HOW WE INTUITIVELY REASON

> By the ratiocination of our mind, we add and subtract in our silent thoughts, without the use of words.[1]
>
> THOMAS HOBBES

§1. In the 17th century, Hobbes stated that we reason by addition and subtraction. Later in that century, Leibniz emphatically agreed:

> Thomas Hobbes, everywhere a profound examiner of principles, rightly stated that everything done by our mind is a computation by which is to be understood either the addition of a sum or the subtraction of a difference. So just as there are two primary signs of algebra and analytics, + and –, in the same way there are, as it were, two copulas. [2]

Historians of logic note that Hobbes thought of reasoning as "a 'species of computation,'" but point out that "his writing contains in fact no attempt to work out such a project." [3] Though Leibniz speaks of the plus/minus opposition of the positive and negative copulas, neither he nor Hobbes say anything about a plus/minus character of other common logical words that drive our deductive judgments, words like 'some', 'all', 'if', and 'and', each of which actually turns out to have an oppositive, positively or negatively charged character that allows us, "in our silent thoughts, without the use of words", to ignore its literal meaning and to reckon with it purely as one reckons with a plus or a minus operator in elementary algebra or arithmetic. Such 'logical constants' of natural language figure crucially in our everyday reasoning. Some years ago, I discovered that the natural logical words we use in everyday reasoning could be reckoned with as one reckons with plus/minus operators in elementary algebra.[4] I found, for example, that 'IS,' 'AND', 'SOME,' and 'THEN,' are "PLUS-WORDS" but that 'ISN'T,' 'NOT,' 'ALL,' and 'IF,' are "MINUS-WORDS" and that they so behave in common inferences. Reasoning with these oppositively charged +/- particles of natural language enables us to reckon with meaningful sentences as easily and as fluently as we reckon with the plus and minus operators of algebraic expressions such as '-(+x-y)' and '+x-y' of elementary algebra or arithmetic. For example. by intuitively reckoning with the +/- logical constants of natural language one is instantly able to infer 'Some$^{\{+\}}$ dogs aren't$^{\{-\}}$ friendly' from 'Not$^{\{-\}}$: All$^{\{-\}}$ dogs are$^{\{+\}}$ friendly':

DISCURSIVELY: Not$^{\{-\}}$: All$^{\{-\}}$ Dogs are$^{\{+\}}$Friendly ≡ Some$^{\{+\}}$Dogs aren't$^{\{-\}}$Friendly

⇓ ⇓ ⇓ ⇓ ⇓

"LOGIBRAICALLY": - (- Dogs + Friendly) = + Dogs – Friendly

We similarly reckon 'All$^{\{-\}}$creatures were$^{\{+\}}$ motionless$^{\{-\}}$' equivalent to 'Not$^{\{-\}}$ a$^{\{+\}}$ creature was$^{\{+\}}$moving'by reckoning that -(+C+M) = -C+(-M) and 'Every$^{\{-\}}$ Voter is$^{\{+\}}$ a Citizen' equivalent to 'Every$^{\{-\}}$ non$^{\{-\}}$-Citizen is$^{\{+\}}$ a non$^{\{-\}}$-Voter' since -V+C = -(-C)+(-V). The +/- calculus works also in propositional logic. Thus just as

if$^{\{-\}}$ p then$^{\{+\}}$ q = not$^{\{-\}}$ both$^{\{+\}}$ p and$^{\{+\}}$ not$^{\{-\}}$ q,

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[1] **The English Works of Thomas Hobbes,** vol. I, W. Molesworth (ed), London, Kessinger. 1839, p.3.
[2] **Leibniz: Logical Papers,** G.H.R. Parkinson (ed, transl), Oxford: Clarendon Press, 1966, p. 3.
[3] William and Martha Kneale, **The Development of Logic,** Oxford University Press, 1960, p. 511.

[4] Cf. **THE LOGIC OF NATURAL LANGUAGE,** Oxford, Clarendon Press.1982

so -p+q = - (+p+(-q))

Hobbes and Leibniz were right about what transpires in ‘ratiocination.’ But because they did not directly apply their +/- thesis to the logical words of natural language that figure in our actual ‘ratiocinations,’ their insight did not lead to the development of an effective ‘research program’ that describes how people mentally reason.

The failure to focus attention on the +/- character of the ‘logical constants’ of natural language was to have a profound effect on the history of logic. In the modern era, Predicate Logic, inaugurated by Frege in 1879 and promoted by Whitehead and Russell early in the 20th century would thoroughly supplant the millennial Term Logic of Aristotle, replacing the natural language of thought by a symbolic language of quantifiers and bound variables that has become the new ‘Logic of the Schools.’

We reason in sentences of our natural language (be it English, Italian, Finnish, or Danish). Soon after I had become convinced that the common logical words that drive our reasoning had the powers of plus/minus operators, I came to believe that we reason instinctively by *unconsciously exploiting* these powers. For example, when we intuitively reckon that ‘Not$^{\{-\}}$ every$^{\{-\}}$ archer will$^{\{+\}}$ hit$^{\{+\}}$ a^{+} target,’ says the same thing as ‘Some$^{\{+\}}$ archer will$^{\{+\}}$ miss every$^{\{-\}}$ target’ and judge these sentences to be ‘logically equivalent’ by distributing the minus word ‘Not’ to its right, changing ‘every$^{\{-\}}$ archer to ‘some$^{\{+\}}$ archer’, ‘hit$^{\{+\}}$’ to ‘hit$^{\{-\}}$’, and ‘some$^{\{+\}}$ target’ to ‘every$^{\{-\}}$ target,’ in effect, instantly transforming ‘ –(-Arrow +Hit +Target)’ to ‘+Arrow+(-Hit)-Target’. The difference between instantly getting from ‘-(-a+h+b)’ to ‘+a +(-h) –b’ in a beginners algebra class, and intuitively getting from ‘Not{$^{-\}}$(Every$^{\{-\}}$ A will$^{\{+\}}$ Hit some$^{\{+\}}$ T)’ to ‘Some$^{\{+\}}$A will$^{\{+\}}$ non$^{\{-\}}$H every$^{\{-\}}$’ in natural language is that when people “silently” reason with logical words like ’not’, ‘is,’ ‘every’, and ‘some,’ or with logically contrary terms like ‘hit’ and ‘miss’, they ignore the discursive meanings of the words and reckon only with their oppositive, +/- characters. Discursively, these *six* words have very different meanings but logically we reckon only with their charges, which are no more than two in number: ‘+’ and ‘-.’

