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# Physics Before Newton

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# The Question of Intensive Magnitudes According to Some Jesuits in the Sixteenth and Seventeenth Centuries

## The Problem of Intensification in Antiquity and the Middle Ages

The problem of the intensification and remission of qualities was a *crux* for philosophical, theological, and scientific thought in the Middle Ages.[1] It was raised in Antiquity with this remark of Aristotle: some qualities, as accidental beings, admit the more and the less.[2] Admitting more and less is not a trivial property, since it belongs neither to every category of being (for instance not to substance or quantity[3]), nor to every quality (it does not apply to geometrical figures, for instance). Rather it applies only to states and dispositions such as virtue, to affections of bodies such as heat and sweetness, and to affections of soul such as anger. However, the property of admitting more and less was a matter of importance for the qualitative physics that had reigned up to about the time of Descartes, a physics which was concerned with concepts such as heat, coldness, lightness, heaviness, and so on.

More precisely, these qualities are such that more or less of them can be attributed to a subject (we can say that some water is more hot or less hot). We may be comparing one thing with another, or a thing with itself at two different moments. In the latter case, we shall say that there is intensification or remission of its quality. It is important to note that this phenomenon is considered to be happening to the same part or parts of the subject.[4] We can for instance exclude the case of propagated heat, which spreads from an extremity of a subject to cold parts of the same subject. Spatial extension, related to the category of quantity, is only accidental when it pertains to a quality.[5] For Aristotle, intensification is solely a re-

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inforcement of a quality in the same place, with the reinforcement not spreading into parts which do not yet possess it. It is a kind of alteration,[6] which nevertheless leads neither to a corruption of the subject,[7] nor to a mutation of the quality into its opposite.[8] It takes place within a zone of indetermination, in which the quality can stay in the subject, and the intensity can vary.

With regard to this variation of intensity, Aristotle left one question unsolved. We can see that a white color can be more intense or less intense in a thing, but this variation of intensity is not so clear for qualities such as justice or health: they always seem to remain identical to themselves, the variation depending only on the subjects which receive them. Justice in itself cannot be more or less just, but one individual can be more just than another. Aristotle's commentators in late Antiquity systematically examined the following problem, a problem raised in Chapter 8 of the *Categories*: does intensification occur in the form itself, or in the subject? Equivalently we can ask: has each form an absolutely determinate and unvarying nature (so that variation only depends on the subject), or do some forms have in their nature a certain "latitude" (so that they can be in themselves realized to a greater or lesser degree?

Medieval thinkers knew something of the Aristotelian commentary tradition (principally Porphyrius *via* Boethius, and since 1268 Simplicius, both on the *Categories*), and treated intensification in their works on the *Categories*, the *Physics* and *De Generatione*. But, for them, it was not only a philosophical problem: it was also theological. Thus Peter Lombard, in his *Sentences* (l. I, d. 17, 2), asked whether charity can be augmented. The problem for him derived from the fact that, in Augustinian manner, he identified charity in the human soul with the Holy Spirit itself (d.14, 2). It was then difficult for him to give an account of the fact that this man is more charitable than that one, or generally speaking, that charity admits of variation. Most of the scholastics did not follow Lombard on this point: for them, charity is a created *habitus*, an accidental form in the human soul. But the question remained: how does charity increase or decrease? A reason for the persistence in raising this question was that charity, as an accidental form, is a quality, and therefore is subject to the same analysis as the other forms which are capable of intensification.[9] Indeed, the problem of charity was discussed with examples borrowed from physical phenomena, and with the same arguments that are found in

commentaries on the *Categories* and on Aristotle's natural philosophy. Thomas Aquinas, for instance, explicitly states that "increase in charity is similar to increase in natural qualities."[10]

The theological problem raised by Peter Lombard had a special influence: it introduced the idea of augmentation. Prompted by Augustine's remark: "*caritas meretur augeri,*"[11] Medievals formulated the question as: "whether charity could be augmented?" (*utrum caritas possit augeri*). Doubtless, the terms augmentation and to augment (*augmentum* and *augere*) as well as diminution and to decrease (*diminutio* and *minuere*) were originally employed, especially by Boethius, in reference to qualities as well as quantities, and they still had the same sense in the twelfth century.[12] Augustine resorted to the notion of "quantity of virtue or potency" (*quantitas virtutis*, or *potentiae*) in opposition to "quantity of dimension or bulk" (*quantitas dimensionis* or *molis*)—a distinction he borrowed from Neoplatonism,[13] and which led to a concept of purely qualitative intensive magnitude: "to be bigger and to be better are one and the same" (*idem est esse maius quod melius*).[14] But later, in the thirteenth century, the word "augmentation" was interpreted as standing for the Aristotelian concept of *auxesis*. This represents a deep transformation of the problem. For Aristotle, there is a strict separation between quality and quantity; consequently the corresponding motions—alteration and augmentation—are quite different. Augmentation (or diminution) can only occur where parts can be distinguished and subsequently added (or subtracted)[15]—in other words: where there is divisibility, and then quantity.[16] Alteration can happen without quantitative change, and augmentation without qualitative change.[17] As we have seen, for Aristotle that sort of change which is a matter of something's becoming more so or less so, occurs in qualities, not in quantities. It is a kind of alteration, and has nothing to do with augmentation and diminution. On his purely qualitative account, the more and the less depend only on the preponderance of a quality over its opposite, an idea formulable in terms of act and potency: the more its contrary is in potency,[18] the more the quality is actualized.[19]

But this solution could not be accepted in the Middle Ages, especially not by theologians: charity, as a divine gift, has no contrary (though, the same objection was raised about a natural phenomenon: namely light, obscurity being only a privation).[20] An alternative to this purely qualitative analysis could be explored analogously with increases in quantities,

literally understanding "augmentation" and "to augment" in the strict sense of *auxesis*, with the help of the revised notion of quantity of power. Although such a magnitude is neither quantifiable nor countable (since it is not properly a quantity), the process of augmentation was made to apply. Augmentation consists in *addition*, as Aristotle had said: "increase is an adjunction of a preexisting quantity."[21] It can be imagined, then, that intensification is produced by the same formal mechanism as augmentation: an addition of something to something. However, intensification will be an addition without quantities: an addition of parts of a quality.[22] This of course presupposes that quality can be divided into parts or degrees.

The point at stake, therefore, was the following: a form is not necessarily a perfectly determined invariant structure, characterized by simplicity. For Aristotle, however, forms are like numbers:[23] something cannot be added to or subtracted from them without changing their specific nature, just as a number is not a mere collection of unities, but a species *per se*. But some Medieval thinkers were of the opinion that at least some forms do have an intrinsic indetermination or latitude, so that additive or subtractive operations on parts can take place in them. These operations would not lead to a change of species, which would be a corruption.

In the thirteenth century, the pure theory of addition was mainly defended by Franciscans. For them, every quality whose intensity is increased acquires something new that it did not possess before. A distinct reality is added to the quality's preexisting degree, thereby creating a new unity ("in being added [to a thing] it is united [with it] and in being united [with it] it is added [to it]"[24]). Their view was developed by the so-called Oxford Calculatores in the fourteenth century, and finally triumphed everywhere, and in particular, as we shall see, among the Jesuits of the sixteenth and seventeenth centuries.

The main opposition to this view came, in the thirteenth century, from Thomas Aquinas. Thomas says that he cannot understand how charity (the reasoning would be the same for any other quality) could be augmented "by addition of charity to charity."[25] For in the operation of addition, a distinct thing is added to another, and a distinction is either specific or numerical. But two cases of charity do not differ in their essence, and numerical difference depends exclusively on the diversity of the subjects in which accidents inhere. Thus a quality is added to a quality only when a subject is added to a subject, and this is not the case here, where we are

talking about increasing charity in a single and unique soul. This argument against the additive theory will be tirelessly adduced by early Thomists and applied to all qualities and subjects. The additive theory, Thomas continues, proceeds from a "false image," which confuses augmentation of qualities with augmentation of bodies. In the latter, quantity is added to quantity. It is clear, then, that Thomas tries to maintain an absolute separation between quality and quantity, despite the fact that he also speaks of "quantity of power." He remains faithful to Aristotle's analysis: alteration is nothing other than an actualization of what was already present in potentiality, with nothing new being introduced.[26] However, Thomas does also remain faithful to other antique sources such as Boethius and to early scholastic sources such as the anonymous twelfth-century *Liber sex principiorum*.[27] He shares with the latter the Neoplatonic presupposition of the existence of a maximum in each hierarchy, and describes intensification as an "*accessus ad terminum*": the quality progresses nearer and nearer to its full actuality along a range of degrees. But this should rather be explained the other way around. As a matter of fact, Thomas's solution is quite close to that of Boethius. A quality (as a form) is not itself subject to variation, only the *qualia*, the qualified things, are intensified or remitted. Instead of genuine Aristotelianism, this solution assumes a "Platonism" of forms. A form is in itself strictly immutable and always identical to itself; consequently, Boethius states, variation can be found solely in the participation of the subjects.[29] However, participation is interpreted by Thomas in terms of the Aristotelian distinction between act and potency. According to him, such a form, if it could exist as separate, would be in its maximum actuality. When received in a subject, however, it enters into a composition with the potency of this subject, which more or less limits its full actuality. In other words, the relation of participation between a qualitative form and subjects admits of different degrees according to the capacity of the latter to receive the form's perfection. Thomas cites here a Neoplatonic axiom: "everything that is received in a thing is received in it according to the mode of the recipient."[30]

However, Thomas's view is not devoid of ambiguity. Although he ascribes variation to participation only, he maintains that one must speak of intensification, or qualitative augmentation, with respect to the essence ("*secundum essentiam*," or "*essentialiter*") of qualities.[31] He even on occasion speaks of latitude (in virtue).[32] In summary, Thomas constantly

refuses to accept the theory of addition and any divisibility of forms into parts, but he is somewhat imprecise regarding what grounds the possibility of variation. It is obvious that the very nature of qualitative forms provides the opportunity for the forms to be more or less received by subjects. By contrast, if the intrinsic constitution of an essence (as in a substantial form) does not permit this, then a diversified participation is impossible. If so, intensification seems to depend on something inside the form itself. Such is the recurrent objection of Franciscans: variation in actuality (or being, *esse*) cannot be understood without latitude in essence.

