

# **Kant's Schematism of the Categories:**

## **An Interpretation and Defense**

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### **Abstract**

Most commentators agree that the Schematism chapter plays a very important role in the *Critique of Pure Reason (CPR)*. But there is little agreement on what role, exactly, the Schematism is supposed to play and how successfully it plays that role. Many commentators consider it a failure. My aim in this paper is to provide an interpretation of the role of the Schematism and a qualified defense of its main doctrines. The topic of the Schematism is the “subsumption” of objects under concepts, as the first sentence announces: “in all subsumptions of an object under a concept [ . . . ]” (A137/B176). Its primary aim is to explain how it is possible for sensible objects to be subsumed specifically under the pure concepts of the understanding (categories). That much is relatively clear. However, “subsumption” admits of two different readings, which previous commentators have not carefully distinguished. In one sense, an object *a* can be said to be subsumed under a concept *F* just in case some subject thinks of *a* that it is *F*. In another sense, though, an object is only said to be subsumed under a concept when it instantiates that concept, e.g. when *a* is *F*. The first notion of subsumption is neutral on the truth of subsumption, whereas, on the second reading, subsumptions as such are true. In this paper I argue that this ambiguity in “subsumption” is the key that unlocks the argument of the Schematism.

### **Introduction**

Most commentators agree that the Schematism chapter plays a very important role in the *Critique of Pure Reason (CPR)*. But that is where the agreement ends. In particular, there is little agreement on what role, exactly, the Schematism is supposed to play and how

successfully it plays that role. Many commentators consider it a failure.<sup>1</sup> My aim in this paper is to provide an interpretation of the role of the Schematism and a qualified defense of its main doctrines.

The topic of the Schematism is the “subsumption” of objects under concepts, as the first sentence announces: “in all subsumptions of an object under a concept [. . .]“

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<sup>1</sup> Prominent Schematism-bashers include Jacobi (“the most wonderful and mysterious of all unfathomable mysteries and wonders” – quoted by Schaper 1964, 270), Schopenhauer (“famous for its profound darkness, because nobody has yet been able to make sense of it” 1911, vol. 1, 533), Adickes (“in my opinion, the section on the Schematism is of no scientific value” Kant 1889, 171), Prichard (“it seems clear that if the first part [of the *Analytic*] is successful the second must be unnecessary” 1909, 246), Kemp Smith (“no such problem as Kant here refers to will exist” 1923, 334), Warnock (“but if this is [. . .] Kant’s question, he fails to answer it” 1948–9, 81), Bennett (“the incoherence of Kant’s problem of category-application is matched by the vacuity of his proposed solution” 1966, 151), Wolff (“the artificiality of the problem and the solution is evident upon reflection” 1969, 207), and Wilkerson (“the Schematism serves no useful purpose and can in my opinion be ignored without loss” 1976, 94). Guyer offers a more mixed verdict: “only in the case of the three relational categories does there seem to be a genuine possibility that any persuasive connection can be drawn” (1987, 175). More recent commentators tend to be more sanguine; see Pendlebury 1995 and Allison 2003, 202–228 for sympathetic reconstructions.

(A137/B176).<sup>2</sup> Its primary aim is to explain how it is possible for sensible objects to be subsumed, specifically, under the pure concepts of the understanding (categories).<sup>3</sup> That much is relatively clear. However, “subsumption” admits of two different readings, which previous commentators have not carefully distinguished. In one sense, an object *a* can be said to be subsumed under a concept *F* just in case some subject thinks of *a* that it is *F*. In another sense, though, an object is only said to be subsumed under a concept when it instantiates that concept, e.g. when *a* is *F*. The first notion of subsumption is neutral on the truth of subsumption, whereas, on the second reading, subsumptions as

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<sup>2</sup> The *Critique of Pure Reason* is cited in the customary fashion: the page number in the 1781 edition (A) followed by the page number in the 1787 edition (B). Kant’s other works are cited by volume and page number in the *Akademie* edition, Kant 1902–.

“*Refl.*” refers to *Reflexionen*, Kant’s unpublished handwritten notes, which can be found in volumes 14–19 of the *Akademie* edition. The *CPR* is quoted from the Guyer & Wood translation (Kant 1998a), with my modifications.

<sup>3</sup> In this essay, I only discuss “determining” or “subsumptive” schemata (by which sensible objects are subsumed under corresponding concepts), the topic of the Schematism chapter of the *CPR*, rather than schematism by analogy or symbol (5:351–2). Neither do I discuss the “schematism” of the moral law (5:68–70), the pure concept of right (23:273–5), the duties of virtue (6:398), or the Ideas of reason and their associated regulative principles (A665/B693, A674/B702, A682/B710), much less the imagination “schematizing without a concept” in aesthetic judgment (A684/B712). A comprehensive account of Kant’s general notion of “schematism,” and its applications in various domains, is a project for future work.

such are true.<sup>4</sup> If we make this distinction precise, we get two different questions that might be the question of the Schematism:

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| (1) The <i>subsumption</i> <sub>1</sub> of an object <i>a</i> under a concept F is the thought that <i>a</i> instantiates F (the thought that <i>Fa</i> ).   | (Q1) How is it possible to think of a sensible object that it falls under a category (where F is a category, to think a thought whose content is <i>Fa</i> )? |
| (2) The <i>subsumption</i> <sub>2</sub> of an object <i>a</i> under a concept F is the object's instantiating that concept, thus the content of the thought involved in the corresponding subsumption <sub>1</sub> ( <i>Fa</i> ) being <i>true</i> . | (Q2) How is it possible for sensible objects to instantiate categories (where F is a category, how is it possible that <i>Fa</i> )? <sup>5</sup>              |

Kant sometimes talks about the “application” (*Anwendung*) of concepts to objects,<sup>6</sup> and some commentators follow him by talking about schemata as “application conditions” for the categories, but “application” admits of the same ambiguity: is satisfaction of the application conditions for concept F sufficiently merely to think of an object as falling under F (to think that *Fa*) or is satisfaction of those conditions sufficient for *a* to instantiate F (for it to be the case that *Fa*)? This distinction between two notions of “application” mirrors precisely the distinction between subsumption<sub>1</sub> and subsumption<sub>2</sub>.

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<sup>4</sup> Since we are concerned only with the subsumption of sensible objects, and hence with objects of cognition, this agrees with Kant's official definition of truth at A58/B82.

<sup>5</sup> Where the expression “*Fa*” abbreviates “object *a* falls under/instantiates concept F.”

<sup>6</sup> E.g. “How, then, is the subsumption of intuitions under pure concepts, the application of a category to appearances, possible?” (A138/B177).

Some previous commentators have focused on the “semantic” question (Q1),<sup>7</sup> others on the “metaphysical” question (Q2),<sup>8</sup> but have not carefully distinguished them.<sup>9, 10</sup>

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<sup>7</sup> E.g. “we are, then, asking in our schematism question [ . . . ] for the rules which specify the conditions under which [whether this or that is a causal connection] could ever be a *question*” (Pippin 1976, 161; emphasis in original); “this raises issues concerning the possibility of intentional content” (Pendlebury 1995, 777; Au’s emphasis); “ ‘applying a category’ then means claiming for a discursive combination objective validity, validity not only ‘for myself, in my present state of perception’ but ‘for everybody, always’” (Longuenesse 1998, 244; Au’s emphasis)

<sup>8</sup> E.g. “the problem is not a new one, but simply Plato’s problem of giving an account of how it is possible to subsume a concrete particular under a thoroughly abstract universal” (Chipman 1972, 43); “the chief objective of the Schematism must therefore be to show how exactly these categories can be instantiated or manifested in our sensible intuitions of objects” (Guyer 1987, 158).

<sup>9</sup> A notable exception to the trend of conflating Q1 and Q2 is W.H. Walsh, whose distinction between a weaker and a stronger sense in which categories might have “sense and significance” roughly parallels my Q1 and Q2 (Walsh 1957, 97). Walsh interprets the problem of the Schematism as Q2, without connecting it, as I do, to Q1. As a result, he is unable to give a satisfactory reconstruction of the argument of the Schematism.

<sup>10</sup> Some scholars even read the Schematism as concerning the epistemic problem of how we know that objects instantiate categories or how we “recognize” them under categories (Krausser 1976; Dicker 2004, 213). I take it to be clear, though, that the epistemic problem is downstream of both Q1 and Q2: to know that an object falls under a concept,

In this paper I will argue that this ambiguity in “subsumption” is the key that unlocks the argument of the Schematism.<sup>11</sup> I agree with many commentators that Kant’s ultimate aim in the Schematism is to explain the real possibility of the subsumption<sub>2</sub> of sensible objects under categories (to answer Q2). He does so by explaining the possibility of the subsumption<sub>2</sub> of sensible objects under schemata. This much is relatively uncontroversial.<sup>12</sup> But Kant’s explanation of the possibility of subsuming<sub>2</sub> objects under schemata will issue in an answer to Q2 only if there is a connection between subsuming<sub>2</sub> an object under a schema and subsuming<sub>2</sub> it under the corresponding category. I think

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I must be able to subsume<sub>1</sub> and subsume<sub>2</sub> it under that concept, but not vice versa. I will ignore the epistemic question in what follows.

<sup>11</sup> The Schematism chapter is not the only text where Kant discusses the mediating representations (schemata) by means of which sensible objects are subsumed under categories; he discusses them elsewhere in the *CPR* (A163/B204, A181/B223–4, A245, A247/B304, A665/B693, A286/B342, A553/B581, A664/B692, A682/B710, A718/B746) in other writings (4:316, 495; 5:68-9; 20:212, 232, 274), and in some unpublished Reflections (*Refl.* 5552, 5933, 6359) and letters (11:316, 12:224–5). But the Schematism is by far the most extensive and informative discussion of transcendental schemata in Kant’s entire corpus.

<sup>12</sup> My strong suspicion is that even the commentators quoted in note 7 would agree that the ultimate aim of the Schematism is to explain the possibility of subsumption<sub>2</sub> (Q2); however, because they do not make explicit how they understand “subsumption” (or “application”) it is hard to say for certain.

this connection is established by means of the notion of subsumption<sub>1</sub> and a connection between subsumption<sub>1</sub> and subsumption<sub>2</sub>, namely:

**Subsumption Link:** if subsuming<sub>1</sub> an object under a concept F is sufficient to subsume<sub>1</sub> that very object under a concept F\* then explaining the real possibility of the subsumption<sub>2</sub> of objects under F is sufficient to explain the real possibility of the subsumption<sub>2</sub> of objects under F\*.

This principle might appear complicated, but, I think, it expresses a straightforward truth.

If I can prove the real possibility of sensible objects instantiating F (being subsumed<sub>2</sub> under F) then I can prove the real possibility that Fx (for at least some sensible object x).

But if the thought that Fx is sufficient for the thought that F\*x (subsuming<sub>1</sub> x under F is sufficient for subsuming<sub>1</sub> x under F\*) then by proving the real possibility that Fx I have

thereby proved the real possibility that F\*x. For example, since <gold> contains

<yellow> as a mark, subsuming<sub>1</sub> an object under <gold> is sufficient to subsume<sub>1</sub> it under <yellow>; consequently, proving the real possibility of an object being subsumed<sub>2</sub>

under <gold> is sufficient to prove the real possibility of an object being subsumed<sub>2</sub>

under <yellow>. If I am right that this principle plays a key role in reconstructing Kant's

argument in the Schematism, this will vindicate my claim that the distinction between

subsumption<sub>1</sub> and subsumption<sub>2</sub> (and thus the distinction between Q1 and Q2) are key to understanding this chapter of the *CPR*.

Where G is a mark of F, subsuming<sub>1</sub> an object under F is analytically sufficient for subsuming<sub>1</sub> under G. Subsumption Link will be of interest only if we can find instances

of non-analytic sufficiency of subsumption<sub>1</sub> under categories.<sup>13</sup> I will argue that this is precisely what the Schematism provides: subsuming<sub>1</sub> an object under a schema is sufficient (though not analytically sufficient because the category is not a mark of the schema) for subsuming<sub>1</sub> it under the corresponding category. Consequently, the task of explaining the real possibility of objects being subsumed<sub>2</sub> under categories is reducible to the task of explaining the real possibility of their being subsumed<sub>2</sub> under the corresponding schemata.

Schemata thus play two crucial roles in Kant's proof of the real possibility of the subsumption<sub>2</sub> of sensible objects under categories. On the one hand, they help establish that:

**Subsumption Sufficiency:** subsuming<sub>1</sub> a sensibly given object under the schema of a concept is sufficient for subsuming<sub>1</sub> that object under that concept.<sup>14</sup>

After some preliminaries (§§1–2), in Part One I reconstruct Kant's theory of schemata, focusing on explaining why Subsumption Sufficiency holds. The guiding thread of my

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<sup>13</sup> Analytic sufficiency will not help for it would require finding concepts *of which the categories are marks*; explaining the possibility of subsuming objects under those “enlarged” concepts would require explaining the possibility of subsuming objects under their marks, the categories, the very problem with which we started. See §2 for more discussion of this issue.

<sup>14</sup> If we read “application” as application<sub>1</sub>/subsumption<sub>1</sub> this corresponds to Kant's claim that “an application of the category to appearances becomes possible by means of the transcendental time-determination which, as the schema of the concept of the understanding, mediates the subsumption of the latter under the former” (A139/B178).



interpretation is Kant's claim that schemata are "transcendental time-determinations." After specifying what notion of "determination" is involved in this claim (§3), I explain why the schemata of empirical concepts (§4) and mathematical concepts (§5) are time-determinations, and why the schemata of categories are transcendental time-determinations (§6). The conclusion of Part One is that, for each kind of concept and their associated schemata, Subsumption Sufficiency holds.

In the much shorter Part Two I explain Kant's justification for:

**Real Possibility:** there is an explanation of the real possibility of the subsumption<sub>2</sub> of sensible objects under schemata of empirical concepts, mathematical concepts, and categories.

In the case of empirical and mathematical concepts I argue (§7) these explanations are nearly direct consequences of the theory of schemata outlined in Part One and some relatively uncontroversial features of Kant's cognitive theory. In the case of categories, though, the explanation is not significantly more difficult. I argue in §8 that the real possibility of subsuming sensible objects under categorical schemata follows almost immediately from Kant's specification of those schemata. Subsumption Link, Subsumption Sufficiency, and Real Possibility allow Kant to conclude that:

**Conclusion:** there is an explanation of the real possibility of the subsumption<sub>2</sub> of sensible objects under categories.

This is precisely the conclusion for which the Schematism, on my interpretation, constitutes an argument. This establishes that, on my reading, the argument of the

Schematism is *valid*.<sup>15</sup> Whether it is *sound* depends on the strength of the arguments Kant gives in favor of the premises. I will argue that Kant's detailed theory of schemata provides a plausible case for the truth of these premises. Thus, I will argue, the Schematism to a large extent succeeds in its aims.<sup>16</sup> But we can only see this if we explicitly distinguish subsumption<sub>1</sub> and the "semantic" question Q1 from subsumption<sub>2</sub> and the "metaphysical" question Q2.<sup>17</sup>

### **Preliminaries**

Before proceeding to the core of the argument in Part One, I clarify some key preliminary notions.

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<sup>15</sup> None of the commentators I am aware of (see Bibliography at the end) reconstruct the Schematism *as an argument*. Even George Dicker, who is otherwise so meticulous in formally reconstructing Kant's arguments, abandons this methodology when it comes to the Schematism (Dicker 2004, 213–224).

