes at face value mathematical fictionalists' claim that they refuse to believe any atomic or existentially quantified mathematical propositions, because such mathematical propositions imply the existence of things they believe do not exist. It seems to me that someone who takes refusal to believe any atomic or existentially quantified mathematical propositions at face value should take an objectivist's refusal to believe ~AC* at face value, and therefore is someone who should deny No Refusal and, for parallel reasons, No Denial.

I am not aiming to make a "gotcha" argument. Clarke-Doane could easily change his mind about what fictionalists claim not to believe. Perhaps he could decide to take on epistemic analyticity as well. At the very least, I want to highlight the sorts of things one may be inclined to endorse that one must give up in order to hold No Denial and No Refusal. It also helps me to point out that his views on what others refuse to believe must be perfectly general. Perhaps we strain to believe that a person could actually be a fictionalist. But No Refusal and No Denial require more than that. It is possible to argue for more restricted views than No Refusal and No Denial. But they are the starting points for any principled rejection of plausible claims about interpreting experts at face value.

Here is the second strategy for preventing denial of AC and refusal to believe ~AC*. It argues that, given the publicity of language, metasemantic pluralism (the "cooperative metasemantics" Clarke-Doane mentions) implies that our beliefs are stable if pluralism is true. Metasemantic pluralism will imply that anyone who affirms 'ZF~C' gets some meaning attached to their use of 'ZF~C'. The best interpretation of 'ZF~C' for all those who use and affirm that syntactic type is ZF~C*. Why? At least someone should be interpreted as believing ZF~C* by affirming 'ZF~C'. A self-aware pluralist, when clarifying exactly what she means, will use asterisks or neologisms like 'set-asterisk' or 'set-star' or 'shmet' instead: in brief, she will use 'ZF~C*' instead. But the vast majority of us will not actually do that, even some self-aware pluralists, and will use 'ZF~C' instead. Language is public: in sharing a language in a linguistic community, we come to follow and to share implicit conventions which assign meanings to the syntactic types we jointly use. Given this, whatever meaning we assign to that syntactic type 'ZF~C' when it is affirmed ought to be the same no matter which speaker of the public language uses and affirms it. Thus, since at least someone should be interpreted as believing ZF~C* by affirming 'ZF~C', *everyone* should be interpreted as believing ZF~C* by affirming 'ZF~C'.

This argument takes the publicity of linguistic meaning too far. Given her pattern of beliefs and behaviors, including her metalinguistic behavior in thinking about and using her language, it is plausible a committed, expert non-pluralist expresses ZF~C, and not ZF~C*, when she affirms

‘ZF~C’. Although generally the sentence types those of us who use a common language utter express the same propositions, there is reason in particular cases to override this general presumption. If I were to decide to be careful in expressing clearly what I am committed to, I have the opportunity and disposition to use, or fail to use, a sentence type other than ‘ZF~C’. If I am a pluralist, I would add an asterisk, or use a neologism such as ‘shmet’ or ‘choiceless sets’, or the like; in short, I am disposed to *clarify myself in words* using ‘ZF~C*’. If I am not a pluralist, I would clarify myself in words *not* using such neologisms, using ‘ZF~C’ instead.

I claim that, as a general schematic rule, the following is plausible—or, at least, is consistent with whatever publicity linguistic meaning has: if an expert in mathematics, or for that matter any legitimate scientific discipline, affirms ‘*S*’ at a given time and, at that time, is disposed to clarify themselves in words with sentence ‘*S**’ (which may be identical to ‘*S*’), then that is strong, although not always sufficient, evidence that ‘*S*’ means that *S**. The public nature of linguistic meaning, it seems to me, ought to be consistent with the ability of a given expert to engage in idiolect variation that is continuous with the language use of the larger linguistic communities of which she is a member. This strikes me as in accord with actual practice. Further, if the division of linguistic labor ties our meanings to some experts’ meanings, it seems experts generally have the right to craft their own.

