Logic and Gambling

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1. Abstract/Overview

This paper outlines a formal recursive wager resolution calculus (*WRC*) that provides a novel conceptual framework for sentential logic via bridge rules that link wager resolution with truth values. When paired with a traditional truth-centric criterion of logical soundness WRC generates a sentential logic that is broadly truth-conditional but not truth-functional, supports the rules of proof employed in standard mathematics, and is immune to the most vexing features of their traditional implementation.

WRC also supports a novel probabilistic criterion of logical soundness, the *fair betting probability criterion (FBP)*. It guarantees that the conclusion of an FBP-valid argument is at least as credible as a conjunction of premises, and also that the conclusion is true if the premises are. In addition, WRC provides a platform for a novel non-probabilistic, computationally simpler criterion of logical soundness – the *criterion of Super-validity* - that issues the same logical appraisals as FBP, and hence the same guarantees.

2 Wager Resolution

2.1 The General Structure of WRC

The scope of WRC is simple wagers with two payoffs – a single amount won and a single amount lost. WRC employs three wager resolution values: W, read as 'won', L, read as 'lost', and K, read as 'neither won nor lost'. There are two semantic (or truth) values, t, read as 'true' and u, read as 'false' and as 'untrue'. WRC incorporates the idea that a gambler wins a wager if and only if (iff) his opponent loses it, and loses it iff his opponent wins it. Wager resolution and semantic values are bridged by the idea that a gambler wins a wager on a hypothesis H if H proves true and its wager-opposite d(H) - the hypothesis that his opponent bets on - proves false. A gambler loses a wager on H if d(H) proves true and H proves false. It follows that no party to a wager on H either wins or loses his bet if H and d(H) have the same truth-value. In this case H has the wager resolution value K.

1.2 The formal Calculus.

The atomic formulae of the languages Λ of WRC are p, q and r with or without integer subscripts. In practical applications we select a language with enough atomic formulae to encode the propositions involved. The monadic sentence operators of WRC are the traditional contradictory generating operator, which is represented by the tilde symbol '~', and the wager-opposite operator, which is represented by 'd'. The binary sentence operators of a language Λ are represented by ‘∩’, ‘∪’, ‘→’, and ‘⊃’. The intended colloquial readings of the first three in order are ‘and’, ‘and/or’ and ‘if .. then _’ and its cognates. *Material conditional* formulae of the form ‘H ⊃ S’ have the same truth and wager resolution conditions as the corresponding disjunction $\sim H \cup S$.

Grouping is accomplished by a standard use of parentheses. Uppercase letters ‘A’, ‘B’, and ‘C’ of the meta-language (with or without integer subscripts) represent formulae that are neither → formulae nor contain ‘→’ sub-formulae, i.e. are *simple*. Uppercase letters from E through J, and M through T of the meta-language (with or without subscripts) are not so restricted.

An interpretation of the formulae of a language Λ of WRC is an ordered pair consisting of a semantic value function s and a wager resolution value function v that are defined over the formulae of Λ. An interpretation <s,v> satisfies the following rules:

i) a) $s(H) = t$ iff $s(\sim H) = u$

ii) If H is an atomic formula, $s(d(H)) = s(\sim H)$;

iii) $v(H) = W$ iff $s(H) = t$ and $s(d(H)) = u$.

iv) $v(H) = L$ iff $s(H) = u$ and $s(d(H)) = t$.

v) $v(H) = K$ iff $s(H) = s(d(H)$

vi) The function v satisfies the following Wager Recursion Matrix (WRM).

	H	S	d(S)	~ S	H ∩ S	H ∪ S	H → S	H ⊃ S
1	W	W	L	L*,K ?	W	W	W	W
2	W	L	W	W*,K?	L	W	L	L
3	W	K	K	L*,K?	W*,K?	W	K* W?	K
4	L	W	L	L	L	W	K	W
5	L	L	W	W	L	L	K	W
6	L	K	K	L	L	L, K*?	K	W
7	K	W	L	L	W*,K?	W	W* K?	W
8	K	L	W	W	L	L, K*?	L* K?	L
9	K	K	K	L	K	K	K	K

Question marks in a cell of WRM indicate that each of the two entries is prima facie plausible; the asterisk to the right of an entry indicates that it is the least problematic of the two and hence is the official entry. Rationales for asterisked entries are presented in the next section

Rules iii) iv) and v) bridge semantic and wager resolution values. Since s has two values, t and u, it follows from i) that s(H) ≠ t iff s(~ H) ≠ u, whence s(H) = u iff s(~ H) = t. It also follows that s(H) = s(~ (~ H)), and that that atomic formulae never have the value K.

The entries for the d-column indicate that v(d(d(S))) = v(S), hence s(S) = s(d(d(S))). Rows 1, 2, 4 and 5 of WRM show that If H and S never have the value K, the only formulae formed from them that have the value K are either → formulae or formulae that have → sub-formulae. A formula that does not have the value K on any interpretation will be described as *non-K*. Simple formulae are non-K, but the converse has exceptions. For example, under WRM ~ (H → S) is never K.

The entries on Rows 1 and 2 of WRM imply that if S is a non-K formula then s(d(S)) = s(~ S) and v(d(S)) = v(~ S) . So the wager-opposite of a formula is its contradictory if the formula is non-K, but not if S can have the value K (because if S is K, then s(S) = s(d(S)).

3. Rationales for Asterisked Entries

3.1. Negation.

We want the entries in the negation column to capture the intuition that a wager on a conjunction of a hypothesis and its contradictory is lost in all cases, e.g., a formula of the form S ∩ ~ S should be L in all cases. Entries for the ∩ column capture the uncontroversial idea that a wager on a conjunction is lost iff a separate wager on at least one conjunct would be lost. Thus if S is L, v(S ∩ ~ S) = L. If S is W, satisfaction of the intuition requires v(~ S) = L. Similarly if S is K satisfaction of the intuition requires an entry of L for ~ S, hence the entry of

L for rows 1 and 3 of the negation column. In light of rule iv) the entry of L for row 3 implies that if S is K, s(~ S) = u, whence by rule i) s(S) = t. This holds for all cases. So a formula that is K is t. It follows from bridge principle iv) that if S is L, as in row 2, then ~ S is t and thus is either K or W. So for the entry on Row 2 we have three alternatives: K, W, or leaving the choice between K or W open. The no choice policy would generate significant computational complexity, and a choosing K would lead to the undesirable result that a wager on the contradictory of a proposition is never won. W is the least problematic entry for Row 2.

The reader may have noticed that the result that a formula is t if it is K has profound consequences for the logic of conditionals. It implies that a conditional formula has the same truth-conditions as the corresponding ⊃ formula, i.e. the same truth-conditions as its *material counterpart*. A reader who is has a negative attitude to this *Material Counterpart Rule* (*MCR*, for short) might consider rejecting the rule that v(S ∩ ~ S) = L in all cases in favor of the weaker rule that if S is K a wager on S ∩ ~ S is K. But this would have the effect of conflating the negation operator with the wager-opposite operator. In addition, there is another compelling intuition that supports the link between K and t: the intuition that a secondary wager on the hypothesis that a primary wager on a proposition will be won is won if the primary wager is won, and is lost otherwise. For, in light of bridge principle iii) '$S \cap \sim d(S)$', $\omega(S)$ for short, encodes the hypothesis that a wager on S will be won. So a wager on the secondary hypothesis is tantamount to a wager on $\omega(S)$. Thus satisfaction of this intuition requires a wager resolution value of L for $\omega(S)$ when S is K. If S is K; then $\sim d(S)$ must be L in order for $\omega(S)$ to be L ; in that case $\sim d(S)$ is u whence d(S) is t and so is S so we have the same general result: s(S) = t if v(S) is K.