<sup>16</sup> To this extent I agree with Allison 2004, 202–228. However, I provide detailed reconstruction of the argument of the Schematism, which Allison does not.

<sup>17</sup> This paper covers most of the main topics, and many of the details, of the Schematism. One topic that is salient by its absence is the *imagination*. Kant writes that "the schema is in itself always only a product of the imagination" (A140/B179). For reasons of space, I omit here my detailed interpretation of imagination and its relation to schemata (and time-determination). Two interpretations of Kant's theory of the imagination that harmonize with some of the points I make here are Sellars 1978 and Young 1988.

## §1. Subsumption

Kant sometimes uses “subsumption” to refer to a relation between concepts: one concept is said to be “subsumed” under a more general concept (e.g. *<human>* under *<animal>*).<sup>18</sup> However, in the Schematism he is concerned with subsumption as a relation between an object and a concept (e.g. subsuming Socrates under *<human>*); the very first sentence of the Schematism chapter begins “in all subsumption of an object under a concept [. . .]” (A137/B176; Au’s emphasis). I will refer to the latter as *object* subsumption and the former as *concept* subsumption. This distinction can be drawn within either subsumption<sub>1</sub> or subsumption<sub>2</sub>. We can distinguish between the thought that an object falls under a concept and the thought that one concept falls under another (subsumption<sub>1</sub>). Likewise, we can distinguish between the fact that an object falls under a concept and the fact that one concept falls under another concept (subsumption<sub>2</sub>).<sup>19</sup> Until Part Two, unless otherwise noted, when “subsumption” appears without a modifier, it always refers to object subsumption<sub>1</sub>.

The logical form of object subsumption is simple: *a* is subsumed under *F*, or, in contemporary symbolism *Fa*. The logical form of concept subsumption is, I take it, the logical form of judgment in general, which Kant specifies in the Jäsche logic as follows: “to everything *x*, to which the concept of body (*a + b*) belongs, belongs also attraction

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<sup>18</sup> For instance, in his discussions of subsumption in syllogistic reasoning: A301/B357, A304–5/B361, A308/B364.

<sup>19</sup> For further discussion of these two different uses of “subsumption” see Longuenesse 1998, 92–97, and de Boer 2016, 451.

(*c*)” (*Jäsche Logic*, 9:111).<sup>20</sup> In contemporary symbolism, the subsumption of concept *F* under concept *G* has this logical form:  $(x)(Fx \rightarrow Gx)$ . In the case of an analytic subsumption (subsuming one concept under a concept of which it is a mark), *G* will be a constituent of *F*; in the case of synthetic subsumption (subsuming one concept under a concept which is not one of its marks), *G* will not be a constituent of *F*, but the logical form of the subsumption will still be  $(x)(Fx \rightarrow Gx)$ . All of the objects subsumed under *F* are also subsumed under *G*. If I am right that the logical form of object subsumption is *Fa* then this means that object subsumption is already “built into” the logical form of judgment (concept subsumption). The role of judgment, after all, is to relate concepts which can be used to subsume *objects*.<sup>21, 22</sup>

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<sup>20</sup> This is specifically the form of synthetic judgment because Kant uses the variables “(*a* + *b*)” and “(*c*)” to indicate that *a* and *b* are not marks of *c*. If we relax that requirement (the mere form of the symbols is non-committal about mark-inclusion) we can use this as the form of judgment in general, both analytic and synthetic.

<sup>21</sup> The same point can be drawn out of the quoted passage: “to everything *x*, to which the concept of body (*a* + *b*) belongs” already involves a universal quantifier “to everything *x*” and an object subsumption “to which the concept of body (*a* + *b*) belongs.” Cf. Longuenesse 1998, 87.

<sup>22</sup> I thus depart from Allison 2004, 212, who takes the problem of subsumption to be the problem of how judgments are possible. On my reading, subsumption is a necessary condition on judgments relating to objects, but one can make judgments (concept-subsumptions) involving concepts under which no object can be subsumed<sub>1</sub>.

What objects are available to us to be subsumed under concepts? Given the logical form of object subsumption, what values can be substituted for  $x$  in the form  $Fx$ ? When I subsume an object under a concept I am not subsuming a general class of objects under that concept (that would be concept subsumption), I am subsuming a specific object. To the question “which object is subsumed”? the answer must always be a single object. This means that subsumption involves a singular aspect as well as a general aspect (the concept under which the object is subsumed).<sup>23</sup> Intuitions, in virtue of being singular representations, are the source of our ability to represent single objects. Consequently, to subsume an object under a concept I must be able to *intuit* that object.<sup>24</sup> If I cannot intuit objects of a certain kind then I may be able to have general thoughts about them (I can engage in concept subsumption involving concepts of them) but I cannot think *de re* of a single one of them that *it* falls under a concept. I cannot engage in object subsumption with respect to that kind of object. The objects we can intuit are what Kant calls “phenomena,” objects of sensible spatiotemporal intuition. Thus, in talking about object

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<sup>23</sup> This does not mean that “universal” object subsumptions are impossible (e.g. “to everything  $x$ , to which the concept of body ( $a + b$ ) belongs”) but that they are parasitic on the ability to subsume individual objects under concepts (“the concept of body ( $a + b$ ) belongs to  $x$ ”).

<sup>24</sup> My interpretation of intuition agrees, in broad outlines, with that of Allais 2015, while remaining neutral on her more controversial claims (e.g. the independence of intuition from understanding). See Stang (2016) for some differences between my reading of transcendental idealism and Allais’s.

subsumption I will always mean the subsumption of spatiotemporal objects under concepts.<sup>25</sup>

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<sup>25</sup> This means that I can think whole judgments involving concepts like *<noumena>* (e.g. *that noumena causally affect us*) but for no noumenon  $x$  can I think a thought of the form  $Fx$ . I anticipate two objections. First, Kant allows us to think about God and *<God>* is not a general concept (there is at most one God) so this might appear to be an object subsumption involving a noumenon (God cannot be intuited by us). That there can be at most one God does not mean, however, that a thought about God has the form  $Fa$  (where “ $a$ ” refers to God); it means that the concept of God is so determinate that it can be instantiated by at most one object. Nonetheless, thoughts about God remain concept subsumptions, not object subsumptions (their logical form is:  $(x)(\text{God}(x) \rightarrow Fx)$ ). Secondly, Kant allows that we I can think of myself as a noumenon. Here we must distinguish between theoretical and practical representation. Within theoretical philosophy I think that all of our thoughts about ourselves as thinking substances underlying our mental lives are concept subsumptions (e.g. “to everything  $x$ , to which the concept *<substance underling my thought-predicates>* belongs, belongs also *<simplicity>*”). In practical cognition, though, we can represent ourselves *de re* as noumena through the awareness of ourselves as bound by the moral law. I take this to involve something like the practical analogue of a singular representation. In this paper I am focusing solely on theoretical contexts where such *de re* noumenal representation is impossible. For a compelling account of the source of our practical awareness of ourselves as noumena, see Schafer Forthcoming.

My discussion up to this point implicitly presupposed that “subsumption” means subsumption<sub>1</sub> but all of the issues I raised can also be raised about subsumption<sub>2</sub>. We can distinguish between facts of the form  $(x)(Fx \rightarrow Gx)$  and those of the form  $Fx$ , and we ask, of those objects  $x$  which we can intuit, how it is possible for a fact of the form  $Fx$  to obtain?

## §2. Categories and Schemata

In trying to understand Kant’s explanation of the possibility of the subsumption (in either sense) of objects under categories it is helpful to think about the negative case where Kant thinks that such subsumption is impossible: non-sensible objects or “noumena.” In the Phenomena and Noumena section Kant writes: “if this condition of judgment (schema) is missing, then all subsumption disappears; for then nothing would be given that could be subsumed under the concept” (A247/B304). Subsuming<sub>1</sub> an object under a category, without its schemata, is impossible. Because schemata relate categories exclusively to sensible objects, this means no subsumption<sub>1</sub> of a non-sensible object under a category (or any concept for that matter) is possible.<sup>26</sup>

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<sup>26</sup> But this does not entail that the subsumption<sub>2</sub> of individual noumena under categories is impossible. It entails that Kant’s strategy for explaining the possibility of the subsumption<sub>2</sub> of sensible objects under categories cannot work for noumena. Since Kant (with his characteristic modesty) takes this to be the only possible such explanation, he takes the subsumption<sub>2</sub> of categories under noumena (their instantiation of categories) to be *inexplicable*.

Kant is not denying, though, that we can subsume *concepts* of non-sensible objects (e.g. <*noumenon*>) under other concepts (e.g. <*non-spatiotemporal*>), even categories (e.g. <*cause-effect*>), for his transcendental theory of experience requires precisely such concept-subsumptions (e.g. the thought that *noumena are non-spatiotemporal* and that *noumena cause the matter of experience*).<sup>27</sup> As so-called “metaphysical” readers of Kant point out, the *CPR* is replete with claims that, in my terminology, are concept-subsumptions involving concepts of non-sensible objects.<sup>28</sup> These “noumenal thoughts” remain available to us, even if we cannot use their constituent concepts to subsume non-sensible objects.

It is clear that the categories as they appear in these thoughts are not *identical* to schemata and, what is more, these thoughts do not even involve application of the corresponding schemata. This has given rise, in the Kant literature, to talk about “unschematized” categories (a term Kant never uses)<sup>29</sup> or “pure” categories (see A248/B304), i.e. categories that appear in thoughts (concept subsumptions) about non-sensible objects and hence do not involve schemata. What, then, is the relation between

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<sup>27</sup> For the former claim, see B307–8; for the latter, see especially 8:215, as well as the texts cited by Adickes 1924, ch. 3.

<sup>28</sup> E.g. Adickes 1924, Aquila 1979, Ameriks 1982, and Allais 2015. The presence of these noumenal thoughts is not proof that the “metaphysical” reading is correct, however. For a non-metaphysical take on, for instance, Kant’s talk of noumena affecting us, see Allison 2004, 64–73.

<sup>29</sup> This term appears to have been introduced into the Kant literature by Paton 1936, vol. 2, 41.



so-called “pure categories” and schemata?<sup>30</sup>

One tempting, but ultimately mistaken, model would take the unschematized or “pure” category to be a *genus* and the schemata to be a *differentia* within the relevant genus, so that the “schematized” category would be something like a *species*. For instance, the genus would be *<cause-effect>* (“something that allows an inference to the existence of something else” – A243/B301<sup>31</sup>), the differentia would be the corresponding

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<sup>30</sup> de Boer 2016 argues that “pure concepts of the understanding” already contain their own schemata (are already “schematized”), while “categories” are these pure concepts, abstracted from their schemata. While I have my doubts about whether this is accurate to Kant’s uses of these terms (de Boer’s reading is hard to square with Kant’s use of “category” at B128, A146/B185, A161/B201–2, A184/B227, A219/B266, A222/B269, and many other passages) even if de Boer is correct, my view can be easily translated into her terms as follows: where I say “unschematized category” substitute “category” *simpliciter*.

<sup>31</sup> A page earlier, at A242/B300, Kant distinguishes between the purely nominal definitions we can give of the categories in abstraction from their relation to sensible objects, and the real definitions that are possible in relation to such objects. In the case of each category he discusses, he identifies the possibility of the real definition of the category in its sensible use with its relation to *time*: quantity: “this how-many-times [*Wievielmals*] is grounded on successive repetition, thus on time” (A242/B300); reality: “reality, in contrast to negation, can be defined only if one thinks of a time (as the sum total of all being) that is either filled by it or empty” (A242/B300); substance: “if I leave out persistence (which is existence at all times)” (A242/B300); cause: “from the concept

schema (“the real upon which, whenever it is posited, something else always follows” – A144/B183), and the “schematized category” would be the conjunction of the two: a cause is a real being (one possessed of an intensive degree of reality) whose existence posits, and from whose existence always follows according to a rule, the existence of a distinct real being, its effect. The problem with the species-genus model is that it fails to explain precisely what Kant wants to explain: the subsumption of a sensibly given object under the category. A differentia is one mark among others of a species-concept; e.g. *<rational>* is one mark, among others, of *<human>* (*<rational animal>*). Likewise, on the species-genus model, the schema is one mark among others of the schematized category. But in order to subsume an object under a concept one must subsume it under *all* of its marks, not merely one of them. The species-genus model fails to explain why, in subsuming the object under the schema (differentia), one thereby also subsumes it under the category (genus). It is akin to explaining the possibility of subsuming an object under *<human>* by citing merely the possibility of subsuming it under *<rational>*.

The defender of the species-genus model might reply that this is part of the definition

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of a cause as a pure category (if I leave out the time in which something follows something else in accordance with a rule)” (A243/B301); contingency: “I would gladly know by what means you intend to cognize the possibility of this not-being, if you do not represent a succession in the series of appearances and in this succession an existence, which follows on the not-being (or conversely), and thus a change” (A243/B301). I take this to mean that the distinction Kant is drawing is between “pure” categories, abstracting from their temporal schemata, and schematized categories. See also A146-7, B186.

of a species-constituting differentia: it can only apply to instances of the genus (e.g. only animals can be rational).<sup>32</sup> But this is to restate my point without responding to it: without a further explanation, the species-genus model fails to explain why the schema is a differentia, in this stronger sense, of that category, i.e. why anything that is subsumed under “the real upon which, whenever it is posited, something else always follows” is thereby subsumed under *<cause-effect>*. And that is precisely what needs to be explained.<sup>33</sup>

The schema cannot be identical to the unschematized category (this would render thoughts involving pure categories impossible), it cannot be a species-constituting differentia, and, more generally, it cannot be one of the marks of the schematized category. What other relation might obtain between schema and category? In this paper I will defend the following proposal: although the schema is not identical to the category, to subsume<sub>1</sub> an object under a schema is sufficient to subsume<sub>1</sub> it under the corresponding category. Schemata are vehicles of object subsumption<sub>1</sub> for categories. This is the principle of Subsumption Sufficiency. Subsumption Sufficiency is secured, on my reading, because schemata are, in Kant’s words, “transcendental time-determinations.” In Part One, I explain what this means, and why it secures Subsumption Sufficiency.

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<sup>32</sup> See Anderson 2014, 123–5.

<sup>33</sup> To call schemata “application conditions” of categories is just to state my question, not to answer it. Why is satisfaction of the application-conditions (schema) of *<cause>* sufficient for subsumption under *<cause>*?

## Part One

### §3. Determination

In exploring Kant's argument for Subsumption Sufficiency, my guiding thread will be his claim at A139/B178 that the schema of a category is a "transcendental time-determination": "hence an application of the category to appearances becomes possible by means of the transcendental time-determination [*transcendentale Zeitbestimmung*], which, as the schema of the concept of the understanding, mediates the subsumption of the latter [appearances] under the former [the category]" (A139/B178). To unpack the notion of transcendental time-determinations we need to understand (i) what it means that schema are "transcendental," (ii) what they have to do with time, and (iii) what it means that they are "determinations." With respect to (i), for now I will merely point out that, in explaining how subsumption<sub>1</sub> of objects under *a priori* concepts is possible, schemata of such concepts explain (on my reading) how subsumption<sub>2</sub> of objects under such concepts is possible, and the possibility of subsumption<sub>2</sub> is a necessary condition for cognition using those concepts (Bxxvi, footnote). Consequently, schemata, insofar as they explain the subsumption<sub>2</sub> of objects under categories, will be transcendental in precisely Kant's technical sense at A56/B80: they will explain (partly) how *a priori* cognition is possible.<sup>34</sup> I will return momentarily to (ii) their relation to time, but I will

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<sup>34</sup> "[. . .] not every *a priori* cognition [*Erkenntnis*] must be called transcendental, rather only those through which we cognize that and how certain representations (intuitions or concepts) are applied merely *a priori* or are possible (i.e. the possibility of cognition or its use *a priori*)." Cf. Allison 2004, 216.

begin by examining (iii), the respect in which schemata are *determinations* (*Bestimmungen*).

