On the persistence of the ether as absolute space

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Abstract

We analyse how the concept of the ether, playing the role of absolute space, is still present in physics. When the problem is considered in the context of classical mechanics, we show that vestiges of absolute space can be found in the standard presentation of inertial systems. We offer an alternative –fully relational– definition of inertial systems which not only eliminates the problem but it further shows that the equivalence principle is just a particular consequence of the No Arbitrariness Principle. In terms of Special Relativity, the non-existence of relative velocities implies a constructive contradiction (their existence is assumed in the construction). The problem is inherited from Lorentz' use of the ether, developed in his interpretation of Maxwell's electrodynamics. In summary, the velocities in the Lorentz transformations must be considered velocities relative to the ether (absolute space) if the theory is not to fall apart for being inconsistent. We discuss the relevance of the phenomenological map, and how previous works have failed to acknowledge that the consistency problem is not in the exposed part of the theory but in the supporting phenomenological map which, rather than being constructed anew, it transports concepts of classical mechanics by habit, without revising their validity in the context of Special Relativity.

1 Introduction

The construction of physics requires to go from the observable to the ideas, often represented in mathematical language, and to return to the observations from theoretically elaborated ideas. The mapping from observations into ideas is the process of ideation [Husserl, 1983] and it is associated with simple and philosophical intuition. There is then a phenomenological projection, Π, (we will call: ideation) in which observations are stripped of particularities that are regarded as irrelevant in answering the questions to be addressed. There exists a lift, Γ, as well; that goes from mathematics to new and old observations (call it *interpretation*). Between ideation and interpretation there are two required consistency conditions

$$\begin{aligned} \Gamma \circ \Pi &= Id_{obs} \\ \Pi \circ \Gamma &= Id_{math} \end{aligned} \tag{1}$$

The first relation says that the interpretation of what was ideated must be the originally observed and the second that a new ideation of interpreted symbols must return the original symbols. These phenomenological pairs of maps have not been studied in any extent as far a we know except for [Margenau and Mould, 1957, Dingle, 1960] where the relevance of the "rule of correspondence" is recognised, although no further progress was done on the subject. It is relevant to notice that the present strict view was followed in the development of physics well into the XIX century. A turning point takes place when Hertz [Hertz, 1893] introduced *free interpretation* to make room for his form of understanding (the *bild* conception [D'Agostino, 2004]). Free interpretation became the dominant approach at the beginning of the XX century. Under Hertz' point of view, equations can be detached from the production process and reinterpreted in several different forms. For what concerns to this work, Einstein [Einstein, 1905] proceeds in a similar way. Such epistemological shift leaves then room for inconsistencies.

A second epistemological change was proposed by Poincaré. Poincaré sought a new foundation of physics based on axioms [Poincaré, 1913a], for Poincaré, the old physics played the role of provider of fundamental beliefs which had to become axioms to make further progress possible. This is, the habits of the trained physicist took the place of intuition in the foundation of physics. It is in this context that Poincaré introduces the Principle of Relativity as a fundamental belief supported by habit.

Hertz' and Poincaré's contributions are both in the direction of (possibly) increasing irrationality in physics. Habits cannot be equated to intuition, much less to Husserl's philosophical intuition. Moreover, to detach the concept from the conceptualisation is to render it void, superficial, mere opinion [Hegel, 2001].

We will first address (Section 2) some basic requirements imposed by intuition on the concept of space, requirements that must be satisfied for otherwise the concept becomes devoid of meaning, this is, it becomes metaphysical.

Poincaré's Principle of relativity is a weak (intuited) form of a stronger and more demanding principle: the no-arbitrariness principle (NAP). The principle states that *no knowledge of Nature depends on arbitrary decisions*, and was recently discussed in [Solari and Natiello, 2018]. Resigning this principle, i.e., admitting arbitrariness as a basic ingredient, is indeed an obstacle in the quest for understanding Nature. The principle delimits the conditions with which knowledge about Nature is to develop. As such, it is prior to physics and experience and it cannot be questioned empirically. The demand for objectivity in our understanding directs us to reject arbitrary practices. However, if it were not possible to achieve a full exclusion of arbitrariness, we must make sure that arbitrariness becomes irrelevant when answering valid questions regarding Nature. In [Solari and Natiello, 2018], the notions of space and time were developed elaborating the concepts of *permanence* and *change*. Starting from the subjective intuitive notion of space we proposed the existence of *inertial frames.* Further, a relational formulation of Newtonian mechanics was presented, indicating that absolute space was not fundamental to Newton's theory. In this work we develop the notion of inertial frame in a fully relational way. We will further elaborate about the concept of space from this perspective in Section 2 We will show in that section how the classical concept of space emerges in this context.