It follows that d(S) does not imply $\sim S$, but the converse holds, hence $\sim d(S)$ and $\sim S$ are logically incompatible.

Finding colloquial readings of the wager-opposite operator and the negation operator that captures the same work they do for non-K propositions and the different work that they do for K propositions is a challenging task. I propose reading formulae of the form d(S) as 'It is not the case that S' and also as 'Its is false that S', and reading $\sim S$ as 'The proposition that S is not true [or is false/untrue] untrue'. If S is correctly represented by a non-K formula $\sim S$ and d(S) such have the same truth and wager resolution conditions (as shown by rows 1 and 2); but if S is a conditional or logically embeds conditionals we should see the difference between the two operators indicated by row 3. The entries for conditionals show this, as explained in section 3.3.

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3.2 Conjunction.

An entry of K in the cells on rows 3 and 7 of the conjunction column, in concert with rule that a wager on a conditional is K when its antecedent proves false, would indicate that a wager on the intuitive tautology represented by $(A \rightarrow A) \cap (\sim A \rightarrow \sim A)$ is never lost but cannot be won (because then it is K on all interpretations). Ditto for the intuitive tautology $(\sim A \rightarrow \sim A) \cap (A \rightarrow A)$. Rejecting this strange in favor of the thesis that wagers on such tautologies is won in all cases requires an entry of W on rows 3 and 7. Another counterintuitive consequence of an entry of K on these rows is shown by the following example. Suppose that a coin is to be tossed three times and that a bet is proposed on the conjunctive hypothesis that if exactly one heads is tossed (A), the first toss will be heads (C) and that if exactly two heads are tossed (B), the first toss will be heads (C). This hypothesis is encoded by a formula of the form $(A \rightarrow C) \cap (B \rightarrow C)$. In light of the relatively uncontroversial entries on rows 1 and 5 of the $\rightarrow$ column and the observation that A and B cannot both be true, if one conjunct is W the other is K. Thus an entry of K on rows 3 and 7 would indicate that a wager on this conjunction is not won. However, the proposition that if A or B, C is typically understood as saying the same thing as the conjunction, hence it should have the same resolution conditions as a wager on the conjunction. In light of the uncontroversial entries for ∪ and → the formula (A ∪ B) → C is W if A and C are W and B is L. Thus an entry of W on rows 3 and 7 is required to capture the intuition that the conditional has the same wager resolution conditions as the conjunction.

3.3 Entries in the → Column

In light of the grammatical work done by 'if' described in standard English dictionaries, that of introducing a subordinate clause of supposition or condition, a proposal of the form 'I will bet you x dollars that S if H' will be interpreted as the proposal of a wager of x dollars on S that is conditional on the truth of H: if H proves false the wager is void, and if H proves true, the wager is won if a wager on S would be won under the circumstances, is lost if a wager on S would be lost under the circumstances, and is neither won nor lost if a wager on S would neither be won nor lost under the circumstances. This is clearly captured by the entries in rows 1-6. and also by the entries on rows 7-9 (in light of the theorem that a formula is t if it is K). A consequence of these wager resolution conditions is that if H is either W or K, v(H → S) = v(S), and v(H → S) = K if H is L.

If H and S are simple then the established practice of resolving wagers on conditionals with simple antecedents and consequents is captured by the entries on 1,2, 4 and 5. For, these entries indicate that a wager on 'If A, C' is won if A and C prove true, is lost if A proves true and C false, and is neither won nor lost when A proves false.

A consequence of these entries is that the formula d(H → S) has the same wager resolution conditions and hence the same semantic value as the *conditional contrary* of H S, i.e., of H → d(S). Thus the conditional contrary of H → S is also wager-opposite of it. A wager on it is won if H proves true and S proves false, is lost if H and S prove true, and is neither won nor lost if H proves false. On the other hand, a wager on ~ (H → S) is won if H proves false and S proves true, and is and is lost otherwise. The wager-opposite of a conditional is not equivalent to its contradictory.

From the standpoint of WRC the criterion of sound encoding is sameness of wager resolution conditions, not sameness of truth conditions. With this in mind, these results are consistent with the readings proposed above. For, in light of the grammatical work done by 'if, statements of the forms 'It is not the case [or false] that if H, S' have the same intuitive wager resolution conditions as corresponding statements of the form 'Under the supposition or condition that that H, it is not the case [or false] that H. The latter does the same work in cooperative exchanges of information as that 'If H, it is not the case [false] that S. Under the established practice of resolving wagers on colloquial conditionals, wagers on 'If H, it is not the case that S' are won if H proves true and S proves false, are lost if H and S prove true , and are neither won not lost if H proves true. On the other hand, wagers on hypotheses of the form ‘The proposition that if H, S’ is not true [false]’ are intuitively won if the proposition that if H, S proves false and are lost otherwise. They are aptly encoded by a formula of the form ~ (H → S), which has these wager resolution conditions.

3.4 Disjunction and MCR. The asterisked entries on rows 6 and 8 of the ∪ column reflect reluctance to judge a wager on a disjunction as lost unless separate wagers on both disjuncts are lost. It also captures the intuition that a wager on an inclusive disjunction is won iff a wager on at least one disjunct would be won.

The entries for H ⊃ S are the wager resolution values of the corresponding entries of ~ H ∪ S.

3.5 McDermott's Wager Resolution Calculus.

In his [1996] M. McDermott explored the links between wager resolution and logic by identifying semantic values with wager resolution values: truth with won, falsehood with lost, and neither won nor lost with a third semantic value X, which he treated both as 'neither won nor lost' and as 'neither true nor false' His calculus of wager resolution/truth-value does not employ a special wager-opposite operator in addition to a negation operator, and none of the alternative treatments of conjunction and disjunction he proposed uniformly matches WRM's. His interesting approach piqued my interest in exploring the constraints placed on semantically bivalent sentential logics by a wager resolution calculus with bivalent bridge rules and a wager-opposite operator.

4. Some Important Definitions and Consequences of WRM. .

4.1 t-validity

Definition: a formal argument form S1, S2 ... Sn ∴ H is *t-valid* iff there is no interpretation on which the premises are all t and the conclusion is u. In light of the identification of K and W with t and L with u this is criterion does the same work as a criterion that treats S1, S2 ... Sn ∴ H as logically sound iff there is no interpretation on which the premises are either W or K and the conclusion H is L.

In light of the link between K and t the truth-conditions of ~, ∩, ∪, and ⊃ formulae rules match the negation, conjunction, disjunction and material conditional operators of classical sentential logic. Hence argument forms that contain just these operators are t-valid under WRC iff they are valid under the traditional truth-centric criterion of logical soundness. This does not hold for arguments that contain conditionals.

