The Formal Ontology of Boundaries

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Abstract

[0] Moving within a realist perspective, we present a general typology of boundaries based primarily on an opposition between *bona fide* (or physical) and *fiat* (or human-demarcation-induced) boundaries. Cutting across this opposition are further oppositions in the realm of boundaries, for example between: complete and incomplete, enduring and transient, crisp and indeterminate, symmetrical and asymmetrical. The final part deals with formal aspects: two axiomatic theories of boundaries are presented, and the need for both is examined in some detail. The resultant framework is shown to have application above all for our understanding of contact, division, and separation.

1. Introduction

[1] Human cognitive acts are directed towards entities of a wide range of different types. What follows is a new proposal for bringing order into this typological clutter. We shall embrace a broadly naturalistic perspective in the sense that we shall presuppose that the objects towards which human cognition is directed are parts of reality. The theory of objects of cognition should be thus at least consistent with the truths of natural science. We presuppose further that a categorial scheme for the objects of human cognition should be *critical* in the sense that it should recognize that cognitive subjects are liable to error, even to systematic error of the sort that is manifested by believers in Atlantis or in the Pantheon of Olympian gods. Thus not all putative object-directed acts should be recognized as having objects of their own. Linguistic and other forms of idealism, as well as Meinongian theories, which assign to each and every referring expression or intentional act an object precisely tailored to fit, yield categorial schemes which fall short of being critical in this sense. We assume in addition that a categorial scheme should do justice to each sort of object on its own terms, and not attempt too hastily to eliminate objects of one sort in favour of objects of other, more favoured sorts in the manner of physicalistic and other forms of reductionism. What follows is a (partial) categorial scheme that is both critically realistic and non-reductionist. Thus it enjoys some of the benefits of linguistic idealism and physicalism, without (or so it is hoped) the corresponding disadvantages of each.

2. Articulations of reality

[2] The focus of our categorial scheme is the region of *extended entities.* Two sorts of extended entity shall be initially distinguished: *objects*, which are extended in space; and *processes*, which are extended in time. Examples of objects are: John and Mary (or perhaps we might better say: John's body and Mary's body), the Moon, the Earth. Objects are possessed of divisible bulk: they can be divided, in reality or in thought, into spatial parts. Examples of processes are: John's life, Mary's current headache, and the sinking of the Titanic. Just as an object (for example a house) may be conceived as being put together or assembled out of spatial parts (the separate bricks), so also a process (for example a sporting competition) may be conceived as being put together or assembled out of temporal parts (the successive heats). Of course, persons and houses are in a sense extended not only in space but also in time. But they cannot be conceived as being built up out of temporal parts in the way in which for example the running of a race can be so conceived. An object is always entirely present at any moment it exists; a process is not. [1]

[3] But objects and processes do not merely have constituent object- and process-parts. They also have boundaries, which contribute as much to their ontological make-up as do the constituents they comprehend in their interiors. Our suggested categorial scheme will thus recognize, in addition to objects and processes, also what we shall call the *outer boundaries* of objects and processes, which we can think of as the infinitely thin extremal slices of such entities in space and in time. The outer boundary of you is (roughly speaking) the surface of your skin. The outer boundaries of processes can be divided into *initial* and *terminal* boundaries, respectively (for example the beginning and the ending of a race).[2]

2.1. Fiat vs. Bona Fide Articulations

[4] What, now, of inner boundaries (the boundaries of the interior parts of things and processes)? Imagine a perfectly homogeneous object, for example a spherical ball made of some perfectly homogeneous prime matter. There is a sense in which no boundaries can be discerned within the interior of such an object: for it seems that the possession by an object of genuine inner boundaries presupposes either some interior spatial discontinuity [3] or some qualitative heterogeneity (of material constitution, colour, texture, electric charge, etc.) among the parts of the object. Thus there are genuine two-dimensional inner boundaries within the interior of John's body in virtue of the qualitative differentiation of the body into organs, cells, etc. There are also genuine one-dimensional inner boundaries discernible on the surface of John's body in virtue of the wrinkles and edge-lines around his warts, eyes, mouth, surgery-scars, etc.

[5] It is clear, however, that we do sometimes speak of inner boundaries even in the absence of corresponding spatial discontinuities or intrinsic qualitative differentiation. Examples are: John's waist and the equator, [4] and if zero-dimensional boundaries are allowed then also: the North Pole, the midpoint of the sun, the centre of mass of John's body. Even in relation to the perfectly homogeneous sphere we can talk sensibly of its upper and lower hemispheres, etc.

[6] Analogously, we can distinguish also two sorts of inner boundary of a process. Examples of genuine inner boundaries -- corresponding to spatio-temporal discontinuity or to intrinsic qualitative differentiation -- might be: the point in the flight of the projectile at which it reaches its maximum altitude and begins its descent to earth, the point in the process of cooling of the liquid at which it first begins to solidify. Examples of inner boundaries of the second sort might be: the boundary between the fourth and fifth minute of the race, John's reaching the age of three. The entities demarcated by such boundaries are not demarcated from their surroundings through any intrinsic features of the underlying reality.

[7] Let us call genuine inner boundaries of the first sort *bona fide* boundaries, inner boundaries of the second sort *fiat* boundaries. [5] This distinction applies not solely to inner boundaries but also to entities which play some of the roles of *outer* boundaries, too. National borders, as well as county- and property-lines, provide examples of fiat outer boundaries in this sense, at least in those cases where, as in the case of Colorado, Wyoming or Utah, they lie skew to any qualitative differentiations or spatio-temporal discontinuities on the side of reality.

[8] But now, once fiat outer boundaries have been recognized, it becomes clear that the *bona fide*-fiat opposition can be drawn not merely in relation to boundaries but in relation to *objects* also. Examples of *bona fide* objects are: John and Mary, the planet Earth. Examples of fiat objects are: all geographical entities demarcated in ways which do not

respect qualitative differentiations or spatio-temporal discontinuities in the underlying territory. Thus Colorado, Wyoming, the United States, the Northern Hemisphere, etc., are fiat objects, as also is the North Sea, whose objectivity, as Frege writes, 'is not affected by the fact that it is a matter of our arbitrary choice which part of all the water on the earth's surface we mark off and elect to call the "North Sea"' (1884, §26).

