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ON THE EVIDENCE OF TESTIMONY FOR MIRACLES: A BAYESIAN INTERPRETATION OF DAVID HUME'S ANALYSIS

BY JORDAN HOWARD SOBEL

The evidence provided by a piece of testimony depends according to Hume not only on the reliability of the testifier but on the credibility of the attested matter considered in itself and apart from this testimony for it. Hume holds that the second dependence is of particular importance in the assessment of extraordinary tales that purport to report marvellous things:

> [When] the fact which the testimony endeavours to establish partakes of the extraordinary and the marvellous – in that case the evidence resulting from the testimony admits of a diminution, greater or less in proportion as the fact is more or less unusual. . . "I should not believe such a story were it told to me by Cato" was a proverbial saying in Rome. . . . The incredibility of the fact, it was allowed, might invalidate so great an authority.[1]

To defend Hume's position on the relevance of antecedent credibilities, and to make plain connections with recent discussions, I offer an articulation of it in formal probability-terms. In Laplace's words, uttered without *explicit* reference to Hume, Hume's position on the evidence of testimony is that of "simple common sense,"[2] and, again in Laplace's words, "the theory of probability is really only common sense reduced to calculus".[3] It is certain that Hume did not have Bayes' Theorem or

[1] *Enquiries Concerning the Human Understanding and Concerning the Principles of Morals by David Hume*, Second Edition, ed. L. A. Selby-Bigge (Oxford, 1902) (cited below as "Hume's Enquiries") p. 113. All references to this work are to "Of Miracles," Section X of Hume's *An Enquiry Concerning Human Understanding* (first published in 1748 under the title, *Philosophical Essays Concerning Human Understanding*).

[2] "[T]he probability of . . . the falsehood of the witness becomes as much greater as the fact attested is more extraordinary. Some authors have advanced the contrary. . . . Simple common sense rejects [their] strange assertion . . . " Pierre Simon, Marquis de Laplace, *A Philosophical Essay on Probabilities*, Second Edition, translated from the sixth French Edition by F. W. Truscott and F. L. Emory (New York 1917) p. 114. Also see p. 17. (This essay first appeared in the second edition, Paris 1814, of Laplace's *Theorie analytique des probabilities*. It was "the development of a lecture on probabilities" that Laplace "delivered in 1795." p. 1)

[3] Laplace, op. cit., p. 196.

anything very like it explicitly in mind.[4] Even so, as David Owen has claimed, "Hume can best be seen as applying a proto-Bayesian argument."[5]

In what follows a Bayesian articulation of Hume's views is first set out with special attention to problems posed by the supposed extreme improbabilities of miracles – according to Hume there is a "proof" against every miracle. Next, objections made by Richard Price that are representative of the main objections made to this day to Hume's position are stated and dealt with. And lastly, certain recent experiments are reported in which persons have been found systematically and without signs of confusion or patent irrationality to discount base-rates and prior improbabilities when assessing reports of witnesses. These experiments suggest that an adequate theory of updating may need to be more complicated than the simple Bayesianism of our initial articulation of Hume's position. This possibility is discussed briefly with special attention again to the case of testimony for miracles, and it is maintained that even given certain possible complications Hume's main conclusions regarding the evidence of testimony for *miracles* would be left intact.

A Bayesian Interpretation of Hume's Argument

1. According to Hume, "the evidence resulting from . . . testimony admits of dimunition, greater or less in proportion as the fact is more or less unusual".[6] Confidence in a reporter, and the improbability of facts he reports, are held to pull in opposite directions. Our project is to make all this precise. My first and main proposal is that we take as a measure of

[4] Why is it nearly certain that Hume did not have Bayes' Theorem in mind? Because first, "Of Miracles" was published in 1748, whereas Thomas Bayes' essay did not appear in print until 1763. Second, there is little evidence that Hume ever read Bayes' essay, for though it is cited by Richard Price in a footnote to his dissertation on historical evidence and miracles, a copy of which Hume acknowledged receiving, we have no evidence that Hume followed up this somewhat obscure reference. (Richard Price, *Four Dissertations*, Fifth Edition (1811) p. 290, n. O to p. 229 of the text.) And third and most important, Bayes' Theorem as we know it, complete with places for possible unequal prior probabilities, is not in evidence in Bayes' essay. It is thus, I think, at least misleading to say that "Price explicitly invokes Bayes's theorem" in his dissertation, and that "it is regrettable that Hume's knowledge of Bayes's theorem has gone unnoticed for so long." David Raynor, "Hume's Knowledge of Bayes' Theorem," *Philosophical Studies* 38 (1980) p. 107.

When Hume wrote "Of Miracles" he had no knowledge of Bayes' essay. He may never have had knowledge of it. He certainly never had knowledge of what we know as Bayes' Theorem. It is possible that *Bayes* had no knowledge of this theorem in its full generality. See note 8 below wherein I elaborate on these points.

[5] David Owen, "Hume, Miracles and Prior Probabilities," presented at the 28th Annual Congress of the Candian Philosophical Association, June 11, 1984 I commented on his paper on that occasion.

[6] "Hume's Enquiries," p. 113.

"the evidence that would result for a positively probable piece of testimony", or, equivalently, of 'the credibility of a positively probable piece of testimony' the ratio of probabilities,

$$P[t(S) \& S]/P[t(S)],$$

wherein S *is a proposition affirming some state of affairs* or fact (for example, that the taxicab involved in the accident was blue), *and* t(S) *is a proposition affirming the existence of a piece of testimony to the effect that* S (for example, that the witness in the dock said that the taxicab involved in the accident was blue). The *numerator* above is the probability that the testimony *exists* (or will exist) – that it has (or will) be given – *and is true*. The *denominator* is the probability that it exists. And my claim, to repeat, is that the *ratio* measures the evidential potential relative to S of this possible testimony. Since this ratio is by definition the conditional probability

$$P[S/t(S)],$$

my proposal is that 'the evidence of testimony' is measured by this conditional probability, that is, by the probability of its *truth* given its *existence*.[7]

I stress that P[S/t(S)] is of course *not* always equal to the reverse conditional probability,

$$P[t(S)/S],$$

which is the probability of this testimony's existence given its truth. It is essential to the subject that these two conditional probabilities not be confused, and it is important to an appreciation of much controversy that it is, given various expressions of them in ordinary English, easy to confuse them. Consider for example the words 'the veracity or truthfulness of Jones in relation to the possible fact that the taxicab involved in the accident was blue' or 'the probability that Jones would speak truly about the possible fact that the taxicab involved in the accident was blue'. "Would, given what?" one might ask. "That the taxicab involved in the accident was blue, *or* that Jones said it was blue?". Depending on the answer to this question, the words mentioned express either something of the form P[t(S)/S] or something of the form P[S/t(S)]!