4.2 Super-Equivalence.

Sameness of truth-conditions is defined as *t-equivalence,* and sameness of wager resolution conditions is *super-equivalence.* Super-equivalence is tantamount to joint t-equivalence and t-equivalence of wager opposites. Proof of this claim: t-equivalence of S and H is incompatible with rows 2,4,6 and 8 of WRM, and t-equivalence of d(H) and d(S) is incompatible with rows 3 and 7. So the t-equivalence of H with S and of d(H) with d(S) is compatible only with rows 1, 5 and 9. In these rows H and S have the same wager resolution value. Conversely, if H and S are super-equivalent then rows 1, 5 and 9 and no others obtain, hence d(H) is t-equivalent to d(S)

The entries on rows 4,5 and 6 show that conditionals are t-equivalent but not super-equivalent to their material counterparts. It will be shown that the lack of super-equivalence drives significant logical wedges between conditionals and their material counterparts – wedges that defang, disarm or dissolve many problematic features of the traditional implementation of MCR.

The entries for negation show that S is t-equivalent but not super-equivalent to ~ (~ S). On the other hand d(d(S)) is super-equivalent to S and hence t-equivalent to it.

4.3 Truth-hypotheses

Intuitively a wager on a hypothesis of the form ‘S is true’ is won if S proves true and is lost otherwise. Wagers on truth-hypotheses are thus either won or lost, never canceled in principle. WRC has the tools to capture the difference between wagers on S and wagers on ‘S is true’: the formula ~ S ∩ d(S) , λ(S) for short, is L iff S is either W or K, hence d(λ(S)) is W and t if S is t, and is L and u if S is u. It is never K – it is a *non-K* formula. Thus, d(λ(A→ C)) has the wager resolution profile of 'The proposition that C if A is true' but not of 'A → C'.

It will be shown in section 5.4 that in concert with the time-honored idea that fair betting odds in favor of a proposition are measures of a warranted degree of confidence in it, a warranted degree of confidence that C if A is typically less than a warranted degree of confidence that it will prove true that C if A: the latter can be quite high while the former is zero. This difference reflects the difference in wager resolution profiles

4.4 Properties of the Wager Opposite operator.

The wager-opposite operator is not truth-functional. For example, S is t-equivalent to ~(~ S), but d(S) is not t-equivalent to d(~(~ S)). For, if S is K, d(S) is K and t, but d(~ (~ S)) is L and u. More generally, substitution of a formula that is t-equivalent to S for S in a formula of the form d(S) may not have the same semantic value as the original. On the other hand, if S is super-equivalent to R, d(S) and d(R) are super-equivalent as well. Since the wager resolution conditions of a formula are a determinate function of the wager resolution conditions of its sub-formulae in situ substitution of a formula for a super-equivalent sub-formula does not change overall wager resolution conditions and thus truth-conditions.

An important example of the non-truth-functional character of the wager-opposite operator is revealed by a comparison of the → column with the ⊃ column of WRM; it shows that d(H → S) is not t-equivalent to d(H ⊃ S) even though H → S is t-equivalent to H ⊃ S. For, d(H → S) is K and hence t on rows 4,5 and 6 whereas d(H ⊃ S) is L and u on these rows. The wager-opposite of a material counterpart of a conditional with non-K antecedent and consequent is a contradictory of it, but a wager-opposite of the conditional is not a contradictory of it.

This logical wedge between conditionals and their material counterparts disarms a major complaint about MCR – that it treats as logically sound colloquial non-sequiturs of the forms ‘It not the case that if H, S, therefore H and not-S’. An example: ‘It is not the case that if it rains the game will be canceled ; therefore, it will rain and the game will not be canceled'. This absurd argument is logically sound under the traditional implementation of MCR because it recommends encoding propositions of the forms 'It is not the case that if H, S' when H and S are simple by a formula of the form ~ (H ⊃ S), which implies both H and ~ S. In his [2001]. Lycan describes examples of this sort as “the worst material implication paradox of all”, and claimed that no treatment of indicative conditionals, in particular, MCR, “can be right that does not explain the outrageously evident invalidity of that inference pattern”. However, WRC's implementation of MCR offers a cogent explanation: it supports encoding propositions that are expressed by sentences of the form 'It is not the case that if H, S' by a formula of the form d(H → S). The latter is super-equivalent to H → d(S) which does not imply either H or ~ S. So if H and S are simple, H → d(S) is t-equivalent to H → ~ S and hence to H ⊃ ~ S, which does not entail H, and does not entail ~ S. For a similar reason, WRC disarms the bogus compliant that MCR per se treats as valid the colloquial non-sequitur 'It is not the case that S if H; therefore H if S'.

Another example of the mistake of treating the wager-opposite of a conditional as its contradictory is Edgington's tongue-in-cheek "favorite proof of the existence of God", which she mistakenly interprets as a counterexample to MCR per se, rather than as a counterexample to its traditional implementation.

> 'If God doesn't exist (not-G) then it is not the case that if I pray (P), my prayers will be answered (A), I do not pray; therefore God exists. (Edgington [2001] p.394).

This argument is an intuitive non-sequitur. But from the standpoint of WRC it is not, as Edgington claimed, an instance of an MCR -valid argument form. Edgington probably assumed that MCR per se warrants encoding the argument by ~ G ⊃ ~ (P ⊃ A), ~ P ∴ G , which is t-valid. However, from the standpoint of

WRC the form of the colloquial argument is correctly represented by $\sim G \rightarrow d(P \rightarrow A), \sim P \therefore G$. And since $d(P \rightarrow A)$ is super-equivalent to $P \rightarrow d(A)$ and hence to $P \rightarrow \sim A$ (because A is simple), the latter may be substituted for $d(P \rightarrow A)$ in situ. The result of such substitution is $\sim G \rightarrow (P \rightarrow \sim A)$, which is super-equivalent to $(\sim G \cap P) \rightarrow \sim A$. The material counterpart of the latter is $(\sim G \cap P) \supset \sim A$, which does not imply G in concert with $\sim P$. (Note that the material counterpart of a *stand-alone* conditional premise may be used for a truth-value analysis of an argument).

4.5 Wager-opposites and Logical Incompatibility

If a hypothesis and its wager-opposite are both true, they will be described as *trivially true*. Some readers may interpret the trivial truth of $A \rightarrow C$ and its wager-opposite $A \rightarrow \sim C$ when A is u under WRC as proof that WRC is flawed because it is incapable of capturing the intuitive sense of opposition between colloquial conditionals and their wager opposites. However, this "proof" ignores potent pragmatic sources of intuitive opposition: since a wager on a hypothesis is won iff a wager on its wager-opposite is lost a wager on a conjunction of $A \rightarrow \sim C$ and $A \rightarrow C$ cannot be won. More generally, $d(S \cap d(S))$, which represents the schema ‘It is not the case that both S and not-S’, is a tautology of WRC. In addition, it will be shown that the credibility of a conditional is low if the credibility of its wager-opposite is high, and vice versa..

