# 8

# Absolute Space and the Riddle of Rotation
## *Kant's Response to Newton*

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All motion is relative, Mach declared, berating Newton's 'error' of letting absolute space ground his dynamics. Reichenbach later scolded him as a crude dogmatist.[1] This snide animus toward Newton is old among relativists: Christian Wolff had blithely dismissed him, with scant regard for his case, as a novice in philosophy.[2] Kant too says that all motion is relative, but knows better than to be smug about Newton's case. Widely seen as a spokesman for Newtonian science, for once he looks deeply taxed by Newton's grounding of dynamics. In *Metaphysical Foundations of Natural Science* (henceforth *MAN*), Kant brings up Newton's thought experiment of the two globes in the *Principia* and notes that, based on it, true rotation can be detected empirically even if the *material* space relative to which the globes allegedly *ought to* move is absent. 'This paradox deserves to be solved', he declares.[3]

[1] Cf. E. Mach, *The Science of Mechanics*, 6th edn., trans. Th. McCormack (LaSalle: Open Court, 1960), 340; H. Reichenbach, 'The Theory of Motion according to Newton, Leibniz and Huygens', in M. Reichenbach (ed. and trans.), *Modern Philosophy of Science* (London: Routledge & Kegan Paul, 1959), 46–66, at 60.

[2] For instance, in a 1741 letter to J. D. Schumacher, head librarian at the Imperial Academy of Sciences in St. Petersburg. Speaking dismissively of Voltaire's attempts to show that Newton had a metaphysics, Wolff adds, 'pure mathematicians are the least skilled in metaphysics, and [when they dabble in it] they are like a mere poet trying to pass judgment on mathematical matters of which he has no clue.' Cf. A.A. Kunik (ed.), *Briefe von Christian Wolff aus den Jahren 1719–1753* (St. Petersburg, 1860), 133f. Unless otherwise noted, all translations are mine.

[3] *MAN*, 'Metaphysical Foundations of Phenomenology', in I. Kant, *Gesammelte Schriften*, ed. Royal Prussian Academy of Sciences (Berlin: G. Reimer, 1902–), iv. 558. All Kant citations are by volume and page number from this edition, hereafter abbreviated as '*GS*'. I cite Kant's *Critique of Pure Reason* as A or B, respectively, followed by page number.

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In this chapter, I examine and answer three related questions: What *is* the paradox? What makes it a paradox? And how does he solve it?

*In nuce*, Kant rejects Newtonian absolute space, and holds instead that motion is relative to matter, not to space itself; but rigid rotation (as in Newton's tied-globes example) violates Kant's injunction: the globes do move, yet *no* motion relative to *matter* occurs. That is the paradox. Its source is Kant's dual move above: the rejection of Newton's absolute space and the commitment to the relativity of motion. Kant's solution is to redefine the meaning of 'motion' so as to claim that, in a rotating body or system, its parts *do* after all move relative to each other. He defines their motion as a 'dynamical relation of matters in space'. Here I explicate Kant's account in detail.

Some years ago, John Earman tackled my last two questions. I trust that the reading I give is more attuned to Kant's actual views on rotation, accounts better for his explicit claims about it, and incorporates the historical backdrop to his solution. Still, it reaffirms Earman's verdict that rotation was 'an especially difficult challenge' for relationists—by showing that Kant was no exception.[4] My reading shares some traits with Martin Carrier's, though I diverge sharply from his assessment of Kant's solution.[5] More generally, I use the thorny case of rotation to convey a sobering message: Kant pays a philosophical price for turning down Newton's absolute space.

Kant's account of rotation, its backdrop, and assumptions are intricate, and so the reading I will offer is convoluted. And, I use that reading to make three points, two philosophical and one exegetical. To give the reader a clear sense of my destination, I offer an outline of the three arguments I will be advancing here. The first is an argument about Kant's engagement with Newton (call it **N**). It is as follows: Newton claims the alternative to his absolute space—viz. relationism about motion—must vindicate the 'properties, causes and effects' of true motion. Kant espouses relationism, but addresses just the 'effects' of true motion. He never tackles its properties and causes, though he should. So, Kant engages with Newton's case for absolute space selectively.

[4] J. Earman, *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time* [*World Enough*] (Cambridge, MA: MIT Press, 1989), 89.

[5] See M. Carrier, 'Kant's Relational Theory of Absolute Space' ['Kant'], *Kant-Studien*, 83 (1992), 399–416. I discuss his interpretation in greater detail at the end of section 4.

My second argument too is about Kant's response to Newton, but from a special vantage point, viz. his account of rotation. (In §4, I explain why I choose this vantage point to examine Kant's relation to Newton.) Call this argument **R**. In a nutshell: A relationist account of rotation compatible with Newton's science would explain rotation as a type of motion relative to a material *inertial* frame. But Kant analyzes rotation as motion with respect to a rotating, hence *non-inertial*, material frame. So, Kant gives metaphysical foundations for a science that is not fully Newtonian.

Lastly, I have an argument about some interpretations of Kant's analysis of rotation; call it **E**. Very briefly: Earman, Carrier, and Friedman have claimed that Kant analyzes rotation as motion relative to an *inertial* frame. But Kant's words entail that rotation is true motion relative to a rotating, hence *non-inertial*, material frame. So, either Kant made a basic mistake or these interpreters cannot be right.

While these figures differ on the details of Kant's solution, they all presume that Kant's solution must be fully compatible with Newton's mechanics. Based on that presumption Kant's handling of rotation appears grossly flawed. Fortunately, my reading does not saddle Kant with a blunder, but this hermeneutic virtue comes at the price of relaxing his Newtonianism; I deem it worth paying. Now let us proceed.

I begin, in §1.1, with a close look at Newton's account of rotation in the *Principia*; in §1.2, I spell out the dual challenge it posed to relationists and their initial reactions to it in Germany. Next, I outline Kant's early theory of motion, a unique kind of relationism (§2). This commitment survives in Kant's analysis of circular motion in the 1780s. In §3.1, I detail his mature account of absolute space and true motion, and in §3.2 his general view of rotation. In §3.3, I explain how rotation for Kant is relative, and how this solves his 'paradox' of rotation by widening his early idea of relative motion. This leads Kant to quietly change his official theory of motion, and it induces some tension in his natural philosophy. I assess these facts in §4. Further, the Kantian analysis of rotation implies that we ought to reconsider most previous construals of Kant, including Michael Friedman's influential interpretation (§4.2). However, my critique of Friedman rests on a reading of Kant that leaves me open to an objection. In §5, I present the objection and answer it.

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Before I proceed, a clarificatory note is in order. In this chapter, I discuss at times the motion of rigid bodies or systems. Now, 'rigid' has a dual sense in mechanics. It may denote rigid motion, i.e. an extended body or system in translation, rotation or both, such that the body or system does not change its configuration: all relative distances between its parts remain the same throughout the motion. Or it may refer to bodies or systems rigid throughout interactions, e.g. collisions or mutual attractions. Such bodies are called rigid because they do not change shape and size even though external forces act on them. In my chapter, I use 'rigid' solely in the first sense.[6]

## 1. THE RIDDLE OF ROTATION

### *1.1 Newton on Rotation*

What exercised Kant is the globes setup in the *Principia*, but Newton used rotation to challenge relationists *twice*. Before I explain his dual challenge, some clarification is needed.

Many interpreters have seen Newton's conflict with his opponents as a debate about whether there are absolute motions or whether all motion is relative. In the 1990s, a new reading of Newton accounted for all his claims in the Scholium and had the added benefit of uncovering the fundamental ambiguity of the terms 'absolute' and 'relative' in early modern theories of motion.[7] Then, 'absolute motion' denoted two distinct things. (1) In one sense, it was just a synonym for *true* motion: most took for granted that, besides their *apparent* motions, bodies also have true, or 'absolute', motions. True motion was meant as a *complete*, or monadic, predicate: a truly moving body moves *tout court*,

[6] I thank Sheldon Smith for pressing me to make my meaning of 'rigid' explicit.

[7] The interpretation of Newton's Scholium, its background and aftermath is in R. Rynasiewicz, '"By Their Properties, Causes and Effects": Newton's Scholium on Time, Space, Place and Motion' ['Newton's Scholium'], *Studies in History and Philosophy of Science*, 26 (1995), 133–53, 295–321. Later, Rynasiewicz spelled out the ambiguities of 'absolute' and 'relative' motion in 'On the Distinction between Absolute and Relative Motion', *Philosophy of Science*, 67 (2000), 70–93. I endorse both his reading of Newton and the distinctions he draws between the various senses of absolute and relative *motion*. N. Huggett also endorses Rynasiewicz's reading of the Scholium, in his 'Essay Review: Harvey Brown's *Physical Relativity* and Robert DiSalle's *Understanding Spacetime*', *Philosophy of Science*, 76 (2009), 404–22.

not relative to this or that; it either truly moves or is truly at rest, but not both; and it has exactly one true motion. Therefore, it has a *unique* quantity of motion. This privileged true motion some called 'absolute', 'real', 'proper', or 'physical'.[8] In the Copernican dispute, *both* sides accepted that bodies have motions that are absolute in this sense.[9] (2) In another sense, 'absolute motion' denotes the idea that true motion *is,* or *consists in*, velocity in immobile space metaphysically distinct from body, thus absolute. Newton articulated this view. His disciples in Britain endorsed it, as did Euler, the giant of Enlightenment dynamics.[10] This second sense comprises the Newtonians' answer to the key question in the controversy: if true motion exists, *what* does it consist in? What is its *nature*, or proper definition?

In turn, 'relative' likewise had two senses. (3) For some, it was their answer to the definitional question earlier. They claimed that a body's *true* motion consists in a special type of motion relative to other *bodies*: e.g. those that surround it; the fixed stars; the center-of-mass in a material system, etc. Descartes, Berkeley, and the young Kant each championed some version of it. (4) Others meant 'relative motion' as the *denial* of the claim that single bodies have any true motions *at all*. For them, relativity of motion meant that any assignment of motion and rest is *arbitrary*. This view assumed that bodies can move *only* relative to other bodies and added that *none* of these motions is privileged as 'true', 'real', or 'objective': true motion does not have a

[8] See, e.g., Mariotte's *Traité de la Percussion ou Choc des Corps* (1679), Definition 3: 'The relative speed of two bodies is that with which they approach or recede from each other, no matter what *their proper velocities* may be', in *Oeuvres de M. Mariotte*, vol. 1 (The Hague, 1740), 3 (my emphasis). And: 'local motion is either from one place in world space to another; or in the relative space of some enclosing container. The former we shall name *real* and *physical* motion, the latter relative motion', in G.A. Borelli, *De vi percussionis* (Bologna, 1667), 3.

[9] Both Aristotelians and Copernicans tacitly accepted that the dispute was about the *true* motion of the Earth and the Sun. There would have been no dispute about the *apparent* motions of these two bodies: the Earth clearly appears to rest; and the Sun plainly appears to move.

[10] Consider: 'Absolute Motion is the Change of absolute Place, and its Celerity is measured by absolute Space', in J. Keill, *An Introduction to Natural Philosophy*, 4th edn. (London, 1745), 78; 'Motion is the change of place; that is, of the part of space which the body occupies, or in which it is extended. The motion is *real* or *absolute*, when the body changes its place in absolute space', in C. MacLaurin, *An Account of Sir Isaac Newton's Philosophical Discoveries* (London, 1748), 101; and also: 'When a body occupies successively one part of this immense [absolute] space after another, it moves; but if it persists continually in the same place, it rests', in L. Euler, *Mechanica*, Tomus I (St. Petersburg, 1736), 2.

subject, because there is *no fact of the matter* as to whether an individual body truly moves or rests. Huygens favored this view for collisions and sought to extend it to rotation; the young Leibniz backed it too, at least for impact mechanics.[11]

To show the conceptual links between these senses, and so to shed light on Newton's argument, I must use some stipulative terms. Call 'completism' the view implied by (1), 'absolutism' the one under (2), 'relationism' the view (3), and call (4) 'relativism'.[12] Absolutism and relationism are competing *metaphysical* accounts of the *definiendum* 'true motion' asserted by completism. Relativism is the denial of completism, therefore it entails that *both* absolutism and relationism are false. Hence, relationism and relativism are logically incompatible views. For that reason, the interpreter must be very careful in dealing with claims that motion is relative.

Most early moderns accepted completism—the idea that bodies have true motions drove both the Copernican controversy and the 'new mechanics'.[13] The real question was, how to *define* that concept: what does true motion *consist* in? Newton is no exception: he takes true motion for granted and claims that it *is* change of absolute place. In our terms, it is velocity in absolute space. In the Scholium, he concurs with his peers 'that each body has a state of true or absolute motion unique to it' and adduces reasons to *define* true motion 'in terms of an absolutely immobile space, distinct from body'.[14] That is, Newton accepts

[11] For the young Leibniz's views on true motion, see, e.g., this note of 1677: 'A remarkable fact: motion is something relative, and one cannot distinguish exactly which of the bodies is moving' (A VI. iv. 360/*LOC* 1970). And a few years later, in 1683: 'For it cannot really be said which subject the motion is in' (A VI. iv. 1465/*LOC* 263). However, by the time of the correspondence with Clarke, Leibniz had come to admit that motion is *not* relativistic, after all: 'I grant there is a difference between an *absolute true motion* of a body, and a *mere relative* change of its situation with respect to another body.' Cf. H.G. Alexander (ed.), *The Leibniz–Clarke Correspondence* [*Correspondence*] (New York: Barnes and Noble, 1970), 74 (emphasis added). To grant that bodies have true motions is precisely to admit that we *can* tell 'which subject the motion is in', contra Leibniz's earlier professions of relativism.

[12] Position (1) is the view that individual bodies have *true*, or *real*, or *proper* motions. I chose 'completism' because on this view the true motions are *complete* predicates of those bodies. To call it 'truism' or 'realism' is misleading, and 'properism' too rebarbative.

[13] Early modern mechanics implies a distinction between merely apparent and privileged states of motion through the law of inertia. If taken to describe the *apparent* motions of bodies, the law fails trivially. The inertial motion or rest that the law assigns to force-free bodies must be understood as distinguished from the ways bodies *appear to us on Earth* to move.

[14] Rynasiewicz, 'Newton's Scholium', 134, 135.

completism and asserts that it must be *explicated* as absolutism. He reveals *expressis verbis* that he is after the nature of *true* motion: how to define it, or what it consists in.[15]

Alas, Newton made a confusing move. He uses 'absolute' as a synonym for 'true' *even before* he has argued that the two kinds of motion are identical. That is, first he starts talking about absolute = true motion *without* prejudging whether it consists in motion in absolute space. Then he concludes that absolute = true motion must *be the same thing as* motion in absolute space. This can give the impression that Newton had been arguing all along for the *existence* of motion in absolute space. Since relationists deny absolute space, they may have thought (incorrectly) that they could subvert Newton's absolute motion simply by attacking absolute space, instead of addressing separately his theory of true motion.