That Pedicate Logic is not Cognitively Veridical

§2. We do not, in real life, reason in the manner of predicate logic. The intuitive judgment that ‘not every archer will hit a target’ entails ‘some archer will miss every target’ is *never* made by regimenting these sentnces as something like ‘Not: for every x, if x is an archer then there is a y such that y is a target and x hits y’ and then, by applying laws of “quantifier interchange” from Modern Predicate Logic **(MPL)** and some laws of propositional logic, arrive (in about eight steps and several minutes) to ‘There is an x such that x is an archer and for every y, if y is a target, then not: x hits y’ .

This (along with other common examples of deductive inference made in the characteristically deliberate manner of MPL) shows that the canonical notation of predicate logic, though it enables us to formally *justify* our intuitive judgment that an inference is valid, casts no light at all on how children (and even adult logicians, outside the classroom) actually reason when they intuitively and *instantly* move mentally from ‘Not$^{\{-\}}$ every$^{\{-\}}$ archer will$^{\{+\}}$ hit$^{\{+\}}$ a target’ to ‘some$^{\{+\}}$ archers will$^{\{+\}}$ miss [non$^{\{-\}}$hit] every$^{\{-\}}$ target’.

In the 20th century, MPL became the standard “logic of the schools,” supplanting the older Aristotelian term logic **(ATL)** that had been standard logic for more than two thousand years, because MPL was able to *formally* justify many common simple valid inferences we *intuitively* make that traditional, pre-Fregean logicians, who were unaware of the oppositive character of the logical particles of natural language, were simply unable to validate. Historically, ATL had reached an impasse;

because it was unaware of the plus/minus character of the logical words it was reckomniong with, it could not plausibly account for many common arguments. For example ATL was unable to provide an acceptable formal proof for an intuitively valid inference like

($\mathfrak{R}$) Every colt is a horse so anyone who rides a colt rides a horse.

Havng learned that all colts are horses, a teenager will *instantly* intuitively judge that ($\mathfrak{R}$) is true. He is however, unable to *justify* his subjective certainty by providing a formal proof that ($\mathfrak{R}$) is valid. By contrast, a formal MPL proof that ($\mathfrak{R}$) is valid, does not rely on intuition; it may take a trained logician 5 to 10 minutes to present a formal MPL proof of ($\mathfrak{R}$)'s validity but he cannot be accused of a rush to judgment. The proof he presents is transparent and objective, not intuitive and subjective. So, despite its slow tempo, MPL became the preferred way to explain the validity of($\mathfrak{R}$).

Nevertheless, MPL has achieved its objective inference power by *changing the subject*, moving us away from the natural language in which we actually think and reason to a non-natural language of quantifiers, bound variables and sentential constructions not found in the variable-free sentences that are the actual vehicles of everyday reasoning. MPL first translates ($\mathfrak{R}$) as

($\mathfrak{R}$*) For any x if x is a colt then x is a horse, ∴ For any y, if there is an x such that x is a colt and y rides x then there is z such that z is a horse and y rides z'

and then, by applying laws of predicate and propositional logic, derives the conclusion of ($\mathfrak{R}$*) from its premise in a series of justified steps, thereby formally validating the intuitive judgment that the conclusion of ($\mathfrak{R}$) follows from its premise.

Though MPL thus displays its ability to formally account for inferences by regimenting ($\mathfrak{R}$) as ($\mathfrak{R}$*), MPL *cannot* be the *actual* method that enables a teenager to move intuitively from 'all colts are horses are animals 'so anyone who rides (owns, feeds, …) a colt, rides (owns, feed, …) a horse, in a split second. The benighted teenager *instantly but correctly* arrives at these judgments in spite of the fact that he can't possibly be reasoning in anything like the way one reasons in predicate logic. For he knows nothing about MPL and even if he did, there is no possible way for anyone to use predicate logic in a way that enables him to arrive at ($\mathfrak{R}$*)'s conclusion as *rapidly* as the teenager, who reasons naturally, arrives at ($\mathfrak{R}$)'s conclusion.

In any case, in explaining how the teenager reasons, we must assume he is reasoning in natural language. For that is the language of thought. A **cognitively veridical logic** (CVL) would conform to and illuminate how the teenager actually thinks (Logic as 'Laws of Thought'). We need a cognitively veridical version of ATL that explains how we all intuitively arrive at the judgment that ($\mathfrak{R}$) is true. Because the main task of a CVL is to clarify something that is already intuitively obvious to us, its formal proofs should elicit an "aha" reaction. *"Aha, so **that's** how we reckon these sentences! **That's** how we so quickly and confidently arrive at this conclusion!"* No student presented by an **MPL** proof of the validity of his intuitive reasoning is ever moved to exclaim. "Oh so *that*'s how I came to that conclusion!"

Consider that anyone who encounters the following two sentences

(A) Every senator is a U.S. citizen. → -S+C

(B) Someone talking to a senator wasn't talking to a U.S. citizen. → +(t+S) - (t+C)

instantly and intuitively judges them to be jointly inconsistent. The inconsistency is "obvious." But what *makes* it obvious? In logic what isntnaxiomatic must be shown true by formal proof. This MENO-like question asks for an account of what it is that we mentally "see" in these sentences that instantly proves that (A) and (B can't both be true. Modern predicate logic (MPL), translating (A) and (B) into the artificial language of quantifiers and variables, is able (in some ten steps and almost as many minutes to provide a formal proof that (A) and (B) (properly formulated in canonical notation) entail overt self-contradictions (of form 'Px&~Px'), something the older (syllogistic) term logic seemed unable to do. Nevertheless, no MPL proof that (A) and (B) are jointly inconsistent explains why a *teenager,* who is untutored in logic and who thinks in the variable-free sentences of his natural language, is *instantly* able to arrive at the judgment that (A) and (B) cannot *both* be true. The teenager must unconsciously be using some quick, algorithmic, method that exposes a inconsistency. ***What method does he use? And what inconsistency does it expose?*** Modern logicians, from Frege to Quine are not concerned with this MENO-type question. They are intent on rationally reconstructing a formal justification of the teenager's intuitive reasoning even if that does not describe or explain how he is actually reasoning.

Just here is where Hobbes's +/- thesis of ratiocination turns out to be so precient and valuable. It turns out that the conjunction, '(A) and^{+} (B)' --- logibraically entails an obvious self-contradiction of form 'Some X is not an X', viz., *'Someone^{+} talking to some^{+} citizen isn't^{-} talking to some^{+} citizen:'* +(t+C)-(t+C).