Taking P[S/t(S)] as a measure of the credibility of a possible piece of testimony, it can be seen how this credibility depends not only on the testifier's reliability, but also on the antecedent probability of what would be the fact attested. It can be seen how it depends on these things in precisely the way Hume would have to do.

[7] I am *not* proposing P[S/t(S)] as a measure of 'the potential evidential *bearing* of this testimony on S'. That, I think, is measured by the signed difference, P[S/t(S)] – P(S).

2. *The Bayes-Laplace Theorem for Incompatible Hypotheses,*

if P(e), P(h), and P(~h) > 0, then

$$P(h/e) = \frac{P(h)P(e/h)}{P(h)P(e/h) + P(\sim h)P(e/\sim h)}$$

is an easy consequence of probability-axioms and the definition of conditional probability.[8]

And this theorem has, as an application to testimony, *The Bayes-Laplace Rule for the Evidence of Testimony*, viz.

if P[t(S)], P(S) and P(~S) > 0, then

$$P[S/t(S)] = \frac{P(S)P[t(S)/S]}{P(S)P[t(S)/S] + P(\sim S)P[t(S)/\sim S]}$$

Hume's ideas can be articulated in terms of this rule. This will be apparent after we have stated and applied to his text a formulation of this rule suggested by reflection on a rule once tentatively proposed by Condorcet.

There can be found in writings of Condorcet the suggestion that when it is known that some witness has testified to some event, the probability that he has told the truth is given by the ratio

$$\frac{vv'}{vv' + ee'},$$

[8] Axioms sufficient to this theorem are (i) that probabilities are positive, $P(p) > 0$, (ii) that necessities have probability one, $\Box p \rightarrow P(p)$, and (iii) that probabilities are additive, $\sim\Diamond(p \,\&\, q) \rightarrow P(p \vee q) = P(p) + P(q)$.

Bayes does not explicitly state this theorem or its many-hypotheses generalization. In his essay he deals with a special case in which prior probabilities are equal: I note that, when equal, prior probabilities 'cancel out' in Bayes' Theorem, and that the theorem I have stated has as a corollary that, if $P(h) = P(\sim h)$, then $P(h/e) = P(e/h)/[P(e/h) + P(e/\sim h)]$.

It has been said, and I believe, that "there is no evidence in the essay that Bayes visualized the general problem: so far as the evidence goes he attacked his problem ab initio and not as a particular species of a broader genus." (Edward C. Molina, "Some Comments on Bayes' Essay," *Facsimiles of Two Papers by Bayes*, ed. W. E. Deming (1940, republished New York 1963) p. lx.) In the problem Bayes deals with, prior probabilities are equal and thus can be ignored as indeed they are by Bayes: they do not have places in his formulas or calculations.

On the evidence, Richard Price – who added an introduction, notes, and appendix to Bayes' essay, and transmitted the whole to *Philosophical Transactions* in 1763 – seems also to have had no clear conception of what we today term "Bayes' Theorem", *complete with places for possibly unequal prior probabilities.* It is salutary to read Price's criticisms of Hume's views with this in mind. We seem to have in his criticisms, objections to Hume's attention to prior probabilities, made by a mathematician who had no clear conception of the role of prior probabilities in "Bayesian calculations"!

Formulations of what we know as "Bayes' Theorem", complete with places for possibly unequal "priors", were I think first made by Laplace. He puts the general rule in words on pages 15–16 of the work cited above in footnote 2. There is evidence that Laplace had the idea of the rule by 1785. Condorcet, after citing Bayes and Price in connection with methods for finding probabilities of future events given laws of past events, writes that "M. de la Place est le premier qui ait traité cette question d'une manière analytique." *Essai sur l'application de l'analyse à la pluralité des voix*, Paris 1785, "Discours préliminaire," lxxxiii.

wherein, Karl Pearson informs us, "*v* and *e* . . . represent the probabilities of the truth and falsity of the event and v' . . . the probability that the witness confirms the truth and e' the probability that he does not."[9] I note that $e = (1 - v)$, and, since it is known that the witness has testified to the event, that $e' = (1 - v')$. Isaac Todhunter, taking some liberty, writes that Condorcet gave the formula,

$$\frac{pt}{pt + (1 - p)(1 - t)},$$

wherein "*p* is the probability of the event itself . . . *t* . . . the probability of the truth of a certain witness."[10] Condorcet was in the end not entirely happy with his formula for the probability of the truth of given testimony. He came to feel that it was apt to lead "to results too far removed from those given by common reasoning."[11] He was particularly concerned with problems concerning the probable truth of reports of numbers drawn in lotteries. But notwithstanding Condorcet's misgivings, and the rather chequered history of his formula,[12] his formula admits of an interpretation – albeit one different from Condorcet's and from that of others who have used and discussed it – under which interpretation it is a theorem of probability theory and is indeed "just the formula" in terms of which to read Hume.[13]

[9] Karl Pearson, *The History of Statistics in the 17th and 18th Centuries*, ed. E. S. Pearson, London 1978, pp. 459–60. "Lectures . . . given at University College London during academic sessions 1921–1933." (Title page.)

See Marie-Jean-Antoine-Nicolas Caritat, Marquis de Condorcet, "Mémoire sur le calcul des probabilités," section five which is titled "Sur la probabilité des faits extraordinaires," *Histoire de l'academie royale des sciences*, 1783, pp. 554–555. (Published in 1786.) Condorcet there writes:

> Supposons maintenant que *u* & *e* representent les probabilitiés de la vérité d'un événement extraordinaire & de la fausseté du même évènement; & qu'en mêmetemps u' & e' expriment la probabilité qu'un témoignage fera ou non conforme à la véritel, & qu'un témoin ait asureé de la vérité de cet évènement."

(I note that Pearson uses '*v*' where Condorcet uses '*u*'.) Also relevant is section six, "Application des principes de l'article précédent à quelques questions de critique," op. cit., 1784, pp. 454–68. (Published in 1787.)

[10] Isaac Todhunger, *A History of the Mathematical Theory of Probability* (New York, 1965) (being a reprint of the first edition of 1865), p. 400.

[11] Pearson, op. cit., p. 461.

[12] See for example John Venn, *The Logic of Chance*, Third Edition, London 1888, pp. 412n and 421n, and Ilkka Niiniluoto, "L. J. Cohen versus Bayesianism," *The Behavioral and Brain Sciences* (1981) p. 349.