[9] Broadly, it is the drawing of fiat outer boundaries in the spatial realm which yields fiat objects. We say broadly, since it seems that there are cases of objects which ought reasonably to be classified as fiat objects whose boundaries involve a mixture of *bona fide* and fiat elements. (Haiti and the Dominican Republic, or the Northern and Southern hemispheres, are examples which spring to mind, but every national boundary will in course of time involve boundary-markers: border-posts, watch-towers, barbed wire fences and the like, which tend in cumulation to convert what is initially a fiat boundary into something more objective). Moreover, there are normally perfectly good reasons, of a non-arbitrary sort, why these and those fiat objects are created rather than others. It was a complex structure of considerations relating to shipping, trade, harbours, climate, markets, etc., which led our ancestors to create the fiat object "North Sea" in a way which could not, just as well, have motivated them to create, say, a "Middle Sea" stretching between the Bermudas, the Azores, and Gotland.[6] Fiat objects thus owe their existence not exclusively to human fiat; real properties of the underlying factual material will standardly be involved also.

[10] Lastly, just as the drawing of fiat outer boundaries in the spatial realm yields fiat objects, so the drawing of fiat outer boundaries in the temporal realm yields (simple and not so simple) fiat processes: the penultimate minute-long segment of Mary's headache, the hour-long portion of Mary's life which began 4 minutes ago, Mary's childhood, the Reagan Years, the Millennium, the Second World War, the Renaissance, etc. All of these are perfectly objective sub-totalities within the totality of all processes making up universal history, even though the spatial reach as well as the initial and terminal temporal boundaries of, for example, the Second World War were, like the spatial boundaries of Indiana or Illinois, decided by fiat. Clearly, complex processes of the mentioned sorts can be divided into sub-totalities in different ways and along a number of different dimensions.

2.2. Boundaries and Cognition

[11] All examples of fiat objects mentioned above are cases where proper parts are delineated or carved out (by fiat) within the interiors of larger *bona fide* wholes. While we can reasonably assume that all genuine objects (John and Mary, the planet Earth) are connected, fiat objects may be scattered: they may be such as to circumclude constituent *bona fide* objects within a larger fiat whole. Polynesia is a geographical example of this sort; other examples might be: the Polish nobility, the constellation Orion, the species *cat.* Following Meinong we might refer to such entities as 'higher-order' (fiat) objects (see Meinong 1899). Objects of this sort may themselves be unified together into further fiat objects (say: the Union of Pacific Island Nations). The fiat boundaries to which higher-order fiat objects owe their existence are the mereological sums of the (fiat and *bona fide*) outer boundaries of their respective lower-order constituents. Set theory is a general theory of the structures which arise when objects are conceived as being united together in this way on successively higher levels *ad libitum*. This theory is of course of considerable mathematical interest. It is however an open question whether there is any theoretical interest attached to such *ad libitum* unification from the perspective of a realist ontology. For the concrete varieties of higher-order fiat objects which in fact confront us are subject always, in their construction, to quite subtle sorts of constraints, which vary from context to context: there is no Union of Cheese-Eating Nations.

[12] To set out the constraints on the drawing of fiat boundaries is a task that is by no means trivial. Here, however, we shall content ourselves with considering what might be the justification for awarding the categories of fiat boundaries and fiat objects so crucial an organizing role in our categorial scheme. Are the geographical and political examples, upon which many of our remarks have been concentrated so far, truly of central ontological importance? To see why this question must be answered in the positive, consider what happens when two political entities (nations, counties, or even parcels of land) lie adjacent to one another. The entities in question are then said to share a common boundary. This sharing or coincidence of boundaries is, we want to claim, a peculiarity of the fiat world: it has no analogue in the world of *bona fide* entities. To see this, it may suffice to imagine that two bodies, say John and Mary, should similarly converge upon each other for a greater or lesser interval of time, for example in shaking hands or kissing. Physically speaking, as we know, a complicated story has to be told in such cases as to what happens in the area of apparent contact of the two bodies, a story in terms of sub-atomic particles whose location and whose belongingness to either one or other of the two bodies are only statistically specifiable: as far as the *bona fide* outer boundaries of John and Mary are concerned, no genuine contact or coincidence of boundaries is possible at all. Certainly every genuine kiss or handshake is such as to involve real physical phenomena (relating to surface tension, fluid exchange, etc.) as well as associated real psychological phenomena (of tactual and emotional feeling, etc.). These are, however, merely such as to provide an appropriate real basis for our fiat demarcation. For in comprehending the apparent contact between the two bodies as a *kiss* or *shaking hands,* our healthy common sense carves out from this congeries of physical and psychological processes conventional and neatly demarcated units. Kisses, handshakes and other, similar entities are to this extent creatures of the fiat world.

2.3. Common Sense, Ontology, and Truth

[13] As should by now be clear, many other denizens of what we might call *common-sense reality* are entities whose existence is tied to the existence of a system of fiat boundaries in the sense described above. [7] Fiat boundaries may be more or less ephemeral. One important motor for the drawing of ephemeral fiat boundaries is perception, which as we know from our experience of Seurat paintings has the function of articulating reality in terms of sharp boundaries even when such boundaries are not genuinely present in the autonomous (which is to say mind-independent) physical world. Fiat boundaries, however, come to be drawn also in virtue of the groupings and refinings of reality which are involved in other cognitive phenomena, such as our use and understanding of natural language. We apprehend the world as consisting of (fleets of) ships, (pairs of) shoes and (ounces of) sealing wax, as well as of (variously demarcated spurts of) bombings, butterings and burnishings, and in each case fiat boundaries are at work in articulating the reality with which we have to deal.[8]

[14] The way in which natural language contributes to the generation of fiat boundaries may also be illustrated by the opposition between mass nouns (such as 'water') and count nouns (such as 'person'). A hungry carnivore points towards the cattlefield and pronounces 'There is cow over there'. How does his pronouncement differ, in its object, from 'There are cows over there'? Not, certainly, in the underlying real bovine material. Rather, it differs in virtue of the special sorts of fiat boundary that are imposed upon this material in the two cases.

[15] Finally, there are objects (deserts, valleys, dunes, etc.) which are delineated not by crisp outer boundaries but rather by boundary-like regions which are to some degree indeterminate. This is not to say that the ontology we are here expounding is ultimately

vague -- that the fundamental categorial scheme should allow for a distinction between crisp and scruffy (fuzzy, hazy, indeterminate) entities, as some have urged (see for instance Tye 1990). Rather, vagueness is a conceptual matter: if you point to an irregularly shaped protuberance in the sand and say 'dune', then the correlate of your expression is a fiat object whose constituent unitary parts are comprehended (articulated) through the concept *dune.* The vagueness of the concept itself is responsible for the vagueness with which the referent of your expression is picked out. Each one of a large variety of slightly distinct and precisely determinate aggregates of molecules has an equal claim to being such a referent.