The pragmatics of everyday cooperative exchanges of information provide additional sources of the sense of opposition. For example, the assertion, without qualification, of a hypothesis S in such an exchange signals, but does not explicitly say, that the speaker judges S to be more credible than the hypothesis that it is not the case that S. In light of the grammatical work done by 'if', to signal that one judges that it is significantly more credible than not that C if A is to signal that that the speaker judges C significantly more credible than not-C on the supposition that A. So asserting that C if A without qualification, followed by an assertion, without qualification, that not-C if A conveys the self-defeating signal that on the supposition that A C is both significantly more credible than not-C and that not-C is significantly more credible than C.

Readers who remain unconvinced that the sense of opposition between an indicative conditional and wager-opposites is not logical may demand a clear example of a conditional and its wager-opposite that are both true on a realistic scenario. The Voting Example presented in the next section is such an example. It is not a one-off in light of the accompanying recipe for constructing such pairs. This recipe makes it clear that the number of such examples is limited only by a lack of imagination.

4.6 The Voting Example

A background assumption of the Voting Example is that Smith and Jones each decide to vote in an upcoming election with two candidates, a republican and a democrat, by independently tossing a fair coin: Smith will vote for the republican candidate if heads is the result of his toss and for the democrat candidate if tails, and Jones will follow the same practice for his toss. Consider the hypothesis that Smith and Jones will both vote for the republican (C) if they vote for the same candidate (A). Smith and Jones agree to a wager in which Smith bets on the hypothesis that if A,C and Jones bets on its wager opposite if A, it is not the case that C. ‘H’ stands for ‘Smith will toss heads ’ and ‘T’ for ‘Jones will toss tails. ‘R’ stands for Smith will vote for the republican candidate, and ‘D’ stands for Jones will vote for the democratic candidate. In light of the background assumptions, the hypothesis that if A, C follows from R which in turn is a consequence of H, Similarly, the hypothesis that if A, it is not the case that C' follows from D which follows from T. Under the time-honored principle of the monotonicity of deductive entailment both conditional hypotheses follow from the conjunction of H and T. So if Smith tosses heads and Jones tails the hypotheses respectively represented by 'If A, C' and 'If A, it is not the case that C' are both true, in which case no party to the wager wins or loses their wager, not because the truth-value of the conditional and its wager-opposite are difficult to ascertain, but because they are both clearly true.

Even if the reader does not find the appeal to monotonicity compelling, he/she should find compelling the thesis that if H and T prove true there are no grounds to treat either conditional as false. This leaves two options, neither conditional has a truth-value (the "nuclear" option) or both are true. The assumption of bivalence is consistent only with the latter. In his seminal discussion of his Riverboat Example, Gibbard [1981] chose the nuclear option on the ground that the intuitive opposition between a conditional and its conditional contrary rules out joint truth. However, once it is seen that the intuitive sense of opposition between 'If A, C' and 'If A, not-C' is plausible explained by the pragmatics of wagering and of conditionals, the joint truth thesis is more reasonable than the nuclear option. Moreover, in light of the foregoing arguments and more to come, the logical infirmities of MCR alleged in the literature are infirmities its traditional implementation, and are largely disarmed by WRC. Add to this the observation that logics that incorporate the joint truth option both capture the traditional rules of demonstrative proof of standard mathematics and the case for joint truth is compelling.

A recipe for constructing examples like the Voting Example : articulate propositions R, D, C, A and plausible background assumptions B such that A and C are simple, R is logically consistent with D, in concert with B R implies that C on the supposition that A, and D implies, in concert with B, that 'It is not the case that C on the supposition that A.

5. Absurd Instances and Credibility

5.1 Absurd Colloquial Instances of t-valid Argument Forms

I have not yet addressed the most serious objection to MCR in the literature – patterns of reasoning with conditional conclusions that are t-valid in concert with MCR but have absurd instances – instances that are obvious non-sequiturs. A list of such argument forms would include, in addition to the so-called "paradoxes of material implication", stalwarts of reasoning with conditionals in standard mathematics and a host of technical disciplines, e.g., Inference to the Contrapositive, the Hypothetical Syllogism, and Strengthening the Antecedent.

First, some definitions. A colloquial argument R1, R2, ... Rn ∴ F is an *instance* of a formal argument form S1, S2 ... Sn ∴ H (Φ, for short) iff for all i, Si correctly encodes Ri and H correctly encodes F. The looseness of the concept of 'correct encoding' is not a problem for our treatment of absurd instances.

A hypothesis is *trivially true* on a possible scenario (or state of affairs) iff it is true and its wager-opposite is also true on it. In light of the link between K and the semantic value t, a conditional is trivially true iff its antecedent is false. A proposition is *non-trivially true* on a scenario if the proposition is true on it and its colloquial wager-opposite is false on it. A proposition is *absurd* if its wager-opposite is true on all realistic scenarios. A colloquial instance of an argument form is *absurd* iff its conclusion is absurd and a conjunction of its premises is non-trivially true on some realistic scenarios, and is thus credible on them.

An absurd instance of Inference to the Contrapositive follows:

> If it rains in Boston tomorrow (B) it will not rain hard in Boston tomorrow (not-H); therefore if it rains hard in Boston tomorrow (H), it will not rain in Boston tomorrow (not-B).

The instance is formally encoded by $B \rightarrow \sim H \therefore H \rightarrow \sim B$. $H \rightarrow B$ encodes a wager-opposite of the conclusion. Since the colloquial wager-opposite of the conclusion represents an analytically true proposition for practical purposes the conclusion is absurd. The premise is non-trivially true on scenarios in which Boston will experience only a few light showers tomorrow. Despite its evident absurdity the instance does not constitute a strict logical counterexample because the conclusion is trivially true if the premise is true. For, on the supposition that H B follows from the true wager opposite, and the contradictory of H follows from B and the premise. Thus H is false if the premise is true whence the conclusion is trivially true if the premise is true. The strategy of deriving the trivial truth of the conclusion via Indirect Proof and Modus Ponens and other noncontroversial rules of proof works for all absurd instances with conditional conclusions. For example, suppose that the argument A → B, B → C ∴ A → C represents an absurd instance of the Hypothetical Syllogisms. The wager-opposite of the conclusion is represented by A → ~ C, and may presumed true on all scenarios on which the premises are true. Under the supposition that A, it follows from the premises and the wager-opposite that both C and ~ C (by two applications of Modus Ponens). So if the premises are t, A is u and the conclusion is trivially true. This analysis does not beg the question by presupposing MCR directly because it is grounded in the entries in WRM for negation. The bottom line is that absurd instances are not strict logical counterexamples to the join of MCR and t-validity

Absurd instances of t-valid argument forms are intuitive non-sequiturs because their conclusions are incredible on all realistic scenarios and a conjunction of the premises is credible on some realistic scenarios. The idea that an instance of a t-valid argument can have a fairly credible premises and in incredible conclusion may appear to incompatible with the theorem that the standard probability of the conditional conclusion of a t-valid argument must be at least as great as the standard probability of a conjunction of premises. But it is not because standard probabilities of truth are not reliable measures of the credibility of conditionals. The following plausible probabilistic treatment of the Rain Example shows this.

