

# Kant's Schematism of the categories: An interpretation and defence

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## Abstract

The aim of the Schematism chapter of the *Critique of Pure Reason* is to solve the problem posed by the “inhomogeneity” of intuitions and categories: the sensible properties of objects represented in intuition are of a different kind than the properties represented by categories. Kant's solution is to introduce what he calls “transcendental schemata,” which mediate the subsumption of objects under categories. I reconstruct Kant's solution in terms of two substantive premises, which I call Subsumption Sufficiency (i.e., that subsuming an object under a transcendental schema is sufficient to subsume it under the corresponding category) and Real Possibility (i.e., that it is really possible to subsume objects under each of the transcendental schemata). These two principles, together with a trivial modal one (the Subsumption-Possibility Link), entail that it is possible to subsume objects under categories; in other words, the argument of the Schematism is *valid*. The main work of the paper consists in reconstructing Kant's arguments for, and explanations of, these premises. I argue that they hinge on Kant's claim that transcendental schemata are “time-determinations,” which I interpret to mean: rules for reflexively representing the temporal relations among our own representational states. On the basis of this reading, I reconstruct Kant's argument for Subsumption Sufficiency, category by category. I also explain why Real Possibility follows almost immediately. Granting Kant the argument up to this point in the *Critique*, the argument of the Schematism is *sound*.

## 1 | INTRODUCTION

Most commentators agree that the Schematism chapter plays a very important role in the *Critique of Pure Reason* (CPR), but that is where the agreement ends. In particular, there is little agreement on what role, exactly, the Schematism is supposed to play and how successfully it plays that role, and many commentators consider it a failure.<sup>1</sup> My aim in this paper is to provide an interpretation of the role of the Schematism and a qualified defence of its main doctrines.

The topic of the Schematism is the “subsumption” of objects under concepts, as the first sentence announces: “in all subsumptions of an object under a concept [...]” (A137/B176).<sup>2</sup> Its primary aim is to explain how it is possible for sensible objects to be subsumed, specifically, under the pure concepts of the understanding (categories).<sup>3</sup> That much is relatively clear. What is significantly less clear is how Kant would answer two questions that arise immediately:

**Q1.** Why does the subsumption of sensible objects under categories stand in need of explanation?

**Q2.** How do schemata explain this subsumption?

The second question becomes even more pressing when we realize that sensible objects (henceforth, simply “objects”<sup>4</sup>) are subsumed under the schemata themselves.<sup>5</sup> This can raise the suspicion that schemata reintroduce the very problem they were meant to resolve. In light of this, the second question ramifies into two further questions:

**Q2a.** Why is an explanation of the subsumption of an object under a schema an explanation of the subsumption of that object under the corresponding category?

**Q2b.** What explains the subsumption of objects under schemata? Why does the very problem the Schematism is intended to solve not arise again at this level?

In this paper, I will attempt to give a unified answer to all of these questions, and thus an interpretation, and qualified defence, of the doctrines of the Schematism chapter.<sup>6</sup> In Section 2.1, I answer Q1 by arguing that the reason the subsumption of sensible objects under categories is problematic is that the former are not “homogeneous” with the latter: that is, the properties of objects represented in intuition are not the properties of objects represented by categories. Some explanation must be given of how it is possible for these objects to be subsumed under (i.e., to instantiate) these concepts.

The rest of the paper is devoted to answering Q2(a) and Q2(b). The heart of my answer consists in a reconstruction of Kant's argument that it is possible, by means of schemata, for objects to be subsumed under categories. I reconstruct not only the argument for this claim, but Kant's explanation of the premises of the argument, and why they together constitute an explanation of the possibility of subsuming objects under categories.

The first premise in the argument is the following:

*Subsumption Sufficiency:* Subsuming an object under a schema is sufficient to subsume it under the corresponding concept.

In the rest of Part One, I reconstruct Kant's theory of schemata. The guiding thread of my interpretation is Kant's claim that schemata are “transcendental time-determinations,” which I understand to mean: rules for self-consciously representing the determinate temporal relations among the states (intuitions) by which objects are given to us. The core idea of my interpretation is that, in representing temporal relations among our states as schemata dictate, we thereby represent the objects of those states as falling under the concepts corresponding to those schemata. After

specifying what notion of determination is involved in this claim (Sections 2.2 and 2.3), I explain why the schemata of empirical concepts (Section 2.4) and mathematical concepts (Section 2.5) are time-determinations, and why the schemata of categories are specifically *transcendental* time-determinations (Section 2.6). The conclusion of Part One is that for each kind of concept and their associated schemata, Subsumption Sufficiency holds. This constitutes my answer to Q2(a).

In the much shorter Part Two, I explain Kant's answer to Q2(b). This consists mainly in reconstructing his explanation for this:

*Real Possibility:* It is really possible for objects to be subsumed under the schemata of empirical concepts, mathematical concepts, and categories.

Kant is concerned with explaining not merely the logical possibility of subsuming objects under categories (their mere logical consistency), but the real possibility of such a subsumption.<sup>7</sup> I argue in Section 3.1 that in the case of empirical and mathematical concepts, the explanations of subsuming objects under schemata are nearly direct consequences of the theory of schemata outlined in Part One and some relatively uncontroversial features of Kant's cognitive theory. In the case of transcendental schemata, the explanation is not significantly more difficult. I argue in Section 3.2 that the real possibility of subsuming sensible objects under categorial schemata follows almost immediately from Kant's specification of those schemata. These two premises – Subsumption Sufficiency and Real Possibility – entail that subsumption of objects under categories is possible only if we assume a further, albeit trivial, principle:

*Subsumption-Possibility Link:* if subsuming an object under a schema is sufficient to subsume it under a concept, then if it is really possible for objects to be subsumed under the schema, it is really possible for them to be subsumed under that concept.

This may appear complex, or even controversial, but it is an immediate consequence of some elementary principles about modality. If  $p$  is modally sufficient for  $q$  (necessarily, if  $p$  then  $q$ ), and  $p$  is possible, then  $q$  is possible. I will not argue for it further here.<sup>8</sup>

These three premises entitle Kant to conclude validly that:

*Conclusion:* It is really possible for objects to be subsumed under categories.

This is precisely the conclusion for which the Schematism constitutes an argument.

This establishes that, on my reading, the argument of the Schematism is *valid*.<sup>9</sup> Whether it is plausibly regarded as *sound* depends on the strength of the arguments Kant gives in favour of the premises. Whether it succeeds in explaining the possibility of subsuming objects under categories depends in turn on whether Kant's premises explain the Conclusion. I will argue that if we grant him the conclusions he takes himself to have proved up to that point in the CPR, Kant's detailed theory of schemata provides a plausible case for the truth of these premises, and they in turn explain the Conclusion. Thus, I will argue, the Schematism to a large extent succeeds in its aim: to explain the real possibility of subsuming objects under categories.<sup>10,11</sup>

## 2 | PART ONE: SUBSUMPTION SUFFICIENCY

### 2.1 | The problem of subsumption

In the second paragraph of the Schematism chapter, Kant succinctly summarizes the problem that chapter is supposed to solve: "Now how is the subsumption of the latter [sensible intuition] under the former [pure concepts of

the understanding], thus the application of the category to appearances possible?" (A137/B176).<sup>12</sup> Kant sometimes uses "subsumption" to refer to a relation between concepts (one concept is said to be "subsumed" under a more general concept (e.g., <human> under <animal>),<sup>13</sup> but in the Schematism, he is concerned with the subsumption of an object under a concept (e.g., the subsumption of Socrates under <human>).<sup>14</sup> After all, the very first sentence of the whole chapter begins: "In all subsumption of an object under a concept [...]" (A137/B176, Au's emphasis). An object is subsumed under a concept when it "falls under" or is "contained under" that concept.<sup>15</sup> To avoid unnecessary prolixity, I will sometimes speak of objects *instantiating* concepts.

The subsumption of objects under categories is taken to be problematic – that is, it stands in need of further explanation – because the intuitions that give us objects are not "homogeneous" with the categories: "Now pure concepts of the understanding, however, in comparison with empirical (indeed in general sensible) intuitions, are entirely inhomogeneous [*ungleichartig*], and can never be encountered in any intuition" (A137/B176). In the immediately preceding paragraph, Kant briefly explains what homogeneity consists in: "In all subsumptions of an object under a concept the representation of the former must be **homogeneous** with the latter, i.e. the concept must contain that which is represented in the object that is to be subsumed under it, for that is just what is meant by the expression 'an object is contained under a concept'" (A137/B176).

From this we can extract a preliminary definition of homogeneity: an object is said to be *homogeneous* with a concept if and only if what is represented in the object is also contained in the concept. But to understand what this means, we need to understand what is represented in objects, what is contained in concepts, and how they can be the same or different.

Concepts are composed of further concepts, called *marks*. For instance, the marks of <gold> are <yellow>, <metallic>, and <metal>. Since we are concerned with the subsumption of intuited objects under concepts, I take Kant's reference to "what is represented in the object" to mean what is represented in the object *by intuition*. And since we are trying to account for the relation between intuited objects and concepts, we have to focus on what intuition *merely as such* represents in its object. Otherwise, if we included something conceptual in what is "represented in the object" – that is, if we included its instantiating some conceptual mark – we would potentially be assuming the very thing we are trying to explain, namely, the subsumption of intuited objects under concepts.

The nature of Kantian intuition has recently become highly controversial. In particular, it has become a highly controversial question what (if any) the content of intuition *as such* is, that is, what (if anything) intuition represents in its object independently of the contribution of the understanding.<sup>16</sup> I cannot hope to address that controversy in this paper. Instead, I will sketch out a reading that, I argue, makes very good sense of the immediate context of the original passage (A137/B176) and motivates the problem of the Schematism. In the rest of this paper, I will argue that this reading of the representational content of intuition also gives Kant a plausible solution to that problem.

Intuitions give us objects. I understand this to mean that intuitions make objects available to us to think *de re* as falling under concepts. Thus, in order to think that *x* is *F*, I need to be able to think *de re* of *x* that *it* is *F*. What allows me to do this is an intuition of *x*; an intuition of a distinct object *y* would allow me to think *de re* of *y* that *it* is *F*. But intuition does not give me the object *x* merely as a "bare particular." If it did – that is, if the representational content of intuition were exhausted by the object itself – then nothing about that representational content could explain why its object falls under one concept (*F*) rather than another (*G*). Instead, the intuition must also represent some properties or features of the object.<sup>17</sup>

Kant calls these features within the representational content of intuition "intuitive marks":

A mark is a partial representation (which), as such (is a ground of cognition). It is either intuitive (a synthetic part): a part of intuition, or discursive: a part of a concept, which is an analytic ground of cognition. (*Refl.* 2286, 16:299–300)<sup>18</sup>

An intuitive mark is a feature of an intuited object that can be represented in the object by intuition: it is a feature of the intuited object as such. It is said to be a "part of intuition" because it can be a spatial or temporal (and either

proper or improper) part of the object; for instance, an object can be intuited as red in part or all over. And it is said to be a “synthetic” part, to contrast it with the analytic inclusion of what are here called “discursive marks” within concepts.<sup>19</sup> Since intuition is singular, intuitive marks are singular as well. What intuition presents is the redness of *this* apple (*redness*<sub>1</sub>); a different intuition of a different apple would present the numerically distinct redness of *that* apple (*redness*<sub>2</sub>).<sup>20</sup> In contemporary terms, intuitive marks are tropes, or individual property instances.<sup>21,22</sup> Discursive marks (marks of concepts), by contrast, are general representations; they are representations of properties as general, that is, as shareable among numerically distinct objects.<sup>23,24</sup> The discursive mark <red> applies to all, and only, objects that have a red intuitive mark (i.e., a red trope).<sup>25</sup> This, as Kant says, is what the homogeneity of an object and a concept consists in: “The concept must contain that which is represented in the object that is to be subsumed under it.” An object with an F intuitive mark falls under the concept <F>. This is just what is meant by the expression “an object is subsumed under a concept.”<sup>26</sup>

With these ideas in place, we can now turn to the second sentence of the Schematism: “Thus the empirical concept of a **plate** has homogeneity with the pure geometrical concept of a **circle**, for the roundness that is thought in the former can be intuited in the latter” (A137/B176).<sup>27</sup> Setting aside the details of how we read this rather obscure (and possibly garbled) sentence, the core idea is that something can be both intuited in an object and thought in a concept. I take this to be a round mark that is intuited in a round object and conceptually represented under the discursive mark <round> in both the empirical concept <plate> and the geometrical concept <circle>. So the roundness in the object is a partial ground of its subsumption under <plate> and under <circle>. This also grounds a kind of “conceptual homogeneity” between <plate> and <circle>; since both of them contain <round> as a mark, any object that is homogeneous with that mark of one concept will be, to that extent, homogeneous with the other concept.<sup>28</sup> The subsumption of objects under concepts is unproblematic insofar as those concepts represent features that can be directly intuited in objects, that is, as long as the discursive marks are <F> where F-ness is some intuitive mark (e.g., roundness, redness).<sup>29,30</sup>

This is why Kant writes, a few lines later: “In all other sciences, where the concepts, through which the object in general is thought, are not so different and heterogeneous [*heterogen*] from those which represent the object in *concreto*, as it is given, it is unnecessary to offer a special discussion of the application of the former to the latter” (A138/B177). Kant is making the simplifying assumption that in sciences other than metaphysics, the concepts are of intuitive features, and thus there is no deep problem in explaining how the subsumption of objects under such concepts is possible. His considered view about “other sciences” is, of course, significantly more complex than this: all sciences, in the strict sense, contain an a priori part, in which a priori concepts (the categories) are applied to sensible objects. In natural science, for example, matter is conceived of as substance.<sup>31</sup> His considered view about empirical concepts is also significantly more complex: subsuming objects under empirical concepts presupposes, and is made possible by, the subsumption of those objects under categories.<sup>32</sup> But Kant wants to focus in the Schematism chapter on explaining the possibility of subsuming objects under categories, so he downplays the way in which that very subsumption is required for natural science and the application of empirical concepts.

To formulate with perfect generality the problem of the homogeneity of concepts and objects, and thus to distinguish with perfect generality between those concepts that are homogeneous with sensible objects and those that are not (e.g., the categories), we would need a perfectly general account of what the intuitive features of objects are. But that is not our task here, for what we are concerned with here is the non-homogeneity of objects and the *pure* concepts (of the understanding), which contain no empirical content whatsoever. Consequently, we can abstract from the empirical intuitive features of objects (i.e., the intuitive features that depend upon the contingent constitution of our sense organs).

Intuition has both a form (space and time) and a matter (sensation). This means that intuition will represent objects as having both spatial features (e.g., roundness) and temporal features (e.g., one state before another), but also sensory features (e.g., redness and blueness). In abstracting from the empirical intuitive features of objects, we abstract from the particular sensory features presented to us in intuition (which depend upon our sense organs, as, e.g., colours depend on vision) but not from sensation altogether (for the fact that our intuition has sensory content

at all depends merely upon its being receptive).<sup>33</sup> I will say that a feature F is a *pure intuitive feature* of outer (or inner) objects just in case F is a determinate of some determinable D, where outer (inner) objects have D solely in virtue of being objects of outer (inner) intuition.<sup>34</sup> For instance, roundness is a pure intuitive feature of outer objects because it is a determinate of *shaped*, and *shaped* is a determinable that objects have solely in virtue of being objects of outer intuition. Likewise, *occurring before* is a pure intuitive relational feature of inner objects, because it is a determinable of *being successive*, and *being successive* is a determinable that objects have solely in virtue of being objects of inner intuition. But no sensory quality (e.g., a colour) is a pure intuitive feature, because it is not solely in virtue of being an object of outer or inner sense that objects have these qualities, but in virtue of being objects of particular senses (e.g., sight). It is not the aim of the Schematism to explain the real possibility of the subsumption of an object with a pure intuitive feature under a concept of that feature. For the purposes of the Schematism, Kant will take this as given.

