Philosophical Mechanics in the Age of Reason

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1 Introduction

This is a book about philosophy, physics, and mechanics in the 18th century, and the struggle for a theory of bodies. Bodies are everywhere: from pebbles to planets, tigers to tables, pine trees to people; animate and inanimate, natural and artificial, they populate the world, acting and interacting with one another. And they are the subject-matter of Newton's laws of motion. At the beginning of the 18th century, physics was a branch of philosophy, tasked with the study of body in general. With an account of body available, the special areas of philosophy (whether natural, moral or political) that presuppose special kinds of bodies (such as plants, animals, and human beings) could proceed assured of the viability of their objects and the unity of their shared inquiries. For all had "bodies" in common. So: What is a body? And how can we know? This is the *Problem of Bodies*, and its contours and depths turn out to be a treasure trove.

How so? For two reasons: the *Problem of Bodies* was foundational for natural philosophy, and it proved surprisingly resistant to solution. As a result, it ensnared a wide range of figures who appealed to a diverse assortment of resources. At the forefront we find familiar figures from the received canon, such as Leibniz, Malebranche, Wolff, Hume and Kant, wrestling with the *Problem of Bodies* alongside others of equal or greater import, like Maupertuis, Musschenbroek, du Châtelet, Bernoulli, Euler, d'Alembert, Boscovich and Lagrange. Attempted solutions drew on matter theory, metaphysics, physics and mechanics; they appealed to a variety of principles metaphysical, epistemological, and methodological; and they simultaneously disputed the appropriate criteria for success. At stake were two central issues of philosophy then: material substance and causation.

In this chapter, we introduce the *Problem of Bodies*, along with the main analytical tools we use for its investigation (sections 2-6). We outline our methods (section 7) and our intended audiences (section 8). Finally, we give a guide to the master argument of this book (section 9), and a preview of our main conclusions (section 10). Inevitably, some of what we say is compressed and may seem somewhat cryptic at first sight, but when read in conjunction with the later chapters it is, we hope, sufficient to anchor the thesis of chapter within the argument of the whole.

2 The Problem of Bodies

The *Problem of Bodies* (hereafter *Body*) is large and unwieldy. And yet, we argue that it has a structure that makes it amenable to analysis, in the form of a goal and four criteria for success. We have inferred them the arguments of the participants in the debate. Here we make them explicit, and we use them to assess purported solutions.

> **Goal:** a single, well-defined concept of body that is simultaneously (i) consistent with an intelligible theory of matter, (ii) adequate for a causal-explanatory account of the motion behaviors of bodies, and (iii) sufficient for the purposes of mechanics.

Any satisfactory solution to *Body* was expected to meet this goal. Moreover, in order to do so, it needed to meet the following success criteria: Nature, Action, Evidence, and Principle.

> **Nature:** Determine the nature of bodies. Ascertain their essential properties, causal powers, and generic behaviors.
>
> **Action:** Explain how bodies act on one another. Give an explanation of how, if at all, one body changes another's state (where specifying the "state" of a body is addressed by Nature).

These first two are metaphysical. The next two are epistemological: they seek to uncover the justificatory reasoning behind Nature and Action.

> **Evidence:** Elucidate the evidentiary reasoning behind Nature and Action. Spell out what counts as evidence for these claims, and what patterns of inference take us to them as conclusions.
>
> **Principle:** Elucidate the constraining principles appealed to in attempting a solution to *Body*, and check that proposed solutions conform. Such principles include the Principle of Sufficient Reason, the Law of

> Continuity, the restriction to contact action, the criterion of clear and distinct ideas, and so forth. Different protagonists understood these principles variously as a priori philosophical requirements, defeasible heuristics, and so forth; but such principles were always in play, whether implicitly or explicitly.

These criteria (NAEP) may be variously interpreted and implemented. Their more precise specification varies from one philosopher to another, and this comprises one element of the debate over *Body*. Excavating and investigating them is a part of what we do in this book. The diversity of views on offer is evidence that *Body* was neither easy to state nor straightforward to solve.

3 Philosophical mechanics

"Philosophical mechanics" is a term of art.[1] We use it to label the framework within which we use the resources described above ("Goal" and "NAEP") to analyze *Body* in the 18th century. This framework is justified by its utility: it stands or falls by the work that it does for us in this book, and that assessment can be made only when we reach the end. Still, here at the outset we can make some remarks that will be helpful.

Simply put, a philosophical mechanics is any project that integrates matter theory with rational mechanics. To motivate the idea, we offer some 17th-century background in Descartes' *Principles of Philosophy* and in Newton's *Principia*. In Descartes, we see the connection of *Body* to collision theory, and from there to philosophical mechanics. In Newton, we find an explicit example of a philosophical mechanics. Reflected in each is an important disciplinary distinction between physics, then a subdiscipline of philosophy; and rational mechanics, then a branch of mathematics. Philosophical mechanics draws on both.

Cartesian origins

In his 1644 *Principles*, Descartes set out to explain all the rich variety of the natural world around us: he sought to provide a complete physics that included everything from planetary motions to the creation of comets, from the

[1] While the term was first used in 1800, in Gaspard Riche de Prony's *Mécanique philosophique,* we adopt it for our own purposes.

formation of mountains to the behavior of the tides, and from earthquakes to magnetism and beyond. His physics begins with his theory of matter; the principal attribute of Cartesian matter is extension, and the parts of matter have shape, size and motion. All change comes about through matter moving in accordance with the laws of nature.