This indicates that a pluralist should be interpreted as believing ZF~C* in uttering ‘ZF~C’, while a non-pluralist would believe ZF~C in uttering ‘ZF~C’ because they would not clarify themselves in the way a pluralist would: they would not seek to add asterisks or coin neologisms. This disposition to clarify what one says is evidence for how one should actually be interpreted, evidence which is outweighed only by something like No Denial and No Refusal or some other paternalistic principle of interpretation. Other interpretive principles may come into play here to outweigh the interpretation which the disposition to clarify in words evinces. For example, fictionalism or ultrafinitism (the thesis that some finite numbers do not, or at least might not, exist) could be argued to be inconsistent with the best interpretation of a speaker, given the way they use mathematical expressions in other contexts such as applications in empirical science or when balancing their checkbooks; so, individuals do not believe them, despite clarifying themselves with utterances such as ‘No, I mean it, there are no numbers, and so 2 + 2 does not equal 4’ or ‘No, I mean it, $2^{1,000,000}$ does not exist’. I am not sure this is correct, but even if correct, this seems to me to be the kind of exceptional case that could outweigh the general principle, given we can find a large body of evidence in their total behavior that they do believe what they deny they believe. I do not see how non-pluralism when it comes to axioms such as AC fits this kind of exceptional case that is sufficient to show pluralism implies stability, since such axioms are to a significant degree isolated from linguistic behavior in empirical applications or daily life. Or, if such axioms do not fit what I portrayed as an exceptional case, I would like an argument. To be sure, experts cannot be interpreted using words they would clarify themselves with if the propositions expressed using those words do not exist. But the proposition that ZF~C exists given pluralism is true; so, this point helps us none here.

One might object that I have not given a systematic metasemantics in response to whatever systematic metasemantics would generate No Denial and No Refusal. That is true, but I do not see it as the basis for an objection to my arguments. The kind of deference to experts’ self-understanding articulated in what I have said are plausible propositions that create burdens to be discharged by anyone such as Clarke-Doane who wishes to argue that our beliefs are stable given pluralism is true. As suggested already, while the points I make fit well with Williamson’s well-known views on expertise and the relation between understanding and belief, they are, I claim, consistent with alternatives. Even if I must endorse a view like Williamson's to make the claims I have made, that is still an interesting result: assessing pluralism's stability requires taking sides in metasemantics.

One might argue that, for the vast majority of us, we could not easily have been such expert, committed non-pluralists. Only those of us who are significantly knowledgeable about mathematics and philosophy, one might argue, could easily be expert, committed non-pluralists. Thus, the instability argument applies to vanishingly few of us.

I am not sure how few of us it applies to. It seems that many people who are reasonably intelligent and who could have easily been privileged enough to have been educated at a university with philosophy departments in analytic philosophy could easily have ended up taking classes in philosophy and then eventually the foundations of mathematics or could have easily read and understood books and articles on these matters. (Or so we academic philosophers dream as we desperately advertise our majors.) At any rate, only a few is enough, since the thesis that our beliefs are safe given pluralism is supposed to apply to just about any inquirer who can be credited with beliefs in foundational axioms, however testimonially or indirectly caused. And I surmise it applies to many readers of this paper. If so, and you think the safety of your beliefs is important, this paper is of interest to you.

Perhaps one might argue at least *pluralists' beliefs* are stable given pluralism is true. If so, so long as we adopt pluralism, our beliefs could not easily be false. However, the response fails, since the instability argument applies with full force: if I believe pluralism and believe AC, then I could easily believe ~AC for the reasons discussed. Worse, even though as a pluralist I also believe ~AC*, I could easily either believe ~~AC* or refuse to believe ~AC*, for parallel reasons: a non-pluralist nearby-me could deny or refuse to believe not only any of the orthodox axioms I actually accept, but also any of non-orthodox axioms in arithmetically sound theories I actually accept. Therefore, if I am a pluralist, my beliefs in both orthodox foundational axioms and non-orthodox arithmetically sound theories are unstable given the instability argument.

Moreover, the instability argument seems to have an interesting consequence: our beliefs become more likely to have easily been false the more expert we could easily be. In foundational mathematics, expertise threatens stability.

Perhaps expertise threatens stability in all theoretical matters. Clarke-Doane may be happy with this thought or something near it, at least not assuming pluralism is true. In (2020a), he endorses what I will call *The Permeation of Philosophy:*

> *The Permeation of Philosophy*: Philosophical beliefs and methods, and therefore controversial beliefs and methods, permeate the beliefs and methods of legitimate areas of inquiry.