[16] Something similar applies to temporal entities: here again we can distinguish crisp and non-crisp *articulations* of a reality that is in itself entirely determinate. Examples of crisp (fiat) articulations are: hours, weeks, months, millenia. Examples of non-crisp articulations: Mary's childhood, dusk, Clinton's campaign, the Renaissance.

3. The problem of contact

[17] At this point we must take a closer look at the fundamental question that arises as soon the ontological status of boundaries is seriously taken into account. A boundary separates two entities, or two parts of the same entity, which are then said to be in contact with each other. How then is this contact to be explained? Take again the case of two adjacent countries, or the case of the boundary separating the sea from the atmosphere. Shall we say, following Brentano, that there are in the latter case *two* boundaries (one of air and one of water) which are exactly co-localized in space and time?[9] Or shall we rather accept Bolzano's "monstrous doctrine" that contact is only possible between one entity with a surface and another without, so that, if water and air are to be in contact, then one or the other (but which?) would have to lack a surface?[10]

[18] We take it that denying the possibility of contact between separate objects or processes would involve too radical a departure from common sense. To be sure, natural language does not distinguish between true topological contact (or connection, as we may also say) and mere physical closeness. We have seen that as far as the bona fide outer boundaries of John and Mary are concerned, no genuine topological contact is possible at all. In general, the surfaces of distinct physical bodies cannot be in contact topologically, though the bodies may of course be so close to each other that they appear to be in contact to the naked eye. This, however, leaves the question open in those cases where the two candidates for contact are not physical objects. At the very least we want to say that every object must be in contact with its complement (i.e., with the entity that results when we imagine this object as having been subtracted from the universe as a whole). But even this is enough to cause problems. This is seen in Peirce's formulation of the puzzle, when he asks: Which colour is the line of demarcation between a black spot and a white background? (1893, p. 98) Similarly, in the temporal realm there arises Aristotle's riddle: At the instant when an object stops moving, is it in motion, or is it at rest? (*Physics,* VI, 234a ff). All of these puzzles serve together to call into question the realist attitude towards boundaries, which have accordingly been assigned to almost total oblivion in the history of metaphysics.

[19] More careful reflection, however, allows us to view the above as the conflict of intuitions concerning the question whether talk of boundaries must always involve a distinction between closed and open entities-i.e., a distinction between entities that do and entities that do not include their boundaries among their constituent parts.[11] We shall argue that the view according to which boundaries exist and yield an open/closed dichotomy (a view that amounts to an ontology based on ordinary topology) is correct

when the relevant varieties of contact involve *bona fide* boundaries. However, fiat boundaries, and the analogue of contact which they involve, call for a different account which dispenses with the open/closed distinction altogether. In fact we shall argue that a combination of these two positions will be needed by any complete account of the formal ontology of boundaries.

3.1. The Open/Closed Opposition

[20] Those who read the demarcation puzzle as a threat to a realist attitude toward boundaries rely -- more or less explicitly -- on an argument that runs like this: (1) Admitting boundaries implies the open/closed distinction. But (2) the open/closed distinction is counterintuitive -- it runs against common sense. (Surely, if we cut an object in half, we are not left with one piece that is closed and another that is not.) Thus (3) we must do without boundaries (and without the open/closed distinction) and regard talk of boundaries as a mere *façon de parler* about other things -- for example (as on standard mathematical treatments of the continuum) about infinite series.[12]

[21] This is the argument we want to reject. To begin with, as we shall show, admitting boundaries does not by itself imply the open/closed distinction; hence the first premiss of the argument is actually false. This will be seen in some detail below, in relation to the theory of fiat boundaries.

[22] Second, the distinction is not by itself at odds with common sense, or so we shall argue; thus the second premiss is false also. Holes, for instance, are bounded from the outside: the boundary of a hole is the surface of its material host.[13]

[23] Third, and more important, the main worry about the open/closed distinction -- that if we cut an object in half, one piece will be closed and the other not -- is grounded on a model of cutting that we find questionable. [14] Topologically, the intuitive feeling that dissecting solids "reveals matter in their interiors" and "brings to light new surfaces," to use Ernest Adams' words (1984: 400), is ill-grounded. Rather, the model we should have in mind, if we wish to understand what happens topologically when a process of cutting takes place, is that of a splitting oil drop. The drop grows longer, slowly but gradually. As it grows, the middle part shrinks and gets thinner and thinner. Eventually the right and left portions split and you have two drops, *each with its own complete boundary.* We have a long, continuous process which suddenly results in an abrupt topological change. There was one drop; now there are two. There was one surface, and this surface eventually separated into two. (Think also of a soap bubble splitting.) There appears to be something mysterious in this process, but the mystery will disappear on a more complete assessment, which will require a step into the territories of kinematics and surface elasticity.[15]

[24] This account reduces the problem of cutting to that of separating two spheres that are connected by one tiny point. But does this really *solve* the problem? Where does the point belong -- to the left sphere or the right one? Our answer is very simple: the point belongs to both -- they overlap. Later we shall explain this better in terms of the notion of a fiat boundary. But even without the formal details pertaining to the latter notion, our explanation can be anticipated. There is a tendency to see the phenomenon of cutting as an intuitively clear process that a boundary-based topological account is incapable of explaining; we are arguing that this is misguided. In particular, the problem of cutting is in fact a highly technical one, and not an intuitive question at all -- it cannot and should not be solved by any innocent pre-theoretical appeal to naive intuitions. There is indeed something deeply problematic about the point of separation, but this is true of every topological change. Consider:

> **1.** Two drops of oil move toward each other until they come into contact. There is a topological catastrophe -- literally -- that now takes place: the topology of the overall configuration they are part of is suddenly altered. Two surfaces merge. *Two* drops become *one.* (Or consider Roy Sorensen's blob (1997). Make it shrink in a continuous, gradual process, ending in an abrupt change. There was a blob -- now it has vanished. There was something -- now there is nothing.)
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> **2.** You drill a hole and break through the other side (of a brick, say). Once again we may speak of a topological catastrophe taking place at the termination of such a process: a sphere becomes a doughnut; the topology of the object undergoes an abrupt, qualitative change. [16] In fact, to make things simpler we may ignore the complications involved in the actual processes of digging or drilling -- processes that involve removal of matter. Just think of a piece of soft plasticine (or a mushy blob) through which you make a perforation just by slowly pressing your finger: there then occurs a constant elastic deformation which terminates when your finger -- *mirabile dictu* -- breaks through to the other side.
>
> **3.** You can also bring a tunnel into existence by different means -- for instance by gluing. Imagine for instance that the blob starts growing a "finger" somewhere. pose the finger continues its growth until it eventually comes round to meet the main body again, forming a sort of handle. At the instant that it does so, the topology of the object changes: we had a sphere; now we have a torus.