Next (Section (3)), we attempt to perform the same task using Minkowski's space. We show that an inconsistency appears violating (1) and furthermore, that NAP is violated as well.

We show that the way out of these problems is to acknowledge that the velocities involved in Lorentz transformations are to be regarded as velocities relative to the ether as originally proposed by Lorentz. This is, we must assume the existence of absolute space. However, the standard use of Special Relativity (SR) corresponds to relational velocities between source and detector such as in the measurement of Doppler shifts. In experiments where changes in a body (source) are followed with delay by changes in other bodies (receptors) a description centred at the source takes a natural special character, as Faraday originally suggested [p. 447, Faraday, 1855].

2 The perception of space and the phenomenological map

The phenomenological map translating observations and relations between observations and symbols is not completely free of conditions. While we usually leave this map to intuition, it can be put to some extent under the supervision of reason, this is, it can be addressed by philosophical intuition. What is of fundamental relevance here is the notion of velocity.

To address the notion of velocity we need to address first the notion of spatial relations. Take e.g., a look at the garden. Leave aside (project out) the moving leaves and birds, and consider those elements that impress us as keeping a constant relation among them. We seek for a universal instruction to move around the garden. We then identify the elements with labels, $i \in [1 \dots N]$ and summarise the instruction of "going from i to j" as x_{ij}. Any moving instruction can be given as a concatenation of moving instructions, this is the most essential condition of space,

$$\begin{aligned} x_{ij} + x_{jk} &= x_{ik} \\ x_{ij} + x_{jj} &= x_{ij} \\ x_{jj} &\equiv 0 \\ x_{ij} &= -x_{ji} \end{aligned} \tag{2}$$

We then realise that $x_{ij} = x_{ij} + x_{jj}$ since x_{jj} is the instruction for remaining at the locus of j, or just "do nothing". We call x_{jj} the *neutral* element, $0 \equiv x_{jj}$. The last line in (2) express the perceived fact that the outcome of staying in one place is the same as going from that place to another and returning. Thus, *returning* becomes the inverse operation of *going*: $x_{ij} = -x_{ji}$.

This is enough for our purposes. We need now to introduce velocities, and hence, we need to introduce change, or its abstract form, time (that time is abstract form of change was already known to Aristotle [Aristotle, 1994–2010]). Our observations may indicate/suggest that the organisation of the garden is not always the same, perhaps because we want to explain where a bird is feeding in the garden. We have then decided that there are things that, for our purposes, are permanent (do not change) such as trunks and stones as well as birds. But the birds cannot be located with "old instructions", the location instructions have to be updated as a function of other perceived changes. Each observer may have his own clock, so we write t_S and understand that all the instructions in Eq. 2 were given for a determined time:

$$\begin{aligned} x_{ij}(t_S) + x_{jk}(t_S) &= x_{ik}(t_S) \\ x_{ij}(t_S) + x_{jj}(t_S) &= x_{ij}(t_S) \\ x_{jj}(t_S) &\equiv 0 \\ x_{ij}(t_S) &= -x_{ji}(t_S) \end{aligned}$$

We finally consider the relation between the rate of change our clock and the rate of change of relative positions,

$$v_{ij}(t_S, \delta) \equiv \frac{x_{ij}(t_S + \delta) - x_{ij}(t_S)}{(t_S + \delta) - t_S}$$

It follows immediately that

$$v_{ij}(t_S, \delta) + v_{jk}(t_S, \delta) = v_{ik}(t_S, \delta)$$

This is, whatever the clock is, the composition law between velocities must be the same that the composition law of the space.

Confronted with this fact, the trained physicist may want to elude its consequences by denying the correctness of the construction, since the request on velocities can be seen by the trained eye as contradicting the transmitted knowledge regarding the addition of velocities in SR. She/he is certainly welcome to do it, but under two restrictions: (a) a substitute phenomenological map must be provided, since otherwise physics would not talk about that what is perceived as real, but it will rather be a formal exercise. And, (b) under the new conceptualisation the adoption of concepts previously learned by habit or arising from any kind of social complicity must be banned, since habit or social complicity only exist within the original construction now being doomed as incorrect.