5.2 Probability and the Rain Example

Suppose that a standard probability function P represents a statistically grounded forecast for tomorrow's weather in Boston. In general 'P(S)' represents the likelihood that S will prove true (because standard probability is probability of truth). Since it is impossible that both H and not-B, $P(H \cap \sim B) = 0$. Suppose that that $P(B) = .5$ and that $P(B \cap \sim H) = .45$; it follows that $P(B \cap H) = .05 = P(H)$, and that $P(\sim H) = .95$. The zero likelihood of $H \cap \sim B$ indicates that $\sim B$ has zero credibility on the supposition that H, i.e., that it is incredible that it will not rain in Boston tomorrow if (on the supposition that) it will rain hard in Boston tomorrow. On the other hand, the premise is intuitively credible because on the supposition that B $\sim H$ is 9 times more likely than H. This credibility inequality holds despite their t-equivalence and the equality of their standard probabilities. Since, $H \rightarrow \sim B$ is true if and only $H \supset \sim B$ is true, and since P is a standard probability function and thus assigns equal numbers to t-equivalent propositions, $P(H \rightarrow \sim B)$ equals $P(H \supset \sim B) = P(\sim H) + P(H \cap \sim B)) = .95 + 0$. All of this .95 probability of truth is probability of trivial truth. Similarly $P(B \rightarrow \sim H) = P(\sim B) + P(B \cap \sim H)) = .5 + .45 = .95$. We may conclude that The standard probability of a conditional is not a reliable measure of its credibility because it may not equal the probability of its consequent on given that or on the supposition that its antecedent is true. A plausible explanation of this phenomenon is that it reflects the difference in wager resolution conditions of a conditional and the hypothesis that it will prove true: a wager on ‘If H, not-B cannot be won (because the conjunction of H and not-B is impossible), and a wager on the hypothesis that if H, not-B will prove true is won if H proves false. This strongly suggests that an apposite measure of the credibility of conditionals, and of propositions in general, is a function of the likelihood of potential wager outcomes.

Gamblers have used such a measure for centuries: fair betting odds in favor - the ratio of the likelihood of winning to the likelihood of losing. In view of bridge principles iii) – v) this ratio equals the probability of non-trivial truth divided by the likelihood of falsehood. For the Rain Example the fair betting odds in favor of the conclusion equal zero, but for the t-equivalent premise they equal .45 ÷ .05 = 9, or 9 to one in gambler's parlance. However, fair betting odds for both the hypothesis that the premise is true and for the hypothesis that conclusion is true equals .95 ÷ .05 = 19 to one, which indicates a very high degree of credence. This is possible because these odds take reflect the likelihood of trivial truth, whereas odds in favor of the conclusion and premise reflect only the likelihood of non-trivial truth. $P(\omega(S) \div P(\lambda(S)$, represents the ratio of the likelihood of nontrivial truth to the likelihood of falsehood; this is the ratio that is of interest to gamblers considering a proposed wager on S.

We have seen that the fair betting odds measure of credibility does not require that t-valid propositions have the same measure of credibility. This disarms D. Edgington's [1995] influential "credibility gap" argument against MCR. The main premise of her argument is the thesis that if a logical theory implies that a conditional has the same truth-conditions as a given proposition, and intellectually competent and probability literate persons can have a high degree of confidence in the given proposition and a low degree of confidence in the conditional, or vice-versa, then this is strong evidence that the theory is wrong. This claim presupposes that for statements that have truth-conditions, i.e., that express propositions, credibility goes by the likelihood of truth. The Rain Example and others of the same kind show that such a person can indeed have a high degree of confidence in the material counterpart of a conditional and a low degree of confidence in the conditional and that this is explained by the fair betting odds measure of credibility. Since this measure is time-honored and cogent Edgington's argument is disarmed.

The case for the cogency of the fair betting odds cum fair betting probability measure is firmed up by the following discussion

5.3 Fair Betting Odds in Favor.

A rationale for the fair betting odds measure of credibility exploits the observation that the maximum ratio of risk to gain an probability savvy and economically prudent gambler G is willing to tolerate in a simple straight up wager Wag on a hypothesis S (*max(S)* for short) is a sound measure of his degree of confidence that S: the higher G's max is, the greater his degree of confidence is S, and the lower max is the weaker is his degree of confidence. If an apposite standard probability function P defined over the possible outcomes of Wag is at hand then P determines G's max for S in concert with Decision Theory, the theory of economically prudent choice among risky and uncertain alternatives. The theory holds that if the sums of money associated with a Wag are small in relation to G's total fortune and $P(\lambda(S))$ is not zero then G's max(S) should equal $P(\omega(S) \div P(\lambda(S)$ - the ratio of the probability that a wager on S will be won to the probability that it will be lost, i.e., fair betting odds in favor of S. For, according to Decision Theory participation in Wag is economically prudent if the expected gain, i.e. potential gain times the probability of winning, is greater than expected loss - potential loss times the probability of losing- and is imprudent if the reverse. So if the posted ratio of potential loss (risk) to potential gain – the posted odds in favor of S - is greater than fair betting odds in favor, then expected loss is greater than expected gain, whence participation would be imprudent. Conversely, if the posted odds are less than fair betting odds, participation is economically prudent because then expected gain is greater than expected loss. If posted odds in favor of S equal fair betting odds in favor of S, $P(\omega(S)) \div P(\lambda(S))$ then the wager is *fair* – fair because expected gain equals expected loss, whence no party to the wager has a advantage over his opponent. So fair betting odds in favor of S constitute a warranted measure of a probability savvy and economically prudent gambler's degrees of confidence that S. Fair betting odds in favor may thus be described as a measure of the credibility of S relative to P.

We have seen that if S can have the value K there can be a large difference between the fair betting odds in favor of S and the fair betting odds in favor of the hypothesis that S is true. The former equals $P(\omega(S) \div P(\lambda(S)$ and the latter equals $P(\omega(S) \cup \kappa(S)) \div P(\lambda(S)$ where $\kappa(S)$ is the probability of cancelation, i.e., the probability of trivial truth. The numerator equals P(S) = the standard probability that S is true relative to P. If S is non-K, $P(\kappa(S)) = 0$.

5.4 Fair Betting Probability/ Fair Betting Quotients

The fair betting odds measure of credibility is numerically unbounded, and thus is computationally less convenient than a normalized version of the odds measure that pairs maximum credibility with a real number, say one, and minimum credibility with zero, and increases when fair betting odds do and decreases when they do. The fair betting probability function P* determined by fair betting of odds does this job. It identifies P*(S) with the maximum ratio of potential loss to the sum of potential loss and potential gain that an economically prudent gambler is willing to tolerate for a wager on S. This ratio is determined by fair betting odds in favor and is traditionally described as the fair betting quotient for wagers on S. Simple algebra shows that if O(S) is the fair betting odds in favor of S, $P^*(S) = O(S) \div [O(S) + 1]$. Thus If $O(S) \geq O(R)$ iff $P^*(S) \geq P^*(R)$. Simple algebra also shows that

$P^*(S) = P(\omega(S) \div[P((\omega(S)) + P(\lambda(S))]$ provided that O(S) is defined. Under this proviso $P^*(S) = P(\omega(S)/(\omega(S) \cup \lambda(S))$, which represents the conditional probability that a wager on S will be won given that it is either won or lost. It also represents conditional probability that S is non-trivially true given that it is either false or non-trivially true. If S is non-K, $P(\omega(S) + P(\lambda(S) = 1$ and $P(\omega(S) = P(S)$ hence $P^*(S) = P(S)$.