To defend his claim that true motion *consists in* change of absolute place, Newton argues that true motion has traits that relationism fails to bear out, whereas absolutism does. The traits are: true motion has certain properties; is caused by forces 'impressed upon the moving body' itself; and induces effects such as 'the forces of receding from the axis of circular motion'. These traits are conditions of adequacy for an acceptable account of the *definiendum* 'true motion'. To be successful, a philosophical theory of motion must vindicate them. Newton's own definition—motion as change of place in absolute space—satisfies these conditions. *Sed contra*, relationism—defining true motion as a special motion relative

[15] In the five paragraphs on properties, causes and effects, Newton uses the vocabulary of defining [*definiri*] and of true motion [*motus verus*] and true rest [*quies vera*]. For instance: 'true rest cannot be defined on the basis of the position of bodies in relation to one another'; 'all contained bodies... will participate in the true motions of the containing bodies'; 'true motion is neither generated nor changed except by forces impressed upon the moving body itself'; 'true motion certainly does not consist [*minime consistit*] in relations' to other bodies; 'true circular motion cannot be defined [*definiri nequit*] by means of such [relative] changes in position'. He also makes clear that he endorses another part of completism, namely, the thesis that each body has a *uniquely assignable quantity* of true motion: 'the truly circular motion of each revolving body is unique [*unicum*], corresponding to a unique endeavor as its proper and sufficient effect'. And, in the last paragraph, having identified true motion with absolute velocity: 'It is certainly very difficult to find out the true motions of individual bodies [*motus veros corporum singulorum*]'. Cf. I. Newton, *The Principia: Mathematical Principles of Natural Philosophy* [*Principia*], I. B. Cohen and A. Whitman (trans.) (Berkeley, CA: University of California Press, 1999), 411–14; and Newton, *Philosophiae Naturalis Principia Mathematica*, 3rd edn. (London: W. & J. Innys, 1726), 8–11 (translation slightly altered).

to other bodies—fails to meet them. Therefore, 'true motion certainly does not consist in relations of this kind'.[16] Granted, in his argument from properties, causes, and effects, Newton singles out Descartes's account as an especially inadequate definition of true motion. Yet it is plain that Newton rebukes relationism in general, not just its Cartesian version.[17] From the triple failure of relationism and success of absolutism, he concludes that his explication alone—to wit, equating true motion with velocity in absolute space—is adequate. Therefore, absolutism is the only correct metaphysical account of completism.[18]

Having argued that true motion must be defined as motion in absolute space, Newton ends the Scholium with a brief account of how the latter may be *measured empirically*. As illustration, he offers a thought experiment: let an observer in empty space come across two globes, 'with a cord connecting them', that *appear* to rotate around their common center-of-mass. By various means, she can ascertain the presence—and measure the strength of—tension in the cord. Generally, this tension was held to correlate with the globes' true rotation, which Newton by now has *identified* with angular speed in absolute space. But, the observer can do more: by applying equal torques 'upon the alternate faces' of the globes, she can find out the true direction of their rotation. Thereby, she will have measured the globes' absolute angular velocity, i.e. rate of rotation in absolute space.[19]

[16] Newton, *Principia*, 412. This, if I read him correctly, is the construal of Newton's argument that Rynasiewicz puts forward in his 'Newton's Scholium'. Robert DiSalle has articulated a different interpretation of Newton's Scholium, most recently in his *Understanding Space-Time* (Cambridge: Cambridge University Press, 2006), 17–47. To my knowledge, DiSalle has not engaged critically with Rynasiewicz's reading of the Scholium.

[17] This is apparent twice. First, his argument from causes shows that *all* body-relationisms are vulnerable to Newton's objection: apply a force to the *reference bodies*, viz. the special frame; then the subject body will have changed its state of motion *without* a force being applied to *it*. This result violates the Newtonian condition that true motion changes *only if* the force is applied to the body. And, it subverts all accounts of true motion as relative to bodies, not just Descartes's. Second, in the bucket experiment—his argument from effects—Newton targets relationism beyond Descartes: at every instant, the water has a unique quantity of true motion, correlated with the unique height to which it ascends. But the water has countless relative motions, corresponding to its innumerable relations to other bodies. Hence, Newton infers, its true motion cannot consist in a relation to other bodies.

[18] I call it a triple failure because it fails to satisfy three *kinds* of conditions—the properties, causes, and effects of true motion. Newton in fact has *six* arguments against relationism: three from properties, two from causes, and one from effects.

[19] I must preempt possible misconceptions. First, one may ask: why is it not enough that the observer *sees* the globes to rotate? It is because Newton makes no assumptions about the

Before we move on, a clarificatory point. We may wonder whether the cord is rigid or not—does it stretch as the globes rotate, or does its length stay unchanged? Newton is silent on this matter. However, it is a historical fact that his opponents—Huygens, Leibniz, and Kant—all took it to be so, and reacted accordingly: Leibniz sought to deny that there are rigid systems in nature, while Huygens and Kant took the globes to spin rigidly, and sought to provide relationist accounts of motion that accommodate that fact. In any event, whether the cord is rigid or deformable is a red herring: even if it does stretch, opponents of absolute space must explain that fact too. The elongation is clearly caused by the globes' true motion, which still awaits a relationist account. This is because, if the cord is deformable, as the globes rotate over time they move *away from each other* along the line *between* them. But, in Newtonian mechanics the globes have instantaneous velocities along the *tangent* to their individual trajectories. To be compatible with Newtonianism, the relationist must vindicate this very fact: it must give a relationist account of the *physical straight line* tangent to the trajectory; it must indicate the matter that individuates that line, or serves to distinguish it from all other straight lines passing through the instantaneous location of the rotating particle. This may look easy, but is extremely difficult. As evidence, I offer Kant's struggle with this task in what follows.

## *1.2 Newton's Dual Challenge*

Against this backdrop, we can now see the challenge with full clarity. The *real* one was Newton's three-pronged argument from the properties, causes, and effects of true motion. It defies relationists—who, just like Newton, grant that true motion is a legitimate concept. They must supply a definition, or metaphysical account, of true motion that vindicates the properties, causes, and effects which Newton details in

observer's own state of motion—she herself may be rotating around some axis, in which case the globes' rotation as it appears to her will *not* in general coincide with their true rotation. Second, note that Newton need not appeal to resources proper to *his* dynamics alone. For example, to measure tension, the observer may cut the cord and attach its resulting ends to the ends of a spring, then use Hooke's insight *ut vis, sic tensio* to infer from the spring's elongation to the strength of the tension. The latter is proportional to Huygens's 'centrifugal force', which yields a measure of the body's angular speed.

the Scholium and construes motion as *actual* kinematic change relative to other *bodies*.

But most of his opponents did not respond to Newton's real challenge. Rather, they *mistook* Newton to have argued merely that his globes display phenomena of rotation best explained as absolute motion. So, Huygens, Leibniz, and Kant singled out rotation as the *only* problem coming from the Scholium. *If* taken that way—*contrary* to Newton's actual intention—the problem of rotation is as follows.

In the Scholium, Newton uses rotation twice, to different ends. First is the bucket experiment. That is a *direct*, explicit challenge. Newton takes it for *granted* that, at every instant, the water has a true state of motion, corresponding to a *unique* quantity of motion, manifested empirically by a dynamical effect of true rotation: the 'forces of receding from the axis of circular motion'.[20] He uses the bucket experiment primarily to refute *one* definition of true motion, the Cartesian.[21] But his conclusion makes clear that he means it to oppose *any* version of relationism, not just Descartes's:

> The truly circular motion of each revolving body is unique, corresponding to a unique endeavor as its proper and sufficient effect, while relative motions are innumerable in accordance with their varied relations to external bodies.[22]

That is, he takes relationists to believe that bodies have true motions, which consist in motions relative to some privileged body (or bodies). Then, with his bucket experiment, Newton defies relationists to *name* the body or system of bodies relative to which the spinning water moves truly, and to get it right: to show that the water's true rotation exactly *correlates* with its relative motion.[23] These are just minimal

[20] Newton, *Principia*, 412.

[21] Howard Stein first argued that Descartes was Newton's main target in the bucket scenario. Newton shows the water's true motion to be *anti-correlated* with its Cartesian construal: when the water is truly at rest, Descartes's account entails that it is truly rotating; and when it reaches its maximum true rotation, Descartes must conclude that it is truly at rest—in *his* sense of 'true motion' and 'true rest'. Therefore, Descartes's account of motion cannot be correct. Cf. H. Stein, 'Newtonian Space-Time', *Texas Quarterly*, 10 (1967), 174–200.

[22] Newton, *Principia*, 413.

[23] This is my explication of Newton's challenge in the passage quoted earlier. The second condition is obviously needed, because the first can be met trivially: one can always name *some* set of bodies as the privileged frame relative to which the water moves. Newton's sharp rebuke of Descartes shows that not just any set of bodies will do.

requirements: the very least a relationist must do if she is to *begin* mounting a response to Newton's case for absolute space. A full response requires addressing his *entire* case from the 'properties, causes and effects' of true motion, not just his appeal to rotation.

This is because the bucket experiment is part of a *threefold, parallel* argument. The argument is that true motion has certain properties, causes, and effects, which Newton's definition—true motion as translation in absolute space—satisfies, whereas Descartes's account (the chief kind of relationism targeted in the Scholium) fails to do so. Therefore, true motion consists in absolute motion, i.e. velocity in absolute space. To blunt Newton's critique, a relationist has two options. One is to argue effectively that the concept of motion required for mechanics does *not have* the properties, causes, and effects Newton ascribes to it and to offer a theory of motion *demonstrably* capable of grounding a nontrivial mechanics that rivals Newton's. Another is to spell out a relationist alternative to Newton's account of true motion and to show that it has the 'properties, causes, and effects' of true motion.[24]

In contrast, the tied-globes scenario is *not* a direct challenge to relationists. Whereas the bucket experiment makes a point in *metaphysics*, the globes setup illustrates a point in epistemology. Its official role is to teach how motion in absolute space may be *measured* empirically, even though just in principle, at this stage in the *Principia*. Still, though not meant to provoke them, some relationists (in particular, Huygens, Leibniz, and Kant) took the *globes* to be Newton's *sole* challenge to their doctrine that true motion is always relative to bodies. Collectively, they failed to take the *bucket* scenario to be *one of three* challenges, as Newton had intended. These relationists either missed Newton's point (about the properties, causes, and effects of true motion) or thought it ineffectual. Perhaps they were riled by his taunting words, as he describes the experiment, that 'both the quantity and the direction of [the globes'] circular motion could be found in any immense vacuum, *where nothing external and sensible existed with which the globes could be compared*'.[25]

[24] If I read him correctly, Earman too argues that the first option above is a requirement for the relationist; cf. Earman, *World Enough*, 65.

[25] Newton, *Principia*, 414 (my emphasis).

*If* taken as a challenge, the globes experiment defies relationists, first, to (1) specify the body or *material* system relative to which the globes rotate truly; and (2) prove that their motion relative to that system correlates with their *true* angular velocity as measurable by dynamical means—Hooke's law, Huygens's results on centrifugal force—available to all, not exclusive to Newton's mechanics. But that is not all. The globes setup forces them to spell out their relationism with greater care. (3) If they explicate the globes' rotation as motion relative to bodies *external* to the globes-system, they must explain how that relative motion could be discovered and measured *even in the absence* of that to which it is relative. (4) If they choose to explicate it as a motion of the globes *relative to each other*, they must explain what makes that rotation a kind of *motion*: what *moves* there—what meaning do they attach to the view that the globes have instantaneous velocities relative to each other?

Reactions in Germany to Newton's *explicit* challenge from rotation were scant. Leibniz ignored the direct problem—viz., to give a relational account of the water's true motion as the bucket spins—and sought instead to deflect what he perceived as the real threat to his position: the *rigid* rotation of Newton's two globes. In the 1690 *Dynamica*, he crafts a philosophical argument meant to conclude that no truly rigid bond exists in nature.[26] From this, he infers that a system *appearing* to rotate rigidly is really in 'relative' motion (*motus respectivus*). It is far less easy to discern Leibniz's exact sense of the relativity of rotation.[27]

[26] GM vi. 509f.

[27] Leibniz appears of two minds on this issue. Sometimes, he implies rotation is relativistic, viz. *not* a kind of *true* motion: this follows from his claim that rotation is subject to the law of the 'equivalence of hypotheses'. Other passages suggest he thinks true rotation can be distinguished from both *apparent* rotation and rotational *rest*—through the *conatus paracentricus* that truly rotating bodies exhibit; cf. GM vi. 510. Commentators have not reached any consensus as to what exactly Leibniz's analysis of rotation amounts to. Earman reads it as relational because one-body rotation supervenes on the motions of its component particles relative to one another; see Earman, *World Enough*, 72. Others claim it is true rotation relative to the 'center-of-mass frame' of the ether filling the Leibnizian plenum of the actual world; see A. Jauernig, 'Leibniz on Motion and the Equivalence of Hypotheses', *The Leibniz Review*, 18 (2008), 1–40; and 'Leibniz on Motion: Reply to Edward Slowik', *The Leibniz Review*, 19 (2009), 139–48, at 142f. E. Slowik strikes a sobering note about the prospects of articulating a fully consistent Leibnizian account of rotation and subtly distinguishes between senses in which rotation may be absolute or relative for Leibniz. Cf. 'Another Go-Round on Leibniz

Wolff, his posthumous acolyte in Germany, never confronts the thorny issue of rotation, nor does he address Newton's threefold argument from properties, causes, and effects. He simply toes the Leibnizian line that motion is relational, viz. a relation between bodies, as it consists in change of place, which allegedly is relative to 'actually coexistent' bodies—a view that Euler crushes in 1748.[28] Perhaps Wolff ignored rotation because, by 1730, impact (a type of interaction where bodies move in straight lines, not curves) had again taken center stage in foundational debates in natural philosophy.[29] Still, that does not absolve Wolff; his cosmology rests on a vortex theory of planetary motion, and *that* raises the question, relative to what do planets really move? Newton had sought to drive home this very point with his bucket experiment: the water in his spinning bucket was an analogue of the Cartesians' ether vortices supposedly driving the planets in orbit.

## 2. KANT'S EARLY RELATIONISM

On the other hand, Wolff's fixation on impact, and failure to explain how motion is relative, *does* help explain Kant's interest in collision as he first outlines a theory of motion. This occurs in his 1758 *New Doctrine of Motion and Rest* (*ND*), a seminal paper in which Kant lays

and Rotation', *The Leibniz Review*, 19 (2009), 131–8. An attempt to take a position on these matters is beyond the confines of my chapter.

[28] Christian Wolff, *Philosophia prima, sive Ontologia* [*Ontologia*] (Frankfurt, 1730), 462f., 494. However, Wolff's own mechanics presupposes a *distinguished standard of rest*, which calls for him to explain which of a moving body's many relations (to other bodies) is the *true* one. Wolff fails to meet that demand. The term 'distinguished standard of rest' denotes a frame relative to which a body either truly moves or rests truly. Wolff needs it to ground his mechanics, for two reasons: (1) his fundamental concept of force is Leibnizian *vis viva*, whose measure is $mv^2$, hence it is a function of a body's *speed*; but, to be an *objective* feature, *vis viva* demands that the *true* speed of bodies be knowable; (2) Wolff also distinguishes between active force, present in *moving* bodies, and 'passive force of resistance', found in bodies at *rest*; again, to be objective, this distinction requires the possibility of distinguishing true motion from true rest. Wolff's metaphysical mechanics is in his *Cosmologia Generalis*, editio nova [*Cosmologia*] (Frankfurt, 1737), §§129–75, 302–502. On Euler's critique of Wolff, see §4.1 in this chapter.