2.1 Descartes' mathematisation of space

The Cartesian view is always the view of an observer, the view that matches our intuitive construction, namely an extrinsic view. In Descartes' method, directions and distances are used instead of giving instructions to move around the garden based upon landmarks. Thus, the instruction that was "walk from i to j, x_{ij}" becomes "from i walk x steps in the direction $\hat{e}_{ij}$ to j", that we annotate $x_{ij} = \hat{e}_{ij}x$. If we now agree to consider only the path from a given reference position (the position of "ego"), all paths consist in concatenations of this kind of instructions. Our intuition tell us more, it tells us that there are only three independent directions, at least as much as we can perceive. Therefore, the space is three dimensional and the mathematical construct is Cartesian space, which not only inherits the rules developed for the concatenation of instructions, but adds new rules based on intuition, such as $x\hat{e} + x'\hat{e}' = x'\hat{e}' + x\hat{e}$ (addition is commutative) and the other rules of vector algebra.

2.2 Subjective and relational spaces

Newtonian mechanics [Newton, 1687] was originally formulated resting on the notion of absolute space. Already with Leibniz the alternative idea of a relational space arose, i.e., a space free of the arbitrariness of an extrinsic reference. How do we construct an intrinsic view (i.e., without external observers)?

Individual subjective spaces contain the arbitrariness of the choice of origin and the choice of references, but "what is real presents characters that are entirely independent of our opinions about them" [Peirce, 1955, p. 18]. This is, the Cartesian space is not real. In contrast, relational constructions as those like x_{ij} we used to introduce this discussion, stand their chance of being real. Yet, the Cartesian view is not completely arbitrary because all arbitrary spaces that we can produce map into each other in a one-to-one form. Thus, the observations in one space need only to be translated into the observations in another space (characterised by different arbitrariness). We say that the descriptions are intersubjective. When the differences between subjective spaces correspond to arbitrary elections that influence the description in a systematic form, as it is the case of the choice of origin and the choice of directions of reference, the set of transformations relating the different descriptions must satisfy conditions of consistency that allow us to move in the set of arbitrary descriptions without contradictions. This is the core meaning of the No Arbitrariness Principle (NAP) [Solari and Natiello, 2018], in short: the set of transformations associated to arbitrary decisions must form a group. Actually, considering it in finer detail, there is a group associated with each subset of equivalent arbitrary decisions, this is: a group for the election of reference point, a group for the election of reference directions, and so on.

If the Cartesian space for N bodies corresponds to $\mathbb{R}^{3N}$and the group of transformations between arbitrary representations (after restricting the choice of directions to orthogonal directions) is $\mathbb{E}(3) = ISO(3) = SO(3) \ltimes \mathbb{R}^3$, then the real space is what results of modding out the arbitrariness: $\mathbb{R}^{3N} / \left(SO(3) \ltimes \mathbb{R}^3\right)$, this is a point for $N = 1$, the real line for $N = 2$ and only for $N \geq 3$ it acquires the characteristics we intuitively assign to relational space by removing from the subjective space $\mathbb{R}^{3N}$ a global orientation and the position of the centre of mass (for example).

2.3 Inertial frames

We look for the view of an inertial frame considered separately from other things.

We begin by considering one body alone in a 3-dimensional, $3d$, universe. For such a body, relative space makes no sense at all. There is nothing else available to consider e.g., relative positions. When we consider two bodies, only a one dimensional universe is conceivable. The distance between the two bodies is the only possibility for geometric change. The only conceivable direction is that from one body to the other.

When we consider three bodies, a distinct difference arises. Let $i, j \in (1, 2, 3)$. We have the relations x_{ij} (oriented distances) and we can consider a large number of different vectors, e.g. the set $\{x_{ij}, dx_{ij} = x_{ij}(t_S + \delta) - x_{ij}(t_S)\}$ (even before formalisation the latter represents a difference between two situations). In a three dimensional space those vectors are not completely arbitrary: some internal relations will become explicit. We are now in the position of considering *inertial* systems, except under singular circumstances. We set

$$\hat{e}_{ij} = \begin{cases} \frac{x_{ij}}{|x_{ij}|} & if \ \frac{d}{dt_S} x_{ij} = 0 \\ \frac{v_{ij}}{|v_{ij}|} & if \ \frac{d}{dt_S} x_{ij} \equiv v_{ij} \neq 0 \end{cases}$$

where $v_{ij} = \lim_{\delta \to 0} v_{ij}(t_s, \delta)$. Notice that in the absence of a mathematically defined vector space, the derivation symbols $\frac{d}{dt}$ indicate a quotient between perceived changes (for some fixed value of δ). The numerator refers to the perception we call position and velocity and the denominator is any other kind of change used as reference, its abstract form being *time*, hence called dt_S, still allowing for clocks of different systems to be non synchronised. The conditions $dx_{ij} = 0$ and $dv_{ij} = 0$ are ultimately a subjective perception of the observer and as such they introduce subjectivity in the description. It is this decision made by the subject what creates the space that can be mathematically represented.