§2.1 Here, in my opinion, is a how a teenager methodically, albeit unconsciously, *instantly* arrives at the **reductio** absurdity that causes him to intuitively
reject the conjunction of (A) and (B):

(Ω)

(A) All$^{\{-\}}$ senators are$^{\{+\}}$ citizens; **-~~S~~+C**

+(B) Someone$^{\{+\}}$ talking to a^{+} senator wasn'$^{\{-\}}$ talking to a$^{\{+\}}$ citizen; **+(t+~~S~~) – (t+C)**

=> (C) Someone$^{\{+\}}$ talking to a$^{\{+\}}$ citizen *wasn't*$^{\{-\}}$ talking to a$^{\{+\}}$ citizen = **+(t+C) - (t+C)**

Because (C) is an obvious self-contradiction, (Ω) is an indirect, logibraic, **cognitively veridical,** proof that *(Σ) ALL SENATORS ARE CITIZENS BUT SOMEONE TALKING TO A SENATOR WASN'T^{-} TALKING TO A^{+} CITIZEN can't possibly be true*.

That the logical constants in the natural sentences we reason with actually ***have*** a +/- character must of course be demonstrated. Sections §3, §3.1, and §4 show how we may proceed to determine the plus-minus character of the basic logical words and particles of the natural language with whose sentences we fluently reason every day.

THAT THE LOGICAL PARTICLES OF NATURAL LANGUAGE *ARE* OPPOSITIVELY CHARGED

§3. We do intuitively regard 'and' and 'is' as positive particles ("plus-words") and 'not,' 'no,' and 'non-' as negative particles ("minus-words").. Although we do not *consciously* reckon with words like 'some', 'every', 'if', and 'then' in a plus/minus way, we find that they, *too*, are oppositively "charged," each having a fixed plus or a minus character that can reckoned with "logibraically" in actual reasoning. Here is how one may determine their +/- values.

We begin by assuming with Leibniz and Hobbes, that the positive copula 'is' is a "plus-word" and that 'not' is a "minus-word." If so, the negative copula, 'isn't'-- the contraction of 'is$^{\{+\}}$' and 'not$^{\{-\}}$'

‾‾ is a minus word. ‘And’ is literally a plus-word. This gives us four oppositively charged natural logical constants --- *‘NOT*$^{\{-\}}$*,’ ‘AND*$^{\{+\}}$ *IS*$^{\{+\}}$*,* and ‘*ISN’T*$^{\{-\}}$’ --- to start with.

If ‘some’ and ‘every’ *are* also oppositive particles, how could we determine which one is positive and which one is negative?[5] Let us begin with ‘some.’ We are regarding ‘is’ as a plus-word, but we do not know whether ‘some’ even has a +/- character. If ‘some’ does have a +/- character, we do not know how to determine whether it is a plus word or a minus word.

The commutative equivalence of ‘some A is B’ to ‘some B is A’ suggests that a sentence of form ‘some A is B’ is to a sentence of form ‘some B is A’ as the algebraic expression ‘+a+b’ is to ‘+b+a’. This suggests that a necessary condition for a correct assignment of a +/- value for ‘some’ is that the law of commutation for addition should equally hold for both the discursive and the algebraic formulas.

We are regarding ‘is’ as a plus-word, and tentatively testing the assumption that some’ has a +/- character. What we *have* is

$$\text{Some}^{\{?\}}\ \text{A is}^{\{+\}}\ \text{B} \quad \equiv \quad \text{Some}^{\{?\}}\ \text{B is}^{\{+\}}\ \text{A}$$

Of the two possibilities for an oppositive, +/-, value of ‘some,’ only a ‘plus’ assignment for ‘some’ preserves commutation:

(i) $\text{Some}^{\{+\}}\ \text{A is}^{\{+\}}\ \text{B} = \text{Some}^{\{+\}}\ \text{B is}^{\{+\}}\ \text{A} \quad \Leftrightarrow \quad +A+B = +B+A$

(ii) $\text{Some}^{\{-\}}\ \text{A is}^{\{+\}}\ \text{B} \neq \text{Some}^{\{-\}}\ \text{B is}^{\{+\}}\ \text{A} \quad \Leftrightarrow \quad -A+B \neq -B+A$

In (i) the assignment of ‘+’ to ‘some’ gives us a corresponding law of commutation for discursive sentences. By contrast , assigning ‘-’ to ‘some’ results in a failure of commutation. showing that ‘some’ is not a minus-word. This does not prove that ‘some’ is a plus-word. For we have not shown that ’some’ and ‘all’ are opposed in a plus/minus way. However, we may tentatively take the commutative behavior of ‘some$^{\{+\}}$..is$^{\{+\}}$’ as an indication that ‘some$^{\{+\}}$…is$^{\{+\}}$’ is plus-like and see whether this casts light on the character of ‘all.’

THAT ‘ALL’ IS A MINUS-WORD, IF ‘SOME’ IS A PLUS WORD

§3.1 Here is what Aristotle says of propositions that begin with ‘every’ or ‘all’:

We say that one term is predicated of all of another, whenever no instance of the subject can be found of which the other term cannot be asserted. (Pr. Anal. 24b: 29-30)

Aristotle interprets ‘All M are P’ as denying that some M are not-P. This is analogous to the way modern logicians define ‘If p then q’ as the denial of ‘both p and not q.’

Having fixed the +/- character of ‘is’ and ‘are’, as ‘ + ’, and of ‘not’ as ‘ – ’ we can now see that the assumption that ‘some’ is a plus-word leads us to determine that ‘all’ is minus-word.

According to Aristotle,

[5] In anwering this question, the approach I used is much like the one taken in propositional logic when one starts with ‘&’ and ‘~’ as primitive sentential operators and goes on to define other sentential operators such as ‘⊃’ ‘v’, and ‘≡’.

$$\begin{array}{ccc} \text{All}^{\{?\}}\ \text{M are}^{\{+\}}\text{P} & \equiv & \text{def. N}\textsc{ot}^{\{-\}}\text{: some}^{\{+\}}\text{M is}^{\{+\}}\ \text{non}^{\{-\}}\text{P} \\ \Downarrow \quad \Downarrow & & \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \\ - \quad M + \quad P & \equiv & \text{def.} \ - \ (\ + \ M \ + \ (-P)\ ) \end{array}$$[6]

The "logibraic" equivalence reveals that *if* 'some' and 'is' are plus-words and 'not is a minus word, then 'all' is a minus-word. It thus suggests that 'some' and 'all' are +/- oppositively charged like 'is' and 'isn't.'