[29] After 1716, Wolff and other *Leibnitii sequaces* strove to reinforce Leibniz's case for *vis viva* as *the* dynamical quantity conserved in interactions. To do so, Wolff focused on straight-line motion—be it uniform translation, as in his 1726 *Principia Dynamica*, or collision between non-rotating bodies, as in *Cosmologia Generalis*.

out a terse yet distinct form of relationism (*GS* ii. 13–25). To see how it differs from its precursors, we must examine its genesis.

Kant motivates his theory with an attack on the view he plans to replace: the *metaphysical* thesis that motion *is*, or *consists* in, change of place. Tacitly but undeniably, he means *true* place, for he is after a concept of *true* motion.[30] He sees two ways to define 'change of place' and is confident he can subvert both. One is with respect to the fixed stars: if a body changes position relative to them, it moves truly. Kant charges that this construal is empirically deficient. Specifically, he invokes findings by James Bradley to claim that the fixed stars are not really at rest, as the view assumes; and that the solar system and the stars move relative to each other, but it is impossible to determine which one truly moves.[31] Therefore, the stars are useless as a global frame for defining the true motions of bodies: 'I ask, in which direction and with what speed [do bodies move relative to the stars]? No one answers me' (*GS* ii. 17).

Next, he turns to a second way to define true places: as the parts of a rigid whole, viz. absolute space. This is recognizably Newton's view, though some Leibnizians embraced it too.[32] Kant protests that, since

[30] Though Kant, in *ND*, often talks simply about 'motion', he seeks the meaning of *true* motion; see, e.g., *GS* ii. 18: even a body that *appears* 'completely at rest relative to other adjacent objects in [a] space has nonetheless a *true* motion with respect to any body that approaches it' (my emphasis).

[31] James Bradley, Astronomer Royal between 1742 and 1763, had discovered terrestrial nutation in 1727. Announcing this new phenomenon, he adds, 'there appears to have been a real change in the position of some of the fixed stars with respect to each other'. But he admits being unable to determine if these changes are due to the solar system's moving toward the stars or the stars moving toward the system, or both moving relative to each other. Deciding between these would be arduous: it requires 'the observations of many ages to determine the laws of the apparent changes even of a single star: much more difficult therefore must it be to settle the laws relating to all the most remarkable stars'. See J. Bradley, 'Letter to Dr. Edmund Halley giving an account of a new-discovered Motion of the Fixed Stars', *Philosophical Transactions*, 406 (1728), 637ff.; and 'A Letter to the Rt. Hon. George Earl of Macclesfield, concerning an apparent Motion observed in some of the Fixed Stars', in S. P. Rigaud (ed.), *Miscellaneous Works and Correspondence of James Bradley* (New York: Johnson Reprint Corp. 1972), 17ff. The first reliable estimates of the solar system's motion relative to the stars came from William Herschel, in the early 1800s.

[32] Jakob Hermann, a follower of Leibniz and protégé of Jakob Bernoulli, had worked with Wolff to defend the cause of Leibnizian *vis viva* in the early eighteenth century. He spells out motion as translation with respect to absolute space; see J. Hermann, *Phoronomia* (Amsterdam, 1716), 1. Kant had read Hermann's tract, as is plain from his youthful *Thoughts on the True Estimation of Living Forces*.

absolute places are insensible, true motion defined as change of absolute place would be undetectable: 'even if I imagined a mathematical space, empty of all creatures, this would not help me in the least. For how am I to distinguish its parts and the various places not occupied by anything corporeal?' (*GS* ii. 17). The objection is not new, but he thinks it sufficient to disqualify absolute space as an *explicans* of true motion. He closes on a dual note, a diagnosis of the flaw in the 'common concept' of motion and a hint at a solution:

> there is something lacking in the expressions 'motion' and 'rest'. I should never use them in an absolute sense, rather always respectively [*respective*]. I should never say that a body rests, without adding with respect to which things it is at rest; and should never say that it moves without also naming the objects with respect to which it changes its relation. (*GS* ii. 18)

So far, Kant's proposed remedy is ambiguous: he might be advocating either relativism, the denial of true motion, or relationism, the defense of true motion as relative to special bodies. He began his essay by refuting two definitions of true motion, which intimates he might be planning to endorse relativism. But he defies that expectation. Instead, he argues for relationism—his specific doctrine that any body has a true motion, *wahrhafte Bewegung*, relative to some special bodies.

Recall that a relationist must designate the material system—the body or bodies—relative to which a body truly moves or rests. In *ND*, Kant claims it is that object with which the body *interacts*. Though a body has countless relations to countless other ones, by interacting with another body it enters a special relation—special, as it results in *dynamical* effects. For Kant, a body's true motion is 'relative', i.e. *relational*, because it is defined by *this* relation. Relations to other bodies have no bearing on its true motion, for they are dynamically irrelevant: 'You will grant that, when we talk about the effect which the two bodies have on each other through impact, the relation to other external things has nothing to do with that' (*GS* ii. 18).[33] If two bodies interact by collision, Kant explains, they are in a relation of approach (*Annäherung*). He sees it as a 'mutual relation' in which each has an individual 'share' (*Anteil*). To quantify a body's share, and thereby its

[33] Kant's paradigm is collision in which all external forces are balanced, e.g. two billiard balls on a frictionless flat table.

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'motion relative to the other', Kant relies on considerations of symmetry and claims that both bodies interact with *equal* true motions:

> tell me if one can infer, from what happens between them, that one is at rest and only the second moves, and also which of them rests or moves. Must we not ascribe the motion to both, namely *in equal measure*? Their mutual approach may be attributed to the one just as much as to the other. (*GS* ii. 18)

At this stage in his analysis, by 'motion' Kant has come to mean quantity of motion, or momentum. Hence, in any two-body impact, the two colliding bodies have equal true momenta, and so their true velocities vary inversely with their masses.

The keen reader who saw Kant rejecting absolute space early in *ND* might think Kant engages with Newton's mechanics. In fact, he works out the foundations for a dynamics of impact along Wolffian lines. This is clear if we inspect what Kant does with his new concept of motion: he uses it to derive two dynamical laws *a priori*; then he takes those laws and shows that they yield the 'rules of motion', a Wolffian term of art for the kinematic rules of velocity exchanges in collisions.[34] Kant's interest in this post-Leibnizian agenda explains why rotation is *not* part of his early doctrine: like Wolff, Kant focuses exclusively on impact between bodies in *straight-line* motion.

To sum up: Kant's early theory of motion is a type of relationism, not relativism. It asserts that any colliding body has a true motion, which consists in the 'active' relation that the body has to the *other body* with which it collides. It is active because, as Kant claims with the Wolffians, in virtue of its true motion a body has 'moving force' whereby it acts, i.e. causes dynamical effects. Numerically, true motion is the body's share in that relation. And, this share has a kinematic

[34] Jean École first showed that Wolff started the project of using two *a priori* laws to derive the kinematic rules of two-body impact; cf. J. École, 'Un essai d'explication rationelle du monde, ou la *Cosmologia generalis* de Christian Wolff', *Giornale di Metafisica*, 18 (1963), 622–50. In a groundbreaking piece, Eric Watkins demonstrated that efforts to derive two *a priori* dynamical laws were a collective program of post-Leibnizian dynamics in Germany; see his 'The Laws of Motion from Newton to Kant' ['Laws of Motion'], *Perspectives on Science*, 5 (1997), 311–48. A development of Watkins' insight is M. Stan, 'Kant's Early Theory of Motion' ['Kant's Theory'], *The Leibniz Review*, 19 (2009), 29–60. Desmond Hogan also emphasizes the post-Leibnizian context of Kant's *New Doctrine*; see 'Kant's Copernican Turn and the Rationalist Tradition', in P. Guyer (ed.), *The Cambridge Companion to Kant's* Critique of Pure Reason (Cambridge: Cambridge University Press, 2010), 21–40.

aspect: a velocity. Therefore, any two colliding bodies have *true velocities*, defined with respect to the center-of-mass (CM) of their impact.[35]

An ambiguity mars Kant's early theory, worth signaling before we proceed, for it crops up again in his Critical years. It is this: Kant oscillates between two accounts of the relation that defines a body's true motion. One spells it out as a relation to the *other body*, with which the subject-body interacts. That relation is privileged, because its relata act on *each other*. The second explicates it as a relation to a *material frame*, in which the system's center-of-mass rests. The force of tradition drives him toward the second: for most early moderns (and Kant too) true motion denotes true velocity, which requires reference to a frame. Yet, by rejecting the 'common concept' of motion as *change of place*, Kant denies the very idea that motion consists in a relation to a frame. He sees his innovation precisely as replacing that idea with the thesis that true motion is a direct relation *between bodies*. Still, Kant seeks a theory of motion so as to ground mechanics and *that* requires a kinematic quantity, which compels him to single out a special frame of reference, just like the precursors he claims to leave behind. This tension in Kant's thought resurfaces in the 1780s to cloud the lucidity of his views on rotation. Another blemish is the lack of a full account of frames of reference, especially of the distinguished frame relative to which colliding bodies have true velocities, as he believes. He looks at direct impact only, in which the bodies' true velocities are on the line between them. Then one point on that line—namely, their common center-of-mass—is enough for his analysis. But, an account of frames needs more than a privileged point; frames also have axes, or 'substantial lines', as some theorists call them.[36] And a way to identify *them* is essential when the focus of theory-grounding moves beyond direct particle collision: e.g. in orbital dynamics, where the true motions are *not* along the line between the bodies but along the tangent to the orbit; or in the mechanics of extended bodies, where the forces acting on them (and the resulting motions they induce) must be resolved into

[35] Daniel Warren appears to read Kant's early theory of motion largely along the same lines as I proposed above; cf. his 'Kant's Dynamics', in E. Watkins (ed.), *Kant and the Sciences* (Oxford: Oxford University Press, 2001), 93–116, at 111, n. 7.

[36] C. Schaefer, *Einführung in die theoretische Physik*, vol. 1 (Berlin: de Gruyter, 1929), 14.

components along three orthogonal axes that must be specified by reference to matter, if one rejects Newton's absolute space.

## 3. TRUE MOTION IN THE CRITICAL DECADE

### *3.1 Absolute Space and True Motion*

Kant's awakening from dogmatic slumber affects his philosophy of physics, yet he also retains a striking allegiance to key pre-Critical ideas, as will appear in what follows. In the 1780s, his theory sees two salient changes, one concerning absolute space, the other the nature of true motion.[37]

We saw Kant in 1758 dismiss out of hand *Newton*'s idea of absolute space. In 1768, having discovered incongruent counterparts, Kant asserted that they prove after all that Newtonian absolute space is real: subsisting 'independently of the existence of all matter'.[38] Yet that conversion was short-lived, for by 1770, Kant had found his own, idealist view of space. And not even during those two short years did Kant use absolute space to do what Newton had done, viz. to define true motion. By the time of *MAN*, absolute space had returned, under a Kantian guise that sets it apart from Newton's. To grasp it, we must first inspect another concept, namely Kant's 'relative space', likewise distinct from Newton's analogous idea. A Kantian relative space is a volume that is *finite*, marked off by sensible *matter*, and *movable* (*GS* iv. 481). Certain bodies move, or can move, 'inside' it; and a relative space too can move 'inside' other, larger relative spaces; these, in turn, can move 'inside' yet larger ones; and so on. For any relative space **A**, Kant explains, we may *suppose* an arbitrarily large relative space **B** enclosing **A**. Now *abstract* from, viz. ignore any specifics about, the matter of **B**; and *assume* **B** to be immobile, though it cannot really be so. We need these two idealizations so as to study mathematically the motion of

[37] An excellent account of the general concept of motion in Kant's Critical years is K. Pollok, 'Kant's Critical Concepts of Motion', *Journal of the History of Philosophy*, 44 (2006), 559–75.

[38] I. Kant, 'Concerning the Ultimate Ground of the Differentiation of Directions in Space' ['Directions in Space'], in D. Walford and R. Meerbote (ed. and trans.), *Theoretical Philosophy, 1755–1770* (Cambridge: Cambridge University Press, 1992), 361–72, at 366; *GS* ii. 378.

bodies 'inside' **A**, of **A** 'inside' **B**, as well as of any other enclosed relative space **C**, **D**, **E**, etc. 'inside' **B**. Thereby, we have turned **B** into a *Kantian* absolute space. This, I submit, is the sense of his claim that absolute space

> denotes every other *relative* space that I can always *think* for myself outside the given relative space, and extend to infinity beyond the given one. That is, I can *think* it as enclosing the latter, and can *assume* the latter to move inside it. This enlarged space is always a *material* one. Still, as I have it only *in thought*, and know nothing about the matter that designates it, I thus abstract from this matter. Consequently, this space is *represented* as a pure, non-empirical, and absolute space. I can compare every empirical space with it, and represent the empirical as movable inside it. Hence it counts as itself immobile. (*GS* iv. 481f.; italics added)

Unlike Newton, Kant denies that his absolute space is a singular concept. This is evident from his admonition against thinking of absolute space as a *single*, *global*, *truly stationary* relative space enclosing *all other* relative spaces:

> By turning [absolute space] into an actual thing, one mistakes the *logical universality* of any space with which I can compare all empirical spaces (enclosed in it) with a *physical universality* of real extent—and thereby one misunderstands reason in its idea. (*GS* iv. 481f.; original emphasis)

To paraphrase, absolute space is universal in this sense: for *any* relative space **A**, one may suppose an arbitrarily large relative space **B**, assume it to be at rest, and use it to describe kinematically the motion of **A** and of *any other* relative space possibly enclosed in **B**. It is *not* universal in the sense of an actually existing relative space **B** larger than all other actual relative spaces.

As his absolute space does not denote a single entity, Kant calls it 'an idea of reason'. I shall not try to explicate his thought behind that term; we already have an interpretation of it, from Michael Friedman.[39] Rather, I will just examine Kant's use of it in one special case—to analyze rotation. Circular motion, it will appear soon, is a stumbling block for interpreters who seek to equate Kant's absolute space and the

[39] Cf. M. Friedman, *Kant's Construction of Nature* [*Kant's Construction*] (Cambridge: Cambridge University Press, 2013).

concept of inertial frame. As a result, even Friedman's construal, elegant as it is otherwise, must be retuned to accommodate this fact.

The other change in doctrine follows from Kant's move to transcendental idealism. Now he censures peers, precursors, and his old self for the error of conceiving true motion as transcendentally real, or 'lying beyond' apparent motion—such that the latter occludes it, as it were, and needs to be filtered out and overcome. Thus, pre-Critical thinkers saw it as their task to find criteria for telling whether a motion is true as opposed to a mere 'illusion' of motion.[40] This project chased a property—true velocity—of things in themselves, exposed since 1781 as unknowable. Instead, Kant says now, we must accept that knowledge of motion *indispensably* starts with apparent motions and seek to find out what is *objective* about *them*: what velocities we may justifiably ascribe to things as perceptual objects. So, a philosophical theory of motion ought to 'specify the conditions whereby the object (the matter) must be determined in one way or another through the predicate of motion'. For Kant, this entails a triple agenda: (i) to defend that all true motion is relative to *matter*; (ii) to explain the role of absolute space, given that motion is *only* relative to matter; and (iii) to argue that all ascriptions of true motion presuppose his modal *categories*: possibility, actuality, and necessity. Thus arises Kantian Phenomenology, the philosophy-guided activity of turning perceptions of apparent motion, or 'phenomena', into theory-laden justified claims about the true motions of bodies, or 'experience'.[41] I will move presently to his account of (ii) and (iii).