But intuitions, even pure ones, do not represent the features of objects in virtue of which they instantiate the categories. Intuition may represent states of an object, but it does not represent their inherence in a substance. Intuition may represent a succession of states in objects but it does not represent their necessary connection. More abstractly, although intuition represents objects that are unities, pluralities, and totalities, it does not represent them *as* unities, pluralities, and totalities; *unity*, *plurality*, and *totality* are not intuitive features of objects that can be directly represented in intuition. Intuition represents objects with real qualities, but it does not represent those qualities as realities: *reality* is not an intuitive feature of objects.<sup>35</sup> The pure concepts of the understanding are not concepts of intuitive marks, nor do they contain such concepts as marks.

This generates the problem of how it is possible for objects to instantiate (i.e., be subsumed under) categories. If the relation between an intuited object and a category is fundamentally different from the relation between an object intuited with the feature *red* and *<red>*, and the latter relation is “just what is meant by the expression ‘an object is contained under a concept’” (A137/B176), then we lack an explanation of how the subsumption of objects under categories is possible at all.

Some readers, especially those committed to what have come to be known as *conceptualist* readings of Kant's theory of intuition, will object that I have interpreted the content of intuition in overly austere terms and worry that, so understood, the project of the Schematism is doomed to fail.<sup>36</sup> While I cannot hope to adjudicate in this paper the debate about the representational content of Kantian intuitions, it surely counts as a merit of my reading if I can show that the Schematism succeeds *even on this minimal understanding of the content of intuition*. In the rest of the paper I will argue that this is the case.

## 2.2 | Subsumption sufficiency: Schemata as mediating representations

Kant's solution to the problem of the subsumption of objects under categories is to introduce “mediating” representations, which he calls “transcendental schemata” (A138/B177). Schemata “mediate the subsumption” of appearances under categories (A139/B178), which I interpret to mean that objects are subsumed under categories by *means* of being subsumed under schemata. But if objects are subsumed under categories by means of being subsumed under schemata, then subsuming them under schemata must be *sufficient* to subsume them under categories. If it were not, then by explaining how subsumption of objects under schemata (their “homogeneity” with objects) is possible, Kant would not thereby have explained the possibility of their subsumption under categories. An explanatory gap would remain. He is thus committed to a principle I originally stated in the Introduction:

*Subsumption Sufficiency:* Subsuming an object under a schema is sufficient to subsume it under the corresponding category.

The main task of Part One is to reconstruct Kant's reasons for holding this. My strategy is to derive this principle from two subordinate principles, the first more substantive and the second fairly trivial. They are:

*Representational Sufficiency*: Representing an object under a schema is sufficient to represent it under the corresponding concept.

*Representation-Subsumption Link*: If representing an object under a schema is sufficient to represent it under a concept, then subsuming an object under the schema is sufficient to subsume it under that concept.

My reconstruction thus rests on drawing a distinction between *representing* an object under a schema or concept and *subsuming* that object under that schema or concept. The difference is this: representing an object under a schema/concept does not require that the object instantiate the schema/concept (since the representation may be inaccurate), whereas subsumption does.<sup>37</sup> The idea of the reconstruction is that representing objects under schemata is sufficient to represent them under categories (Representational Sufficiency).<sup>38</sup> Given the Representation-Subsumption Link, Subsumption Sufficiency follows immediately. In effect, I have “factored” Subsumption Sufficiency into a substantial component (Representational Sufficiency) and a relatively trivial one (the Representation-Subsumption Link).

The Representation-Subsumption Link may seem complicated or controversial, but I take it to express a straightforward truth. To subsume an object under a concept or schema is to represent it correctly under that schema or concept, that is, to represent it under the schema or concept that it instantiates. But if representing an object under a schema is sufficient for representing it under a concept, if I do the former correctly then it follows that I do the latter correctly. Otherwise, in representing the object under the schema I would be doing something incorrect: I would be representing it under a concept that by hypothesis it does not instantiate.<sup>39</sup>

Schemata can mediate objects with categories: schemata are said to be “homogeneous” (*gleichartig*) with both (A138/B177). It is worth pausing here to understand this “two-sided” homogeneity of schemata, with categories on one side and objects on the other. The homogeneity of schemata with objects, like the homogeneity of concepts of intuitive features with objects having such intuitive features (Section 2.1), means that schemata represent features that objects can possess merely as objects of sensibility. This means that there is an explanation of the real possibility of subsuming objects under schemata; my reconstruction of that claim (Real Possibility) occupies Part Two (Sections 3.2 and 4). The homogeneity of schemata with categories, however, I understand to mean, once again, that subsuming an object under a schema is sufficient for subsuming it under the corresponding category (Subsumption Sufficiency). I will now turn to the more substantive premise in my reconstruction of the argument for that principle, namely, Representational Sufficiency.<sup>40</sup>

### 2.3 | Representational sufficiency: Schemata as time-determinations

In exploring Kant's argument for Representational Sufficiency, my guiding thread will be his claim at A139/B178 that the schema of a category is a “transcendental time-determination”: “Hence an application of the category to appearances becomes possible by means of the transcendental time-determination [*transcendentale Zeitbestimmung*], which, as the schema of the concept of the understanding, mediates the subsumption of the latter [appearances] under the former [the category]” (A139/B178). To unpack the notion of transcendental time-determinations, we need to understand (i) what it means to say that schemata are “transcendental,” (ii) what they have to do with time, and (iii) what it means to say that they are “determinations.” With respect to (i), for now I will merely point out that in explaining how representing objects under a priori concepts is possible, schemata of such concepts explain (on my reading) how subsumption of objects under such concepts is possible. Furthermore, the possibility of subsumption is a necessary condition for cognizing objects under those concepts (Bxxvi, footnote). Consequently, schemata, insofar as they explain the subsumption of objects under categories, will be transcendental in precisely Kant's technical sense at A56/B80: that is, they will explain (partly) how a priori cognition is possible.<sup>41</sup> I will return momentarily to

(ii) their relation to time, but I will begin by examining (iii) the respect in which schemata are *determinations* (*Bestimmungen*).

We should distinguish at least two different senses of “determination” that are relevant to Kant's investigations. Each of these is a sense that the German word *Bestimmung* and its cognates (*bestimmen*, *bestimmt*, *Bestimmtheit*, *unbestimmt*, *Unbestimmtheit*, etc.) can have.<sup>42,43</sup> First, *Determination<sub>S</sub>* is the semantic notion of a representation representing its object as determinately having or lacking a property (i.e., being determinate with respect to that property). Conversely, *Indeterminacy<sub>S</sub>* is the notion of a representation that represents its object as neither having nor lacking a given property. Conceptual representation, being general, is typically indeterminate<sub>S</sub> to some degree.<sup>44</sup> *Determination<sub>S</sub>* is a gradable notion: one representation can be more determinate<sub>S</sub> than another in virtue of representing its objects as possessing a wider range of properties (e.g., a species concept is more determinate<sub>S</sub> than a genus concept). Second, *Determination<sub>M</sub>* is the metaphysical notion of an object determinately having or lacking a property (being determinate with respect to that property). It is the notion involved in the “principle of complete determination,” according to which every object is fully determinate with respect to every property (it either determinately has it or determinately lacks it).<sup>45</sup> Conversely, *Indeterminacy<sub>M</sub>* is the notion of an object neither determinately having nor determinately lacking a property (which may or may not be possible). *Determination<sub>M</sub>* is also gradable: one object may be more determinate<sub>M</sub> than another in virtue of being determinate<sub>M</sub> with respect to a wider range of properties. There are other philosophically important meanings of “determination” and its cognates, but they will not be significant for our investigation.<sup>46</sup>

While *determination<sub>S</sub>* and *determination<sub>M</sub>* are distinct notions, they are closely connected in Kant's philosophy. Because “the conditions of the possibility of the experience of objects [i.e., determinately<sub>S</sub> representing them as, e.g., substances and causes] are the conditions of possible objects of experience [i.e., their determinately<sub>M</sub> being substances and causes]” (A493/B521), empirical objects can be determinate<sub>M</sub> (at least with respect to the range of properties representable in experience) only insofar as we can determinately<sub>S</sub> represent them.

Having made some preliminary distinctions among different notions of determination, we must now ask: In which (if any) of these two senses are schemata transcendental time-determinations? Subsumption involves correctly representing an object under a concept, that is, determining<sub>S</sub> it in a way that captures how it is determined<sub>M</sub>. As Kant writes: “The concept must contain that which is represented in the object that is to be subsumed under it, for that is just what is meant by the expression ‘an object is contained under a concept’” (A137/B176). Subsumption thus involves both the semantic and the metaphysical notion of determination. My strategy has been to “factor” the Subsumption Sufficiency premise into two premises: Representational Sufficiency and the Representation-Subsumption Link. Representational Sufficiency, which is my focus here in Part One, concerns subsumption<sub>S</sub>: to represent an object under a concept or schema is to determine<sub>S</sub> it. *Determination<sub>M</sub>* will thus become relevant when I turn in Part Two from mere representation and *determination<sub>S</sub>* to reconstruct Kant's explanation of the real possibility of subsuming objects under schemata (Real Possibility) and thus under categories (via the Subsumption-Possibility Link from the Introduction). Until further notice, “(in)determinate/determinacy/determination” will refer to the relevant forms of *determination<sub>S</sub>*.

I can now offer a first-pass characterization of my interpretation of schemata as transcendental time-determinations: schemata explain the possibility of determinately representing a sensibly given object under a category. They do so by determinately representing the specifically *temporal* features of objects.<sup>47</sup> Why representing the temporal features of objects should have anything to do with determinately representing them under categories is one of the main things I shall have to explain in this paper.

Schemata can mediate between pure intellectual concepts and sensibly given objects, writes Kant, because they are “homogeneous” with both (A138/B177). I have already discussed above their homogeneity with categories. The complete account of their homogeneity with sensible objects – that is, Kant's explanation of the possibility of subsuming objects under schemata – is the topic of Part Two. In building up to that account, however, I want to focus on Kant's claim that schemata are homogeneous with sensible objects because “time is contained in every empirical representation of the manifold” (A139/B178). It is important to realize that “contained in” (*enthaltten in*) cannot mean

“contained in the content of” because, as Kant reminds us earlier in the same paragraph, time is the form of inner sense,<sup>48</sup> and thus outer-sense intuitions are not *as such* intuitions of objects *as temporal*.<sup>49</sup> In fact, Kant will have to do some very intricate work in the Analogies and the Refutation of Idealism to demonstrate both the possibility and the necessity of representing outer objects as also being *in time*.<sup>50</sup> At this point, in the Schematism, Kant can only assume that representations of outer objects are *in time* (because all representations, as states in us, are in time).<sup>51</sup> Hence it might have been more perspicuous if Kant had written: “every empirical representation of the manifold” is contained in time (A139/B178).

Schemata mediate categories with appearances because, as *time*-determinations, they are “homogeneous” with appearances.<sup>52</sup> But if appearances are in time (at this point in the *CPR*) only in the indirect sense that the sensible representations by which those appearances are given to us (intuitions) are in time, then schemata mediate categories with appearances by mediating categories with intuitions of appearances. Time is the “medium” of this mediation because all representations, as states of ours, are in time. Schemata, as mediating representations, are specifically *time-determinations*, because they further determine objects in respect of temporal features. But which objects? At this point in the *CPR* it cannot be that they immediately determine outer objects in respect of time, for Kant has not yet built the machinery for doing so (primarily, the Analogies).<sup>53</sup> Instead, schemata immediately determine the inner states by which such objects are given to us (intuitions, which *qua* states in us are in time). Specifically, they determine the temporal properties of those states.<sup>54</sup>

As a first approximation, then, we can say that schemata are transcendental time-determinations insofar as: (i) they allow us to determine objects with respect to categories by allowing us to represent given objects under them; and (ii) they do so by determining temporal relations among the states by which these objects are given to us (intuitions). The question is: why, in doing (ii), do schemata also do (i)?

## 2.4 | Empirical schemata as time-determinations

Schemata of categories are transcendental time-determinations. In building up to a more complete characterization of what that means, I am going to start by considering the schemata of empirical concepts, a topic Kant introduces and briefly discusses at A141/B180. Categorial schemata are mediating representations that make possible the representation of given objects under categories, while schemata of empirical concepts (henceforth, empirical schemata) make possible the representation of given objects under empirical concepts.<sup>55</sup> Hence empirical schemata are not transcendental but, as schemata, they are still time-determinations. This is indirectly confirmed in the text of the Schematism itself: “[An empirical concept] always refers immediately to a schema of the imagination as a rule for the determination of our intuition, in accordance with a certain general concept” (A141/B180). A “rule for the determination of our intuition” is, I take it, a rule for determining intuitions as temporally ordered states in us, rather than a rule for determining the object of that intuition (representing it under a concept), though of course Kant needs to show that the former will explain the latter.

But what does applying an empirical concept to an object (especially an outer object) have to do with representing our intuition of that object as temporally determinate? Kant's example of a dog is not especially helpful: “The concept of a dog designates a rule, according to which my imagination can draw the shape [*Gestalt*] of a four-footed animal, without being restricted to any particular shape [*Gestalt*] that experience may have offered me or any possible image [*Bild*] which I can present *in concreto*” (A141/B180). One thing that is notable about this example is that Kant has, in effect, “mathematized” the schema of the concept of a dog by thinking of it as a schema for drawing a shape in space, albeit an empirically specified shape rather than a purely geometrical one; he has partially assimilated it to the schema of a geometrical concept. However, I think Kant's example of the schema of the concept <dog>, if thought through more fully, contains the materials to understand better not only schemata of empirical concepts but mathematical and categorial schemata as well.