[13] One wonders how Condorcet arrived at his formula. He gives it, as Todhunter observes, "with very little explanation." (loc. cit.) A "derivation" which may well have been Condorcet's own way to the formula can be found in Francis Y. Edgeworth's "Probability", *The Encyclopedia Britannica (Eleventh Edition)* (Cambridge, 1911) volume XXII, p. 383, paragraph 48.

To facilitate our Bayesian interpretation of Hume's argument we recast the Bayes-Laplace Rule: *The Hume-Condorcet Rule for the Evidence of Testimony*,

if P[t(S)], P(S), and P(~S) > 0,

$$p = P(S),$$

and

$$r = \frac{P[t(S)/S]}{P[t(S)/S] + P[t(S)/{\sim}S]},$$

then

$$P[S/t(S)] = \frac{pr}{pr + (1-p)(1-r)},$$

is equivalent to the Bayes-Laplace Rule. (The only principle of probability on which this equivalence depends is one that yields P(~S) = [1 − P(S)].) In this rule, *r* is defined in a way that makes it a measure of *the reliability of the testifier relative to S*: *r* varies as such a measure should according to what are taken to be the testifier's propensities to error and deception in connection with S. Two things matter to that reliability – one thing is how likely it is that the testifier would on this occasion testify that S supposing

Edgeworth begins with a plausible formula for *the probability that concurring independent witnesses* have spoken the truth:

$$\frac{tt'}{tt' + (1-t)(1-t')},$$

wherein *t* and *t′* are *truthfulness measures for these witnesses*. This rule has some initial plausibility. Its final assessment, not undertaken here, would depend on the exact interpretations of the just underlined phrases. For the credibility of a piece of testimony given by a *single* witness, Edgeworth treats 'nature' as if it were a concurring independent witness, and allows all evidence, other than the evidence of this piece of testimony, that bears on the attested fact to determine a truthfulness measure for nature qua concurring witness to this fact!

Regarding this "derivation" it is significant that the formula displayed for the probability that concurring independent witnesses have spoken the truth is plausible only for concurring *independent* witnesses. "The application of probabilities to testimony proceeds upon two assumptions," the second of which is "that the statements of witnesses are *independent* in the sense proper to probabilities." (ibid.) Presumably the intended independence is 'relative to S', and testimonies t(S) and t(S)' are to be independent in this sense if and only if P(t(S)'/[t(S) ∧ S]) = P[t(S)'/S]. But testimony to some fact on the one hand, and what one takes to be other evidence relevant to this fact on the other, are of course often *not* independent for one 'relative to this fact'. *That* they are generally not independent in this way in a community seems, indeed, a condition to its being a *language* community. Edgeworth's 'derivation' of Condorcet's formula is un-Bayesian and seems unsound.

that S did obtain, P[t(S)/S], and the other thing is how likely it is that he would on this occasion misrepresent in *this* way, by testifying to S, supposing that S did *not* obtain, P[t(S)/~S]. These things are involved in the factor *r* in the way required given its role as a measure of the testifier's reliability.

Other words exist for what the factor *r* measures. Indeed, this factor recommends itself as a measure of *the evidence of "the testimony considered apart and in itself,"* the consideration that Hume held was in competition with the improbability of the fact considered apart and in itself, which is (1 − p) in the Hume-Condorcet Rule (the *improbability* of the fact being the *probability* of its *negation*). "The evidence of a possible piece of testimony to S, considered apart and in itself" should depend as *r* does on the likelihood that the possible testifier would testify to S supposing that S obtained, and the likelihood that he would testify to S even though it did *not* obtain.

For a possible derivation of *r* as a measure of "the evidence for a fact S of the existence of testimony, t(S), to it *considered apart and in itself*", one could begin with the observation that this evidence should be what the evidence of the existence of this testimony would be, if the fact, S, were as probable as not, so that its probability made no *difference* to the evidence of existence of this testimony. The derivation could then conclude with the observation that, when P(S) = .5, prior probabilities P(S) and P(~S) are equal and cancel out in the Bayes-Laplace Rule, so that by that rule P[S/t(S)] = r. (It is perhaps noteworthy first that while *r* does not in general equal P[t(S)/S], it does equal this conditional probability in any case in which P[t(S)/S] = P[~t(S)/~S]; and second that this inequality will *hold* if (i) it is certain that there is only one alternative S′ to S (for example, that the taxicab was either blue or green), (ii) it is certain that the testifier will testify either to S′ or S, and (iii) the testifier is considered an equally accurate reporter of S′ and S. Of course often there are many alternatives to S, and P[~t(S)/~S] is much greater than P[t(S)/S].)

According to Hume, the credit to be placed in a piece of possible testimony depends on two things: the evidence of this testimony, considered apart and in itself; and the probability of that fact considered apart from this testimony and in *itself.* In the Hume-Condorcet Rule for the Evidence of Testimony, *r* measures the first of these things, and *p* measures the second. As promised, Hume's general position on testimony has been given expression in formal probability-terms; it has, to use Laplace's words, been "reduced to calculus". What remains is to relate Hume's general position on testimony to the special case of testimony for miracles. This requires some work.

3. According to Hume a miracle would be a violation of a law of nature, and since there is a "proof" for every law of nature, there is a "proof" against every miracle. Furthermore, he tells us that "proofs" give rise to "highest certainties" and "last degrees of assurance."[14] A literal representation in standard probability theory of these ideas would include the rule that if M is a *miracle* then $P(\sim M) = 1$ and thus $P(M) = 0$. But if miracles are absolutely improbable, then the Bayes-Laplace and Hume-Condorcet rules are not applicable to evidence of testimony for miracles: it is a condition for these rules that $P(S) > 0$ only under this condition is $P[t(S)/S]$, a conditional probability that occurs in each of these rules, defined. A simple solution to this small problem could consist in the following rule, which *is* applicable even to absolutely improbable propositions. *A Rule for the Evidence of Testimony for Miracles*:

if $P[t(M)] > 0$, then

$$P[M/t(M)] = \frac{P[M \ \& \ t(M)]}{P[M \ \& \ t(M)] + P[\sim M \ \& \ t(M)]}.$$