Another solution, ascribed to Geoffrey of Fontaine, will be explored, most likely starting out from a Thomist understanding of the problem.[33] For him, every form is indivisible, but some can lose their imperfect existence and acquire a more perfect one: a "more curved curve" and a "greater heat" mean a more perfect curve and a more perfect heat. On this view, intensification is a succession of more and more perfect realizations of a form that remains the same regarding essence or species, although not remaining numerically the same: the previous form, of a lesser degree, is corrupted, and replaced by a new form of greater degree. Hence, there is no composition of new forms with prior forms, or of essential parts with a number of preexisting parts. At each moment, there is just one simple form, wholly new and wholly destroyed when replaced by the next one, so that, under no circumstance, is there addition or accumulation, but only renewal. Therefore, this process of change cannot be intrinsically described as quantitative: here, addition is not even a metaphor. We may only relate each form to an extrinsic measure. The same theory is put forward by Walter Burleigh in the early fourteenth century.[34] But as we already said, the additive theory will be the most effective and will compel recognition even outside the Franciscan Order. No longer regarded as intrinsically immutable and indivisible, qualitative forms will be regarded instead as subject to processes of construction and deconstruction part by part, and to calculation (although not in our modern sense).

### The Jesuits and the Variation of Intensity

The situation is still the same at the end of the sixteenth century. I intend now to examine some discussions among the Jesuits concerning the variation of intensity. The first interesting point is that, although they are professed Thomists, they subscribe to the additive theory. The second is

that they are unable to solve certain serious difficulties concerning continuity. Their failure clearly shows that their inquiry into intensification was predestined to be a dead-end, and had to be undertaken anew on another basis, with the help of new mathematical tools and new conceptions of continuity.

I shall take into account here only a few landmarks in the development of Jesuit thought. More specifically, the authors I shall consider are: Francisco Toledo (1532–1596), one of the first great teachers at the Collegium Romanum, Francisco Suárez, who does not need to be introduced, Rodrigo de Arriaga (1592–1667), one of the most famous scholars in the seventeenth century, and finally Silvestro Mauro (1619–1687), the well-known Aristotelian commentator and the last important teacher at the Collegium Romanum.[35]

## The Triumph of the Additive Theory

In his forty-sixth *Disputatio metaphysica*, Suárez, describing the *status quaestionis*, correctly states that the point is whether or not a qualitative form is divisible into parts.[36] If not, it will have to be granted that variation consists in the fact that a form, always identical with itself, affects the subject either more or less. But how can we explain that the same unchanging quality is able to inhere in a subject following different modes, i.e., more or less perfectly?

The solution of Giles of Rome (according to which there are degrees not in essence, but in existence, more exactly degrees of *esse in subiecto*, being in a subject) does not solve the problem, because he does not answer whether, in order to grow, the single and unique *esse* must be composed of parts. Other Thomist interpretations of the intensification of an indivisible form, namely by processes such as "to be extracted by degrees from the potency of the subject" (*magis et magis educi de potentia subjecti*) or "to be more and more united to the subject" (*magis ac magis uniri subjecto*), are, according to Suárez, insufficient and in need of further explanations.[37] Moreover, the second process ("to be more and more united to the subject") falls under the following objection (as does the theory of succession, as we shall see): if each union is indivisible, it is totally and simultaneously produced by an indivisible mutation, and intensification will be, for the same reasons, a non-continuous succession. This is im-

possible, at least in Aristotelian physics, because alteration, like every motion, is continuous,[38] i.e., it is divisible into disjoined parts but not actually divided: it has unity of itself (*per se*) and is not a discrete entity.

Thus, the old metaphysical explanations, in terms of potency and act, or of the opposition of contraries, appear unsatisfactory. By elimination of all other solutions (we shall later examine Suárez's criticism of the theory of succession), it is clear for Suárez that a quality capable of intensification cannot be indivisible in its entity, but must have a "latitude of parts," by which it can either be totally inherent in a subject, or be partially inherent, according to a greater or a lesser part.[39] But he specifies that those parts are not absolutely similar to each other and so not interchangeable.[40] They have an intrinsic order of subordination, so that the introduction of a subsequent part will necessarily require the introduction of all the preceding ones. In other words, this part, by its own nature, is the first degree, this one the second, this the third, and so on. Otherwise, in remission no degree could be designated as the one which must be the first to disappear when the contrary quality is introduced, which is needed for the process to begin: there would be no reason why this degree rather than that one had to be first expelled. Either all degrees of coldness would be evicted, or none, for if they are all identical, each of them has the same repugnancy or compatibility with the degree of heat which is introduced. Hence, the process must begin by a degree that is by nature the first to disappear.

For Suárez, insofar as the preexisting form is really perfected by a new action of the agent, undoubtedly its acquisition of some perfection is due to the real addition of something, which can only be something of the very entity of the form ("*aliquid entitatis formae*"). But he must recognize that this addition is quite special.[41] That part of a formal entity which was added cannot have actually preexisted before this action. For, either it would have preexisted in the same subject, or in another. If it were in the same subject, there would not be addition of something new (as Thomas pointed out). If it were in another subject, then an accident would migrate from one subject to another, which is impossible. Therefore, what is added did not actually preexist, but is actualized precisely when intensification is in progress. Still, it has to come from somewhere. Now, at least in an Aristotelian ontology, a cause does not formally create something in its effect but transmits actuality and "awakens" something which was lying

potentially in the effect. Hence, Suárez concludes that what is added to the form was contained in the form itself, and is brought out to actuality by an "extraction" (*eductio*). It may equally be said that intensification is an increasing extraction of the form itself out of the potency of the subject, in proportion to its parts or degrees.

But we may also find it rather strange to call this process an addition, for the form receives here nothing it did not already possess: the part of the "entity" that was "added" to its actuality was potentially in itself; thus what is "added" was lying within itself. We might indeed speak of degrees of intensification, inasmuch as the potentiality of the form is progressively converted into actuality. We might perhaps speak of parts of an actualization, by analogy with parts of space in the trajectory of a local motion; or of fractions, by analogy with augmentation, and say, for instance, that one eighth, two eighths, etc., of its full actuality are realized. But the difference is that in a real augmentation, as in the filling of a container, you really add, i.e., you bring from the "outside" some parts which were not present; even if the subject had the potential to receive them, they are not merely extracted from its own resources. Nevertheless, Suárez believes that to have more or less entity cannot be understood other than by a sort of addition. This makes it clear that the old concept of participation no longer has any real significance. It is no longer enough to say that the subject participates to a higher degree in a form: the greater actuality which the form gains must be explained in terms of parts and addition.

In the next century, Mauro will note that all major Jesuit thinkers, first Toledo,[42] then Suárez and Vazquez (although they were so often adversaries), and then Rubio, Arriaga, "*et alii ex nostris communiter*", follow the solution of the Franciscans (and later of the Nominalists).[43] Mauro is quite aware that Thomas, after denying that charity is augmented by addition, concluded:

> augetur solum per hoc quod subjectum magis ac magis participet caritatem, id est secundum quod magis reducitur in actum illius, et magis subditur illi; hic enim augmenti modus est communis cuiuslibet formae quae intenditur. (*Summa theologiae* IIa IIae q. 24 a. 5)

But he prefers to draw from the preceding article (q. 24 a. 4 ad 3m), where Thomas states that in this augmentation charity stretches out "according to its essence" (*secundum essentiam*) or "in essence" (*essentialiter*). Mauro

(like many commentators before him) takes this passage to amount to the admission by Thomas of a latitude in some qualitative forms. But he is wrong in concluding that the whole of his theory does not in fact disagree with Thomas's, even if the words differ. His view is actually quite divergent from that of Thomas, since he concedes what the Franciscans had always claimed and Thomas and the early Thomists had always denied, namely that forms can be compounded.[44] In fact, Thomas intended to say that, in intensification, an accidental form acquires more and more actuality in the subject, but he surely did not mean that there are parts in its essence. For him, forms are simple, "like numbers." In the mentioned passage he writes against those who say that charity solely augments "according to its rootedness in the subject, or according to fervour" (*secundum radicationem in subiecto, vel secundum fervorem*). In contrast to them, he concedes that intensification to some extent concerns the qualitative essence because the nature of an accidental form is to be inherent in a subject, so that for it to be more inherent is to develop its nature, to increase something essential for it. But to augment "according to its essence" is not somehow to augment essence itself with new parts of quiddity.

Nevertheless, Mauro thinks that when Thomas rejects the additive theory, he is only speaking against an addition in the strict sense, in which added elements are first independent, and could become again independent. But in an addition in the loose sense, an existing element is intensified by receiving a complement, which did not previously exist and could not exist separately (it would not make sense, says Mauro, to suppose that the part of heat which corresponds to intensification could be separated and exist elsewhere—there is no transfer of calories of the sort we know today, but rather actualization of the patient by the agent). Just like Suárez, Mauro claims that this complement is not brought from "outside": rather, it is extracted from the potentiality of the subject ("*ex potentia subjecti educatur major et major calor*") in virtue of the action of a warming cause. This is enough, according to Mauro, to permit him to speak of the composition of "essential parts" in quality: "it is increased by (acquiring) more and more new parts of the essence of heat, and not only (by an increase) in existence, or in unity, or in rootedness."[45]

But this typically leads Mauro to make a comparison between intensification and extension. In extension, too, "new parts" of a quality are

extracted from the quality (*eductio*) in order to affect directly other parts of the subject. Likewise, in what is properly intensification, "new parts" of a quality are extracted from it, but they stay in the same part of the subject. This comparison with extension and quantity is finally for Mauro the easiest, the most "appeasing" explanation for the mind to accept[46]—but the one that Aristotle and Thomas precisely tried to avoid.

Of course, Mauro knows the famous sentence: "forms are like numbers." But he replies that when a number is augmented by the addition of another number, it changes in species as *this* number (*in ratione numeri*), but it does not change in nature (of number in general). Likewise, an intensified quality changes "according to intensive magnitude" (*in ratione magnitudinis intensivae*), but not in nature: a heat of 8 degrees is of the same species of heat as a heat of 4 degrees, though it is of a different species regarding intensive magnitude, just as—again the same comparison—heat on the surface of two palms together has the same nature as heat on the surface of one palm, but is different regarding extension.[47]

## The Problem of Continuity

### *Suárez and the First Instant*

Let us now go back to Suárez and examine some problems concerning continuity in intensification.

