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A KANTIAN REPLY TO BOLZANO'S CRITIQUE OF KANT'S ANALYTIC-SYNTHETIC DISTINCTION

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Summary

One of Bolzano's objections to Kant's way of drawing the analytic-synthetic distinction is that it only applies to judgments within a narrow range of syntactic forms, namely, universal affirmative judgments. According to Bolzano, Kant cannot account for judgments of other syntactic forms that, intuitively, are analytic. A recent paper by Ian Proops also attributes to Kant the view that analytic judgments beyond a limited range of syntactic forms are impossible. I argue that, correctly understood, Kant's conception of analyticity allows for analytic judgments of a wider range of syntactic forms.

Introduction

Although he praised Kant's (re)discovery of the analytic-synthetic distinction as having great significance in the history of philosophy,[1] Bolzano was sharply critical of how Kant drew that distinction. His criticisms include:

(1) The distinction, as drawn by Kant, is unclear. (*NAK*, 34)

1. In *Neuer Anti-Kant* Příhonský writes: "Wir bemerken, daß, obgleich wir die Eintheilung der Urtheile in analytische und synthetische für eine der glücklichsten und einflußreichsten Entdeckungen halten, die je auf dem Gebiete der philosophischen Forschung sind gemacht worden, es uns doch scheinen wolle, als wenn sie von Kant nicht mit dem nöthigen Grade von Deutlichkeit aufgefaßt worden sei" (*NAK*, 34; cf. *NAK*, xxii). I say rediscovery because Bolzano recognizes that the distinction is inchoately present in Aristotle and Locke, and is made by Crusius in much the same way as Kant (*WL*, 87). I refer to the *Neuer Anti-Kant* (Příhonský 1850) by '*NAK*' followed by page number. I refer to Bolzano's *Wissenschaftslehre* by '*WL*,' followed by volume number, paragraph number and the page numbers in (Bolzano 1837). I refer to the *Kritik der reinen Vernunft* (*KrV*) in the usual fashion by giving the page number in the 1781 edition (A) followed by the page number in the 1787 edition (B). I refer to the Akademie edition of Kant's *Gesammelte Schriften*, (Kant 1900–) by '*Ak.*,' followed by volume and page number. Where applicable, I indicate which translation I have used. '*JL*' refers to the *Jäsche Logik*.

(2) The distinction, as drawn by Kant, inappropriately relies on the metaphor of the 'containment,' or 'inclusion' [*Enthaltensein*] of one concept in another. (*NAK*, 34)

(3) The distinction, as drawn by Kant, only applies to judgments within a narrow range of syntactic forms, namely, universal affirmative judgments. Kant cannot account for judgments of other syntactic forms that, intuitively, are analytic. (*WL* II, 88)

(4) Kant ignores the semantically interesting class of sentences, those whose truth-value is unaffected by substitution of one or more of their terms. (*NAK*, 35-36)

(5) Kant fails to prove there are synthetic judgments in mathematics. (*NAK*, 36)

(6) Kant's 'containment' criterion for analytic judgments is too broad; it includes judgments that, intuitively, are not analytic. E.g. [king of Macedonia] is contained in the concept [father of Alexander, King of Macedonia] but, intuitively, the judgment <The father of Alexander, King of Macedonia, is King of Macedonia> is not analytic.[2] (*NAK*, 35)

(7) Kant is wrong to identify what is 'contained' in a concept with what is 'thought' in the concept; the latter is a purely psychological matter of which other ideas I contingently associate with the concept. (*WL* II, 89)

The list above is not exhaustive and there are other criticisms. This paper is part of a larger project of offering a comprehensive interpretation of the Kantian analytic-synthetic distinction that answers all of them. Here however, I will focus on (1)–(7), and especially on (3). I will argue that, at least on this point, Bolzano is mistaken. Although Kant does not emphasize this point, his theory of analytic judgment has the resources to account for judgments of a wide variety of syntactic forms other than universal affirmative categorical judgments, that is, judgments of the form <All As are B>. I will propose a more charitable reading of Kant's texts that supports attributing to him the view that there can be analytic judgments of other syntactic forms. In doing so, I will indirectly address points (1) and (2), and to a lesser extent (4), (6), and (7). Point (5) I will not address at all

2. I use square brackets […] to refer to concepts and angle brackets <…> to refer to judgments. Thus, '[A]' refers to the concept expressed by A, and <p> refers to the judgment expressed by the sentence p. In the context of Bolzano's philosophy, square brackets refer to *Vorstellungen an sich* and angle brackets refer to *Sätze an sich*.

other than to point out that, correctly understood, Kant's theory of analytic judgments is especially vulnerable on that point. By arguing that Kant can consistently classify as analytic a wider class of judgments than Bolzano, and many contemporary readers of Kant, assume, I am arguing that it is in principle harder for Kant to show that mathematical and natural scientific judgments do not fall within that wider class of analytic judgments.

In section one I begin my interpretation of Kant's analytic-synthetic distinction by offering an interpretation of the passage in the Introduction to *KrV* (A6/B10) where Kant introduces the distinction and appears to give a definition of what it is for a judgment to be analytic. I then go on to explain why it would be a problem for Kant's epistemological project in the *Kritik* if his definition of analyticity did not apply to judgments outside a narrow range of syntactic forms. I then examine several of Bolzano's examples of judgments that, intuitively, should count as analytic, but which do not pass the relatively narrow definition given in the Introduction to *KrV*. In section two I turn to the other main discussion of analyticity within the *KrV*, the section titled "On the supreme principle of all analytic judgments" (A150-3/B189-3). I argue that the meaning of that passage depends upon how Kant conceives of the principle of contradiction, and the relation between that principle and the principle of identity. In the third section, I argue that Kant thinks of these as two principles that immediately entail one another, and, on that basis, in the fourth section I offer an interpretation of the "supreme principle" section on which Kant, contra Bolzano, has a conception of analytic judgments that includes judgments of a much wider range of syntactic forms.

Bolzano's claim—that Kant's notion of analyticity is restricted to judgments within a narrow range of syntactic forms—is also put forth by Ian Proops in an influential recent paper (Proops 2005). Despite significant differences in their readings of Kant, Bolzano and Proops both attribute to Kant the view that analytic judgments only have a restricted range of syntactic forms; while Bolzano appears to read Kant as restricting analyticity to judgments of the form <All (A+B)s are B>, Proops correctly recognizes that Kant recognizes analytic judgments of the form <No (A+B)s are ~B>, but, on his reading, all analytic judgments must be either of these two forms. Whereas Bolzano takes this to be a criticism of Kant's view, Proops is neutral on whether Kant is wrong on this point. The Bolzano/Proops thesis will be my stalking horse throughout this paper.[3]

3. Two recent papers by Lanier Anderson (Anderson 2004, 2005) vindicate the idea that

1. *The problem of analytic judgments with alternate syntactic forms*

In the *Kritik der reinen Vernunft* Kant introduces the analytic-synthetic distinction in the following famous passage:

> In all judgments in which the relation of a subject to the predicate is thought, (if I consider only affirmative judgments, since afterwards the application to negative ones is easy,) this relation is possible in two different ways. Either *the predicate B belongs to the subject A as something that is (covertly) contained in this concept A*; or *B* lies entirely outside the concept *A*, thought to be sure it stands in connection with it. In the first case I call the judgment **analytic**, in the second **synthetic**. Analytic judgments (affirmative ones) are thus those in which the connection of the predicate is *thought through identity*, but those in which this connection is thought without identity are to be called synthetic judgments.[4] (*KrV*, A6–7/B10–11; my italics)

This passage has been the focus of a great deal of critical scrutiny, so before continuing I want to explain how I interpret it.