There is, however, a further more serious problem with the suggested representation in standard probability theory of Hume's ideas about miracles. It leaves no room for a possibility Hume envisaged and indeed stressed – namely that the falsehood of certain testimony for a miracle might *itself* be a miracle. On the standard analysis suggested above, if M asserts a miracle, then $P(M) = 0$, $P(\sim M) = 1$. But then, if it is positively probable that a certain piece of testimony to M should exist so that $P[t(M)] > 0$, then $P[\sim M \ \& \ t(M)] > 0$, and it *cannot* be a miracle that this piece of evidence should exist though it is false. Furthermore, on the analysis being tested, no testimony for a miracle could have any credibility if if M would be a miracle then $P(M) = 0$ and $P[M/t(M)] = 0$. But according to Hume, though there is a "proof" against every miracle, testimony *can* suffice to establish one if its falsehood would be an even greater miracle, which

[14] "Hume's Enquiries," pp. 114 and 110. It has been written that "Hume . . . continually employs the term 'miracle' . . . to signify anything that is highly *improbable* and *extraordinary*." Richard Whately, *Historic Doubts Relative to Napoleon Buonaparte*, New York 1853, p. 57n. (A putative *reductio ad absurdum* of Hume's critique of the evidence of testimony for miracles, "this work first appeared in the year 1819." p. 8) In fact, however, Hume equates the extraordinary with the marvellous ("Hume's Enquiries," p. 113), and contrasts the miraculous with the *merely* marvellous: "Let us suppose that the fact . . . instead of being only marvellous, is really miraculous." ("Hume's Enquiries," p. 114.) A miracle would be a "violation of the laws of nature", against which there are always full proofs that, if unopposed by counter-proofs, give rise highest certainties for denials of miracles, and make miracles themselves not only highly improbable but improbable *"in the extreme"*. "It's a *miracle*!" said Father Dominic, though "the photocopier by Xerox" was merely marvellous in Hume's terms.

Hume supposed it might conceivably be.[15]

Hume believed in degrees of "proof", in superior and inferior "highest certainties" and "last degrees of assurance", and in the possibility of viewing things as greater and lesser miracles.

> The proof against a miracle . . . is of that *species* or *kind* of proof, which is full and certain when taken alone . . . but there are degrees of this species, and when a weaker proof is opposed to a stronger, it is overcome. (From a letter written no later than 1762.)[16]

'Proofs', according to Hume, give rise to last degrees of assurance *in the absence of 'counter-proofs'*, but not always (Hume might have thought not ever) in the presence of 'counter-proofs', in which cases net *non*-extreme degrees of assurance can be produced. One way to accommodate these ideas of Hume's in an interpretation in quantitative probability-terms is to employ a "nonstandard" probability theory in which probabilities are "hyperreal" ("non-standard real") numbers.[17] That indeed seems the best theoretical environment for an interpretation of Hume's ideas *in mathematical probability-terms*.

For a quantitative gloss on Hume's concept of a miracle, we say that M asserts what would in a person's view be a miracle if and only if M is logically possible and there is what Hume would term a "proof" for this person against M. *And*, availing ourselves of the resources of a nonstandard probability theory, we say that there *is* a "proof" for a person against M if, for this person,

$$P(M) < i,$$

for some positive infinitesimal *i*, and that such an inequality holds for a person if and only if there is, for this person, a "proof" against M and no

[15] Hume did once write that "upon the whole . . . it appears that no testimony for any kind of miracle can ever possibly amount to a probability, much less to a proof." These words can be found in editions of "Of Miracles" prior to 1768. (See David Hume, *An Inquiry Concerning Human Understanding*, ed. C. W. Hendel (Indianapolis 1955) p. 137, n. 11.) But even in these early editions he began the following paragraph with a clear disclaimer:

> I beg the limitation here made may be remarked, when I say that a miracle can never be proved so as to be a foundation of a system of religion. For I own that otherwise there may possibly be miracles . . . of such a kind as to admit of proof from human testimony, though perhaps it will be impossible to find any such in all the records of history.

Hume, in the edition of 1768, changed the misleading words quoted above to "it appears that no testimony for any kind of miracle has ever amounted to a probability much less to a proof."

[16] Ernest Campbell Mosner, *The Life of David Hume*, Second Edition (Oxford 1980) p. 293.

[17] See James K. Henle and Eugene M. Kleinberg, *Infinitesimal Calculus*, Cambridge, Mass. 1979, chs. 1–4, for an introduction to the theory of hyperreal numbers.

"proof" for M. (A positive infinitesimal is a positive hyperreal number that is less than every "real (hyperreal)" number. There is an isomorphism between the real numbers and a subset of the set of hyperreals, and a "real (hyperreal)" number is a member of that subset.)

According to Hume a person views a logical possibility as a miracle if and only if he views it as a violation of a law of nature, and so views it as a *natural impossibility*. We have such views. Hume considers them to be philosophically suspect and incapable of fully face-saving empirical analysis, but he thinks that there are natural and indeed irrepressible views even for enlightened philosophic critics. (There is, he might say, a sense in which "we cannot do without them": though we may not for any theoretical or practical purposes *need* them, we cannot psychologically *avoid* them in our ordinary thinking.) The proposal I am making is that such "views" (scare-quotes in deference to Hume's philosophic suspicions) be accorded distinctive treatment in representations of credence-states. Only such "views" will be assigned positive infinitesimal probabilities. Similarly, I suggest that only "views" of things as *natural necessities* should be assigned probabilities that are "infinitely close" to 1. (For hyperreals n and m, n is "infinitely close" to m if and only if $|n - m|$ is less than some infinitesimal. We shall let '$\simeq$' abbreviate 'is infinitely close to'.)

This treatment accords to 'views' of natural impossibilities and necessities a kind of resilience that seems appropriate. For any positively probable E that does not in a person's view express a natural impossibility, if M does in this person's 'view' express a natural impossibility *against* which there is thus a proof, and *for* which there is *not* a 'proof', then

$$P(M/E) \simeq P(M).$$

'Proofs' when unopposed by 'proofs' are being held to give rise to extraordinary probabilities from which no amount of ordinary conflicting evidence can significantly detract, though they are not absolutely fixed and immune to significant diminution which can be called for by opposing 'proofs'.[18] It is being suggested that the 'presumption', to use Bishop Butler's term, generated by an unopposed 'proof' against what in a person's view would be a miracle and a violation of a law of nature, is to differ not merely in degree from the strong presumption that lies antecedently against most ordinary facts, but in *order* of degree.[19]

[18] When Hume contrasts proofs with probabilities and writes that proofs are "such arguments from experience as leave no room for doubt or opposition," I take him to be contrasting proofs with 'ordinary probabilities' and to have in mind opposition by ordinary, non-proof, arguments from experience. (The quoted words are from "Of Probability" in "Hume's Enquiries." p. 73)

[19] Bishop Joseph, Butler, *The Analogy of Religion*, with an introduction by Ernest C. Mossner (New York: Frederick Ungar Publishing Co., 1961), pp. 147–8.