[25] Stories like these all involve something genuinely problematic. And there is nothing wrong with this. Topological mystery is all around us. It is a basic ingredient of our general conceptual scheme. And our point is -- the splitting puzzle and the puzzles of demarcation are just a sign of this very same phenomenon.

[26] Of course, the mystery strikes only insofar as it applies to the ideal domain of continuous, homogeneous bodies. From the perspective of the physical sciences, ordinary physical objects are not continuous and do not have boundaries of any kind -- at least, not boundaries of the sort countenanced by our unreflected view of the world. If the solid bodies of common sense are replaced by intricate systems of subatomic particles, speaking of a body's surface is like talking of the "flat top" of a fakir's bed of nails.[17] Surfaces become merely imaginary entities enveloping smudgy bunches of hadrons and leptons, and their exact shape and properties involve the same degree of arbitrariness as those of any mathematical graph smoothed out of scattered and inexact data. Then the mysterious moments of topology lose all their mystery. You make a tunnel by removing the last molecule or atom. And you split two things when you pull apart the last two molecules or atoms in such a way as to create a gulf between them. No mystery is left. But what follows from this? Not that we should give up talk of boundaries (and topological talk) altogether. For even if we wish to stay close to the ontology of the physical sciences, the fact remains that space is most naturally regarded as an open-ended continuum within which things are free to move and events to occur. Reasoning about the regions of such a coordinate system (as opposed to the entities that occupy them) is typically a matter of continuous reasoning. And if topological talk is deemed inadequate with respect to the entities of atomic physics, one still needs it when it comes to the regions of space and time occupied by the putative objects of ordinary discourse.

3.2. Static vs. Dynamic Demarcations

[27] We do not know exactly how this intuitive explanation can be extended to account also for other forms of dissection -- breakage, for instance. Here, however, a complete picture is not necessary. All we wish to emphasize is that the assumption that dissection will always leave two parts, one of which is closed while the other is open, may be reasonably challenged in those cases where it would seem to yield unreasonable results. Other cases may be less clear, but so are our intuitions. There is a complicated kinematic story to tell in each and every case.

[28] We still have Peirce's puzzle, however. And here the puzzle seems truly problematic, for in this case the demarcation is perfectly static. Take any entity **x**. Does the boundary of **x** (or any piece thereof) belong to **x** or to its complement? Does the boundary inherit the properties -- for example, colour properties -- of **x** or of its complement? There is no kinematic story to tell here. But how can we answer these questions without selecting one or other term at random and thus contravening the principle of sufficient reason?

[29] Again, some cases are clearer than others. For instance, material objects such as stones or bars of soap are in unproblematic fashion the owners of their boundaries -- their surfaces, in fact. Thus, where a complement meets an object of this sort, the complement itself will be open. We may also, as we just saw, argue on the other side that immaterial bodies such as holes are not the owners of their boundaries: these belong to the material bodies that are their hosts. Thus, where the two meet, the complement (host) is closed and the entity (hole) is open. Again, this is unproblematic, since the hole is itself a part of the host's open complement. But even such simple cases may give rise to certain dilemmas. For consider a typical hole -- a hollow, say, such as the Grand Canyon or the interior of an egg-cup. The hole is here in contact with the host; but as we already mentioned above, there are also some regions of its boundary -- corresponding to the opening of the hole facing up towards the sky -- that are not thus in contact. They are "free". Or, if you prefer, they are in contact with an immaterial, airy body. The question then is: Where do we place the boundary corresponding to those regions? Within the hole? Within the complement? Both answers seem unintuitive. And either choice would be arbitrary.

[30] Part of the problem here lies in the trade-off between our theory of objects (and processes) and our theory of boundaries. The latter must explain what it means for two things to be connected, but it does not need to give a full explanation of the underlying metaphysical (or physical) grounds. Thus, whether the boundary between hole and complement belongs to the hole or to the complement -- or whether the boundary is of the fiat or *bona fide* type -- is a question which might be answered specifically by a theory of holes, not by a general theory of boundaries. By the same token, we might admit that Peirce's puzzle and Aristotle's temporal analogues are truly problematic and yet see them as extrinsic to our present concerns. They are questions which pertain to the specific ontology of colour patches, of motion and rest, not to the general ontology of boundaries.

[31] This is not the whole story, however. For consider again the cutting of a solid object. We argued that the cutting does not bring to light a new surface. But, of course, we can *conceptualize* a new, potential surface right there where the cut *would* be. In fact, we can conceptualize as many "potential" boundaries as we like -- two-, one-, even zero-dimensional boundaries. As we have seen, our ordinary description of the world very often and quite naturally makes reference to fiat boundaries of this sort, even in the absence of any corresponding discontinuity or qualitative heterogeneity among the parts of the objects or processes involved. And here the open/closed distinction seems to face a real problem. For in the case of fiat boundaries there is *no fact of the matter* that can port their belonging to one or the other of two adjacent entities. Hence we cannot defer the solution to a theory of the extended entities at issue. The boundary demarcating the right and left hemispheres of a sphere of homogeneous stuff is not only hard to assign to either half. It *cannot* be

assigned, no matter what our theory of spheres might look like. And we cannot simply say that it belongs to neither, treating both halves as semi-open entities. The right and left hemispheres use up the whole sphere by definition -- no boundary can be left as a thin, unowned slice *between* them.

3.3. Coincidence of Fiat Boundaries

[32] It is here that the peculiarity of fiat boundaries comes into play. Fiat boundaries are in a sense potential in that they do not actually separate anything from anything -- they do not mark any actual discontinuity. The categorial distinction between fiat and *bona fide* boundaries is thus absolute. Cutting the Earth in half would not bring the equator to light in such a way that one and the same entity would be transformed from fiat to *bona fide* status. Rather, it would yield two Earth-halves, each enveloped by a closed connected surface, in such a way that the equator itself is gone forever. Fiat boundaries are not the boundaries that *would* envelop the interior parts with which they are associated in case those parts were brought to light by separating the remainder. Wherever you have fiat boundaries in a physical object, you might generate *bona fide* boundaries in the corresponding places. But the former never *turn* into the latter -- at most, they *precede* them in time.