It is a basic article of Kantian lore that Kant gives several apparently different criteria for analyticity.[5] The passage quoted above contains two of them; another we will see later in the next section.[6] They are:

Kantian analytic judgment include judgments of a wider range of syntactic forms (e.g. disjunctive judgments). However, I do not have space in this paper to fully explain why I reject Anderson's view. For a telling, but not decisive, objection to Anderson's interpretation, see Proops (2005, 600).

4. Translations from *KrV* are taken, with occasional minor modifications, from Kant (1997).

5. See Van Cleve (1999, 17–21) for a helpful comparison of the different criteria.

6. There is also Kant's statement, immediately after the passage quoted, that analytic judgments are judgments of "clarification" [*Erläuterungurtheile*] while synthetic judgments are "judgments of amplification" [*Erweiterungsurtheile*] (A7/B11). Proops (2005) reads this as a distinct criterion on analyticity, in contrast with the containment-or-identity criterion of A6/B10. On this point I think Proops goes astray. He assumes that Kant's characterization of analytic judgments as judgments of clarification rather than amplification entails that analysis of a concept never reveals judgments not previously known to me. Admittedly, this is suggested by Kant's claim that "nun ist hieraus klar: daß durch analytische Urtheile unsere Erkenntnis nicht erweitert werde" (A7). But this sentence is removed in the B edition, and even in the A edition it continues: "sondern der Begriff, den ich schon habe, aus einander gesetzt, und mir selbst verständlich gemacht wurde." But, presumably, if a concept is made 'verständlich' to me, I come to know judgments I did not previously know, and the first part of the sentence only supports Proops' reading if 'erweitern' means provide us with knowledge of judgments we did not previously know. But '*erweitern*' is precisely the word Kant uses to characterize synthetic judgments in the first place [*Erweiterungsurtheile*], so the sentence from the A edition provides no support to Proops' interpretation not already found in the characterization of synthetic judgments as '*Erweiterungsurtheile*.' A more plausible reading, and one that does not conflict with

Containment criterion: A judgment <All As are B> is analytic if and only if the predicate concept [B] is contained in the subject concept [A].

Identity criterion: A judgment <All As are B> is analytic if and only if the connection of the subject [A] and the predicate [B] is 'thought through identity.'

I refer to these as criteria because I want to prescind for now from assumptions about which, if either, is definitional of analyticity for Kant, and which, if either, is simply a necessarily true bi-conditional. Both criteria have their problems—it is at least as unclear how one concept can be contained in another as it is how their connection can be 'thought through identity'—but first I want to focus on the containment criterion.

As we saw earlier, Bolzano objects to the Containment criterion that it inappropriately relies on a metaphor (*NAK*, 34f.). I think this is mistaken. Kant is not using 'containment' metaphorically; he means quite literally that in an analytic judgment the predicate is a part of the subject.[7] In other words, I think the Containment criterion should be understood—at least provisionally—as follows:

Containment criterion: A judgment <All As are B> is analytic if and only if the predicate [B] is a part of the subject [A.]

This criterion faces the immediate problem—Bolzano's point (6) from above—that it is too wide: it counts as analytic some judgments that, intuitively, are not analytic. For instance, since [bachelor] is part of [son of a bachelor] it follows that <Every son of a bachelor is a bachelor> is analytic, which it surely is not.[8] What this shows is that the notion of 'part' in the Parthood-criterion needs to be restricted; only certain parts [B] of a concept [A] are such that <All As are B> is analytic. This would be a serious problem

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the Containment-or-identity criterion given a few sentences earlier, is that analytic judgments clarify concepts by bringing to our awareness marks that were contained in them (and thus were contained in our thoughts involving those concepts all along), and thereby allow us to know new judgments, but do not 'amplify' our knowledge because they only call to our attention the logical consequences of judgments we already knew. Furthermore, the idea that an analytic judgments cannot constitute a new piece of knowledge cannot be squared with Kant's insistence that, for all we know, there may always be more analytic marks to be discovered in an empirical concept (*JL* §103, *Ak.* 9:142); cf. *Ak.* 24:272, 915, 757.

7. Lapointe (2011, 20ff) makes a similar point.

8. This is my replacement for Bolzano's 'King of Macedonia' example.

if the Containment Criterion were Kant's ultimate view about what it is for a judgment to be analytic. I will argue in the final section of this paper that this is not the case, so the problem Bolzano points to—not all of the parts of a concept are analytic marks of the concept —does not cut very deeply.

One might object to the Containment criterion that talk of one concept being a part of another concept can only be metaphorical because parthood is only a relation among spatiotemporal particulars. The exact ontological status of concepts for Kant is somewhat unclear, but, I take it, it is quite clear they are not spatiotemporal particulars, so if parthood were only a relation between spatiotemporal particulars, then this objection would be sound. However, Kant himself does not restrict parthood to spatiotemporal objects. In various passages of the *KrV* and elsewhere Kant discusses the mereological structure of non-spatiotemporal things in themselves, and contrasts it with the mereological structure of spatiotemporal appearances.[9] He also consistently denies that things in themselves are parts of appearances. In none of these passages does Kant deny that things in themselves can stand in parthood relations with other things in themselves (*Ak.* 8:201–10). So, while it may be true that parthood is a relation that can only hold among spatiotemporal objects (although I have my doubts), this is not Kant's view.

To return to the Identity criterion, the meaning of that criterion depends upon the meaning of the phrase "thought through identity." If "thought through identity" just means *identity*, then the Identity criterion should be read as follows:

> *Identity criterion*: A judgment <All As are B> is analytic if and only if [A]=[B].

However, the Identity criterion, so understood, stands in obvious tension with the Containment criterion, because the Containment criterion entails

9. In the *KrV*, the Second Antinomy (A434–443/B462–471); *Prolegomena* §13 (*Ak.* 4:286). Some readers might object to my attributing views about the mereological structure of things in themselves to Kant on the basis of the Second Antinomy. Isn't the point of the point of the Antinomies to show that each of these opposing arguments is fallacious? While I don't have the space to fully defend my reading of the Antinomies, I will note that Kant's view is that the Antithesis of the Second Antinomy is fully correct *if we limit it to objects of sensible intuition*, and the Thesis of the Second Antinomy is fully correct *if it expresses how we would most rationally think about things in themselves*, although we cannot know its conclusion to be true. All I am claiming in the body of the text is that Kant does not regard mereological relations (composition, simplicity, etc.) as *ipso facto* restricted to spatial objects.

that judgments like <All gold is metal> are analytic while the Identity criterion entails that they are not. Thus, it might seem that Kant gives two incompatible criteria on analyticity in the course of the two sentences from the Introduction to *KrV* quoted earlier.