For the remainder of this section, we take 'M' to express what in a person's view would be a miracle, a miracle *for* which there is *not* a "proof" – that is, M is to be an event against which there is a 'proof' that is *not* opposed by a 'counter-proof', at least not yet. And we take 't(M)' to express the existence of a piece of testimony for M such that for the person in question P[t(M)] > 0. By an application of the Rule for the Evidence of Testimony for Miracles, it follows from conditions imposed above on probabilities of miracles that, for an infinitesimal that is less than or equal to P(M),

$$P[M/t(M)] = \frac{i}{i + P[\sim M \ \& \ t(M)]}.$$

There are two possibilities relevant to this ratio. Either P[~M & t(M)] is not an infinitesimal, or it is one. If it is not an infinitesimal, then

$$P[M/t(M)] \simeq 0,$$ [20]

and everything considered the testimony would lack "all" (more strictly, "nearly all") credibility.

If P[~M & t(M)] *is* an infinitesimal, then the falsehood of the testimony – that it should exist though false, [~M & t(M)] – would *itself* be a miracle, and, supposing that P[~M & t(M)] = i′,

$$P[M/t(M)] = \frac{i}{i + i'}.$$

Everything in *this* case would thus depend on which would be the greater

[20] Take as a premise that P[~M & t(M)] is *not* an infinitesimal, and suppose, for purposes of an indirect argument, that for an infinitestimal *i*,

$$\frac{i}{i + P[\sim M \ \& \ t(M)]} \not\simeq 0.$$

Then, for some non-infinitesimal *n* that is less than 1 and greater than 0,

$$\frac{i}{i + P[\sim M \ \& \ t(M)]} = n,$$

and so

$$P[\sim M \ \& \ t(M)] = i \ [\frac{(1 - n)}{n}].$$

But then P[~M & t(M)] *is* an infinitestimal, which completes the indirect argument from the premise, P[~M & t(M)], to the near inequality, P[M/t(M)] ≃ 0, of the text. (If *n* is a 'real hyperreal' *r*, then [(1 – r)/r] is a 'real hyperreal' and the product i[(1 – r)/r] is an infinitesimal by Theorem 4.1 of Henle and Kleinberg, op. cit., p. 34. If *n* is a 'non-real hyperreal' *h*, then, since *n* is not an infinitesimal, for some 'real hyperreal' *r*: h > r; so [(1 – h)/h] < [(1 – r)/r]; and so i[(1 – h)/h] is smaller than the infinitesimal i[(1 – r)/r], and is thus itself an infinitesimal.)

miracle: that the testimony should exist and be *true* so that $i' = P[\sim M \& t(M)] > P[M \& t(M)] = 1$; *or*, that the testimony should exist and be *false*, so that $i = P[M \& t(M)] > P[\sim M \& t(M)] = i'$. If that the testimony should exist and be *true*, then

$$P[M/t(M)] < .5.$$

If that the testimony should exist and be *false*, then

$$P[M/t(M)] > .5.$$

In this last case in which "the falsehood of testimony would be more miraculous," than would be its truth, and *only* in this case, can testimony for a miracle "pretend to command . . . belief."[21] Hume does not give an example of such testimony. He strongly suspected that there never have been any actual examples – that "it will be impossible to find such in all the history."[22] From absence of even a fanciful example of such testimony we should, I think, conclude that he found it impossible to *imagine* one. But he evidently felt that he had not only not proved that such testimony was quite *impossible*, but that such testimony was in fact a *possibility*, a logical possibility that he was obliged to concede as he unequivocally does.

Richard Price: "the man who . . . stood up to David Hume."[23]

According to Price, "the turning point in Mr. Hume's argument is . . . the principle, that no testimony should engage our belief, except the improbability in the falsehood of it is greater than that in the event which it attests."[24] Price objects to this principle: he maintains "that improbabilities *as such* do not lessen the capacity of testimony to report truth."[25] We may begin by conceding that Hume is committed to the stated principle at least for the special case in which it is given that the testimony at issue exists. For presumably testimony $t(S)$ should engage belief only if $P[S/t(S)] > .5$, and if it is given that this testimony exists so that $P[t(S)] = 1$, then

[21] "Hume's Enquiries," p. 116.

[22] "Hume's Enquiries," p. 127. Hume should have been nearly certain that there has never been such testimony. If $t(M)$ asserts the existence of such testimony to M, then both $P[M \& t(M)]$ and $P[\sim M \& t(M)]$ would be infinitesimals, *and so P[t(m)] itself would be an infinitesimal* and the existence of the testimony affirmed by $t(M)$ would *itself* be a miracle!

[23] Pearson, op. cit., p. 378. Leslie Stephen reviews arguments in George Campbell's *Dissertation on Miracles*, 1762, which book was "long considered to be the ablest reply to Hume," p. 398. Leslie Stephen, *History of English Thought in the Eighteenth Century, Third Edition*, New York 1949, Vol. 1, pp. 398–401. Price's objections, thought more carefully stated than Campbell's, are for the most part the same in outlines and main thrusts.

[24] Richard Price, "Dissertation IV on the Importance of Christianity, the Nature of Historical Evidence, and Miracles," op. cit., p. 234.

[25] Price, op. cit., p. 238.

$P[S/t(S)] > .5$ if and only if $P(S) > P[{\sim}S \,\&\, t(S)]$, that is, if and only if the probability of the event is greater than that of the testimony's falsehood, or the *im*probability of this falsehood is greater than that of the event. It must also be allowed that, speaking somewhat loosely, there is a sense in which 'improbabilities as such cannot lessen capacities of testimony to report truth': they cannot lessen capacities of *testifiers* to report truth. The issue is whether Price's indicting words are true in any sense that is damaging to Hume.

Thinking that he was making a case against Hume, Price could ask us to suppose that it is given, or that we have learned from experience, that *The Toronto Star* reports truth two times in three "across the board" and without regard to antecedent improbabilities of facts reported. Then, Price could observe, even when *The Star* reports a very improbable event, as it is after all in the business of doing, it "communicates the probability of 2 to 1 to the event." "Evidence *generally* right ought to be received as being so, notwithstanding improbabilities by which we have found it not to be affected."[26] "A *given probability* of testimony communicates itself always entire to an event,"[27] and does so quite regardless of the event's antecedent improbability.