We should distinguish at least four different senses of “determination” that are relevant to Kant’s investigations. Each of these is a sense the German word “*Bestimmung*” and its cognates (*bestimmen*, *bestimmt*, *bestimmtheit*, *unbestimmt*, *unbestimmtheit*, etc.) can have.<sup>35,36</sup>

*Determination-1* is the semantic notion of a representation representing its object as determinately having or lacking a predicate (being determinate with respect to that predicate). Indeterminacy<sub>1</sub> is the notion of a representation that represents its object neither as having nor as lacking a predicate. Conceptual representation, being general, is typically indeterminate<sub>1</sub> to some degree.<sup>37</sup> Determination<sub>1</sub> is a gradable notion; one representation can be more determinate<sub>1</sub> than another in virtue of representing its objects as possessing a wider range of predicates (e.g. a species-concept is more determinate<sub>1</sub>

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<sup>35</sup> One difference between English and German is that German does not have different words for “determined” and “determinate”; both correspond to “*unbestimmt*.”

<sup>36</sup> Stang 2012 contains a preliminary version of these distinctions.

<sup>37</sup> A concept is completely determinate<sub>1</sub> just in case, for every predicate F, it either contains F or ~F. Our concepts are typically incompletely determinate<sub>1</sub>, the possible exception being <God>, for it is completely determinate<sub>1</sub> at least with respect to God’s intrinsic properties. Conceptual representation, in general, is not completely determinate<sub>1</sub>. For extensive discussion of complete determination<sub>1</sub> see Kant’s lectures (28: 413, 503, 554, 630, 722–4, 779, 1156) and *Reflexionen* (*Refl.* 5760, 5783, 5784, 5786, 6245, 6255, 6290, and 6322) on metaphysics.

than a genus-concept). Determination<sub>1</sub> will play a key role in the argument of this paper; I will sometimes refer to it as “semantic” determination (likewise “semantic” indeterminacy is indeterminacy<sub>1</sub>).

*Determination-2* is the metaphysical notion of an object determinately having or lacking a predicate (being determinate with respect to that predicate). It is the notion involved in the “principle of complete determination,” according to which every object is fully determinate with respect to every predicate (it either determinately has or determinately lacks it).<sup>38</sup> Indeterminacy<sub>2</sub> is the notion of an object neither determinately having nor lacking a predicate (which may or may not be possible). Determination<sub>2</sub> is gradable; one object may be more determinate<sub>2</sub> than another in virtue of being determinate<sub>2</sub> with respect to a wider range of predicates. Determination<sub>2</sub> will also play a key role in the argument of this paper; I will sometimes refer to it as “metaphysical” determination (likewise, “metaphysical” indeterminacy refers to indeterminacy<sub>2</sub>).

*Determination-3* is also a metaphysical notion, closely tied to the notion of a determining ground. An object is said to be determined<sub>3</sub> with respect to a predicate just in case there is a determining ground of its possessing that predicate.<sup>39</sup> A determining ground of an

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<sup>38</sup> See Kant’s discussion of this principle at A573/B601, as well as Stang (2012).

<sup>39</sup> A determining ground is one that not only brings about its consequence but does so necessarily: it is impossible that a determining ground, in the very same context, would “posit” a different consequence (or fail to posit any consequence). The notion of a determining ground (and its difference from a merely sufficient ground, which may posit different consequences in different contexts) is important primarily in the context of debates about free will, in particular, whether the will is a *determining* ground of its acts,

object's possession of a predicate is an entity such that, necessarily, if the same background conditions obtain, it grounds the possession of that predicate by that object. Determination<sub>3</sub> will not be further discussed in this paper (though it has a claim equal to determination<sub>2</sub> to be called "metaphysical" determination). I mention it only to distinguish it from other notions of determination.

*Determination-4* is an epistemic notion. An object is determined<sub>4</sub> with respect to a predicate by an agent just in case that agent knows that object has or lacks that predicate.<sup>40</sup> An object is said to be indeterminate<sub>4</sub> with respect to a predicate when it is epistemically open, for an agent, whether or not the object has that predicate. Determination<sub>4</sub> will not be further discussed in this paper. I mention it only to distinguish it from other notions of determination.

While determination<sub>1</sub> and determination<sub>2</sub> are distinct notions, they are closely connected in Kant's philosophy. Because "the conditions of the possibility of the

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or merely a *sufficient* ground. The distinction is originally due to Crusius, in his 1763 work *Dissertatio de usu et limitibus principii rationis determinantis, vulgo sufficientis* (Crusius 1964, vol. 4, §§I, III, XX). Kant notes this distinction with approval in the pre-Critical *Nova dilucidatio* (1:398), although he later distances himself from it in his metaphysics lectures (28:14, 30, 54, 401, 491; 29: 808, 809, 815, 819). For critical discussion see Hogan (2009) and Stang (2019).

<sup>40</sup> Determination<sub>4</sub> could be extended to include other epistemic states less demanding than knowledge (e.g. Kant's notion of *Glaube*). However, since this notion of determination will play a minimal small role in this paper, I will leave it at that. See Chignell (2007) for an account of different kinds of epistemic states in Kant.

experience of objects [determinately<sub>1</sub> representing them as, e.g. substances and causes] are the conditions of possible objects of experience [their determinately<sub>2</sub> being substances and causes] (A493/B521), they are determinate<sub>2</sub> (at least with respect to the range of predicates representable in experience) only insofar as we determinately<sub>1</sub> represent them.

Having made some preliminary distinctions among different notions of determination, we must now ask: in which (if any) of these four senses are schemata transcendental time-determinations? On my reading (see Introduction) the primary role of the Schematism is to establish a thesis about the subsumption<sub>2</sub> of objects under categories (Conclusion) by means of a premise about subsumption<sub>1</sub> (Subsumption Sufficiency). Subsumption<sub>1</sub> involves determinacy<sub>1</sub>: to subsume<sub>1</sub> an object under a concept is to determine<sub>1</sub> that object with respect to that concept. To subsume<sub>2</sub> an object under a concept, by contrast, that concept must represent the object as having features the object determinately<sub>2</sub> has.<sup>41</sup> Determination<sub>2</sub> will thus become relevant when we reconstruct Kant's argument for the claim that it is really possible for objects to be subsumed<sub>2</sub> under schemata (Real Possibility) and thus under categories (via Subsumption Link). While determination<sub>3</sub> (having a determining ground) is not strictly *irrelevant* it is decidedly secondary to this project.<sup>42</sup> So is determination<sub>4</sub> (being epistemically determined). The

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<sup>41</sup> See the opening of the Schematism at A137/B176.

<sup>42</sup> For one thing, one of the categories <cause-effect> is the category by which we think of one object being determined<sub>3</sub> by another; for another, the determinacy<sub>1</sub> (content) of our representations determines<sub>3</sub> (grounds) objects as determinate<sub>2</sub> (having properties) and



question of how we *know* objects to fall under a set of concepts (determining<sub>4</sub> them with respect to those concepts) cannot arise unless we assume that those concepts can be representationally determinate<sub>1</sub> with respect to properties sensible objects have (determination<sub>2</sub>).<sup>43</sup>

I can now offer a first-pass characterization of my interpretation of schemata as transcendental time-determinations: schemata explain the possibility of determinately<sub>1</sub> representing a sensibly given object under a category (subsuming<sub>1</sub> it under the category). They do so by determinately<sub>1</sub> representing the specifically *temporal* features of objects.<sup>44</sup>

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thus allows us to determine<sub>4</sub> (know) them *a priori*. (I promise the reader this is the only sentence in this paper in which all four notions of determination are involved!)

<sup>43</sup> More indirectly, Kant makes clear that schemata make possible cognition (*Erkenntnis*) of objects using categories rather than knowledge (*Wissen*) that they fall under categories. While there is, as of yet, no consensus among scholars what “cognition” (*Erkenntnis*) is, there is a growing awareness that it is crucial to distinguish it from knowledge (*Wissen*); see Smit 2000, Hanna 2006, Watkins and Willaschek 2017, and Schafer Forthcoming. Kant’s investigation into the possibility of cognizing objects using categories is not thereby an investigation into the possibility of knowing (determining<sub>4</sub>) that objects fall under categories.

<sup>44</sup> By contrast, Henry Allison gives the following definition: “we may preliminarily characterize a transcendental time-determination as a rule-governed product of the figurative synthesis (a “transcendental product of the imagination”), which exhibits in a non-discursive manner the form of unity conceptually expressed in a category” (Allison

Why determinately<sub>1</sub> representing the temporal features of objects should have anything to do with determinately<sub>1</sub> representing them as falling under categories (subsuming<sub>1</sub> them under categories) is one of the main things I shall have to explain in this paper. *Until further notice, “(in)determinate/determinacy/determination” will refer to the relevant forms of determination<sub>1</sub>.*

At the risk of introducing yet a further distinction, I think it will aid our comprehension of Kant’s discussion if we distinguish, within determination, what I will call the *agent* of determination (the determining representation) and the *patient* of determination (the determined object). For instance, as I progressively determine my concept of matter (e.g. through physical investigation), my concept becomes further determinate and is thus an agent by which I further determine matter, the patient of determination. The agent of determination in our case is two-fold: the schema (it determinately represents the temporal features of objects) and the category (we are explaining how it comes to be able to determinately subsume objects). But what is the patient?

Schemata can mediate between pure intellectual concepts and sensibly given objects, Kant writes, because they are “homogenous” with both (A138/B177). They are homogenous with sensibly given objects because “time is contained in every empirical representation of the manifold” (A139/B178). It is important to realize that “contained in” (*enthalten in*) cannot mean “contained in the content of” because time is the form of

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2004, 215). I do not think this characterization is false, but I think my characterization is more explanatory.

inner sense<sup>45</sup> (as Kant reminds us earlier in the same paragraph) and thus outer sense intuitions are not *as such* intuitions of objects *as temporal*.<sup>46</sup> In fact, Kant will have to do some very intricate work in the Analogies and the Refutation of Idealism to demonstrate both the possibility and necessity of representing outer objects as also being *in time*.<sup>47</sup> At this point, in the Schematism, before he has done that work, Kant can only assume that representations of outer objects are *in time* (because all representations, as states in us, are in time).<sup>48</sup> Hence it might have been more perspicuous if Kant had written: “every empirical representation of the manifold [is contained in time]” (A139/B178).

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<sup>45</sup> “Time is nothing other than the form of inner sense, i.e. of the intuition of ourselves and our state [*Zustand*]” (A49/B50).

<sup>46</sup> “Time cannot be a determination of outer appearances: it belongs neither to shape nor location etc., but rather determines the relation of representations in our inner state” (A33/B49–50).

<sup>47</sup> I read Kant’s discussion of “time determination” in the Analogies (A183/B226, A185/B228, A188/B231, A215/B262, A217/B264) and the Refutation (B275–8), continuous with my reading of the Schematism, as concerning the conditions of the possibility of determinately<sub>1</sub> representing the temporal relations of outer objects (permanence, succession, simultaneity), which, given that these objects are mere appearances, are at the same time conditions of the possibility of their determinately<sub>2</sub> standing in those temporal relations.

<sup>48</sup> See A34/B50–1. My reading is in some tension with the conclusion of that paragraph: “from the principle of inner sense I can say entirely generally: all appearances in general, i.e., all objects of the senses, are in time, and necessarily stand in relations of time.” This

Schemata mediate categories with appearances because, as *time*-determinations, they are “homogenous” with appearances. But if appearances are in time (at this point in the *CPR*) only in the indirect sense that the sensible representations by which those appearances are given to us (intuitions) are in time, then schemata mediate categories with appearances by mediating categories with intuitions of appearances. Time is the “medium” of this mediation because all representations, as states of ours, are in time. Schemata, as mediating representations, are specifically *time-determinations*, because they further determine objects in respect of temporal features. But which objects? What is the patient of determination here? It cannot be, at this point in the *CPR*, that they immediately determine outer objects in respect of time, for Kant has not yet built the

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can be read as making either a weak or a strong claim about the temporality of all appearances *überhaupt* (including outer ones): (i) all appearances, even outer ones, are in time in the minimal sense that our intuitions of them are in time (weak), or (ii) all appearances, even outer ones, not merely our intuitions of them, are in time (strong). On the weak reading, Kant is entitled to this claim at this point in the *Aesthetic*, but the weak reading does not fit the text very well (Kant emphasizes that it is the *objects* of representation, not merely the states in us by which we represent them, that are in time). The strong reading is more natural to the text, but it is a *non sequitur*, for it does not follow from what Kant says earlier in the paragraph. I think the sentence must be read in the strong way, but as anticipating an argument Kant will give later, in the *Analogies* and the *Refutation of Idealism*. Bader 2017 offers a fascinating, and very different, account of why outer objects are in time, but one not very closely tied to the letter of the text.

machinery for doing so (primarily, the Analogies).<sup>49</sup> Instead, schemata immediately determine the inner states by which such objects are given to us (intuitions, which *qua* states in us are in time). Specifically, they determine the temporal properties of those states.<sup>50</sup>

At a first approximation, then, we can say that schemata are transcendental time-determinations insofar as: (i) they allow us to determine<sub>1</sub> objects with respect to categories by allowing us to subsume<sub>1</sub> sensibly given objects under them; (ii) they do so by determinately<sub>1</sub> representing temporal relations among the states by which these objects are given to us (intuitions). The question is: why, in doing (ii), do schemata also do (i)?

#### **§4. Empirical Schemata as Time-determinations**

Schemata of categories are transcendental time-determinations. In building up to a more complete characterization of what that means, I am going to start by considering the schemata of empirical concepts, a topic Kant introduces and briefly discusses at A141/B180. Categorical schemata are mediating representations that make possible the subsumption of given objects under categories, while schemata of empirical concepts (henceforth, empirical schemata) make possible the subsumption of given objects under

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<sup>49</sup> And the schemata will themselves play an important role in showing how the time-determination of outer objects is possible, for the Analogies involve the time-schematized relational categories as. See A181/B223–4.

<sup>50</sup> On this point I agree with Allison 2004, 217.

empirical concepts.<sup>51</sup> Hence empirical schemata are not transcendental but, as schemata, they are still time-determinations. This is indirectly confirmed in the text of the Schematism itself: “[an empirical concept] always refers immediately to a schema of the imagination as a rule for the determination of our intuition, in accordance with a certain general concept” (A141/B180). “A rule for the determination of our intuition” is, I take it, a rule for determining intuitions as temporally ordered states in us, rather than a rule for determining the object of that intuition (subsuming<sub>1</sub> it under a concept), though of course Kant needs to show that the former will explain the latter.