But, as the following passage from Kant's metaphysics lectures makes clear, Kant understands the Containment and Identity criteria as equivalent:

> All analysis is nothing other than the consciousness that a certain mark is contained in the concept of the thing. The mark that is contained in the concept of the thing is partially identical with the [concept] of the thing. Every analytic mark is identical, not with the whole concept, but rather only with a part. For instance, 'every body is extended,' is really judged through identity. (*Metaphysik Mrongovius*, *Ak*. 29:789)

When Kant says in the Introduction to *KrV* that in an analytic judgment the relation of the predicate to the subject is thought 'through identity' he means that the predicate is identical to a (perhaps improper) part of the subject-concept. There is no conflict, therefore, between the Containment and Identity criteria.[10] However, there does appear to be a conflict between the passage just quoted from the Mrongovius lectures and Kant's Critical view. In that passage, Kant denies that judgments in which subject and predicate are identical—which elsewhere calls 'identical' judgments and 'tautologies'—are analytic, but in a number of published writings in the Critical period he seems to accept that identical judgments are analytic (*KrV* B17; *Prolegomena* §2, *Ak.* 4:269). Since the Mrongovius passage occurs in an unpublished lecture transcript, while the published passages support the claim that identical judgments are analytic, I will assume that the view expressed by Kant in *KrV* is that identical judgments are analytic.

We have now seen how to combine the Containment and Identity criteria into a single criterion:

> *Containment Criterion*: A judgment <All As are B> is analytic iff the predicate [B] is identical to a proper or improper part of [A].

I have formulated these two criteria as criteria on the analyticity of judgment having the form <All As are B>. This raises the natural question,

10. Similarly, Bolzano is mistaken when he complains that Kant's definition of analytic judgments in *JL* as judgments "whose certainty rests on identity of concepts" applies only to identical judgments like <Man is man> (*WL* II, 87).

(how) does the analytic-synthetic distinction apply to judgments of other syntactic forms? I did this because in the crucial passage from the Introduction to *KrV* Kant is careful to restrict the range of this criterion: to judgments with one subject and one predicate (categorical judgments), and affirmative ones at that. He claims that the "application to negative ones [judgments] is easy" (A6/B10). Bolzano correctly points out that the Containment criteria, as formulated at A6/B10, only applies to judgments within a limited range of syntactic forms, but fails to note that Kant himself explicitly states that, as formulated, it is not supposed to apply more widely. Bolzano concludes from this that Kant cannot allow for analytic judgments of different forms. In the remainder of this section I want to focus on whether Kant is right about that, whether the 'application' of these criteria to negative, and other kinds of judgments, is as easy as Kant says it is.

Bolzano offers the following as examples of forms of judgments that, intuitively, are analytic, but fail the Kantian criterion because they are not of the right form:

(a) <A, which is B, is A.>
(b) <Every A is either B or ~B.> (*NAK* 35; *WL* II, 88)

Correctly understood, none of these is a clear counter-example to the Containment criterion, although the first does pose some problems. First I will explain why judgments of these forms are not problematic for Kant, and then I will explore other judgmental forms that do pose very serious problems for Kant's theory of analytic judgments.

Since Kant clearly intends the Containment criterion given above to apply in the first place to universal judgments—let us hope that the "application" to particular and singular judgments is "easy"—I will read (a) as:

(a) <All As, which are B, are A.>

This judgments can be read in either of two ways: either the relative clause 'which is B' is a component of the subject-concept [A], or it is not and expresses merely a separate judgment. Therefore (a) can be read either as:

(a*) <All (As which are B) are A.>
(a†) <All As are B, and are A.>

But (a*) clearly satisfies the Containment criterion on analyticity given above: the predicate [A] is a part of the subject [As which are B]. On this reading, (a*) is analytic. However, (a†) does not satisfy the Containment criterion because it does not have only one predicate: it is what we would now call the conjunction of the judgment <All As are B> and the analytic judgment <All As are A.> Whether this is analytic will depend upon whether conjunctions of judgments can be analytic; as any students of Kant's logic knows, Kant does not have a logical function of conjunction of judgments, so there may be serious problems in his giving a criterion for whether (a†) is analytic or not. (Intuitively, (a†) is analytic if and only if its first conjunct, <All As are B>, is analytic.) But this shows that the original judgment (a) is problematic for Kant, not in virtue of the relative clause 'which is B,' but in virtue of the fact that Kant does not recognize conjunctions of judgments as a separate logical form. Bolzano may ultimately be right that Kant cannot account for the analyticity of judgments of the form of (a), but not for the reasons he thinks. At the end of the paper, I will return to (a†) and argue that, charitably understood, Kant does have an account of analyticity of judgments of that form.[11]

This brings us to the second judgment form that, according to Bolzano, could not count as analytic on Kant's criterion:

(b) <All As are B or not-B.>

Bolzano is right that this does not satisfy Kant's criterion on analyticity, but this is not a weakness of Kant's view because Kant has his own philosophical reasons for thinking that (b) is not analytic. If all judgments of the form (b) were analytic, then it would be analytic that every object is fully determinate with respect to every contradictorily opposed pair of predicates [B] and [~B]. In the opening of the Transcendental Ideal section of *KrV* (A571–573/B599–601) Kant explicitly distinguishes that principle, which he calls the 'principle of complete determination,' from the principle that "of every two contradictorily opposed predicates only one can apply to" an object, which "rests on the principle of contradiction." This principle is:

11. See the discussion of this example in Siebel (2011, 110). I share Siebel's puzzlement at Bolzano's distinction between <A, which is B, is B> and <A, which is B, is A>. In the paper I've tried to go a little further in understanding the barriers Bolzano might have seen to the analyticity of the latter judgment.

(b*) <No A is B and not-B.>

It might be thought that (b) and (b*) are logically equivalent, but for Kant they are not. I do not have the space here to fully explain why they are not equivalent for Kant, but it suffices for our purposes to point out that, while (b*) is a logical principle for Kant, (b) is not. On Kant's view, logic does not determine whether objects are fully determinate with respect to every pair of contradictorily opposed predicates. Kant wants to leave room for the *logical* possibility that there is some object that falls *neither* under [B] *nor* under [~B].[12]

Abstracting for a moment from the particular examples Bolzano uses, his point is that the Containment criterion only applies to universal affirmative judgments. It would appear that Kant's criterion on analyticity does not even apply to judgments of any of the following forms:

(c) <No (A+B) is ~B>, e.g. <No gold is non-metallic>.[13]
(d) <No ~B is (A+B)>, e.g. <No non-metallic thing is gold.>
(e) <All ~B are ~(A+B)>, e.g. <All non-metallic things are not gold>
(f) < No A is (B + ~B)>, e.g. <No metallic thing is gold and not-gold.>

The Containment criterion states that a judgment is analytic if and only if the predicate concept is a proper or improper part of the subject concept. But in (c) through (f) the predicate is neither a proper nor improper part of the subject-concept. The prospect for allowing analytic judgments of different-syntactic forms looks hopeless, given the Containment criterion.

But recall that Kant explicitly restricts the Containment criterion to affirmative subject-predicate judgments, and (implicitly) to universal ones at that. But he also says that the "application" to—by which I take him to mean the extension of that criterion, since, as we have seen, the criterion itself is not applicable—to negative judgments is "easy". In the other main text on the analytic-synthetic distinction in *KrV*, "On the supreme prin-

12. I explore this issue in much greater detail in Stang (forthcoming). Kant does describe the principle of excluded middle ((b) in the main text) as "springing" from the principle of contradiction and the principle of identity (MM, *Ak.* 29:791) but I don't think that he means by this that it is analytic (as those principles are).

13. I follow Kant in writing negated particular judgments using 'No.' So <No A is B> is the negated particular judgment <~(Some A is B)>.

ciple of all analytic judgments" Kant writes: "if the judgment is analytic, whether it be negative or affirmative, its truth must always be able to be cognized sufficiently in accordance with the principle of contradiction" (A151/B190). This would appear to be Kant's application of the definition of analyticity to negative judgments.[14] In the next section I offer a detailed look at that text. Proops (2005) argues that the "supreme principle" section extends Kant's conception of analyticity to judgments of form (c). I will argue in the next section that, on Proops' reading of that text, judgments of form (d)–(f) do not count as analytic on Kant's conception. In the remainder of this section I want to explain why it would be a serious problem for Kant if judgments of forms (c)–(f) were not analytic.