As to item (i), Kant thinks his case for it is clear. All judgment about true motion must start with apparent motions and only matter moving relative to matter can be perceived. But *Newtonian* space is not 'given in the appearance', so motion in *it* is unknowable: that space 'can neither be perceived in itself nor in its consequences, viz. motion in absolute space'.[42] The conclusion is premature, however. True, we cannot

[40] *MAN*, *GS* iv. 555. Next quotation, ibid.

[41] For an account of how Kant's Phenomenology fits in with his larger argument in *MAN*, see E. Watkins, 'The Argumentative Structure of Kant's *Metaphysical Foundations of Natural Science*', *Journal of the History of Philosophy*, 36 (1998), 567–93.

[42] *MAN*, *GS* iv. 481. See also ibid., 559: 'For motion to be given even as just an appearance, an empirical representation of space is required, relative to which the movable may change its relation. But the space that has to be perceived must be material.'

perceive absolute space; Newton granted it too. But, it is not yet shown that we cannot experience *motion in absolute space*. Newton's point was precisely that true centrifugal effects (and inertial accelerations in general) are consequences of motion in absolute space. To block Newton's conclusion, Kant must show that he has an account of true rotation (and the centrifugal effects it engenders) that does not need absolute space to support it.[43] Only then can he afford to claim, 'absolute space is nothing, and no object at all', hence that *all* motion is relative to matter.

### *3.2 Rotation, Absolute Space, and the Categories*

Though he says that all motion is relative to matter, Kant also insists that '*all* motion and rest must be *reduced to absolute space*' so as to become knowledge (*GS* iv. 560, my italics). By that reduction, he means a two-stage activity. First, any given apparent motion is to be assigned to its proper kind in a taxonomy that Kant provides. In it, the number of kinds is fixed by his categories of modality: true motions are either possible or actual or necessary—so his epistemology proves indispensable for a theory of motion rightly conceived. The criteria for belonging to a definite class are *dynamical*: the presence or lack of 'moving forces', which some rashly call 'Newtonian'.[44] Second, once the apparent motion has been categorized, it must be re-described so as to yield a 'determinate concept of experience', i.e. knowledge of a definite true velocity, *if* there is one. The direction and quantity of a

[43] Earman, *World Enough*, 77, makes the same point, it seems to me.

[44] E.g., Carrier, 'Kant', 404. Said of forces, 'Newtonian' is equivocal. It may mean the kind that Newton treats in Book One of the *Principia*, viz. one class of *impressed* forces: centripetal force acting on a *particle*. Or it may denote impressed force in general, namely Second-Law forces acting on any volume element of matter (not just on particles or the center-of-mass of extended bodies, as in the *Principia*) and compatible with the Third Law, i.e. always accompanied by an (equal and opposite) reaction force on the *source* of the action. The 'moving forces' of Kant's official theory of motion are neither. First, his forces are not impressed, but *inherent* in bodies as they move; and are proportional to their true *velocities*, not accelerations, as Newtonian mechanics claims. Second, the 'centrifugal forces' in Kant's account of rotation are *not* Newtonian interactions, as the Third Law dictates; they arise only in rotating frames. This is a momentous fact, and I discuss its relevance at some length in §3.3. Daniel Warren perceptively notes that the 'moving forces' of Kant's theory of collision—i.e. the forces which bodies have simply by having 'necessary' motions, in his Phenomenology—are not Newtonian; cf. Warren, 'Kant's Dynamics', 111, n. 7.

body's true velocity correlates with the direction and strength of its 'moving force', *just in case* it has one. (For instance, a non-interacting body in uniform translation has no such 'moving force', hence no true velocity.) Often, this kinematic re-description will mandate us to refer a body's true velocity to a 'space' *other* than the relative space 'inside' which the apparent motion was initially given, or perceived. This other space is a Kantian absolute space, viz. an arbitrarily large volume we may *assume* to be stationary. The re-description completes the 'reduction to absolute space'.

In this official account, true circular motion—with rotation, or spin, as a species—belongs in a class of its own, the group of actual motions.[45] Rotation is *actual*, because we ascertain its presence by perception, to which true circular motion 'is appended in accordance with empirical laws', as the Analytic of Principles teaches.[46] An example of such perception is the pull on our hand we feel as we twirl around a stone tied to a string; or the tension we would feel if we pinched the cord between Newton's globes. And, rotation is *true* motion, because a body truly in circular motion 'has moving force by means of its motion', as Kant's *a priori* mechanics decrees. From this, he proves Theorem II of the Phenomenology, namely that:

> The circular motion of matter, as distinguished from the opposite motion of the space, is an *actual* predicate of it. But the opposite motion of a relative space, taken instead of the body's motion, is not an actual motion of the latter. Rather, if taken to be such, it is mere illusion. (*GS* iv. 536)

With this result, Kant aims to justify our practice of judging that there exists an objective distinction between *merely apparent* and *true* circular motion—for example, when we say that the stars merely appear to move around us but do not truly do so.

The proof is not perfect, as a commentator has noted.[47] In addition, an obscurity weakens the theorem greatly. Based on it, Kant claims, we

[45] *MAN*, *GS* iv. 557f. Strictly speaking, any circular motion (of an extended body) *is* a rotation, around some axis. Here, I use 'rotation' and 'spin' *stricto sensu* for the case in which the instantaneous axis of rotation is internal to the body. Kant's paradigm is one-body spin around an axis of geometric symmetry.

[46] B 274, in I. Kant, *Critique of Pure Reason*, ed. and trans. P. Guyer and A. Wood (Cambridge: Cambridge University Press, 1998), 326.

[47] See K. Pollok, *Kants* Metaphysische Anfangsgründe der Naturwissenschaft. *Ein kritischer Kommentar* [*Kommentar*] (Hamburg: Felix Meiner, 2001), 484. Pollok objects, correctly, that

are right to assert that—if the proper dynamical criteria obtain—circular motion is a predicate of the body, not of the relative space. But it is unclear which relative space Kant has in mind. On the one hand, if he means the space relative to which a circular motion is 'given in the appearance', the theorem proves too much: often that space itself truly rotates around some axis. (Consider a merry-go-round that an observer at rest on Earth ascertains to spin relative to her.) So, it is *wrong* to claim that only the body moves truly but not the space, as his theorem urges. On the other hand, if he means that true circular motion is relative to a space *other* than the one in which an observer perceives it, the theorem says too little: Kant's relative spaces are all marked off by matter, so his theorem is incomplete without an account of the matter designating those spaces relative to which circular motion is true motion. Because this is the more charitable construal, I endorse it here and hope that his 'reduction to absolute space' will fill the gap in this reading. At any rate, it must be clear that, for reasons internal to Kant's doctrine, the space relative to which *all* rotation is true motion *cannot* be the 'fixed stars'.[48]

In his Theorem II, Kant explicates true circular motion as 'relative', i.e. *relational*: it consists in a 'change of external relations in space' (*GS* iv. 558). He means them as kinematic relations, but he is ambiguous as to what exactly they amount to. The phrase 'change of external relations in space' implies a change of relative distance or

for Kant's proof to be complete, he ought to have also established that the 'space' relative to which the body is in true circular motion does not *dynamically affect* the body's motion; and that Kant merely asserts, without proof, that the motion of the relative space is 'mere illusion', not true motion.

[48] By that, I mean a global frame of reference defined by any four stars (not all in the same plane). Three reasons militate against crediting Kant with the view that all circular motion is true motion relative to the stars (as Berkeley seems to have claimed). First, already in the 1750s he knows from Bradley that the stars have proper motions, i.e. there are (slow) changes in the relative distances between them; see §2. Second, Kant's own cosmology entails that the stars *cannot* really be at rest: his *Allgemeine Naturgeschichte* teaches that all celestial bodies have formed from nebulae which slowly acquired angular momentum over time, hence rotate truly. Third, in his 'Metaphysical Foundations of Phoronomy', Kant asserts confidently that we 'are unable to assign, in any experience at all, any fixed point relative to which we might determine what motion and rest are to be absolutely. For everything given to us this way is material, hence movable, and so—as we are not acquainted with any outermost limit of possible experience in space—is perhaps in actual motion too, without our being able to perceive this motion' (*GS* iv. 488). Clearly, this inability to establish true rest would extend to the stars too, on his view.

position. So, for a body moving on a curve, true circular motion would denote its true velocity along the *tangent* to the curve. But, as he proves in his Theorem II, Kant spells out circular motion as a 'change of a change in these external relations in space', and a 'continual arising of new motions' (ibid., 557). This entails that the quantity of true circular motion is the orbiting body's true acceleration, directed *toward* the force center keeping the body in orbit. Later in the Phenomenology, Kant compounds this ambiguity, by adopting a third view, in which true spin is a change of relations directed *away* from the axis of rotation (as we will see in §3.3). At any rate, oscillating between three accounts is not Kant's real problem here. The greater difficulty is this: it is not enough for Kant to claim that rotating matter has an instantaneous endeavor to move along the tangent to the trajectory. He must *ground* that claim in his relationist foundation. That is, he must give an account of that tangent physical line *without* relying on Newton's absolute space—by *naming the underlying matter* that distinguishes the tangent line from *all other* lines passing through the point of tangency.

However, Kant's official definition given earlier—true circular motion as a change in relation between parts of matter—quickly faces a problem: Newton's globes move truly, but *not in Kant's* official sense of 'changing external relations in space'. For one, Kant takes the globes to spin rigidly, such that the cord between them—and so their kinematic relation to one another—does *not* change. For another, the globes do not change any kinematic relations to some material system outside them either: *ex hypothesi*, no such system is present as they rotate in empty space. Thus, the mere possibility of Newtonian globes spinning rigidly causes a paradox:

> Thus, a motion—which is a change of external relations in space—can be given empirically, even though this space is not itself given, and is no object of experience. This is a paradox deserving to be solved. (*GS* iv. 558)

There are really two sources of paradox here. One is epistemological. In Kant's Phenomenology, true motion is *always known from apparent* motion: one has to start with motion 'given in the appearance', then apply categories of modality and dynamical criteria, so as to decide what true motion, if any, is present there. But Newton's globes setup defies that key thesis: we know that the two globes truly rotate, yet—

paradoxically—'nothing external and sensible exists there with which the globes could be compared', in Newton's words. Another source is metaphysical, having to do with the *nature* of true motion as Kant defines it. Rotation is a species of circular motion, which *consists in* a 'change of external relations in space', his Theorem II proclaims. But, in Newton's setup, the external space is *not* given in the appearance, thus it 'is no object of experience', Kant freely admits. It follows that, *contra* Kant, rigid rotation *cannot consist in* a change of relations to an external space, as the Phenomenology asserts it should. That is the paradox. How does Kant solve it?

### *3.3 Rotation as 'Relative Motion'*

On my reading, Kant solves it in the chapter's final section, entitled 'General Note on Phenomenology'. His solution is in three stages. First, he offers an analogue to Newton's globes, and shows the reader how true rotation *in empty space* could be detected. Second, he denies that the rotation thus detected *consists in* motion relative to *Newtonian* absolute space. Third, Kant gives a relationist analysis of that rotation to establish that rotation is *relative to matter* after all, so it does not need Newton's absolute space to explain it. Let us now examine these stages more closely.

Kant's first move is to replicate the globes setup in the *Principia* under even stricter constraints: instead of a *two*-body system, as Newton employed, Kant offers *just one* body doing precisely what the globes do: spin rigidly around the center-of-mass. That body is a *fictive* Earth, a hypothetical body idealized to match Newton's scenario in all *relevant* respects: Kant assumes his 'Earth' to be arbitrarily far away from the stars; rigid, not deformable; and devoid of perturbing factors, e.g. winds or air currents. Though tacit, these three restrictions are needed to make it a proper analogue of Newton's globes, which do have the three features noted earlier. They also make Kant's 'Earth' *unlike* the real Earth on which we live. But why resort to a contrived example, we may ask—why not address directly Newton's globes, the source of his woes? I think Kant flaunts his fictive 'Earth' because it allows him a *tour de force* to parade the merits of his Phenomenology—by showing that he can take a setup even more challenging than Newton's globes and *still* account for it in terms of his Critical theory of motion. There is solid textual evidence that Kant takes his Earth example to be an analogue to Newton's globes. For example, in the Note to Theorem II, he brings up the globes setup in *Principia*, and says, 'There we are taught that the circular motion of two bodies around a common center (hence *also the axial rotation of the Earth*) can be known even in empty space' (*GS* iv. 558). Then, as he discusses the nature of rotation, Kant says, 'We can *imagine*, *for instance*, the Earth as rotating on its axis *in infinite empty space*, and display this motion also in experience' (ibid., 561). Lastly, in a footnote to the same thought, Kant refers again to Newton's skillful use of centrifugal effects to 'show how the reality of [the globes'] motion *in empty space*, together with its direction, could still be found in experience'. And he adds, 'I have endeavored to show *the same thing*, under somewhat different circumstances, for the case of the Earth rotating on its axis' (ibid., 562, my italics). Thus, Kant strongly indicates that, in his Earth scenario, he seeks to replicate Newton's experiment with the two globes.

With his fictive Earth in place, Kant next teaches how we could detect whether it spins truly or is at true rotational rest. He offers two thought experiments, structurally very similar. In one, an observer on the 'Earth' projects *vertically upward* a stone initially at rest.[49] As the stone rises up against gravity and then falls back freely, it is deflected *westward* relative to the observer on the 'Earth' rotating under the stone. In the second thought experiment, the same stone is dropped from rest in a deep symmetrical tunnel whose axis coincides with the local radius of the 'Earth'; the stone will veer *eastward*. Both *Gedankenexperimente* rely on the same prediction, viz. the deflection of a body—the stone, in his examples—as it falls freely *in vacuo* near the surface of an 'Earth' rotating 'on its axis from west to east'. If the 'Earth' did *not* rotate, no deflection would be seen. Assume we *do* detect a deflection. Then we may rightfully 'conclude' (*schliessen*) that the 'Earth' *truly* spins eastward

[49] In the *Akademie* edition of Kant's *MAN*, the editor Alois Höfler reads Kant's two experiments as structurally identical, i.e. as both involving a stone being *dropped* from rest (above and below ground, respectively). See *GS* iv. 648. Recently, K. Pollok denounced this as an editorial error and corrected Höfler's reading. Pollok contends that Kant's first test requires that the stone be projected upwards, unlike its counterpart in the second test. Cf. Pollok, *Kommentar*, 494. I thank Michael Friedman and an anonymous referee for pressing me on this point.

(though it appears at rest); and 'both perceptions [of deflection] will be sufficient to prove the reality of this motion' (*GS* iv. 561f.)