These observations are supposed to be an embarrassment to Hume, but in fact they are all quite consistent with his position. If, as is often the case, the credibility of a piece of testimony is given to a person's satisfaction notwithstanding the improbability of the fact attested, then this person need not attend further or again to this improbability in order to satisfy himself regarding the testimony's credibility. If the credibility of this testimony is given to his satisfaction, he need not, to be satisfied, attend further or again to the testifier's reliability either. But then these are merely wordy tautologies. Though the improbability of the fact attested *is* relevant to the credibility of given testimony (as distinct from the reliability of the testifier, or his "capacity to report truth" in the case), when interested in this credibility one is of course not doomed to take into account the improbability of the fact attested not just once, but *again and again*.

Price's main objections depend for the most part on a conceptual confusion. He was, it seems clear, in the grip of that "persistent cognitive illusion" in which credibilities of pieces of testimony are confused with

[26] Ibid.
[27] Price, op. cit., p. 243.

capacities of testifiers to testify truly.[28] For the rest, Price's case against Hume consists of allusions to calculations that he thought would dramatize the supposed error of allowing prior improbabilities to affect credibilities of testimony. He supposed that since almost every fact is antecedently improbable, taking prior improbabilities into account would render useless all testimony, and all *sense* for that matter.

> TESTIMONY is truly no more than SENSE at second-hand: and improbabilities . . . can have no more effect on the evidence of the one, than on the evidence of the other.[29]

Suppose that a reliable reporter has said that the number drawn from some lottery was 79. Price implies that if one were to calculate as Hume prescribes and take into account the great antecedent improbability of this number's being drawn, then, notwithstanding the reporter's reliability, one would not believe him. Let us by spelling out the case in a natural way see in detail that Price was wrong. Suppose possible draws are from numbers 1 through 1,000, and that for each number n in this range, the probability, P(n), that it will be drawn, is .001. Suppose it is certain that the reporter has said of some number or other in this range that it was drawn. Let the "veracity" of the reporter in this case be such that for each n in this range, $P[t(n)/n] = .9$, and by implication $P[\sim t(n)/n] = .1$. And suppose that, whatever number had been in fact drawn, it is as likely that the reporter would have erred in favor of one number as another: for example, had 78 been drawn (perhaps it was), it is as likely that he would have misreported that 79 had been drawn as it is that he would have misreported that 998 had been drawn, and similarly for every pair of possible misreports. I am assuming that the reporter is not particularly prone to misreport in favor of 79, as he would be if he had "some interest in choosing number 79,"[30] and similarly for every other number. Our assumption is that for any two distinct numbers n and n' in the lottery's range, $P[t(n')/n] = .1/999$: the .1 probability of not reporting a number drawn correctly is, we are supposing distributed completely and evenly among the 999 possible misreports of it. What credit, according to the Bayes-Laplace/Hume-Condorcet Rule, would be due to the report that 79 had been drawn?

[28] Persi Diaconis and David Freedman, "The Persistence of Cognitive Illusions," *The Behavioral and Brain Sciences*, 1981, pp. 331 and 332 n. 1. Price does distinguish between "the *capacity* of testimony to report truth" and "the *credit* of testimony," and allow that prior improbability of a reported fact *can* affect the latter. But I think that he did not understand exactly *how* it could affect the latter, and that he was only very *close* to bringing under control the distinction that is necessary and largely sufficient to an appreciation and *acceptance* of Hume's position.

[29] Price, op. cit., p. 240.

[30] Laplace, op. cit., p. 111.

$$P[79/t(79)] = \frac{.001(.9)}{.001(.9) + P(\sim 79)P[t(79)/\sim 79]}$$

$$\begin{aligned} P[\sim 79)P[t(79)/\sim 79] &= P[\sim 79 \,\&\, t(79)] \\ &= \sum_{n:\ 0<n<1000\ \&\ n\#79} P[n \,\&\, t(79)] \\ &= \sum_{n:\ 0<n<1000\ \&\ n\#79} P(n)P[t(79)/n] \\ &= 999[.001\ (.1/999)] \\ &= .001(.1) \end{aligned}$$

$$P[79/t(79)] = .9$$ [31]

The important thing about reports of lotteries, the thing that tends to lend credibility to them, is that there are many ways in which draws can be misreported *and* no reasons favouring one misreport over another so that it is generally *very* unlikely that one should be misreported in any *particular* way: In the above case, for example, though $P[\sim t(78)/78] = .1$, $P[t(79)/78] = .1/999$. In this, reports of lotteries are like reports of the ordinary run of unremarkable possible facts, and *unlike* reports of what would be marvellous, wondrous, or miraculous happenings, especially ones that would be of religious significance. For example, if T is the statement that a bridge hand just dealt but not turned up contains thirteen spades, and O is a statement to the effect that it is of some particular mixed and unremarkable character, then even if for some person $P(T) = P(O)$, it could be that for this person $P[t(T)/\sim T] > P[t(O)/\sim O]$. We realize that persons both enjoy believing marvellous tales, and are prone to relate them with innocent relish. Persons also can have motives for relating miracles that would have religious significance. Persons can have motives for concocting such tales. These observations are made by Hume in Part II of "Of Miracles." Calculating credibilities of reports in a manner that takes into account, as Price would *not* have done, the prior improbability of the draw reported, results, in the kind of case considered, in a credibility that equals the reporter's "veracity" in the case, as Price would have it *do*.[32] Similar considerations explain the credibility of the ordinary run of reports

[31] Laplace gets the same value by a somewhat different calculation. Laplace, op. cit., pp. 109–110. Diaconis and Freedman present a somewhat more general treatment of reports of lottery draws. Op. cit., p. 334 n. 1. Price does not explain why he thinks that Hume is committed to rejecting as incredible all reports of lottery-draws. He does not spell out the calculations that he thinks Hume's position validates.

[32] "The veracity of the witness" is defined in a case if and only if in this case, for any S and S′ such that $P(S)$, $P(S')$, $P[t(S)]$, and $P[t(S')] > 0$, $P[t(S)/S] = P[t(S')/S']$. In the case in the

despite the antecedent improbabilities of many facts that are in the ordinary run reported, which improbabilities are often great. One thing that, on my reading of Hume, here distinguishes what, in a person's view, would be miracles for which there are at least not yet 'proofs', is that *their* improbabilities are to be *extraordinarily* great. They are to have *infinitesimal* probabilities. Another thing that distinguishes many reports of would-be miracles is that these reports are relatively likely even given that they are false.