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<sup>51</sup> There is a debate in the secondary literature (see Pendlebury 1955; Chipman 1972; Pippin 1976; Guyer 1987; Matherne 2015) on whether empirical concepts are identical to their schemata. Even if, contra Chipman 1972, it is impossible to possess an empirical concept without being able to subsume a sensible object under it, it does not follow that they are identical, merely that they are necessarily co-occurring. The view that empirical concepts are their own schemata fits poorly with Kant’s principal discussion of them: “Even less does an object of experience or an image of it ever reach the empirical concept, rather the latter is always related immediately to the schema of the imagination, as a rule for the determination of our intuition in accordance with a certain general concept. The concept of a dog signifies [*bedeutet*] a rule in accordance with which my imagination can specify the shape of a four-footed animal in general [. . .]” (A141/B180). If an empirical concept is its own schema then, by the second sentence, it signifies [*bedeutet*] itself, which is rather awkward.

But what does applying an empirical concept to an object (especially an outer object) have to do with representing our intuition of that object as temporally determinate? Kant's example of a dog is not especially helpful: "the concept of a dog designates a rule, according to which my imagination can draw the shape [*Gestalt*] of a four-footed animal, without being restricted to any particular shape [*Gestalt*] that experience may have offered me or any possible image [*Bild*] which I can present *in concreto*" (A141/B180). One thing that is notable about this example is that Kant has, in effect, "mathematized" the schema of the concept of a dog by thinking of it as a schema for drawing a certain shape in space, albeit an empirically-specified shape rather than a purely geometrical one. He has partially assimilated it to the schema of a geometrical concept. However, I think Kant's example of the schema of the concept <dog>, if thought through more fully, contains the materials to understand better, not only schemata of empirical concepts but mathematical and categorial schemata as well.

Subsuming a sensibly given object under the concept <dog> is thinking of that object as a dog. But, as Kant is fully aware, to think of an object as a dog is not merely to think of it as having a certain shape.<sup>52</sup> The concept <dog> is not the concept <dog-shaped object>, for an object could be a dog-shaped cat or a dog-shaped pile of wood. A dog is an object with a certain kind of history, a certain kind of life. In an Aristotelian vein, we could say that a dog is a being with a certain nature, one that dictates how it changes and develops over the course of time and in the right environment. The form of the dog is an inner principle of change by which it assimilates nutrients from the environment,

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<sup>52</sup> If it were, Kant could have saved himself a lot of trouble in the Critique of Teleological Judgment!

maintains its internal constitution, grows into maturity, decays, and dies. These are considerations relevant to answering the question “what is a dog?” Kant, however, is concerned with the semi-transcendental question, “what is it to represent something as a dog?”<sup>53</sup> But we can transform these object-oriented answers into answers to the Kantian question by internalizing them within representation: to represent an object as a dog is to represent it *as* having a certain kind of history determined by an inner principle of change, etc. Thus, in a Kantian vein we could say: representing an object as a dog is representing it as temporally determinate in certain ways (standing in specific temporal relations to its past and future stages). If you are representing something in front of you as having just come into existence and as about to go out of existence then you are not representing it as a dog (you may be representing it as a momentary dog-stage). The schema of an empirical concept like <dog> might, then, plausibly be thought of as a time-determination.<sup>54</sup>

But in saying all of this we will not yet have explained, or made any more plausible, the idea that schemata for empirical concepts are rules for representing *our inner states*

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<sup>53</sup> “Semi-transcendental” because transcendental cognition *sensu strictu* concerns how *a priori* cognition is possible (see A56/B80).

<sup>54</sup> The considerations in this paragraph, and the rest of this section, are similar to the views of Sellars 1976 and Matherne 2015. But whereas Sellars focuses on the case of empirical concepts (and only makes a few remarks on schemata for pure categories), and Matherne is primarily concerned with images rather than schemata, I extend this broadly “phenomenological” interpretation to all schemata for all concepts whatsoever. See the following three sections for details.



as temporally determinate. Why should representing an object as a dog involve representing my very representation of the dog (a state of mine) as standing in some determinate range of temporal relations, rather than representing the object of that representation (the dog itself) as so standing? To return to a theme of the previous section, time is the form of inner sense; space, the form of outer sense. I take this to mean that temporal content is not “built into” intuitions of outer objects. Outer sense intuitions of objects do not represent temporal features merely in virtue of being the kind of representations they are (by contrast, inner sense intuitions represent temporal features solely in virtue of being the kind of representation they are).<sup>55</sup> I can represent outer objects as having temporal features but this is not an ultimate representational fact; it stands in need of an explanation, a specification of its ground. We cannot simply say that we *do* represent the object (the object of an outer sense intuition) as having certain temporal features; we must say what is in virtue of which we are doing so. Kant’s answer to this question—in virtue of what are we representing an outer object as temporally determinate?—involves our representing our representation of that object (a state in us) as standing in corresponding temporal relations to other representations (other states in us). In particular, to represent an object as a dog is to represent one’s present intuition of it as standing in determinate temporal relations to other representations: actual and merely possible past and future intuitions of the very same object.<sup>56</sup> To represent the

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<sup>55</sup> See A33/B49–50 (quoted above).

<sup>56</sup> The idea that representing an object requires representing a relation among the vehicles of representation themselves (intuitions, in this case) may seem baroque, but

object as a dog, for instance, is to represent one's present intuition of it as a representation of *the very same* object as, say, one's previous intuition of it as a puppy, and one's future intuition of it as an older dog. It is, more minimally, also to represent one's current actual intuition as being of the same object as a non-actual but merely possible *simultaneous* intuition of it from a different spatial perspective.<sup>57</sup> If you are representing your current intuition as being of an object that could not be represented from a different perspective, or could not have been represented in the past, or in the future, you are not representing that object as a dog (you may be representing it as your visual image of a dog).<sup>58</sup>

Kant does not draw out these consequences of his example of the schema for *<dog>*. In fact, there are reasons he could not have, for his considered view in the third *Critique* is that representing something as a dog, an organized living thing, involves representing it as having a kind of internal purposive organization not accounted for by the transcendental theory of cognition in the first *Critique*.<sup>59</sup> So there are reasons to think Kant may have chosen a poor example, even by his own lights. But all of the points I have drawn out from *<dog>* could also have been drawn out from one of Kant's most famous examples, *<cinnabar>*. The difference is that cinnabar does not undergo (much) internal change and a sample of cinnabar does not need to have any particular past or

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consider that the natural alternative (simply representing temporal relations among the objects) is not available to Kant, given that time is the form of inner, not outer, sense.

<sup>57</sup> This point lies at the heart of Sellars 1976.

<sup>58</sup> A similar point is made by Pendlebury 1995, 790.

<sup>59</sup> See *Critique of Judgment* §65, 5:295.

future as long as it maintains the same inner material constitution and causal powers that make it cinnabar. The temporal determinacy required to represent cinnabar is thus more flexible (less determinate, one might say)<sup>60</sup> than what is required to represent something as a dog. Just as with the case of the dog, we can transpose a purely object-oriented account of what is it to be cinnabar (i.e. to have a certain causal history and causal power) into an account of what it is to represent something as cinnabar. To represent an intuited object as cinnabar is to represent one's present intuition of it as a representation *of the same object* as possible past intuitions of it, as a representation of the same object as possible experiences in which it passes the tests for mercury oxide, etc.

In this section I have reconstructed Kant's reasons for two key claims:

**Schema<sub>emp</sub>**: the schemata of empirical concepts are rules for determinately<sub>1</sub> representing temporal relations among our inner states, in particular, the intuitions by which empirical objects are given to us.

**Subsumption Sufficiency<sub>emp</sub>**: subsuming<sub>1</sub> a sensibly given object under the schema of an empirical concept (as specified above) is sufficient for subsuming<sub>1</sub> that object under that concept.

In the next section, I will reconstruct Kant's reasons for parallel claims about geometrical concepts.

Intuitions are states of ours that represent objects. In contemporary terms, we can distinguish intuitions as *vehicles* of representation (states) from the *content* of those intuitions, which for expository purposes I will take to be their objects (though I am not assuming that objects exhaust the content of intuition). My discussion of empirical

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<sup>60</sup> Determination<sub>1</sub> is gradable (see Section 3 above).

schemata will arouse in some readers the suspicion that my account falls afoul of this distinction. For on my account, schemata are determinate<sub>1</sub> representations of relations among inner states, representational *vehicles*. Why, in representing these relations among vehicles do I thereby represent something about the *content* of those vehicles (their objects)? Why do I represent the contents of these vehicles (their objects) as subsumed<sub>1</sub> under the corresponding concept?<sup>61</sup>

This objection can be raised at every stage in my account of schemata, so it is worth responding to it here at the outset. I think this objection assumes an implausibly strong distinction between representing (relations among) vehicles and representing (relations among) contents. To take a linguistic example, names are vehicles and (let us assume, for the sake of simplicity) their contents are their bearers. If “Brian” is a name coined at  $t_1$  to refer to some perceptually demonstrated person,<sup>62</sup> and “Alex” is coined in the same way at  $t_2$ , in representing that “Brian” and “Alex” are co-referring terms, I am representing (if we now disquote the names) that Brian/Alex exists at  $t_1$  and at  $t_2$ . I have represented a relation among contents (objects, in this case) by representing a relation among representational vehicles. If, instead, I had represented “Brian” (vehicle) as co-referential with a host of possible names coinable through perceptual demonstration from  $t_1$  to  $t_2$ , then I would have represented Brian as existing during this period in virtue of

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<sup>61</sup> Thanks to Colin McLear for pressing me on this.

<sup>62</sup> We may have to build in the assumption that a name coined through a perceptual demonstrative (let “Brian” refer to *that* guy) at a time must apply to an object that exists at that time. Regardless of what fixes are needed, the larger point holds: we can represent relations among contents by representing appropriate relations among vehicles.

representing a relation among an actual name (“Brian”) and merely possible names (vehicles).<sup>63</sup> Likewise, if I represent my current intuition of an object as co-referential with possible intuitions during a period of time T, I am representing the object of my intuition as persisting through that period. I am subsuming<sub>1</sub> the object of my intuition (its content, in this simplified story) under a concept (*<persists through T>*). So if the worry is that by representing a relation among the vehicles (intuitions) by which empirical objects are given to me, I am not representing anything about their objects (not adding additional content to my representation of the object), then the objection is based on a tempting fallacy.

#### §5. Mathematical Schemata as Time-determinations<sup>64</sup>

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<sup>63</sup> The role of the modalities is complicated here, and this is reflected in the Kant’s schemata for modal categories. However, for reasons of space, I leave my account of modal schemata out of this paper.

<sup>64</sup> Béatrice Longuenesse emphasizes the role of schemata in the understanding’s “affection” and thereby “determination” of inner sense (Longuenesse 1998, 246). However, Longuenesse never fully clarifies the meaning of “determination” here, i.e. the way in which schemata are transcendental time-determinations and how this explains the subsumption of objects under concepts. My account of schemata fills this gap in Longuenesse’s reading. But, unlike Longuenesse, I do not, in this paper, give an account of the relation of schemata to logical functions. That must await future work.

I noted earlier that Kant's discussion of the schema for <dog> quasi-mathematizes this concept by treating it as the concept of a dog-shaped object. This suggests that Kant will be on stronger grounds in schematizing mathematical concepts. However, the idea that schemata in general are time-determinations faces the obvious objection that mathematical objects (e.g. triangles) are not in time,<sup>65</sup> so it is unclear what time-determination has to do with the possibility of subsumption in mathematics.

In the Schematism, Kant uses two arithmetical examples and one geometrical example:

[. . .] on the contrary, when I merely think of a number, which may be 5 or 100, this thinking is more the representation of a method for representing in an image a collection [*Menge*] appropriate to the concept (e.g. a thousand) than [it is] the image itself. (A140/B180)

The scheme of the triangle can exist nowhere else than in thoughts and designates a rule for the synthesis of the imagination in respect of pure shapes [*Gestalten*] in space. (A141/B180)

[. . .] the pure **schema of magnitude** (*quantitatis*), however, is **number**, which is a representation which collects together [*zusammenbefaßt*] the successive addition of unit to (homogeneous) unit [*Einem zu Einem*]. (A142/B182)

This leads some commentators on the Schematism to refer back to Kant's discussion of "drawing a line in thought" and "representing a certain number to myself" in the A

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<sup>65</sup> A point of which Kant was well aware; see his letter to Schultz, November 25, 1788 (10:556).

Deduction (A102; cf. B154).<sup>66</sup> Kant's point in the A Deduction, roughly, is that one cannot draw a line in thought without mentally "reproducing" the previously drawn parts of the line.<sup>67</sup>

In drawing a line, or constructing a triangle in thought, one must not mentally "lose track" of the prior moments of one's construction. But if we think through the consequences of that point, we see a structure very similar to the structure I uncovered earlier in empirical schemata. A schema for a mathematical concept allows us to subsume an object under a mathematical concept, for that is what schemata in general do: they make subsumption possible.<sup>68</sup> For Kant, there is a close connection between mathematical subsumption and mathematical construction: to subsume an object under the concept *<triangle>* is to represent it as the result of a possible act of construction.<sup>69</sup> This is why the schema of *<triangle>* is a rule for constructing a triangle, just as the schema of a number *n* is a rule for generating a collection of *n* elements (see the passages quoted above). So if we can uncover a structure of temporal determination within the act

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<sup>66</sup> E.g. Longuensee 1998, 249–257.

<sup>67</sup> "Drawing a line in thought" is Kant's own phrase but it refers, I take it, to what he elsewhere refers to as construction "in pure intuition" (A713/B741). I will continue to use the former expression, however, since I do not have the space in this paper to discuss Kant's complex doctrine of construction in pure intuition.

<sup>68</sup> A138/B177.

<sup>69</sup> I say "possible act of construction" for presumably I can see a figure in a geometrical proof and cognize it as a triangle without myself having to construct it, as long as I represent it as constructible.

of constructing the triangle we will have found such a structure in the act of subsuming an object under the concept *<triangle>*.

The canonical construction of a triangle ABC has three parts: (1) using a ruler, construct a line segment AB of arbitrary length; (2) using a compass, construct a circle of arbitrary radius around one endpoint of length  $l$  (A); and (3) using a compass, construct another circle with radius  $l'$  (where  $|l - \overline{AB}| < l' < |l + \overline{AB}|$ ) around the other endpoint (B). Let C be the point where the circles intersect; the sides of the triangle are the line segments AB, BC, and AC. This is represented graphically in Figure I.