To represent an object under the concept <dog> is to think of that object as a dog. But as Kant is fully aware, to think of an object as a dog is not merely to think of it as having a certain shape.<sup>56</sup> The concept <dog> is not the

concept <dog-shaped object>, for an object could be a dog-shaped cat or a dog-shaped pile of wood. A dog is an object with a certain kind of history, a certain kind of life. In an Aristotelian vein, we could say that a dog is a being with a certain nature or form, one that dictates how it changes and develops over the course of time and in the right environment, and that the form of the dog is an inner principle of change by which it assimilates nutrients from the environment, maintains its internal constitution, grows into maturity, decays, and dies. These are considerations relevant to answering the question “What is a dog?” Kant, however, is concerned with the semi-transcendental question “What is it to represent something as a dog?”<sup>57</sup> But we can transform these object-oriented answers into answers to the Kantian question by internalizing them within representation: to represent an object as a dog is to represent it as having a certain kind of history determined by an inner principle of change, and so forth. Thus, in a Kantian vein, we could say: to represent an object as a dog is to represent it as temporally determinate in certain ways (i.e., as standing in specific temporal relations to its past and future stages). If you are representing something in front of you as having just come into existence and as about to go out of existence, then you are not representing it as a dog. (You may be representing it as a momentary dog-stage.) The schema of an empirical concept like <dog> might then plausibly be thought of as a time-determination.<sup>58</sup>

But in saying all of this, we will not yet have explained, or made any more plausible, the idea that schemata for empirical concepts are rules for representing *our inner states* as temporally determinate. Why should representing an object as a dog involve representing my very representation of the dog (a state of mine) as standing in some determinate range of temporal relations, rather than representing the object of that representation (the dog itself) as so standing? To return to a theme of the previous section, time is the form of inner sense, and space is the form of outer sense. I take this to mean that spatial content, but not temporal content, is “built into” intuitions of outer objects. Outer-sense intuitions of objects do not represent temporal features merely in virtue of being the kind of representations they are; by contrast, inner-sense intuitions represent temporal features solely in virtue of being the kind of representation they are.<sup>59</sup> I can represent outer objects as having temporal features, but this is not an ultimate representational fact: it stands in need of an explanation, that is, a specification of its ground. We cannot simply say that we *do* represent the object (the object of an outer-sense intuition) as having certain temporal features; we must say in virtue of what we are doing so.

Kant's answer to this question – In virtue of what are we representing an outer object as temporally determinate? – involves our representing our very representations of that object (states in us) as standing in corresponding temporal relations to other representations (other states in us). In particular, to represent an object as a dog is to represent one's present intuition of it as standing in determinate temporal relations to other representations: actual and merely possible past and future intuitions of the very same object.<sup>60</sup> To represent the object as a dog, for instance, is to represent one's present intuition of it as a representation of *the very same* object as, say, one's previous intuition of it as a puppy, and one's future intuition of it as an older dog. It is, more minimally, also to represent one's current actual intuition as being of the same object as a non-actual but merely possible *simultaneous* intuition of it from a different spatial perspective.<sup>61</sup> If you are representing your current intuition as being of an object that could not be represented from a different perspective, or could not have been represented in the past, or in the future, you are not representing that object as a dog (though you may be representing it as your visual image of a dog).<sup>62</sup>

Kant does not draw out these consequences of his example of the schema for <dog>. In fact, there are reasons he could not have, for his considered view in the third *Critique* is that representing something as a dog, an organized living thing, involves representing it as having a kind of internal purposive organization not accounted for by the transcendental theory of cognition in the first *Critique*.<sup>63</sup> So there are reasons to think Kant may have chosen a poor example, even by his own lights. But all of the points I have drawn out from <dog> could also have been drawn out from one of Kant's most famous examples, <cinnabar>. The difference is that cinnabar does not undergo (much) internal change, and a sample of cinnabar does not need to have any particular past or future as long as it maintains the same inner material constitution and causal powers that make it cinnabar. The temporal determinacy required to represent cinnabar is thus more flexible (i.e., less determinate)<sup>64</sup> than what is required to represent something as a dog. Just as with the case of the dog, we can transpose a purely object-oriented account of what it is to be cinnabar

(i.e., to have a certain causal history and causal power) into an account of what it is to represent something as cinnabar. To represent an intuited object as cinnabar is to represent one's present intuition of it as a representation of *the same object* as possible past intuitions of it, as a representation of the same object as possible experiences in which it passes the tests for mercury oxide, and so forth.

In this section, I have reconstructed Kant's reasons for two key claims:

*Schema<sub>emp</sub>*: The schemata of empirical concepts are rules for determinately<sub>s</sub> representing temporal relations among our inner states, in particular, the intuitions by which empirical objects are given to us.<sup>65</sup>

*Representational Sufficiency<sub>emp</sub>*: Representing a sensibly given object under the schema of an empirical concept (as specified above) is sufficient for representing that object under that concept.

In the next section, I will reconstruct Kant's reasons for parallel claims about geometrical concepts.

Intuitions are states of ours that represent objects. In contemporary terms, we can distinguish intuitions as *vehicles* of representation (states) from the *content* of those intuitions, which for expository purposes I will take to be their objects (though I am not assuming that objects exhaust the content of intuition – see Section 2.1). My discussion of empirical schemata will arouse in some readers the suspicion that my account falls afoul of this distinction, for, on my account, schemata are determinate representations of relations among inner states, that is, representational *vehicles*. Why, in representing these relations among vehicles, do I thereby represent something about the *content* of those vehicles (their objects)? Why do I represent the contents of these vehicles (their objects) as falling under the corresponding concept?<sup>66</sup>

This objection can be raised at every stage in my account of schemata, so it is worth responding to it here at the outset. I think this objection assumes an implausibly strong distinction between representing (relations among) vehicles and representing (relations among) contents. To take a linguistic example, names are vehicles and (let us assume for the sake of simplicity) their contents are their bearers. If “Brian” is a name coined at  $t_1$  to refer to some perceptually demonstrated person,<sup>67</sup> and “Alex” is coined in the same way at  $t_2$ , then in representing that “Brian” and “Alex” are co-referring terms, I am representing (if we now disquote the names) that Brian/Alex exists at  $t_1$  and at  $t_2$ . I have represented a relation among contents (objects, in this case) by representing a relation among representational vehicles. But if I had instead represented “Brian” (vehicle) as co-referential with a host of possible names coinable through perceptual demonstration from  $t_1$  to  $t_2$ , then I would have represented Brian as existing during this period in virtue of representing a relation between an actual name (“Brian”) and merely possible names (vehicles).<sup>68</sup> Likewise, if I represent my current intuition of an object as co-referential with possible intuitions during a period of time  $\Delta t$ , I am representing the object of my intuition as persisting through that period; in other words, I am representing the object of my intuition (its content, in this simplified story) under the concept *<persists through  $\Delta t$ >*. So if the worry is that by representing a relation among the vehicles (intuitions) by which empirical objects are given to me, I am not representing anything about their objects (i.e., I am not adding additional content to my representation of the object), then the objection is based on an exaggeration of the vehicle-content distinction.

## 2.5 | Mathematical schemata as time-determinations

I noted earlier that Kant's discussion of the schema for *<dog>* quasi-mathematizes this concept by treating it as the concept of a dog-shaped object. This suggests that Kant will be on stronger grounds in schematizing mathematical concepts. However, the idea that schemata in general are time-determinations faces the obvious objection that mathematical objects (e.g., triangles) are not in time,<sup>69</sup> so it is unclear what time-determination has to do with the possibility of subsumption in mathematics.

In the Schematism, Kant uses two arithmetical examples and one geometrical example:

[...] when I merely think of a number, which may be 5 or 100, this thinking is more the representation of a method for representing in an image a collection [*Menge*] appropriate to the concept (e.g. a thousand) than [it is] the image itself. (A140/B180)

The scheme of the triangle can exist nowhere else than in thoughts and designates a rule for the synthesis of the imagination in respect of pure shapes [*Gestalten*] in space. (A141/B180)

[...] the pure **schema of magnitude** (*quantitatis*), however, is **number**, which is a representation that collects together [*zusammenbefaßt*] the successive addition of unit to (homogeneous) unit [*Einem zu Einem*]. (A142/B182)

This leads some commentators on the Schematism to refer back to Kant's discussion of "drawing a line in thought" and "representing a certain number to myself" in the A Deduction (A102; cf. B154).<sup>70</sup> Kant's point in the A Deduction is, roughly, that one cannot draw a line in thought without mentally "reproducing" the previously drawn parts of the line.<sup>71</sup>

In drawing a line, or constructing a triangle in thought, one must not mentally "lose track" of the prior moments of one's construction. But if we think through the consequences of that point, we see a structure very similar to the structure I uncovered earlier in empirical schemata. A schema for a mathematical concept allows us to represent an object under a mathematical concept, for that is what schemata in general do.<sup>72</sup> For Kant, there is a close connection between mathematical subsumption and mathematical construction: to represent an object under the concept <triangle> is to represent it as the result of a possible act of construction.<sup>73</sup> This is why the schema of <triangle> is a rule for constructing a triangle, just as the schema of a number  $n$  is a rule for generating a collection of  $n$  elements (see the passages quoted above). So if we can uncover a structure of temporal determination within the act of constructing the triangle, we will have found such a structure in the act of representing an object under the concept <triangle>.

The canonical construction of a triangle ABC consists of three stages: (a) using a ruler, construct a line segment AB of arbitrary length; (b) using a compass, construct a circle of arbitrary radius around one endpoint of length  $l$  (A); and (c) using a compass, construct another circle with radius  $l'$  (where  $|| - \overline{AB} || < l' < || + \overline{AB} ||$ ) around the other endpoint (B). Let C be the point where the circles intersect; the sides of the triangle are the line segments AB, BC, and AC. This is represented graphically in Figure 1.

The straightforward reading of A102 is that one will not have constructed a circle if, at stage 2, one forgets stage 1 (likewise for stage 3 and stages 2 and 1). But a stronger claim is warranted: one is not constructing a triangle at stage 2 if one is not aware with regard to one's construction of the circle with radius AB that it is part of a construction of a triangle, completing the construction of AB (stage 1) and to be completed by constructing a circle around B

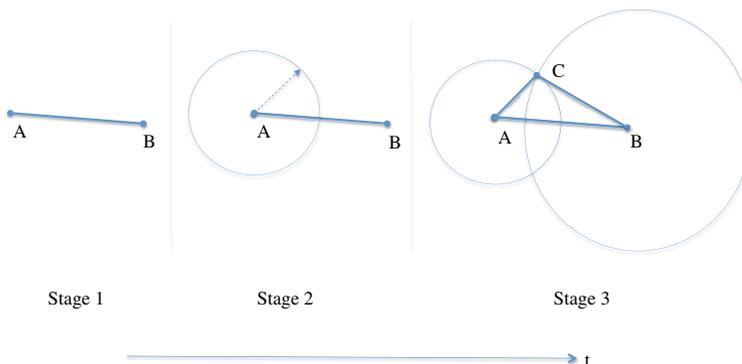
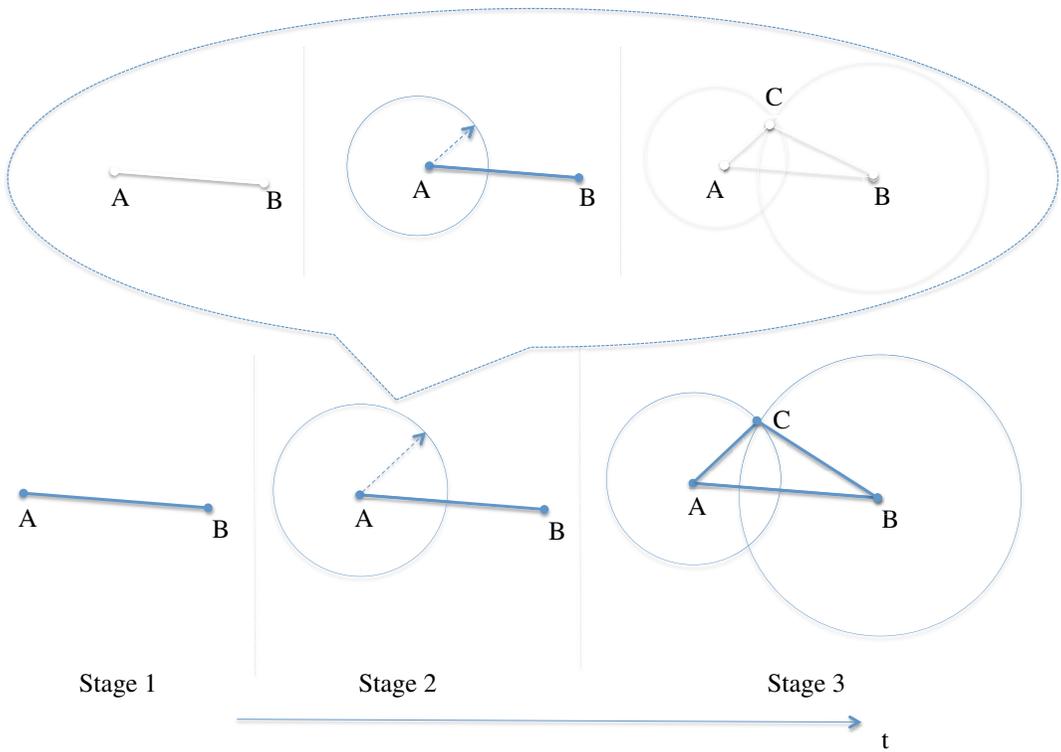


FIGURE 1 Stages of constructing a triangle



**FIGURE 2** Schema for constructing a triangle

(stage 3); one is not constructing a triangle at stage 2 if one is not aware of it as completing stage 1 and to be completed by stage 3; and one is not constructing a triangle at stage 3 unless one is aware that this is the completion of stages 1 and 2, and so forth. We can represent this graphically as the idea that stage 1 must contain a representation of Figure 1 as a whole, as well as its own place in that process, and likewise for the next stages. I have attempted to convey this in Figure 2. More precisely, in constructing a triangle one must be conscious at each stage of the construction that one's present constructive act stands in determinate temporal relations to past and future acts of construction. This means that construction involves not only object-directed consciousness (i.e., consciousness of the object being constructed) but consciousness of one's own constructive act, that is, consciousness of one's own representations. Only a being capable, at stage 2, of representing that very representation as part of some larger constructive procedure could construct a triangle. In other words, construction requires self-consciousness, that is, consciousness of one's representations *as such*.<sup>74</sup>

In the case of mathematical concepts, I take it that it is clearer than in the case of empirical concepts like *<dog>* that what we are representing as temporally determinate is our own representations. This is clear because the object represented (the triangle) is not in time and has no being outside our construction of it. The only items here that could be represented as temporally determinate are the representations that make up our complex constructive act.<sup>75,76</sup>

The rule for constructing a triangle is what Kant identifies as the schema of *<triangle>*. I have argued that this rule can be understood as a rule for representing the determinate temporal relations among various representations. This explains, I take it, the sense in which even mathematical schemata are time-determinations. They are specifically *transcendental* time-determinations because they explain the possibility of a priori cognition: the a priori construction of a triangle.<sup>77</sup> Instead of turning to the case of arithmetical schemata, in the next section I explain why schemata of the pure concepts of the understanding are transcendental time-determinations.

In this section, I have reconstructed Kant's reasons for two key claims:

*Schema<sub>geo</sub>*: The schemata of geometrical concepts are rules for determinately<sub>S</sub> representing temporal relations among our inner states, in particular, the states involved in constructing a corresponding object.

*Representational Sufficiency<sub>geo</sub>*: Representing an object under the schema of a geometrical concept (as specified above) is sufficient for representing that object under the concept.

In the next section, I will reconstruct Kant's reasons for parallel claims about the categories.