Hume, in a letter to Price concerning the latter's dissertation on miracles, wrote:

> I own to you, that the Light, in which you have put this Controversy, is new and plausible and ingenious, and perhaps solid. But I must have some more time to weigh it, before I can pronounce this Judgment with Satisfaction to my self.[33]

I know of no evidence that Hume communicated with Price again on this subject. It is likely that on reflection Hume found that he was still satisfied in the main with his argument on "Of Miracles," even though he had nothing useful to say to Price that would help to clarify the controversy and resolve their differences. Hume had good reasons to be satisfied with his argument even in the face of Price's criticisms and even if he was not able to state them in clear and persuasive controverting terms. It is simply not credible that the apparent good sense of Hume's essay should imply the contradictions and absurdities that Price claimed it did. Despite Price's superior mathematics, Hume was I think the more solid thinker, and the better intuitive Bayesian.

Tversky and Kahneman's Taxicabs[34]

Subjects in recent experiments have been found to ignore or discount initial probabilities when coming to conclusions on the basis of testimony. They have been found in some cases to accord to witness's reliabilities what, from a simple Bayesian perspective, would be inordinate

text, the reporter's veracity is taken to be .9. Note that while the reporter's *veracity* is in this case the same as the credibility of his report, it is not the same as his *reliability* in the sense in which I use this term. In this case his reliability relative to 79 is $(.9/[.9 + 1/(10(999))]) = .999889...$ The reporter's 'reliability' relative to 79 reflects not only his 'veracity' relative to 79, but his lack of bias towards 79, and the great improbability of his misreporting 79 if some other number is drawn: r here depends not only on $P[t(79)/79]$ but also on $P[t(79)/{\sim}79]$, which latter conditional probability is, in the case, $\cdot 0001001\ldots$

[33] Raynor, op. cit., p. 105.

[34] A. Tversky and D. Kahneman, "Causal Thinking in Judgment under Uncertainty," *Basic Problems in Methodology and Linguistics*, ed. R. Butts and J. Hintikka (Dordrecht, 1977).

weights. One possible reaction to these experiments would be to think that in them persons tend to make mistakes. Another would be to at least suspect that simple Bayesianism is not a completely adequate theory of rational updating – that simple Bayesianism is insensitive to dimensions of credence-states that are important to the range of rational responses in some cases. After describing experiments of two related sorts, two sorts in which subjects' responses are interestingly *different*, I consider what Hume, speaking in formal probability-terms, might make of these experiments.

Subjects in one set of experiments are told that 85 taxicabs in a town are green and that the rest, of which there are 15, are blue. Subjects are also told that one of these 100 taxicabs was involved in an accident at night. If without having any further information to go on a subject were asked for the probability, expressed as a decimal, that the taxicab involved in the accident was blue, he would of course have to say, ".15." Subjects however, were told more. They were told that a witness to the accident has reported that the taxicab involved in it was blue, that a court has tested this witness for his ability to identify green and blue taxicabs by presenting him with equal numbers of the two kinds, and that in these tests the witness got each colour right 80% of the time. The subjects were then asked for the probability that the taxicab involved in the accident was blue.

One assumes that before being told what the witness said, the subjects would have revealed the following probabilities and conditional probabilities: $P(B) = .15$, $P[t(B)/B] = .8$, and $P[t(B)/{\sim}B] = .2$. (Regarding this last probability, explanations of the case will ensure that, before being told what the witness reported, the subjects are sure that the taxicab was either blue or green, and that the witness had reported either that it was blue or that it was green, so that $P[t(B)/{\sim}B] = (1 - P[t(G)/G])$.) One thus assumes that before being told what the witness said,

$$P[B/t(B)] = \frac{.15(.8)}{.15(.8) + .85(.2)} = .41$$

A simple Bayesian, on receiving the witness's report that the taxicab was blue, would update his probability for its being blue from .15 to .41. The median response of subjects in the experiment was, however, .8, and there was little variation from this value. It is as if, when *updating* initial probabilities for the taxicab's being blue, these subjects in fact *ignored* them and set them aside.

There was another set of experiments. These were like the experiments already described except that subjects, instead of being told what was *the ratio of green taxicabs to blue ones in the town*, were informed of the ratio of green taxicabs to blue ones *involved in accidents* in the town – they were told that *this* ratio was 85 to 15 (and were not explicitly told anything about the other ratio). In these experiments responses varied widely. That is one contrast with experiments of the first set. And, there was another contrast: The median response was .55, and so much closer to the simple Bayesian response of .41, than was the median response of the first experiments.

What might Hume make of these two sets of results? He might of course take the simple hard line and say, "I have never claimed that people are always rational and wise, or that they always proportion their beliefs as they should to the evidence. People are prone to confusions, and to irrational prejudices concerning kinds of evidence. Witness the responses of subjects in those experiments." He could, however, take a more conciliatory line and concede that responses of subjects in these experiments were not necessarily unreasonable. And he might elaborate his position on testimony to make room for this conciliation. He might allow that simple Bayesian assessments are not always mandatory, and even so continue to insist that when testimony is for a *miracle*, proper assessments will need to take into account *their* improbabilities, and do so in much the way that has been described. Let me indicate in somewhat more detail how such a measured response to the base-rate experiments might go.

> Of course (one might say) persons are not always perfectly rational and wise. People are often confused, and are sometimes inconsistent in their beliefs and incoherent in their degrees of belief. Furthermore, people are often *inexact* in their degrees of belief. However, unlike inconsistency and incoherence, inexactness is not a mode of irrationality but rather a natural and rational condition for persons who are variously limited in their capacities to store and process data, and who are possessed of variously imperfect and limited data.
>
> Consider a person who while limited in data and capacities is otherwise perfectly rational. As a simplification, pretend that "logical omniscience" is a part of perfect rationality, and so consider a person who is quite certain of every logical necessity. How might the total credence-state of this person be represented? The main thing to say is that it might be better represented by a many-membered *set* of probability functions than by any single probability function.[35] Suppose it would be. Even so we could speak of the person's probabilities for a propositions – his "singular probabilities" for propositions – meaning by his "singular

[35] This idea can be found in Richard Jeffrey's "Probability and the Art of Judgment," Section 14, *Experiment and Observation in Modern Science*, eds. P. Achinstein and O. Hannaway, forthcoming. Also see Bas C. van Fraassen's "Belief and the Will," *Journal of Philosophy* 81 (1984) pp. 251–2.

probability" for a proposition an average, perhaps a somehow weighted average, of his probabilities for it. A person's "singular probabilities" would thus have both *quantities* and *qualities*, the latter being functions of the distributions of, as well perhaps as the "significance" or relative "weights" of, the probabilities averaged. Qualities of "singular probabilities" would correspond to what some might term the "weights" or "degrees of ambiguity" of evidential bases for propositions,[36] and to the confidence a person had in his various "singular probabilities", displayed perhaps in his readiness to accept bets based on them.