Definition 1. Two bodies are *relatively inertial* if there exists a reference frame, a constant vector a, and a scalar b such that $\dfrac{db}{dt_S}$ is constant and

$$\begin{aligned} \hat{e}_{ij} \times (x_{ij} \times \hat{e}_{ij}) &= a \\ \hat{e}_{ij} \cdot x_{ij} &= b \\ \frac{d}{dt_S} \hat{e}_{ij} &= 0 \end{aligned}$$

where we always have

$$x_{ij} = a + b\hat{e}_{ij}$$

If we force the inertial system to have a reference point in the relatively inertial set, we then get the standard definition of inertial system. In such a case, the study of inertial systems becomes a self-referencing approach. Thus, the standard view of inertial frames is the one we have when we cannot remove the observer from the scene. It is indeed a view much closer to our intuition.

Definition 2. The reference frame of 1is called "inertial frame".

Lemma 3. *The inertial frames referring to a set of relatively inertial bodies have as group of arbitrariness the translations (which may be time dependent) and the (time independent) rotations composed as a semi-direct product group.*

Proof. The instantaneous relative position of N bodies is invariant under $ISO(3)$ as explained in Subsection 2.2. With relative positions, the arbitrariness of the origin of coordinates cancels out even when it changes as a function of time. In contrast, a change in the arbitrary choice of orthogonal directions of reference as a function of time will make $\frac{d}{dt_S}\hat{e}_{ij} \neq 0$ in the definition 1, hence it will break the concept of relatively inertial set. □

The introduction of inertial sets deserves a detailed discussion. In subsection 2.2 we introduced the instantaneous space for the description of the relational problem of N bodies. Starting from subjective space, $\mathbb{R}^{3N}$, we arrived to a relational space where the orbits of the points by the action of the group $E(3) = ISO(3)$ were identified, this is: $\mathbb{R}^{3N}/E(3)$. The description of the evolution in time in the relational space is then represented by a function of time, $\mathbb{R}$, into $\mathbb{R}^{3N}/E(3)$. Thus, each trajectory is given by a function $F(t) : \mathbb{R} \mapsto \left(\mathbb{R}^{3N}/E(3)\right)$ and the set of functions will be called $\mathcal{F}$. The definition 1 uses relative positions, x_{ij}, which are invariant under changes of the origin of the (subjective) coordinates, hence, these mathematical objects are invariant under the action of $T(3)$, the group of translations, and only rotations in $E(3)$ may change them. Then, from the original group of arbitrariness, $E(3)$, we are left with the effective action of $E(3)/T(3) \sim SO(3)$ (a result that can be intuited as well). Since we have to make such a choice for every time when considering a trajectory, the group of arbitrariness associated to the set $\{x_{ij}(t)\}$ consists of (continuous and twice differentiable) time-dependent rotations. Continuity and differentiability is requested because we have to deal with velocities in the definition. Because the translation of the subjective origin of coordinates does not intervene in the definition of the inertial set and inertial frame, we can allow any arbitrariness to such a collective translation.

The standard introduction of inertial frames is made in terms of subjective spaces by selecting one such reference system in which the N bodies are described as having coordinates $x_i(t) = x_i(0) + v_i t$. This view is restrictive in excess and it is equivalent to arbitrarily adding an extra body, name it 0, representing the origin of coordinates. Thus, the set of relatively inertial bodies is augmented in one, and the positions are $x_i(t) \equiv x_{0i}(t)$. The transformations among inertial frames in the standard setting, the Galilean group, arise as a consequence of this arbitrary restriction. This extra arbitrariness will have to be compensated later (patched) with a new axiom, the "equivalence principle".

Remark 4. The inertial class is larger that the class usually considered as inertial, because it contains accelerated systems as those considered in Einstein's "equivalence principle" (stating the "complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system"[Einstein, 1907]) and beyond them, any kind of global time-dependent displacement of the frame and the objects under study. The "equivalence principle" [1]is then not an independent principle but just a consequence of NAP and its derived concept of relatively inertial.

Two separate sets of bodies can be inertial and yet no inertial frame may be available for all the bodies to pertain to a relatively inertial set (i.e., they belong in different equivalences classes of Lemma 3).

The meaning of inertial is that, if we are going to consider a set of bodies as isolated from the rest of the universe, in the absence of interactions among them we expect the set to be relatively inertial and a class of inertial frames to exist. Any departure of the trajectory of the bodies making them not free, i.e., breaking the relatively inertial condition, must be attributed to interactions (reciprocal actions) among them.