## 2.6 | Transcendental schemata as time-determinations

I have argued that schemata in general are time-determinations, that is, rules for (self)-consciously representing one's representations (inner states) as standing in determinate temporal relations. In this section, I want to apply this analysis to the primary case of schemata, that is, the schemata of pure concepts of the understanding (categories). Kant's discussion of the schemata of individual categories at A142–145/B182–184 has a recursive structure, insofar as later sets of categories are schematized in terms of previous sets. Consequently, I will discuss the schemata of the categories in the order in which they appear in the Table of Categories itself: quality, quantity, and relation. For reasons of space, I forgo discussion of the schemata of modal categories.<sup>78</sup>

### 2.6.1 | Quantity

In my discussion of mathematical schemata earlier, I focused on geometrical schemata to the exclusion of Kant's other main mathematical example, arithmetical schemata for concepts of numbers. I did so because numerical schemata arise in the course of Kant providing schemata for the categories of quantity, and thus deserve a separate treatment:<sup>79</sup>

[...] the pure **schema of magnitude** (*quantitatis*), however, is **number**, which is a representation that collects together [*zusammenbefaßt*] the successive addition of unit to (homogeneous) unit [*Einem zu Einem*]. Number is thus nothing other than the unity of the manifold of a homogeneous intuition in general [*überhaupt*], through which I generate [*erzeuge*] time itself in the apprehension of the intuition. (A142/B182)<sup>80</sup>

It is notable that Kant does not give individual schemata for each of the categories of quantity – <unity>, <plurality>, and <totality> – but instead gives a single schema for quantity in general, which he calls “number” (*Zahl*).<sup>81</sup> I take this to mean that each of the categories of quantity is to be schematized, and thus given conditions of application to objects, in virtue of its role within a unified act of numbering or counting.

The structural moments of such an act of counting are relatively easy to articulate. To assign a whole number *n* to an object, and thus to think of it as an object with a *quantitas* (i.e., a determinate limited magnitude) is to identify a *unit* of numbering. This can be as simple as picking some part of the object as the unit to be counted or as (relatively) complex as picking some standard length as the unit.<sup>82</sup> I will focus on the simple case of numbering the parts of an object. First, we identify some part of the object as the unit of counting; for instance, given a book, we can count either chapters or pages. After that, we successively distinguish and enumerate the parts of the object that are conceptually homogeneous with the chosen unit; for instance, having chosen to count the pages in the *CPR*, we successively distinguish each of the pages in the book. The process ceases when we have counted every part of the object, when no parts remain that were not distinguished and enumerated at any previous stage.

The structural moments of a whole number  $n$  as the quantity of an object, considered as a collection of parts, relative to a choice of unit, are given in Figure 3.

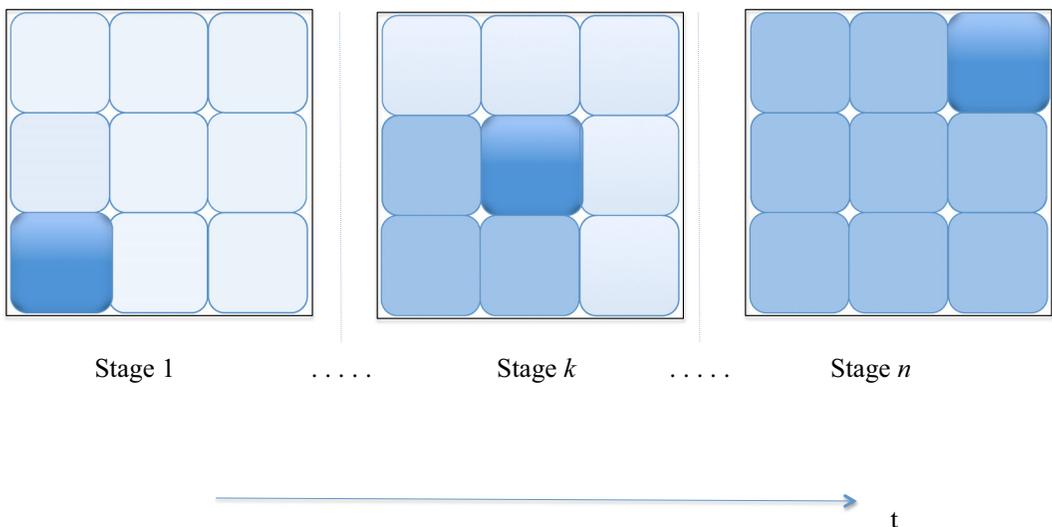
As with the geometrical example from Section 2.4, the straightforward way of reading Kant's discussion of reproduction in the A Deduction (A102) is that at each stage of this process one must not "forget" the previous stages. In line with my discussion of the geometrical example, however, I think we can say something stronger: in order for any of the stages distinguished in Figure 3 to be parts of a mental act of counting an object, one must represent the entire process represented in Figure 3 as well as the relation between the current stage and the rest of the process. One must represent one's current representation as the enumeration of the  $k$ -th part, completing the enumeration of the first to  $(k - 1)$ -th and to be completed by the  $(k + 1)$ -th to  $n$ -th parts. In so representing the relation of one's current representation to previous and past representations, one is representing one's representation as standing in determinate temporal relations. This is the arithmetical version of a point we saw in geometric construction in the previous section. Represented graphically, it is the requirement that at each stage distinguished in Figure 3, the whole of Figure 3 must be represented as well as one's current position within that process. Figure 4 is an attempt to illustrate this point.

Earlier I said that each of the categories of quantity is to be schematized by its role in this unified act of quantitatively determining a magnitude. In particular, when we specify some part as the unit of counting, we subsume that object under the category *<unity>*; at each stage in the enumeration, we subsume the relevant sub-collection enumerated at that stage under *<plurality>*; and when we complete the process we subsume the whole collection under *<totality>*, a single object composed of a plurality with a determinate magnitude with respect to the unit (in this case,  $n$ ).<sup>83</sup> Subsuming objects under these categories just is representing the intuitions in which they are given as components of this complex self-conscious act of time-determination.<sup>84</sup>

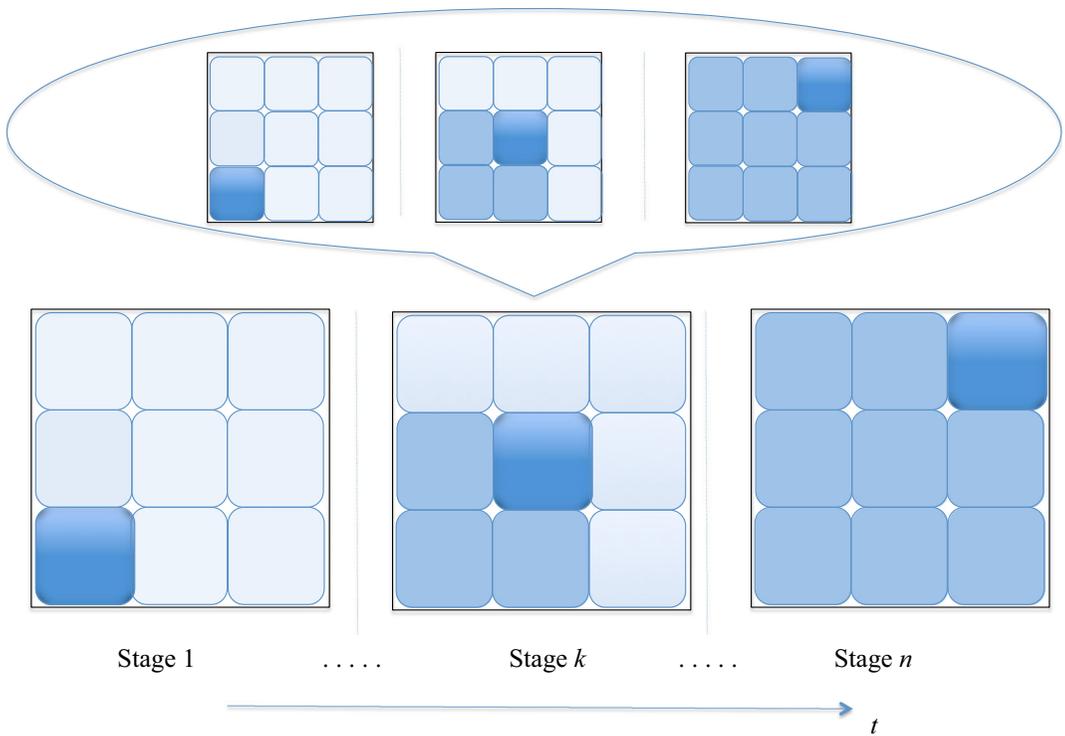
In this section I have reconstructed Kant's reasons for two key claims:

*Schema<sub>quant</sub>*: The schema of the categories of quantity is a rule for determinately<sub>s</sub> representing temporal relations among our inner states, in particular, the states involved in representing a sensibly given object as a determinate<sub>s</sub> magnitude.

Because there is only one schema for three different categories of quantity, the individual categories are schematized by their role in the complete schema specified above. Consequently, to represent an object under the relevant moment or aspect of the complete schema is to represent it under the corresponding category:



**FIGURE 3** Stages of counting to  $n$



**FIGURE 4** Schema for counting to  $n$

*Representational Sufficiency<sub>quant</sub>*: Representing a sensibly given object under the schema of quantity (as specified above) is sufficient for representing that object under quantity as such (i.e., representing it as having a quantity); representing an object under a specific moment of that schema is sufficient for representing it under the corresponding category.

In the next section, I will reconstruct Kant's reasons for holding parallel principles about quality.

## 2.6.2 | Quality

Kant's initial characterization of the first two categories of quality, *<reality>* and *<negation>*, is deceptively simple:

Reality, in the pure concept of the understanding, is that which, in general, corresponds [korrespondiert] to a sensation. Thus, [it is] that whose concept contains in itself a being in time, while negation is that whose concept represents a non-being (in time). The opposition of the two occurs, consequently, in the distinction of one and the same time, as either a filled or an empty time. (A143/B183)

The distinction Kant here draws between filled and empty time relies on the distinction between form and matter. Time is the form of inner sense (thus, the form of our inner states as intuited by us) but the matter that time

“en-forms” is the very inner states (representations) we intuit in temporal order. The distinction between <reality> and <negation> is the difference between matter en-formed by time (“filled” time) and pure form without en-formed matter (“empty” time). This means that the representational states we are concerned with are no longer pure intuitions, but perceptions: empirical intuitions with a sensory matter that we consciously apprehend (B162, B207).<sup>85</sup> The concept of <reality> is to be applied to whatever in objects “corresponds to” this sensory matter in us, which Kant identifies with “thinghood” (*Sachheit*) and which he will later be in a position to identify as causal powers in spatiotemporal objects, once he has introduced the categories of relation.<sup>86</sup>

The difference between the positive presence of some sensory matter in us and its absence (as well as between the presence and the absence of the corresponding reality in the object) is continuously gradable, not discrete:

Now, every sensation has a degree or magnitude, by which it can more or less fill the same time, i.e. inner sense in respect of the same representation of an object, until it ceases in nothing (= 0 = *negatio*). Consequently there is a relation and connection between, or, better, a transition from, reality to negation, which makes every reality representable as a quantum. (A143/B183)

Thus, even if the complete absence of sensory matter is representable, we can still think of the total absence of reality in general as the limit of a process of continuous decrease in reality. What is more, we can think of the absence of a specific reality (corresponding to a specific kind of sensory matter, e.g., redness) as an achieved limit.<sup>87</sup>

Kant explicitly gives the schema for quality only in the rest of the paragraph: “[...] the schema of a reality, as the quantity of something insofar as it fills time, is precisely this continuous and uniform generation [*Erzeugung*] of [that reality] in time, as one decreases the sensation over time, which has a determinate degree, until its complete disappearance, or one gradually increases from zero to the magnitude of the sensation” (A143/B183). Just as with the categories of quality, Kant does not give a different schema for each category of quality (<reality>, <negation>, and <limitation>) but gives a single schema, which I take to be the schema for the whole moment of quality. Again, as with quantity, Kant specifies the schema as a single complex mental act (“this continuous and uniform generation”). I take it that, in parallel with quantity, the individual categories of quality are schematized insofar as they are identified with structural moments of that complex mental act, so I will analyze that complex mental act and isolate the relevant moments.

The complex mental act is the representation of the gradual diminution of a sensory matter of a particular kind to the limit case of total absence of that kind of sensory matter. Unlike the case of quantity, this mental act does not have discretely separable parts; there are infinitely many “degrees” that separate the presence of a given sensory matter from its total absence (= 0). To represent it as having a determinate magnitude, we must represent it as *generable* (i.e., possibly generated) from smaller magnitudes: we must represent the sensory matter in us as the limit of some possible continuous process of increasing magnitude, and as the beginning of some possible process of continuous increase (assuming no sensory reality has an absolute maximum).<sup>88,89</sup> This means that in order to represent our present sensory matter (<reality>), we must self-consciously represent it as standing in various relations to possible past and future representations: it is the limit of some possible past process of continuous increase (<reality>), the beginning of some possible future process of continuous increase (<limitation>), and the beginning of some possible future process of continuous diminishment whose limit is its absence (<negation>).<sup>90</sup> In doing so, we represent what corresponds to sensation in the object as reality, as limitation, and as negation.<sup>91</sup>

So again we have the recursive structure we have seen already in the case of empirical, mathematical, and quantitative schemata. To represent the object of perception as falling under a category (i.e., as subsuming the object under the category) is to self-consciously represent that perception as standing in determinate temporal relations to possible past, present, and future perceptions. In each case, we have the structure of some temporally complex mental act (e.g., experiencing an empirical object over time, constructing a geometrical figure, counting), such that every stage in that act must be self-consciously represented as a stage in an actual or possible such act in order to constitute a stage in an act of that type.<sup>92</sup>

In this section I have given an account of Kant's reasons for two key claims:

*Schema<sub>qual</sub>*: The schema of the categories of quality is a rule for determinately<sub>S</sub> representing temporal relations among our inner states, in particular, the states involved in representing sensory matter as continuously increasable or decreasable.

Because there is only one schema for three different categories of quality, the individual categories are schematized by their role in the complete schema specified above. Consequently, to represent an object under the relevant moment or aspect of the complete schema is to represent it under the corresponding category:

*Representational Sufficiency<sub>qual</sub>*: Subsuming<sub>S</sub> the perception by which an object is given under the schema of quality (as specified above) is sufficient for subsuming<sub>S</sub> that object under quality as such (i.e., representing it as having a quality); subsuming an object under a specific moment of that schema is sufficient for subsuming<sub>S</sub> it under the corresponding category.