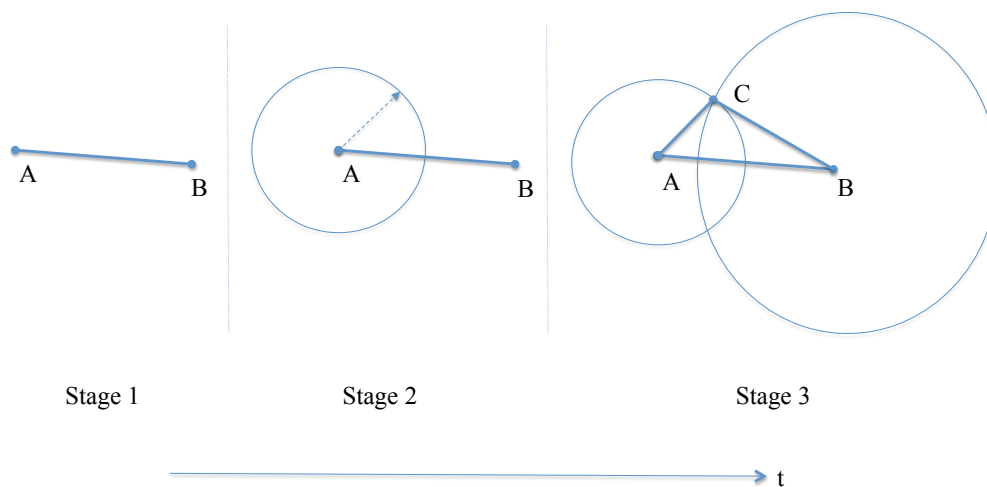


Figure I

The straight-forward reading of A102 is that one will not have constructed a circle if, at stage 2, one forgets stage 1 (likewise for stage 3 and stages 2 and 1). But a stronger claim is warranted: one is not constructing a triangle *at stage 2* if one is not aware of one's construction of the circle with radius AB that it is part of a construction of a triangle, completing the construction of AB (stage 1) and to be completed by



constructing a circle around B (stage 3); one is not constructing a triangle at stage 2 if one is not aware of it as completing stage 1 and to be completed by stage 3; and one is not constructing a triangle at stage 3 unless one is aware that this is the completion of stages 1 and 2, etc. We can represent this graphically as the idea that stage 1 must contain a representation of Figure I as a whole as well as its own place in that process; likewise for the other stages. I have attempted to convey this in Figure II. More precisely, in constructing a triangle one must be conscious at each stage of the construction that one's present constructive act stands in determinate temporal relations to past and future acts of construction. This means that construction involves not only object-directed consciousness (consciousness of the object being constructed) but consciousness of one's own constructive act, consciousness of one's own representations. Only a being capable, at stage 2, of representing that very representation, as part of some larger constructive procedure, could construct a triangle. Construction, in other words, requires self-consciousness: consciousness of one's representations *as such*.<sup>70</sup>

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<sup>70</sup> Consider the following objection: a computer draws a triangle, and the representational content of the state of the computer that corresponds to Stage 1 is causally relevant (it is not erased from working memory) to the representational content of the state of the computer that corresponds to Stage 2, etc. But the computer lacks self-consciousness, let us assume. Has the computer not constructed a triangle? It has not, I would answer. Constructing a triangle is *a priori* presenting to oneself an object corresponding to the concept (A713/B741), which is cognition (A50/B74) and cognition, according to Kant, requires self-consciousness (B137). The computer may have generated an *image* of a

In the case of mathematical concepts, I take it that it is clearer than in the case of empirical concepts <dog> that what we are representing as temporally determinate is our own representations. This is clear because the object represented (the triangle) is not in time and because it has no being outside our construction of it. The only items here that could be represented as temporally determinate are the representations that make up our complex constructive act.<sup>71, 72</sup>

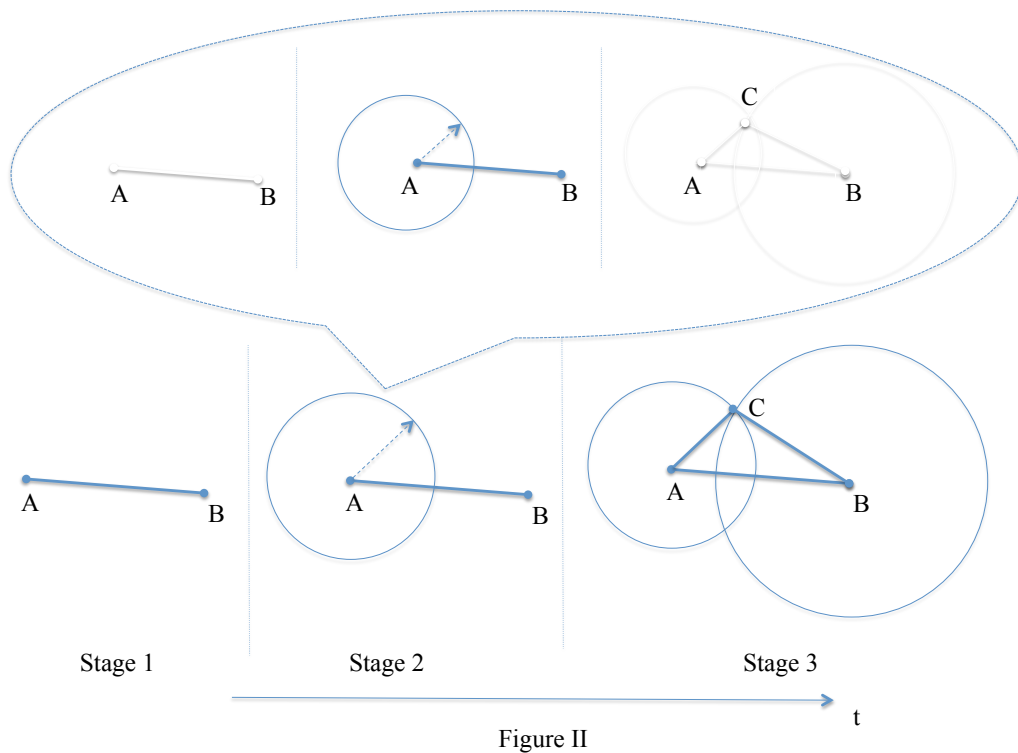
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triangle, but an image is not a schema (A140/B179). Schemata are products of self-consciousness (A142/B181), and without schemata mathematical cognition and construction are impossible. A non-self-consciousness machine cannot engage in mathematical construction.

<sup>71</sup> One might point to the marks on paper, or lines drawn in the sand, as the relevant objects. Two points about this: (i) lines drawn on paper are not geometrical lines except insofar as they being used as illustrations of a geometrical construction, so the point in the main text still holds; (ii) Kant holds that we can perform constructions without drawing physical figures, “through mere imagination, in pure intuition” (A713/B741).

<sup>72</sup> This also shows how to put to rest the concern, expressed by some commentators, that the Schematism may be incompatible with the General Remark to the System of Principles (B288–94), where Kant claims that “in order to understand the possibility of things in accordance with the categories, and thus to establish the objective reality of the latter, we do not merely need intuitions, but always outer intuitions” (B291). The Schematism concerns the temporal relations among intuitions we must determinately represent in order to subsume the objects of those intuitions under concepts; the General Remark concerns the spatial character of the objects of those intuitions. There is no

The rule for constructing a triangle is what Kant identifies as the schema of *<triangle>*. I have argued that this rule can be understood as a rule for representing the determinate temporal relations among various representations. This explains, I take it, the sense in which even mathematical schemata are time-determinations. They are specifically *transcendental* time-determinations because they explain the possibility of *a priori* cognition: *a priori* construction of a triangle.<sup>73</sup> Instead of immediately turning to the case of arithmetical schemata, in the next section I explain how schemata of the pure concepts of the understanding are transcendental time-determinations.



In this section I have reconstructed Kant’s reasons for two key claims:

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tension here.

<sup>73</sup> See Kant’s definition of “transcendental” at A56/B80.

**Schema<sub>geo</sub>**: the schemata of geometrical concepts are rules for determinately<sub>1</sub> representing temporal relations among our inner states, in particular, the states involved in constructing a corresponding object.

**Subsumption Sufficiency<sub>geo</sub>**: subsuming<sub>1</sub> an object under the schema of a geometrical concept (as specified above) is sufficient for subsuming<sub>1</sub> that object under the concept.

In the next section, I will reconstruct Kant's reasons for parallel claims about the categories.

## §6. Pure Schemata as Time-determinations

I have argued that schemata in general are time-determinations, rules for (self)-consciously representing one's representations (inner states) as standing in determinate temporal relations. In this section I want to apply this analysis to the primary case of schemata, the schemata of pure concepts of the understanding (categories). Kant's discussion of the schemata of individual categories at A142–5/B182–4 has a recursive structure, insofar as later sets of categories are schematized in terms of previous sets. Consequently, I will discuss the schemata of the categories in the order in which they appear in the Table of Categories itself—quality, quantity, and relation. For reasons of space, I forego discussion of the schemata of modal categories.<sup>74</sup>

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<sup>74</sup> The Table of Categories derives, of course, from the Table of Logical Functions of Understanding in Judging (A76–80/B102–106). Consequently, we should expect a relation between schemata and logical functions of judging, which is directly confirmed by *Refl.* 5933: “the Schematism shows the conditions under which an appearance is

(i) *Quantity*

In my discussion of mathematical schemata earlier, I focused on geometrical schemata to the exclusion of Kant's other main mathematical example, arithmetical schemata for concepts of numbers. I did so because numerical schemata arise in the course of Kant providing schemata for the categories of quantity and thus deserve a separate treatment.<sup>75</sup>

[. . .] the pure **schema of magnitude** (*quantitatis*), however, is **number**, which is a representation that collects together [*zusammenbefaßt*] the successive addition of unit to (homogeneous) unit [*Einem zu Einem*]. Number is thus nothing other than the unity of the manifold of a homogenous intuition in general [*überhaupt*], through

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determined in respect of a logical function" (18:392; cf. A79/B104–5, A94/B128). The most complete attempt to relate the schemata systematically to logical functions of judging is Longuenesse 1998, ch.9–11. Longuenesse does not discuss the modal schemata, however. I tackle modal schemata in work currently in preparation.

<sup>75</sup> Immediately before this passage, Kant writes "the pure image of all magnitudes (*quantorum*) for outer sense is space; that of all objects of the senses in general is time" (A142/B182). I am not giving an account of images, so I will not attempt to explain that puzzling claim here. The account of images in Matherne 2015 is in line with my interpretation of schemata, but does not discuss the "pure images" of space and time

which I generate [*erzeuge*] time itself in the apprehension of the intuition.

(A142/B182)<sup>76</sup>

It is notable that Kant does not give individual schemata for each of the categories of quantity—<unity>, <plurality>, <totality>—but instead gives a single schema for quantity in general, which he calls “number” (*Zahl*).<sup>77</sup> I take this to mean that each of the categories of quantity is to be schematized, and thus given conditions of application to sensible objects, in virtue of its role within a unified act of numbering or counting. The structural moments of such an act of counting are relatively easy to articulate. To assign a whole number *n* to a sensible object, and thus to think of it as an object with a *quantitas*, a determinate limited magnitude, is to identify a *unit* of numbering. This can

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<sup>76</sup> Kant’s terminology here should be remarked. He schematizes the concept “magnitude” (*Größe*) and identifies it with “*quantitas*.” A magnitude is what Kant elsewhere calls a “*quantum*,” which he defines as follows: “Quantum is one thing, in which there is quantity [*Quantum: est unum, in quo est quantitas*]” (28: 21). So a *quantum* is an intuitively given object in which there is *quantitas*, i.e. in which many other objects are also posited (“*quantitas: determination entis, quoties sit positum—Ibid*; cf. A163/B204). When the quantum has determinately many parts, when the answer “how many?” has a determinate answer (given a specification of a unit-part), then it can be numbered and the schema given in the body of the paper applies. Infinite quantities (*quanta* with infinite *quantitas*), like space and time, cannot be numbered; their *quantitas* exceeds any number, so the schematization I supply below does not apply. See Longuenesse 1998, 264–271, and Sutherland 2004, 427–435, for a discussion of *quantum* and *quantitas*.

<sup>77</sup> Longuenesse 1998, 253–4, also makes this point.

be as simple as picking some part of the object as the unit to be counted or as (relatively) complex as picking some standard length as the unit.<sup>78</sup> I will focus on the simple case of numbering the parts of a thing. First we identify some part of the object as the unit of counting. For instance, given a book we can count either chapters or pages. After that, we successively distinguish and enumerate the parts of the object that are conceptually homogenous with the chosen unit. For instance, having chosen to count the pages in the *CPR*, we successively distinguish each of the pages of the book. The process ceases when we have counted every part of the object, when no parts remain that were not distinguished and enumerated at previous stages. The structural moments of a whole number  $n$  as the quantity of an object, considered as a collection of parts, relative to a choice of unit, is given in Figure III.

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<sup>78</sup> The reason the second case is more complex is it requires representing the unit as itself something that is quantitatively determinate, so it depends upon a prior act of quantitative determination. For ease of exposition, in the main text I focus on the case of simply counting the parts of an object.

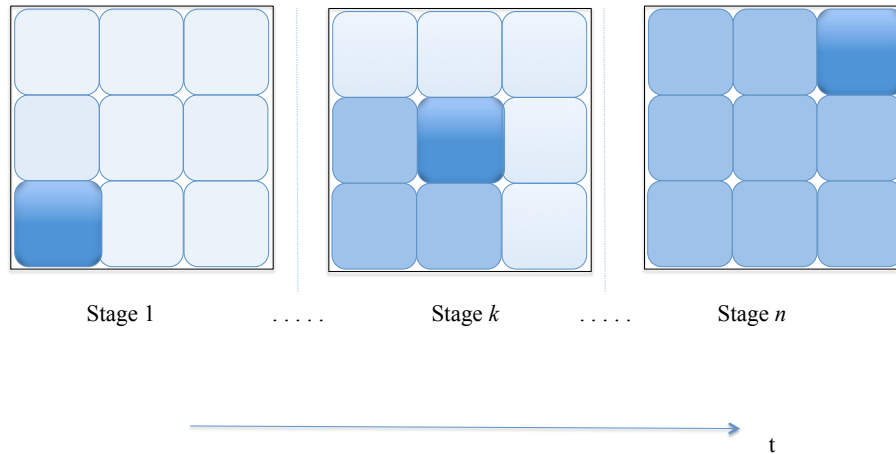


Figure III

As with the geometrical example from section 4 the straightforward way of reading Kant's discussion of reproduction in the A Deduction (A102) is that at each stage of this process one must not "forget" the previous stages. In line with my discussion of the geometrical example, though, I think we can say something stronger: in order for any of the stages distinguished in Figure III to be parts of a mental act of counting an object, one must represent the entire process represented in Figure III as well as the relation between one's current stage and the rest of the process. One must represent one's current representation as the enumeration of the  $k^{\text{th}}$  part, completing the enumeration of the 1 to  $(k-1)^{\text{th}}$  and to be completed by the  $(k+1)$  to  $n^{\text{th}}$  parts. In so representing the relation of one's current representation to previous and past representations, one is representing one's representation as standing in determinate temporal relations. This is the arithmetical version of a point we saw in geometric construction in the previous section. Represented graphically, it is the requirement that at each stage distinguished in Figure



III, the whole of Figure III must be represented as well as one's current position within that process. Figure IV is an attempt to illustrate this point.