[33] The open/closed account for *bona fide* boundaries is thus not affected by those demarcation puzzles raised by the possibility of drawing fiat boundaries at will. But how do we account for the ownership of fiat boundaries as such? Which hemisphere does the equator belong to? The answer we want to consider is that in this case the right thing to say is that the two hemispheres actually *share* the equator. The equator belongs to both. Or, more precisely, each hemisphere has its own equator, and the two equators *coincide* (i.e., have the same spatial location).

[34] This suggestion draws on Brentano's view, which in fact regards the possibility of coincidence as a distinguishing feature of all boundaries: Brentanian boundaries are *located in* space-time, but they do not *occupy* space-time; they can therefore be perfectly co-located one with another. [18] This means that for Brentano there is actually no need for the open/closed opposition, since topological connection is due to boundary coincidence. However, we do not need to embrace this account as a general theory of boundaries. Thus we may not wish to go as far as saying that if a white and a black surface are in contact with each other, then a white and a black line coincide (see Brentano 1976: 41 and also 1924: 357ff). For we have seen that the demarcation puzzle is not a problem for the general theory of boundaries when the demarcation is due to a genuine qualitative discontinuity (a *bona fide* boundary). Rather, we want to regard Brentano's theory as a theory of what goes on when two (potential) parts of an actual entity are separated by fiat. It is when it comes to the notion of contact induced by fiat boundaries that coincidence relations become relevant. We can still speak of the equator as a single thing. But, strictly speaking, such a thing is to be recognized as being made up of two perfectly coinciding fiat boundaries bounding the Northern and the Southern hemisphere, respectively.

4. Formal developments

[35] We then have two complementary boundary theories. According to the first, more classical theory, Bolzanian in spirit, contact is only possible between two entities one of which is open and the other closed in the relevant area of contact. The second family of boundary phenomena (fiat boundaries) turns on a contrary insight, due to Brentano, according to which what is above all characteristic of a continuum is the possibility of a

coincidence of boundaries. The two theories are not completely in disagreement. For instance, both *bona fide* and fiat boundaries arguably share a fundamental property: they are ontologically parasitic on (i.e., cannot exist in isolation from) their hosts, the entities they bound. This is a common feature that an overall theory of boundary phenomena should emphasize. On the other hand, the two theories yield different notions of contact, and so should be kept distinct. In this final section we shall attend to the task of providing a precise formulation of the theories, starting with their common core and moving then to the two needed supplements.

4.1. The Core Theory

[36] The fundamental ontological property of boundaries was given a clear formulation by Brentano himself (who in turn elaborated on Aristotle's sketchy remarks in the *Physics* and the *Metaphysics*): if something continuous is a mere boundary then it can never exist except in connection with other boundaries and except as belonging to a continuum of higher dimension (see Brentano 1976 part I). There are, in reality, no isolated points, lines, or surfaces. Boundaries are in this respect comparable to universal forms or abstract structures (for example the structure of a molecule as this is realized in a given concrete instance), as also to shadows and holes.[19] This must be said of all boundaries, including those which possess no dimension at all, such as spatial points and moments of time and movement: a cutting free from everything that is continuous and extended is for them, too, absolutely impossible. (There is no death without life; no reaching of the top without ascension.)

[37] All the mentioned types of entities share further the fact that they license certain sorts of ontological inference (*if* there is a boundary/ structure/ hole/ shadow having these and those properties, *then* there is a host having these and those properties). We cannot infer to any specific host, however. Thus it cannot be said of any definite continuum that a boundary is dependent on *it:* that which a boundary is dependent on can be designated rather only via a general term: what is required by a boundary is, Brentano says, "not this or that particular continuum, but any continuum of the appropriate kind" (1933: 56; translation corrected). For while no boundary can exist without being connected with a continuum, "there is no specifiable part, however small, of the continuum, and no point, however near it may be to the boundary, which is such that we may say that it is the existence of *that* part or of *that* point which conditions the boundary." In short, the continuum is specifically dependent on its boundary, but the boundary is not in this same sense dependent on its continuum; it is only generically so.[20]

[38] It is of course impossible to do justice to these distinctions without resorting in some way to modal notions. However, we shall attempt in what follows to embed the dependent nature of boundaries at least into a basic non-modal mereological (more generally, mereotopological) framework. Our aim will be illustrative, so we shall not be too concerned with the question of what sort of formal mereological theory is most adequate for this purpose. We shall, however, try to be rather specific as concerns the question of how such a mereological background can be integrated with a theory of boundaries (of the *bona fide* and fiat sorts, respectively).

4.2. Mereology

[39] For simplicity, we shall assume a standard extensional mereological framework constructed around the primitive *is a part of,* which we symbolize here by means of '**P**'.[21] (Intuitively, we take '**P(x,y)**' to be true when **x** is any sort of part of **y**, including an improper

part, so that **P(x,y)** will be consistent with **x**'s being identical to **y**.) If we define proper parthood and overlap in the usual way:

DP1 **PP(x,y) := P(x,y) & ¬P(y,x)**

DP2 **O(x,y) := (Ez)(P(z,x) & P(z,y)),**

then the axioms for this mereological background can be formulated as follows:[22]

AP1 **P(x,x)**

AP2 **(P(x,y) & P(y,x)) --> x = y**

AP3 **(P(x,y) & P(y,z)) --> P(x,z)**

AP4 **PP(x,y) --> (Ez)(P(z,y) & ¬O(z,x))**

AP5 **(Ex)(*f*x) --> (Ey)(z)(O(y,z) <--> (Ex)(*f*x & O(x,z))).**

Thus, parthood is axiomatized as a reflexive, antisymmetric, and transitive relation (i.e., a partial ordering) by AP1-AP3. In addition, AP4 ensures that the result of removing a proper part always leaves a remander, whereas AP5 guarantees that for every satisfied property or condition ***f*** (i.e. every condition ***f*** that yields the value true for at least one argument) there exists an entity, the sum or fusion, containing among its parts all the ***f***-ers.[23] This entity will be denoted by ***s*x*f*x** and is defined contextually as follows:

DP3 ***s*x(*f*x) := *i*y(z)(O(y,z) <--> (Ex)(*f*x & O(x,z))).**

With the help of this operator, we can immediately define a corresponding operator for arbitrary products (of overlapping entities): the product of a class of ***f***-ers is simply the sum of their common parts:

DP4 ***p*x(*f*x) := *s*z (x)(*f*x --> P(z, x)).**

Other useful notions are also easily defined. In particular, we shall have use for the following quasi-Boolean operators of sum, product, and complement:

DP5 **x+y := *s*z(P(z, x) v P(z, y))**

DP6 **x · y := *s*z (P(z, x) & P(z, y))**

DP7 **~x := *s*z (¬O(z, x))**

4.3. The Theory of Bona Fide Boundaries

[40] Let us now proceed to the formulation of the basic principles for boundaries. We shall begin with the theory of bona fide boundaries, which effectively corresponds to an ontology based on ordinary, Bolzanian topology; we shall then move on to the Brentanian theory for fiat boundaries.