As he puts it:

> Despite the conventional wisdom that philosophy is controversial while the sciences are not, it might be more accurate to say that all areas of inquiry are controversial, in the sense that should be of interest to epistemologists, because an explicit formulation of an area's findings will generally be up to its ears in controversial philosophy. (2020a, p. 63).[8]

Given the Permeation of Philosophy, it is plausible that, the closer one gets to expertise in some area of inquiry, the more likely it is that one's beliefs are easily false. Rather than deferring to others, one will be more likely to form well-supported opinions on controversial matters about

[8] See also (Clarke-Doane 2023, p. 822).

foundational questions in one's discipline. One will be then more likely to easily have had different such opinions, since one is more able to see the plausibility of the variety of viewpoints at issue. One might think that pluralism about a subject matter saves one here. But my argument so far indicates that it does not, unless one wishes to buy strong metasemantics. To the extent to which the Permeation of Philosophy is true, expertise to that extent threatens stability in any area, whether or not pluralism in that area is true.

6. Specifying a Method

Recall that a belief in a foundational mathematical axiom *P* is *safe* if one of the following is true: (i) there is no nearby world in which I believe a false answer to the question "*P*?", and (ii) in any nearby world where I believe a false answer, I do not use the method I actually used to form my answer. The second argument I will consider defends (ii) for our beliefs in foundational axioms. In short, it denies *nearby sameness of method:* it argues one's method in all nearby worlds where one believes falsely are not the same as in the actual world. Actually, it does something different, and weaker: it concludes that, if *one is a pluralist*, one's method in all nearby worlds where one believes falsely is not the same. The argument does not need the truth of pluralism, but it does need the believer to be a pluralist.

Here is the argument. The only method a pluralist actually uses to form foundational mathematical belief is this: determine whether or not a theory is arithmetically sound. Call this method *M*. Any world in which the pluralist believes falsely—in which she is a non-pluralist—is a world in which she uses a different method than *M* in addition to *M*, and thus uses a different total method than *M*, since the non-pluralist in those worlds must also use another method in addition to *M* to judge that at least one arithmetically sound theory is false. Therefore, the pluralist's method is not the same in any world in which she believes falsely. Therefore, the pluralist's beliefs are safe.

Note why I assumed being a pluralist, and not pluralism. It is hard to see how a method just any given individual follows could merit this defense of (ii) in application to foundational mathematics; at least the method the pluralist *prima facie* follows seems to help us here.

But this argument fails once we think about the method many of us actually follow in believing foundational theories. Arguments for axioms involve what we might very well call philosophical considerations. At the very least, they are arguments for mathematical axioms that contain at least some premises that themselves are not mathematical axioms. Such premises, or premises for those premises in turn, or somewhere not far along the finite chain of premises, seem to me to be aptly called 'philosophical'. The way many of us come to believe in the truth of these axioms is on the basis, I claim, of some amalgam of plausibility judgments plus deference to others. Perhaps the most informative thing to be said about this method is that it involves the weighing of plausibility judgments in light of a variety of arguments and principles, resulting in plausibility judgments, perhaps different from those before or with different degrees of plausibility than before, mixed in with deference to others to some degree. (One might call this *reflective equilibrium*, but the term 'reflective equilibrium' sometimes is reserved for something more specific). These points seem to follow from the Permeation of Philosophy, depending upon how strenuously we interpret 'permeate'.

So, while a pluralist believes the biconditional that a given theory is true if and only if it is consistent and arithmetically sound, the right-hand side of this biconditional does not tell the full story of the typical committed pluralist's thinking. For one, it does not explain her belief in "orthodox" theories: ZFC, to fix ideas. While she expands her beliefs beyond ZFC, she likely came to believe ZFC before endorsing pluralism, and likely due to some combination of deference or weighing plausibility judgments. Moreover, it seems important to explain why she holds theories such as ZF~C* that we use the fact that she finds it plausible that every consistent (or arithmetically sound) theory is true for this or that philosophical reason. The philosophical argument she would endorse for pluralism is relevant to explaining why she believes ZF~C*, and therefore is a part of her method for believing it. Therefore, I would not say the typical pluralist follows only *M* in believing foundational axioms. In fact, it seems to me we can be convinced a pluralist follows only *M* only if we are dealing with a pluralist of a rather unreflective sort: one who believes just arithmetically sound theories but who is unable to give a principled reason as to why. I suggest that, at best, a pluralist of a rather unreflective sort may have safe beliefs by using *M*. Other pluralists, I suggest, follow an amalgam of *M* and philosophical methods in forming their belief in foundational axioms. They follow *some* method that includes how they detect arithmetically sound theories *plus* whatever methods, or submethods they use to believe orthodox theories, *plus* whatever methods, or submethods, they use to believe in the truth of non-orthodox arithmetically sound theories, which would produce a bevy of plausibility judgments that smack of philosophy. To summarize this, I will say they follow *an M+*. I will not commit to there being just one such *M+*: the individuation of methods is a tricky matter, especially when dealing with plausibility judgments (Clarke-Doane agrees, as I will note in Section 7).