The Spanish Jesuit criticizes the solution ascribed to Geoffrey of Fontaines (which, as he points out with perspicacity, is related to Thomas's rejection of addition). Instead of composing degrees of existence, as Giles of Rome had done, Geoffrey conceives of intensification as being a succession of more and more perfect occurrences of the same specific form. But this answer seems to Suárez as incredible as that of Giles. There are against this solution two decisive and constantly reiterated objections.

First, during a remission of heat, the more intense quality would be corrupted and a less intense produced. What would be the cause of this fact? Necessarily the cooling agent. That means that coldness, destroying the more intense heat, would produce the less intense heat, and so a quality would produce its contrary.[48]

The second objection is that it would be impossible for alteration to be continuous.[49] Suppose that an accident of heat of 2 degrees is destroyed and replaced by an accident of heat of 3 degrees. What was the duration

of the former accident in the subject? If it lasted for some time, then alteration ceases during this interval, and so was not continuous. If it lasted for no interval of time but only for an instant, then we shall have to ask again about the subsequent accident of heat: what was its duration? This cannot be only an instant, because two instants are never immediately consecutive. Rather, an instant is nothing but a boundary which separates two periods of time.[50] Therefore it must have lasted for some time, a period limited by a beginning and an end, in which, as mentioned above, alteration ceased. So, a successive generation and corruption of qualitative forms would imply discontinuity (which is inconsistent with motion for Aristotle, as we have already noted), since at least one form out of two will have to remain for a moment without changing. If we supposed an intermediate degree between two steps, we would have to also suppose another intermediate between this intermediate and one of the steps, and thus we should proceed to the infinite. That means that in an alteration there would be an actual infinity of successive intensities, of complete, indivisible, really distinct qualities—which is unintelligible. As this infinity would be discrete, it could not be traversed in a finite time,[51] and the process of alteration could not reach its terminus.

It is important that the account of continuity in intensification be comparable to that of the continuity of motion. But the continuity of motion rules out the possibility that motion might consist of a series of jumps from one successive actual state to another. Similarly, says Suárez, although we may conventionally assign eight or ten degrees to a given intensification,[52] the latitude of an intensive quality is not compounded of a definite number of indivisible degrees; rather it is only potentially divisible into parts (without limit), exactly as in the case of a line or any other continuum.[53] Therefore, it is not, he claims, difficult to understand how intensity can be continuous. Taking any definite part of quality, a smaller one can be selected, which can be acquired in a proportionally smaller part of time, and so the whole intensification is effected along a continuous succession. A quality does not jump from the first degree to the second, but traverses all the possible intermediate states, just as a moving body, in local motion, passes through all places without being completely actualized in any of them.

However, Suárez has now to discuss the main reasons why some scholars, such as (Paolo Barbo) Soncinas, a Dominican master of the

fifteenth century, had, against Geoffrey, defended the view that alteration cannot be continuous.[54] They argued that a quality, in the very beginning, is introduced with some entity by an indivisible mutation and in an instant; therefore, it is afterwards increased in the same way, i.e., discretely.[55]

We then meet here the classic puzzle about the initial moment (*"incipit"*). Transition from nothing to something cannot be progressive, there must be a beginning, i.e., for qualities a minimum of intensity above zero, which cannot exist except instantaneously. If we admitted something smaller than this minimum, there would never be a determined starting point, because we could again and again suppose something smaller. So, there is a first intensity which, as small as it is, is given at one go. As a matter of evidence, we can hardly imagine that fire begins to warm water, without producing something, a determined effect. We would perhaps say today that there must be a first quantum of radiant energy, so to speak, which excites some atoms of the heated body, or that there must be a first excitation of those atoms. Here, in terms of ancient physics, what is produced is an accidental qualitative form. Like every form, substantial or accidental, this form has a peculiar entity, a "definite mode of being" (*definitum modum essendi*). So, as soon as the cause is beginning to act, for example the fire to warm, it cannot but produce its proper effect, which is to provoke in the subject the emergence of an accidental form of heat; and that form of heat, as a quality, has a certain intensity, i.e., has a certain first degree, which is given all at once, and which is the foundation for the next.

That argument seems nearly irrefutable, and Jesuits had to take it into account, even though, as Suárez notes, important commentators like Soto (*I Phys.* q. 4 a. 4 concl. 5) or the *Conimbricenses* (*I Phys.* cap. 4, q. 1) thought that every marked part in the latitude can be produced and conserved as smaller, and separately, i.e., independently of any further intensification.[56] The core of the problem, then, is whether or not there exists a minimum which a quality must have in order to exist. Soncinas alleges that there is indeed a sort of atom of intensive magnitude, a "particle of entityhood" (*particula suae entitatis*). Toledo seems to be of nearly the same opinion. But, according to Soncinas, this minimum is an absolutely indivisible, non-intensifiable entity, so that when affecting the subject, it does not tolerate any variation. Toledo's position is more complex. He has recourse to an explanation borrowed from the "Calcula-

tor Suyset,"[57] that is: Richard Swinshead, the Oxford scholar of the mid-fourteenth century, whose *Liber Calculationum*, printed first in Padua (1477) and then reprinted in Pavia (1498) and Venice (1520), much influenced Renaissance thinkers. According to Toledo's reading, a distinction should be made between "degree of form" (*gradus formae*) and "gradual part" (*pars gradualis*). A "degree of form" is the minimum intensity a quality must have in order to subsist: if it were less, it could not actually exist in a subject.[59] As such, it would be a "part of degree," which can be as little as you wish, but which exists only potentially as united with the other parts of the degree with which it forms a continuum.[60] In other words, a degree can be divided into an infinity of parts, but their multiplicity is no more than potential and can never be actualized, whereas distinctions between degrees are actual insofar as a degree can have autonomous being. So, in intension as in extension,[61] there are minimal parts, which constitute unities, but which are not simple.[62] Hence, an "intensive magnitude" is first really divisible into degrees, and each of these are divisible into parts of degrees, which are not solely imaginary and can be considered as existing, but only as united with others from which they are only potentially distinguishable.[63] Toledo concludes that intensification or remission occurs by acquisition or loss of those "parts of degrees."[64] In intensification, the same parts which were in the less intense form remain in the more intense, and are united with the new ones which are acquired (and inversely in remission).[65] So the process is continuous, and does not move forward as if degrees were suddenly produced in separate moments like substantial forms,[66] though each degree is a minimal datum of quality, and though there is such an inceptive degree. Gradual parts are progressively gained (or lost), just as in the extension of a form, as, for instance, when fire is spreading through a body. It is progressing from minimum to minimum, but those minima are all joined together and gained little by little.[67]

The model for alteration is, then, here again local motion, but we may wonder whether Toledo does not vainly try to reconcile two opposite requirements: namely, continuity of movement and the existence of minimal data. In summary, his position is complex because, on the one hand, wanting to save continuity, he thinks that alteration cannot be made of indivisible and separate states, and so he considers each degree to be divisible (but not in separable parts). On the other hand, because of the

commencement (*incipit*) problem, he admits that, like any permanent thing, an intensive quality must begin through some "first being" (*primum sui esse*), i.e., some minimum intensity, produced in an instant. Hence, he proposes a compromise, but it scarcely makes sense that parts of degrees be really acquired one after an other if they cannot exist separately.

Let us now see if Suárez can do any better. In this question, arguments *pro et contra* are equally valuable, and he shares his hesitation with the reader: "none of these propositions can be demonstrated with evidence."[69] However, the second thesis (that there are no qualitative minima) appears to be better suited to the very nature of alteration, which is according to him continuous; so it is "more probable."

But we must note two important qualifications. First, there are in qualities minima and maxima, under or above which substantial forms cannot subsist.[70] In order to be in a subject, a substantial form requires a certain disposition of that subject, which depends on accidental forms. For instance, an animal can live only within a latitude of heat in its body: under a definite degree, it will die, the soul no longer being able to inform this matter. So, *in concreto* there are often indeed minima and maxima of qualities, since accidental forms always exist in relation to a substantial form. But this consideration is of little significance, since we are speaking here of qualities in themselves. Independently of any extrinsic circumstances, the question is whether, by their own nature, they have minima and maxima.

At this point, Suárez introduces a distinction—which may seem quite *ad hoc*. He suggests that a quality can have two states.[71] One is its "natural and perfect state." In order to be in this state, a quality has to reach a certain intensity in a latitude comprised between two boundaries. Outside these boundaries, it will fall in an "imperfect state"—a state that might be understood as one in which the quality is no longer predominant and no longer characterizes the subject. So, from the first point of view, a quality does have a minimum (and a maximum), but as soon as it becomes imperfect, going below this minimum, there is no longer a definite boundary to its smallness: it can become more and more imperfect, tending towards nothingness. Therefore, according to Suárez, heat is truly heat only when it reaches a certain degree, below which it is not only imperceptible for us, but also degraded in itself: it is a case of heat which has lost the fullness of the nature of heat. Nevertheless, it is present even when

coldness predominates. Thus, since smallness can be infinitely divided, i.e., since the continuum of imperfection can be infinitely prolonged, it seems to be implied that a quality never completely disappears however intense its contrary may have become: for instance, a thing which becomes dark remains somewhat white, it has a tiny degree of white, but the latter is not a "perfect" white. But this appears to deny the status of potential being, in which a quality is supposed to be finally reduced to the point of elimination when its contrary predominates. By contrast, in the present supposition, quality, though it has almost vanished, still enjoys an actualized being (unless the "imperfect state" is an intermediary state between potential and actual being, but Suárez does not throw out any hints which might suggest that he admitted such a possibility; in any case, the concept of "imperfect state" cannot be identified with that of potential being—otherwise, why should Suárez have introduced the former?).