Note that judgments of form (f) are not categorical judgments; they are disjunctive judgments. Since the Containment criterion, as given at A6/B10, applies only to judgments with one predicate and one subject, it is not at all clear that it can apply to disjunctive judgments, which have multiple predicates, or hypothetical judgments, which have multiple subjects and multiple predicates. On Proops' reading, the "supreme principle" section does not alter the picture; the expanded criterion Kant gives there still applies only to categorical judgments. This is problematic for several reasons. First of all, there is significant textual evidence that Kant held that there are analytic judgments with non-categorical form, although that evidence is mainly confined to unpublished student transcripts of his lectures on metaphysics. The clearest evidence occurs in the von Schön transcripts of Kant's lectures from the late 1780s:

> According to its form the principle of contradiction is a categorical judgment, and is therefore the judgment of all analytic judgments only insofar as they are categorical [*nur in so fern sie kategorisch sind*] and expresses in complete generality that a predicate may not contradict the subject; it determines merely the relation of the predicate to the subject. 'To any subject, [at most] one out of [a pair of] contradictory predicates applies' [*Cuilibet subjecto pradicatorum contradictoriorum competet unum*] is the principle of disjunctive judgments and the principle of hypothetical judgments is: every judgment has a ground. Therefore we have, as highest principles of all analytic judgments, the principle of contradiction, a principle of division and one of the connection of the conditioned with the condition. We cannot have more, because there are no

14. This is confirmed by the fact that in the A edition Kant writes "denn auf die verneinende ist die Anwendung leicht" (A6) while in the B edition he adds 'nachher' (B10); I take 'nachher' to be a reference to the "Supreme Principle" section, where Kant shows how to apply his Criterion to negative judgments.

> further relations of cognition [of subject and predicate] in judgments. These judgments are all analytic. (*Metaphysik von Schön*, *Ak*. 28:478)[15]

Aside from textual considerations, though, there are overwhelming systematic reasons why Kant needs to accept analytic judgments of a wide range of syntactic forms, including, (c)–(f). If, for example, a judgment of the form of (f) is not analytic, then it is either *a priori* or *a posteriori*. If it is *a posteriori*, this means it can only be known through experience. Intuitively, though, that is absurd and would constitute a one line *reductio* of Kant's epistemology. It must, therefore, be *a priori*.

If it is synthetic *a priori*, though, according to Kant's transcendental epistemology, it is made true by the forms of experience: space, time and the categories. Not only is this highly counter-intuitive, it also leads to problems for Kant's system. If it is *a priori* cognizable by us, then the *KrV* provides an account of our ability to cognize it, since Kant is quite clear that he believes that the *KrV* provides a systematic account of all our synthetic *a priori* knowledge. At the beginning of the 'System of all principle of pure understanding' Kant writes:

> Now our task is to exhibit in systematic combination the judgments that the understanding actually brings about *a priori* subject to this critical warning, for which our table of the categories must doubtless give us natural and secure guidance. For it is precisely these whose relation to possible experience must constitute all pure cognitions of the understanding *a priori*, and whose relation to sensibility in general will, on that very account, display all transcendental principles of the understanding completely and in a system. *A priori* principles bear this name not merely because they contain in themselves the grounds of other judgments, but because they are not themselves grounded in higher and more general cognitions. (*KrV*, A148–9/B178–9)

The *Kritik* does not contain an inventory of every *a priori* cognition of which we are capable—for that, we must wait for the 'system' for which the *Kritik* is merely a necessary propadeutic (*KrV*, A13/B27). But the *Kritik* must present systematically and completely all principles of *a priori* cognition, that is, all of the fundamental *a priori* cognitions from which other *a priori* cognitions are derived and which are not themselves grounded in 'higher and more general cognitions.' If judgments of the form (c)-(f) are not *analytic* then they are not grounded in the 'supreme principle' of analytic judgments. But then where in the *KrV* does Kant

15. Cf. *Ak*. 28: 522-3, and 28: 544.

account for our knowledge of such judgments? That the system of the *KrV* might not successfully ground all of our synthetic *a priori* knowledge is not in itself implausible; what is implausible is that Kant could have been unaware that something so basic as our *a priori* knowledge of <No A is B and not-B> could not be accounted for within his system. The same points apply to judgments of forms (c)–(e). In other words, the cost of attributing to Kant a conception of analytic judgment that precludes judgments of these forms from being analytic is attributing to Kant either an absurd philosophical position—that judgments of these forms are *a posteriori*—or one that violates one of the basic systematic aims of the *KrV*, namely, to systematically account for the sources of all *a priori* cognition. Minimal interpretive charity favors the contrary view, that Kant had a conception of analyticity wide enough to embrace judgments of syntactic forms other than universal-affirmative-categorical and universal-negative-categorical.[16]

The other systematic reason why Kant must accept analytic judgments of a wide range of syntactic forms, other than (c)-(f), is the anti-Leibnizian thrust of the analytic-synthetic distinction. One of Kant's main points in drawing the analytic-synthetic distinction is to argue that there are synthetic *a priori* truths, i.e. necessary truths that are not analytic. This is supposed to rebut the view—held in common by Leibniz, Wolff and Baumgarten—that the negation of a necessary truth entails a contradiction, i.e. that all necessary truths are logically necessary (Baumgarten 1757, §7, §101; Wolff 1962, I–2, §36).[17] Kant holds, on the contrary, that there are non-contradictory judgments that are nonetheless not 'really possible,'

16. Thus it is surprising to read Ian Proops claim: "I have argued that Kant settled for a non-exhaustive classification of judgments [into analytic and synthetic]. Some commentators have found this idea hard to accept. Their desire to find an exhaustive classification in the first *Critique* has inclined them toward reading Kant's discussion of the principle of contradiction at A151/B190–1 as containing a further characterization of analyticity in epistemic terms. Interestingly, however, such a tradition of interpretation does not appear to have originated in serious Kant scholarship" (Proops 2005, 610). While it may not have originated in "serious Kant scholarship," it does originate both in an appreciation of the systematic aims of the *KrV* and, more straightforwardly, in the text itself. Recall how Kant originally introduces the distinction: "In allen Urteilen, worinnen das Verhältnis eines Subjekts zum Prädikat gedacht wird […] ist dieses Verhältnis auf *zweierlei Art* möglich. *Entweder* […]; *oder* […]" (A6/B10).

17. What I mean here by 'necessary' corresponds to Leibniz's 'strict' or 'metaphysical' necessity; among texts available to Kant in 1781, see the Fifth Letter to Clarke, *New Essays* (IV.ii.1, IV.vii.11, IV.xxix.9), *Theodicy* §174, *Monadology* §33, as well as the unpublished texts 'Primary truths' (Leibniz 1989, 98-101), 'On Freedom' (Leibniz 1989, 96), and 'First Truths' (Leibniz 1969, 267-271).

or, equivalently, that not all necessary truths are logically necessary. His analytic-synthetic distinction needs to allow him to formulate this anti-Leibnizian point; it needs to allow him to formulate, in his own technical terminology of 'analytic' and 'synthetic' judgments, the claim that some judgments that are not logically necessary are nonetheless 'really necessary.' To do this, he needs, as much as possible, his notion of 'analytic' judgments to line up with what Leibniz would call 'truths of reason': judgments whose negations logically entail a contradiction. In other words, if Leibnizian 'truths of reason' with alternate syntactic forms (e.g. (c)–(f) from above) do not count as Kantian analytic judgments, then in claiming that there are really necessary truths that are not analytic Kant would be making a claim Leibniz would accept.[18] Drawing the analytic-synthetic distinction in such a way would neuter the anti-Leibnizian thrust of the *Kritik* from its opening pages. There are, therefore, overwhelming reasons, both textual and systematic, for thinking that Kant must have a conception of analytic judgments that includes judgments with alternate syntactic forms. In the rest of the paper I will attempt to determine precisely what that conception is.