Remark 5. Inertial frames are not equivalent to absolute space, they are a device used to organise our description of the mechanical world.

Lemma 6. *Relatively inertial is an equivalence relation i.e., a relation $\sim$ such that for three bodies A, B, C it holds that $A\sim A$, $A\sim B \Rightarrow B\sim A$, $A\sim B$* and *$B\sim C \Rightarrow A\sim C$.*

Proof. $A \sim A$ since whenever $x_{ii} = 0$ and for arbitrary $\hat{e}$ we have $a = 0$ and $b = 0$ in Definition 1. If a pair a, b exists such that $A \sim B$ it follows that $B \sim A$ with the associated pair $-a, b$. The third relation follows from vector addition rules. We have

$$\begin{aligned} x_{AB} &= a + b\hat{e}_{AB} \\ x_{BC} &= a' + b'\hat{e}_{BC} \\ x_{AC} &= x_{AB} + x_{BC} \\ &= a + a' + b\hat{e}_{AB} + b'\hat{e}_{BC} \end{aligned}$$

Further, $b\hat{e}_{AB} + b'\hat{e}_{BC} = c + t_S \left(\frac{db}{dt}\hat{e}_{AB} + \frac{db'}{dt}\hat{e}_{BC}\right)$, where c is some constant vector and the constant quantity in parenthesis is either zero or some other constant nonzero vector. In the latter case, letting $\lambda = ||\frac{db}{dt}\hat{e}_{AB} + \frac{db'}{dt}\hat{e}_{BC}||$ and $\hat{e}_{AC} = \frac{1}{\lambda}\left(\frac{db}{dt_S}\hat{e}_{AB} + \frac{db'}{dt}\hat{e}_{BC}\right)$ we get $x_{AC} = (a + a' + c) + \lambda t_S \hat{e}_{AC}$. □

[1]There exist references in the literature to a "weak equivalence principle" stating that inertial mass is identical with gravitational mass. However, this statement is more a social construction than a basic ingredient of mechanics. The concept of (linear) momentum –or quantity-of-motion– is not directly intuited but rather ideated. It was first ideated by Newton, who developed the concept of force as well. While the gravitational mass of a body is part of our intuitions, the inertial mass is a later construction. The emergence of two concepts of mass appears to be the outcome of a social process: The *instruction* of physics (i.e., to learn physics passively rather than through personal (re)discovery). A consequence of passive learning is the creation of habits. Subsequently, habits are confused with intuitions. In this way, the concept of force was "naturalised" by habit (in the sense that forces are treated as observables, rather than as meta-observables –ideated–). In this process, an inertial-mass appears, relating force and acceleration. A detailed construction of mechanics [Solari and Natiello, 2018] shows that the concept of inertial mass rests completely on the concept of gravitational mass. Therefore, it makes no sense to distinguish them.

The notion of inertial is associated to freedom, i.e., to *not* being subject to the influence of other bodies in a persistent form, i.e., independence of other bodies implies that the law of motion cannot make reference to any circumstance of them [Solari and Natiello, 2018]. In turn, persistence requires the identification of something that is not changing, this is, a time derivative being zero. The law of motion for two relatively inertial bodies must read

$$\frac{dx_{ij}}{dt_S} = \phi(x_{ij})$$

for some function ϕ of the relative coordinate, being independent of other bodies and persistent. But the change must be independent of the proximity of the companion body for, after all, they do not influence each other. Thus, ϕ cannot depend on relative position and hence, $\phi(x_{ij}) = v_{ij} = const.$

3 Special Relativity

The traditional presentation of SR rests on two explicit principles, namely the *relativity principle* (the laws of physics are the same regarded from any inertial reference frame) and the constancy of the velocity of light, C, regardless of the movement of the source (as seen from any inertial system). Einstein [Einstein, 1905] presents and completes the Lorentz transformations between two systems with (constant) relative velocity v, as the well-known *Lorentz boosts*. In their general form, the boosts read:

$$\begin{aligned} T_v X &= \Big(\gamma_v(x - vt) + (1-\gamma_v)\hat{v} \times (x \times \hat{v}), \gamma_v(t - \frac{v \cdot x}{C^2})\Big) \\ &= \Big(\gamma_v\left((x \cdot \hat{v})\,\hat{v} - vt\right) + \hat{v} \times (x \times \hat{v}), \gamma_v(t - \frac{v \cdot x}{C^2})\Big) \end{aligned}$$

with $\gamma_v^{-1} = \sqrt{1 - \frac{v^2}{C^2}}$ (Lorentz, Poincaré and Einstein worked out the special case where the direction of v is aligned with $\hat{e}_1$, for an orthogonal reference system where $x = \sum_{i=1}^{3} x_i \hat{e}_i$). Some additional specifications are not fully stated but consistently used. We state them as *claims*, allowing for the possibility that they may need improvement:

Claim 7. If v_{BA} is the relative velocity of B with respect to A, then, reciprocally, $v_{AB} = -v_{BA}$.

