habit of flagging up its use—rather as analysts or topologists who need the Continuum Hypothesis draw attention to points where it is needed. Thus theorems based on CFSG may either be read as unconditional results (by those who believe in it) or as results into whose assumptions the hypothesis that CFSG is correct must be added. We win either way.

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Experimental mathematics in the 1990s: A second loss of certainty?

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In most traditional accounts, experiments — one of the corner-stones of modern natural sciences — have had no place in mathematics. However, during the 1990s, with the advent of high-speed computers and sophisticated software packages a new experimental flavour was brought to parts of mathematics leading to the gradual formation of a branch of so-called "experimental mathematics" with its own research problems, methodology, conferences, and journals. The purpose of this paper is to situate the institutionalization of experimental mathematics in discussions within the mathematical community during the 1990s.

Despite early success in 1976 with the computer-assisted proof of the Four Colour Theorem, the full impact of the computer on mathematical practice was not felt until the mid-1980s. In 1985, when a new Cray-2 supercomputer was being installed at the University of Minnesota at Minneapolis, a group of remarkable geometers including Benoît Mandelbrot, David Mumford and William Thurston began work on a proposal for a *Geometry Supercomputing Project* to be funded by the NSF. That project would explore the power of computers for "visualization as a tool for experimentation, exploration, and inspiration in research" [9, p. 11].

Members of the project were instrumental in founding the journal *Experimental Mathematics* in 1991 with David Epstein and Silvio Levy as its editors. The journal was devoted to publishing experiments, new theorems, algorithms, practical issues, computer programs, a program column, and surveys and miscellanea [6, p. 1]. In introducing the journal, the editors alluded to a possible division of labour between hypotheses and proofs that would later be taken up with more force by Arthur Jaffe and Frank Quinn in their suggestion for a "theoretical" mathematics [8]. As the editors of *Experimental Mathematics* explained, the journal "was founded in the belief that theory and experiment feed on each other, and that the mathematical community stands to benefit from a more complete exposure to the experimental process. The early sharing of insights increases the possibility that they will lead to theorems; an interesting conjecture is often formulated by a researcher who lacks the techniques to formalize a proof, while those who have the techniques at their fingertips have been looking elsewhere" [6, p. 1]. Eight years later, in the opening issue of 2000, the same editors could celebrate the "maturity of the journal" [5, p. 1]: The journal's output had grown by 30% between 1992 and 1999 and would increase from 420 pages annually in 1999 to 640 pages a year from 2000. Thus, the journal established itself and the experimental approach to mathematics on the horizon of mathematical publishing in the 1990s.

At Simon Fraser University in Vancouver, another group formed in 1993 around the brothers Peter and Jonathan Borwein at the *Centre for Experimental and Constructive Mathematics* (CECM). That group has focused more on symbolic algebra and the use of computational methods in number theory. In a paper published in the *Mathematical Intelligencer*, the group announced their definition of the field: "*Experimental Mathematics* is that branch of mathematics that concerns itself ultimately with the codification and transmission of insights within the mathematical community through the use of experimental [...] exploration of conjectures and more informal beliefs and a careful analysis of the data acquired in this pursuit" [4, p. 17]. Thus, they also argued for a more inclusive view of mathematics and envisioned experimental mathematics as a dual dialectic between the computer and the human mathematician and between experiments and proofs [3, p. viii].

Among the results obtained by researchers affiliated with the group at the CECM is the so-called *PSLQ algorithm* which can be used for interactive, computerized searches for integer linear combinations of mathematical constants; see also [10]. It takes as its input a vector of high-precision real numbers $(x_1, \dots, x_n) \in \mathbb{R}^n$ and after a specified number of iterations produces either a very good suggestion for a non-trivial integer linear combination $(m_1, \dots, m_n) \in \mathbb{Z}^n$, such that $\sum_{k=1}^{n} m_k x_k \approx 0$ with high precision or a lower bound on the coefficients.

Members of the CECM group put the PSLQ algorithm to use in proving a remarkable formula which allowed the computation of individual hexagesimal digits of π without the computation of the previous ones. The authors described their process as applying ideas generalized from similar expressions for $\log 2$ and "a combination of inspired guessing and extensive searching using the PSLQ integer relation algorithm" [2, p. 905]. The CECM group would advocate searching for traditional proofs of conjectures such as those obtained from the first case of the PSLQ algorithm; and for the above-mentioned formula such a proof could be found. It relied on yet another use of computers in performing standard calculations that go into the the lemmas; such uses are now widespread and largely uncontroversial.

However, discussions emerged within the mathematical community over the need for traditional proofs of the more complicated computer-generated insights. Taking his inspiration from the new use of computers in visualization and proof, the science journalist John Horgan wrote an article entitled "The Death of Proof" for the *Scientific American* in 1993 [7]. There, Horgan captured the new dilemma of mathematics in the subtitle: "Computers are transforming the way mathematicians discover, prove and communicate ideas, but is there a place for absolute certainty in this brave new world?" and he suggested that the notion of proof was becoming an anachronism in mathematics.