But we can do more than simply suggest. The oppositive character of 'some+' and 'every-', like that of 'is' and 'isn't, is confirmed in valid inferences such as obversions, contrapositions, and other common logical equivalences that involve these four logical words:
Obversion: not every A is B ≡ some A isn't B ⇔ -(-A+B) = +A-B
Contraposition: every A is B ≡ every non-B is non-A ⇔ -A+B = -(-B)+ (-A)
Conversion: some A is non-B ≡ some non-B is A ⇔ +A+(-B) = +(-B)+A

Syllogistic reasoning provides additional confirmation of the +/- oppositive nature of 'all' and 'some':

A syllogism (or sorites) is an argument that has as many terms as it has sentences. We get further confirmation of the +/- oppositive natures of 'all' and 'some' by remarking that every valid syllogism satisfies two necessary and sufficient logibraic conditions:

(1) The Summation Condition
Its conclusion must be equal to the sum of the premises.

(2) Its Mood must be "regular." Only *two* moods are regular:

Mood (i): *Every proposition of the syllogism is universal*, being equivalent to a proposition of form '-(±X)+(±Y).'
Mood (ii) *The conclusion is particular and it has a* SINGLE *particular premise, i.e., a premise of* form '+(±X)+(±Y)'

E.g.,

All{-}M are{+}P and{+} No{-} S are{+} non{-} M, hence All{-}S are{+} P
[-M+P] + [-(S+(-M)] => -S+P

All{-}M are{+}P and{+} Some{+} M are{+} S hence Some{+} S are{+} P
[-M+P] + [+M+S] => +S+P
No{-} non{-}B is A and{+} No{-} B isn't{-} C and{+} Every{-}D is{+} C => No{-} A is{+}D
[-((-B)+A)] + [-(B-C)] + [-D +C] => -(A+D)

All{-} M are{+}P and{+} All{-} S are{+} M and{+} Some{+} S are{+} Q => Some{+} P are{+} Q
[-M+P] + [-S+M] + [+S+Q] => [+P+Q]

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[6]The definitional equivalence of ' All S are P' and 'not: some S are not P' is logibraic: -S+P = def.-(+S+(-P)). Scientific generalizations are universal propositions. That 'all is a minus-word' is the logical ground of Karl Popper's thesis in *Der Logik der Forschung* that one cannot inductively confirm an empirical proposition of form 'All S are P;' one can only falsify it.

THE VALIDITY CONDITION

A syllogism is valid if and only if its mood is "regular" and its conclusion is equal to the sum of its Premises

'IF' and 'THEN'

§3. 'And' is a plus word and 'not' is a minus word, so the logibraic transcription of 'not both p and not q' is '-(+ p+ (-q)). 'Not both p and not q' is often regarded as the definiens of 'If p then q'. So regarded, the definitional equivalence,

if p then q =def. not(both p and not q),

may be logibraically expressed as

-p+q = def. -(+p+ (-q))

in which ''if' is seen to be a minus-word and 'then' a plus-word.

That 'if' and 'then' actually behave this way in our deductive judgments is illustrated by such common inferences as the following:

1. If p then q ∴ if not q then not p
   -p + q ∴ -(-q) + (-p)

2. If p then q and if q then r, ∴ if p then r
   [-p+q] + [-q+r] ⇒ -p + r

PROPOSITIONAL LOGIC: 'IF' AND 'THEN'

§4. 'And' is a plus word and 'not' is a minus word, so the logibraic transcription of 'not both p and not q' is '-(+ p+ (-q)).' 'Not both p and not q' is often regarded as the *definiens* of 'If p then q'. So regarded, the definitional equivalence,

if p then q =def. not(both p and not q),

may be logibraically expressed as

-p+q = def. -(+p+ (-q)),

whose *definiendum* shows ''if' to be a minus-word and 'then' a plus-word.

That 'if' and 'then' actually behave that way in propositional logic illustrated in and confirmed by such common inferences as the following:

1. If p then q ∴ if not q then not p
   -p + q ∴ -(-q) + (-p)

2. If p then q and if q then r, ∴ if p then r
   [-p+q] + [-q+r] ⇒ -p + r

Note the difference between expressing the equivalence of 'if p then q' to 'not: both p and not q' in the conventional notation of modern symbolic logic as

$$p \supset q \equiv \sim(p \,\&\, \sim q)$$

and expressing it “logibraically” as

$$-p+q \;=\; -(+p + (-q)).$$

Expressed logibraically the equivalence of ‘if p then q’ to ‘not: both p and not q’ is perspicuous as an algebraic truism. Expressed symbolically, we use a truth table to show that both sides have the same truth values.

Among the basic inference patterns in standard propositional logic are the principles known as MODUS PONENS, MODUS TOLLENS. Here is how they are represented in the modern notation of symbolic logic:

MODUS PONENS	MODUS TOLLENS
$p \supset q$	$p \supset q$
$\underline{p}$	$\underline{\sim q}$
$\therefore\ q$	$\therefore\ p$

In standard logic, the validity of these inferences is shown by truth tables. Represented in +/- notation their validity is *logibraically perspicuous* :

MODUS PONENS	MODUS TOLLENS	HYPOTHETICAL SYLLOGISM
$-p+q$	$-p+q$	$-p+q$
$\underline{p}$	$\underline{\quad -q}$	$\underline{-q+r}$
q	$-p$	$-p \quad +r$

§5. THE LOGICAL AND THE EXTRA-LOGICAL: THE FORMATIVE AND MATERIAL ELEMENTS

SCHOLASTIC logicians distinguished between the ‘syncategorematic,’ formative elements of a sentence that determine its logical form, and the ‘categorematic,’ material elements that carry its material content. The material elements of traditional (Aristotelian and Stoic) logic are terms and propositions; the formative elements of natural language sentences are term connectives like ‘every$^{\{-\}}$ . . . is$^{\{+\}}$’ and ‘some$^{\{+\}}$ . . . is$^{\{+\}}$’ and propositional connectives like ‘if$^{\{-\}}$ . . . then$^{\{+\}}$’ and ‘both$^{\{+\}}$ . . . and$^{\{+\}}$’. The following is a very partial but representative list of some basic natural formatives that we logibraically reckon with many times a day:

‘SOME’(‘SEVERAL,’ ‘A’..), ‘IS’ (‘WAS,’ ‘WILL BE,’ etc.), ‘BOTH,’ ‘AND’, and ‘THEN’ are “PLUS- words;”

‘EVERY,’(‘ALL,’’ANY’..), ’NOT,’ (‘NO,’ ‘AIN’T,’ ‘UN-,’ etc.), and ‘IF’ are “MINUS- words.”[7]

Russell somewhere says that a good notation is like a live teacher. Here are two examples of what the +/- notation can teach us about the logical constants of natural language:

[7] Readers will find accounts of the +/- logic in the author’s *The Logic of Natural Language* (Oxford, Clarendon Press, 1982), chapter 9; in “Predication in the Logic of Terms,” *Notre Dame Journal of Formal Logic* 31 (1990): 106–26 ; “The World, the Facts, and Primary Logic,” *Notre Dame Journal of Formal Logic*, 34 (1993): 169–82; and in *An Invitation to Formal Reasoning* (Aldershot, Ashgate, 2000), (with G. Englebretsen).