### 2.6.3 | Relation

Matters are significantly more complicated with the schemata of the categories of relation, for here (for the first time) Kant gives a separate schema for each category. For reasons of space, I will discuss only <substance>, which Kant schematizes as follows: “The schema of substance is the persistence of the real in time, i.e. the representation of it as a substratum of empirical time-determination in general, which remains while everything else changes [wechselt]” (A144/B183).<sup>93</sup>

Kant's discussion here is complicated by the fact that he is anticipating a significant amount of material that will not be developed until later, in the Analogies (especially the first Analogy), some of which appears as a parenthetical remark after the sentence quoted above. Representing outer objects as “outer” (not transcendently outer but “empirically” outer; cf. A373) means representing them not only as the objects of my states but as objects in space that other subjects can access through their perceptions as well. We are thereby said not only to “perceive” objects but to “experience” their “existence,” which I take to mean: representing them as existing independently of our perceptions of them.<sup>94</sup> Kant's argument in the Analogies, which is partly anticipated here, is that we do this by representing the states of those objects as having a temporal order that is in principle different from the temporal order of the perceptions (which are states in us) by which they are given to us.<sup>95</sup> In effect, we externalize time by “projecting” it onto outer objects. Time has three structural aspects or “moments”: unity (there is only one time; all states in time are temporally related), succession (states are ordered by an irreflexive, asymmetric, transitive relation of priority), and simultaneity (some states are neither before nor after one another).<sup>96</sup> The unity of time, combined with its successive and simultaneous structure, entails that any two states in time are either simultaneous or successive. Kant's argument in the first Analogy is that in order to represent “outer time” (the time of outer objects) as unitary, we must represent an absolutely persisting subject – namely, substance – of which the states changing in time are mere accidents.<sup>97</sup>

Kant's task in the Schematism is not to make that argument but to show how it is possible for sensible objects to be substances and accidental modifications in the first place (i.e., to subsume them under the category <substance-accident>). We can think of the difference between the tasks of the Schematism and the first Analogy as the difference between, respectively, explaining how it is possible to subsume objects under <substance> (Schematism) and arguing that it is necessary to do so (first Analogy).<sup>98</sup>

The schema of <substance> relies on the schema of the categories of quality: “The schema of substance is the persistence of *the real* in time” (A144/B183). In order to represent spatial objects as substances we must represent them as having realities, that is, the features in them that correspond to the sensory matter of our representations;

this requires the whole complex machinery explored in the previous subsection. We further represent these objects as substances when we present their reality as persisting at all times, as the ultimate “substratum” in virtue of which states of outer objects are temporally determinate with respect to one another. As Kant writes near the end of the first Analogy, “Substances are the substrate of all time-determination” (A188/B231).<sup>99</sup>

I read “determination” here as determination<sub>M</sub>, such that substance is the “substratum” of temporal determination in the sense that it is what makes it possible for outer objects to be determinately<sub>M</sub> temporally related to one another.<sup>100</sup> My reconstructive strategy is first to distinguish representation from subsumption, and thus to distinguish determination<sub>S</sub> from determination<sub>M</sub>, in order to explain why representing objects under schemata is sufficient for representing them under categories (Representational Sufficiency) and then, via the Representation-Subsumption Link, to use this to explain the subsumption of objects under those categories. Thus, while Kant is already engaged directly with subsumption and determination<sub>M</sub> in the Schematism, at this stage in my reconstruction I am still dealing with the semantic level of representation and determination<sub>S</sub>. In Section 3.2 I will explain why this has immediate consequences for subsumption and determination<sub>M</sub>.

Yet again, in the schema of <substance>, we have the structure of reflexive self-conscious temporality that we have seen in the other cases. To represent an object as a substance is to represent it as possessing a reality (corresponding to the sensory matter in my representations) that is present at all times. This means that it is the very same substance that has been experienced in the past and will be experienced again in the future. Obviously, in order to experience it as present at all times, I do not need to have ever experienced it before or to ever encounter it again; I need only represent it as *possibly* experienced in the past and *possibly* experienced in the future. I also need to represent it as possibly experienced in the *present*, and not merely for the trivial reason that in representing it as *actually* experienced now, I trivially represent it as *possibly* experienced now; the reason is rather that I must represent it as a persisting object in space that could have been experienced by me in different ways now. (It could have been perceived from a different spatial vantage point, for example.) In representing it as possibly experienced in the past and possibly to be encountered again in the future, I am, for reasons parallel to those given in Section 2.4, representing my current experience of it as related to possible and actual past and future experiences. In other words, I am representing it as determinately temporally related to possible past and future representations. It is something that was not experienced in the past but could have been, which will be experienced in the future under certain conditions, and so forth. Just as we “transposed into representation” the Aristotelian considerations about the possible life of a dog, we can transpose purely object-oriented considerations about the possible spatiotemporal trajectory of a substance into considerations about possible and actual past and future experiences that would be experiences of the same substance.<sup>101</sup>

Just as in the other cases, we have a relatively simple account of the mental act of representing an object under <substance> – namely, thinking of it as persisting at all times – and then we have the more complex structure in which that structure is, so to speak, embedded in every one of its own stages. More precisely, for any stage in the cognition of a substance to be a state in such a cognitive act, one must be self-consciously representing that stage as a determinate stage in a sequence of possible and actual past, present, and future experiences of the same object.<sup>102</sup> If one’s representation of the object has this complicated, recursive temporal structure, then one has succeeded in representing the object under the category <substance>.

In this section I have given an account of Kant’s reasons for two key claims:

*Schema<sub>sub</sub>*: The schema of <substance> is a rule for determinately<sub>S</sub> representing the temporal relations among our inner states, in particular, the states by which a temporally persistent object is given to us.

*Representational Sufficiency<sub>sub</sub>*: Representing a sensibly given object under the schema of <substance> (as specified above) is sufficient for representing the object under that concept.

In concluding Part One, I hope to have established the following for each of the kinds of concepts considered here and their associated schemata:

*Schema*: The schema of a concept is a rule for determinately<sub>S</sub> representing the temporal relations among our inner states, in particular, the inner states by which an object is given to us.

*Representational Sufficiency*: Representing a sensibly given object under the schema of a concept is sufficient for representing it under the corresponding concept.

My argument for the latter principle has been by cases. In my defence, this is precisely how Kant proceeds in the Schematism: he provides schemata for the categories, case by case. So the argument for Representational Sufficiency in its full generality will be incomplete until we have explained how each of the remaining categories (<cause-effect>, <reciprocal action>, and the modal categories) are transcendental time-determinations for which Representational Sufficiency holds. For reasons of space I cannot do that here, but I hope that it is reasonably clear how my account can be extended to the remaining categories.

Representational Sufficiency, combined with the relatively trivial Representation-Subsumption Link from Section 2.2, entails:

*Subsumption Sufficiency*: Subsuming a sensibly given object under a schema is sufficient to represent it under the corresponding concept.

While Representational Sufficiency has been my stalking horse in Part One, my ultimate aim all along was to establish Subsumption Sufficiency for empirical and geometrical concepts, as well as categories. That principle, together with my argument for it over the previous four sections, also constitutes my answer to question Q2(a) from the Introduction.

## 3 | PART TWO

### 3.1 | Real possibility for empirical and geometrical concepts

The ultimate aim of the Schematism is to explain the possibility of the subsumption of objects under categories. Combined with Subsumption Sufficiency, for which I have now argued at length, the fairly trivial Subsumption-Possibility Link effectively reduces this problem to the problem of explaining the possibility of subsuming objects under schemata. In this section and the next I will reconstruct Kant's explanation of why subsumption of objects under schemata is possible, that is, the Real Possibility premise from the Introduction (and his answer to Q2(b) from the Introduction). This completes Kant's explanation of why subsumption of objects under categories is possible.

Recall the notion of homogeneity from Section 2.1: an object and a concept are homogeneous just in case the sensible marks (features) intuited in the object are discursively represented by the concept; in other words, an object intuited with an F mark is homogeneous with <F>. An object is subsumed under a concept just in case its sensible features are recognized as singular instances of the discursive marks contained in that concept. The problem of subsuming objects under categories is that the sensible features in objects are not homogeneous with categories. Intuitions do not represent objects as F, where <F> is a category (or even a mark of a category). A feature F of an object is a pure intuitive feature just in case it is a determinate of some determinable D, where D is grounded in the very form of intuition.<sup>103</sup> Categories, although they are pure concepts, are not concepts of pure intuitive features.

The machinery reconstructed so far issues in an easy explanation of the subsumption of objects under empirical concepts. In Section 2.4 I specified the schemata of empirical concepts, focusing on Kant's example of the schema of the concept <dog>. That schema involved representing a sensibly given object as dog-shaped and representing temporal relations among the intuitions by which that object is given. I take it that the homogeneity of dog-shapedness with <dog-shaped> is unproblematic, for the former feature is a determinate of a determinable (in this case, shaped) that objects possess solely in virtue of the form of (outer) intuition. Likewise, the temporal relations among the states

by which the object is given to us are themselves intuitive features, that is, determinate specifications of the determinable form of inner states in general (time). But my articulation of the schema of the empirical concept <dog> (Kant's example) made use of a relation among intuitions that, some readers will object, is not an intuitive feature at all: the relation of two intuitions co-referring, that is, being intuitions of one and the same object.

Some readers will remain sceptical about whether *having the same object* (e.g., the same dog) is an intuitive relational feature of intuitions. I address that scepticism in the next section. Assuming my argument there is successful, there is no further difficulty in explaining how subsuming sensible objects under these schemata is possible: sensible objects can stand in these relations solely in virtue of being objects of intuition.<sup>104</sup> Given Subsumption Sufficiency<sub>emp</sub> (see Section 2.4) and the Subsumption-Possibility Link (see the Introduction), it follows that subsuming objects under those empirical concepts is really possible.

Geometric schemata (see Section 2.5) represent spatial features of their objects as well as temporal relations among the stages of the procedure by which the object is constructed. But the temporal relations they represent (stage 1 is *before* stage 2, which is *before* stage 3; see Figure 2) are intuitive features: they are determinate specifications of determinable features that objects have solely in virtue of being intuited. The very form of time determines that inner states stand in an asymmetric relation of succession (A31/B47), so in representing the determinate temporal relations among the parts of a constructive procedure, a geometric schema is not representing anything more than intuitive relational features.<sup>105</sup> Given Subsumption Sufficiency<sub>geo</sub> (Section 2.5) and the Subsumption-Possibility Link (Introduction), it follows that subsuming objects under these empirical concepts is really possible.

What is problematic – what the entire machinery of schemata is meant to explain – is the real possibility of a sensible object being subsumed under a concept that is not merely a concept of some feature that is a specification of the form of intuitable objects in general. I complete that explanation in the next section.

### 3.2 | Real possibility for categories

Immediately after the discussion of the plate example and the introduction of the notion of homogeneity (see Section 2.1), Kant writes:

Now it is clear that there must be a third [thing], which is homogeneous with the category, on the one hand, and with the appearance, on the other, and which makes possible the application of the former to the latter. This mediating representation must be pure (without anything empirical) and yet, on the one hand, **intellectual**, and, on the other, sensible. Such [a representation] is the **transcendental schema**. (A138/B177)

The question now is how to understand the homogeneity of a schema with the category on the one hand, and a sensible object on the other. With respect to the former, Kant is not especially helpful: “Now a transcendental time-determination is homogeneous with the **category** (which constitutes its unity) insofar as it is **universal** and rests on a rule *a priori*” (A138/B177). But since all schemata are universal *a priori* rules, this does not explain why a schema is homogeneous, specifically, with the category of which it is the schema. However, we already have at hand an account of the homogeneity, specifically, of schemata and the categories that they schematize, namely, Subsumption Sufficiency. Subsuming an object under a schema is sufficient for subsuming it under the corresponding concept. Combined with the Subsumption-Possibility Link, this entails that explaining the possibility of subsuming an object under a category reduces to the more tractable problem of explaining the possibility of subsuming it under the corresponding schema. So the problem that remains is explaining the homogeneity of objects and schemata.

On the homogeneity of schemata with appearances, Kant has this to say:

[The schema] is, on the other hand, homogeneous with the appearance insofar as time is contained in every empirical representation of the manifold. Thus an application of the category to appearances

will be possible by means of the transcendental time-determination, which, as the schema of the concept of the understanding, makes the subsumption of the latter under the former possible. (A138–139/B177–178)

Drawing on the example of the plate and my account from Part One of schemata as transcendental time-determinations, a schema will be “homogeneous” with sensible objects if the temporal relations determinately<sub>s</sub> represented in the schema are specifications of the relevant form of intuition itself, that is, time. If they are, then the content of the schema will stand to pure intuitable features in the same unproblematic relation that <round> stands to roundness in objects.

I will first consider the schema of quantity. Referring back to Figure 4, the schema of a determinate magnitude represents temporal relations among individual stages of a process of enumeration, that is, individual representational acts of identifying parts in a collection (the collection being enumerated). These temporal relations are what I have called “pure intuitive features”: relations of *before* and *after*. Earlier stages of the enumeration are “completed” by later stages: the  $k$ -th stage is the  $k$ -th stage of the enumeration of a collection of  $n$  objects only if it follows previous stages of enumerating the first to  $(k - 1)$ -th parts and is followed by later stages of enumerating the  $(k + 1)$ -th to  $n$ -th parts, *and* the subject of enumeration is self-consciously aware of these temporal relations. Again, I take these temporal relations to be pure intuitive features that are unproblematically homogeneous with their conceptual representation by the relevant schema. This explains why it is possible to subsume objects under quantitative schemata.

Without repeating my whole account of the schema of quality, we can see relatively easily why this schema refers only to pure intuitive features of objects and our representations of them. The schema of quality is the self-conscious representation of the matter of one's perception (i.e., its sensory content) as being both continuously decreaseable to its total absence (= 0) and arbitrarily increaseable. This requires representing one's perception of the object as standing in a certain relation to possible past and future perceptions, namely, the relation of having a greater or lesser degree of that sensory reality in them. The temporal relations are clearly what I have called “pure intuitive features” and thus generate no inhomogeneity and no problem of subsumption. But the specifically sensory relations of possessing more or less of a particular kind of sensory matter are also (relational) intuitive features. Solely in virtue of being objects of inner sense, our inner states possess a matter. That is to say, it follows from the form of inner sense that inner states possess matter, but the form alone does not determine which matter they possess. Likewise, it follows from the very form of space that intuited objects have some shape or other, but the form alone does not determine which shape they possess. And just as it follows from the very form of space that one spatial object can be larger than another, and this relation of *largeness* is continuously gradable (there is no “next largest” object); it follows from the form of inner sense that the sensory matter in one state can be “greater” than the sensory matter in another state and that this relation of “greater” is continuously gradable. Thus, in representing relations of continuous gradability in the sensory content of our inner states, the schema of <reality> represents exclusively those features that objects and the sensory states by which they are given are able to possess solely in virtue of the form of our (inner) intuition. Again, I take these to be pure intuitive features that are unproblematically homogeneous with the relevant schemata. This explains why it is possible to subsume objects under qualitative schemata.

The schemata of geometrical concepts, and quantitative and qualitative categories, are to some extent the easy case for Kant's theory. For these are concepts of features that objects have solely qua intuited: they are spatial (geometrical), denumerable (quantitative), and possess continuous degrees of reality corresponding to the reality in sensation (qualitative). The hard case for Kant will be the categories of relation.<sup>106</sup> Since in Section 2.6 I explained only the schema of <substance>, I will here limit myself to explaining why the subsumption of objects under that schema is unproblematic. In future work I will extend this account to the other relational schemata (<cause-effect> and <interaction>).

Recall the schema that Kant gives for <substance>: “The schema of substance is the persistence of the real in time, that is, the representation of it as the substratum of empirical time-determination in general, which therefore remains when everything else changes [*wechselt*]” (A144/B183). As I observed in Section 2.6, <substance> is

explicitly schematized in terms of a prior category, namely, <reality>. In the context of explaining the subsumption of objects under schemata, this recursive structure takes on an added significance, for it means that if it is explicable why objects can be subsumed under the schema of <reality>, then to the extent that <substance> is schematized in terms of <reality>, to that extent subsumption under <substance> is also explicable.

But there is more to the schema of <substance> than merely this reference back to <reality>: “The schema of substance is the *persistence* of the real in time” (A143–144/B183, Au’s emphasis). The question now becomes: What explains the homogeneity of “persistence” with the realities in objects? What explains the possibility of subsuming real objects under the concept <persistence>?