These are contentious claims. Some think we use a faculty of intuition of mathematical objects, or grasp the content of mathematical concepts or conventions, or some other such thing, as our actual and nearby counterfactual method. However, that we follow an *M+*, I take it, is something approaching the default position in contemporary philosophy of mathematics, and at the very least is a position Clarke-Doane seems to endorse in endorsing the Permeation of

Philosophy. (Clarke-Doane's discussion in Chapter 2 (2020a) nicely capitulates the default nature of this position).

Since we follow *an M+*, not only does the strategy given above for making pluralists' beliefs safe fails, but it also makes it difficult to establish (ii). It seems quite tricky to look at the plausibility weighing we engage in and decide that, in one world I follow this *M+* but in another I follow that *M+*. It is even trickier to do this in a uniform way for all pluralists to establish the safety of all of their beliefs. There is a simple way: we could decide that there is just one *M+*. But then that risks pluralists' beliefs being *unsafe*. Suppose we type mathematical method broadly and decide there is just one *M+*: the amalgam of plausibility weighing constitutes a single method. For our beliefs to be unsafe, the negations of both (i) and (ii) above must both hold: my beliefs must be both unstable and there must be nearby sameness of method. So, it may very well be that pluralists' beliefs are unsafe given (i) is false since it may very well be that pluralists follow just one *M+*. And (i) very well may be false, given the plausibility of the interpretations of nearby non-pluralists that only strong metasemantics can override. And notice these points apply to just about anyone: if pluralists use just one *M+*, so do almost all of us.

Clarke-Doane, in his replies to critics in a symposium on (2020a), objects to an argument that might seem similar to mine, but which is not my argument. He says: "our belief that *P* may be safe *even if our belief in a philosophical theory that implies that P is unsafe*" (2023, p. 822). That is: if someone believes *p* and believes a philosophical theory *q* that, together with other things she believes, implies *p*, it does not follow one's belief that *p* is unsafe given her belief in *q* is unsafe. Using Clarke-Doane's example from (Clarke-Doane 2023): one could believe there is a squirrel in the yard using perception and also believe in a mereological theory which, together with existence of an arrangement of squirrel-making and yard-making simples, implies there is a squirrel in the yard; the possible unsafety of one's belief in the mereological theory does not bear on the safety in one's belief in the squirrel in the yard.

I am not making the inference Clarke-Doane correctly challenges. My claim is that, when we think about the methods a pluralist uses in the actual world to believe both orthodox theories, it is the weighing of plausibility judgments which explains why she believes what she does. A pluralist, except those of an unreflective sort, who believes ZFC and ZF~C* does so, I claim, on the basis of the plausibility weighing that smacks of philosophical considerations. Unlike perception of everyday objects, our beliefs in the foundations of mathematics do not in fact rely on some philosophically neutral, or even mostly neutral, faculty like perception we can cleanly separate from philosophical considerations. We defer to mathematicians or philosophers, who give their own pastiche of plausibility considerations that smack of philosophy, or we indulge in these considerations ourselves.

7. Concluding Upshots

Let me draw some upshots from the above arguments. I argue above that, unless one endorses strong metasemantics, expertise threatens stability. If, in addition, an individual expert follows just one *M*+ in the actual world and nearby worlds, it follows that expertise not only threatens stability but also threatens safety unless strong metasemantics is endorsed. This is an interesting, even curious, result.