(i) Unlike ‘SOME’ and ‘ALL’ which are words of quantity, ‘NO’ is not a word of quantity (in the sense that zero is a number) but a denial of propositional scope whose meaning is ‘it is not the case that.’ The ‘no’ of ‘no$^-$ S is$^+$ P’ denies the proposition that follows it.. We transcribe it as ‘-(S+P)’ an abbreviated form of ‘-(+S+P)’ [NOT$^-$: SOME$^+$ S IS$^+$ P].

(ii) ‘OR, ’ does not appear on the list of primitive plus/minus words; unlike ‘AND,’ and ‘IF,’ ‘OR’ is neither a plus-word nor a minus word but a composite of two basic minus words. Logibraically, ‘p or$^{\{?\}}$q’ is ‘p if$^{\{-\}}$ not$^{\{-\}}$q.’ If English had a contraction for ‘if not,’ ‘OR’ would literally have the meaning ‘IFN’T.’ ‘Or’ is a combination of two minus-words that cannot be reduced to or contracted to a ‘+’ . Thus ‘p OR q ’ logibraically transcribes as ‘p –(-q).’ In this respect, ‘OR’ is not like other composites such as ‘isn’t’ and ‘won’t,’ which are contractions of ‘+, -’ that can safely be transcribed as minus words. ‘Or’ irreducibly transcribes as ‘-, -’ belying the saying that “ two negatives always make a positive.”

$$\text{p or q} \ \text{=def.} \ \ \text{p if}^{\{-\}} \text{ not}^{\{-\}}\text{q} \ \Rightarrow \ \text{p-(-q).}$$

Having established the +/- character of the formative elements, I soon became convinced that we intuitively reason by reckoning with the natural formatives as plus or minus operators, unconsciously reading ‘some’ as ‘some$^{\{+\}}$’, ‘all’ as ‘all$^{\{-\}}$’, ‘if’ as ‘if$^{\{-\}}$’, ‘then’ as ‘then$^{\{+\}}$’, ‘and’ as and$^{\{+\}}$ ‘is’ as ‘is$^{\{+\}}$.’ Admittedly, it is more than a little odd that we should all our lives be reasoning mentally by unconsciously exploiting the oppositive +/- character of familiar logical words like ‘all’, ‘some’, ‘is’, ‘not’, and ‘if’ without ever becoming aware of doing so. This hypothesis needs to be empirically tested. One may expect that cognitive science, and more specifically, cognitive psychology, will find ways to falsify it if it is in fact false. I am however confident that the +/- hypothesis will resist falsification because it offers the most reasonable explanation of why we are mentally as adept at reasoning with the logical constants of natural language as we are adept at reckoning with the plus and minus operators of elementary algebra and arithmetic. Ratiocination is something that takes place in real time in real minds reasoning in sentences of their natural language. Originally an inspired conjecture of Thomas Hobbes, plus/minus discursive reasoning will, I expect, be found to be a psychological reality. Of course cognitive scientists must themselves first become aware of the hypothesis that the logical constants of natural language are oppositive functors. That will take some time since (“full disclosure”), despite repeated efforts to attract attention to the Plus/Minus Hypthesis, I must report that I have failed to make it well-known even to academic logicians. But perhaps cognitive scientists, unlike many practitioners of MPL, are more open to regarding reasoning as what actually takes place in our ratiocinations.

THE PLUS/MINUS HYPOTHESIS

§6. The Plus/Minus Hypothesis provides a ‘best explanation’ for our ability to reason discursively with the sentences of natural language with the same confidence, speed and ease that we reckon with simple formulas of elementary algebra and arithmetic. We are all natively endowed with some rudimentary algebraic know-how. It stands to reason that sapient social beings who had evolved to the point of communicating in a descriptive natural language would eventually hit on the most expeditious and economical way to reason deductively: *by instinctively (unconsciously) exploiting the plus/minus character of its ‘logical constants’*

Even children reason logibraically. Having just learned that all$^-$ snakes are$^+$ reptiles, a bright child can immediately conclude that every owner of a snake is an owner of a reptile. She realizes that it can’t possibly be true that someone who owns a snake ***doesn’t*** own a reptile. For she may have reasoned thus:

Θ

(1) all^{-} snakes are^{+} reptiles; $-\cancel{S}+R$

but^{+}, (suppose) (2) some owner of a snake isn’t an owner of a reptile; $+(O+\cancel{S})-(O+R)$

∴ (3) some owner of a reptile isn’t an owner of a reptile; $+(O+R)-(O+R)$

This shows that accepting (2) along with (1) leads to absurdity. So she rejects (2) in favor of its negation, ‘-(O+S)+(O+R),’ viz., Every owner of a snake is an owner of a reptile.

Consider an example of De Morgan,

Θ

Θ Every horse is an animal, so every tail of a horse is a tail of an animal.

A teenager intuitively makes the inference Θ almost instantly. And, in fact, presenting a formal logibraic proof of its validity takes less than a minute. Here is one form of a logibraic proof that Θ is valid:

To the given premiss, ‘Every horse is an animal’, we may add the tautology ‘Every tail of an animal is a tail of an animal.’ This give us the argument *A*:

A

(1) Every^{-} horse is^{+} an animal;	-H+A	Premise
+ (TP) Every^{-} tail of an^{+} animal is^{+} a tail of an^{+} animal;	-(t+A)+(t+A)	Taut. Premise
∴(2) Every^{-} tail of a^{+} horse is^{+} a tail of an^{+} animal;	-(t+H)+(t+A)	(1) +(TP)

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§6.1 Being unaware that ‘some’ and ‘every’ are opposed to on another as ‘+’ to ‘-‘, contmporary philosophers of language tend to believe that traditional term logic cannot account formally account for Θ and contempory philosophers of language and they celebrate Frege’s victory over Aristotle. Here is Michael Dummett’s laudatory judgment that opens Duimmett’s book on Frege:

> For all the subtlety of the earlier systems, the analysis of the structure of the sentences of huan language which is afforded by modern logic is, by its capacity to handle multiple generality, shown to be far deeper than they were able to attain… If he [Frege] had accomplished only this he would have rendered a profound service to human knowledge.[8]