In order to treat P* as a function whose domain is the same as that of P, we treat P*(S) as 1 if $P((\omega(S)) + P(\lambda(S))] = 0$. In this case 1 does not represent maximal credibility. Fair betting quotients were labeled 'probabilities' by the 17th and 18th century pioneers of the theory of mathematical probability. I will follow this tradition. Formalists may complain that since the standard probability that S is a measure of the likelihood of its truth, and P*(S) is primarily a measure of the likelihoods of winning or losing a wager on S, it is not correctly described as a probability that S. However, P*(S) is equal to P'(S) where P' is a standard probability function derived from P by conditionalization on $\omega(S) \cup \lambda(S)$. So not only is P*(S) is formally a probability that S , it is correctly described as the practically relevant standard probability that S for the appraisal of proposed wagers on S.

This observation has relevance to the widely accepted interpretation of the thesis that given a standard probability function P P(C/A) is a correct measure of the probability that 'C if A' (presuming that A and C are simple), i.e., should equal P(A → C) because 'the probability that C if {on the supposition that or given that A is correctly represented by P(C/A). My label for this thesis is the Limited Conditional Probability Rule, LCPR, for short. Since standard probability is probability of truth this thesis is incompatible with MCR except for some trivial cases, e.g. when P(C) = P(A ∩ C). For this reason D.K. Lewis [1976], who argued against trashing MCR, proposed treating P(C/A) as the "assertability" of the proposition that C if A. Although the link between the concept of assertability and degree of credibility is not direct, it is reasonable to assume that intuitive credibility measured by P(C/A) "rubs off" onto degree of assertability. Frank Jackson, who similarly endorsed MCRC, replaced Lewis's degree of assert*a*bility by a concept of degree of "assert*i*bility". Jackson treated an assertibility version of Adams's criterion of probabilistic validity as a complementary criterion of logical soundness that roots out some – and I emphasize 'some' blatant non-sequiturs that are instances of MCR + t-valid argument forms. The reason for 'some' is that Adams's probability logic does not treat formulae that have conditional sub-formulae.

On the other hand, under the fair betting probability measure of credibility P(C/A) is aptly described as a measure of the credibility that C if A, and , for good measure , is properly described as the practically relevant standard probability that 'C if A'. probability that C if A relative to given P and also as the practically relevant p. In other words, $P^*(A \rightarrow C) = P(C/A)$ if $P(A) > 0$. For, $P^*(A \rightarrow C) = P(\omega(A \rightarrow C)/(\omega(A \rightarrow C) \cup \lambda(A \rightarrow C))$, which equals $P(A \cap C) \div \{P(A \cap C) + P(A \cap \sim C)] = P(A \cap C) \div P(A) = P(C/A)$. So under the fair betting odds criterion of credibility, the fair betting probability that C if A is P(C/A). Moreover, $P^*(A \rightarrow C)$ is properly described as a probability because it equals $P'(A \rightarrow C)$ where P' is the standard probability function derived from P by conditionalization on A. The rationale for the label 'limited' is that If S and H are not simple, then $P^*(S \rightarrow H)$ may not equal P(H/S). For example $P^*(A \rightarrow (B \rightarrow C)) = P(A \cap B \cap C) \div P(A \cap B) = P(C/A \cap B)$, which equals $P(B \rightarrow C/A)$ only if $P(A \cap \sim B) = 0$.

Since the hypothesis that C if A will prove true is typically not super-equivalent to the hypothesis that C if A, two fair betting probabilities are associated with 'C if A': $P^*(A \rightarrow C)$, which equals P(C/A) if $P(A) > 0$, and the fair betting probability that $A \rightarrow C$ is true, which equals $P^*(d(\lambda(A \rightarrow C)))$. Since d(λ(A → C) is non-K its fair betting probability equals P(d(λ(A → C))) = P(A ⊃ C) = P(A → C). Thus P*(A → C) equals the fair betting quotient/ fair betting probability for wagers on A → C, and P(A → C) equals the fair betting quotient/fair betting probability that A → C is true.

More generally, it is proved in Section 7.2 that P*(S) = P(S) only if P(ω(S) ∪ λ(S)) equals either 1or zero or P(ω(S), and is less than P(S) when neither of these “trivial” cases holds.

5.6 Some Salient Properties of Fair Betting Probability Functions

P* is not formally a standard probability function because it doesn't satisfy the additivity condition and may not assign the same number to t-equivalent propositions. However, it assigns the same number to super-equivalent propositions.

The theorem that P*(d(S) = 1- P*(S) is proved below

Fair betting probability functions are more nuanced than their parent standard probability function. For, if S can have the wager resolution value K, there may be no proposition R in the domain of P such that P(R) = P*(S). On the other hand, for every S in the scope of P there is a formula R, in particular d(λ(S)), such that P*(R) = P(S).

6 The Fair Betting Probability Criterion of Logical Soundness and the Criterion of Super-validity.

6.1 The fair betting probability measure supports a novel MCR-compatible CRED criterion of logical soundness that complements t-validity. Definition: Suppose that S1, S2, ... Sn, and H are formulae of a language Λ of WRC. Then S1, S2, ... Sn $\therefore$ H is *fair betting probability valid (FBP-valid for short)* iff

$P^*(S1 \cap S2 \cap .. \cap Sn) \leq P^*(H)$ for all standard probability functions P defined over Λ. It is proved in Section 7.3 that FBP-validity implies t-validity.

6.2 Some Examples:

Under FBP The rule of Conditional Proof does not hold. Some arguments that are cited in defense of MCR, e.g., by Jackson [1987] are not FBP-valid. For example, 'Either A or C, therefore if not-A then C'. If $P(C/\sim A) = 0$, $P(A \cup C)$ can be non-zero. On the other hand, the argument 'It is true that either A or C, therefore it is true that if not-A then C is FPB valid because the conclusion does not have the wager resolutions of the conditional conclusion: its P* value equals $P(A) + P(\sim A \cap C)$ which equals the standard probability of the premise, i.e., $P(A \cup C)$. In general, the "truth" version of a t-valid argument is FPB valid because P*(S is true) = P(S).