2. *The non-contradiction criterion*

Kant's discussion of the non-contradiction criterion in the "On the Supreme Principle of all Analytic Judgments" is sufficiently important to merit quoting in its entirety:

> Now the proposition that no predicate pertains to a thing that contradicts it is called the principle of contradiction, and is a general though merely negative

18. Some readers might reply that, since Leibniz took all judgments to be ultimately reducible to categorical judgments, Kant would not be blunting his critique of Leibniz by ignoring analytic judgments of alternate syntactic forms. Some evidence that Leibniz contemplated the reduction of hypothetical to categorical propositions is offered in (Proops 2005, 595); see also (Mates 1986, 58). However, Proops offers no evidence that Leibniz took disjunctive propositions to be reducible to categorical ones. Even if Leibniz did in fact envisage reducing all propositions to those with categorical form, Kant's objection to Leibniz would still be significantly philosophically weaker if it ignored the possibility of analytic judgments with non-categorical form: it would then no longer be an objection to the Leibnizian view that all necessary truths are (what Kant would call) analytic, but only to the conjunction of that view and the reductionist view just described. Kant would then not be targeting the heart of Leibniz's modal theory, but a dispensable commitment of that theory (i.e. that all propositions are ultimately categorical in character).

criterion of all truth, but on that account it also belongs merely to logic, since it holds of cognitions merely as cognitions in general, without regard to their content, and says that contradiction entirely annihilates and cancels them.

But one can also make a positive use of it, i.e., not merely to ban falsehood and error (insofar as it rests on contradiction), but also to cognize truth. [α] *For,* ***if the judgment is analytic****, whether it be negative or affirmative, its truth must always be able to be cognized sufficiently in accordance with the principle of contradiction.* [β] *For the contrary of that which as a concept already lies and is thought in the cognition of the object is always correctly denied, while the concept itself must necessarily be affirmed of it, since its opposite would contradict the object.*

Hence we must allow that the **principle of contradiction** to count as the universal and completely sufficient **principle of all analytic cognition**; but its authority and usefulness does not extend beyond this as a sufficient criterion of truth. For that no cognition can be opposed to it without annihilating itself certainly makes this principle into a *conditio sine qua non*, but not into a determining ground of the truth of our cognition. Since we now really have to do only with the synthetic part of our cognition, we will, to be sure, always be careful not to act contrary to this inviolable principle, but we cannot expect any advice from it in regard to the truth of this sort of cognition. (*KrV*, A150-2/B189-191; italics and bracketed letters by author)[19]

In the first paragraph, Kant points out that the principle of contradiction (PC) is only a negative criterion on truth: any judgment that violates it is false, but some non-contradictory judgments are false or merely groundless. He then goes on to claim that some judgments can be cognized through the PC alone. Specifically, if a judgment is analytic, whether it is affirmative or negative, it can be cognized through the PC alone. Fully understanding what this means requires understanding how Kant understands the PC. The closest that Kant comes to explaining his conception of the principle is given in sentence [β].

Kant here describes the PC as governing the case of affirmative analytic judgments of the form <All (A+B) is B> as well as negative analytic judgments of the form <No (A+B) is ~B>. This suggests that the Non-Contradiction Criterion just consists in the following:

Non-Contradiction Criterion: A judgment is analytic if and only if (i) it is affirmative and has the form <All (A + B)s are B>, or (ii) it is nega-

19. Cf. the discussion of this passage by Bolzano/Příhonský (*NAK*, 91).

tive and has the form <Not (A+B) is not-B>, where [A+B] stands for a concept in which the concept [B] is contained.

If this is all that Kant has in mind in this passage, then the Non-Contradiction Criterion adds little to the Containment Criterion. What it adds is an 'extension' to the case of negative judgments; it can accommodate negative judgments of the form <No (A+B) is not-B>. But it cannot accommodate analytic judgments of forms (d)–(f) from section one. This is how Proops (2005) reads the "supreme principles" passage; if that is the correct reading then Proops is correct in holding that all that passage adds to A6/B10 is an extension of the definition of analyticity to judgments of the form <No (A+B) is not-B>.

But there is another way of reading this passage. The first italicized sentence [α] suggests the following:

Cognition through PC Criterion: A judgment <*p*> is analytic if and only if <*p*> can be cognized through the principle of non-contradiction alone.

Initially, Kant formulates this criterion by writing "if the judgment is analytic […]" ; this suggests that where I have written 'if and only if' I should have written 'only if.' But he goes on to write that "we must allow the principle of contradiction to count as the *universal* and *completely sufficient* principle of all analytic cognition" (my emphasis). I take this to mean that only analytic judgments can be cognized through the PC alone, which means that cognizability through the PC alone is both a necessary and sufficient condition for being an analytic judgment.

How might the Cognition through PC Criterion differ from the Non-Contradiction Criterion? Assuming (as seems plausible) that all of the judgments that pass the Non-Contradiction Criterion are cognizable through the principle of contradiction (since they are substitution instances of it, either in its 'affirmative' guise or its 'negative' guise), the two criteria are distinct if and only if there are judgments that can be cognized through the PC alone but which do not have the form <All (A+B) is A> or <No (A+B) is not-B>. But, intuitively, that is precisely the case with judgments with alternate syntactic forms from the previous section ((d)–(f)). Intuitively, those judgments can be cognized through the principle of non-contradiction because the denial of any of them entails a contradiction, although none of them are mere substitution instances of the PC.

This seems like the right conception of analyticity, not merely 'intuitively,' but also because it seems like a promising start for reconstructing the class of our *a priori* cognitions that do not rely on the synthetic *a priori* principles of experience.[20] But is there any ground for thinking that Kant had a conception of what it is to 'cognize a judgment through the principle of contradiction' other than merely recognizing it as a substitution-instance of that principle, either in its positive or its negative guise? In the next section, I will argue that, in fact, careful attention to Kant's texts supports attributing this view to him.

3. *The unity of the principle of contradiction*

In the passage on the "Supreme Principle of all Analytic Judgments" quoted earlier, Kant claims that all analytic judgments can be sufficiently cognized through the principle of contradiction, and this accounts for analytic judgments of the form <All (A+B) are B> and <No (A+B) is not-B>. However, this is puzzling, because only the latter seems to be an instance of the principle of contradiction in the strictest sense, while the former is an instance of what Kant calls the 'principle of identity.' How can the idea that there is a *single* highest principle of all analytic judgments be squared with this apparent duplicity in their application?