The second qualification is again a distinction to be made between divisible parts of a continuum and the indivisibles which have the functions of bonds and boundaries (*"indivisibilia continuantia et terminantia"*).[72] According to Aristotle, we may consider a point on a line to separate two parts, but also to be a junction between them and thereby to constitute the continuity of the line: it is a common boundary, the beginning of one part and the end of another.[73] The same is true of an instant in time.[74] Similarly, argues Suárez, in an accident of heat of 8 degrees, there is an "indivisible of heat" which establishes a continuity with the accident of heat of 7 degrees. This eighth degree begins just when the motion of intensification which culminates in it has been achieved, i.e., in that instant, that "indivisible" of time, where it is true to say: now there is no more intensification[75] and immediately before there was. So, in that instant, something is added which perfects and, so to speak, constitutes the heat of 8 degrees. This "something" cannot be anything other than the indivisible term of that heat. All its divisible parts have been acquired during all the divisible parts of the time before. The indivisible comes to be through the ultimate achievement of the heat of 8 degrees, not in an interval of time but rather in an instant, so that we may say: from now on, there are 8 degrees, and immediately before, there were not.[76] So far we find that Suárez agrees with Aristotle, who stated that there is a first, indivisible moment when the motion is achieved and the mobile has changed.[77]

But Aristotle added that there is no starting point of change, no first instant where the subject begins to change.[78] He thought that as soon as it is changing, it is in the moving condition, which is continuous and has no indivisible parts. So, before each moment we might point to as the first, we could point to a preceding one, defining a smaller interval of time. For Aristotle, then, a first instant of achieved motion is possible because it is not itself a change but a boundary of change; but it is not possible to find a first instant of motion as such, or else there would be a discontinuity when motion would pass from this indivisible to the next moment.

It is here that Suárez has to diverge from Aristotle, since he admits the necessity of there being a first intensity produced by the cause. He could perhaps claim that for Aristotle it is only in respect to the subject and the time that there is no first moment in change, because of their divisibility. In our present case, however, where what is changing is this or that attribute of the subject, and, specifically, a quality, the situation is different, since qualities are marked by indivisibility. However, Aristotle here means that qualities are indivisible *per se* (a thesis which was abandoned by most of the scholastics from the end of the thirteenth century; Suárez himself thinks, as we have seen, that a quality can be considered as composed of parts). A quality is only divisible *per accidens* insofar as its subject is divisible into parts. From this point of view, there is no first moment of alteration: we cannot say that this part of the subject is the first to be changed, since we can find a smaller part, and so on indefinitely. But, according to Aristotle, neither is there, in quality itself, a first part to be intensified: not because we could always designate a smaller part, but because there are no parts at all. So, although in alteration quality is at once present at a definite moment, by a sudden mutation[80] (it can be still more and more intensified), this is not a first instant in the sense which is defended by Suárez, i.e., an instant when a first minimal qualitative part is realized.[81]

In order to prove the existence of such a first part, Suárez reasons by analogy.[82] In the same way that there is an "ultimate terminating indivisible" (*ultimum indivisibile terminans*), so there is at the lower range of latitude an indivisible first term where intensification begins.[83] All finite continua are enclosed by two boundaries. In successive things, these boundaries are extrinsic because they do not simultaneously exist with the whole. For instance, a local motion does not exist as such when it reaches

its boundary, which is the instant when it ceases. In non-successive things, such as substances or accidents like quality in itself, these boundaries are intrinsic, because they actually exist with the whole, just as in the case of the two boundaries of a line (this is consistent with the additive theory, where preceding qualitative parts are conserved throughout the process).

Hence Suárez proposes another distinction—which may be judged somewhat *ad hoc*. When we say that there is no minimum in a quality, we are talking about its divisible latitude, which alone is quality in the proper sense.[84] The first indivisible is only a "commencement of quality" (*initium qualitatis*): although it is not yet really the quality, it is indeed something of the quality, by which the latter can be said to begin its existence. If we accept such an expedient (i.e., if we assume together these two propositions: quality is already present and is not yet present, but not from the same point of view), then the commencement problem seems to be solved, or rather dissolved, with some satisfaction of both of the opposing requirements. On the one hand, when the agent initiates an action, something is indeed immediately actualized in the patient with this first step: there is an effective beginning with a determinate effect. On the other hand, quality in the proper sense remains infinitely divisible, the indivisible being only a boundary, and the process of acquisition of parts being continuous.

However, a difficulty remains. Such an indivisible commencement, since it is a boundary, appears to be naturally unable to exist by itself, separately from following parts. So it marks the beginning of the existence of the quality, but at the same time some other parts are given, a certain intensity of quality, and this could lead to the acknowledgment that Soncinas is right.

Suárez answers that we must make a distinction between a genuine first moment and an intensification properly described. The latter requires that there already exists some quality to be intensified. In other words, intensification does not proceed from nothing to something, but rather from something to something more. Thus, when it begins, some quality has already been produced in a certain degree with an intrinsic indivisible term of that degree; then, a superior degree can be added (and this is the intensification). Therefore, this superior degree, although it is a permanent thing, does not begin through some "first being" (*primum sui esse*), but from its own "ultimate non-being" (*ultimum non esse*). The latter concept was introduced in the Middle Ages as a response to the impossibility of a

first instant in change. When we speak of the beginning of a motion, what we refer to is in fact the last instant of non-motion. There is no first instant at which Socrates is running, but there is a last instant at which he is not running. This view can be found in many treatises *De primo et ultimo instanti* (Burley's being one of the most famous). However, this rule was true only for successive entities like motion, the boundaries of which are extrinsic. The inception of permanent entities was generally considered to be intrinsically limited by a first instant of being. Here, Suárez seems to apply the concept of "ultimate non-being" to a non-successive entity, namely a qualitative part. But for him the concept applies, because when intensification is properly described, intensity is progressively acquired, "part by part."[86] In his view, intensification does not begin with a determinate first part of quality, since we can always imagine a smaller part. Nor does it begin with some indivisible, since properly described intensification starts when there already is some part of quality, which according to Suárez is an indivisible (and there cannot be two immediately successive indivisibles since an indivisible is a boundary). This term is not corrupted by the intensification because, as we saw, during intensification no positive entity is destroyed in the remitted quality. Further, another indivisible cannot be immediately added, since in a continuum two immediately successive indivisibles can never be found. Therefore, in intensification, that indivisible which was first "terminating" for the preexisting part, becomes "continuating" with respect to the part which is now added (and which can be as small as we wish).

Concerning the genuine first instant of a quality, Suárez says that we must consider how the relevant cause of the intensification concretely acts.[87] If a natural agent encounters no resistance, it instantaneously acts as intensely as it can. By itself, its acting power is not restricted to an "indivisible" of the quality, but is determined to immediately exert the maximum efficiency which is compatible with the receptive capacity of the patient.[88] So, it produces a part of a quality (or perhaps the whole), but not a minimum; it is then a "first being." Generally speaking, an agent operates "not in a mathematical way, but in a physical one": it does not produce spots or patches of quality, but degrees, which we can mentally divide, but which are really given at once as wholes, supposing, again, that the subject offers no resistance (as air does when it receives illumination).

Now, if a natural agent encounters some resistance in the subject, its action will be progressive, and subsequently will be decomposed along a succession of qualitative parts, not only with respect to the extension, but also intensively inside each single spatial part of the subject. Explaining how this begins is the hard part. Suárez resorts to the same concept as above, and again answers that, in its first introduction, quality begins by an "ultimate non-being," rather than by a "first being," although this "indivisibile beginning" is produced by mutation in the last instant of non-intensification.[89] Surely, if the agent can produce some effect, its power is at least a little bit stronger than the resistance of the patient, and then it produces a certain part of quality. But it does not follow from this that there is a minimum in quality itself, since this part could be infinitely smaller in another subject. So, we may concede that something is instantaneously produced ("*aliquid simul fieri in instanti*"), but, because of the diversity of strength in agents and of resistance in subjects, that "something" can be smaller and smaller in intensity. It follows that it has the same sort of inception as motion and as intensification when the latter is properly described. We cannot designate a first instant of existence and a minimum degree which would be valid for this or that quality in any circumstances. We can only point to its "ultimate non-being," i.e., to the last instant when there is still an absolute lack of this quality. Afterwards, the agent introduces a "first indivisible of quality" which is sufficient to overcome the resistance of the patient. But this indivisible is variable according to each case, so we cannot assign a general value of minimal intensive magnitude. Moreover, this indivisible is not a permanent entity but is "in progress" (*in fieri*), it has the transitory nature of motion, and, as a boundary, is immediately followed by a "something of the quality" (*aliquid qualitatis*).[90]

*Arriaga and the Division of Space, Time and Motion*

In this way Suárez tried to avoid the dilemma between continuity and beginning by means of a distinction between a general minimum and a particular indivisible. But the latter, as a first effect of the agent, appears to be in reality something more than a mere boundary. Therefore, despite Suárez's efforts, some Jesuits in the seventeenth century will take the opposite view: according to Arriaga and his master, Pedro Hurtado de Mendoza, neither augmentation nor alteration are continuous.