[41] We shall symbolize the primitive boundary relation by 'B', reading 'B(x, y)' as "x is a (bona fide) boundary for y". We say "boundary for", rather than of, to avoid a too narrow interpretation of boundaries as maximal boundaries. (In general, any boundary for

something is a boundary of some part or parts thereof.) The notion of a maximal boundary of x is then immediately defined, using AP5, as the sum of all boundaries for x:

DB1 **b(x) := *s*z(B(z, x)).**

[42] The basic axioms for 'B' can now be given as follows:

AB1 **B(x, y) --> B(x, ~y)**

AB2 **(B(x, y) & B(y, z)) --> B(x, z)**

AB3 **(P(z, x) & P(z, y) --> (P(z, b(x · y)) <--> P(z, b(x)+b(y))).**

Equivalently, we could set:

AB1' **b(x)= b(~x)**

AB2' **b(b(x)) = b(x)**

AB3' **b(x · y) + b(x+y) = b(x) + b(y).**

These correspond to the standard axioms for topological boundaries. In fact, if we define the operator for topological closure in the obvious way, as always yielding the sum of an entity with its maximal boundary:

DB2 **c(x) := x+b(x),**

then AB1-AB3 (AB1'-AB3') are easily seen to be tantamount to (the mereologized version of) the familiar Kuratowski axioms:

TB1 **P(x, c(x))**

TB2 **P(c(c(x)), c(x))**

TB3 **c(x+y) = c(x) + c(y).**

[43] We thus have a straightforward reformulation of much of standard topology based on mereology instead of set theory. In particular, AB1 assumes boundaries to be always symmetrical, in the sense that every boundary of an entity is also a boundary of the entity's complement -- if that complement exists. This ensures that boundaries are always connected to the things they bound, where connection is defined as follows:

DB3 **C(x, y) := O(c(x), y) v O(c(y), x).**

Accordingly, if we define adjacency as external connection, i.e., connection without overlap:

DB4 **A(x, y) := C(x, y) & ¬O(x, y),**

and if we define closed and open entities in the obvious way:

DB5 **Cl(x) := (z)(B(z, x) --> P(z, x))**

DB6 **Op(x) := (z)(B(z, x) --> P(z, ~x)),** then we immediately infer from the above that two entities can be adjacent only if they are not both closed or both open:

TB4 **A(x, y) --> (Cl(x) --> ¬Cl(y)) & (Op(x) --> ¬Op(y)).**

Thus, contact between two closed entities is not possible if contact is understood in terms of connection. The contact between John and Mary when they shake hands or kiss is something which falls outside the orbit of topology; it requires a different (e.g. metric) account.

[44] Here is a list of further theorems that can be proved from AB1-AB3:

TB5 **C(x, x)**

TB6 **C(x, y) --> C(y, x)**

TB7 **B(x, y) <--> (z)(P(z, x) --> B(z, y))**

TB8 **P(x, y) --> (z)(C(z, x) --> C(z, y))**

TB9 **(x)(*f*x --> B(x, y)) --> B(*s*x(*f*x), y).**

[45] The last two of these theorems are especially noteworthy. TB8 highlights the main connection between mereological and topological notions. There are mereotopological theories which also assume the converse of TB8, with the effect of reducing mereology to a part of topology.[24] By contrast, the possibility that topologically connected entities bear no mereological relationship to one another leaves room for a much richer taxonomy of basic mereotopological relations and is therefore preferable

[46] At this point, we can get closer to a standard topological structure in various ways by strengthening the set of relevant axioms as desired. In particular, we obtain a structure corresponding to that of a topological space in the usual sense (modulo the mereological rather than set-theoretical basis) by imposing the analogues of the usual closure conditions:

AB4 **(Cl(x) & Cl(y)) --> Cl(x+y)**

AB5 **(x)(*fx* --> Cl(x)) --> (z=*p*x(*f*x) --> Cl(z)),**

or, equivalently:

AB4' **(Op(x) & Op(y)) --> (z=x·y --> Op(z))**

AB5' **(x)(*f*x --> Op(x)) --> Op(*s*x(*f*x))**

(In AB5 and AB4', the consequent is in conditional form due to the need to take account of the absence of a null individual.)

[47] This gives us a basic reformulation of standard topological ideas which we take to provide an adequate account of the theory of contact yielded by *bona fide* boundaries. We now wish to go further and capture the Aristotelian-Brentanian idea that boundaries are "parasitic" entities. This thesis -- which stands opposed to the ordinary set-theoretic conception of boundaries as, effectively, sets of points, each one of which can exist though all around it be annihilated -- has a number of possible interpretations. One general statement of the thesis would assert that the existence of any boundary is such as to imply

the existence of some entity of higher dimension which it bounds. Here, though, we must content ourselves with the formulation of a simpler thesis, to the effect that every boundary is such that we can find an entity which it bounds and which is such as to have interior parts.[25] To this end, we define the relational predicate of *interior parthood:*

DB7 **IP(x, y) := P(x, y) & ¬O(x, b(y))**

we define also, for convenience, the predicate *is a boundary:*

DB8 **Bd(x) := (Ey)B(x, y)**

We can then write:

AB6 **Bd(x) --> (Ez)(B(x,z) & (Ew)IP(w,z))**

[48] This is not very strong, however. For as it turns out, we always have **B(x,y) --> B(x,y+w)** for any arbitrary w that is separate from (i.e., not connected to) the closure of **y**. Thus AB6 is satisfied by choosing **w** open (so that **IP(w, w)**) and setting **z** equal to the scattered object **x+w**, which trivializes the thesis.