In his pre-Critical work of 1755, *Principiorum primorum cognitionis metaphysicae nova dilucidatio*, Kant explicitly claims that there are *two* basic logical principles: the principle of contradiction and the principle of iden-

20. Siebel (2011) quotes with approval Künne's claim that "unlike 'true' and 'necessary,' the world 'analytic' is a philosopher's term of art. Memories of doctrines associated with this term (be they Kantian, Fregean, Carnapian or whatever) should not be mistaken for pre-theoretical 'intuitions' concerning analyticity. There simply are no such intuitions one could appeal to" (Künne 2006, 219). I think Künne and Siebel are mistaken on this point; as Grice and Strawson point out "there is no need to appeal only to tradition; for there is also present practice. We can appeal, that is, to the fact that those who use the terms 'analytic' and 'synthetic' do to a very considerable extent agree in the applications they make of them. They apply the term 'analytic' to more or less the same cases, withhold it from more or less the same cases, and hesitate over more ore less the same cases. This agreement extends not only to cases which they have been taught so to characterize, but to new cases. In short, 'analytic' and 'synthetic' have more ore less established philosophical use; and this seems to suggest that it is absurd, even senseless, to say that there is no such distinction. For, in general, if a pair of contrasting expressions are habitually and generally used in application to the same cases, where these cases do not form a closed list, this is a sufficient condition for saying that there are kinds of cases to which the expressions apply; and nothing more is needed for them to mark a distinction" (Grice and Strawson 1968, 142f.).

tity, and neither can be derived from the other. The principle of identity is the basic principle of all affirmative truths, Kant there contends, while the principle of contradiction is the basic principle of all negative truths. One possibility would be that Kant retains, within the Critical period, this dichotomy of basic logical principles, and the Non-Contradiction criterion should really read:

> *Non-Contradiction Criterion*: The judgment <p> is analytic if and only if it is a substitution instance of *either* the principle of contradiction *or* the principle of identity.

Aside from its displeasingly disjunctive character, this principle fails to do justice to Kant's ambivalence in the Critical period about whether the PC and the PI are one or two principles. In some texts he refers to 'the principle of contradiction or identity'—suggesting there is one principle, than can either be formulated in terms of contradiction or in terms of identity —while in others he refers to them as separate principles.[21]

This ambivalence strongly suggests that Kant's considered view is that there is some sense in which the PI and the PC are the same principle, and some sense in which they are not. He criticizes Baumgarten for attempting to derive the PC from the PI:

> In §11 the author seeks to prove the principle of identity through the principle of contradiction. One cannot, however, prove it in this fashion, because the principle of contradiction, which is supposed to prove the principle of identity, already exhibits [the principle of identity], and would thus be used to prove itself. (*Metaphysik Vigilantius*, *Ak.* 29:964)

Or, as Kant expresses the same point more tersely but perhaps more clearly: "The author wishes to prove the principle of identity from the principle of contradiction; but the ground [*Ursach*] is itself the principle of identity, which he does not observe (*Metaphysik Herder*, *Ak.* 28:11). The term Kant uses here is '*beweis*en.' But '*Beweis*' is a technical term for Kant; it does not refer to just any logically valid argument, but to one in which the premises are the grounds of the conclusion, that is, a logically valid argument in which the premises are more basic than, and therefore can

21. There are a number of texts in which Kant writes as though they are two principles (E.g. *Ak.* 24:477, 28:9, 24:279, 28:544; and *Refl.* 4655), and a comparable number of texts in which Kant writes as though they are one principle (E.g. *Refl.* 3741, 4275, 4477, 4872, 5176, 5176; and *Ak.* 28:523).

explain, the conclusion (*Wiener Logik*, *Ak.* 24: 892). But this is not the case with Baumgarten's putative '*Beweis*' of the PC from the PI because the PI is no more basic than the PC and therefore cannot explain why the PC holds. In fact, Kant holds that there can be no '*Beweis*' strictly speaking of either the PC or the PI; they are basic principles that cannot be grounded in anything more fundamental (*Logik Blomberg*, *Ak.* 24:278f.).

Kant denies that the PI can be '*bewiesen*' from the PC, but he admits that the PC follows from the PI and vice versa:

> The proposition or principle of identity is valid for affirmative judgments just as the principle of contradiction is valid for negative judgments. Fundamentally, one can regard both of these principles as one; for when I posit one, the other follows immediately [*so folgt aus diesem auch schon das andere*.] The principle of identity is already conceived in the principle of contradiction. (*Metaphysik Pölitz*, *Ak.* 28: 544)

The PC entails the PI and vice versa. Unordered materials from the K_1 manuscripts include an illuminating passage on the same point, which Erdmann prefaces with the comment, "Especially illuminating is the proof [*Begründung*], missing in the later work, that the principle of identity and the principle of contradiction do not need to be distinguished by transcendental reflection":

> […] judgments, according to their forms, are divided into the affirmative and the negative, for affirmative [judgments] stand under the principle of identity and negative [judgments] stand under the principle of contradiction. Nonetheless, there is no distinction in content between the affirmative and the negative. Here we are not concerned with the form, as in logic, but with the content, so among the two principles [of identity and contradiction], we choose one and make it the general principle, and it makes no difference which one we choose, except that the principle of contradiction makes more of an impression when we show that the opposite [of a judgment] is false. It makes more of an effect when one shows that [the judgment] is an absurdity and that it contradicts itself, than when one merely proves an identity. (*Ak.* 28:1519)

The PC and the PI have different logical forms, because one is affirmative and one is negative, but they present the same content. Consequently, it makes no difference which one we choose as the most general principle of analytic judgments; the other can be derived from it. The only difference is presentational: the principle of contradiction makes a stronger impression.

This idea, of two principles entailing one another and thus presenting the same content under different logical forms, recalls the discussion of 'equivalent' judgments in the *Jäsche Logik*: "they are to be regarded as a mere substitution of words that signify one and the same concept, where the judgments themselves also remain unaltered as to form" (*JL*, *Ak.* 9:116). Kant's example of a pair of equivalent judgments is:

(1) Not all men are virtuous.
(2) Some men are not virtuous.

He says of these judgments that they "have the same form"—which cannot strictly be true because (1) is a negative universal judgment and (2) is a particular infinite judgment—and that they "say one and the same thing." Their equivalence consists in the fact that they can be 'immediately converted' into one another. Putting this together with the discussion of the PC and the PI—and discounting the sloppy claim in the *JL* that equivalent judgments have the same form—I conclude that Kant's view is that the PC and the PI are equivalent judgments, in the sense discussed in the *Jäsche Logik*.

Kant goes on to make clear that universal affirmative judgments can always be 'contraposed,' that is, immediately converted into equivalent negative judgments. Thus,

(1) <All bodies are extended.>

can be immediately converted to

(2) <No non-extended thing is a body.>

And, since negative universal judgments can themselves be 'converted'—their predicate and subject can be reversed *salva veritate*—(2) is equivalent to

(3) <No body is non-extended.>

Thus, (1), (2) and (3) are 'equivalent' judgments, judgments that "say the same thing" although they instantiate different logical forms. But note that (1) has the form <All (A+B) is B> and (3) has the form <No (A+B) is not-B> In other words, we have shown that, on Kant's theory of equivalent

judgments, the PI in the former formulation is equivalent to the PC in the latter formulation. This allows us to eliminate the displeasingly disjunctive character of the Non-Contradiction Criterion, using the notion of equivalent judgments:

> *Non-Contradiction Criterion*: A judgment <p> is analytic if and only if <p> is equivalent to a substitution-instance of the PC.