This suggests an argument for safety of belief of experts given pluralism is true, the third and final strategy I consider for defending stability and hence safety given pluralism, which goes as follows. Expertise threatens safety, because experts use just one *M*+ and expertise threatens stability. Experts in a legitimate, scientific subject area such as mathematics are to be interpreted as having safe beliefs. One reason, perhaps, is that unsafety undermines all-things-considered justification, and experts in a legitimate subject area have all-things-considered justified beliefs. A more schematic reason one could insert and develop is that unsafety is an epistemically bad property of beliefs that experts in a legitimate scientific subject matter do not have. To interpret experts as having safe beliefs, we should interpret them as not refusing to believe, and not denying, true arithmetically sound theories when understanding and uttering sentences syntactically in conflict with true arithmetically sound sentential sets. So, experts' beliefs are stable, and *a fortiori* safe. This argument can be modified to talk about only some subset of experts, depending upon how plausible one thinks each premise is in application to a given subset of experts. (For example, you might think only some experts have all-things-considered justified beliefs). Call this the *charity to safety argument*, as it is based on a principle of charitable (perhaps also paternalistic?) interpretation toward the epistemic good standing of experts.

I myself do not find the charity to safety argument very plausible, and it needs more spelling out. I register it because it is yet another way to argue for charitable interpretation of mathematical belief in a way relevant to the considerations we have been entertaining so far, and so I think it is worth mentioning. Perhaps it also articulates some philosophers' intuition that mathematicians' beliefs are safe.

This brings us to another pair of upshots. Clarke-Doane brings up the safety of pluralism in the course of his investigation of the *Reliability Challenge:* as he interprets it, the challenge inspired by (Benacerraf 1973) and (Field 1989) that (1) our mathematical beliefs are unjustified if we cannot explain their reliability and (2) certain forms of mathematical realism leave us unable to explain our mathematical beliefs' reliability. Safety, he suggests, provides the best interpretation of the reliability to be explained: on his view, the concept of safety is the only viable option we have for interpreting the Reliability Challenge among the available interpretations of the Reliability Challenge. He notes that unsafety is often thought of as undermining the justification of our beliefs. Given his views of the unsafety of mathematical beliefs given objectivism and their safety given pluralism, he suggests mathematical realism paired with objectivism cannot meet the Reliability Challenge while realism paired with pluralism can. However, he is explicit that he refrains from endorsing the thesis that unsafety undermines justification. Thus, while he thinks safety is the only viable option for the success of the Reliability Challenge, he demurs from asserting it succeeds. His stated reason is that establishing it requires wading into the *generality problem* in epistemology.[9] Although the problem is usually addressed to reliabilist theories of justification, he thinks of it more generally as a problem facing any thesis about the justification of belief that crucially relies on the individuation of methods. The problem is that, of the large number of methods that fit our behavior and mental states at a time when one's belief is formed, it is difficult to discern in a principled way what method guided the formation of our belief that is relevant to its justification.

What Section 6 above indicates is that the generality problem, so generally understood, raises a problem for the thesis that our beliefs are safe given pluralism, independent of its bearing on justification. Unless we buy strong metasemantics, we will need to individuate methods to defend pluralism's safety. Another upshot of my paper is that, if Clarke-Doane's view that safety is the only viable option for the success of the Reliability Challenge is correct, the Reliability Challenge is in hot water, if not unsound.

[9] See (Conee and Feldman 1998).

In conclusion, we can vouchsafe the safety of our beliefs given pluralism if we in addition adopt a substantive conception of what our beliefs or methods are really like. In a way, this is not surprising. The thesis about the safety of our beliefs is a claim about how we think and talk in our actual and nearby counterfactual practices, not some ideal practice where we believe pluralism is true. Pluralism by itself is a thesis about how things are. It stands to reason, then, that pluralism only bears upon how we think and talk if we have a conception of how we think and talk that matches up with how pluralism says things are. Many philosophers find pluralism plausible because it gives us the best conception of the mathematical universe; Clarke-Doane is one such philosopher.[10] For these philosophers, I suggest it is better to focus on this justification and to set appeals to safety aside.[11]

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[10] Prominently Hamkins (2012). Clarke-Doane cites Hamkins approvingly in (2020a, pp. 162–63) and (2023, pp. 823–24), suggesting that the arbitrariness in privileging one set theory over another is a consideration in favor of pluralism.

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