Now Kant moves to the second stage. Recall that, from the tension in the cord, Newton computed the globes' centrifugal endeavor to recede, which he took to be proportional to their *motion in Newtonian absolute space*. Kant asserts that such dynamical effects—the tension in Newton's cord, the deflection of a falling stone in his analogues—prove *solely* that true, or actual (*wirklich*), rotation is present. But that true rotation *is not*, or does *not consist in*, motion in Newton's absolute space, he adds emphatically:

> here, it is about the true (real) *motion*, which however does not *appear* as such. Hence, it could be taken for *rest*, if we wanted to judge merely based on empirical relations to space. That is, it is about the *true motion* as distinguished from *illusion* [*Schein*]—not about it as absolute motion in contrast to relative. (*GS* iv. 561)

Kant moves resolutely to block Newton's account of the globes' true spin, because it flatly contradicts a key tenet of his Phenomenology: that 'matter can only be thought to move or rest *relative to matter*, but never with respect to mere space devoid of matter' (*GS* iv. 559, my italics). Still, Kant does not just state baldly that the globes, and his 'Earth', are in 'relative motion'. Rather, he tries to prove it—by giving a *relationist* analysis of the 'Earth's' spin, as he should. That analysis is Kant's answer to Newton, and his solution to the 'paradox' of spin. The solution, as the third and last stage of the reduction to (Kantian) absolute space, completes his account of rotation.

Kant presages this final stage with a hint of his full solution. Prior to introducing his 'Earth' scenarios, he announces that circular motion 'exhibits a continual dynamical change of relations of matter *in its space*' (*GS* iv. 561). He lets the reader know he will analyze true rotation as a special relation between parts of the rotating body, or system. The full analysis is a long, intricate passage:

> But this motion, though it is *no change of relation to empirical space*, is however no absolute motion. Rather, it is a *continual change of the relations of parts of matter to one another*, though represented in absolute space. Hence really it is *only relative*, and, precisely because of that, true motion. This claim rests on the representation of the mutual, continual *withdrawal* of each point of the Earth (outside its axis) from the part lying opposite it *across the diameter*, equidistant from the

> center. This motion is real in absolute space, because through it, the diminution of the distance we may think between the parts—which gravity, acting alone, would induce in the body—is constantly compensated [*ersetzt*]. This would happen without any dynamical repulsive cause, as we can see from Newton's example in the *Principia* (1714), p. 14. Hence the decrease in distance is compensated through real motion—except that this motion is referred to the space contained inside the moved matter (i.e. *its center*), and not to the external space. (*GS* iv. 561–2; italics added)

I move now to elucidate his cryptic solution: how is rotation 'relative'? In what sense is it motion? And what is the 'space' to which it must be referred?

Rotation is relative in the sense of *relational*: true rotation consists in 'a continual change in the relations of parts of matter to one another'. The parts between which it is a relation are any two 'opposite parts'. By that, Kant means any two portions of matter lying in the same plane, perpendicular to the axis of rotation, *on the straight line intersecting the axis*. Such opposite parts could be either two bodies (in a spinning rigid system) or two finite parts (*Theile*) of a single spinning body.[50] It is a relation between *these* parts because they are 'dynamically related' to each other *in a new way*, if the body (or system) rotates truly. If the body is at rotational rest, its 'opposite parts' are related merely by the forces underlying the constraints: gravity, cohesion, etc. But if it rotates, *new* forces arise to relate any two 'opposite parts' in it. They are the 'centrifugal forces' inherent in matter orbiting around some center. In virtue of such forces, truly rotating matter acquires an 'endeavor to recede' from the axis of rotation. Kant takes this endeavor to be radial, or directed *straight away* from the axis, *not* tangential. It follows that, for any two 'opposite parts' of equal mass in a rotating body, their centrifugal forces are equal and *contrary*. The equality and *co-linearity* of these forces, Kant thinks, justifies him in defining true rotation as a relation between the 'parts of matter' in which they inhere.

This might explain why rotation is relational, but what makes it *motion*? In what sense do the parts of a (rigidly) spinning body move, for Kant? Based on his Theorem II, we would expect him to explicate

[50] Thus, in his own example it is not really his 'Earth' that rotates. Kant's analysis entails that, properly speaking, what rotates is the 'opposite parts' relative to one another; and, in Newton's setup, the globes rotate relative to each other.

rigid rotation as some type of *actual* kinematic change in respect to a privileged 'relative space' external to the body. This is all the more needed as Theorem II had omitted to locate that relative space by means of material markers, which his Phenomenology demands.

But he does not take that route. Instead, he makes an abrupt *volte-face* and says that rotation is *not* motion relative to an external space; his theory of motion shifts gears abruptly, so as to cope with Newton's challenge. Rotation, Kant implies now, is *latent* motion: in a body rotating truly—e.g. his 'Earth'—any two 'opposite parts' *would* move relative to *each other*, were it not for the rigid constraint.[51] Dissolve the constraint and the parts' latent motion would turn into *actual* change of relative distance between them. Or, turn off mentally the forces responsible for the constraint and then we could perceive the parts' relative motion as an actual increase of their relative distance—the predictable outcome of removing the constraints.

Now, to describe this kind of motion Kant does not use the term 'latent' explicitly. The term is my coinage, as is Carrier's 'virtual motion'. But, I submit that it captures Kant's thought accurately. For one, he knows that, having analyzed rotation as motion of the parts relative to each other, he cannot say that they move so *actually*. When a rigid body spins, its parts do not change relative distance. So, they are *at actual rest* relative to each other, as Kant openly acknowledges: rotational motion 'could be taken for *rest*, if we wanted to judge merely based on empirical relations to space' (*GS* iv. 561). It would be sheer incoherence for Kant to say that rotation is actual motion kinematically *identical* to actual rest. He avoids self-contradiction by meaning rotation as a latent type of motion. Secondly, Kant shows that he takes the rotating parts to be in latent motion by switching to the subjunctive mood: the parts truly move because they *would* in fact recede from each other, provided the constraints *were* removed. Their mutual receding 'would happen without any dynamical cause', or essential force of matter. That is to say, the parts would recede because of their latent motion away from each other, not because of any mutual forces of

[51] Martin Carrier, with whose reading mine overlaps on this issue, calls it 'virtual motion'—Carrier, 'Kant', 410f. I chose to call it 'latent' to avoid confusion: in classical dynamics, 'virtual' is a technical term—e.g. as in the Principle of Virtual Velocities—whose meaning does not coincide with Carrier's 'virtual' motion.

repulsion that any two chunks of matter *essentially* exert on each other. This, I submit, is what he means when he claims his 'Earth' has a true motion that we should represent—i.e. think—as 'the mutual, continual *withdrawal* of each part of the Earth (outside its axis) from the part lying opposite it across the diameter, equidistant from the center' (*GS* iv. 561). Though not actual, this relative withdrawal *would* occur, as Kant makes clear by using the subjunctive. And it would occur *only* if the body truly rotates. As proof, switch off mentally the 'essential-dynamical' forces between parts in a non-rotating body: *no* relative motion between them would ensue. To Kant, proving results by counterfactually turning off forces is a familiar device. He makes key use of it in his *Dynamics*, in the Balance Argument for the existence of fundamental forces, and later, in the *Opus postumum*: 'If the attraction of the internal cohesion in matter were suddenly to cease completely, matter would extend itself infinitely, and, if repulsion ceased, matter would coalesce into one point.'[52]

But what is the *direction* of that relative motion of the parts in a rotating body? In Kant's analysis, it is *radially* away from each other, i.e. along the straight line on which the two 'opposite parts' and their center of rotation lie. This emerges from his laconic claim that the parts' rotation consists in their 'mutual, continual *withdrawal* of each part . . . from the part opposite it', and that 'this motion is referred to the space inside the moved matter, i.e. *its center*' (*GS* iv. 561–2). So, any two truly rotating 'opposite parts' have a latent motion relative to each other, along the line between them, hence *normal* to their individual trajectories.

Before moving on, let us pause to clear up an ambiguity and note an absolutely crucial fact. For Kant, the true motion of a rotating body (or system) is directed *radially* away from the axis of rotation, because it generates the 'centrifugal forces' whereby the rotating parts strive to pull away from each other. However, the phrase 'centrifugal force' is ambiguous. In one sense, it may denote a *Newtonian* force: the Second-

[52] See Kant, *Opus postumum*, ed. and trans. E. Förster with M. Rosen (Cambridge: Cambridge University Press, 1993), 20 (*GS* xx. 409). Sheldon Smith insightfully uncovers the argumentative role of Kant's device of mentally turning off forces, in 'Does Kant Have a Pre-Newtonian Picture of Force in the Balance Argument? An Account of How the Balance Argument Works' ['Balance Argument'], *Studies in History and Philosophy of Science*, 44 (2013), 470–80.

Law force that a body A in orbit, subject to a centripetal acceleration induced by a body B, exerts on *the body B*, which caused the impressed force on A. This 'centrifugal' force (improperly so called) is directed toward A, in an *inertial* frame. An example: the Sun's impressed force on the Earth is centripetal, as it makes the Earth 'seek' the center from which the force emanates; by the Third Law, the Earth exerts on the Sun an equal and opposite force, sometimes called 'centrifugal'. In another sense, it may denote the *Huygensian* force whereby a body in circular motion, observed in a *rotating* frame, strives to escape radially away from the axis of rotation.[53] Kant's idea of centrifugal force is identical with Huygens's, not Newton's. This is because Kant analyzes rotation as 'latent' motion between the parts along the line between them. *But, that line itself truly rotates, and so does the frame of which it is an axis.*[54]*And, a rotating frame is not Newtonian.* This seems to have escaped all previous commentators. Some paper over this key fact with appeals to 'Kant's embedding absolute space' or 'non-rotating CM-frames'.[55] Yet it is doubtful that there are any in his 'Earth' scenario. All frames definable there *by appeal to matter*, as Kant urges, are truly rotating, hence so is the Kantian 'absolute space' to which actual rotation must

[53] If I understand him correctly, D. Bertoloni Meli has argued that Newton used 'centrifugal force' in both senses, at different stages in his intellectual development; see his 'Inherent and Centrifugal Forces in Newton', *Archive for History of Exact Sciences*, 60 (2006), 319–35. The phrase 'centrifugal force' was still being used indiscriminately in both senses as late as the early 1900s; see L. M. Hoskins, 'Review of *Technical Mechanics* by E. R. Maurer', *Science*, 21 (1905), 302–6.

[54] That is to say, the line rotates over time with respect to Newtonian absolute space or the class of Galilean frames defined by material bodies (if there are any). The line is common to all the rotating frames that have it and the body's instantaneous axis of rotation as two of their three rigid axes.

[55] Carrier, 'Kant', 411, 414. It is unclear to me how exactly Carrier means those phrases. Kant introduces 'Kantian absolute space' in his Phoronomy, where it underwrites the 'geometric', or purely kinematic, modeling of (straight-line) motions. *Ipso loco*, he is explicit that, to have objective content, his brand of absolute space always refers to volumes identified by reference to *matter*: 'this enlarged [absolute] space is always a *material* one'. Yet in Kant's 'Earth' example, his rotating body is *ex hypothesi* the sole material entity present—surrounded on all sides by extension *ganz leer*, completely empty of matter. All there is to anchor Kant's absolute space in matter—thus turning it into a material frame—are the parts of the spinning 'Earth'. I fail to see how they could ground any inertial, non-rotating frame. Appealing to a point—the rotating body's center-of-mass—solves nothing: to identify a frame univocally, one needs *four* non-coplanar points; but any two points outside the axis of Kant's 'Earth' will be truly rotating, and so is the straight line on which they lie.

be 'reduced' in order for the 'appearance of motion' to be turned into a 'determinate concept of experience'.

To make my point vivid, imagine Huygens's observer in *De vi centrifuga*, except that the man sits at the center of the wheel, not on the circumference as Huygens had him. Let the man hold two globes of equal mass, one in each hand, hanging from ropes of equal length. Now apply two equal and opposite torques on the circumference of the wheel. By the time the wheel has reached a constant speed of rotation, the man feels the globes pulling away from him, stretching his arms out along the line of his shoulders, *normal* to the circumference. These equal and opposite pulls are the 'moving forces' arising in any two 'opposite parts' of a spinning body, according to Kant's analysis.[56] The forces are 'moving' because they're associated with a certain kind of relative motion. To illustrate, let the man let go of the globes simultaneously, as the wheel turns evenly. As soon as he lets go, he will see them fly away from him *and each other*. It is important to note that, in the first few instants after their release, the man will see the globes leave his hands *at right angles* to the circumference. (If he keeps watching, he could see them slowly *curve* sideways, along a cycloid.) The key fact here is that these pulls and motions will be apparent only to an observer *at rest on the rotating* wheel. This is in fundamental contrast with the experience of a second observer watching this process from outside the wheel, as he sits at rest in Newtonian absolute space. The second, 'Newtonian' observer will see the first man *rotate* with the wheel. And, he will see the globes upon their release fly off along the *tangent* to the circumference, then continue *uniformly in a straight line*. Kant ends up analyzing spin as a type of relative motion as it would appear to the first, 'Huygensian' observer.

Therefore, Kant's analysis entails that a body's true rotation consists in the 'latent' motion of its parts with respect to a *rotating* frame, which is *not* inertial. As to its *quantity*, the latent motion of an 'opposite part' equals its 'velocity in the first instant' after its counterfactual release from the rigid bond, i.e. the part's Huygens-acceleration in the rotating

[56] Cf. Christian Huygens, *De vi centrifuga* (1703) [*Vis centrifuga*], in *Oeuvres complètes de Christiaan Huygens*, vol. XVI (The Hague: M. Nijhoff, 1929), 255–301. The strength of these Huygens–Kant centrifugal forces is $mv^2/R$ each, where $m$ is the mass of the sphere, $v$ is its angular speed, and $R$ is the distance from the axis of rotation.

frame.[57] This key fact merits all the typographic emphasis I can give it: *according to Kant, truly rotating matter has an endeavor to recede, defined in respect to a rotating frame. That endeavor generates its Huygens-centrifugal 'moving' force. But, rotating frames and Huygens-centrifugal forces are in tension with Newton's mechanics: the frames are non-inertial and the forces do not obey Newton's Third Law.* This establishes my argument **R**, as I called it in the introduction.

Against this backdrop, Kant's 'reduction of circular motion to absolute space' is as follows. One starts with an appearance, viz. an instance of either apparent circular motion or apparent rest. One then tests to see if centrifugal effects obtain; if they do, one may infer that true circular motion is present. To describe it rigorously, one identifies pairs of 'opposite parts', the straight lines between them and the axis of rotation. One extends *in thought* the line between any two opposite parts, in an arbitrarily large *mental* 'grid' of straight lines, and *assumes* the said line to be stationary in this grid. One quantifies the parts' true motions as 'latent' velocities away from each other, along that line—presumably, by appeal to Huygens's theorems relating centrifugal force and instantaneous velocity. The 'reduction to absolute space' is now complete.

With the details of Kant's analysis in place, we can see how, in his attempt to make do without Newton's absolute space, he falls back on his early relationism, which he tries to extend to rotation. First, just as he did with straight-line motion in the 1750s, Kant explicates rotation as 'relative' because it consists in a relation between 'parts of matter' related to *each other* by means of 'moving forces'. Second, the true motion of rotating parts, for him, is commensurate with the quantity of the centrifugal *forces* they have in relation to each other—just as earlier, the true motions of colliding bodies were commensurate with their 'moving *forces* relative to one another'. Both types are 'moving forces' in the same sense: they are forces of motion, i.e. they arise because the bodies that have them truly move. Third, in both impact and spin, the parts of matter have true motions along the *line between them*, relative to a *point* on this line: the center-of-mass in the early doctrine, the center of rotation in the Critical account of circular motion.