Even if we formulate the PC strictly in the form <No (A+B) is ~B>, this will deliver the result, not only that judgments of the form <All (A+B) are B>, but also that judgments of form (d)–(e) from section one are analytic:

(d) <No ~B is (A+B)>, e.g. <No non-metallic thing is gold.>
(e) <All ~B are ~(A+B)>, e.g. <All non-metallic things are not gold.>

Given Kant's rules of immediate inference (outlined in *JL,* §44–55), these are all equivalent to <No (A+B) is ~B.> Consequently, this constitutes a significant advance over Proops (2005). Strictly speaking, judgments (d) and (e) have neither of the forms countenanced by Proops for analytic judgments, <All (A+B) is B> and <No (A+B) is ~B>. Thus, strictly speaking, no judgment of form (d) or (e) can be analytic, on Proops's reading. The way to accommodate judgments (d)–(e) in the Non-Contradiction Criterion is via the notion of 'equivalent judgments,' but, as I argue in the next section, this is the thin end of the wedge. Once we allow judgments equivalent to substitution instances of the PC to count as analytic, the door is open to include analytic judgments of a much wider range of syntactic forms.

4. *Cognition through the PC*

I have used Kant's discussion of judgments that are equivalent—those that can be immediately converted into one another—in the section on 'Immediate Inferences of the Understanding' in the *Jäsche Logik* to understand the relation between the PC and the PI. What started us down the path was the idea that there might be more to a judgment being 'cognizable through the' PC than its merely being a substitution instance of the PC. And we have seen that this is so: cognizing that a judgment is equivalent to the PC although not itself a substitution instance of that principle is,

I argued, a way of cognizing a judgment 'in accordance with' [*nach*] the PC, and judgments that can be so cognized are analytic. Consequently, as we saw in the previous section, the Non-Contradiction Criterion can be understood as wider than it might initially seem, and admits analytic judgments of a (slightly) wider range of syntactic forms.

But a natural continuation of this idea allows us to extend the range of analytic judgments even further. Recall the 'Cognition through PC' Criterion from earlier:

> *Cognition through PC Criterion*: A judgment <*p*> is analytic if and only if <*p*> can be cognized through the principle of contradiction alone.

In what sense is a judgment that satisfies the Non-Contradiction Criterion cognized through the PC? The natural answer is that the judgment is either a substitution instance of the PC—in which case it is cognized by being subsumed as a case under the general principle that is the PC—or it is equivalent to a substitution instance of the PC—in which case it is inferred from the PC. Judgments of the latter kind are cognized 'through the PC' in the sense that they are inferred from the PC. Consequently, judgments that are not equivalent to PC substitution-instances but which are entailed by such substitution-instances are 'cognized through the PC' in exactly the same sense, and should be counted as analytic.

What this means depends, however, upon what notions of entailment and inference are involved. Fortunately, though, Kant's texts contain a natural answer. The discussion of 'equivalent' propositions in the *Jäsche Logik* that I used in the previous section to motivate the idea that the PC and the PI are equivalent is embedded in a larger discussion of 'immediate inferences of the understanding.' Equivalent propositions are defined as judgments that immediately entail one another. Immediate inferences are 'immediate' in the sense that they have only one premise. The rules of immediate inference are the conversion rules by which we transform premises of syllogisms into premises that have the forms recognized by the traditional syllogistic figures. Kant describes the immediate inferences of the understanding as follows:

> §44. *Peculiar nature of inferences of the understanding*. The essential character of all immediate inferences and the principle of their possibility consists simply in an alteration of the *mere form* of judgments, while the *matter* of the judgments, the subject and predicate, remains *unaltered*, *the same*.

> *Note 1.* By virtue of the fact that in immediate inferences only the form of judgments is altered and not in any way the matter, these inferences differ essentially from all mediate inferences, in which the judgments are distinct *as to matter* too, since here a new concept must be added as mediating judgment or as middle concept (terminus medius) in order to deduce the one judgment from the other. (*JL*, *Ak.* 9:115)

Here Kant points out, correctly, that the point of a conversion rule is to alter the form of the judgment—so that it can be used as a premise in a syllogism form—while not altering the 'matter' of the judgment, intuitively, 'what the judgment says.' Immediate inferences are to be distinguished from mediate inferences, syllogisms, in virtue of the fact that the conclusion of a syllogism always combines one concept from the major premise and one concept from the minor premise, via the middle term (which does not appear in the conclusion). So the conceptual 'matter' of a syllogistic conclusion is always different than its conceptual 'input.' This is not the case with immediate inferences.

I have suggested that if a judgment is cognized through the PC in virtue of being equivalent to a substitution instance of the PC, then it is cognized through the PC in virtue of being immediately inferred from the PC. Consequently, any judgment immediately inferred from a substitution instance of the PC is cognized through the PC, and thus, according to the "Supreme principle" section, should be counted as analytic. In other words,

> *Immediate Inference*: A judgment <p> is cognized through the PC if it is either a substitution instance of PC or immediately inferred from a substitution instance of the PC.

This is a sufficient condition on cognition through the PC, and thus, by the Cognition through PC Criterion, on a judgment's being analytic. I will argue that it is not a necessary condition; there are other ways of cognizing a judgment through the PC than immediately inferring it from (a substitution instance of) the PC.

I noted earlier that Kant's definition of immediate inferences as inferences with only one premise admits of a crucial exception, which Kant explains in this passage:

> With inferences of the understanding a *judicium intermedium* [intermediate judgment, or middle premise] may also be made, to be sure, but then this mediating judgment is merely *tautological*. As, for example, in the immediate

> inference, All men are mortal, *some men are men*, hence some men are mortal, the middle concept is a tautological proposition. (*JL*, *Ak.* 9:115)

Tautological judgments, as Kant here defines them, are just instances of the form, <A is A>. But notice that in this passage Kant claims that the judgment <All men are moral> *immediately entails* <Some men are mortal>, even though this inference requires as a premise the tautology <Some men are men.> But this inference is clearly a syllogism. But this means that the rules of syllogistic inference are themselves admissible as rules of immediate inference provided that at least one of the premises is a tautology. Consequently, the rules of syllogistic inference are themselves included among the rules of immediate inference, provided those syllogistic rules are applied to pairs of premises at least one of which is a tautology (e.g. <Gold is gold.>).

This means that we can amend the Immediate Inference Criterion to read:

> *Immediate Inference*: A judgment <p> is cognized through the PC if it follows by immediate inference from a substitution instance of the PC, or by syllogistic inference from a substitution instance of the PC and a tautology.

By definition, immediate entailment is not transitive: if <p> immediately entails <q> and <q> immediately entails <s> it does not follow that <p> immediately entails <s> (although it might). So if there were a judgment <p> that followed by immediate inference from <q>, where <q> followed by an immediate inference from a substitution instance of the PC, <p> would not necessarily follow immediately from that substitution instance. But clearly <p> would be cognizable 'according to' [*nach*] the PC alone, for the only starting points for our cognition of <p> is a substitution instance of the PC; everything else is done through immediate inferences (including the special case of syllogistic inference with a tautological premise). Thus, the 'Immediate Inference' principle is too restrictive, and needs to be reformulated as:

> *Inference*: <p> is cognized through the PC iff <p> follows by a series of inferences from a subsitution instance of the PC, where each inference is either an immediate inference in sensu stricto (i.e. a one premise transformation rule), or is a syllogistic inference with one tautological premise.