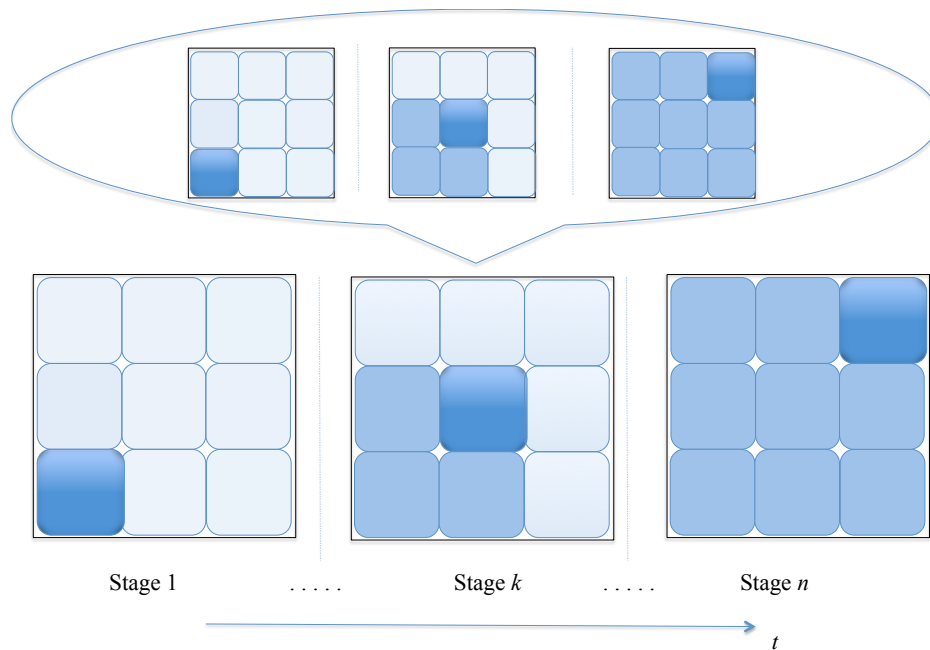


Figure IV

Earlier I said that each of the categories of quantity is to be schematized by its role in this unified act of quantitatively determining a magnitude. In particular, when we specify some part as the unit of counting we subsume that object under the category *<unity>*; at each stage in the enumeration we subsume the relevant sub-collection enumerated at that stage under *<plurality>*; and when we complete the process we subsume the whole collection under *<totality>*, a single object composed of a plurality with a determinate magnitude with respect to the unit (in this case,  $n$ ).<sup>79</sup> Subsuming objects under these

<sup>79</sup> In the *Prolegomena* (4:303) Kant identifies unity, plurality, and totality, with, respectively, measure (*Maß*), magnitude (*Größe*), and the whole (*das Ganze*). These correspond to the structural moments of the act of enumeration described in the main

categories just is representing the intuitions in which they are given as components of this complex self-conscious act of time-determination.<sup>80</sup>

In this section I have reconstructed Kant's reasons for two key claims:

**Schema<sub>quant</sub>**: the schema of the categories of quantity is a rule for determinately<sub>1</sub> representing temporal relations among our inner states, in particular, the states involved in representing a sensibly given object as a determinate magnitude.

Because there is only one schema for three different categories of quality, the individual categories are schematized by their role in the complete schema specified above.

Consequently, subsuming an object under the relevant moment or aspect of the complete schema is subsuming it under the corresponding category:

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text. To represent an object as a unity is to represent it as a *measure* of enumeration in some larger collection. To represent an object as a plurality is to represent it as a *magnitude*, as something with *quantitas* with respect to a unit/measure. But it is not yet to represent it as a totality, for not all of the parts have been enumerated. Representing an object as a totality is representing it as the *whole*, the place where enumeration ends. If you are enumerating an infinite magnitude (space, time) at every step you are representing it as a plurality without having enumerated all of its parts, i.e. you are not yet representing it as a totality/whole. Representing space and time as totalities requires starting with the whole and then proceeding to the parts/limitations; that involves a different schema. But I am limiting myself to number as the schema of determinate, finite *quantitas* (see above).

<sup>80</sup> This is my detailed specification of what Longuenesse 1998 refers to as the “understanding [affecting] inner sense” (259) in the case of quantity.

**Subsumption Sufficiency<sub>quant</sub>**: subsuming<sub>1</sub> a sensibly given object under the schema of quantity (as specified above) is sufficient for subsuming<sub>1</sub> that object under quantity as such (representing it as having a quantity); subsuming an object under a specific moment of that schema is sufficient for subsuming<sub>1</sub> it under the corresponding category.

In the next section, I will reconstruct Kant's reasons for holding parallel principles about quality.

(ii) *Quality*

Kant's initial characterization of the first two categories of quality, *<reality>* and *<negation>*, is deceptively simple:

Reality, in the pure concept of the understanding, is that which, in general, corresponds [*korrespondiert*] to a sensation. Thus, [it is] that whose concept contains in itself a being in time, while negation is that whose concept represents a non-being (in time). The opposition of the two occurs, consequently, in the distinction of one and the same time, as either a filled or an empty time. (A143/B183)

The distinction Kant here draws between filled and empty time relies on the distinction between form and matter. Time is the form of inner sense (thus, the form of our inner states as intuited by us) but the matter that time “en-forms” is the very inner states (representations) we intuit in temporal order. The distinction between *<reality>* and *<negation>* is the difference between matter en-formed by time (“filled” time) and pure form without en-formed matter (“empty” time). The concept of *<reality>* is to be applied to whatever in objects “corresponds to” this representational matter in us, which Kant identifies with “thing-hood” (*Sachheit*) and which he will later be in a position to

identify as causal powers in spatiotemporal objects, once he has introduced the categories of relation.<sup>81</sup>

The difference between the positive presence of some sensory matter in us and its absence (as well as the presence and absence of the corresponding reality in the object) is continuously gradable, not discrete:

Now every sensation has a degree or magnitude, by which it can more or less fill the same time, i.e. inner sense in respect of the same representation of an object, until it ceases in nothing (=0 =*negatio*). Consequently there is a relation and connection between, or, better, a transition from, reality to negation, which makes every reality representable as a quantum [. . .] (A143/B183)

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<sup>81</sup> “If one regards this reality as cause (whether of the sensation or of another reality in appearance, e.g., an alteration), then one calls the degree of reality as cause a ‘moment,’ [. . .] But I touch on this here only in passing, for at present I am not yet dealing with causality” (A168–9/B210). I take this to mean that we cannot represent the intensive degree of reality in our sensation as caused by a corresponding intensive degree of reality in the affecting object until we have the machinery of the Analogies in place. In this respect, Kant’s original statement of the principle of the Anticipations (A166: “in all appearances sensation and the real, to which it corresponds in the object, have an intensive magnitude, i.e. a degree”) is more precise than its replacement in the B Edition (B207: “in all appearances the real, which is an object of sensation, has an intensive magnitude, i.e. a degree”). In the A edition Kant has separated the claim that sensation has intensive magnitude from the further claim that the corresponding object does; in the B edition only the claim about the object is retained.

Thus, even if the complete absence of sensory matter is representable, we can still think of the total absence of reality in general as the limit of a process of continuous decrease in reality. What is more, we can think of the absence of a specific reality (corresponding to a specific kind of sensory matter, e.g. redness) as an achieved limit.<sup>82</sup>

Kant only explicitly gives the schema for quality only in the rest of the paragraph: “[. . .] the schema of a reality, as the quantity of something insofar as it fills time, is precisely this continuous and uniform generation [*Erzeugung*] of [that reality] in time, as one decreases the sensation over time, which has a determinate degree, until its complete disappearance, or one gradually increases from zero to the magnitude of the sensation” (A143/B183). Just as with the categories of quality, Kant does not give a different schema for each category of quality (<*reality*>, <*negation*>, <*limitation*>) but gives a single schema, which I take to be the schema for the whole moment of quality. Again, as with quantity, Kant specifies the schema as a single complex mental act (“this continuous and uniform generation“). I take it that, in parallel with quantity, the individual categories of quality are schematized insofar as they are identified with structural moments of that complex mental act, so I will proceed to analyze that complex mental act and isolate the relevant moments.

The complex mental act is the representation of the gradual diminution of a sensory matter of a particular kind to the limit case of total absence of that kind of sensory

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<sup>82</sup> I talk about specific realities because I take Kant to be arguing, not only that sensation in general has a degree (one perception can be more intense than other), but that specific kinds of sensation do as well (one perception can be more intense in respect of one quality than another, and less intense in respect of another).

matter. Unlike the case of quantity, this mental act does not have discretely separable parts; there are indefinitely many “degrees” that separate the presence of a given sensory matter from its total absence (=0). To represent it as having a determinate magnitude we must represent it as *generable* (possibly *generated*) from smaller magnitudes: we must represent the sensory matter in us as the limit of some possible continuous process of increasing magnitude, and as the beginning of some possible process of indefinite continuous increase (assuming no sensory reality has an absolute maximum).<sup>83, 84</sup> This means that in order to represent our present sensory matter (<*reality*>) we must self-consciously represent it as standing in various relations to possible past and future representations: it is the limit of some possible past process of continuous increase, the beginning of some possible future process of continuous increase (<*limitation*>), and the beginning of some possible future process of continuous diminishment whose limit is its absence (<*negation*>).<sup>85</sup>

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<sup>83</sup> We ourselves do not actively generate the magnitude of sensation, for sensation is the passive result of affection; see Longuenesse 1998, 300.

<sup>84</sup> The temporal direction is a heuristic device; we can just as well represent our present sensory matter as the product of a decrease, and the beginning of a further process of decrease.

<sup>85</sup> To put this in familiar Humean terms, to represent a given shade of blue as a determinate gradable hue we do not need to actually represent the “missing shade” of blue (and then generate the given shade out of it); we must be able to represent that missing shade (and represent the given shade as generated out of it).

So, again we have the recursive structure we have seen already in the case of empirical, mathematical, and quantitative schemata: representing the object of an intuition as falling under a category (subsuming the object under the category) is self-consciously representing that representation as standing in determinate temporal relations to possible past, present, and future representations. In each case, we have the structure of some temporally complex mental act (experiencing an empirical object over time, constructing a geometrical figure, counting), such that every stage in that act must self-consciously be represented as a stage in an actual or possible such act in order to constitute a stage in an act of that type.<sup>86</sup>

In this section I have given an account of Kant's reasons for two key claims:

**Schema<sub>qual</sub>**: the schema of the categories of quality is a rule for determinately<sub>1</sub> representing temporal relations among our inner states, in particular, the states involved in representing sensory matter as continuously increasable or decreasable.

Because there is only one schema for three different categories of quality, the individual categories are schematized by their role in the complete schema specified above.

Consequently, subsuming an object under the relevant moment or aspect of the complete schema is subsuming it under the corresponding category:

**Subsumption Sufficiency<sub>qual</sub>**: subsuming<sub>1</sub> a sensibly given object under the schema of quality (as specified above) is sufficient for subsuming<sub>1</sub> that object under quality as such (representing it having a quality); subsuming an object under a specific moment of that schema is sufficient for subsuming<sub>1</sub> it under the corresponding category.

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<sup>86</sup> This is my more detailed specification of what Longuenesse 1998 refers to as “affection of inner sense by the understanding” in the categories of quality (298).

(iv) *Relation*

Matters are significantly more complicated with the schemata of the categories of relation, for here (for the first time) Kant gives a separate schema for each category. For reasons of space, I will discuss only *<substance>*,<sup>87</sup> which Kant schematizes as follows: “the schema of substance is the persistence of the real in time, i.e. the representation of it as a substratum of empirical time-determination in general, which remains while everything else changes [*wechselt*]” (A144/B183).

Kant’s discussion here is complicated by the fact that he is anticipating a significant amount of material from the Analogies (especially the first Analogy), some of which appears as a parenthetical remark after the quoted sentence. Representing outer objects as “outer” (not transcendently outer but “empirically” outer; cf. A373) means representing them not as states of mine but as objects in space that other subjects can access through their states. Kant’s argument in the Analogies, which is partly anticipated here, is that we do this by representing those objects as having a temporal order in principle different from the temporal order of the inner states by which those objects are given to us.<sup>88</sup> We,

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<sup>87</sup> The schema of *<cause-effect>* cannot be satisfactorily discussed here, for the relation of time and modality in that case is especially complicated. I discuss this issue further in work currently in preparation.

<sup>88</sup> See B219. The Analogies have spawned a vast literature, to which I cannot hope to do justice here. Two highlights are Guyer 1987, 207–278, and Watkins 2005, 185–229.

Whereas Guyer adopts an “epistemic” reading of the Analogies, on which they uncover the conditions of our knowledge of objective time (1987, 258–9), Watkins opts for a



in effect, externalize time by “projecting” it onto outer objects. Time has three structural aspects or “moments”: unity (there is only one time; all states in time are temporally related), succession (states are ordered by an irreflexive, asymmetric, transitive relation of priority) and simultaneity (some states are neither before nor after one another).<sup>89</sup> The unity of time, combined with its successive and simultaneous structure, entail that any two states in time are either simultaneous or successive. Kant’s argument in the 1<sup>st</sup> Analogy is that in order to represent “outer time” (the time of the states of outer objects) as unitary we must represent an absolutely persisting subject, substance, of which the states changing in time are mere accidents.<sup>90</sup>

Kant’s task in the Schematism is not to make that argument but to explain what it is in virtue of which we are able to so much as *think* of an outer object as substance or as an accidental modification of substance (subsumption<sub>1</sub>) and how it is possible for such

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“metaphysical” reading, on which they uncover the grounds of objective temporal relations obtaining in the first place (2005, 200–1), which itself make our knowledge of objective temporal relations possible. However, I opt for what I call a “semantic” reading: the Analogies uncover the conditions of the possibility of so much as determinately representing (determining<sub>1</sub>) temporal relations among outer objects. (Some remarks in Friedman 1992, 46–7, suggest a similar view.) Given Kant’s transcendental idealism, objective time determination<sub>1</sub> makes possible objective time determination<sub>2</sub>, which makes possible objective time determination<sub>4</sub> (this is Watkins’s point). But I do not have the space to spell this out here.

<sup>89</sup> A177/B219.

<sup>90</sup> B225.

objects to *be* substances and accidental modifications (subsumption<sub>2</sub>). We can think of the difference between these tasks as the difference between explaining how it is *possible* to subsume (in both senses) outer objects under *<substance>* (Schematism) and arguing that it is *necessary* to do so (1<sup>st</sup> Analogy).<sup>91</sup>

The schema of *<substance>* relies on the schema of the categories of quality: “the schema of substance is the persistence of the real in time” (A144/B183). In order to represent spatial objects as substances we must represent them as having realities, the features in them that correspond to the sensory matter of our representations. This requires the whole complex machinery explored in the previous sub-section. We further represent these objects as substances when we present their reality as persisting at all times, as the ultimate “substratum” in virtue of which states of outer objects are temporally determinate with respect to one another. “Substances,” Kant writes near the end of the 1<sup>st</sup> Analogy, “are the substrate of all time-determination” (A188/B231).<sup>92</sup>

I want to make two comments on this. First, in describing substance as the “substratum” of time-determination Kant does not mean “substrate” in the traditional sense in which it is synonymous with substance (that would result in the tautology that

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<sup>91</sup> Likewise, the Transcendental Deduction proves the necessity of subsuming (in both senses) sensible objects under categories, while the Schematism explains the possibility of doing so (in both senses). Similar points are made by Pendlebury 1995, 779; Dahlstrom 1984, 41; and Symington 2011, 289.

<sup>92</sup> See A183/B226, A185/B229.

substances are substances)<sup>93</sup> but in the broader sense of “that which makes possible,” a sense manifestly present in his description of time (which, as pure sensible form, lacks reality, hence substantiality) as itself a “substratum” (B224). Thinking of spatial objects as accidental modifications of absolutely permanent substance is representing this substance as the “substratum” of temporal determination because that substance is the background against which temporal determination is representable. Secondly, “determination” here once again has the semantic sense of determination<sub>1</sub>: substance is the “substratum” of temporal determination in the sense that it is what make it possible to determinately<sub>1</sub> represent states of outer objects as temporally related to one another.<sup>94</sup> Kant is not yet concerned with the conditions under which we can *know* (determine<sub>4</sub>) the temporal states of objects, nor the conditions under which they can be determined<sub>3</sub> (the necessary effect of a cause)—for that requires the categories of *<cause-effect>* and *<reciprocal action>* (which come next in Kant’s discussion)—nor is he yet concerned (in

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<sup>93</sup> The non-tautologous claim, which Kant does make in the Schematism, and for which he argues in the Analogies, is that the persistent (substance) is the “substratum” in which that which changes (*wechselt*) (accidents) inhere.