This claim appears explicitly in [Einstein, 1905]. Equivalently, $v_{AB} + v_{BA} = 0$. Further,

Claim 8. If v_{BA} is the relative velocity of B with respect to A, then B is transformed from A by a Lorentz boost,

This claim appears on [Ch. 16, Vol. i, Feynman et al., 1965] (among other places), where it is stated that the *correct* transformations between systems moving with relative velocity v are Lorentz transformations) and

Claim 9. For the addition of two velocities, the following law holds,

$$u \oplus v = \frac{1}{1+\frac{u \cdot v}{C^2}} \left((u \cdot \hat{v})\, \hat{v} + v + \frac{1}{\gamma_v} \hat{v} \times (u \times \hat{v}) \right)$$

where v is the velocity of an object in a given system S_1,u is the velocity of S_1 relative to another system S_0 and $u \oplus v$ is the velocity of the object in system S_0. The law is proved in [Einstein, 1905] for collinear velocities, while the "general form" is in e.g., [Jackson, 1962, p. 531, eq. 11.31] (the lhs is not explicitly written by Jackson; the expression in the rhs might be interpreted from the text as $v \oplus u$, but this is unimportant for the sake of the argument).

Lorentz boosts do not form a group. Hence, the expected behaviour of coordinate transformations as given by the intuitions of Classical Mechanics, are not met. The following result summarises the situation:

Theorem 10. *Claims 7, 8, 9 are inconsistent, i.e., the three claims together cannot be generally valid.*

Proof. Consider three bodies a, b, c with constant velocities $u, v, -w$ relative to a system S_0. The velocities are the slope of the world-lines $a = T(u)(0,0,0,t)^\dagger$ (i.e., the spatial components of a divided with the temporal coordinate), and similarly for b and c. By Claims 7 and 8, world-lines a and b can be transformed to the system at rest with c by a Lorentz transformation $T(w)$, since they are given in S_0 and $v_{cS_0} = -w$ is known (and therefore $v_{S_0c} = w$). Hence, $a_w = T(w)T(u)(0,0,0,t)^\dagger$ and $b_w = T(w)T(v)(0,0,0,t)^\dagger$. Except for the collinear case, $T(w)T(u)$ is not a Lorentz boost but a general element of the whole *Poincaré group* (also called the *homogeneous Lorentz group*; for the sake of the problem the group of 4×4 transformation matrices P such that $PM = MP$, where M denotes the Minkowski metric). We note on passing that Lorentz transformations do not form a group. Instead, we have $T(w)T(u) = T(w \oplus u)R(w,u) \neq T(w \oplus u)$, for some non trivial 3×3 spatial rotation matrix R. Therefore, if Claim 9 is true, then Claim 8 is false since $R \neq Id$ and Claim 7 is false since from the slope of a_w it holds that $w \oplus u = \frac{1}{1+\frac{w \cdot u}{C^2}} \left((w \cdot \hat{u})\, \hat{u} + u + \frac{1}{\gamma_u} \hat{u} \times (w \times \hat{u}) \right)$ while $u \oplus w = \frac{1}{1+\frac{u \cdot w}{C^2}} \left((u \cdot \hat{w})\, \hat{w} + w + \frac{1}{\gamma_w} \hat{w} \times (u \times \hat{w}) \right)$ and their sum is nonzero outside the exceptional case of collinearity, i.e., $u \times \hat{w} = 0$. In fact, if there exists a relative velocity, then Claim 8 is false independently of Claim 9, since there is no X whatsoever such that $T(w)T(u) = T(X)$ for non collinear w, u. □

Moreover, the following results also hold:

Corollary 11. *(No NAP class of inertial systems) If Claim 9 is true, then the relative velocity between a and b depends on the observer.*

Proof. The four-vector whose slope gives the velocity of b relative to a as seen by the system c is $T(-(w \oplus u))b_w = R(w,u)T(-u)T(v)(0,0,0,t)^\dagger$, while the same relative velocity as seen by S_0 is given by $T(-u)b = T(-u)T(v)(0,0,0,t)^\dagger$. Hence, every observer w has a different opinion about the relative velocity between a and b. □