Arriaga's argument is the following:[91] Suppose that fire, applied to a log, acts continuously, that is, in each instant; then it will have burnt up the log in the twinkling of an eye. As a matter of fact, like an infinity of points in a line, there are, in the twinkling of an eye, an infinity of instants, so that, even if the fire is weak, and even if the patient is resisting, it will attack the wood an infinity of times and so it should be in a position to consume it entirely, even in this very short interval. The premise is that, for Arriaga, the given instants are not necessarily only virtuals or boundaries. He is tempted to dismiss the Aristotelian thesis as to the potential infinite divisibility of time, and to believe that no continuum is actually undivided. If there were no really distinct parts, they could never be really separated.[92] Conversely, suppose two separate volumes of water are joined: they remain really distinct parts. If they did not, they would have lost their intrinsic essence, since they were distinct from each other by their own essences before the mixing.[93] Therefore they are joined but distinct, just as soul is distinct from body although it is joined to it.[94] Whereas Suárez hesitates between the Aristotelian conception of the indivisible as a boundary, and the physical necessity of the indivisible as a first part of quality, Arriaga pleads for a rehabilitation of Zenonists, atomists and others who thought that any finite continuity is compounded of actually distinct, finite, indivisible parts[95] (parts which are distinct but not separated: they are joined, and this makes the continuity). The same holds of space, or of time which is made of indivisible instants.[96] Likewise, there are in the latitude of a quality, indivisible intensive parts.[97] For instance, as long as a body, on being warmed, has not yet reached its maximum temperature, it maintains one final degree of cold, which will disappear suddenly, and not by a progressive remission.[98] In such continuities, motion can be discontinuous in the following sense: It can progress, definite part after definite part, or more frequently, block after block of parts. It can occupy this portion (of space or of a latitude), then stop and rest, afterwards occupy a new portion, and so on.[99] For example, according to Arriaga, if A is two paces removed from a source of noise and B is three paces removed, then they hear the sound at the same time; but not if A is two paces removed and B one thousand.[100] This is supposed to prove that sound alters the medium (the air) portion by portion, i.e., *via* whole portions of space. Two and three paces are included in the same portion, not two and a thousand. The extension of a form, Arriaga

concludes, is not absolutely continuous, but interrupted by pauses, during which it prepares the next section to be occupied.[101] Similarly, an intensification progresses by alternating qualitative jumps with stops. Nothing prevents several indivisible parts of quality from being acquired at one time: this does not imply that their number is infinite, for there are not constant mutations, but only mutations at determinate moments, and only a finite number of them.[102] This is also the reason why, in alteration, fire does not consume the log in the twinkling of an eye: the log burns piece by piece, in a finite sequence, and with some pauses. More generally, this is true of any motion because of its non-instantaneous nature: "slowness of motion cannot be correctly understood without short delays."[103] Arriaga explains that it may happen that a heavy stone exerting pressure on a beam of wood causes it to break not immediately but only after some years: the reason, he thinks, is that the action of the stone (its pressure on the wood) is not continuous, but has some periods of rest![104] Therefore, a natural agent, constantly applied, can interrupt its action inside the same motion. Likewise, a feather falling down keeps the same weight, and air remains the same; however, we can see that its fall sometimes stops, and then goes on.[105] Arriaga also knows that water, drop after drop, will finally wear away the surface of a stone in such a way as to create a hole; but according to him the first drops do not take away the slightest part of matter because we do not perceive any matter being taken away.[106] Such ordinary experiences, according to him, compel one to admit "short delays" or pauses.[107] Therefore, the same cause does not always produce the same effect, an agent does not continuously act along one and the same motion.[108] One will conclude that, although Arriaga may have been well-known for his dialectical skill, his was not a great scientific mind.

*Mauro Between Addition and Succession*

More reasonably, Silvestro Mauro leads us back to the impossibility of the view that alteration not be a continuous motion.[109] He repeats Suárez's argument:[110] why should an agent stop acting at some moment and afterwards act again, since the agent is not supposed to be a voluntary and free cause, but rather a natural one, which necessarily acts in accordance with its nature? His opponents will perhaps answer that the agent ceases to communicate its actuality to the patient but continues to prepare the still unaffected parts of the latter. However, this reverts to the conces-

sion that it continually acts, since the patient is continually being disposed in each part of time which separates two intensifications. If not, one can then ask once more why this disposing action is interrupted and should afterwards start again. If, as it is the case, it does not cease, how could the action of intensification itself not be continuous?

Therefore, as soon as the agent acts, an alteration takes place, and at the same time the subject is disposed while its form receives new essential parts of quality. As a matter of fact, Mauro also objects to the various Thomist statements to the effect that it is simpler to admit qualitative parts, and we have already seen that he defends an additive theory. However, according to him, although that theory is true for qualities like heat, light, impulse, etc., it is not true for motions, figures and vital acts. As far as the latter phenomena are concerned, Mauro takes into account the idea of succession. And it is noteworthy that for him the main advocate of the theory of succession is not Geoffrey of Fontaines (or Walter Burley, or Toledo), but Durand of St Pourçain, the dissident Dominican of the early fourteenth century. Durand proposed a somewhat different version, which is of some interest because it provides an answer to one of the objections that Suárez will make.

Durand differs from Geoffrey in this: he admits an intrinsic latitude in those forms capable of intensification, a divisibility of their essence into degrees, so that their intensification does not depend only on participation by the subject.[112] But he agrees with Geoffrey in claiming that there cannot be an addition of a quality to a quality, or an informing of a quality by a quality, and that there cannot be two qualities of the same species in the same subject. However, Durand pays special attention to the task of placing continuity on firm theoretical ground. He is quite aware that it can be raised as an objection to the succession theory that there would be, in a definite process of intensification, an infinity of actually different forms, and that it would be impossible to reach the term of this alteration. He also knows that, in the succession theory, it is not proper to say that a form is intensified: each single form disappears as soon as it appears, since it is immediately replaced by a more intensified form, until the end of the change. Instead of an intensification of one and same form, one should therefore rather speak of a succession of different forms, though they differ only numerically and in intensity (not in nature or species).

Hence Durand states that, in intensification, the agent successively actualizes not exactly new, more perfect, forms, but more accurately parts of the same total form which is finally acquired. But those parts are not amassed as in the addition theory. Each of them, being more perfect than the preceding one,[113] overcomes and replaces it. However, they are all partial regarding the final perfection of the form, the *telos* of this motion, of which they are successively approximating states. Durand tries to guarantee in this way the continuity of the motion, and the unity of the intensified form. It remains the same insofar as it is its own parts which are successively acquired. It is one, not by indivisibility, but by continuity of its parts.[114] That form has then exactly the transitory nature of any form in any movement: until rest is reached, it is a "flowing form" (*forma fluens*).[115] In local motion, this accidental form is place (*ubi*): each point of the trajectory is successively covered, but the moving body never stays in one place except for that place where its motion terminates. It has left its *terminus a quo*, and is going to its *terminus ad quem*, passing through every intermediate state in between. In its turn, each intermediate state is a *terminus a quo* for the next step (which plays the role of a temporary *terminus ad quem*), so that the moving subject never stops in it, but is, as Aristotle said, passing over it. Thus, motion is divisible and yet continuous. Similarly, we find in intensification nothing other than a flow of just this sort, an unceasing passing away through new states, until an end is reached. This allows Durand to escape the objection based on the notions of infinity and discontinuity: transitory states are distinct only potentially (hence, there is no actual infinity), and they are not introduced *via* instantaneous mutations.[116]

As Mauro concedes, Durand's view could explain all intensifications perfectly well if it could only explain how two contrary qualities act upon each other.[117] But here lies the weakness of his solution. Let us consider two contrary qualities, heat and coldness. Each of them acts on the other: at the same time that it reduces its contrary, its contrary reduces it. For instance, a very hot hand, A, and a very cold hand, B, come into contact. A reciprocal action takes place: hand A warms hand B, and hand B cools hand A. This reciprocal action would be impossible, claims Mauro, if qualities were changed by succession of remitted or intensified forms, because this would imply a reciprocal causality of efficient causes, which

would have a mutual priority of nature. The heat in hand A is the efficient cause reducing the cold in hand B. Suppose, with Durand, that this decrease is a production of a lesser cold replacing a greater cold. Then, the heat in hand A will be the cause of the lesser cold in hand B. But, inversely, the cold in hand B will be the cause of the lesser heat in hand A. Therefore, they cause and transform each other in the same instant, and this is contradictory: the cause must precede its effect, since to act efficiently is to act by one's own existence, but here this existence would precisely depend on the thing on which it is supposed to act.

For that reason, Mauro rejects Durand's solution, and adopts the theory of composition of parts, at least for qualities of the third species.[118] But the case is different for augmentation of motions, figures and vital acts. There, acquired parts are not collected, but successively actualized. For instance, when we say that a motion accelerates, then in reality a faster motion succeeds a slower one, just as in an increasingly bent bow a more pronounced curve succeeds a less pronounced one.[119] We cannot ascertain ("*mihi est imperceptibile*") that the bow retains the previous curve it had, and only receives some new complement of curvature to make a greater one, in the same way as someone increases his fortune by adding 5 coins to the 100 coins that he already has and that he keeps. This augmentation is an addition; but a more pronounced curve is a new curve in comparison with the previous one. Similarly, an eye does not see more accurately the same object by keeping the preceding vision and adding a degree. Such a vital act is an "intentional motion" towards an object and its increasing is intensive; it does not imply quantity at all: it is perfective. This means that it elevates form precisely above matter and quantity, to a more perfect act. Thus a more precise vision is a new vision. Might we also say that an increase of desire is an addition of a part of the desire to preceding parts? The paradigm for a gradation in perfection cannot be quantitative, but strictly qualitative.

These reflections on sensitive and mental acts are of considerable interest. Much later, Henri Bergson will address similar criticisms towards "psychophysics," which was an attempt to quantify this field. According to him, it is false to assert that a sensation augments in proportion to the action of its cause. In reality, the same sensation does not remain after being increased, but there is a succession of qualitatively different sensations.[120]

But, according to Mauro, what fits psychic or intentional motions also fits natural motions. This view, however, had already become untenable when he wrote. Galileo had already applied a sort of addition theory to uniformly accelerated motion: the increase of speed (*"intensio velocitatis"*) is proportional to length of time (*"fit juxta temporis extensionem"*), each new equal degree of speed (*"momentum velocitatis"*) being added in each new equal part of time.[121] Moreover, when saying that it is not the same movement that is accelerated, but there is a succession of different faster movements,[122] Mauro seems unable to conceive the change of the same phenomenon, i.e., its continuity through the different values of a variable (velocity). Even if, following Durand, he alleges that intermediary states are not actually but only potentially separate, he still cannot save continuity. The problem is the same for his thesis concerning other qualities (of third species) which admit an intrinsic composition. There too, new parts of essence are unceasingly introduced without any interruption; and each of those parts must be considered as something real because of the reality of the action of the agent. But if so, we may ask how all these parts deliver the required continuity. On the one hand, if they are only potential, what is the consistency of the supposed reality of each of them? What will be, at each moment, the effect of the agent that is acting constantly? On the other hand, if they are actual, then some minimum has to exist, otherwise we would again have an infinite division; but then, how could the motion not be discrete, being compounded of a succession of minimal data, part by part?