But we can go further than this. If some judgment follows, not from a single substitution instance of the PC but from several substitution instances thereof, by a series of immediate inferences and syllogistic inferences, then that judgment itself is cognized according to the PC alone. After all, our cognition of the judgment is based solely on instantiating the PC and then performing immediate inferences on it, and deriving consequences using syllogistic reasoning. The only basis for objecting to this is that, at this stage, the syllogistic rules may be applied in cases where neither of the premises is a tautology, for instance:

(1) All gold is metal. [equivalent to a substitution instance of the PC]
(2) All metal is conductive. [equivalent to a substitution instance of the PC]
(3) ∴ All gold is conductive. [By (1) and (2) by syllogism]

Although neither of these premises are tautologies in the strict sense of *JL* §44, I see no reason to deny that if we cognize (3) by deriving it from (1) and (2) we have cognized (3) according to the PC alone. Thus, I propose the following as the final and definitive criterion for 'cognition through the PC alone' in the sense of A151/B190:

> *Inference*: A judgment <p> is cognized through the PC iff <p> is derived by a series of immediate inferences and syllogisms from a set of premises, each of which is a substitution instance of the PC.

But, plugging this into the 'Cognition through PC' Criterion on analyticity, we get:

> *Cognition through PC Criterion*: A judgment <p> is analytic iff there is a series of immediate inferences and syllogisms from a set of premises, each of which is a substitution instance of the PC, whose conclusion is <p>.

Because tautologies (<A is A>) and simple containment truths (<(A+B) is B>) are immediately equivalent to substitution instances of the PC, this is equivalent to allowing the premises to be either substitution instances of the PC or tautologies or simple containment truths. From the perspective of this criterion on analyticity, the Containment-or-identity criterion of A6/B10 applies and *was only intended to apply* to the special case of judg-

ments that are straight-forward instances of the PI, that is, those that have the form <All (A+B) are B.> Since most of Kant's examples of analytic judgments are of this form, the Containment-or-identity criterion is sufficient for most of his philosophical purposes, but by no means exhausts his conception of analyticity.

I began by talking about several of Bolzano's criticisms of how Kant draws the analytic-synthetic distinction; I have focused on his third criticism —that it applies only to judgments within a narrow range of syntactic forms—but along the way, I have indirectly touched on several of his other criticisms (e.g. that it rests on the metaphor of inclusion). However, my interpretation, if correct, makes one of Bolzano's critiques—that Kant never adequately proves that mathematical judgments are synthetic —harder to answer. If analytic judgments have only a limited range of syntactic forms, then Kant can prove that mathematical judgments are not analytic merely by proving they do not have these forms. However, if my Inference Criterion is the correct interpretation, Kant has to prove the more ambitious claim that mathematical judgments like 7+5=12 do not follow by logical inference from any number of Containment-truths and substitution instances of the PC. Whether, and how, Kant can prove this lies beyond the scope of this essay.[22]

For now, I would like to conclude by making two related points. First of all, to return to an observation I made earlier in the paper, Kant's analytic-synthetic distinction does not occur in a vacuum. Instead, it is made within a very specific argumentative context: he wants to deny the Leibnizian/Wolffian view that all necessary truths are derivable from definitions plus logical laws. Consequently, it is incumbent upon him to formulate his own view in such a way as to enable him to formulate precisely his disagreement with Leibniz. If Kant's analytic judgments are in fact a proper subset of what Leibniz calls "truths of reason" then in claiming that there are necessarily true non-analytic judgments (the synthetic *a priori*) Kant is making a claim Leibniz need not deny. Specifically, if Kant's analytic judgments are restricted to the limited range of syntactic

22. Thus, one of the advantages of interpretations like those of Anderson (2004, 2005) and Proops (2005) is that they make clear how Kant could have taken himself to have proven that arithmetic is synthetic, given how brief his actual arguments for this conclusion are. In future work I will argue that in this respect Anderson is a false friend to Kant; he equips Kant with a short, powerful argument for the synthetic nature of arithmetic at the cost of burdening him with an indefensible conception of analyticity.

forms that both Bolzano and Proops claim they are, then in claiming that there are necessary judgments that are not analytic Kant is not expressing a disagreement; of course Leibniz would accept there are necessary truths that do not belong in that limited class of judgments, because Leibniz has to accept that there are necessary truths of a vastly wider range of syntactic forms, e.g. <No A is B and not-B.> If Kant were to limit the analytic in this way, he would be drawing the analytic-synthetic distinction in such a way as to make one of his central anti-Leibnizian points moot.

If, on the other hand, Kant has the wider conception of analyticity that, I have argued, he does, then he is in a perfect position to express his disagreement with Leibniz. The Cognition through PC Criterion, in its final formulation above, fits remarkably well with Leibniz's definition of 'truths of reason' in *New Essays* and other texts—'propositions that follow from definitions and identities'[23]—since definitions (e.g. <(A+B) is B> and identities (e.g. <A is A>) will be what Kant would consider substitution-instances of the principle of identity (PI). Clearly, Kant and Leibniz will differ over the definition of certain concepts, hence about which truths have the form <(A+B) is B>, and they may differ over the rules of inference, but this will not be a difference over what it is to be an analytic judgment, or 'truth of reason.' Furthermore, the set of Kantian analytic judgments and Leibnizian truths of reason may not overlap completely, but this will be due to the differences in the logical rules that are built into these definitions. This leads me to my second point.

I started out by discussing Bolzano's claim that Kant's definition of analyticity applies only to judgments with a narrow range of syntactic forms, and with a contemporary reading of Kant that seems to vindicate this charge, Proops (2005). I have argued that, properly understood, Kant's

23. *New Essays* IV.xviii.9, 'The Source of Contingent Truths' (Leibniz 1989, 98), 'On Freedom' (Leibniz 1989, 69), Fifth Letter to Clarke (Leibniz 1969, 696), Monadology §31–33 (Leibniz 1969, 646). In some of these passages Leibniz says: p is 'strictly' necessary iff ~p entails a contradiction, in other places he says: p is 'strictly' necessary iff p can be demonstrated from identities and definitions. These are equivalent. It is relatively clear that the latter definition in terms of demonstration is equivalent to the Kantian Cognition through PI criterion. The former definition, in terms of contradiction-entailment, is equivalent because, if ~p entails a contradiction, then the negation of that contradiction, which is a substitution instance of the PI, entails p. Conversely, if p can be demonstrated from a finite set S of PI substitution instances, then from ~p we can demonstrate the negation of the conjunction of the elements of S. Assuming we are allowed to introduce PI substitution instances at any stage in a demonstration (as I argued in the text), this means we can demonstrate from the negation of the conjunction of the elements of S, using a disjunctive syllogism, the negation of a PI substitution-instance. This shows that the Kantian and the Leibnizian criteria are equivalent.

conception of analyticity includes analytic judgments of a wider range of syntactic forms. However, the definition of analyticity given may not actually allow for analytic judgments of all the syntactic forms Kant would want to recognize. Whether this is true or not depends upon the strength of the logical rules built into the definition of analyticity. The final point I want to make is that if Kant's definition of analyticity does not extend to judgments of some syntactic form that, intuitively, should be counted as analytic, the fault is not to be laid at the door of his definition of analyticity or his philosophical intentions in distinguishing analytic from synthetic judgments. The fault lies in Kant's logic. Kant simply does not possess a logic powerful enough to generate all of the analytic judgments from the simple containment truths, i.e judgment of the form <All (A+B) is B>. So, in the end, Bolzano and Proops may actually be right, but they will be right for different reasons than they thought. It isn't that Kant intentionally limited analyticity to judgments within a narrowly circumscribed range of syntactic forms; he simply did not possess a sufficiently powerful logic to derive all of the analytic judgments from substitution instances of the principle of identity.[24]

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