[57] By 'Huygens-acceleration' I mean the part's acceleration radially away from the axis of rotation, just as Huygens quantifies it in *De vi centrifuga*.

These continuities are no accident. Kant also keeps his early account of direct impact—and offers it again, with minor changes, as the archetype of 'necessary motions', the third class of true motions in his Phenomenology. It was only natural for him to give a unitary account of motion, in collisions *and* rotations, as grounded in relations between interacting parts of matter.

## 4. ASSESSMENTS

### *4.1 The Cost of Victory and Kant's Newtonianism*

Kant's official theory of motion is a species of relationism, as he claims that bodies have true motions that consist in relations between the bodies themselves. As I explained in §1.2, Newton's real challenge to relationists was his five arguments from the properties, causes, and effects of true motion. Euler in 1748 had restated one of Newton's objections to relationism, the 'argument from causes'. In the essay on handedness, Kant shows that he is *au fait* with Euler's 1748 arguments that absolute space is needed to ground the Law of Inertia. Two of them, echoing Newton's assault on relationism, are virtually unanswerable; though aimed at the Wolffians, they hit at Kant's doctrine too.[58] He fails to respond or to explain his refusal to link a fresh acceptance of absolute space with his theory of motion. Instead, he accuses Euler of

[58] In *Réfléctions sur l'Espace et le Tems*, Euler charges that, if we define true rest and motion with respect to some special frame individuated by other *bodies*—as the 'Leibniz–Wolffian' school did—the Law of Inertia fails. Let a body be at rest in that frame; apply a force to the *frame*. On the relationist view, it follows that the *subject body* has been truly accelerated, even though *no* force was applied to *it*—contrary to the Law. To keep the body at true rest relative to the (now moving) frame, we must apply an *unbalanced* net force on the *body*, once again violating the Law. Therefore, Euler infers, 'the idea of place, such as the Mathematicians conceive it, cannot be explicated by a body's relation to other bodies, either near or remote'. This closely parallels Newton's 'argument from causes', which Euler now wields to refute Wolff's relationist account of true motion. His other argument points out that a body on which the net force is balanced *will generally fail* to move uniformly in a straight line (as the Law of Inertia predicts) if its motion is referred to a frame defined by other bodies. It is because most bodies, *including* those making up the privileged frame, are accelerated or rotating; relative to *them*, the trajectory of an inertially-moving body will be a *curve*, not a straight line, as the Law dictates. He concludes, 'from this it is plain that the identity of direction, an essential predicate in the general principles of motion, simply cannot be explicated by means of the relation, i.e. the order of coexisting bodies'. Cf. L. Euler, 'Réfléxions sur l'Espace et le Tems' ['Réfléxions'], *Histoire de l'Académie Royale des Sciences*, Année MDCCXLVIII (1750), 324–33.

ignoring 'the no less serious difficulties' we supposedly face if we represent the Law of Inertia '*in concreto*, using the concept of absolute space'.[59] Kant, therefore, ignores most of the critique, which leaves him vulnerable to Newton's objections from the properties and the causes of true motion.[60] Instead, Kant tackles *only* Newton's argument from the effects of true motion, i.e. the challenge of rotation. Even then, he ignores the real objection—the spinning bucket example—and takes rigid spin, as in the globes setup, to be the only difficulty. This establishes my argument **N**, as I called it in the introduction.

Then, given his selective view of rotation as the sole challenge for his relationism, recall from §1.2 that he must (1) specify the material frame relative to which a rotating system truly moves; (2) locate the correct frame, i.e. give a relationist account of the system's true velocity; (3) explain why we can detect true rotation even in the absence of external frames; (4) analyze spin as a type of relative motion, or matter moving relative to matter, as orthodox relationism has it.

Kant answers these challenges as follows. (1) If a system rotates, any two 'opposite parts' move truly, relative to each other. Their true motion is with respect to the 'space inside' the system: the frame defined by the instantaneous axis of rotation and the straight line between the respective opposite parts. (2) Any two opposite parts have *latent* true velocities away from each other, *along* the line between them. Here, Kant's doctrine diverges sharply from Newtonianism. In Newton's mechanics, those parts have true velocities tangent to their trajectories, hence *perpendicular* to the line between them. Kant seems unable or unwilling to account for this fact. To ground tangential velocities, he would need to locate *inertial* frames. Absent Newton's absolute space, only such frames can give objective meaning to the phrase 'instantaneous velocities on parallel lines in opposite directions'. However, Kant takes a different route: he analyzes circular motion in a

[59] Kant, 'Directions in Space', 366.

[60] To see why, consider a direct impact between two bodies A and B—Kant's paradigm of interaction in 1758, and his archetype of 'necessary motions' in the Phenomenology. In Kant's account, both A and B before impact have true velocities with respect to their common center-of-mass (CM). Now apply a force to *B*. The system's CM is displaced as a result. It follows that the (Kantian) true motion *of A* has been changed too, even though *no force* was applied to A itself. Kant's account of necessary motions runs afoul of the Law of Inertia and Newton's demand that a body's true motion can *only* be altered by forces applied to *it*.

*rotating, non-inertial* frame. As a result, he fails to vindicate a key Newtonian element.[61] (3) Since for Kant true rotation is *not* relative to frames 'outside' the spinning system, we can establish its presence even if such frames are absent. (4) Rigid spin is no *actual* kinematic change. Still, it is latent change: the parts of matter would move relative to each other, were they not constrained to keep a fixed distance.

But Kant's solution comes at a price. Tacitly, he changes his theory of motion, relaxing his old relationism in crucial ways, which strains his Newtonianism.

First, Kant ends up having to equivocate on the meaning of 'relative'. Of the three classes in his Phenomenology—possible, actual, and necessary motions—the last two are relative in the sense of *relational*: they consist in relations between portions of matter (rotating or colliding) having a unique, determinate, objective quantity of motion. But the first class, possible motion, is 'relative' as *relativistic*: single bodies in uniform translation have *no* true motion and *no* unique velocity. That is why, in judging about such bodies, *all* ascriptions of motion, including rest, 'are equally valid' of them, hence asserted 'through mere choice', or arbitrarily (*GS* iv. 556).[62] But this tenet is the core of relativism about motion, not relationism. They are contrary views, artificially conjoined by Kant's equivocal use of 'relative'. Remove

[61] This is crucial for assessing the relation of Kant's theory of motion to Newtonianism. It remains an indisputable fact that, with respect to an inertial frame (as Newton's absolute space is), the rotating 'opposite parts' of Kant's 'Earth' have instantaneous velocities in opposite directions along *parallel* lines, *tangent* to their respective trajectories, *perpendicular* to the line between the parts. That is just what Newton measures in his globes experiment. But Kant chooses not to account for those velocities. It is true that, in both Theorem II and the Earth passages, Kant mentions that a body in circular motion has an endeavor along the tangent. However, to just mention it is insufficient; in order to count as a Newtonian, Kant must *ground* it from his own resources. That is, he must give a relationist account of those tangent lines. More precisely, Kant must designate two material points that individuate each tangent, or serve to distinguish it from every other physical line passing through the point of tangency. The material components of the rotating system are *incapable* of securing that—for all the material lines definable from them alone likewise rotate in Newtonian absolute space. Only inertial frames anchored in matter external to the rotating system could ground those tangent lines. But, for reasons I have expounded at length, Kant does not identify such external frames. So, he fails to account for rotating matter's tangential velocity. Therefore, he is not a Newtonian. I thank an anonymous referee for pressing this point on me.

[62] Obviously, according to Theorems II and III of Kant's Phenomenology, it is *not* arbitrary which true velocities we ascribe to bodies falling under their legislation.

the ambiguity and Kant's 'all motion is relative' translates as 'some motions have a subject, some do not'.

Second, he is forced to change the meaning of 'motion'. The essence of relationism, his professed doctrine, is that all true motion is *actual* kinematic change—of linear or angular distance—between portions of matter over time: 'matter can only be thought to move or rest relative to matter, but never with respect to mere space devoid of matter' (*GS* iv. 559). But the challenge of Newton's globes forces Kant to give up that credo and accept that some true motion (e.g. rigid spin) is a merely *latent*, or 'virtual' kinematic change. So, he quietly admits that certain true motions 'could be taken for *rest*, if we wanted to judge merely based on empirical relations to space' (*GS* iv. 561). Though Kant is not explicit about it, his change in meaning is a major concession to Newton. Before Kant's Phenomenology, *all* relationists shared the central tenet that no body could be said to move truly unless 'the situation of other bodies, with respect to it, will be changed consequently'.[63] Kant, who had subscribed to orthodox relationism in his youth, gives up on the tenet above so as to solve the riddle of rotation.

Third, Kant keeps his old ambivalence about the sort of relation that motion is. His absolute space, he asserts, is needed so that 'inside' it 'all motion of what is material can count as merely relative to each other, [to wit,] as alternatively-mutual motion' (*GS* iv. 559f.). His meaning is that, against Kantian absolute space as a kinematic backdrop, our judging about true motion should take the form of one of three kinds of either/or claims: 'alternative', 'disjunctive', or 'distributive'. Each kind, he says, is a type of judgment about some motion being relative to *matter*. But the matter to which it is relative varies, as we see if we look closely. The first two kinds spell it out as a relation to a material *space*, or finite frame; yet the third kind (the distributive judgment) analyzes it as a relation to another *body*, not space. His pre-Critical indecision about whether motion in impact is a relation to a body or to a frame rears its head again. One may respond that his vacillation is just apparent, as sometimes Kant calls body 'material space', hence motion is always a relation to a material space.[64] But this answer is insufficient. Even for Kant, a body differs crucially from a

[63] As Leibniz declares to Clarke, in his Fifth Letter, §53; Alexander, *Correspondence*, 74.

[64] I thank an anonymous referee for pressing me on this issue.

material frame: a body's relation to another body is dynamically *active*, viz. it results in the exercise of (mutual) forces. But a relation to a frame is dynamically *inert*: it is just geometric-kinematic, since the frame cannot exert a force on the body.

Fourth, though Kant succeeds in solving the riddle of rotation, wrestling with it strains his theory of motion. Under the serene façade of his presentation, tensions lurk—because, to address Newton's challenge, Kant had to adopt some positions that conflict with the rest of his metaphysical foundations of physics. Remarkably, these new positions greatly dilute the Newtonianism of his Phenomenology. Thus, one source of tension is that now it is hard to tell how Kant would analyze planetary motion, a key case of true motion in *Principia*. Three possibilities suggest themselves. According to the first, based on his Theorem II he might explicate it as each body *singly* being in true orbital motion relative to an external material 'space'. But, as I noted earlier, Kant leaves open how to identify that space, or what material marks individuate it in nature. Presumably, on this account, the quantity of true motion would be orbital *velocity*. This is the account that Kant *ought* to give, based on his broad Newtonianism about planetary interactions. But then the account would conflict sharply with his official analysis of rotation. Moreover, Kant would face the challenge, explained earlier, of giving a purely relationist account of the tangent straight line along which an orbiting body has a (Newtonian) velocity. The second possibility is, he might argue that it is a case of 'necessary' motions, by analogy with his account of true motion in two-body impact. If so, his analogy entails that two orbiting bodies have equal and opposite motions *toward* each other. Therefore, the unique quantities of their true motions must be the bodies' *centripetal accelerations* in the *inertial* frame in which their common center-of-mass rests. A third possibility is that he could take his analysis of rotation as a guide and claim the true motions of mutually orbiting bodies to be along the line between them *away* from each other. Quantitatively, that would be their *Huygens-centrifugal* accelerations in the *rotating* frame defined by their axis of spin and their line of centers. Which path Kant would take makes a non-trivial difference; orbiting planets would have different actual trajectories, depending on his chosen account: a planet moving in an ellipse according to the second possibility describes a circle based on the third construal. Kant offers scant help to the interpreter struggling to clarify this key issue. Hence, painful hermeneutic choices must be made on Kant's behalf, and the overall shape of one's construal of his Phenomenology will differ accordingly.

Fifth, it turns out that Kant does not analyze rotation as motion relative to an *external* frame, as his Theorem II requires. Instead, he defines it as actual motion of the 'opposite parts' relative to each other. Presumably, Kant's real reason for this about-face is the need to account for Newton's globes, which could truly rotate even if *no* 'relative space' external to them actually existed. I offer this conjecture as an alternative explanation, because Kant's stated reason is baffling. As I have explained, the gist of his Theorem II is that true and merely apparent rotations (e.g. the Earth's real spin vs. the Earth *appearing* stationary relative to the stars, which *appear* to rotate around it) are objectively different. This difference rests on objective dynamical criteria—centrifugal and Coriolis effects—present in truly rotating systems yet absent from those that merely seem to rotate. Kant uses precisely these criteria to infer true rotation in his twin experiments with the stone. At that point, one naturally expects him to invoke his Theorem II and conclude, rightfully, that the experiments prove his 'Earth' to be in true rotation relative to an *external* 'relative space' that he has yet to specify. But, inexplicably, Kant claims that the spin of his 'Earth' is *not* the kind of true motion codified by Theorem II: 'Hence, [the Earth's] rotation should not be regarded as external-relative either.' Allegedly, it is because analyzing the Earth's spin as relative to an external space is 'entirely equivalent' to the contrary description, viz. the stars spinning around the Earth at rest. But that is precisely the claim that Theorem II was *meant to block*: these two analyses may be *kinematically* equivalent, but they are *dynamically* distinct, as provable by experiments. If Kant's justification above were correct, it would make his two experiments completely irrelevant! But, naturally he thinks they *are* relevant—and yet he does not reach for Theorem II to explain rotation. This raises the puzzling question of what work is left for that Theorem to do in Kant's Phenomenology: what class of true motions still fall under its scope?

Newton's treatment of circular motion in the Scholium was a stark warning about the high cost of tackling rotation without his absolute space. Kant must have thought the cost affordable. He sets out as a relationist trying to tame straight-line motion in impact. In 1786, as he expands his theory of motion, he makes the fateful decision to keep relationism. But that theory, in its original form, was not strong enough to grapple with rotation, so Kant stretches it as he needs: motion is now relativistic, now relational; now actual, now latent; now true velocity, now true acceleration; now in Galilean frames, now in rotating ones.

### *4.2 Hermeneutic Reappraisals*

It is time to reveal how my reconstruction compares with or relates to previous construals. My reading of Kant's view of rotation differs markedly from that of Earman. Earman expects Kant to be a consistent relationist, and so he hopes to find an analysis of rotation as *actual* kinematic change relative to an *inertial* frame. Thus Earman construes, on Kant's behalf, rigid spin as actual motion of the body relative to the rest frame of the ether suffusing the body yet not dragged along by it as it spins.[65] But Kant's account does not conform to Earman's expectations. Not only is Kant clear that the body spins in a space 'completely empty', he also spells out rotation as *latent* motion in a *non-inertial* frame. Earman's construal would let Kant stay faithful to orthodox, univocal relationism about motion, always a valuable feature in an interpretation. Unfortunately, his construal has no direct textual support, being entirely conjectural.