It’s indeed true that Aristotle’s terminist logic of natural language had provided no clear way to handle arguments with relational sentences. De Morgan pointed this out, as had Leibniz before him. But the only remedy a traditional term logic *needs* for formally accounting for valid inferences involving relational sentences is the awareness, which Dummett lacks, that the logical constants of natural language are **oppositive +/-** functors that can be reckoned with by exploiting their oppositive plus/minus values. A plus/minus calculus of natural language is as deductively powerful as the predicate calculus, *only very much simpler and expeditious.* The +/-character of the common logical particles renders traditional term logic more efficient, quicker and easier than MPL. Moreover, a traditional term logic, whose formative elements are reckoned with logibraically has the decisive advantage of being

[8] M. Dummett, **Frege, Philosophy of Language, Harvard University Press 1982 pp. xxi -xxii**

cognitively veridical, in contrast to Modern Predicate Logic, which offers only a rational reconstruction of the real-life inferences people, using “Natural Language, ” which is very unlike the symbolic language of quantifiers and bound variables that is taught in the universities by instructors of logic.[9]

The logical grammar of traditional Aristotelian logic that Fregeans find deficient in inference power is the grammar of natural language, of which Dummett says:

It is of the greatest importance for the understanding of Frege, to grasp [that] how matters stand with regard to natural language ... was a totally unsatisfactory state of affairs, revealing a defect of natural language that *must be remedied in any properly constructed language such as Frege’s own symbolic language*.[10] (my emphasis)

In my opinion, what is revealed is not a defect of natural language but a defect in the attitude of logical theorists who baselessly assume that an uninitiated population that intuitively reasons every day with the variable free sentences of its natural language cannot be reasoning properly because it lacks the prosthetic aid of “a properly constructed” symbolic language furnished with a mechanism of quantifiers and bound variables that, in Quine’s words, has been “made for logic” by logicians.

Dummett rightly points out that “the most general lesson which Frege derived form his discovery [of quantification] was a certain disrespect for natural language.” Being unaware of the +/- character of the logical particles of natural language, Dummett believes that the older, Aristotelian, logic is inferentially weak because the logical syntax of natural language lacks the inferential power conferred on MPL by its apparatus of quantifiers and bound variables. But that leaves unexplained the fact that a benighted public somehow intuitively reasons efficiently even with the relational, “multiply general” sentences of natural language sentences that are variable-free.

People may not be aware of how they are reasoning but they are certainty not reasoning in the way prescribed by any mathematical logician’s concept-script. Replacing the sentences of natural argumentations by the well-formed formulae of predicate logic as a “remedy” for the alleged logical deficiencies of natural language is no more a “proper” remedy for what ailed traditional logic than extracting a set of healthy, natural teeth and replacing them with artificial implants would be a proper remedy for toothache.[11]

THE LOGIBRAIC FORM OF CATEGORICAL SENTENCES

§8. Aristotle’s logic is a logic of natural language—the language of thought and everyday reasoning. The general logibraic form of a categorical sentence in natural language is:

YES^{+}/N0^{-}: Some^{+}/Every^{-} X^{+}/non-X^{-} is^{+}/isn’t^{-} Y^{+}/non-Y^{-}:
+/- (+/- +/- X +/- +/-Y)

A categorical sentence that is fully explicit logibraically has five +/- formatives. The first is the judgment sign of affirmation or denial, ‘Yes^{+}’/ ‘No^{-},’ for ‘It is^{+}/isn’t^{-}’ the case that…’ . The second is the sign of particular or universal quantity. ‘Some^{+}/Every^{-}’,. Next is the positive or negative quality of

[9] For a discussion that compares the expressive and deductive powers of MPL and the algebraic version of traditional logic, see my article, Predication in the Logic of Terms,” Notre Dame Journal of Formal Logic 31 (1990): pp. 106–26 PP. 106-126 .
[10] Dummett, **Frege: Philosphy of Language,** p 166
[11] Cf. Dummett, ibid. pp 20-21.

the subject term. The fourth is the sign for the positive or negative copula, ‘is^{+}’/ ‘isn’t^{-}.’ The fifth represents the positive or negative quality of the predicate term. For example, ‘ the full logibraic transcription of the affirmative statement, ‘Some Residents are non-Citizens’ is ‘+(+(+R)+(-C)).’ We would normally transcribe this more briefly as
‘+R+(-C),’ suppressing the positive sign of affirmative judgment and the implicit + sign that qualifies a term as positive. Similarly, we would simply transcribe ‘All Residents were Citizens’ as ‘-R + C.’ In what follows I suppress the plus-signs of term quality but sometimes retain the plus sign of affirmative judgment, sometimes transcribing ‘All Residents were Citizens’ as
‘-R+C’ and sometimes as ‘ +(-R+C).’

We can logibraically determine the existential ‘valence’ of a sentence by looking at the first two signs of its logibraic form. If the sign of judgment and the sign of quantity are the *same*, --- both being plus or both minus --- the sentence is particular in “quantity” and its existential valence is positive. If the signs of judgment and quantity *differ*, the sentence is universal and its existential valence is negative. For example, the first two signs of ‘+(- N+C’)’ [all Natives are Citizens] and ‘-(+N+C)’ [not: some Natives are citizens] *differ*. So both sentences are, universal and existentially negative. The judgment and quantity signs of ‘+(+ Billionaires +Citizens)’ [some billionaires are citizens’] and of ‘–(- Billionairs are Citizens)’ [‘not all billionaires are citizens’] are the *same*. So both these sentences are particular and existentially positive.

Sentences that have the same valence are said to be *covalent*. *Divalent* sentences differ in “valence,” one being particular and existentially positive, the other being universal and existentially negative.

The Conditions of Logical Equivalence:

Two categorical propositions are logically equivalent
if and only if
they are logibraically equal and logibraically covalent.

Both conditions must obtain. By the principle of equivalence divalent sentences cannot be logically equivalent. For example, ‘all natives are citizens’[(+(-N+C)] and ‘some citizens aren’t natives’ [+(+C-N)] are logibraically equal, but because they differ in quantity they are divalent and not logically equivalent.

THE DICTUM DE OMNI

§9. The governing principle of syllogistic inference in traditional term logic is the Dictum de Omni. The D.O. sanctions the logibraic way of inference. According to the D.O.:

What’s true of every$^{\{-\}}$ M is true of whatever is$^{\{+\}}$ an M.