[49] A dependence thesis of the required strength must impose on **z** in AB6 at least the additional requirement of being self-connected (being all of a piece). This predicate can be defined in agreement with ordinary usage, according to which an entity is connected if does not amount to the sum of two disconnected parts:

DB9 **Cn(x) := (y)(z)(x=y+z --> C(y, z))**

We can now amend AB6 to the following thesis affirming, for connected boundaries, the existence of connected wholes which they are the boundaries of:

AB6' **(Bd(x) & Cn(x)) --> (Ez)(Cn(z) & B(x,z) & (Ew)IP(w, z))**

[50] AB6' is still too weak, if we wish to capture the intuition to the effect that boundaries in the real material world are boundaries of *things*. For we then require at least the further requirement to the effect that the entity **z** in question is the object bounded and not its complement. By AB1 every boundary behaves symmetrically in relation to the object and its complement. As we have seen, however, from the perspective of common sense the boundary of, say, this stone is much more intrinsically connected to the stone than it is to the rest of the universe. To capture this notion formally would require an adequate formal account of things, which we can characterize briefly as three-dimensional material entities which are at the same time maximally connected. Thus John's (undetached) arm is three-dimensional and material, but it is not a thing; and similarly the scattered whole consisting of your arm and this pen is three-dimensional and material; but it, too, is not a thing (see Smith 1997). More generally, where ***f*** is any condition, we define the notion of an '***f***-component', or maximally connected ***f***. For values of **x** such that **Cn(x)** we set:

DB10 **mcf(x) := *s*y(*f*y & Cn(y) & P(x, y)).**

The ***f***-component of **x** is the maximal connected ***f*** containing **x**. We can then prove:

TB10 **z = mcf(x) --> (y)((*f*y & Cn(y) & P(z,y)) --> y = z).**

[51] Components are, if one will, those natural units from out of which the world is built (see Smith 1991). Such natural units can be found not only in the realm of three-dimensional materal things, but also, for example, in the temporal dimension (salutes,

weddings, lives, are natural units in the realm of processes). To deal with these matters, here, however, as also with the concepts of dimension (edge, surface) and with the relations between natural units and their underlying stuffs, all of this would lead us too far.

4.4. The Theory of Fiat Boundaries

[52] We shall now orient ourselves around the theory put forward by Brentano. In contrast to the classical topological account, this theory leaves room for the possibility that certain boundaries be *asymmetrical* (so that we might in certain circumstances talk of 'oriented boundaries'). That is, certain boundaries may, on this view, be boundaries only in certain directions and not in others. Brentano takes this to hold for all boundaries but, as we said, we shall embrace it only for fiat boundaries.

[53] The Brentanian theory may be formulated by taking as primitive the concept of *coincidence.*[34] This is to be understood intuitively as a relation that obtains between two boundaries whenever they have exactly the same spatial location. However, coincidence, as we shall here use the notion, is also to be understood as the sort of thing that pertains exclusively to boundaries. Extended bodies do not coincide (not even with themselves); nor do they coincide with the spatial regions they occupy. Other sorts of coincidence may be contemplated, thus for example of *the road from Athens to Thebes* with *the road from Thebes to Athens*, of *Bill Clinton* with *the President of the United States*, of the *mind* and its *brain*, of *this clumsily carved statue* and *this lump of bronze*. Here, however, potential generalizations of the theory of coincidence along these lines are left out of account.

[54] The basic axioms for coincidence -- which we symbolize by '#' -- assert that this relation is symmetric and transitive:

A#1 **x#y --> y#x**

A#2 **(x#y & y#z) --> x#z.**

Thus, coincidence is conditionally reflexive:

T#1 **x#y --> x#x.**

To this we add a further summing principle to the effect that, if two entities coincide with two further entities, then the mereological sum of the first two coincides with the mereological sum of the second two:

A#3 **(x#y & w#z) --> x + w # y + z.**

[55] We may also need to add mixed mereological postulates to guarantee at least weak monotonicity and closure under general sum:

A#4 **P(x,y) & y#z --> Ew(P(w,z) & x#w)**

A#5 **((Ey)(*f*y) & (y)(*f*y --> x#y)) --> x#*s*y(*f*y).**

Thus, in particular, if x coincides with both y and z, then it coincides also with the sum of y and z:

T#2 **(x#y & x#z) --> x # y + z.**

From A#5 we can prove also that, for satisfied predicates ***f*** and **μ**, concidence of all instances implies coincidence of corresponding sums:

T#3 **(x)(y)[(*f*x & μx) --> x#y] --> *s*x(*f*x) # *s*y(μy).**

[56] Finally, we must adopt an axiom to the effect that parts of self-coincidents self-coincide:

A#6 **x#x --> (y)(PP(y,x) --> y#y).**

This guarantees that if we now define fiat boundaries as those entities which may enter the coincidence relation:[27]

D#1 **Bd*(x) := x#x,**

then every part of a fiat boundary is itself a fiat boundary:

T#4 **(Bd*(x) & P(z, x)) --> Bd*(z).**

[57] We are now ready to define Brentanian connection -- connection by fiat boundary. The idea is that this form of connection obtains between two adjacent entities whenever their fiat boundaries coincide at least in part. To this end, let us define the relational concept of a fiat boundary for an entity (the fiat analogue of 'B'). Fiat boundaries are necessarily boundary parts -- proper parts of the entities they bound:[28]

D#2 **B*(x, y) := Bd*(x) & PP(x, y).**

We can then define the sort of connection that is induced by fiat boundaries as follows:

D#3 **C*(x, y) := O(x, y) v (Ez)(Ew)(B*(z, x) & B*(w, y) & z#w).**

Note that the difference between this and the notion of connection defined for bona fide boundaries (DB3) comes to light only in the case of adjacent entities. If the boundary through which adjacent entities are connected is a bona fide boundary, then this by definition bounds one entity from the inside and the other from the outside. (See again TB4, which effectively represents the Bolzanian view of external contact.) If by contrast it is a fiat boundary (i.e., a matter of coincident Brentanian boundaries), then each entity is bounded, as it were, by its own fiat boundary.

[58] At this point, we can define the fiat analogues of the fundamental principle of ontological dependence. This is done simply by taking Bd* and B* in place of Bd and B in the formulations (AB6 and AB6') given in the previous section. In particular, the fiat analogue of AB6' becomes:

A#7 **(Bd*(x) & Cn(x)) --> (Ez)(Cn(z) & B*(x,z) & (Ew)IP(w, z)).**

[59] Thus, the theory of fiat boundaries preposes, in an important sense, the theory of bona fide boundaries (through the notions of interior parthood, **IP**, and connectedness, **Cn**). This is reasonable, for we have seen that bona fide boundaries contribute as much to the ontological make-up of objects and processes as do their extended constituents. By contrast, the demarcations induced by fiat boundaries are not grounded in any intrinsic features of the underlying reality, and correspond only to cognitive phenomena such as those induced by our use and understanding of natural language. Fiat boundaries are in this sense superficial only.

5. Concluding remarks

[60] We need not go any further in the detailed formulation of the theory. Our task was to give an indication of the sort of formal machinery that is involved in the development of a general theory of boundaries, and distinguishing fiat from bona fide boundaries. For this, the above sketchy outline should suffice.