In other words, the description of motion does not comply with NAP. Further,

Corollary 12. *Relative velocity is not well defined.*

Proof. If there exists a relative velocity, it is observer dependent, i.e., nothing that relates exclusively to the involved bodies and it does not comply with the phenomenological demand of reciprocity in Claim 7. □

Finally, it is a quite remarkable issue that there exists no prescription in SR to *measure* relative velocities from arbitrary inertial systems. The coordinate transformations systematically assume that the velocities in some reference frame S_0 are given (known). Everything else arises by application of transformation formulas, with no measurements involved. In other words,

Corollary 13. *At most one system of reference relates to all others systems moving with constant relative velocity v through Lorentz transformations.*

This privileged system S_0 is the absolute space. Only for the privileged system, the bodies (and other reference systems) can be described as moving relative to it with velocity v, with other reference frames oriented parallel to the chosen frame in S_0. Lorentz transformations [Lorentz, 1904] were constructed having the ether as absolute reference, after adapting Maxwell equations to the ether (this is, after disregarding Faraday's insights and experiments on relative motion). Fields, velocities and electromagnetic sources are described from this reference. It is then not surprising that the ether is still present (in the form of a privileged system), even if we restrain from using the word. Textbooks will simply start by stating that a second system moves with velocity v with respect to the initial one, leaving in oblivion that the specification is incomplete except when one of the systems is Absolute Space. This habit corresponds with our intuition rooted in classical mechanics. Since habit soon becomes invisible, the difficult part of the critical thinking that is required to understand the foundation of such claim is left for the student to perform without help: the need of critical thinking is not even suggested.

3.1 The phenomenological map

The origin of the exposed inconsistency lies already in Einstein's original paper [Einstein, 1905]. Einstein makes clear the use of two postulates: (i) The principle of relativity, that represents an habit developed along Newton's mechanics (very much in the same form of elevating habits to postulates proposed by Poincaré [Poincaré, 1913b]) and (ii) the constancy of the velocity of light. We find on Part I, Section 2,

$$velocity = \frac{light\, path}{time\, interval}$$

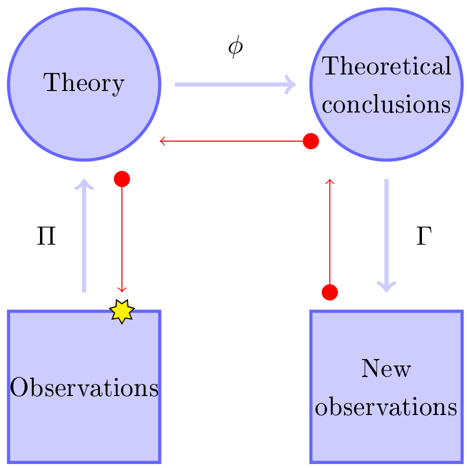

Figure 1: Phenomenological map, Π, theoretical elaboration ϕ and interpretation Γ. The red circuit shows how unexpected observations (failed predictions) propagate "backwards", forcing improvements on Π.

Since in classical mechanics both quantities in the quotient are independent of the frame of reference (Galilean transformations are the result of this requisite, see [Solari and Natiello, 2018]) there would be no need to postulate that such velocity is constant. Yet, Einstein produces a construction (a characteristic rods-and-clocks construction to avoid abstraction; resting upon habits) in which the velocity of light transforms as subjective velocities do, i.e., light is assumed to move body-like. This hypothesis is hidden. Hidden as well is the hypothesis "there exists a (relative) velocity". A few lines further we read "We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod."

The existence of the velocity v is not discussed, it rests on our daily experience and our training (habituation) as physicists. Hence, the existence of the concept "velocity" is not considered an hypothesis. If we raise it to what it really is, we find (as we just proved) that there is no theory unless we introduce the ether, since a basic requirement of a theory is to be consistent.

The theoretical construction must be clarified. Figure 1 shows a basic sketch of a theoretical construction. The phenomenological map Π (a projection) produces symbols and relations among symbols from observations. The map Π cannot be mathematised and rests upon intuitions and the same can be said about the interpretation Γ (a lift) that takes symbols into observables. Any theory of the observable must specify these maps. Thus, if we say: A moves with respect to B with velocity v_{AB} we must indicate which observation is behind the statement and since the statement makes no reference to any other thing than A and B we must show that the construct is independent of the observer. These are requirements of consistency that we have called NAP. Whenever new observations conflict with predictions in Γ, the conflict "propagates" back and triggers an improvement or extension of Π with broader range of validity.

All the existing discussions (see for example [Mocanu, 1986, Ungar, 1988, 1989, Ungar et al., 2005]) with reference to the rotations $R(u, w)$ have been performed following a detailed inspection of ϕ missing that the problem is not there but rather in the hidden maps. It is the phenomenological map what is faulty.