By the D.O., when Ψ is said to be true every$^{\{-\}}$ M in one premise and ‘is$^{\{+\}}$ an M’ is said to be true of something in a second premise, the ‘middle term,’ M, occurs negatively in the first premise and positively in the second premise. When we add the first premise to the second, the negative middle term of the first premise cancels the positive middle term of the second premise, replacing it by Ψ. Logibraically:

$$\begin{array}{rl} & \{\Psi\}(-M) \\ & \underline{\ldots+M} \\ \therefore & \ldots\Psi \end{array}$$

E.g., In syllogisms (i) and (ii), **M** is ‘**M**ammals’ and **Ψ** is ‘**W**arm-blooded’

i.	ii.
All **M**ammals are **W**arm-blooded	All Mammals are **W**arm-blooded
<u>All Dolphins are **M**ammals</u>	<u>Some Sea Creatures are **M**ammals</u>
∴All Dolphins are **W**arm-blooded	Some Sea Creatures are **W**arm-blooded

The D.O. also applies to relational arguments. Take de Morgan’s ‘Tail of a Horse’ inference:

(Δ) Every horse is an animal, so every tail of a horse is a tail of an animal.

Applying the D.O., we may prove (Δ) valid by an indirect argument showing that affirming its premise but denying its conclusion entails a self-contradiction. For suppose it’s true of every horse that it is an animal but also true of some horse that its tail isn’t a tail of an animal. Since by the first premise, ‘is an animal’ is true of every horse, it must, by the D.O., also be true of whatever is a horse. So it would be true of a horse whose tail is not a tail of any animal that that horse is an animal whose tail is not a tail of an animal. That comes out clearly if we logibraically conjoin [add] the premise of (Δ) to the denial of its conclusion. If we do that we cancel the middle term, ‘horse’ and arrive at an overtly self - contradictory conclusion (of form ‘some X is not an X’), viz., that some tail of an animal is *not* a tail of an animal.

(1) $-\cancel{H}+A$: Every horse is an animal; ‘Is an animal’ is true of every horse.
<u>+(2) $+(t+\cancel{H}) - (t+A)$; Some tail of a horse isn’t a tail of a animal;</u> It’s true of some horse that its tail is not a tail of an animal.
∴(3) $+(t+A) - (t+A)$; So, it’s true of some animal that its tail is not a tail of an animal.

§10. Predicate logic is taught by teachers who never ask themselves how their benighted students have been intuitively but correctly reasoning since they were children. That question was raised by Plato in the MENO but it has not been satisfactorily answered. In my opinion, Hobbes had somehow divined the correct answer.

It is because we intuitively reason *logibraically, without being aware of doing so* that Plato’s MENO question of how we actually reason has remained something of a mystery. It is not however, the sort of mystery one can resolve by the kind of rational reconstruction of reasoning that replaces the natural language in which we actually reason by a “properly constructed” symbolic language possessed of inference powers that a logic of natural language is mistakenly assumed to lack. Nor is it the sort of mystery that must await unraveling by a mature neuroscience that will one day present us with a description of the brain activity that takes place in deductive reasoning. The puzzling nature of our intuitive deductive reasoning can be resolved now. After all, we reason deductively many times a day, and how we reason can be exposed in the way a conjurer’s magical trick can be exposed and explained to mystified spectators who then become made privy to the conjurer’s secret. When a conjurer’s methods are disclosed and explained, the exposure elicits an “Aha!” of re-cognition. The mystery is dispelled; “Aha! So that’s how he does it!”

In the case of deductive reasoning, it is we ourselves who are the (unwitting) conjurers as well as the mystified, bewildered spectators. A child that has learned that all snakes are reptiles and is told that someone bitten by a snakes was not bitten by a reptile, immediately concludes that this can't be true. She reasons logibraically but since she is unaware that she she reasons that way, she is unable to justify her conclusion.

Similarly, we are all instantly certain that 'Every tail of a horse is a tail of an animal' follows from 'Every horse is an animal' but are unable to say *why* we are certain. We're sure that 'Some boy loves every girl' entails 'every girl is loved by some boy' but don't know by what method we intuitively come to that conclusion. In all such cases, there is logical method to our instantaneous, intuitive reasoning, and when we learn that we are intuitively reckoning logibraically, we exclaim "Aha! So that's how we're doing it!"

Frege had an ingenious idea of how we *could* be reasoning. Though he believed that we could and should be reasoning the quantifier- bound-variable way, he probably knew better than to claim outright that people were *actually* reasoning that way. Being unable to say how – without the mechanism of quantifiers binding variables --- they could possibly be reasoning effectively and fluently in natural language, Frege changed the subject and the script, and offered a rational reconstruction of their reasoning in a "properly constructed" symbolic language.

Referring to Frege's ability to formally justify the validity of relational arguments with relational sentences, Dummett says, "Frege had solved the problem which had baffled logicians for millennia by ignoring natural language." He says:

Modern logic stands in contrast to all the great logical systems of the past . . . in being able to give an account of sentences involving multiple generality, an account which depends upon the mechanism of quantifiers and bound variables.… If he [Frege] had accomplished only this, he would have rendered a profound service to human knowledge. [12]

Dummett's historical judgment misprizes Aristotle's legacy and grossly overestimates Frege's. Newton's scientific physics did properly supplant Aristotle's physics, but, unlike his Physics, Aristotle's logic of natural language is not unscientific and Frege is no Newton.

§11. Frege's predicate logic did revolutionize the teaching of logic. But not all revolutions are progressive. Hobbes, who was interested in how we actually reason (and *not* in how how one *could*—more 'scientifically'—be reasoning) rightly divined that we mentally reason the +/- way. What I've been calling a 'logibraic' system is basically just an Aristotelian term logic that uncovers and exploits the oppositive, +/-, character of the 'logical constants' of natural language ---the universal "language of thought." Intuitively and instinctively exploiting the +/- character of the natural constants, enables any evolved rational animal possessed of descriptive language and some rudimentary algebraic know-how, to reason safely and correctly.   Frege, like Aristotle before him, was unaware of the +/- powers of the natural logical constants. But unlike Aristotle, Frege was srongly opposed to "psychologism" and to thinking of logic as a theory of everyday ratiocination in natural language. Aristotle would not have been surprised to find that we have evolved to reason naturally and "silently," the +/- way by reckoning with the additive and subtractive logical particles of natural language. That way of reasoning is strikingly simple and vastly more efficient than anything devised by mathematical logicians who "do logic" scientifically in a constructed symbolic language that is not the language of actual ratiocination.