To answer this question, we must understand what the persistence of substance amounts to.<sup>107</sup> As other commentators have noted,<sup>108</sup> in the first Analogy Kant makes two *prima facie* distinct claims:

1. In all empirical change, substances persist.<sup>109</sup>
2. In all empirical change, the quantity of substance is neither increased nor diminished.<sup>110</sup>

These claims are logically distinct. That substances persist through all change does not entail that their quantity neither increases nor decreases; it does not even tell us what the “quantity” of substance refers to. It cannot refer to the number of substances – that is, how many substances there are – for the mere conservation of the number of substances is compatible with massive changes in their quantity of reality (e.g., their mass or causal power). Even more problematically, while Kant’s argument for (1) is not without its problems, the argument for (2) appears to be missing altogether.<sup>111,112</sup>

The solution to the apparent ambiguity in Kant’s conclusion, as well as the apparent lack of an argument for (2) in the first Analogy is that (1) and (2) amount to the same thing for Kant: the persistence of substances across time just is the persistence of their quantity, where this quantity is to be identified with the quantity of their reality (i.e., the quantity of reality present in them). Kant concludes the summary of the Proof in the B Edition as follows: “That which persists, in relation to which all temporal relations of appearances can be determined, is substance in the appearance, i.e., the real in the appearance, which as the substratum always remains the same. Since this, therefore, cannot change in existence, its quantum in nature can also be neither increased nor diminished” (A182/B225). In the first sentence Kant claims that what persists through all change is substance. If we understand this to mean (1) that substances are what persist through all change, the second sentence is a non sequitur: that substances cannot “change in existence” (i.e., they cannot come into or go out of existence) does not entail that their quantity (i.e., how much reality, that is, how much substance, there is in them) cannot change.<sup>113</sup> However, it does follow directly if all there is to the persistence of substances is the persistence of the quantity of their reality. The ambiguity of the first Analogy, both in its conclusion and in its proof, is resolved if we understand (1) via (2). The persistence of substances just is the persistence of the quantity of substance, which is the quantity of reality in them.

This is the key to explaining the possibility of subsuming objects under the schema of substance. We have already schematized <reality>, and part of that schema was the representation of realities as intensive magnitudes, that is, as possessing degrees. If we can subsume objects under the concept of <reality> (i.e., the concept of an intensive degree of reality), then we can subsume them under the concept of having the same or different degrees of reality – that is, we can represent reality *R* in object *x* at time  $t_1$  as the same quantity of reality in object *y* at time  $t_2$ . But this alone is not sufficient. We need one additional stage to ensure that we are representing *x* and *y* as being a single numerically identical locus of a certain quality of reality, as opposed to representing them as, for instance, distinct objects that happen to have the same degree of reality (e.g., two objects of equal mass). If we further represent *x* and *y* as joined by a continuous spatiotemporal path, then we are representing them as the same substance. We are representing  $x (= y)$  as a persisting locus of reality in appearance that exists at both  $t_1$  and  $t_2$ .<sup>114</sup>

This is the object-oriented account of representing an object as a persisting substance. To transpose it into representation is to represent perception  $p_1$  at time  $t_1$  as a perception of the same substance as perception  $p_2$  at time  $t_2$ . But, again, the recipe for doing this follows almost immediately from the schemata we have already discussed. If

we can represent a perception as a perception of an object with reality of a certain degree, then we can represent two perceptions as being perceptions of objects with equal (or different) degrees of reality. To represent those perceptions as perceptions of the very same substance (rather than distinct substances that happen to have the same degree of reality), we must represent those perceptions as perceptions of an object with a spatiotemporally continuous path. If we combine these representations (representing  $p_1$  and  $p_2$  as perceptions of an object with the same degree of reality, and as representations of an object with a spatiotemporally continuous path), then we are representing them as perceptions of numerically the same substance. The schematization of *<reality>*, if successful, explains why it is possible to represent perceptions in the first way (i.e., to represent  $p_1$  and  $p_2$  as perceptions of an object with the same degree of reality). But note that the second representation (representing  $p_1$  and  $p_2$  as perceptions of an object with a continuous spatiotemporal path) is a representation of a pure intuitive feature: perceptions, just in virtue of being intuitions, can represent objects as being joined by a continuous spatiotemporal path.

Representing an object as a substance involves representing it as permanently present, that is, as existing at all times. This means representing that past and future perceptions, at any time, might be perceptions of the very same substance. This requires representing two kinds of relation: (i) purely temporal relations among possible and actual perceptions (before, after) at any time; and (ii) the relation of co-referring, that is, of being perceptions of one and the same substance (i.e., one and the same spatiotemporally continuous quantity of substance). I take it that it is now clear that (i) involves what I have been calling “pure intuitive features,” that is, features that objects can possess solely in virtue of the form of intuition. Solely in virtue of the form of inner sense, the states (i.e., intuitions) by which objects are given to us can be before, after, and simultaneous with one another. So the subsumption of our perceptions of objects under (i) is unproblematic. In the previous paragraph I explained why the homogeneity of the representation of two perceptions in the second relation follows from my account of the schema of *<reality>*. I conclude that the real possibility of subsuming an object under the schema of *<substance>* stands in no need of further explanation. By Representational Sufficiency<sub>sub</sub> and the Subsumption Link, it follows that the real possibility of subsuming a sensible object under *<substance>* is also thereby explained.

My example of a schema for an empirical concept, *<dog>*, involves representing two perceptions as co-referring, as being perceptions of the same object. Now that we have explained how subsuming objects under *<substance>* is possible, it is easy to “piggyback” on that explanation. Dogs are not substances (since they do not persist absolutely, much to the chagrin of dog lovers everywhere), but relatively persistent accidental modifications of substance (matter). Thus, to represent two perceptions as perceptions of the same dog is to represent them (i) as both perceptions of a dog, and (ii) as perceptions of the same quantity of substance, joined by a continuous spatiotemporal path (iii) that remains a dog at every time between the two perceptions. Finally, to represent a perception as the perception of a dog is just to represent it as standing in this *of-the-same-dog* relation to possible and actual past and future perceptions, not at all times, but for some amount of time appropriate to the lifespan of a dog.<sup>115</sup>

## 4 | CONCLUSION

I began with what is, I argued, the claim implicit in Kant's idea that schemata “mediate” the subsumption of objects under categories:

*Subsumption Sufficiency:* Subsuming an object under a schema is sufficient to subsume it under the corresponding concept.

I then “factored” this claim into two further claims that together entail it, the first of which is fairly trivial, the second more substantive:

*Representation-Subsumption Link:* If representing an object under a schema is sufficient to represent it under a concept, then subsuming an object under the schema is sufficient to subsume it under that concept.

*Representational Sufficiency*: Representing an object under a schema is sufficient to represent it under the corresponding concept.

I then reconstructed in detail Kant's reasons for holding Representational Sufficiency for empirical and geometrical concepts, and more crucially, for the categories of quantity, quality, and one category of relation, <substance>. These constitute his reasons for, and his explanation of, Subsumption Sufficiency with respect to these concepts. I then reconstructed Kant's explanation of:

*Real Possibility*: It is really possible for objects to be subsumed under the schemata of empirical concepts, mathematical concepts, and categories.

In the Introduction I also gave a brief argument in favour of the trivial modal principle that:

*Subsumption-Possibility Link*: If subsuming an object under a schema is sufficient to subsume it under a concept, then if it is really possible for objects to be subsumed under the schema then it is really possible for them to be subsumed under that concept.

Together with Subsumption Sufficiency and Real Possibility, this entails that:

*Conclusion*: It is really possible for objects to be subsumed under categories.

Since, on my reading, the primary aim of the Schematism is precisely to explain the real possibility of the subsumption of objects under categories, it follows that if we grant Kant the three previous premises (one of which, I have argued, is nearly trivial) then the Schematism succeeds. I have further attempted to vindicate Kant's entitlement to the two substantial premises, Representational Sufficiency and Real Possibility. I have argued that these are plausible claims for Kant to make, given the transcendental theory of cognition for which he has argued up to this point in the *CPR*. Thus, my conclusion must be appropriately qualified: to the extent that the previous arguments of the *CPR* are sound, the Schematism is as well.

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## ENDNOTES

<sup>1</sup> Prominent Schematism bashers include Jacobi ("the most wonderful and mysterious of all unfathomable mysteries and wonders"; quoted by Schaper, 1964, p. 270), Schopenhauer ("Famous for its profound darkness, because nobody has yet been able to make sense of it"; Schopenhauer, 1911, vol. 1, p. 533), Adickes ("In my opinion, the section on the

Schematism is of no scientific value"; Kant, 1889, p. 171), Prichard ("It seems clear that if the first part [of the Analytic] is successful the second must be unnecessary"; Prichard, 1909, p. 246), Kemp Smith ("No such problem as Kant here refers to will exist"; Kant 1923p. 334), Warnock ("but if this is [...] Kant's question, he fails to answer it"; Warnock, 1948–1949, p. 81), Bennett ("The incoherence of Kant's problem of category-application is matched by the vacuity of his proposed solution"; Bennett, 1966, p. 151), Wolff ("The artificiality of the problem and the solution is evident upon reflection"; Wolff, 1963, p. 207), and Wilkerson ("The Schematism serves no useful purpose and can in my opinion be ignored without loss"; Wilkerson, 1976, p. 94). Guyer offers a more mixed verdict: "Only in the case of the three relational categories does there seem to be a genuine possibility that any persuasive connection can be drawn" (Guyer, 1987, p. 175). More recent commentators tend to be more sanguine; see Pendlebury (1995) and Allison (2004, pp. 202–228) for sympathetic reconstructions.

- <sup>2</sup> The *Critique of Pure Reason* is cited in the customary fashion: by page number in the 1781 edition (A) followed by the page number in the 1787 edition (B). Kant's other works are cited by volume and page number in the *Akademie* edition (Kant, 1902). The *Reflexionen* (abbreviated as *Refl.*), Kant's unpublished handwritten notes, can be found in volumes 14–19 of the *Akademie* edition. The *CPR* is quoted from the Guyer and Wood translation (Kant, 1998a), with slight modifications where noted.
- <sup>3</sup> In this paper, I discuss only "determining" or "subsumptive" schemata (by which sensible objects are subsumed under corresponding concepts). I do not discuss the "schematism" of the moral law (5:68–70), the pure concept of right (23:273–275), the duties of virtue (6:398), or the Ideas of reason and their associated regulative principles (A665/B693, A674/B702, A682/B710), much less the imagination's "schematizing without a concept" in aesthetic judgment (5:287). A comprehensive account of Kant's general notion of "schematism," and its applications in various domains, is a project for future work.
- <sup>4</sup> I do not of course mean to deny the existence of non-sensible objects (noumena), but merely to restrict attention to sensible ones (phenomena), which are the focus in the Schematism chapter.
- <sup>5</sup> As Kant says in the Analogies: "Appearances must not be subsumed under the categories per se, but only under their schemata" (A181/B223). By "not subsumed under the categories per se" I take Kant to mean: not subsumed under the categories directly, but only by means of schemata. This has the effect of restricting the categories, insofar as they figure in the Principles, to appearances. De Boer (2016) argues that "pure concepts of the understanding" already contain their own schemata (are already "schematized"), while "categories" are these pure concepts, abstracted from their schemata. I have my doubts about whether this is faithful to Kant's use of these terms – de Boer's reading is hard to square with Kant's use of "category" at B128, A146/B185, A161/B201–202, A184/B227, A219/B266, A222/B269, and many other passages – but even if de Boer is correct, my view can easily be translated into her terms as follows: where I say "unschematized category" substitute "category" *simpliciter*.
- <sup>6</sup> The Schematism chapter is not the only text where Kant discusses schemata as mediating representations by means of which sensible objects are subsumed under categories; he discusses them elsewhere in the *CPR* (A163/B204, A181/B223–24, A245, A247/B304, A665/B693, A286/B342, A553/B581, A664/B692, A682/B710, A718/B746) in other writings (4:316, 495; 5:68–69; 20:212, 232, 274), and in some unpublished Reflections (*Refl.* 5552, 5933, 6359) and letters (11:316; 12:224–225). But the Schematism is by far the most extensive and informative discussion of transcendental schemata in Kant's entire corpus.
- <sup>7</sup> Recall Kant's definition of (real) possibility as "whatever agrees with the formal conditions of experience" (A218/B266). The task of the Schematism is to prove that it is compatible with the form of experience that intuition gives us objects that can instantiate the categories. For more on real possibility, see Stang (2016).
- <sup>8</sup> This follows immediately from the axiom of distribution (K):  $\Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$ .
- <sup>9</sup> None of the commentators I am aware of (see Bibliography at the end) reconstruct the Schematism as an argument. Even George Dicker (2004), who is otherwise so meticulous in formally reconstructing Kant's arguments, abandons this methodology when it comes to the Schematism (pp. 213–224).
- <sup>10</sup> To this extent I agree with Allison (2004, pp. 202–228); however, I provide here a detailed reconstruction of the argument of the Schematism, which Allison does not do.
- <sup>11</sup> This paper covers most of the main topics, and many of the details, of the Schematism. One topic that is notable for its absence is the *imagination*. Kant writes that "the schema is in itself always only a product of the imagination" (A140/B179). For reasons of space, I omit here my detailed interpretation of imagination and its relation to schemata (and to time-determination). Two interpretations of Kant's theory of the imagination that harmonize with some of the points I make here are those of Sellars (1978), and Young (1984) and (1988).
- <sup>12</sup> As my parenthetical remarks indicate, strictly speaking what Kant is asking about is the possibility of the subsumption of intuitions (*Anschauungen*) under pure concepts of the understanding, which he then equates with the application of

categories to appearances. I take this to be an instance of Kant's well-known tendency to sometimes use *Anschauung* to refer to the object of intuition (what is intuited), rather than the act of intuiting itself. See Sellars (1967, p. 437; 1968, p. 8).