## Conclusion

Ancient thought, then, runs into a problem that modern physics will be able to solve only on a new basis and with new tools: how to create continuity from discontinuity? We know that Newton in the *Principia* introduces the concepts of resisting force and centripetal force as discontinuous forces.[123] He supposes that there is a unique instantaneous impulse at the beginning of each particle of time. But now he has to explain the continuity of forces, such as gravitation, which act without interruption. To this end he claims that one can take those parts of time to be as small as one wishes, i.e., infinitely small. With this claim, intervals converge to continuity. In other words, the solution requires an infinitesimal calculus, with the decision to consider as null the smallest differences,

when passing to the limit. This appears still more clearly in further formulations employing the analytic method, as for instance when Varignon defines concepts of instantaneously accelerating force and velocity. In an accelerated motion, we may consider that, in an infinitely small part of time, an added quantity does not modify the former quantity ($v + dv = v$). Thus in each instant, velocity is taken to be uniform, although it is constantly increasing in regard to duration. It is as if the continuous motion consisted of an infinity of determinate states, so that we can tell at each instant what the velocity of a moving body is: it has a definite value. This allows us to escape the dilemma which confronted Aristotelian thought. Either it fixed actual intermediary states, in which case there was a succession but no continuity or motion, or it considered intermediary states as transitory and potential, like places (*ubi*) in local motion, where the subject is fully actual in none of them. But if so, what about the actuality that whiteness must have as an effective determination of a body, which in each instant, as a derivative, is actually white in this or that degree? When Christian Wolff, in his *Ontologia*, re-examines the question of intensification and remission in the light of the modern theories, he avoids both difficulties.[124] He considers intensive magnitudes as "quantities of qualities," compounded of degrees, capable of addition and subtraction. But these parts do not exist independently as separate entities: they are "imaginary." However, an "imaginary being" is not a chimera: it is a "substitute" (*vicarium*) for real being which we may use in order to understand things more clearly, as mathematicians do. Like the infinitesimal differences, these imaginary parts disappear in the end: they may be as numerous and as small as desired, and they are consistent with the unity and continuity of motion, which permit the calculation of variation at each instant.

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## NOTES

1. Classical studies on this topic are: A. Maier, "Das problem der intensiven Grösse," in *Zwei Grundprobleme der Scholastischen Naturphilosophie*, 2 Auf., Roma: 1951; E. Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," in *Archives d'Histoire Littéraire et Doctrinale du Moyen Age*, XL (1973), pp. 223–83; J. E. Murdoch

and E. Sylla, "The Science of Motion," in D. C. Lindberg (ed.) *Science in the Middle Ages*, University of Chicago Press, 1978, pp. 206–64. I beg to mention also my own paper: "Plus ou moins: le vocabulaire de la latitude des formes," in J. Hamesse and C. Steel (ed.), *L'Elaboration du vocabulaire philosophique au Moyen Age*, Turnhout: Brepols, coll. "Recontres de Philosophie médiévale" n°8, 2000, pp. 437–88.

2. *Categoriae* 8, 10b 27–29.

3. This is not a paradox: a length of five meters is not more or less long than itself or another length of five meters.

4. "[I]dem secundum idem ex tali fiat magis tale," as Silvestro Mauro concisely put it (*Quaestiones philosophicae*, Paris: 1876, t.II, q. XXVII, p. 546).

5. Cf. *Cat.* 6, 5b 7, for whitness in a surface.

6. *Physica* V 2, 226b 1–2, VII 2, 244b 6–12.

7. Cf. *Phys.* VII 3, 246b 15–17.

8. Cf. *De Generatione* I 4, 319b 12–14, 25–26.

9. Aristotle himself, as we have just seen, included affections of the soul in the third species of quality, which tolerates the more and the less.

10. "[A]ugmentum caritatis simile est augmento qualitatum naturalium" (*Scriptum super libros Sententiarum*, 1. I d. 17 q. 2 a. 1 [ed. Mandonnet, Paris: 1929], t. I, p. 416).

11. *Epistola 186 (ad Paulinum)*, 3.10 (CSEL t. 57, p. 53). Cf. P. Lombard, *Sententiae* 1. II d. 26, 2, 3 (Grottaferrata: 1971, t. I, p. 472).

12. See examples in my paper: "Plus ou moins: Le vocabulaire de la latitude des formes," pp. 455–58.

13. See J.-L. Solère, "D'un commentaire l'autre: l'interaction entre philosophie et théologie au Moyen Age, dans le problème de l'intensification des formes," in M.-O. Goulet (ed.), *Le Commentaire entre tradition et innovation* (Paris: Vrin, 2000), pp. 415–16.

14. *De Trinitate* VI, 8.

15. *De Gen.* I 5, 321a 4–5, 20–21, 321b 13.

16. *De Gen.* I 5, 320a 28–29, 320b 30–31. Augmentation and diminution therefore presuppose a change of place, but not an alteration (id., 320a 17–19).

17. *Cat.* 14, 15a 17–31.

18. *Phys.* V 2, 226b 2–8, VII 4, 229a 2–3, *Top.* III 5, 119a 27–28.

19. *De Gen.* II 7, 334b 7–9, 20–26.

20. See, for instance, S. Bonaventura, *Sent.* I d. 17, p. II art. un. q. II. Contrast however Augustine, *Liber 83 qu.*, 36: "Nutrimentum eius (caritatis) est imminutio cupiditatis: perfectio, nulla cupiditas." Furthermore, this objection even seems to be inadequate in regard to Aristotle's physics, which sees coldness as nothing but a privation of heat (*De Gen.* I 3, 318b 17), but as playing the role of a contrary (cf. *Phys.* V 2, 225b, 226b 3, 6–7): blended with heat, it lessens the actuality of the latter. One should rather say with Henry of Ghent that contrariety is not the cause of intensification/remission, but, to the contrary, that the possibility of intensification/remission in a form having a latitude is the reason why it can be blended with a contrary (*Quodlibeta* IV, q.15, Paris, 1518, f° CXXV r°, S).

21. Cf. *De Gen.* I.5, 320b 30–31.

22. See, for instance, Henry of Ghent, *Quodl.* IV, q. 15, f° CXXVIII v° O: "Et quemadmodum ibi quod maius est mole, continet in se minus pluries, vel semel, vel aliquam eius partem secundum partes molis, consimiliter maius virtute in se continet minus secundum partes perfectionis."

23. Cf. *Metaphysica* VIII 3, 1043b 33–1044a 9; V 14; 1020b 7–8. "Formae sunt sicut numeri" as the Scholastics concisely say.

24. "[A]ddendo unitur et uniendo additur," S. Bonaventura, *Questions disputées* ("*De caritate*"), ed. P. Glorieux (Paris: Editions franciscaines, 1950), p. 6. Cf. *Phys.* VII 2, 245a 12–13.

25. *Sent.* I, d. 17 q. II a. II.

26. *Phys.* IV 9, 217a 33–34.

27. Ed. L. Minio-Paluello and B. G. Dod, in *Aristoteles latinus*, I, 6–7 (Bruges-Paris: 1966).

28. *In Cat.*, III, PL 64, 257 C-D. See also Martianus Capella, *De nupt.*, ed. A. Dick and J. Préaux, Teubner, 1. IV, § 370, and Pseudo-Augustinus, *Categoriae decem*, = *Categoriarum Paraphrasis themistiana*, ed. L. Minio-Paluello, *Aristoteles latinus*, I 1–5, p. 164, § 132.

29. "ipsas quidem habitudines nulla intensione crescere, nec diminutione decrescere putat, sed *eorum participantes posse sub examine compositionis venire*, ut de his magis minusve dicatur. . . . " (*In Cat.*, 257 B-C). Cf. Porphyrius, *In Cat.*, ed. Busse, CAG IV 1, p. 138, 4–6, and *Isagoge* XIV 2, XXIV 4, XXVI 2.

30. "Omne quod recipitur in aliquo, recipitur in eo per modum recipientis": cf. *Liber de causis*, IX 99, XIX 157–58, XIX 170, XXIII 177–79.

31. *Sent.* I, d. 17 q. II a. 1, *Quodl.* IX q. 6, *Sum. Theol.* IIaIIae qu. 24 a. 4 ad 3m.

32. *De virt.* q. 1 a. 11 ad 16m.

33. *Quodlibeta* II q. 10, IX q. 11, XIV q. 5. Thomas clearly rejected an older version of the succession theory (*Sent.* I, d. 17 q. II a. 1), also mentioned by Bonaventura (*Sent.* I d. 17 p. II art. unicus q. 1): when we say that days grow longer in the spring, we do not mean that the same day is first shorter and then longer, but that a longer day follows a shorter one; likewise, augmentation of charity signifies that a lesser charity is followed by a more intense one. Thomas ceases to mention this theory after his *Commentary on Sentences*, as if it had been definitely refuted. However, there is an analogous theory regarding the problem of substantial form. Because he thinks that a unique form at one time gives the whole of the quiddity, Thomas finds himself agreeing with Avicenna that the growth of a living being consists in "partial generations" (*In De Gen.*, I, cap. III lect. 8), i.e., that in each new stage the preceding imperfect form is destroyed and replaced by a more perfect one. Regarding qualitative accidental forms, he on one occasion speaks of the "successio alterationis, secundum quod aliquid idem est albius vel minus album" (*In Phys.* VI, lect. 7 n° 8). This means that a quality is said to be intensified when the relation of participation between it and a subject successively takes different degrees.

34. *Tractatus de formis* (Munich: Bayerische Akademie der Wissenschaften, 1970); *Tractatus de intensione et remissione formarum* (Venice: 1496).

35. For the context, see U. Baldini, *Saggi sulla cultura della Compagnia di Gesù (secoli XVI–XVII)*, Padova: CLEUP Editrice, 2000 (especially ch. VIII, on physics), and *Legem impone subactis. Studi su filosofia e scienza dei Gesuiti in Italia 1540–1632* (Roma: Bulzoni Editore, 1992).

36. Sectio I, § 3 sq., p. 753b sq. (*Opera omnia*, t. XXVI [Paris: Vivès, 1866]).

37. Id., § 13 sq., p. 756b sq.

38. Cf. *Phys.* V 4, 228a 20 sq., VI 4, 234b 10 sq.

39. Id., § 35, p. 764a.

40. Id., § 34, p. 763b.

41. Sectio III, § 3, p. 770a sq.

42. *Commentaria una cum quaestionibus in VIII libros Aristotelis de physica auscultatione* (Lyon: 1598), p. 437: "consentio cum D. Thoma, in intensione fieri maiorem

radicationem, sed tamen hanc fieri per additionem gradus alicius de novo producti"; *ibid.*: " . . . argumenta Scoti, quae sunt optima. . . . "

43. *Quaestiones philosophicae*, op. cit., t.II, q.XXVII, p. 546. Arriaga more generally says: "Communis iam inter recentiores Philosophos sententia docet, intensionem consistere in additione novae entitatis qualitatis ad praecedentem, remissionem vero e contrario in amissione alicuius partis illius qualitatis" (*Cursus philosophicus* [Lyon: 1669], De gener., disp. III, sect. I, subsect. 3a, p. 640a).