Contrast the way a practitioner of MPL shows that

(3) Every boy envies some owner of a canine

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[12] M. Dummett, **Frege: Philosophy of Language,** 2nd edn., London: Duckworth, 1981, p. xxxii.

follows from (1) Every dog is a canine and (2) some boy envied an owner of a^{+} dog by painstakingly deriving (3*)$\exists y(\text{Envies}_{xy} \,\&\, \exists z(\text{Canine}_z \,\&\, \text{Owns}_{yz}))$ from the premises,
(1*) $\forall x(\text{Dog}_x \supset \text{Canine}_x)$ and (2*) $\forall x(\text{Boy}_x \supset \exists y(\text{Envies}_{xy} \,\&\, \exists z(\text{Dog}_z \& \text{Owns}_{yz})))$,
to the way our teenager can instantly derive (3) from (1) and (2) logibraically:

$$\begin{array}{rl} & (1) \quad -\cancel{D}+C \\ \text{and}^{+} & \underline{(2)\ -B+E+(O+\cancel{D})} \\ \therefore & (3)\ -B+E+(O+C3. \end{array}$$

CONSTRUCTIVISM IN MODERN LOGIC

§12. Frege's rational reconstruction of reasoning introduced the non-natural, concept-script that was to become the canonical symbolic language for the next century. As the Kneales say about Frege's contribution to logic: "The use of quantifiers to bind variables is the main distinguishing feature of modern logical symbolism and the device which gives it superiority… over ordinary language. 1879 is the most important date in the history of the subject."[13]

Though **MPL** has successfully supplanted the traditional logic of natural language the widely accepted belief that it rightly did so because it is inferentially superior to its millennial predecessor, is mistaken. People actually reason in natural language, which is variable free. Aristotelian logicians may not have known how to account for relational inferences, but they never lost sight of the MENO problem of *how people are intuitively reasoning and they never stopped looking for ways to solve it*.

Frege "solved" the problem of arguments that involving multiple general sentences but he did so by by *changing the subject and the language of everyday ratiocination, including reasoning with relational sentences*. No one, outside a logic classroom, actually reasons in the language of predicate logic. Indeed, even people trained in quantificational logic rarely reason by using quantifiers and variables. Despite this, Frege's predicate logic and logical grammar of quantification and bound variables has thoroughly supplanted Aristotle's logic of 'ordinary language' as the reigning contemporary "Logic of the Schools."

Quine, who played a large role in promoting predicate logic as the new standard logic is always fully aware of the tension between Frege's constructionist approach to logic and the traditional Aristotelian approach. But he doubles down squarely on the side of Frege. The grammar of MPL is, in Quine's unvarnished and unabashed characterization, "an artificial grammar designed by logicians …that we tendentiously call standard," using a "made for logic," grammar[14] that "facilitates logical inference" by methods no one in real life uses when inferring conclusions from premises in natural language. Quine candidly acknowledges that most people find the use of an artificial grammar disconcertingly irksome and cumbersome. Nevertheless, he uncompromisingly maintains that the adoption of MPL as 'standard logic' is *scientifically indispensable* to facilitate logical inference. He says:

"All of austere science submits pliantly to the Procrustean bed of predicate logic. Regimentation to fit it . . . serves not only to facilitate logical inference, but to conceptual clarity." [15]

[13] ***The Development of Logic*, Oxford: Oxford University Press, 1960, p. 511. 1879 is the year Frege's *Concept-Script* appeared.**

[14] Cf. W.V. Quine, **Philosophy of Logic**, Prentice Hall (1970), pp.35-36.
[15] W.V. Quine, **Quiddities,** Cambridge: Harvard University Press, (1987), p. 158.

However, there can be no scientific or pedagogical reason to submit to the procrustean bed of reasoning in the predicate logic manner if, in fact, we intuitively facilitate logical inference by reckoning with the logical constants of natural language as +/-operators. There is nothing unscientific about using elementary algebra in naturally reasoning with the logical particles of our language. The "made-for-logic," mathematical notation of the symbolic is language of MPL is certainly more "austere" than the +/- notation for the logic of natural language can in no acceptable sense be regarded as more scientific.

LOGIC AND COGNITIVE SCIENCE

§12.1 Academic Logic faces a future that will increasingly be shaped by the empirical findings of cognitive science on how we actually reason. Cognitive scientists have yet to become aware that we may all be reasoning in a simple algebraic manner. When they become aware of the +/- hypothesis, they will be looking for ways to confirm or disconfirm the hypothesis that we routinely reason the plus/minus way. I believe they will devise such ways and find that we do actually reason logibraically. That finding will have momentous consequences.

Here, I hopefully expect, is what may very well transpire:

Focusing on our everyday reasoning with the sentences of our variable-free natural language, cognitive psychologists will discover that even children reason fluently and competently by exploiting the oppositive, +/- character of the logical constants of natural language. That experiential finding will flatly overturn the currently accepted view that Aristotle's logic has been definitively superseded by Frege's scientific logic, whose symbolic language with its made-for-logic grammar of quantifiers and bound variables renders MPL capable of elegantly dealing with all kinds of relational arguments that are mistakenly assumed to be beyond the inference powers of any logic of natural language.

It will soon become generally known that we mentally reason very well by intuitively reckoning logibraically with the logibraic particles of natural language. It will then be obvious that there is no justifiable reason to teach Logic in a constructed non-natural language that renders this essential subject uninviting and intimidating to most adults and altogether inaccessible to all children.

A FINAL (PEDAGOGICAL) NOTE

§13. For nearly a hundred years, Predicate Logic, a deductively powerful constructionist system of Logic that does not pretend to describe how people actually reason in their own natural language, has become the 'standard logic' taught in the schools. Traditional Aristotelian logic used to be at the center of the school curriculum. It no longer is. The technical grammar of quantifiers and bound variables of the Predicate Logic that replaced it, is much too difficult to be taught in the lower schools. Even in the universities and the public at large, the revolutionary new logic is not a subject many find attractive. The unforeseen and very unhappy consequence of Frege's "austerely scientific" revolution in the teaching of logic is that whole generations of younger students, who are *intuitively* adept at deductive reasoning, are no longer taught formal logic at all, a state of affairs that is pedagogically unjustifiable.

Teaching a "user-friendly," logic of natural language that casts light on our intuitive mode of reasoning would quickly restore logic to its traditionally central, place in the educational curriculum. Logic would again be accessible to younger students and again be popular with a knowledgeable and interested adult public. The urgently needed first step on a counter-revolutionary road back to Aristotle is to teach logic in a way that comports with how people actually reason. In reactivating that classical approach to logic, one is better guided by the inspired suggestion of Hobbes that we reckon algebraically with the logical particles of natural language, than by the ingenious constructivist approach of Frege, who devised a technically admirable symbolic logical language that is not the language of thought in which we fluently reason.

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