- <sup>13</sup> See for instance his discussions of subsumption in syllogistic reasoning (A301/B357, A304–305/B361, A308/B364).
- <sup>14</sup> For further discussion of these two different uses of “subsumption,” see Longuenesse (1998, pp. 92–97) and de Boer (2016, p. 451).
- <sup>15</sup> Kant typically talks about “subsumption” in ways that suggest it is something we actively do: we subsume objects under concepts. My interpretation may be thought to elide that active aspect of Kant's account, but this concern is misplaced. Recall that we are concerned solely with sensible objects, hence appearances: the possibility of an object falling under a concept F is the possibility of our experiencing it as falling under F (A158/B197). Since experience is partly spontaneous (i.e., it involves acts of synthesis on our part), the “passive” instantiation of a concept by an object always involves the possibility of an active “subsuming” of that object under that concept by a subject.
- <sup>16</sup> For an overview of the *status controversiae*, see McLear (2021).
- <sup>17</sup> In this paper I use “property” and “feature” interchangeably.
- <sup>18</sup> Cited by Smit (2000). I have learned a great deal from Smit's paper, and my interpretation in this section, on “what is represented in the object” by intuition, is especially influenced by it. See below for more.
- <sup>19</sup> I take it that every concept is at least potentially a discursive mark, that is, a mark of more complete concept that contains the original concept as a part; for example, <red> is a mark of <red firetruck>.
- <sup>20</sup> This is not to say that intuition represents the redness as the redness of (i.e., inhering in) the apple. That requires the capacity to predicate the redness of the apple (which is due to understanding not sensibility; see B141–142) and the category “inherence and subsistence” (A80/B106).
- <sup>21</sup> I remain uncommitted here on whether intuitive marks are also individuated by the times at which they are had (e.g., whether the redness of the apple at one time is numerically distinct from the qualitatively identical redness of the very same apple at a later time).
- <sup>22</sup> For an overview of contemporary discussions, see Oliver (1996).
- <sup>23</sup> As Kant makes clear in numerous metaphysics lectures, he is a conceptualist about universals: multiply instantiable properties (e.g., *redness*) exist only as the contents of (general) representations, or concepts. So there is no universal REDNESS present in every red object; rather, there are individual intuitive marks (*redness*<sub>1</sub>, *redness*<sub>2</sub>) in individual objects and a general concept (<red>), formed by a discursive intellect like ours, which subsumes all such objects. We can coherently talk about the general property *redness* only as the content of that concept. See 28:422, 503, 560, and 636.
- <sup>24</sup> Discursive marks are “analytic grounds of cognition” because each mark analytically contained in the concept is a partial ground of cognizing an object under that concept (e.g., cognizing something under <yellow> is a partial ground for cognizing it under <gold>), and the complete set of marks is the complete ground (cognizing something as yellow, malleable, and metallic is a complete ground for cognizing it as gold).
- <sup>25</sup> More precisely: red intuitive marks are marks that are similar, to a specified degree, to some designated intuitive mark, namely, the mark from which I abstract the general concept <red>. The qualification “similarity to a specific degree” allows us to distinguish between concepts of fully determinate shades (i.e., of marks wholly similar to, that is, qualitatively indistinguishable from, a given mark) and concepts of colour-determinables like *red* (i.e., of marks similar to some degree to a given mark), and everything in between.
- <sup>26</sup> Houston Smit (2000) puts it perfectly: “On Kant's view, to subsume an intuition under a concept [just is] to recognize an intuitive mark as a singular instance of a discursive mark” (p. 259).
- <sup>27</sup> “So hat der empirische Begriff eines Tellers mit dem reinen geometrischen eines Zirkels Gleichartigkeit, indem die Rundung, die *in dem ersteren* gedacht wird, sich *im letzteren* anschauen läßt.” Hans Vaihinger suggested that the underlined phrases should be switched, so the subordinate clause reads: “indem die Rundung, die *in dem letzteren* gedacht wird, sich *im ersteren* anschauen läßt” (Vaihinger, 1900, p. 458). But even this is unsatisfactory, for an empirical concept like <plate> can no more be intuited than a pure geometrical concept can. Kemp Smith follows Vaihinger's reading in his translation (Kant, 1929, p. 180); Timmerman notes Vaihinger's reading without endorsing it (Kant, 1998b, p. 239); the issue goes unremarked by Adickes (Kant, 1889, p. 173) and Guyer and Wood (Kant, 1998a, p. 271).
- <sup>28</sup> Note that in the quoted sentence Kant asserts a homogeneity between the concept of a plate and the concept of a circle.
- <sup>29</sup> This raises the question of how it is possible to subsume various intuitive marks under a general concept, that is, to recognize them as singular instances of that concept, even in the empirical case (sensible features). This capacity to subsume objects correctly under rules is what Kant calls the power of judgment (*Urteilkraft*) in the CPR, which is the topic

of the Introduction (A132–A136/B171–B175) to the Transcendental Doctrine of the Power of Judgment (also known as the Analytic of Principles). Kant's view is that this capacity rests, at bottom, on “mother-wit, the lack of which cannot be made good by any school” (A133/B172). I take this to mean that we have a capacity to generalize from certain basic sensory features, that is, to form general concepts from singular instances of them. Kant does not think that this stands in need of any further explanation.

- <sup>30</sup> Chipman (1972) is after something similar with his talk of “elementary sensory components” (p. 39).
- <sup>31</sup> The paradigm example of this is the pure part of natural science, given in the *Metaphysical Foundations of Natural Science*. The subsumption of objects under the a priori concepts involved in pure natural science does, of course, stand in need of explanation; this is why the CPR explains the possibility not only of metaphysics but also of natural science (B17, A171–172/B213). However, even on Kant's considered view, a version of his A138/B177 point holds: pure natural science does not produce antinomial conflicts in reason, and so does not stand in the same need of critique that metaphysics does. Furthermore, our possession of apodictically certain knowledge in natural science (e.g., Newtonian physics) shows that, whatever the explanation, natural science is possible (4:295). Metaphysics cannot say the same.
- <sup>32</sup> This means that, until we have completed the project of the Schematism, the possibility of subsuming objects under empirical concepts remains in some sense unexplained. But to reiterate a point from the previous note, the application of empirical concepts does not produce antinomial conflicts, and our possession of empirical knowledge shows that, whatever the explanation, the application of empirical concepts is possible.
- <sup>33</sup> This is why we can “anticipate” (i.e., prove a priori) the intensive and continuous character of sensation, but not its particular qualities (e.g., colour, taste, etc.); see A175/B216.
- <sup>34</sup> I am not assuming that determinates are *fully* determinate, but only that they are more determinate than determinables. For instance, *round* is more determinate than *shaped*, but it is not as fully determinate as *circular*.
- <sup>35</sup> See Section 2.6 for what more is required to represent a sensory quality as a reality.
- <sup>36</sup> See, for example, Grüne (2011) and Land (2012).
- <sup>37</sup> Some commentators read the Schematism as concerned with explaining the possibility (in my terms) of representing objects under categories; see, for example, Pippin (1976, p. 161), Pendlebury (, p. 777), Longuenesse (1998, p. 244). I agree with Chipman (1972, p. 43) and Guyer (1987, p. 158) that the aim of the Schematism is to explain the possibility of objects instantiating categories. Unlike them, however, I think that the key to reconstructing the Schematism as an argument lies in notionally distinguishing representation from subsumption and, via the Representation-Subsumption Link, using the former to explain the latter. Walsh (1957, p. 97) makes a distinction between a weaker and a stronger sense in which categories might have “sense and significance,” which roughly parallels my distinction between representation and subsumption. However, Walsh interprets the problem of the Schematism – correctly – as concerning subsumption, but without connecting it, as I do, to mere representation; as a result, he is unable to give a satisfactory reconstruction of the argument of the Schematism.
- <sup>38</sup> It should be noted, however, that what I have called “representing” an object under a schema or concept can also naturally be expressed with the word “subsumption.” Arguably, Kant sometimes uses “subsumption” in precisely this instantiation-neutral sense. So my distinction between “representing” and “subsuming” can also be understood as a distinction between two different senses of “subsuming”: one that requires instantiation and one that does not. However, since I think Kant usually uses “subsumption” in a sense that requires instantiation, I have instead chosen to make a distinction between *representing* an object under a schema or concept and that object's being *subsumed* under that schema or concept. Thanks to Daniel Smyth for pressing me to express my view without subscripts.
- <sup>39</sup> While it appears here in Kantian garb, the Representation-Subsumption Link is really the trivial claim that if the representational content of one thought (that the object falls under the schema) entails the representational content of another thought (that the object falls under a concept), the conditions under which the latter thought is true are a subset of the conditions under which the former thought is true.
- <sup>40</sup> Some scholars read the Schematism as concerning the epistemic problem of how we know that objects instantiate categories or how we “recognize” them under categories (Krausser, 1976; Dicker, 2004, p. 213). However, I take it to be clear that the epistemic problem is downstream of the question of subsumption: in order for the epistemic question to even arise, it must be in principle possible for objects to instantiate categories. I will ignore the epistemic question in what follows.
- <sup>41</sup> “[...] not every a priori cognition [*Erkenntnis*] must be called transcendental, rather only those through which we cognize that and how certain representations (intuitions or concepts) are applied merely a priori or are possible (i.e., the possibility of cognition or its use a priori).” Compare Allison (2004, p. 216).
- <sup>42</sup> One difference between English and German is that German does not have different words for “determined” and “determinate”; both correspond to *bestimmt*.

- <sup>43</sup> See Stang (2012) for more on this distinction.
- <sup>44</sup> A concept is completely determinate<sub>S</sub> just in case, for every predicate F, it either contains F or  $\sim$ F. Our concepts are typically incompletely determinate<sub>S</sub>, the possible exception being <God>, for it is completely determinate<sub>S</sub> at least with respect to God's intrinsic properties. Conceptual representation is in general not completely determinate<sub>S</sub>. For extensive discussion of complete determination<sub>S</sub> see Kant's lectures (28:413, 503, 554, 630, 722–724, 779; 1156) and unpublished notes (*Refl.* 5760, 5783, 5784, 5786, 6245, 6255, 6290, 6322) on metaphysics.
- <sup>45</sup> See Kant's discussion of this principle at A573/B601, as well as Stang (2012).
- <sup>46</sup> For instance, an object can be said to be “determined” when there is a ground that fully explains why it has some predicate rather than another (a so-called *determining* ground). See Kant's pre-Critical *Nova dilucidatio* (1:398), as well as his lectures on metaphysics (28:14, 30, 54, 401, 491; 29:808, 809, 815, 819); for critical discussion see Hogan (2009) and Stang (2019). Likewise, “determination” can also have an epistemic sense: I determine an object with respect to a predicate when I know that object to have that predicate. Finally, *Bestimmung* can have the meaning of vocation, or destiny (e.g., Fichte's 1800 work *Die Bestimmung des Menschen*), a sense lacking in the English “determination” and its cognates.
- <sup>47</sup> By contrast, Henry Allison (2004) gives the following definition: “We may preliminarily characterize a transcendental time-determination as a rule-governed product of the figurative synthesis (a ‘transcendental product of the imagination’), which exhibits in a non-discursive manner the form of unity conceptually expressed in a category” (p. 215). I do not think this characterization is false, but I do think my characterization is more explanatory.
- <sup>48</sup> “Time is nothing other than the form of inner sense, that is, of the intuition of ourselves and our state [*Zustand*]” (A49/B50).
- <sup>49</sup> “Time cannot be a determination of outer appearances: it belongs neither to shape nor location and so forth, but rather determines the relation of representations in our inner state” (A33/B49–50).
- <sup>50</sup> I read Kant's discussion of “time determination” in the Analogies (A183/B226, A185/B228, A188/B231, A215/B262, A217/B264) and the Refutation (B275–278), continuous with my reading of the Schematism, as concerning the conditions of the possibility of determinately<sub>S</sub> representing the temporal relations of outer objects (permanence, succession, simultaneity), which, given that these objects are mere appearances, are at the same time conditions of the possibility of their determinately<sub>M</sub> standing in those temporal relations.
- <sup>51</sup> See A34/B50–51. Note that my reading is in some tension with the conclusion of that paragraph: “From the principle of inner sense I can say entirely generally: all appearances in general, that is, all objects of the senses, are in time, and necessarily stand in relations of time.” This can be read as making either a weak or a strong claim about the temporality of all appearances *überhaupt* (including outer ones): (i) all appearances, even outer ones, are in time in the minimal sense that our intuitions of them are in time (weak); or (ii) all appearances, even outer ones and not merely our intuitions of them, are in time (strong). On the weak reading, Kant is entitled to this claim at this point in the Aesthetic, but the weak reading does not fit the text very well: Kant emphasizes that it is the *objects* of representation, not merely the states in us by which we represent them, that are in time. The strong reading is more natural to the text, but it is a non sequitur, for it does not follow from what Kant says earlier in the paragraph. I think the sentence must be read in the strong way, but as anticipating an argument Kant will give later, in the Analogies and the Refutation of Idealism. Bader (2017) offers a very different account of why outer objects are in time.
- <sup>52</sup> I return to homogeneity in Section 4, where I explain the subsumption of objects under schemata.
- <sup>53</sup> Moreover, the schemata will themselves play an important role in showing how the time determination of outer objects is possible, for the Analogies involve the time-schematized relational categories. See A181/B223–224.
- <sup>54</sup> On this point I agree with Allison (2004, p. 217).
- <sup>55</sup> There is a debate in the secondary literature on whether empirical concepts are identical to their schemata: see Pendlebury (1995), Chipman (1972), Pippin (1976), Guyer (1987), and Matherne (2014). Even if, contra Chipman (1972), it is impossible to possess an empirical concept without being able to subsume a sensible object under it, it does not follow that they are *identical*, but only that they are necessarily co-occurring. The view that empirical concepts are their own schemata fits poorly with Kant's principal discussion of them: “Even less does an object of experience or an image of it ever reach the empirical concept, rather the latter is always related immediately to the schema of the imagination, as a rule for the determination of our intuition in accordance with a certain general concept. The concept of a dog signifies [*bedeutet*] a rule in accordance with which my imagination can specify the shape of a four-footed animal in general” (A141/B180). If an empirical concept is its own schema, then by the second sentence, it signifies (*bedeutet*) itself, which is rather awkward.
- <sup>56</sup> If it were, Kant could have saved himself a lot of trouble in the Critique of Teleological Judgment!
- <sup>57</sup> “Semi-transcendental” because transcendental cognition *sensu stricto* concerns how a priori cognition is possible (see A56/B80).