Martin Carrier does see the peculiar trait that, for Kant, rotation is latent motion, or 'virtual' (as he prefers to call it). I fully endorse that part of his reading. Yet he does not pause to note that Kant's 'virtual' motions are with respect to rotating frames. That makes a critical difference to the rest of Carrier's construal. Following Friedman (whom I discuss next), he submits that Kant's absolute space underpins a procedure of 'successive embedding of reference frames'.[66] This supposed embedding aims to get us to the unique quantity that expresses the Kantian true motion of any body: its true velocity, or true rate of *actual* change (of position) relative to a unique *inertial* frame. But Kant does not measure true spin by that sort of quantity. Consequently, Carrier's alleged successive embedding must grind to a halt when it gets to rotating bodies.

[65] Earman, *World Enough*, 76–8. [66] Carrier, 'Kant', 403–4.

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Moreover, Carrier sees correctly that Kant's Critical view of rotation shares striking (though accidental) similarities with Huygens's analysis of rotation as 'relative'. But three blind spots mar Carrier's insight. Huygens has *two* accounts of rotation, not one—a vivid reminder of how difficult circular motion was for Newton's opponents. On the first, rotation is 'relative' because it consists in change of direction (by any part of a spinning system) with respect to what Huygens calls 'bodies relatively at rest', i.e. a class of frames defined by sets of bodies which are kinematically unconstrained and keep the same relative distance over time. Such frames will be themselves non-rotating, therefore inertial, hence compatible with Newton's Corollaries V and VI. This seems to be Huygens's *official* analysis of rotation. On the second account, less represented in Huygens' *Nachlass*, rotation is 'relative' much like Carrier reads it, viz. 'virtual' motion of the parts relative *to each other*, not to Huygensian 'bodies relatively at rest'. Moreover, Carrier also remarks that by 'true motion' Kant means true velocity in an inertial frame, just like all early moderns except Huygens.[67] But this is correct only with respect to Kant's account of necessary motion (in collisions). As we saw earlier, for Kant, the quantity of true motion in rotations is not true velocity. Lastly, a defect of Carrier's parallel between Kant and Huygens is that the Dutchman means motion to be 'relative' in the sense of *relativistic*: for him, all motion is relational, but bodies have no true motions. This sets him worlds apart from Kant, who *does* hold that true motion exists. Regrettably, an extended discussion of these fascinating but difficult topics is beyond what I can afford in this chapter.[68]

Michael Friedman has given the leading account of Kant's Phenomenology in the last two decades.[69] He notes the rejection of Newton's

[67] See Carrier, 'Kant', 411f.

[68] For Huygens's later views on motion and relativity, see his fragments known as the *Codex Huygens 7A*, superbly in G. Mormino (ed.), *Penetralia Motus: la fondazione relativistica della meccanica nell'opera di Chr. Huygens* [*Penetralia Motus*] (Firenze: Nova Italia, 1993).

[69] Friedman first offered his reading in 1986, then expanded it as 'Metaphysical Foundations of Newtonian Science' ['Foundations'] in his *Kant and the Exact Sciences* (Cambridge, MA: Harvard University Press, 1992), 136–64. He restated that reading, more concisely, in 'Philosophy of Natural Science', in P. Guyer (ed.), *The Cambridge Companion to Kant and Modern Philosophy* (Cambridge: Cambridge University Press, 2006), 303–41. I must emphasize that, here, I engage only with Friedman's reading of Kant in the above two pieces. Lately, Friedman's views on these matters have evolved significantly; see his *Kant's Construction*.

absolute space and argues that Kant resolved to let Newton's dynamical laws *define* true motion, by using them to specify 'a privileged frame of reference for describing the true motions' in a system of interacting bodies.[70] 'The common center of gravity of all matter', Friedman expounds, lies ultimately at the origin of that frame, presumably inertial. Though the frame is beyond our empirical reach, Kant teaches how to get ever closer to it, Friedman explains: his Phenomenology outlines a 'constructive procedure for finding better and better approximations'. Allegedly, it parallels Newton's long argument, in Book III of the *Principia*, on how the true motions of planets may be inferred from their apparent motions and the impressed forces on them. As Friedman sees it, Kant's procedure is in three steps, corresponding to his three modal categories and types of true motion: possible, actual, and necessary. In the first step, we *assume* the Earth to be at rest and take *apparent* motions relative to *it* as 'possible', or possibly true. In the second, we assume the acceleration of bodies falling on Earth to be true and use it to 'discern the true or actual rotation of the Earth with respect to the fixed stars'.[71] As I understand Friedman, in the third step his Kant shows the Earth (and the planets) to have a centripetal acceleration toward the CM-frame of the solar system. By Newton's *Lex Tertia*, which Friedman's Kant invokes, so does the Sun. Therefore, the planets and the Sun have Kantian 'necessary motions' relative to each other. He reads the Earth-passages in Kant's 'General Note on Phenomenology' as proposing distinct experiments for different purposes. The second, viz. Kant's tunnel test, aims to prove the daily rotation of the actual Earth relative to the stars. The first is structurally different, Friedman alleges; it demands that the stone be given a lateral velocity.[72] Its supposed aim is to be an analogue of Newton's moon test (in Book III, Prop. IV of the *Principia*), which links terrestrial gravity and the Earth's centripetal acceleration.[73]

My reading of Kant on rotation suggests the second step in Friedman's construal needs a fundamental rethinking and his account of

[70] Friedman, 'Foundations', 143; next quotation, ibid., 144.

[71] Ibid., 146.

[72] I am not sure how Friedman believes we should read the setup of Kant's first experiment. In 'Foundations', 146, n. 15, Friedman has the test stone rise vertically; on 151–2, he urges us to suppose Kant projecting the stone horizontally, thus turning it into a small satellite.

[73] Friedman, 'Foundations', 152.

Kant's Earth passages needs more support. First, as I claimed, Kant's two falling-stone experiments share the same aim: to detect rigid rotation in empty space. Second, Kant's tests are an analogue to a thought experiment in Newton's Scholium to the Definitions, not his physical argument in Book III. Kant's body is a hypothetical 'Earth' spinning in empty space, not Newton's Earth in the solar system. Third, Kant analyzes that spin as the motion of 'opposite *parts*' in his 'Earth' relative to *each other*, not as the *body*'s rotation relative to the *stars*. So, we must also reassess exactly what that privileged frame, in Friedman's second step, is supposed to be. Fourth, we must take a fresh look at the quantity and direction of the true motions at that step. Once we do that, we discover (as I have shown throughout §3.3) that Kant takes the *true* motion of the 'Earth' to be relative to a *non*-inertial frame. But this is the very opposite of what he *ought to* say, if Friedman were right.

Kant's actual views at this juncture are wholly at odds with those that Friedman attributes to him. This sharp disparity requires explanation, because it shows Kant, inadvertently, in a bad light: allegedly, he set out to ground a central result of Newtonian mechanics, but to do so he chose a type of frame *incompatible* with that mechanical theory. And so, Friedman's otherwise very appealing exegesis has the unpalatable consequence that Kant commits a serious mistake because of an elementary confusion: he cannot tell the difference between inertial and non-inertial frames. But surely this outcome must count against Friedman's reading. On mine, as I have explained, Kant *is* aware of his choice of frame, because it alone lets him harmonize his account of rotation with the central idea of his Phenomenology, namely that all true motion is a relation between parts of matter in a 'community of interaction' through the forces they acquire simply in virtue of being in true motion. This concludes my argument **E**, as I called it in the introduction.

## 5. KANT'S EARTH: AN OBJECTION

As Friedman reads it, Kant aims to show the Earth to be a rotating frame, thus not quite inertial, hence a poor choice for defining the true motions of bodies. Rather, our quest for that special frame must go on *in perpetuum*, reaching ever farther, beyond the Earth. This reading hinges on a key claim: that Kant's aim with the tunnel experiment is to evince the actual Earth's rotation. Contra Friedman, I claimed above that it is not; rather, Kant runs two scenarios about a *fictive* Earth-like analogue observed under ideal conditions. His 'Earth' is *unlike* the actual one in that it is supposed as far away from the stars as we wish, rigid, and without an atmosphere. This, I submit, makes Kant's setup a *Gedankenexperiment*, not a description of a procedure to be carried out on the actual Earth. Moreover, it puts Kant's tests on a par with Newton's thought experiment with the two globes, which Kant had set out to account for explicitly.

But my claim faces an objection: Friedman's construal of Kant's intention, at least in the tunnel-test passage, seems natural, while mine is rather counterintuitive. In fact, his analysis looks inevitable if seen against the historical backdrop of tests to detect terrestrial rotation. In what follows, I outline a brief survey of those tests to strengthen Friedman's case. Then I argue that, despite its initial appeal, we should resist his reading and adopt mine instead.

Soon after Copernicanism broke out, its adepts sought to verify its attendant claim that the Earth truly rotates daily. This led to a string of 'mechanical proofs' for terrestrial rotation. Some were based on a common idea. Let a system move under terrestrial gravity. The known laws of this force will entail a definite trajectory relative to an inertial frame, e.g. a stationary Earth. If the Earth rotates, the system's observed trajectory will diverge measurably from its predicted one. Thus, as if trying Kant's first experiment *avant la lettre*, around 1638 Mersenne fired cannons vertically upward, to see if test projectiles would fall back deflected, as he thought they should if the Earth spins. Some failed to return, leading Descartes to speculate that 'the force of the shot, driving them very high up, removes them so far from the center of the Earth that they lose all weight'.[74] In 1679, Newton

[74] Descartes to Mersenne, July 13, 1638 (AT ii. 224). On Mersenne's tests, cf. P. Acloque, *Histoire des expériences pour la mise en évidence du mouvement de la terre* [*Histoire*] (Paris: CNRS, 1982), 15. In the late 1600s, Stefano degli Angeli and James Gregory claimed (falsely) that Descartes too had tried the same sort of test that Mersenne had performed—on which Alexandre Koyré comments in the vernacular: 'Needless to say, Descartes never made such a stupid experiment.' Cf. A. Koyré, 'A Documentary History of the Problem of Fall from Kepler to Newton', *Transactions of the American Philosophical Society*, 45 (1955), 329–95, at 358, n. 141. However, the idea is not silly per se: the test would work *in principle*, i.e. if all perturbing factors could be controlled. Then, a projectile fired vertically to an altitude of 10 km at the equator would land *west* of the exit location by about 0.73 m; see W. Ferrel, *A*

shared with Hooke 'a fansy' of his that a 'heavy body' initially resting at some height be 'let fall', claiming that it 'will shoot forward to ye east side', in the first documented prediction of eastward deflection.[75] The Royal Society commissioned Hooke to carry out Newton's idea; Hooke dropped small iron balls from nine meters and saw them land everywhere, some as far as ten times what he expected (for that height, theory predicts a mere 0.6 mm of deflection). He claimed to have found, on average, a *southeasterly* deviation, not just toward east, as Newton predicted.[76] The Society thought Hooke's results inconclusive and the issue became dormant. Gianbattista Guglielmini took it up anew in 1791; from a height of 78 meters in the Asinelli Tower at Bologna, he let fall 16 balls and claimed to have seen a southeasterly deflection. Advised by Laplace, he redid his trials and reinterpreted their results to support eastward deviation alone.[77] Benzenberg in Germany carried out trials eerily evocative of Kant's tunnel scenario: in 1802, he did 16 drops from 76 meters inside St Michael's Tower in Hamburg; then in 1804, he let metal spheres fall from 85 meters down a mineshaft at Schlebusch.[78] Reich released metal balls from 158.5 meters in the Three Brothers mineshaft at Freiberg in 1831 and found an eastward deflection of about 28.3 millimeters.[79] Like Guglielmini and Benzenberg before him, he too saw an extra deviation southward,

*Popular Treatise on the Winds* (New York, 1889), 88. My interpretation of Kant's first test (a stone projected vertically upward) has the added merit that it escapes Koyré's censure: being a mere *Gedankenexperiment*, it is not beset by the insurmountable technical difficulties of Mersenne's actual test.

[75] Newton to Hooke, November 28, 1679, in H. W. Trumbull (ed.), *The Correspondence of Isaac Newton*, vol. 2 (Cambridge University Press, 1960), 301.

[76] See Acloque, *Histoire*, 17, and J. Lohne, 'Hooke versus Newton', *Centaurus*, 7 (1960), 6–52, at 30–45.

[77] J. Gapaillard, *Et pourtant, elle tourne! Le mouvement de la terre* [*Mouvement*] (Paris: Seuil, 1993), 234; J. G. Hagen, *La rotation de la terre. Ses preuves mécaniques anciennes et nouvelles* [*Rotation*] (Rome: Tipografia Poliglotta Vaticana, 1911), 10f.; J. Fr. Benzenberg, *Versuche über das Gesetz des Falls, über den Widerstand der Luft und über die Umdrehung der Erde* [*Versuche*] (Dortmund, 1804), 250–89. Guglielmini's experiments are recounted and analyzed in D. Bertoloni Meli, 'St Peter and the Rotation of the Earth: The Problem of Fall around 1800' ['Rotation'], in P. M. Harman and A. Shapiro (eds.), *The Investigation of Difficult Things* (Cambridge: Cambridge University Press, 1992), 421–48.

[78] Benzenberg, *Versuche*, 301–60, 403–32; a careful account is in Bertoloni Meli, 'Rotation', 431–46.

[79] F. Reich, *Fallversuche über die Umdrehung der Erde* [*Umdrehung*] (Freiberg, 1832), 45; Acloque, *Histoire*, 26f.

inexplicable from theory. To decide whether this deflection is real or due to imperfect procedure, Edwin Hall at Harvard's Jefferson Lab in 1903 let a large set of pellets fall from 23 meters under carefully controlled conditions. He did find a minute deviation southward, which Hagen refused to accept.[80]

In parallel with these experiments, theoretical studies arose to interpret or correct them. They aimed at exact descriptions of the dynamics and path of falling bodies as seen from a rotating Earth and at relating the time of fall to the length of eastward deflection. Euler and d'Alembert took the first steps, out of sheer theoretical interest.[81] Then Laplace offered his own solution to help Guglielmini; and Benzenberg sought mathematical help from Olbers, who relayed the request to Gauss, the only one in Germany then able to tackle the problem formally.[82] Later in France, the kinematics of apparent motion in rotating frames found renewed attention from Coriolis, Poisson, and Serret, a father of modern differential geometry.[83]

This historical sketch makes Kant's falling-stone tests appear as yet another episode in a long saga, thereby supporting Friedman's account. To dissuade the reader of that thought, I offer four arguments.[84]

[80] Cf. E. H. Hall, 'Do Falling Bodies Move South?' ['Falling Bodies'], *Physical Review*, 17 (1903), 179–90, 245–54; and Hagen, *Rotation*, 23–9.

[81] Cf. J. d'Alembert, 'Sur le mouvement des corps pesans, en ayant égard a la rotation de la Terre autour de son axe', in *Opuscules mathématiques*, vol. 7 (Paris, 1780), 315–70; L. Euler, 'Recherches sur le mouvement des corps célestes en general', *Histoire de l'Académie Royale des Sciences* (Berlin), Année MDCCXLVII (1749): 93–143.