44. The answer to the following question: Can forms be intrinsically limited or not?, is telling. For Thomas, actuality of form was only limited by its reception in potentiality of subject: if a qualitative accident such as heat could exist without being supported by a subject, it would *ipso facto* exist in its maximum degree. Of course, Suárez holds the opposite view, close to the Scotist conception of essence (sect. I, § 36, p. 764): an intensified and a remitted form are distinct not only in their mode of reception in the subject, but in their own entity (which makes them able to affect the subject). God could create outside any subject two unequal accidents of heat, one of 8 degrees, having the totality of perfection and entity of heat, the other of 1 degree, having only a part. If then they affected a subject, the former would give it 8 degrees of heat, the latter 1. Therefore, such forms are different by their intrinsic constitution.

45. "[A]ugetur . . . secundum novas et novas partes essentiae caloris, et non solum secudnum existentiam, vel unionem, vel radicationem" (*Quaestiones philosophicae*, id., p. 547).

46. Id., p. 549.

47. Id., pp. 550–51.

48. "[P]rofecto haec opinio indigna videtur philosopho," says Toledo (*Commentaria . . . in Phys.*, op. cit., p. 436). It has also sometimes been objected against Geoffrey that there is no reason why the agent, in introducing a more intense form, would have to destroy the preceding form, for where the latter belongs to the same species it is not opposed to it. However, Arriaga remarks that this objection is not convincing, because there is an example of such a phenomenon—and we again find an analogy with the growth of embryos: a mother's action destroys the preceding incomplete substantial form (in spite of the fact it is not opposed to it), when it actualizes the following one (*Cursus philosoph.*, De Gener., disp. III, sect. I, subsect. I, § 5, p. 638a).

49. Cf. Suárez, *Disp. Met.* XLVI, sect. I, § 9, p. 755b.

50. Cf. *Phys.* VI 1 231b 6 sq., VI 3, 233a 33 sq.

51. Cf. *Phys.* VI 2, 233a 21 sq., VI 9–10, VIII 8, 263a 4–263b 9.

52. This traditional scale seems to have a medical origin: see J. E. Murdoch and E. Sylla, "The Science of Motion," loc. cit., p. 232.

53. Id., §§ 12–13, p. 773a–b. Cf. *Phys.* III 1, 200b 20, VI 1–2, VI 4.

54. Id., sect. III, § 6, p. 771a.

55. The second degree, which is added to the first, is proportional to it and equally indivisible, since the agent cannot act differently and the subject is disposed in the same manner. Such successive indivisible mutations cannot together form a continuity.

56. Sect. IV, § 5, p. 777b. Their main argument still lies in a comparison with extension, which is justified by the fact that alteration must be continuous. As Suárez puts it: "ergo quaelibet pars minor et minor, quae potest existere in tota qualitate, posset etiam per se existere. Haec enim sola est ratio ob quam secundum extensionem quaelibet pars lineae, quantumvis minor et minor, quantum est ex se, potest per se existere sine aliis, quia ab eis

non pendet, et idem est de quocumque caloris minimo secundum extensionem: ergo idem est de intensione" (id. § 6, p. 778a; cf. *Phys.* VI 1, 231b 16).

57. *Commentaria . . . in Phys.*, op. cit., lib. IV c. IX, q. XII, p. 434.

58. See J. E. Murdoch and E. Sylla, "Swineshead, Richard", in C. C. Gillispie (ed.), *Dictionary of Scientific Biography* (New York: Charles Scribner's Sons, 1976); W. A. Wallace, in *Prelude to Galileo* (Dordrecht: D. Reidel, 1981), pp. 78–90 ("The *Calculatores* in the Sixteenth Century").

59. Id., p. 434.

60. "Pars vero gradualis formae est, quicquid formae quod non potest ob eius remissionem actu per se esse, sicut in extensione quantitatis sunt particulae ita minimae, ut non possint esse per se, nisi cum aliis continuae, ita in forma intensa sunt caloris particulae ita remissae ut non sint unquam per se, et (q)uas vocamus partes graduales. Unde in uno gradu sunt plures graduales partes, quia est divisibilis in infinitum in potentia" (*ibid.*).

61. Significantly, Toledo points out that the concept of latitude includes both extension ("quantitas extensionis") and intension ("quantitas intensionis') of a form. He then enumerates several sorts of latitude.

62. Suárez summarizes Toledo's position, which appears to be an intermediate position, as follows: "in latitudine intensiva qualitatis nulla possit signari pars, quae non sit ex partibus composita, et ideo in ipsa totali entitate nulla sit signabilis pars ita parva, quin alia minor signari possit. Nihilominus tamen signari possit aliqua pars tam parva, seu tam remissa, ut non possit minor seu remissior conservari, aut esse, quasi disjuncta et separata ab aliis partibus, quod aliis verbis dici solet, non dari minimum gradum remissionis inexistentem in tota qualitate, dari tamen minimum separatim existentem, quia qualitas sub minori intensione fieri aut conservari non potest" (sect. IV § 3, p. 777b sq.).

63. " . . . talis magnitudo dicitur intensionis, cuius partes, quae actu possunt existere per se in subiecto, gradus dicuntur, quae vero non possunt per se existere nisi cum aliis coniunctae, quae inexistentes dicuntur, et in potentia graduales vocamus partes' (Toledo, p. 434).

64. Id., p. 438.

65. "[I]n hoc consentio cum Scoto," says Toledo. One should however note that for him, too, the alteration process is improperly conceived as an addition. As for Suárez and Mauro, what is added did not exist before being added; rather, it is drawn out of what receives the addition: it was potentially in it, and is then actualized (p. 438).

66. "[N]on proceditur per gradus, ut in singulis instantibus singuli gradus producantur, sed per partes graduales" (id., p. 438).

67. "[P]ostquam ignis in primo minimo est introductus, postea successive extenditur per minima coniuncta, nec alia minima subit, nisi paulatim" (id., p. 438).

68. Cf. *Commentaria una cum quaestionibus in librum De Generatione et corruptione Aristotelis* (Lyon: 1598), 1. I q. VI, p. 56: "vix intelligi potest, quod detur forma aliqua permanens, et incipiat, nec detur pars primo producta, cum agens sit certae et determinate virtutis. Ista etiam sententia est Auer. expressa 8. Physic. comment. 23. & Iandun. libr. 5. q. 15. & Aegid. q. 25. huius. . . . Tertia conclusio, post primum istum gradum, et minimum non inducitur calor, nec quoad intensionem nec quoad extensionem nisi successiue continue. Et ratio est: quia iam posteriores gradus adueniunt primo, et minima posteriora sunt primo continua. . . . "

69. *Disp. Met. XLVI*, sect. IV, § 7, p. 778a.

70. *Ibid.*

71. *Ibid.*

72. Id., §§ 8–9, p. 778b sq.

73. Cf. Phys. IV 11, 220a 9 sq. Id., V 3, 227a 10–12: there is continuity between two things when they come in contact in such a way that the boundary of each thing in fact become one and the same (i.e., a point).

74. *Phys.* IV 11, 220a 5 sq., IV 13, 222a 10 sq., VI 3.

75. Cf. *Phys.* VI 3, 234a 24 sq. (in an instant, there is neither motion nor rest), and VI 5, 235a 30–236a 7.

76. Cf. *Phys.* VI 3, 233b 33–234a 3.

77. *Phys.* VI 5, 235b 32s sq.

78. *Phys.* VI 5, 236a 13 sq.

79. *Phys.* VI 5, 236a 35–236b 6, 235b 17–18.

80. *Phys.* VIII 8, 263b 9–264a 6.

81. Thomas Aquinas gives his own answer (*In Phys.* VI, lect. 7 n° 8). He explains that intensification, too, determines an accidental division of quality, since it depends on infinitely various participations by the subject. So, although the form is indivisible, its actualization in a subject is progressive, continuous, and does not have a first instant, exactly as is the case with any motion.

82. Aristotle's view (there is no first instant of motion) was contested in the same way already by Theophrastus, according to Simplicius, as remarks P. Pellegrin in his French translation of the *Physics* (Paris: Flammarion, 2000), p. 330 n. 2.

83. Sect. IV, § 10, p. 779a: "Nec potest afferi ulla ratio ob quam latitudo intensiva qualitatis habeat indivisibilia continuantia, et unum terminum terminatem ex parte unius extremi, et non habeat aliud ex alia parte."

84. *Ibid.*: "quae tantum meretur nomen qualitatis."

85. Id., § 12, p. 779b.

86. "[S]icut non repugnat rem permanentem non fieri simul totam, sed partem post partem, ita etiam non repugnat incipere per ultimum non esse" (id., § 11, p. 779a–b).

87. Id., § 13 sq., p. 779b sq.

88. Regarding the spatial extension of its influence, it acts "uniformiter difformiter," that is: producing more and more remitted parts of quality. When it reaches the extremity of the "sphere of its activity," it cŏmmunicates to the remotest part of the subject the qualitative minimum of its activity, below which it can no longer act. This minimum is not the smallest part of the quality in itself, since a voluntary agent like God could discern and produce a smaller part.

89. Cf. Toledo, *Comm. . . . in De Gen.*, I q. VI, p. 56: "Nec propterea [i.e., production of a first part of quality] existimes quod nos ponimus in motu primum mutatum esse: nam iste gradus, et hoc minimum primum mutatione instantanea producuntur, nec sunt quicquam motus, sed sunt ultimum non esse motus, quia tunc non est motus sed immediate post illud erit: et ita est intelligendum Arist. lib. 6 Physi. text. 55 & 56, quamvis non negarem illic magis dicere quod non datur primum: nam docet simpliciter quod ex albo incipit nigrum per motum, et non per mutationem. . . . "

90. Sect. IV, § 17, p. 781a.

91. *Cursus philosophicus*, Lyon: 1669, De Gener., disp. II, sect. III, subsect. III, § 39, p. 628b. This work was first published in 1632, but Arriaga's positions were disputed, especially by other Jesuits like Oviedo. In further editions he included many new remarks and sometimes reshaped his text (I thank Jacob Schmutz for having called my attention to these differences). He did not, however, change his mind on the present topic. I shall quote both editions of 1632 (Antwerpen) and 1669 (the last one).