- <sup>58</sup> The considerations in this paragraph, and the rest of this section, are similar to the views of Sellars (1978) and Matherne (2015). But whereas Sellars focuses on the case of empirical concepts (and only makes a few remarks on schemata for pure categories), and Matherne is primarily concerned with images rather than schemata, I extend this broadly “temporal-phenomenological” interpretation to all schemata for all concepts whatsoever. See the following three sections for details.
- <sup>59</sup> See A33/B49–50 (quoted above).
- <sup>60</sup> The idea that representing an object requires representing a relation among the vehicles of representation themselves (intuitions, in this case) may seem baroque, but consider that the natural alternative (simply representing temporal relations among the objects) is not available to Kant, given that time is the form of inner sense, not of outer sense.
- <sup>61</sup> This point lies at the heart of Sellars (1978).
- <sup>62</sup> A similar point is made by Pendlebury (1995, p. 790).
- <sup>63</sup> See *Critique of Judgment* §65 (5:295).
- <sup>64</sup> Determination<sub>s</sub> is gradable (see Section 2.3 above).
- <sup>65</sup> Béatrice Longuenesse (1998) emphasizes the role of schemata in the understanding’s “affection” and thereby the “determination” of inner sense (p. 246). However, she never fully clarifies the meaning of “determination” here, that is, the way in which schemata are transcendental time determinations and how this explains the subsumption of objects under concepts. My account of schemata fills this gap in Longuenesse’s reading, but unlike Longuenesse, I do not in this paper give an account of the relation of schemata to logical functions. That must await future work.
- <sup>66</sup> Thanks to Colin McLear and Daniel Smyth for pressing me on this point.
- <sup>67</sup> We may have to build in the assumption that a name coined through a perceptual demonstrative (let “Brian” refer to *that* person) at a time must apply to an object that exists at that time. But regardless of what fixes are needed, the larger point holds: we can represent relations among contents by representing appropriate relations among vehicles.
- <sup>68</sup> The role of the modalities is complicated here, and this is reflected in Kant’s schemata for modal categories. However, for reasons of space, I leave my account of modal schemata out of this paper. I discuss modal schemata in detail in forthcoming work.
- <sup>69</sup> A point of which Kant was well aware; see his letter to Schultz of November 25, 1788 (10:556).
- <sup>70</sup> E.g., Longuenesse (1998, pp. 249–257).
- <sup>71</sup> “Drawing a line in thought” is Kant’s own phrase but it refers, I take it, to what he elsewhere refers to as construction “in pure intuition” (A713/B741). I will continue to use the former expression, however, since I do not have the space in this paper to discuss Kant’s complex doctrine of construction in pure intuition.
- <sup>72</sup> A138/B177. This is a point that would be more naturally expressed using the language of “subsumption”; see the Introduction for why I have not used that term here.
- <sup>73</sup> I say “possible act of construction,” for presumably I can see a figure in a geometrical proof and cognize it as a triangle without having to construct it myself, as long as I represent it as constructible.
- <sup>74</sup> Consider the following objection: a computer draws a triangle, and the representational content of the state of the computer that corresponds to step 1 is causally relevant (it is not erased from working memory) to the representational content of the state of the computer that corresponds to step 2, and so forth. But the computer lacks self-consciousness, let us assume. Has the computer not constructed a triangle? It has not, I would answer. Constructing a triangle is a priori presenting to oneself an object corresponding to the concept (A713/B741), which is cognition (A50/B74), and cognition, according to Kant, requires self-consciousness (B137). The computer may have generated an *image* of a triangle, but an image is not a schema (A140/B179). Schemata are products of self-consciousness (A142/B181), and without schemata mathematical cognition and construction are impossible. A non-self-consciousness machine cannot engage in mathematical construction. Thanks to Colin McLear for pressing me on this point.
- <sup>75</sup> One might point to the marks on paper, or the lines drawn in the sand, as the relevant objects. Two points about this: (i) lines drawn on paper are not geometrical lines except insofar as they are being used as illustrations of a geometrical construction, so the point in the main text still holds; (ii) Kant holds that we can perform constructions without drawing physical figures, “through mere imagination, in pure intuition” (A713/B741).
- <sup>76</sup> This also shows how to put to rest the concern, expressed by some commentators, that the Schematism may be incompatible with the General Remark to the System of Principles (B288–294), where Kant claims that “in order to understand the possibility of things in accordance with the categories, and thus to establish the objective reality of the latter, we do not merely need intuitions, but always outer intuitions” (B291). The Schematism concerns the temporal relations among

intuitions that we must determinately represent in order to subsume the objects of those intuitions under concepts, whereas the General Remark concerns the spatial character of the objects of those intuitions. There is no tension here.

<sup>77</sup> See Kant's definition of "transcendental" at A56/B80.

<sup>78</sup> The Table of Categories derives, of course, from the Table of Logical Functions of Understanding in Judging (A76–80/B102–106). Consequently, we should expect a relation between schemata and logical functions of judging, which is directly confirmed by *Refl.* 5933: "The Schematism shows the conditions under which an appearance is determined in respect of a logical function" (18:392; cf. A79/B104–105, A94/B128). The most complete attempt to relate the schemata systematically to logical functions of judging is that of Longuenesse (1998, Chap. 9–11). Longuenesse does not discuss the modal schemata, however. I account for modal schemata in work currently in preparation.

<sup>79</sup> Immediately before this passage, Kant writes: "The pure image of all magnitudes (*quantorum*) for outer sense is space; that of all objects of the senses in general is time" (A142/B182). I am not giving an account of images, so I will not attempt to explain this puzzling claim here. The account of images given by Matherne (2015) is in line with my interpretation of schemata, but does not discuss the "pure images" of space and time.

<sup>80</sup> Kant's terminology here should be noted. He schematizes the concept <*magnitude*> (*Größe*) and identifies it with *quantitas*. A magnitude is what Kant elsewhere calls a *quantum*, which he defines as follows: "Quantum is one thing, in which there is quantity [*Quantum: est unum, in quo est quantitas*]" (28:21). So a *quantum* is an intuitively given object in which there is *quantitas*, that is, one in which many other objects are also posited ("quantitas: determinatio entis, quoties sit positum – 28:21"; cf. A163/B204). When the *quantum* has determinately many parts, when the question *How many?* has a determinate answer (given a specification of a unit part), then it can be numbered and the schema given in the body of the paper applies. Infinite quantities (*quanta* with infinite *quantitas*), like space and time, cannot be numbered; their *quantitas* exceeds any number, so the schematization I supply below does not apply. See Longuenesse (1998, pp. 264–271) and Sutherland (2004, pp. 427–435) for discussion of *quantum* and *quantitas*.

<sup>81</sup> Longuenesse (1998, pp. 253–254) also makes this point.

<sup>82</sup> The reason the second case is more complex is that it requires representing the unit as itself something that is quantitatively determinate, so it depends upon a prior act of quantitative determination. For ease of exposition, in the main text I focus on the case of simply counting the parts of an object.

<sup>83</sup> In the *Prolegomena* (4:303) Kant identifies unity, plurality, and totality with, respectively, measure (*Maß*), magnitude (*Größe*), and the whole (*das Ganze*). These correspond to the structural moments of the act of enumeration described in the main text. To represent an object as a unity is to represent it as a *measure* of enumeration in some larger collection. To represent an object as a plurality is to represent it as a *magnitude*, as something with *quantitas* with respect to a unit/measure. But it is not yet to represent it as a totality, for not all of its parts have been enumerated. Representing an object as a totality is representing it as the *whole*, the place where enumeration ends. If you are enumerating an infinite magnitude (space, time), at every step you are representing it as a plurality without having enumerated all of its parts; that is, you are not yet representing it as a totality/whole. Representing space and time as totalities requires starting with the whole and then proceeding to the parts/limitations, but that involves a different schema; I am limiting myself to number as the schema of determinate and finite *quantitas* (see above).

<sup>84</sup> This is my detailed specification of what Longuenesse (1998) refers to as the "understanding [affecting] inner sense" (p. 259) in the case of quantity.

<sup>85</sup> See Tolley (2020).

<sup>86</sup> In this respect, Kant's original statement of the principle of the Anticipations (A166: "In all appearances sensation and the real, to which it corresponds in the object, have an intensive magnitude, that is, a degree") is more precise than its replacement in the B edition (B207: "In all appearances the real, which is an object of sensation, has an intensive magnitude, i.e., a degree"). In the A edition Kant has separated the claim that sensation has intensive magnitude from the further claim that the corresponding object does; in the B edition only the claim about the object is retained: "If one regards this reality as cause (whether of the sensation or of another reality in appearance, e.g., an alteration), then one calls the degree of reality as cause a 'moment' [...]. But I touch on this here only in passing, for at present I am not yet dealing with causality" (A168–169/B210). I take this to mean that we cannot represent the intensive degree of reality in our sensation as caused by a corresponding intensive degree of reality in the affecting object until we have the machinery of the Analogies in place.

<sup>87</sup> I speak of *specific* realities because I take Kant to be arguing not only that sensation in general has a degree (i.e., one perception can be more intense than other), but that specific kinds of sensation do as well (i.e., one perception can be more intense in respect of one quality, and less intense in respect of another).

<sup>88</sup> We ourselves do not actively generate the magnitude of sensation, for sensation is the passive result of affection; see Longuenesse (1998, p. 300).

- <sup>89</sup> The temporal direction is a heuristic device; we can just as well represent our present sensory matter as the product of a decrease and as the beginning of a further process of decrease.
- <sup>90</sup> To put this in Humean terms, to represent a given shade of blue as a determinate gradable hue we do not need to actually represent the “missing shade” of blue (and then generate the given shade out of it); we need merely be able to represent that missing shade (and represent the given shade as generated out of it).
- <sup>91</sup> As Kant is at pains to emphasize throughout the Anticipations, not only sensation but the object of perception has an intensive magnitude of reality. See the statement of the Principle in both editions, at B207–208, A167–168/B209, A169/B210–211, A171/B212, A172/B214, and A174/B216.
- <sup>92</sup> This is my more detailed specification of what Longuenesse (1998) refers to as “affection of inner sense by the understanding” in the categories of quality (p. 298).
- <sup>93</sup> The schema of <cause-effect> cannot be satisfactorily discussed here, for the relation of time and modality in that case is especially complicated. On my reading, the schema of <cause-effect>, while more complicated, does not in fact pose a harder, but rather an easier, theoretical challenge than that for <substance-accident>. So in avoiding <cause-effect> but discussing <substance-accident>, I am actually focusing on the *hard case* for Kant, at least on my reading.
- <sup>94</sup> B218–219.
- <sup>95</sup> See B219. The Analogies have spawned a vast literature, to which I cannot hope to do justice here. Two highlights are Guyer (1987, pp. 207–278) and Watkins (2005, pp. 185–229). Whereas Guyer adopts an “epistemic” reading of the Analogies, on which they uncover the conditions of our knowledge of objective time (pp. 258–259), Watkins opts for a “metaphysical” reading, on which they uncover the grounds of objective temporal relations obtaining in the first place (2005, pp. 200–201), which itself make our knowledge of objective temporal relations possible. However, I opt for what I call a “semantic” reading: the Analogies uncover the conditions of the possibility of so much as determinately representing (determinings) temporal relations among outer objects. A similar view is suggested by Friedman (1992, pp. 76–77) and sketched by Rödl (2012, pp. 113–127). Given Kant’s transcendental idealism, objective time determinations makes possible objective time determination<sub>M</sub>, which in turn makes possible knowledge of objective temporal relations. (Watkins makes a similar point.) But I do not have the space to spell this out here.
- <sup>96</sup> A177/B219.
- <sup>97</sup> B225.
- <sup>98</sup> Likewise, the Transcendental Deduction proves the necessity of subsuming (in both senses) sensible objects under categories, while the Schematism explains the possibility of doing so (in both senses). Similar points are made by Pendlebury (1995, p. 779), Dahlstrom (1984, p. 41), and Symington (2015, p. 289). Given standard assumptions, necessity entails possibility, but this does not render the Schematism redundant: for even if Kant has proven in the Deduction that subsuming objects under categories is necessary for experience, he has not yet explained why this is possible.
- <sup>99</sup> See A183/B226, A185/B229.
- <sup>100</sup> “All appearances are in time, in which, as substratum (as persistent form of inner intuition), both simultaneity as well as succession can alone be represented. The time, therefore, in which all change of appearances is to be thought, lasts and does not change; since it is that in which succession or simultaneity can be represented only as determinations of it” (B224–225). See also A186/B229 (“Everything would disappear that alone can represent the unity of time”) and A188/B231 (“It is this very thing that persists that makes possible the representation of the transition from one state into another”).
- <sup>101</sup> For expository purposes I here treat “experience” as a count noun, whereas Kant’s considered view is that there is only one experience (A110). Thus, what I say about “experiences” in this passage should be understood as shorthand for claims about “perceptions insofar as they belong to one and the same universal experience” (A110). See Stang (2018) for details.
- <sup>102</sup> This is my detailed specification of what Longuenesse (1998) refers to as the “understanding affecting inner sense” (p. 332) in the case of <substance>.
- <sup>103</sup> Since I am concerned exclusively with pure intuitive features, I typically omit the qualification “pure.” The account of how, and whether, concepts of empirical intuitive features (e.g., concepts of colours) are homogeneous with objects, lies outside the scope of this paper.
- <sup>104</sup> Readers might object that schemata represent intuitive relational features of representations, not of objects; for example, co-reference is a relation of intuitions, not of their objects. So why is subsuming (in either sense) the representations under the schemata sufficient to subsume the objects? For the answer to this question, see my discussion in Section 2.4.

- <sup>105</sup> It might seem that, on my account, schemata represent more than merely temporal relations, unproblematically allowed for by the very form of time (succession), for I spoke of the stages of a constructive procedure as “completing one another” and “to be completed by another.” This kind of language might suggest that schemata represent something like teleological relations among our representations, and teleological relations are definitely not what I have called “intuitive features.” But the “completion” language was not meant to connote teleology, but rather self-referentiality: in order to be constructing a triangle at step 2, one must self-consciously represent one’s current representation as part of a larger constructive procedure, composed of a *prior* step 1, and a *later* step 2. This does not require that, in any problematic sense, step 1 be “for the sake of” step 2, or step 2 “for the sake of” step 3, but only that step 1 be step 1 of a construction only if it is followed by step 2, and so forth. Since the relations thus represented among our representations are purely temporal relations, the unproblematic “homogeneity” of the schema with its object is secured. Thanks to Colin McLear for pressing me on this point.
- <sup>106</sup> In Kant’s terminology, the mathematical categories (quality, quantity) represent objects merely as intuited, while the dynamical categories (relation, modality) represent the “existence” of those objects; see A160/B199. Thanks to Daniel Smyth for pressing me to clarify this point.
- <sup>107</sup> My answer depends upon a reading of the first Analogy that I can only sketch here.
- <sup>108</sup> For example, Wolff (1969, p. 251), Bennett (1966, p. 200), Strawson (1966, pp. 126–133), Van Cleve (1979, p. 158), Allison (2004, pp. 244–246).
- <sup>109</sup> A188/B231. In fact, almost all of Kant’s claims in the first Analogy are about the persistence of substance (*Substanz*) not substances (*Substanzen*). “Substance” is most naturally read as a mass noun, while “substances” is a count noun. This is further support for the interpretation I offer in the main text: the persistence of the quantity of substance (mass noun) is primary for Kant, and the persistence of substances (count noun) is defined as the persistence of the quantity of reality in substance.
- <sup>110</sup> B224.
- <sup>111</sup> The sole argument occurs in this sentence: “Since [substance] cannot change in existence, its quantum in nature can also be neither increased nor diminished” (B225). My point here in the main text is that if the antecedent (substance cannot change in existence) merely means that substances cannot come into, or go out of, existence, this sentence is a non sequitur (though perhaps not one of numbing grossness).
- <sup>112</sup> Of the commentators cited in note 108 above, Wolff, Bennett, and Strawson think (2) is a non sequitur, while Van Cleve and Allison try to supply an argument. Contra Allison, however, the “indeterminacy” of the transcendental concept of substance is not the key to the argument; contra Van Cleve, the quantity of substance cannot depend on the number of its parts, for all substances are arbitrarily and continuously divisible into parts.
- <sup>113</sup> The parts of substances are also substances (substances are infinitely divisible) but an increase in the quantity of a substance would not be an increase in the number of its parts. Substances are continuous (i.e., have continuum-many parts), regardless of their quantity (i.e., the quantity of their reality: see main text). This is part of Kant’s refutation of “mathematical and mechanical students of nature” in the Anticipations (A173/B215).
- <sup>114</sup> I take this sketch of an account to be in essential agreement with the much more developed one given by Friedman (2013, pp. 319–323).
- <sup>115</sup> Naturally, the persistence of dogs does not require the persistence of the substance (matter) of which they are composed, for as living things they undergo continuous change of material parts. But this is simply a downstream consequence of an observation I made earlier: Kant’s example of schematizing the empirical concept <dog> is poorly chosen, for by his own analyses in the third *Critique*, the transcendental machinery of the CPR cannot explain how the subsumption of objects under concepts of living things is possible. What I have explained is, effectively, the subsumption of objects under the concept of a quantity of matter that continuously takes on the shape of the various stages of a dog’s life.

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