A deliberate provocateur, the Rutgers mathematician Doron Zeilberger suggested in 1994 that "[a]s wider classes of identities, and perhaps even other kinds of classes of theorems, become routinely provable, we might witness many results for which we would know how to find a proof (or refutation); but we would be unable or unwilling to pay for finding such proofs, since 'almost certainty' can be bought so much cheaper" [11, p. 14]. Continuing the argument that mathematics was discovering new lands and extending great frontiers, Zeilberger suggested: "I can envision an abstract of a paper, c. 2100, that reads, 'We show in a certain precise sense that the Goldbach conjecture is true with probability larger than 0.99999 and that its complete truth could be determined with a budget of $10 billion"' [11, p. 14]. Such provocation was met with fierce reactions, and George Andrews expressed the thoughts of a more conservative part of the community when he wrote: "Zeilberger has proved some breathtaking theorems [...]. However, there is not one scintilla of evidence in his accomplishments to support the coming '...metamorphosis to nonrigorous mathematics.' [...] [H]e has produced exactly no evidence that his Brave New World is on its way" [1, p. 17]. Such discussions thus touched upon the epistemology of mathematics: It was obvious that so-called experimental methods could provide new heuristics for generating mathematical hypotheses, but whether new experimental methods also be allowed into the justificatory parts of mathematics was a very controversial issue, indeed, within the community.

In conclusion, the previous description has illustrated that to the protagonists of experimental mathematics in the 1990s, experimental mathematics was characterized not by a specific subject matter of mathematics, but rather by a technology (the computer), a somewhat vaguely specified methodology (the experiment) and a vision for an infrastructure (the electronic dissemination).

Based on these analyses, the development of experimental mathematics in the 1990s is not fruitfully analyzed within a disciplinary setting: Despite the developments of infrastructure and institutionalization, experimental mathematics remained cross-disciplinary in its subject matter, and its methodology and technology is increasingly integrated in most branches of mathematical research.

Instead, it is clear that efforts were made during the late 1980s and 1990s by the protagonists of experimental mathematics to promote an experimental approach as a style for doing mathematics. During that period, research institutions and journals were established, and software was developed to facilitate the methodology of interactive experimentation. However, aspects of that style were contested within the mathematical community and in the broader scientific and intellectual milieu. In particular, discussions about the conception of proof went to the core of the mathematical enterprise and an immediate reaction on the part of experimental mathematics was to confine the experimental approaches to the realm of heuristics and still demand traditional proofs. Such discussions over the potential epistemic roles of experiments in mathematics are still active within circles of experimental mathematics and within the community interested in the so-called *philosophy of mathematical practice*.

Some of the philosophical parts of this talk are being published in [10], whereas other parts are being prepared for publication.

References

[1] Andrews, G. E (1994). , *The death of proof? Semi-rigorous mathematics? You've got to be kidding!*, *The Mathematical Intelligencer* 16(4), 16–18.
[2] Bailey, D., P. Borwein, and S. Plouffe (1997). *On the rapid computation of various polylogarithmic constants. Mathematics of Computation* 66(218), 903–913.
[3] Borwein, J., D. Bailey, and R. Girgensohn (2004). *Experimentation in Mathematics: Computational Paths to Discovery.* Natick (MA): A K Peters.
[4] Borwein, J., P. Borwein, R. Girgensohn, and S. Parnes (1996). *Making sense of experimental mathematics. The Mathematical Intelligencer* 18(4), 12–18.
[5] Epstein, D. and S. Levy (2000). *To the reader. Experimental Mathematics* 9(1), 1.
[6] Epstein, D., S. Levy, and R. de la Llave (1992). *About this journal. Experimental Mathematics* 1(1), 1–3.
[7] Horgan, J. (1993). *The death of proof. Scientific American* 269(4), 74–82.
[8] Jaffe, A. and F. Quinn (1993). *"Theoretical mathematics": Toward a cultural synthesis of mathematics and theoretical physics. Bulletin of the American Mathematical Society* 29(1), 1–13.
[9] Marden, A. (1997). *Fred Almgren and the Geometry Center. Experimental Mathematics* 6(1), 11–12.
[10] Sørensen, H. K. (2010). *Exploratory experimentation in experimental mathematics: A glimpse at the PSLQ algorithm.* In B. Löwe and T. Müller (Eds.), *PhiMSAMP. Philosophy of Mathematics: Sociological Aspects and Mathematical Practice*, Number 11 in Texts in Philosophy, pp. 341–360. London: College Publications.
[11] Zeilberger, D. (1994). *Theorems for a price: Tomorrow's semi-rigorous mathematical culture. The Mathematical Intelligencer* 16(4), 11–14, 76.

On the Identities of Algebra in the 19th Century

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It is our aim to question whether algebra can be considered as a mathematical "discipline" during the 19th century or whether algebra took on much more varied and changing identities than the ones which can be described by resorting to a single category such as the one of‘ "discipline". In short, we are referring to the category "discipline" as identifying a corpus of specialized knowledge which resorts to institutionalized practices of transmissions and to a group of actors who are identifying themselves as "specialists". This category must be considered as a dynamical one: as a result of the actions of the groups of experts, the definitions and delimitations of disciplines are in constant evolution. The use of the adjective "disciplinary" in expressions such as Kuhn's "disciplinary matrix" or Bourdieu's "disciplinary habitus" usually aims at taking into account both the social dimension and the cognitive or epistemological aspects of this category. Even though we cannot go into any further detail on the uses of the category "discipline", these preliminary remarks are meant to highlight that, when wondering about the history of mathematical disciplines, it would be highly artificial to distinguish between internal and external approaches. If, indeed, one would consider Algebra as an immanent discipline for the purpose of a historical investigation, such an investigation would not only result in cutting slices of the mathematics of the past through a retrospective glance, but it would also miss the various social mechanisms of