4 Discussion

In terms of classical mechanics we have shown that it is possible to introduce inertial frames and sets of inertial bodies without introducing an Absolute Space, complementing the work already presented in [Solari and Natiello, 2018]. We have also shown that the new approach includes a larger variety of reference systems making the equivalence principle just a particular case of NAP and its consequences.

We have highlighted the relevance of the phenomenological map as part of a theory of the real. Its absence makes any formulation something less than a theory. In discussing SR, we have indicated a severe and fundamental inconsistency in its formulation at this level, one that cannot be cured by analytic reason, but requires critical thinking. The observed problem on the definition of relative velocity has been known at least since 1914 [Silberstein, 1924]. It is currently clear [Gilmore, 1974] that the elements of the Poincaré-Lorentz group can in general be expressed through a coset decomposition as $P = TR$ where T is a Lorentz boost and R a rotation. However, the role of R in an expression such as $P = T(w)T(u) = T(w \oplus u)R(w, u)$ and its role concerning conventional (spatial) changes of coordinates is not the same. There is no conflict in attaching meaning to the velocities, but the conflict arises when we attempt to adscribe to $R(w, u)$ (also called the *Thomas rotation*) the same meaning as usual spatial rotations, since rotations have been considered from the beginning as associated to the arbitrariness of selecting reference directions in space, which is independent of relative motion. The conclusion is that the velocities in the Lorentz transformation refer to a particular reference system, which was the ether in Lorentz [Lorentz, 1904]. The ether faded away from physics texts by (approximately) 1930.

When interpreting relativistic experimental results, such as the relativistic Doppler effect [Kaivola et al., 1985, Mandelberg and Witten, 1962] it is not the ether what plays the role of a privileged frame but rather the frame of the emitting body. Certainly, the frame of the emitting body can be distinguished from all other frames which detect the electromagnetic signal. The distinction emitting vs. receiving is real, but the distinction among the frames of different receptors is arbitrary. Thus, it is admissible that the transformations from the frame of the source to the frame of a receiver could be different than those that go from one receiver to another. This use of Special relativity is strictly Electromagnetic (EM). The idea send us back to the liminal work by Faraday on "vibrating rays" [Faraday, 1855, p. 447].

The actual use of SR indicates that not all the frames are equivalent, the equivalence is constructed only by considering that the light jumps from the emitting body to space and then it travels body-like through space. The centre of the conflict is this construct, call it *substantialism*, that imposes to the non-observable (the actions) the description of the observable (the bodies). It has the practical advantage of allowing for intuitions and habits in the form of analogies. Its cost is that it may be fundamentally wrong as it does not follow from reason but rather from our limitations [Natiello and Solari, 2019].

Historically, absolute space had been rejected by the early XIX century. However, it returns to some extent with Maxwell [2]. The position of Lorentz is bolder. Although he writes "That we cannot speak about an absolute rest of the aether, is self-evident; this expression would not even make sense" [Lorentz, 1895, p. 2], he subsequently adopts the ether as an absolute reference frame. The materiality of the ether was a requisite for Lorentz. The view of Einstein was different. For him [Einstein, 1924]: "It is usually believed that aether is foreign to Newtonian physics and that it was only the wave theory of light which introduced the notion of an omnipresent medium influencing, and affected by, physical phenomena. But this is not the case. Newtonian mechanics had its 'aether' in the sense indicated, albeit under the name 'absolute space' [...] The Maxwell-Lorentz theory eventually influenced our view of the theoretical basis to the extent that it led to the creation of the special theory of relativity. It was recognised that the equations of electromagnetism did not, in fact, single out one particular state of motion, but rather that, in accordance with these equations, just as with those of classical mechanics, **there exists an infinite multitude of coordinate systems in mutually equivalent states of motion**, providing the appropriate transformation formulas are used for the spatial and temporal coordinates" (emphasis added). The highlighted statement is false in SR, (Corollary 11: there is no inertial class, there is no equivalence relation). Einstein legitimation of the "ether" rests on a constructive necessity of Newtonian mechanics, which, according to his view needs "absolute space". In this work we have shown that there is no such necessity. Absolute space is no more than a pedagogical tool which simplifies the access to mechanics making a direct connection with intuition.

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[2]In [[600], Maxwell, 1873] Maxwell writes "Let x', y', z' be the coordinates of a point referred to a system of rectangular axes moving in space, and let x, y, z be the coordinates of the same point referred to fixed axes", what makes evident he is considering absolute space, since "at rest" does not refer to any particular reference.

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