92. "[Q]uod seipso ab alio non distinguitur, nec per potentiam Dei absolutam distingui possit: ergo si lignum bipalmare v.g. actu est indivisible, et carens partibus distinctis, repugnat omnio, illud posse dividi in plures partes" (*Cursus Phil.*, 1632 ed., Phys., disp. XVI, sect. II, § 14, p. 459a).

93. Id., 1632 ed., § 18, p. 459a: "Distinctio unius partis ab alia, est aliquid intrinsecum ipsi parti, est enim praedicatum essentiale, fundans negationem alterius."

94. Id., 1669 ed., sect. II, § 17, p. 557a (1632 ed., § 15).

95. "Exterminanda ergo est haec sententia a Philosophis bene et clare discurrentibus, ac ponendae sunt actuales partes in continuo inter se realiter actu distinctae. . . . Celebris est sententia Zenonis, Pythagorae, Leucippi, omniumque Stoicorum docentium, continuum finitum componi ex indivisibilibus finitis. Eam defendunt recentiores multi et graves e nostra Societate" (1669 ed., sect. II, § 25, 558a-b; 1632 ed., § 20, p. 460b). But Arriaga finally judges this question to be aporetic, some objections being unsolvable within Zeno's framework as within Aristotle's. The later has however for him the weight of "auctoritas," and it enjoys also a stronger position concerning mathematical arguments (id., 1669 ed., sect. XIII, § 240, p. 592b; 1632 ed., § 256, p. 492b). See M. Mráz, "Das infinitum actuale in Arriagas Cursus Philosophicus," and J. Polívka, "Arriagas Kontinuumlehre," in T. Saxlová and S. Sousedík, *Rodrigo de Arriaga Philosoph und Theologe*, (Prague: University Karlovy, 1998).

96. Id., 1669 ed., sect. VII, § 74, p. 566b (1632 ed., § 82 p. 469a).

97. Cf. 1669 ed., disp. III sect. IV, § 91, p. 650a.

98. "[A]ntequam introducatur ultimum indivisible caloris, datur iam in eo passo primum indivisible frigoris" (1632 ed., De ortu, disp. III sect. 4).

99. Of course, Arriaga rejects the distinction, which he judges to be obscure, between beginning "per primum sui esse" and beginning "per ultimum sui non esse" (1669 ed., Phys., disp. XVI, sect. XIII, p. 590b). For him, even successive entities start with their "primum sui esse."

100. Cf. 1669 ed., De Gen., disp. II, sect. III, subs. I § 27, p. 626a-b (cf. subsect. III, § 43, p. 629a; 1632 ed., § 26 p. 526a).

101. Id., 1669 ed., § 30, p. 626b (1632 ed., § 30, p. 526b): "Hinc constat, augmentationem necessario interrumpi per morulas, quibus interim disponitur de novo nova pars materiae ad subsequentem augmentationem."

102. However, according to Arriaga in the first edition, it would not be absurd to hold that their number be infinite: "respondeo nullum esse absurdum, si infinita puncta intensionis categorematice producantur: pars enim maior Aristotelicorum concedit in continuo puncta, seu per modum unionis, seu per modum extremi; multique dicunt, ea esse infinita simpliciter: et certe, ut suo loco ostendi, tenentur ea vocare infinita categorematice. Ut enim ostendi ibi, non sunt infinita solum ex additione quae eis fieri potest, quale est infinitum syncategorematicum (omnia enim actu sunt in eo continuo) sed sunt infinita, quia extrahendo ex his quae ibi sunt actu, non possunt successive exhauriri, quod proprium est infiniti categorematici" (§ 105, p. 552b). In the 1669 ed., the same view is attributed to an opponent ("respondebis. . . . "), and now Arriaga deems this answer to be unsatisfactory: "Nihilominus haec solutio videtur difficilis, quia revera facit intentionem absolute et simpliciter infinitam" (disp. III, sect. VI, § 104 p. 652a0.

103. *"non recte intelligi tarditatem motus sine morulis"* (id., 1632 ed., § 41, p. 528b). Cf. 1669 ed., Phys., disp. XVI, sect. XI, § 210–211, p. 587b: "Igitur acquiesco Galeno lib. I de Dignoscendis pulsibus cap. I Vallesio lib. 3 Controu. Medic. cap. 8 et Recentioribus communiter, qui licet sententiam Aristotelis de continuo defendant, arbitrantur tamen

merito, in eo differre motum velocem a tardo, quod hic interrumpitur per nonnullas morulas, velox autem non discontinuatur. Unde licet in unico instanti nec aquila nec testudo possint acquirere de nouo, nisi unicum punctum spatii, tamen in dimidia hora plus acquirit aquila quam testudo, quia haec non mouetur, nisi per sex aut octo instantia illius dimidiae horae, aquila vero movebitur per triginta. . . . " Those "morulae" are of course imperceptible, since we perceive a motion as continuous.

104. Id., 1669 ed., De Gen., disp. II sect. III, subsect. IV § 48, p. 630a (1632 ed., § 45, pp. 528–29).

105. Id., 1669 ed., § 51, p. 630b.

106. Id., 1669 ed., § 59, p. 632a (1632 ed., § 49, p. 529b0. Cf. on the contrary Toledo, *Comm. . . . in De gen.*, I q. VI, p. 54; "suppono unum principium certum: quando agens est debite approximatum passo, necesse est, quod agat in ipsum statim."

107. Id. 1669 ed., § 56, p. 631a (1632 ed., § 46).

108. Id., § 58: "neque illud axioma, *idem semper manens, idem semper natum est facere idem*, verum est in toto rigore.'

109. *Quaestiones philosophicae*, op. cit., t. II, q. XXVI.

110. *Disp. Met. XLVI*, sect. III, § 7 sq., p. 771b sq.

111. *Quaest. phil.*, op. cit., t. II, q. XXVII, pp. 543–44.

112. *In Sententias commentaria*, Antwerpen: 1561, 1. I d. 17 q. V, §§ 23–28.

113. It does not only bring a complement of perfection (2 added to 5 makes a higher number, but is not by itself higher than 50: it is in and by itself more perfect than any of the preceding parts).

114. "Tenendum est ergo quod forma intenta et remissa acquisitae per motum possunt esse partes unius formae numero, et quae non est una indivisibilitate, sed continuitate suarum partium quae non sunt simul, sed successive et una adveniente alia desinit esse" (id., q. VII, § 39). "Rationes principales quae adductae sunt ad probandum quod forma intensa et remissa non sunt eadem forma secundum numerum concedendae sunt pro quanto probant quod forma intensa et remissa mutuo se expellunt tanquam partes unius continui successivi . . . , sed non probant quin tota forma cuius illae sunt partes sit una numero continuitate' (id., § 41).

115. Cf. Toledo, *Comment. . . . in De Gen.*, 1. I q. VI, p. 56: "motus alterationis est in termino ad quem: unde dealbatio est albedo fluens, vel potius fluxus quo ipsa fluit".

116. "Dicendum est quod sicut in quolibet motu sunt infinitae partes motus in potentia et universaliter in quolibet continuo, sic in forma quae per motum successive et continue acquiritur, sunt infinitae formae partiales quae in sua continuitate constituunt unam totalem formam secundum quod tota successio motus acquiritur, tamen secundum nullam earum est actu mutatum esse, nisi secundum finalem gradum. . . . " (id., § 41). " . . . forma remissa quae ante motum erat in subiecto sub esse fixo abiicitur in instanti, sequens tamen forma non introducitur in instanti sed in tempore, sicut enim per motum localem contactus mobilis cum alio corpore secundum sui ultimum solvitur in instanti, quia erat secundum indivisibile, processus vero mobilis ad quemcumque locum est successivus, quia omnis locus quantitatem habet, sic recessus subiecti a forma remissa quae prius erat in subiecto sub esse fixo et indivisibili [fit] in instanti, acquisitio vero formae sequentis per motum cum habeat latitudinem continuititatis, fit in tempore" (id., § 41].

117. *Quaest. phil.*, op. cit., q. XXVII, pp. 548–49.

118. "[N]on fit [subiectum] magis calidum per hoc quod acquirat aliam formam, sed per hoc quod acquirat alias partes ejusdem formae, quae cum partibus antecedentibus constituunt unam formam magis intensam" (id., p. 551).

119. Cf. *Phys.* VII 9, 217b 1–6. Mauro goes so far as to speak of "mutatio" for the passage from one state to a higher one (p. 548).

120. *Essai sur les données immédiates de la conscience*, in *Œuvres* (Paris: P.U.F., 1970), pp. 31–50. See J.-L. Solère, "Plus ou moins: le vocabulaire de la latitude des formes," loc. cit., pp. 466–70.

121. *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, in *Opere* (Firenze: 1890–1909), t. VIII, pp. 197–98. Cf. *Juvenilia*, id., t.I, pp. 116–17. See P. Galluzzi, *Momento. Studi galileiani*, Roma: Edizioni dell'Ateneo & Bizarri, 1979, and M. Clavelin, *La Philosophie naturelle de Galilée*, 2nd ed. (Paris: Albin Michel, 1996), pp. 101, 126, 285–330.

122. "Cum enim motus acceleratur, est manifestum quod motui tardiori succedit alius motus velocior," "motus intenditur per mutationem" (*Quaest. phil.*, op. cit., q. XXVII, p. 548).

123. L.I sect. II prop. I, L.II prop. II.

124. *Philosophia prima sive ontologia*, in *Gesammelte Werke*, II Abteilung, Band 3 (Hildesheim: G. Olms, 1962), §§ 747–60. See J.-L. Solère, "Plus ou moins: Le vocabulaire de la latitude des formes," loc. cit., pp. 473–78.