Under FBP The rule of Conditional Proof does not hold: the FPB validity of 'S, A $\therefore$ C does not imply the FBP-validity of S $\therefore$ A $\rightarrow$ C

All t-valid argument forms with absurd instances are not FPB-valid. This is because the conditional conclusion of an FBP-valid argument is at least as credible as the credibility of a conjunction of premises, so if the conclusion has zero credibility and a conjunction of premises has non-zero credibility the argument is not FBP valid. The following examples show this. Inference to the Contrapositive is not FBP-valid. For, $P^*(A \rightarrow C) = P(C/A)$ and $P^*(\sim C \rightarrow \sim A) = P(\sim A/\sim C)$. The latter can be zero when $P(C/A)$ is non-zero, e.g. when $P(\sim C \cap \sim A) = 0$ and $P(C \cap A) > 0$. The celebrated paradoxes of material implication $\sim A \therefore A \rightarrow C$ and $C \therefore A \rightarrow C$, are not FPB valid. For since A is simple, $P^*(\sim A) = P(\sim A)$ and $P^*(A \rightarrow C) = P(C/A)$. The latter can be zero when $P(\sim A)$ is not zero. Similarly $C \therefore A \rightarrow C$ is not FPB valid because $P^*(C) = P(C)$, which can be non-zero when $P(C/A)$ is zero.

Consider, for example, the Hypothetical Syllogism. To test for FBP-validity we have to first determine a formula for the values of $P(\omega[(A \rightarrow B) \cap (B\rightarrow C)]$ and $P(\omega[(A \rightarrow B) \cap (B\rightarrow C)] \cup \lambda[(A \rightarrow B) \cap (B\rightarrow C)]$. WRM shows that $\omega((A \rightarrow B) \cap (B\rightarrow C))$ is W iff $B \cap C$ is W (note that B and C are simple), and hence is t-equivalent to $B \cap C$. So its standard probability equals $P(B \cap C)$. WRM also shows that the disjunction of $\omega((A \rightarrow B) \cap (B\rightarrow C))$ with $\lambda((A \rightarrow B) \cap (B\rightarrow C))$ is t-equivalent to $A \cup B$. So $P^*((A \rightarrow B) \cap (B\rightarrow C)) = P(B \cap C) \div P(A \cup B$ if $P(A) > 0$. Then $P^*(A \rightarrow C) = P(A \cap C) \div P(A) = P(C/A)$, so the question is whether $P(C/A)$ can be less than $P(B \cap C) \div P(A \cup B)$. Suppose that $0 < P(A) < 1$ and

$P(A \cap C) = 0$; then $P^*(A \rightarrow C) = 0$. This is compatible with $P(B \cap C) > 0$ and $P(A \cup B) > 0$. Thus the argument is not FBP valid.

On the other hand, the t-valid argument form $A \rightarrow B, (A \cap B) \rightarrow C \therefore A \rightarrow C$ - *the Restricted Hypothetical Syllogism* - is FBP-valid, and doesn't have absurd instances. WRM shows that the wager resolution value of a conjunction of the premises is W iff A and B and C are t, and is either W or L iff A is t. So the P* value of a conjunction of premises equals $P(A \cap B \cap C) \div P(A)$ and the P* value of the conclusion is $P(A \cap C) \div P(A)$, provided that $P(A) > 0$. If $P(A) = 0$, both equal 1 by convention. Thus $P^*(A \rightarrow C) \geq P^*((A \rightarrow B) \cap (A \cap B) \rightarrow C$ for all P.

The logical appraisals of the above arguments match the appraisals provided by Ernest Adams's Criterion of Probabilistic Validity [1975], [1998]. However, as noted earlier, unlike FPB validity, Adams's criterion is limited in scope: it does not treat most arguments that contain formulae with conditional sub-formulae. It does not guarantee that the conclusion of a probabilistically valid argument in Adams's sense is at least as credible as a conjunction of premises. Adams constructed a concept of quasi-conjunction, but he gave it no role in the determination of logical soundness. The guarantee Adams's criterion provides is that if n is the number of premises, and p is the average credibility-cum-"probability" of the premises, and c is the credibility-cum-"probability" of its conclusion then if the argument is probabilistically valid, $c \geq 1 - n + np$. For example if n is 2 and p is .75 then $c \geq .5$. Not particularly reassuring.

Facility with probabilistic computations of the sort required to test for FBP-validity above is limited to persons who have studied mathematical probability. A non-probabilistic criterion that yields the same logical appraisals as FBP-validity would be useful for most users. The criterion of Super-validity outlined in the next section does this job. A proof of the equivalence of FBP validity and Super-validity is presented in section 7.3

6.2 Super-validity.

Definition: an argument of the form S1, S2,.. Sn ∴ H is *super-valid* iff it is t-valid and so is d(H) ∴ d(S1 ∩ S2 ∩.. ∩ Sn).

For example, A → C ∴ ~ C → ~ A not super-valid because d(A → C) is super-equivalent to A → ~ C and d(~ C → ~ A) is super-equivalent to ~ C → A, which does not imply A → ~ C. The "paradoxes of material implication are not super-valid because d(A → C), which is super-equivalent to A → ~ C implies neither d(~A) nor d(C).

The Hypothetical Syllogism is not super-valid because d(A → B) ∩ (B→ C) is L and hence u when A is u and B and C are t whereas in this case d(A → C) is K and hence t. On the other hand, the restricted Hypothetical Syllogism is super-valid because $A \rightarrow B, (A \cap B) \rightarrow C \therefore A \rightarrow C$ is t-valid, and there is no interpretation on which d(A → C) is t and d((A → B) ∩ ((A ∩ B)→ C)) is u

A simple informal reductio argument that a super-valid argument form cannot have absurd instances starts with the supposition that S1, S2,.. Sn ∴ H represents a colloquial instance of a super-valid argument form. Then there is a realistic scenario on which the wager opposite of the colloquial conclusion is true and the conjunction of its premises is non-trivially true. Suppose that the proposition represented by H is absurd. Then an interpretation that models such a scenario assigns t to d(H) and treats S1 ∩ S2 ∩.. ∩ Sn) as non-trivially true, i.e., treats its wager opposite as L and u. But then d(H) ∴ d(S1 ∩ S2 ∩.. ∩ Sn) is doesn't represent a t-valid argument, hence the original argument is not super-valid.

Like FPB-validity Super-validity is non-monotonic For example, S ∴ S is trivially super-valid but S, H ∴S may not be super-valid. For, suppose that S is K on interpretation ξ and that H is W on ξ; then S ∩ H is W and d(S ∩ H) is L and u on ξ, whereas d(S) is K and t. Hence d(S) ∴ d(S ∩ H) is not t-valid.

Super-validity, as well as FPB validity, disarms the claim that absurd instances show that MCR is untenable. For, these criteria of logical soundness are consistent with MCR, and allow no absurd instances of t-valid argument forms with conditional conclusions. Once it is clear that the standard probability of a conditional may not be an apposite measure of its credibility there is no reason to expect that a criterion of logical soundness whose focus is necessary truth preservation also should guarantee necessary credibility preservation for arguments that involve conditionals.

When our primary goal in inquiry is to discover what propositions must be true if others are true, t-validity is the criterion of choice; when it is to discover what propositions are both true and credible credible if others are then either FPB-validity or Super-validity does the job.

7. Appendix: Some Promised Proofs.

7.1 Theorem: $P^*(d(S) = 1 - P^*(S)$ provided that $P(\omega(S) \cup \lambda(S)) > 0$. Proof: $\omega(d(S))$ is super-equivalent to $\lambda(S)$ and $\lambda(d(S))$ is super equivalent to $\omega(S)$. Simple algebra is all that is needed to complete the proof.