Turning to the special topic that has been before us – the evidence of testimony – certain elaborations and qualifications are now in order. When qualities of relevant "singular probabilities" are equal, assessments of credibilities should proceed as has been maintained. When unequal, 'singular priors' should be discounted only if their qualities are inferior to the qualities of 'singular probabilities' that determine witnesses' reliabilities.[37] They should be ignored or discounted *dramatically* only when they are markedly inferior. One consequence of these last points is that always when testimony is for a *miracle* against which there is a 'proof' that is not opposed by a 'counter-proof', 'singular priors' are fully relevant, since a person's 'singular priors' for and against such miracles will, it seems plausible to maintain, be not only of extreme *quantities*, but of *highest qualities* – probabilities averaged of such miracles, given their unambiguous inconsistency with what one takes to be the natural and necessary order of nature, will be concentrated closely around the average value. Indeed it could be that it is this that marks the most important difference between

[36] See L. Jonathan Cohen, "Author's Response," *The Behavioral and Brain Sciences* (1981) pp. 365–6, and Daniel Ellsberg, "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics* (1961).

[37] Qualities of relevant 'singular probabilities' — qualities of 'priors', and of 'likelihoods' that enter into a reporter's reliability measure — *can* be equal. Suppose, for example, that I know that a well-mixed urn contains 50 white balls and 50 black ones, and that Alice and Betty have observed a random draw from this urn. Suppose that in my view Alice and Betty are in this kind of case equally reliable reporters. Let Alice tell me that the ball drawn was white. And then let Betty contradict her, and tell me that it was black. Shall I after this second report ignore my then prior probability for the ball's being black — ignore, that is, Alice's testimony — and believe to the order of Betty's reliability that the ball was black? Shall I believe (with due reservations) Betty rather than Alice just because Betty spoke last?! Surely not. At this point *qualities* on the one hand of 'prior', and on the other hand of 'liklihoods' that enter into my measure of Betty's reliability, can be (and so far would seem to be) *equal*, and consequent to Betty's testimony in which she contradicts Alice's testimony, I should be back to square one as far as my opinions concerning the draw ball's colour (though I perhaps should reconsider my opinions concerning the reliabilities of Alice and Betty).

ordinary events, and what in a person's view would be a miracle, and that it is not that probabilities for miracles are apt to be of extraordinarily low and infinitesimal quantities, but that they are apt to be extraordinarily 'concentrated' and 'focused', and of highest quality.[38]

What more can be said of cases in which relevant "singular probabilities" are not more or less equal in quality? One possibility is that *no more* can be said than that in these cases it may not be unreasonable to discount "singular priors",[39] or *other* relevant "singular probabilities", depending on which 'singular probabilities' are of lower quality.[40]

Another possibility, however, is that more can be said, and that when qualities are not equal a rational updater will calculate using not only quantities but also qualities of relevant 'singular probabilities' in accordance with suitably ramified Bayesian principles, discounting probabilities of inferior qualities more or less depending on the details and degrees of their inferiorities.

I like the second measured response that I have indicated that Hume might make to the base-rate experiments. I wish I could say more along its lines, in particular, more about the averages that are to yield "singular probabilities", more about assessments and measures of their qualities, and more about calculations that would involve both quantities and qualities of

[38] I believe that the positive part of this idea was proposed by L. Jonathan Cohen in discussion following the presentation of this paper on August 29, 1986 during the Hume Conference held in Edinburgh under the joint auspices of the Institute for Advanced Studies in the Humanities, and the Hume Society.

[39] Cf. Cohen, ibid., p. 366.

[40] In so far as there is legitimate controversy regarding applications of Bayesian principles to testimony, it concerns not the relevance of prior probabilities as such, but the relevance of probabilities, "priors" and others, of various qualities. Discussion has concentrated on "priors" because in many cases they are of different qualities and have different bases than do "likelihoods" that enter into reliability measures, and because not infrequently these differences seem to tell against "priors", their qualities seeming inferior and their base such as to render them less relevant, or "less directly relevant". Subjects in the second experiment who had, one assumes, higher quality "priors" than did subjects in the first experiment, would have had yet higher quality "priors" if they had been told that 85 to 15 was the ratio of green taxicabs to blue ones involved in accidents *at night in the part of the town in which this accident took place, and within a month of the day on which it took place*, and would have had much *lower* quality "priors" if they had been told instead only that every taxicab in the town was either green or blue, and that the ratio of green taxicabs to blue ones *in the world* was 85 to 15. Persons who had nothing else to go on than one or another of these ratios would presumably agree in the quantity of their "singular probabilities" for B, the proposition that the taxicab involved in an accident in the town on that recent night was blue, but persons possessed of information at the extremes of relevance indicated would presumably, and reasonably one feels, *differ* in their readiness to base bets on their "singular probabilities" – they would, while agreeing in the quantity of this singular probability, differ, perhaps markedly, in their confidence in it, and in its quality.

"singular probabilities", supposing there *is* more that can be said along these several lines. In fact, I can only express without proof or argument the sense that, notwithstanding the complications generated by recognition of a 'qualitative dimension', simple Bayesianism will be found to provide the central structure for the best theory of rational updating. I think that even if the best theory is considerably more complicated than simple Bayesianism, it will yield for the assessment of testimony for *miracles*, the thing of main concern to Hume himself of course, essentially the simple Bayesian analysis provided in the first section of this paper.

Conclusion

It was I suppose clear from the start *that* – though it may now be somewhat clearer *why* – if anyone were to tell you that a man had died and come back to life you had better not believe him. "The statement that a man has been raised from the dead would," Leslie Stephen wrote, "prove that its author was a liar,"[41] – or at any rate, to temper the phrase on Stephen's behalf, the speaker of an untruth. More fully and precisely, it would prove this to anyone who, before news of the statement, thought the thing would be a miracle and a natural impossibility, and did not then think that the existence of this testimony would, if it is false, be not only also a miracle and a natural impossibility, but an even *greater* miracle. It would prove this to such a person even if he unwisely failed to draw this conclusion, and to proportion his beliefs regarding the statement's *author* to the evidence of his testimony.

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[41] Stephen, op. cit., p. 341.