[82] Cf. P. S. Laplace, 'Mémoire sur le mouvement d'un corps qui tombe d'une grande hauteur', in *Bulletin de la Société Philomatique de Paris*, 3 (1803), 109–15; C. Fr. Gauss, 'Fundamentalgleichungen für die Bewegung schwerer Körper auf der rotierenden Erde' (1804), in Gauss, *Werke*, Band 5 (Hildesheim: Olms, 1973), 495–503; Bertoloni Meli, 'Rotation', 433–8.

[83] See G. G. Coriolis, 'Mémoire sur les équations du mouvement relatif des systèmes de corps', *Journal de l'Ecole Polytechnique*, 15 (1835), 142–54; S.-D. Poisson, 'Sur le mouvement des projectiles dans l'air, en ayant égard a la rotation de la terre', *Journal de l'Ecole Polytechnique*, 18 (1838), 1–69; J. A. Serret, *Théorie du mouvement de la terre autour de son centre de gravité* (Paris, 1860).

[84] There is, in addition, some circumstantial evidence against Friedman. No German expositor of physical proofs for the Earth's diurnal rotation—Benzenberg, Olbers, Reich, Bessel, Hagen—saw fit to cite Kant as a precursor of such proofs, though they clearly knew his work. Benzenberg compares the fate of Copernicanism with that of 'the Kantian system', ignored at first, then met with fierce resistance, then embraced by all—Benzenberg, *Versuche*, 246. Bessel knows Kant's *Allgemeine Naturgeschichte*, and applauds it as a worthy defender of Copernicus and Newton, despite its occasional 'reveries'—F. Bessel, *Die Beweise für die Bewegung der Erde* (Berlin, 1871), 20. This suggests that, contra Friedman, they did not read his Earth-passage as describing tests to detect the actual Earth's rotation, but rather as he meant

First, if Kant indeed referred to the actual Earth's rotation, he would not have failed to bring up *another* rotational motion of our Earth: nutation, a regular wobbling of the terrestrial axis. Kant knew about it from a 1728 paper by Bradley, to which he alludes in his *New Doctrine*. In a paper read at the Berlin Academy, Euler explained it as caused by small differences, under the action of lunar gravity, in the orientation of the Earth's axis as it revolves around the Sun.[85] By 1762, nutation was established fact, and a voluminous compendium of Wolffian physics records it.[86] In a discussion of the actual Earth's true motion, leaving out nutation requires an explanation. But this omission is predictable if, as I have claimed, Kant works with an idealized 'Earth' in empty space: *no* tidal forces from the Moon and Sun would arise to cause terrestrial nutation there.

Second, whereas the actual Earth neither is rigid nor rotates alone in a vast emptiness, Kant is clear that his Earth is *rigid* and it spins alone in *empty* space. This is plain from his words that, in his setup, *no* kinematic change occurs: 'neither *the relation between parts of the Earth* nor the Earth's relation to space outside it changes (phoronomically, or in the appearance)' (*GS* iv. 561, my italics).[87] In addition, Kant says *explicitly* that he means his 'Earth' to replicate 'under somewhat changed conditions' Newton's globes (ibid., 562). But *that* setup was a rigid system spinning in empty space. Hence, so is Kant's 'Earth'. So, it is not deformable; therefore, it cannot be the actual Earth.

Third, reading the Earth-passage as Friedman asks would leave intact an obvious deep gap in Kant's theory of motion: the *nature* of rotation,

them: as thought experiments with an idealized setup. Though telling, this is admittedly weak evidence.

[85] L. Euler, 'Recherches sur la précession des equinoxes et sur la nutation de l'axe de la terre', *Histoire de l'Académie Royale des Sciences et Belles Lettres* (Berlin), Année MDCCXLXIX (1751), 289–325.

[86] M.Chr. Hanov, *Philosophia Naturalis, sive Physica Dogmatica*, vol. I (Halle, 1762), 256.

[87] One might counter that, on a deformable Earth in torque-free spin around some axis of rotation, no kinematic change would occur either. I respond that a deformable Earth would undergo such change—viz., a deformation, or actual change of relative distance between the parts—as a result of being accelerated from rest to some angular velocity. Of course, such change between the parts would only be observable over longer time-spans. But I think that Kant *does* usually take the 'long view' about the relative motions ensuing from or being associated with changed dynamical relations between portions of matter. For instance, he takes the 'long view' in the Balancing Argument from his *Dynamics*, as I learned from Smith, 'Balance Argument'.

which he knows to be a challenge. I have claimed that Kant solves it, as best he can, in the Earth-passage. And, I have explained how he 'reduces' rotation to *Kantian* absolute space. My account thus obviates the need for Friedman's formerly-indispensable reading. Keeping his account of the Earth-passage would burden Kant with a failure to explain how rotation *in general* is a type of relative motion. Friedman might reply that, for Kant, all rotation is relative to the stars. But that proposal has little support in the text and is at odds with Kant's words that it is a motion of the parts *relative to each other*. And it fails to address Kant's 'paradox' that we *can* detect rotation even if *no* stars are present. My reading both fits with Kant's *ipsa verba* and credits him with a solution to the 'paradox'.

Fourth and decisively, if Kant really meant them as tests to discover the actual Earth's diurnal rotation, his choice would be terribly unwise. Here is why. Kant had *three other, much more reliable* ways to prove terrestrial spin. One relies on equatorial bulging, an effect of rotation in deformable bodies. Newton, Huygens, and Jakob Hermann had predicted the effect from distinct dynamical theories. In the 1730s, French expeditions to Lapland and Peru had confirmed the Earth to be oblong, i.e. bulging at the equator and flattened at the poles.[88] Their results were beyond doubt, proved by two independent methods, geodetic and gravimetric.[89] Kant could not have failed to know

[88] Maupertuis led the 1736–7 expedition to Lapland, accompanied by Clairaut, Camus, Le Monnier, the Abbé Outhier, and Anders Celsius. La Condamine and Bouguer, with Godin and two Spanish naval officers, were on the longer, misfortune-ridden 1735–45 expedition to Peru. For an account of Maupertuis's activity in Lapland, see D. Beeson, *Maupertuis: An Intellectual Biography* (Oxford: The Voltaire Foundation, 1992), 110–16; his techniques and results are explained in I. Todhunter, 'On the Arc of the Meridian Measured in Lapland', *Transactions of the Cambridge Philosophical Society*, 12 (1871), 1–27. Todhunter recounts the French expedition to Peru in Chapter XII of *A History of the Mathematical Theories of Attraction and the Figure of the Earth from the Time of Newton to that of Laplace* [*Theories of Attraction*], vol. 1 (London, 1873).

[89] On a spherical body, as a non-rotating Earth should be, one degree of latitude has the same length at all latitudes. In contrast, on an oblong, rotating Earth, one degree of latitude near the poles will be *longer* than one degree close to the equator (flattening at the poles results in a lengthening of meridians around them). Geodetic methods can measure these differences in the length of one degree on the local meridian. Both expeditions established that conclusively; at their respective location, the length of a one-minute arc of meridian differed from its length in Paris (longer in Lapland, shorter in Peru). Gravimetric methods rely on the fact that, on a rotating (hence oblong) Earth, the poles will be closer to the center than the equator, and so the local strength of gravity (a function of distance to the center) will vary accordingly—weaker at the equator, stronger at the poles. At these locations, two equally

about the two French expeditions; all Europe had been waiting with bated breath to hear their outcome, soon rendered into major languages for greater impact, in works easily available in Germany.[90] And Kant knew of Maupertuis's claim that *all* rotating bodies, even stars, bulge at the equator if they are not rigid.[91] Another way to prove the real Earth's rotation is from a Coriolis effect on winds: in the northern hemisphere, winds are deflected eastward. Kant knew it; in fact, in 1756 he proudly explains that fact as induced by the Earth's rotation—independently, it appears, of Hadley's 1735 similar claim.[92] And, Kant was in a position to know of a third proof of diurnal rotation. In our hemisphere, cannonballs fired northward along the meridian are deflected eastward, as they should be if seen by an observer at rest on a rotating Earth. In 1745, Euler proved it as a theorem.[93] Though Kant might not have known Euler's result, he plainly knew of the first two proofs.

Given that these two proofs were established fact in Kant's time, his choosing a falling-stone test to show that the *real* Earth rotates appears quite naïve. Next to those proofs, his experiment is highly uncertain,

long pendulums will complete one oscillation in different times (the times are in proportion to *g*, the local acceleration of gravity). Again, both French expeditions used pendulums to detect differences in local gravity. 'The result is that a pendulum which oscillates in a second at Paris will make 59 more oscillations in 24 hours at Pello [in Lapland] than at Paris.' Todhunter, *Theories of Attraction*, 99.

[90] Maupertuis's 1738 *La Figure de la Terre determinée par les observations faites au cercle polaire* was published in German as *Figur der Erden*, trans. S. König (Zürich, 1741); and in Latin as *Figura Telluris*, trans. A. Zeller (Leipzig, 1742).

[91] This is in Maupertuis's paper 'Sur les figures des Corps Célestes', published in the *Memoires* of the Royal French Academy of Sciences for the year 1734. In the 1755 *Universal Natural History*, Kant invokes Maupertuis to claim that distant nebular galaxies 'appear elliptical when seen sideways, due to their great flattening caused by their rotational impulse [*Drehungsschwung*]' (*GS* i. 254).

[92] Kant offers this account in his *New Remarks for the Explanation of Winds*, *GS* ii. 489–503. For Hadley's claim, see 'On the Cause of the General Trade Winds', *Philosophical Transactions of the Royal Society*, 34(1735): 58–62. Two scholars argue that Kant discovered the Coriolis deflection of winds independently; see M. Jacobi, 'Immanuel Kant und die Lehre von den Winden', *Meteorologische Zeitschrift*, 20 (1903), 419–21; and M. Schönfeld, *The Philosophy of the Young Kant* (Oxford: Oxford University Press, 2000), 78.

[93] Euler proved it in his annotated translation of Robins's *New Principles of Gunnery*; cf. L. Euler, *Neue Grundsätze der Artillerie* (Berlin, 1745), 686, Theorem VII: 'Beside the force of gravity deflecting cannonballs downward in their flight, these are often driven sideways by another force, either left or right.' Euler's tract was no obscure tome; it proved so popular and superior to the original that Hugh Brown translated Euler's greatly expanded version back into English as *The True Principles of Gunnery Investigated and Explained* (London, 1777).

technically daunting, and reliant on forbidding mathematics.[94] Gauss, the prince of mathematicians in the late Kant's Germany, was deeply skeptical of such attempts, in which the margin of error is *five times* as large as the expected result.[95] From single trials, as Kant proposes, *nothing* can be inferred reliably. Bluntly put, if we read Kant as proposing two tests meant for the real Earth, he would appear artless and injudicious. In contrast, Kant's choice becomes insightful if seen as a test on a hypothetical *rigid* Earth spinning in *empty* space and *devoid* of an atmosphere, as I have suggested. On that 'Earth', the two experiments discussed are impossible.[96] There, Kant's proposed tests, though still not easy, would be the *only* option. My reading claims precisely that Kant's 'Earth' is rigid, distant from the stars, and lacks an atmosphere—just as it should, if he means it as an analogue of Newton's globes. In contrast, Friedman's Kant appears reckless and ignorant—a damaging indictment, if true. Fortunately, we can avoid that accusation, if we adopt my reading.

These considerations, I believe, speak against Friedman's construal of Kant's 'Earth'-passage and lend support to my thesis that Kant there uses a fictive Earth-like body to grapple with Newton's challenge, not to justify Newton's celestial dynamics. In any event, if the reader is unconvinced by my arguments, my other point still stands: Kant analyzes rotation as motion with respect to a non-inertial frame, and

[94] If a single stone is dropped once, as Kant's description suggests, it is *extremely* unlikely that it would visibly fall eastward, as he predicts. Initial conditions (e.g. small oscillations before release, deviations of the release mechanism from the direction of true local gravity) and perturbing factors (e.g. imperceptible air currents) greatly influence the falling stone's trajectory; as a result, a single stone could land *anywhere*, as Hooke, Guglielmini, Benzenberg, and Reich discovered with dismay. To prove eastward deviation on the real Earth, one needs a long *series* of drops, then a statistical analysis of results to infer an *average* deflection, i.e. a fact about a set. Then the found deviation must be compared with its expected value from theory. Computing the latter is highly non-trivial. Guglielmini and Benzenberg could not do it and had to ask Laplace and Gauss, the supreme mathematicians of their time, to help them describe analytically the trajectory of the falling test object. For an account of the technical and formal difficulties involved, see Hall, 'Falling Bodies', and Bertoloni Meli, 'Rotation'.

[95] See Gauss's letter to Olbers, March 4, 1803, in C. Schilling (ed.), *Wilhelm Olbers. Sein Leben und seine Werke*, Band II (Berlin, 1900), 131; and Bertoloni Meli, 'Rotation', 440.

[96] This is because the first proof assumes that the actual Earth is deformable. A rigid Earth would *not* bulge at the equator and be flattened at the poles. Further, the geodetic method used in the second proof is impracticable if the stars are not visible: to measure the length of an arc of meridian at different locations, one needs to establish latitude, and *that* requires observations of star positions.

*that* is a powerful objection to Friedman's account of absolute space, true motion, and commitment to Newtonianism in 'Metaphysical Foundations of Phenomenology'.

## 6. CONCLUSION

Whether early or late, Kant never chooses to anchor his theory of motion in Newton's absolute space, resolving instead to explicate true motion as a special kind of kinematic change relative to matter. Then his fateful choice requires him to explain in what sense true rotation is relative. Kant wrestles at length with this problem. To tame it, he builds on his pre-Critical doctrine and defines rotation as a motion of 'parts of matter' spinning relative to each other, in a new sense of 'motion'. This solution allows him a certain unity of conception in the Phenomenology: both actual and necessary motions consist in 'dynamical', i.e. force-inducing relations between 'parts of matter in space'. But there is a key difference: the forces that accompany true rotation, for Kant, are centrifugal, hence *non*-inertial. This creates a new problem for his system: his Laws of Action and Reaction do not underwrite such forces. Further, Kant's centrifugal forces are non-Newtonian, and so his views on rotation bring him closer to Huygens than to Newton.

Moreover, Kant chooses to address Newton's case for absolute space on his own terms, not Newton's. That is, he selects a single issue—rotation—to engage with, ignoring the rest of Newton's argument. This presses upon interpreters the need to elucidate the exact sense in which Kant is a philosophical spokesman for Newtonian science. There is no doubt that the mature Kant has metaphysical reasons to refuse absolute space as Newton conceived of it. Still, if Kant indeed aims to ground Newtonian science philosophically, he must tackle the Briton's complex argument that only *his* absolute space can vindicate the 'properties, causes, and effects' of true motion. I have shown that, in the Phenomenology, Kant responds just to Newton's argument from effects. Explaining Kant's selective engagement with Newton's theory of motion is now a task for future exegesis.

Lastly, my account puts me at odds with three major readers of Kant on rotation. My account is preferable to Earman's because it has direct support from Kant, unlike his. And it is superior to Carrier's and

Friedman's because it avoids making Kant look confused—which they do, inadvertently. True, my reading reveals tensions between Kant's analysis of rotation and Newtonian commitments elsewhere in his system. However, I submit that an interpretation that uncovers difficulties is, *ceteris paribus*, better than one that saddles Kant with shocking mistakes.[97]

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