<sup>94</sup> “All appearances are in time, in which, as substratum (as persistent form of inner intuition), both simultaneity as well as succession can alone be represented. The time, therefore, in which all change of appearances is to be thought, lasts and does not change; since it is that in which succession or simultaneity can be represented only as determinations of it” (B224–5). See also A186/B229 (“everything would disappear that alone can represent the unity of time”) and A188/B231 (“it is this very thing that persists that makes possible the representation of the transition from one state into another”).

the Schematism) with the conditions under which states in space acquire determinate<sub>2</sub> temporal relations, though all of these notions of determination will play a role in Kant's complete story (especially the second and the third Analogy). He is concerned, in the schema of <substance>, with the conditions under which we can subsume<sub>1</sub> outer objects under determinate concepts of their temporal relations, i.e. the conditions under which it is possible to *determinately represent* (determine<sub>1</sub>) temporal relations in space.

Yet again, in the schema of <substance>, we have the structure of reflexive self-conscious temporality that we have seen in the other cases. To represent an object as a substance is to represent it as possessing a reality (corresponding to the sensory matter in my representations) that is present at all times. This means that it is the very same substance that has been encountered in the past and will be re-encountered in the future. Obviously, to represent it as present at all times I do not need to have ever encountered it before or to ever encounter it again; I need only represent it as *possibly* encountered in the past and *possibly* encountered in the future. I also need to represent it as possibly encountered in the *present*, and not merely for the trivial reason that in representing it as actually encountered now I trivially represent it as possibly encountered now; it is because I must represent it as a persisting object in space that could have been represented by me in different ways now, e.g. that could have been intuited from a different spatial vantage point. In representing it as possibly encountered in the past and possibly to be re-encountered in the future I am, for reasons parallel to those given in Section 4, representing my current representation of it as related to possible and actual past and future representations. In other words, I am representing it as determinately temporally related to possible past and future representations. It is something that was

not encountered in the past but could have been, that will be encountered in the future under certain conditions, etc. Just as we “transposed into representation” the Aristotelian considerations about the possible life of a dog, we can transpose purely object-oriented considerations about the possible spatiotemporal trajectory of a substance into considerations about possible and actual past and future representations that would be representations of the same substance.

Just as in the other cases, we have a relatively simple account of the mental act of representing an object under *<substance>* (thinking of it as persisting at all times) and then we have the more complex structure in which that structure is, so to speak, embedded in every one of its own stages. More precisely, for any stage in the cognition of a substance to be a state in such a cognitive act, one must be self-consciously representing that stage as a determinate stage in a sequence of possible and actual past, simultaneous, and future experiences of the same object.<sup>95</sup> If one’s representation of the object has this complicated, recursive temporal structure, then one has succeeded in subsuming the object under the category *<substance>*.

In this section I have given an account of Kant’s reasons for two key claims:

**Schema<sub>sub</sub>:** the schema of *<substance>* is a rule for determinately<sub>1</sub> representing the temporal relations among our inner states, in particular, the states by which a temporally persistent object is given to us.

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<sup>95</sup> This is my detailed specification of what Longuenesse 1998 refers to as the “understanding affecting inner sense” (332) in the case of *<substance>*.

**Subsumption Sufficiency**<sub>sub</sub>: subsuming<sub>1</sub> a sensibly given object under the schema of *<substance>* (as specified above) is sufficient for subsuming<sub>1</sub> the object under that concept.

In concluding Part One, I hope to have established, for each of the kinds of concepts considered here and their associated schemata:

**Schema:** the schema of a concept is a rule for determinately<sub>1</sub> representing the temporal relations among our inner states, in particular, inner states by which an object is given to us.

**Subsumption Sufficiency:** subsuming<sub>1</sub> a sensibly given object under the schema of a concept is sufficient to subsume<sub>1</sub> it under that concept.

My argument for this principle has been by cases. In my defense, this is precisely how Kant proceeds in the Schematism: he provides schemata for the categories, case by case. So the argument for Subsumption Sufficiency in its full generality will be incomplete until we have explained how each of the remaining categories (*<cause-effect>*, *<reciprocal action>*, and the modal categories) are transcendental time-determinations for which Subsumption Sufficiency holds. For reasons of space I cannot do that here, but I hope that it is reasonably clear how my account can be extended to the remaining categories.

## **Part Two**

### **§7. Homogeneity**

The ultimate aim of the Schematism, I have argued, is to explain the possibility of the subsumption<sub>2</sub> of objects under categories (to answer Q2 from the Introduction).

Combined with Subsumption Sufficiency, which I have argued for at length, Subsumption Link has the effect of reducing this problem to the problem of subsuming<sub>2</sub> objects under schemata. In this section and the next I will reconstruct Kant's explanation of why subsumption<sub>2</sub> of objects under schemata is possible, the Real Possibility premise from the Introduction. This completes Kant's explanation of why subsumption<sub>2</sub> of objects under categories is possible, his answer to Q2.

Kant begins the Schematism with a humble example meant to illustrate the "homogeneity" between objects and the concepts under which they are subsumed:

In all subsumptions of an object under a concept the representation of the former must be homogenous [*gleichartig*] with the representation of the latter, i.e. the concept must contain that which is represented in the object that is to be subsumed, for that is what the expression "an object is contained under a concept" means [*bedeutet*]. Thus the empirical concept of a plate is homogenous with the pure geometrical [concept] of a circle, in that the roundness that is thought in the former can be intuited in the latter.<sup>96</sup>

(A137/B176)

Since we are concerned in this section with, finally, explaining subsumption<sub>2</sub> let us assume that is what Kant has in mind by "subsumption" in this passage.<sup>97</sup>

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<sup>96</sup> "So hat der empirische Begriff eines Tellers mit dem reinen geometrischen eines Zirkels Gleichartigkeit, indem die Rundung, die in dem ersteren gedacht wird, sich im letzteren anschauen läßt."

<sup>97</sup> In fact, the difference between subsumption<sub>1</sub> and subsumption<sub>2</sub> is already implicitly contained in the passage itself. When Kant writes "that is what the expression 'an object is contained under a concept' means [Au's emphasis]" he is talking about subsumption<sub>1</sub>:

This passage bears the mark of having been hastily written. First of all, the syntax in the second sentence is confused. To what does “the latter” (*letzteren*) refer? Grammatically, the natural answer is “the pure geometric [concept] of a circle,” but this makes little sense. Concepts cannot be intuited; what can be intuited are the *objects* that fall under concepts, so we might read “the latter” (*letzteren*) as referring to “a circle.”<sup>98</sup> But, philosophically, this makes little sense: Kant would then be explaining the subsumption of a purely geometric circle (the object of the pure geometrical concept of a circle) under the empirical concept of a plate. More plausible is Hans Vaihinger’s suggestion to reverse “the former” and “the latter”: “in that the roundness, that is thought in the latter can be intuited in the former.”<sup>99</sup> But even that is unsatisfactory, for an

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when I say that an object is contained under a concept, I am subsuming<sub>1</sub> it under that concept (that is what the expression means), but I may be doing so incorrectly (it may not be subsumed<sub>2</sub> or even subsumable<sub>2</sub> under that concept). The meaning of the sentence “an object is contained under a concept” does not guarantee that the sentence is true (that the object is contained/subsumed<sub>2</sub> under that concept). So here, at the very beginning of the Schematism, we have an implicit difference between thinking that an object falls under a concept (subsumption<sub>1</sub>) and the fact that object does fall under that concept (subsumption<sub>2</sub>).

<sup>98</sup> This is even more awkward in German, because “a circle” (*eines Zirkels*) is in the genitive. But an awkward reading is preferable to an incoherent one, on which Kant is saying that the *concept* of a circle can be intuited.

<sup>99</sup> “*indem die Rundung, die in dem letzteren gedacht wird, sich im ersteren anschauen läßt*” (Vaihinger 1900, 458). Kemp Smith follows Vaihinger’s reading in his translation



empirical concept like *<plate>* can no more be intuited than a pure geometrical concept!

But the problems with this passage do not stop there. In the context of the first sentence, it is clear that Kant is here interested in the homogeneity of (representations of) objects and the concepts under which they are to be subsumed<sub>2</sub>. But, taken at grammatical face value, and even given Vaihinger's proposed reading, Kant explicates his example, in the second sentence, in terms of the homogeneity of two *concepts*: the empirical concept *<plate>* and the pure geometric concept *<circle>*. I think the best way to make Kant's example make sense, both within his own theory and specifically as an example of object subsumption<sub>2</sub>, is to read him as talking about the homogeneity between the intuition of a round object, an object subsumed<sub>2</sub> under the empirical concept *<plate>*, and the pure geometric concept *<circle>*. So the second sentence might be rendered more perspicuously (albeit with some violence to the text) as follows: "thus the empirical representation [rather than concept] of a plate has homogeneity with the pure geometrical [concept] of a circle, in that the roundness that is thought in the latter [the concept *<circle>*] can be intuited in the former [the empirical representation]." The object is homogeneous, specifically, with one mark of *<round>*, one of the marks of *<plate>* (this is how I read Kant's reference to the "roundness that is thought" in the empirical concept of a plate).

This example relies on there being something that can be intuited in the object (its roundness) that is also thought in a concept (*<round>*). The possibility of the

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(Kant 1909, 180); Timmerman notes Vaihinger's reading without endorsing it (Kant 1998*b*, 239); the issue goes unremarked by Adickes (Kant 1889, 173) and Guyer & Wood (Kant 1998*a*, 271).

subsumption<sub>2</sub> of the object (the plate) under the concept (<round>) consists in the possibility of that very same feature being intuited in the object and represented by the concept. Or, to put the point at a slightly higher level of generality, there are intuited features of objects that are also represented by concepts. It is notable that Kant takes this kind of homogeneity to stand in need of no further explanation; we do not need an explanation of the subsumption<sub>2</sub> of a round object under <round>. Instead, he uses it to motivate the search for a similar kind of homogeneity, a homogeneity between objects and categories, which ultimately is revealed to be a homogeneity between objects and schemata.

The reason that Kant can take such homogeneity (between roundness in the intuited object and <round>) to be unproblematic is that the feature in question (roundness) is a feature of objects purely qua intuited.<sup>100</sup> Roundness, after all, is a spatial feature, that is to say, a specification of the form of outer objects in general (space). I will call such features of objects *intuitive features*.<sup>101</sup> A feature F is an *intuitive feature* of outer (inner)

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<sup>100</sup> Hence I am not bothered by the fact that Kant equates the subsumption of an object under a concept with the homogeneity of the *representation* of the former with the latter (see A137/B176 quoted above). The relevant representation of an object is an intuition, and I am talking about features of objects solely qua intuited. So in this case, there is an especially close connection between the object and our representation of it.

<sup>101</sup> What I call “intuitive features” corresponds to what Smit 2000 calls “intuitive marks”. Smit’s idea is that “marks” (partial grounds of cognition) can be either intuitively or discursive. Intuitive marks are singular features of objects, now referred to as *tropes*. Discursive marks are general representations of what is common to those intuitive marks.

objects just in case F is a determinate of some determinable D, where outer (inner) objects have D solely in virtue of being objects of outer (inner) intuition.<sup>102</sup> For instance, roundness is an intuitive feature of outer objects because it is a determinate of *shaped*, and *shaped* is a determinable that objects have solely in virtue of being objects of outer intuition. Likewise, *occurring before* is an intuitive relational feature of inner objects, because it is a determinable of *being successive*, and *being successive* is a determinable that objects have in solely in virtue of being objects of inner intuition.<sup>103</sup> So the real possibility of an intuitable object with an intuitive feature being subsumed<sub>2</sub> under the concept of such a feature is not problematic. This is why Kant writes, a few lines later: “In all other sciences, where the concepts, through which the object in general is thought,

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For instance, the roundness of the plate is an intuitive mark, while *<round>* is a discursive mark. Kant spells this out in a *Reflexion*: “A mark is a partial representation (which), as such (is a ground of cognition). It is either intuitive (a synthetic part): a part of intuition, or discursive: a part of a concept, which is an analytic ground of cognition” (*Refl.* 2286, 16:299–300; see Smit 2000, 254–9). Whereas Smit characterizes intuitive marks as marks “found” in space and time, I impose a more informative criterion: they must be determinates of a determinable grounded in the very nature of the form of the relevant faculty (outer sense or inner sense).

<sup>102</sup> I am not assuming that determinates are *fully* determinate, but merely that they are more determinate than determinables. E.g. *round* is more determinate than *shaped*, but it is not as fully determinate as *circular*.

<sup>103</sup> Chipman 1971 is after something similar with his talk of “elementary sensory components” (39).

are not so different and heterogenous [*heterogen*] from those which represent the object in concreto, as it is given, it is unnecessary to offer a special discussion of the application of the former to the latter” (A138/B177). Kant is making the simplifying assumption that in sciences other than metaphysics, concepts are concepts of intuitive features, and thus there is no deep problem in explaining how the subsumption<sub>2</sub> of objects under such concepts is possible. His considered view about “other sciences” is, of course, significantly more complex than this.<sup>104</sup>

I have constructed some machinery for explaining the subsumption<sub>1</sub> of objects under categories but we can apply this machinery to give an easy explanation of the subsumption<sub>2</sub> of objects under empirical concepts. In §4 I specified the schemata of empirical concepts. These will involve both intuitive features of objects and complex temporal relations among the intuitions by which objects are given to us. But these temporal relations are themselves intuitive features: they are determinate specifications

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<sup>104</sup> Kant’s considered view is that all sciences, in the strict sense, contain an *a priori* part, in which *a priori* concepts (the categories) are applied to sensible objects. The paradigm example of this is the pure part of natural science, given in the *Metaphysical Foundations of Natural Science*. The subsumption<sub>2</sub> of objects under the *a priori* concepts involved in pure natural science does, of course, stand in need of explanation; this is why the *CPR* explains not only the possibility of metaphysics, but also of natural science (B17, A171-2/B213). However, even on Kant’s considered view, a version of his A138/B177 point holds: the subsumption<sub>2</sub> that primarily stands in need of explanation in other sciences is the subsumption<sub>2</sub> he is about to explain, the subsumption<sub>2</sub> of objects under categories.

of the determinable form of inner states in general (time).

But my articulation of the schema of the empirical concept <dog> (Kant's example) made use of a relation among intuitions that, some readers will object, is not an intuitive feature: the relation of two intuitions co-referring, i.e. being intuitions of one and the same object. Is co-reference an intuitive feature? Solely in virtue of being intuitions (whether of outer or inner sense), intuitions have objects. As such, intuitions can have the same or different objects. This means that the very form of intuition, regardless of whether it is outer or inner, grounds a determinable relational feature *having the same or different objects*, of which one determinate is *having the same object*. By the definition of intuitive feature, it follows that having the same object is an intuitive relational feature of intuitions. Some readers will remain skeptical about whether *having the same object* is an intuitive relational feature of intuitions; I address that skepticism in the next section.