[61] Let us conclude by emphasizing again the two main points of our story. We have argued that the notion of a boundary plays a fundamental role in a categorial scheme that aims at being both critically realistic and non-reductionist. True, from the perspective of the physical sciences, ordinary physical objects are not continuous and do not have boundaries of the sort countenanced by common sense. But even if naive boundary talk is deemed inadequate with respect to the entities of atomic physics, one still needs it when it comes to the regions of space and time occupied by the objects of ordinary discourse.

[62] Moving within this realist perspective, we have seen further that the basic typology of boundaries involves an opposition between bona fide (or physical) and fiat (or human-demarcation-induced) boundaries. Many of the problems in analytic metaphysics connected with the common-sense notions of contact and separation can be resolved in an intuitive way by recognizing this bicategorial nature of boundaries. Bona fide boundaries yield a notion of contact that is effectively modeled by classical topology; the analogue of contact involving fiat boundaries calls for a different account, based on the intuition that fiat boundaries do not port the open/closed distinction on which classical topology is based.

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Notes

* Thanks are due to the National Center for Geographic Information and Analysis in

Buffalo, which supported Smith's work on this paper.

[1] This is not uncontroversial, and various authors have defended four-dimensional ontologies in which objects are presented as having temporal parts (see e.g. Heller 1984). Moreover, it is not only objects as we understand them which have spatial parts: thus certain extended accidents of objects, for example a two- or three-dimensional expanse of colour, can be seen as built up out of spatial parts also. See the discussion of such cases in Smith 1997 (under the heading "accidentals").

[2] This classification can be traced back to (Ingarden 1964: 102ff).

[3] See Casati and Varzi 1994, Ch. 6, "The Natural History of Discontinuities".

[4] Of course, since the equator has a mathematical definition, the question of its ontological status is part-and-parcel of the larger question of the existence and status of mathematical objects in reality.

[5]This terminology was first introduced in (Smith 1994, 1995a).

[6]We owe this point to Wojciech Zelaniec.

[7]The theory of fiat boundaries is thus a contribution to the formal theory of the common-sense world of the sort that is set out in (Hobbs and Moore 1985) and (Smith 1993, 1995b).

[8] There are also what we might call negative fiat objects, as in 'John cut a way through the brush', 'John dug a tunnel under the road', and so on. The question can be raised whether all holes are to be counted as fiat objects in this sense, or whether there are any *bona fide* holes. Superficial hollows seem to belong to the fiat category: part of their boundary„that part which demarcates them from the remainder of the complement of their material hosts„is a fiat boundary. By contrast, an internal cavity (such as a bubble inside a wheel of Swiss cheese) has a genuine boundary around the entirety of its circumference, and seems therefore to qualify as a *bona fide* entity. (See Varzi 1996b.)

[9] See especially (Brentano 1976). Further references will be given below.

[10] See especially (Bolzano 1851: section 66). The epithet "monstrous doctrine" is Brentano's (1976: 146).

[11] An entity may include parts of its boundary, but not all of it, and therefore qualify as partly closed and partly open. In the following we shall ignore the complications that arise in such cases and speak of partly open objects as being open *tout court.* We shall restore the strict terminology in the formal developments of Section 4.

[12] This position is well exemplified in recent literature by Tony Cohn and his AI group (whose main concern, however, is not ontology but efficient qualitative reasoning). See e.g. (Randell, Cui, and Cohn 1992: especially 394Ü95). For a survey of related positions see (Varzi 1996a, 1997).

[13] See (Casati and Varzi 1994). Also in the temporal realm it is not hard to think of entities that do not include their own boundaries. For example, the ordinary classiÞcation of event

types (Vendler 1957) may be viewed as introducing a distinction between non-closed processes (activities such as *Mary's swimming*), and closed processes (accomplishments such as *Mary's crossing of the Channel*), depending on whether the relevant boundary, or achievement (*Mary's reaching of the French coast*), is included in the entity to which reference is made. See (Jackendoff 1991) and (Pianesi and Varzi 1996a, 1996b).

[14]The arguments that follow draw on (Varzi 1997).

[15] See (Davis 1993) for some work in this direction.

[16] We may also describe this process as a sort of by-product: one makes a perforation by doing something else„for instance by initially deforming the object to produce a depression, then a deep hollow, and then a deeper and deeper one, until one ends up on the other side. The qualitative difference, the mysterious moment of perforation, marks the terminus of the process. (One can also start from both sides. For instance, one typically digs a tunnel by digging a cave that gets longer and longer until one breaks through to the other side, or by digging two caves that eventually meet and fuse. Think of the British and the French workers finally joining under the Channel a few years ago.)

[17] The phrase is from (Simons 1991: 91).

[18] Brentano's views have been examined in detail by Chisholm in a number of papers (see especially 1984, 1992/3). See also (Smith forthcoming), which provides a detailed formal theory. On the distinction between location and occupation, see (Casati and Varzi 1996).

[19] Brentano even went so far as to identify boundaries as special sorts of universals: "Because a boundary, even when itself continuous, can never exist except as belonging to something continuous of more dimensions (indeed receives its fully determinate and exactly specific character only through the manner of this belongingness), it is, considered for itself, nothing other than a universal, to which as to other universals more than one thing can correspond" (1976: 12).

[20] See (Smith 1992: section 10), for further discussion of this generic dependence of boundaries upon their hosts.

[21] For an overview of classical extensional mereology and its variants, see (Simons 1987).

[22] Here and in the sequel, initial universal quantifiers are to be taken as understood, and variables are to be conceived intuitively as ranging over all entities, spatial or temporal, extended or boundary-like.

[23] (Varzi 1994) provides an extended treatment of the problems here at issue.

[24] These theories go back to (Whitehead 1929). See e.g. (Clarke 1981) for an influential formulation, embraced also by Cohn and his followers.

[25] See (Varzi 1997) and (Smith 1993, forthcoming) for the more general formulation.

[26] Our formalization is based on Chisholm's rich and compressed account in (1992/93) and on Smith's developments in (forthcoming).

27 If coincidence were understood broadly so as to hold of other entities besides boundaries, then '**Bd***' (or '**B***' below) would have to be taken as primitive and the conditional corresponding to the left-to-right direction of D#1 would turn into an axiom.

28 There is a form of *de re* necessity involved in this claim which is lost in the purely extensional definition given in D#2. See (Smith forthcoming) for further details.

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http://www.phil.indiana.edu/ejap/1997.spring/smithvarzi976.html