7.2 Proof of The General Triviality Result.

The theorem states that for all standard probability functions P defined over the formulae of WRC and every formula S in the scope of P, $P(S) > P^*(S)$ if $0 < P(\omega(S) \cup \lambda(S)) < 1$ and $P(\lambda(S)) > 0$, and that equality obtains otherwise. Proof: $\kappa(S)$ is shorthand for $d(\omega(S) \cup \lambda(S))$, which is W and t if S is K, and L otherwise. The proof employs two Lemmas: a) the result that since S is t-equivalent to $\omega(S) \cup \kappa(S)$ and $\omega(S)$ and $\kappa(S)$ are logically incompatible, $P(S) = P(\omega(S) + P(\kappa(S)$, and b) the result that $P(\omega(S) \cup \lambda(S) \cup \kappa(S)) = 1 = P(\omega(S) + P(\kappa(S) + P(\lambda(S))$.

Cases 1, 2 and 3 establish the equality part of the theorem. Case 1: $P(\omega(S)) + P(\lambda(S))$ equals zero. In this case $P^*(S) = 1$ by convention, and It follows from Lemma b that $P(\kappa(S)) = 1$, whence $P(S) = 1$. Case 2: $P(\omega(S) \cup \lambda(S)) = 1$. Then $P(\kappa(S)) = 0$ and $P^*(S) = P(\omega(S)) = P(S)$. Case 3: $P(\lambda(S)) = 0$. If $P(\omega(S)) = 0$, we are back in case 1, so suppose that $P(\omega(S)) > 0$. Then $P^*(S) = 1$ and $P(\omega(S) + P(\kappa(S)) = P(S) = 1$.

The inequality part of the theorem. Case 4 : $0 < P(\omega(S) \cup \lambda(S)) < 1$ and $P(\lambda(S)) > 0$. Then $P((\kappa(S) > 0$. Suppose, for the sake of argument, that $P^*(S) \geq P(S)$. Then $[P(\omega(S)) \div (P(\omega(S) \cup \lambda(S))] \geq P(\omega(S) + P(\kappa(S)$. It follows that $(P(\omega(S)) \geq [P(\omega(S))P(\omega(S) \cup \lambda(S))] + [P(\kappa(S))P(\omega(S))] + [P(\kappa(S))P(\lambda(S))]$. $P(\kappa(S))P(\omega(S)) = P(\omega(S)(1 - (P(\omega(S) \cup \lambda(S))$ by Lemma b). It follows that $P(\omega(S) \geq P(\omega(S) + P(\kappa(S))P(\lambda(S))$. But this is impossible because the second term in the sum is greater than zero under case 4. So $P(S) > P^*(S)$. Q.E.D.

7.3 An argument is FBP-valid iff it is super-valid. A corollary is that FBP-validity implies t-validity.

Proof of the 'if ' part. Suppose that S1, S2 , ...Sn ∴ H is super-valid. Let 'S' stand for S1 ∩ S2 ∩ ... ∩ Sn. Then S ∴ H is t-valid and d(H) ∴ d(S) is t-valid. It is to be shown that for all probability functions P defined over the formulae of a language that includes the formulae of S1, S2 , ,Sn and H, $P^*(H) \geq P^*(S)$.

Let 'P' stand for such a standard probability function. In addition to the lemmas of the previous proofs we make use of the theorem that an argument is t-valid iff every standard probability function defined over a language in which the argument is formulated assigns a probability number to the conclusion that is greater than or equal to the number it assigns to a conjunction of premises Thus if $S1, S2, \ldots Sn \therefore H$ is super-valid $P(H) \geq P(S)$ and $P(d(S)) \geq P(d(H)$. It follows from the first inequality that $P(\omega(H)) + P(\kappa(H)) \geq P(\omega(S)) + P(\kappa(S))$ which implies that $1- P(\lambda(H)) \geq 1 - P(\lambda(S))$ and hence that $P(\lambda(S)) \geq P(\lambda(H))$. Similarly, it follows from $P(d(S)) \geq P(d(H)$ that

$P(\omega(d(S))) + P(\kappa(d(S))) \geq P(\omega(d(H)) + P(\kappa(d(H)))$. Since $\omega(d(S))$ is super-equivalent to $\lambda(S))$ and $\kappa(d(S))$ is super-equivalent to $\kappa(S)$, this inequality implies that $P(\lambda(S)) + P(\kappa(S) \geq P(\lambda(H)) + P(\kappa(H)$, which implies that

$1 - P(\omega(S) \geq 1- P(\omega(H))$, whence $P(\omega(H)) \geq P(\omega(S)$.

Case 1. Both $P(\omega(S) + P(\lambda(S)$ and $P(\omega(H) + P(\lambda(H)]$ are non-zero and are less than 1. It follows from these inequalities that $O(S)$ and $O(H)$ are defined and that $O(H) \geq O(S)$. It follows by simple algebra that

$O(H) \div [O(H) + 1] \geq O(S) \div [O(S) + 1]$, whence $P^*(H) \geq P^*(S)$.

Case 2 $P(\omega(H) + P(\lambda(H)] = 0$. whence $P^*(H) = 1$ by convention and is $\geq P^*(S)$ regardless of whether $P(\omega(S) + P(\lambda(S)$ is zero or not

Case 3 $P(\omega(H) + P(\lambda(H)$ is > 0, and $P(\omega(S) + P(\lambda(S)$ is zero in which case $P^*(S) = 1$ by convention. Then since $P(\lambda(S) \geq P(\lambda(H)$, $P(\lambda(H) = 0$, in which case $P^*(H)$ equals 1.

Proof of the second part: if S1, S2 , ...Sn $\therefore$ H is FPB valid it is super-valid. It is sufficient to prove the contrapositive: if an argument is not super-valid it is not FPB valid. Case a) : S $\therefore$ H is not t-valid. Then there exists a P such that $P(H) = P(\omega(H)) + P(\kappa(H))$ is less than $P(S) = P(\omega(S)) + P(\kappa(S))$. Among the class of such P there is a function such that $P(\kappa(S)) = 0 = P(\kappa(H)$. So $P(\omega(H)) < P((\omega(S))$ and $P(\omega(S) + P(\lambda(S)$ and $P(\omega(H) + P(\lambda(H) = 1$. It follows that $P^*(H) < P^*(S)$ whence S1, S2 , ...Sn $\therefore$ H is not FBP-valid.

Case b) d(H) $\therefore$ d(S) is not t-valid. Then there is a P such that $P(\omega(d(H))) > P(\omega(d(S)))$, and $P(\kappa(d(H)))$ and $P(\kappa(d(S)))$ are zero, in which case $P((\kappa(H))$ and $P(\kappa(S)$ also are zero. Since $P((\omega(d(H))) = P((\lambda(H))$ and $P(\omega(d(S))) = P((\lambda(S))$, $= P((\lambda(H)) > P(\lambda(S))$ whence $P(\omega(H)) < P(\omega(S))$. It follows that $O(H) < O(S)$ whence $P^*(H) < P^*(S)$ and the argument is not FBP-valid. Q.E.D.

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