The schemata of empirical concepts represent intuitive features of the representations by which empirical objects are given to us: intuitive temporal relations and relations of co-reference. So there is no difficulty in explaining how subsuming<sub>2</sub> sensible objects under these schemata is possible; sensible objects can stand in these relations solely in virtue of being objects of intuition.<sup>105</sup> Given Subsumption Sufficiency<sub>emp</sub> (see §4) and Subsumption Link, it follows that subsuming<sub>2</sub> objects under those empirical concepts is

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<sup>105</sup> Readers might object: schemata represent intuitive relational features of representations, not of objects (e.g. co-reference is a relation of intuitins, not of their objects). Why is subsuming (in either sense) the representations under the schemata sufficient to subsume the objects? For the answer to this question see my discussion on p. 35–36.

really possible.

Geometric schemata (see §5) represent spatial features of their objects as well as temporal relations among the stages of the procedure by which the object is constructed. But the temporal relations they represent (stage 1 is *before* stage 2, which is *before* stage 3; see Figure II) are intuitive features: determinate specifications of determinable features objects have solely in virtue of being intuited. The very form of time determines that inner states stand in an asymmetric relation of succession (A31/B47), so in representing the determinate temporal relations among the parts of a constructive procedure, a geometric schema is not representing anything more than intuitive relational features.<sup>106</sup> Given Subsumption Sufficiency (see §6) and Subsumption Link<sub>geo</sub>, it follows

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<sup>106</sup> It might seem that, on my account, schemata represent more than merely temporal relations, unproblematically allowed for by the very form of time (succession) for I spoke of stages of a constructive procedure as “completing one another” and “to be completed by another.” This kind of language might suggest that schemata represent something like teleological relations among our representations, and teleological relations are definitely not what I have called “intuitive features.” But the “completion” language was not meant to connote teleology, but, instead, self-referentiality: to be constructing a triangle at stage 2, one must self-consciously represent one’s current representation as part of a larger constructive procedure, composed of a *prior* stage 1, and a *later* stage 2. This does not require that, in any problematic sense, stage 1 is “for the sake of” stage 2, or stage 2 “for the sake of” stage 3, but only that stage 1 is only stage 1 of a construction if it is followed by stage 2, etc. Since the relations thus

that subsuming<sub>2</sub> objects under these empirical concepts is really possible.

What is problematic, what the entire machinery of schemata is designed to explain, is the real possibility of a sensible object being subsumed<sub>2</sub> under a concept that is not merely a concept of some feature that is a specification of the form of intuitible objects in general. In the next section I complete that explanation.

### §8. Real Possibility

Immediately after the plate example, Kant writes:

Now pure concepts of the understanding, however, in comparison with empirical (indeed in general sensible) intuitions, are entirely unhomogenous [*ungleichartig*] and can never be encountered in any intuition. (A137/B176)

I take this to mean that categories are not concepts of intuitive features, because they are not concepts of features that are specifications of the form of intuitible objects as such (unlike *<round>* or *<occurs before>*). Kant goes on:

Now it is clear that there must be a third [thing], which is homogenous with the category, on the one hand, and with the appearance, on the other, and which makes possible the application of the former to the latter. This mediating representation must be pure (without anything empirical) and yet, on the one hand, **intellectual**, and, on the other, sensible. Such [a representation] is the **transcendental schema**.

(A138/B177)

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represented among our representations are purely temporal relations, the unproblematic “homogeneity” of the schema with its object is secured.

The question now is how to understand the homogeneity of a schema with, on the one hand, the category, and, on the other, a sensible object. With respect to the former, Kant is not especially helpful: “now a transcendental time-determination is homogeneous with the **category** (which constitutes its unity) insofar as it is **universal** and rests on a rule *a priori*” (A138/B177). But since all schemata are universal *a priori* rules this does not explain why a schema is homogenous specifically with the category of which it is the schema. However, we already have at hand an account of the homogeneity, specifically, of schemata and the categories that they schematize: Subsumption Sufficiency. Subsuming<sub>1</sub> an object under a schema is sufficient for subsuming<sub>1</sub> it under the corresponding concept. Combined with Subsumption Link, this entails that explaining the possibility of subsuming<sub>2</sub> an object under a category reduces to the more tractable problem of explaining the possibility of subsuming<sub>2</sub> it under the corresponding schema. So the deeper issue is the homogeneity of objects and schemata.

On exactly this point, Kant writes:

[the schema] is, on the other hand, homogenous with the appearance insofar as time is contained in every empirical representation of the manifold. Thus an application of the category to appearances will be possible by means of the transcendental time-determination, which, as the schema of the concept of the understanding, makes the subsumption of the latter under the former possible. (A138–9/B177–8)

Drawing on the example of the plate and my account of schemata as transcendental time-determinations from Part One, a schema will be “homogenous” with sensible objects if the temporal relations determinately<sub>1</sub> represented in the schema are specifications of the relevant form of intuition itself: time. If they are, then the content of the schema will



stand to intuitable features in the same unproblematic relation that *<round>* stands to roundness in objects.

I will first consider the schema of quantity. Referring back to Figure IV, the schema of a determinate magnitude represents temporal relations among individual stages of a process of enumeration, individual representational of acts of identifying parts in a collection (the collection being enumerated). These temporal relations are what I have called “intuitive features”: relations of *before* and *after*. Earlier stages of the enumeration are “completed” by later stages: the  $k^{\text{th}}$  stage is only the  $k^{\text{th}}$  stage of the enumeration of a collection of  $n$  objects if it follows previous stages of enumerating  $1 \dots (k-1)^{\text{th}}$  parts and is followed by later stages of enumerating the  $(k+1)^{\text{th}} \dots n^{\text{th}}$  parts *and* the subject of enumeration is self-consciously aware of these temporal relations. Again, I take these temporal relations to be intuitive features unproblematically homogenous with their conceptual representation by the relevant schema. Given Subsumption Sufficiency<sub>emp</sub> (see §4) and Subsumption Link, it follows that subsuming<sub>2</sub> objects under theses concepts is possible. This explains why it is possible to subsume<sub>2</sub> objects under quantitative schemata.

Without repeating my whole account of the schema of quality we can see relatively easily why it refers only to intuitive features of objects and our representations of them. The schema of quality is the self-conscious representation of the matter of one’s intuition, its sensory content, as being continuously decreasable to its total absence (=0) and arbitrarily increasable. This requires representing one’s intuition of the object as standing in a certain relation to possible past and future intuitions, namely the relation of having a greater or lesser degree of that sensory reality in them. The temporal relations

are clearly what I have called “intuitive features” and thus generate no inhomogeneity and no problem of subsumption<sub>2</sub>. But the specifically sensory relations of possessing more or less of a particular kind of sensory matter are also (relational) intuitive features. Solely in virtue of being objects of inner sense, our inner states possess a matter. That is to say, it follows from the form of inner sense that inner states possess matter (but of course it does not follow which matter they possess). Likewise, it follows from the very form of space that intuited objects have some shape or other, but not which fully determinate shape they possess. And just as it follows from the very form of space that one spatial object can be larger than another, and this relation of *largeness* is continuously gradable (there is no “next largest” object), it follows from the form of inner sense that the sensory matter in one state can be “greater” than the sensory matter in another state and that this relation of “greater” is continuously gradable. Thus, in representing relations of continuous gradability in the sensory content of our inner states, the schema of <reality> represents exclusively features that objects and the sensory states by which they are given are able to possess solely in virtue of the form of our (inner) intuition. Again, I take these to be intuitive features unproblematically homogenous with their relevant schemata. This explains why it is possible to subsume<sub>2</sub> objects under qualitative schemata.

The schemata of geometrical, quantitative, and qualitative concepts are, to some extent, the easy case for Kant’s theory for these are concepts of features objects have solely qua intuited: they are spatial (geometry), denumerable (quantitative), and possess continuous degrees of sensory content (qualitative). The hard case for Kant will be the categories of relation. Since in Section 6 I only explained the schema of <substance> I

will here limit myself to explaining why the subsumption<sub>2</sub> of sensible objects under that schema is unproblematic. I hope it will be clear to the reader how to extend this account to the other relational schemata (<cause-effect>, <reciprocal action>).

Recall the schema that Kant gives for <substance>: “the schema of substance is the persistence of the real in time, that is, the representation of it as the substratum of empirical time-determination in general, which therefore remains when everything else changes [*wechselt*]” (A144/B183). As I observed in §6, <substance> is explicitly schematized in terms of a previous category, <reality>. In the context of explaining the subsumption<sub>2</sub> of objects under schemata, this recursive structure takes on an added significance, for it means that, if it is explicable why objects can be subsumed<sub>2</sub> under the schema of <reality> then, to the extent that <substance> is schematized in terms of <reality>, to that extent subsumption under <substance> is also explicable.

But there is more to the schema of <substance> than merely this reference back to <reality>. Representing an object of intuition as a substance also involves representing it as permanently present, as existing at all times. This means representing that past and future intuitions, at any time, might be intuitions of the very same objects. This requires representing two kinds of relations: (i) purely temporal relations among possible and actual intuitions (before, after) at any time, and (ii) the relation of co-referring, of being intuitions of one and the same object. I take it that it is now clear that (i) involves what I have been calling “intuitive features,” features that objects can possess solely in virtue of the form of intuition. Solely in virtue of the form of inner sense, the states (intuitions) by which objects are given to us can be before, after, and simultaneous with one another. So the subsumption<sub>2</sub> of our representations of objects under (i) is unproblematic. It calls for

no additional explanation because it involves no “heterogeneity” between a sensible object and an intellectual concept. In the previous section I argued that co-reference is also an intuitive relational features of intuitions. Since the schema of *<substance>* involves representing *reality* and the possibility of *co-referring* intuitions occurring at *any time*, and all of these are either features, the subsumption<sub>2</sub> of which has already been explained, or intuitive features (or so I have argued), I conclude that the real possibility of subsuming<sub>2</sub> an object under the schema of *<substance>* stands in no need of further explanation. By Subsumption Sufficiency<sub>sub</sub> and Subsumption Link it follows that the real possibility of subsuming<sub>2</sub> a sensible object under *<substance>* is also thereby explained.

I anticipate some resistance to the claim that co-reference is an intuitive feature of intuitions. In particular, some readers will object that this is incompatible with Kant’s procedure in the Analogies, in which he argues that in order to represent a temporal order in spatial objects distinct from the subjective temporal order of our perceptions of them, we must use the relational categories of *<substance-accident>*, *<cause-effect>*, and *<reciprocal action>*.<sup>107</sup> But this does not conflict with my interpretation at all. By claiming that co-reference is an intuitive relational feature of intuitions I am claiming, not that we can represent it without concepts (of course we cannot), but that the subsumption<sub>2</sub> relation between two co-referring intuitions and *<co-reference>* requires no further explanation. Perhaps the worry, then, is not that my view is incompatible with the Analogies, but that it evacuates them of any significance, for it might seem that all of

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<sup>107</sup> See Tolley 2017 for a discussion of the distinction between intuition and perception (*Wahrnehmung*). For the sake of space, I will not go into that difference here.

the significant work has already been done in the Schematism, on my reading. But I do not think this correct either. On my reading, the role of the Schematism is to explain how it is really possible for sensible objects to be subsumed<sub>2</sub> under the categories, including the relational categories. The role of the Analogies is to argue that it is *necessary* to subsume<sub>2</sub> sensible outer objects under the relational categories in order to represent them as standing in temporal relations in principle distinct from the temporal relations of our perceptions of them. Thus, on my reading, a substantial difference between these two sections remains.

Now is also perhaps the time to respond to an objection that some readers will have wanted to raise since the Introduction. I have explained why subsumption<sub>1</sub> and subsumption<sub>2</sub> of sensible objects under categories is possible, namely, by means of subsumption<sub>1</sub> and subsumption<sub>2</sub> of sensible objects under schemata. But why doesn't the same problem arise at the level of schemata? Don't we need a "third" representation to mediate the subsumption of sensible objects under schemata (thus generating an infinite regress)? No, I claim, because the source of the "need" for schemata was not a general demand for a mediating representation between sensible objects and representations under which they are subsumed (in either sense); it was the fact that categories are not homogenous with sensible objects. Kant then provides schemata for empirical concepts and mathematical concepts. While these concepts are not merely concepts of intuitive features, it is relatively easy to schematize them in terms of intuitive features. But no schema or other mediating representation is required *or even possible* for the subsumption (in either sense) of a sensible object with an intuitive feature under the concept of that intuitive feature (e.g. the roundness of the plate and <round>). Here we

hit explanatory bedrock: our capacity to think of an object with an intuitive feature under a concept of that feature (to subsume<sub>1</sub> it) and the real possibility of such an object falling under such a concept (its being subsumed<sub>2</sub>).<sup>108</sup> I have argued that the problem of the subsumption<sub>2</sub> of categories under schemata ultimately reduces to the problem of subsuming<sub>2</sub> them under schemata specified in terms of sensible features. That is to say, it reduces to no problem whatsoever.

### §9. Conclusion

I began with what I take to be the uncontroversial, if somewhat unfamiliar, truth that:

**Subsumption Link:** if subsuming<sub>1</sub> an object under a concept F is sufficient to subsume<sub>1</sub> that very object under a concept F\* then explaining the real possibility of the subsumption<sub>2</sub> of objects under F is sufficient to explain the real possibility of the subsumption<sub>2</sub> of objects under F\*.

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<sup>108</sup> With respect to subsumption<sub>1</sub> I take Kant's point in the immediately preceding section, "On the transcendental power of judgment" to be that the capacity to (reliably) subsume<sub>1</sub> sensible objects under concepts of their intuitive features is the properly functioning power of judgment itself, and stands in need of no further explanation. For instance, a student who cannot reliably subsume<sub>1</sub> sensible objects under concepts of their intuitive features cannot be instructed by further rules (which, by hypothesis, he would not reliably apply) but only by examples, and if the power of judgment is not functioning (if the student lacks "mother wit") then even these will not succeed. See A133–4/B173–4.

The idea was this: if I can prove the real possibility of sensible objects instantiating F (being subsumed<sub>2</sub> under F) then I can prove the real possibility of the truth of the thought Fx (for at least some sensible objects). But if the thought that Fx is sufficient for the thought that F\*x (subsuming<sub>1</sub> x under F is sufficient for subsuming<sub>1</sub> x under F\*) then by proving the real possibility that Fx I have thereby proved the real possibility that F\*x.

Then I reconstructed Kant's argument for the following claim, with respect to various categories and their schemata:

**Subsumption Sufficiency:** subsuming<sub>1</sub> a sensibly given object under a schema is sufficient for subsuming<sub>1</sub> that object under the corresponding category.

Finally, I argued that, with respect to that very same range of schemata:

**Real Possibility:** there is an explanation of the real possibility of the subsumption<sub>2</sub> of sensible objects under schemata.

From these three principles it follows that:

**Conclusion:** there is an explanation of the real possibility of the subsumption<sub>2</sub> of sensible objects under categories.

Since, on my reading, the primary aim of the Schematism is precisely to explain the real possibility of the subsumption<sub>2</sub> of objects under categories, it follows that, if we grant Kant the three previous premises (one of which, I have argued, is nearly trivial) then the Schematism succeeds. I have further attempted to vindicate Kant's entitlement to the two substantial premises, Subsumption Sufficiency and Real Possibility. I have argued that these are plausible claims for Kant to make, given the transcendental theory of cognition for which he has argued up to this point in the *CPR*. Thus, my conclusion must be

appropriately tempered: to the extent that the previous arguments of the *CPR* are sound, the Schematism is as well.<sup>109</sup>

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