Body, as it occurs in *Principles*, concerns the parts of matter, for these are Descartes' bodies. His laws of nature take parts of matter—or bodies—as their subject-matter. The first issue is whether (and if so, how) he succeeds in giving a viable account of bodies prior to his introduction of the laws of nature. For, he claims that matter is divided into parts by means of motion, but he also defines motion by appeal to the parts of matter, generating an undesirable circle. This issue was widely appreciated at the time, and has been much discussed since.[2] If we set it aside, and presume that Descartes has "parts of matter" available, a second issue then comes to the fore. While the laws of nature supposedly take these bodies as their subject-matter, Cartesian bodies seem not to have the properties and qualities demanded by the laws. The only properties that his matter theory secures for bodies are shape, size and motion. Now as we move through the exposition of his laws, we find Descartes appealing to "stronger" and "weaker" bodies; "hardness"; "yielding" and "unyielding" bodies; the "tendency" of bodies to move in a straight line; and the "force" of a body, which is nothing other than the "power" of a body to remain in the same state. But it is far from clear that these are reducible to shape, size and motion.[3]

The issue is pressing because of the role of bodies in Cartesian physics. He aims to explain all of the material world by means of bodies moving in accordance with the laws of nature. But if his philosophy lacks the resources for his matter theory to yield bodies, his physics cannot get off the ground.

A necessary condition on a viable solution to *Body*, within Descartes' system, is that the resulting bodies are capable of undergoing collision. This is because all change in his world takes place through *impact* among the parts of

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[2] See Garber 1992, 181; Brading 2012; and references therein. One upshot is a tendency among Cartesians towards mind-dependent bodies, as seen in both Desgabets and Régis, for example. Lennon (1993, 25) writes of Régis: "Individual things result from our projection of sensations on otherwise homogeneous and undifferentiated extension. On this view individual things are what Malebranche and Arnauld took to be the representations of things."

[3] Impenetrability, solidity, and hardness are among the properties of body that some thought Descartes was not entitled to (think, for example, of Locke's discussion of body). The question of whether a notion of force must be added (think, for example, of Leibniz) persists long into the 18th century, as we will see in later chapters of this book.

matter, moving in accordance with his three laws of nature. Collisions therefore lie at the heart of Descartes' physics, and Descartes supplements his laws with seven rules of collision. The rules, like the laws, appeal to his prior theory of matter: to the essential attribute of matter (extension), and its modes (shape, size, and motion). To sum up: collision theory is foundational for Cartesian physics, and it combines two elements: matter theory and rules of collision.

The centrality of collisions is not confined to Descartes' philosophy. For anyone pursuing "mechanical philosophy" in the 17th century, impact was the *only* kind of causal process by which change comes about in the material world. Moreover, even for philosophers who, in the wake of Newton, sought to move beyond "mechanical philosophy" by endowing bodies with additional "forces," collisions remained an important means of action and interaction among bodies. As a result, collision theory was foundational for natural philosophy in the late 17th century. Moreover, following Descartes, any adequate collision theory was required to combine a theory of matter with rules of collision: we call this a *philosophical mechanics* of collisions.

The integration of philosophical physics with rational mechanics

Projects in philosophical mechanics seek to meet the demands of both physics and rational mechanics. What do we mean by this?

Early modern physics retained the Aristotelian aim of seeking the most general principles and causes of natural things, and of their changes. The primary subject-matter of physics was *bodies*: the role of physics was to provide a causal account of the nature, properties and behaviors of bodies in general. Frequently, the term "physics" was used interchangeably with "natural philosophy." This reflects the fact that early modern physics was a sub-discipline of philosophy, practiced by self-professed philosophers who retained responsibility for, and authority over, the account of body in general. When other areas of philosophy (such as those treating specific kinds of bodies) and other disciplines (such as mechanics) presupposed bodies, they did so with the presumption that physics succeeds in providing an account of bodies in general.

The term "mechanics," on the other hand, had several senses, but here we use it with one particular connotation, current at the time and broadly familiar from present-day usage. Specifically, we are interested in *rational mechanics*, namely, the mathematical study of patterns of motion in space and of mutual rest. It was a descriptive approach that represented mechanical attributes (mass, speed, force, and the like) as measurable quantities. Its inferences were subject to laws of motion and equilibrium conditions, respectively, functioning as constraints on admissible conclusions. Put modernly, rational mechanics pursued deductive schemas for moving from values of relevant parameters to integrals of motion or to differential equations relating these parameters. At first, its representational framework was heavily geometric, but through the 1700s algebraic methods increasingly supplant the earlier reliance on synthetic geometry. Our use of the term "rational mechanics" is one that came to dominate by the end of the 18th century, and it can be found explicitly a hundred years earlier in the Preface to Newton's *Principia* (see below).[4]

By "mechanics" we mean, from here on (unless stated otherwise), rational mechanics. We take the term from the *Principia*. In the Preface to the first edition, Newton offered a taxonomy of mechanics in which he divided "universal mechanics" into three: practical mechanics, rational mechanics, and geometry. For our purposes, the key points are as follows. Like geometry, rational mechanics is mathematical and exact: it suffers from none of the imperfections of practical mechanics. Unlike geometry, however, rational mechanics goes beyond the treatment of magnitudes to include motions and forces. Rational mechanics is the "science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever, and of the forces required for any motions whatever."[5]

The term "mechanics" today typically denotes some *branch* of physics (e.g. classical mechanics, quantum mechanics, statistical mechanics, and so forth). At the beginning of the 18th century, this was not the case: physics and mechanics were distinct fields.[6] Unlike mechanics, physics was largely qualitative and, as we have said, practiced by philosophers. Mechanics, on the

[4] The relation between "mechanical philosophy" and "mechanics" is under explicit negotiation and evolution during the 17th and 18th centuries. Prior to Newton's book, a central tenet of all versions of the "mechanical philosophy" is the commitment to contact action as the only means by which one body acts on another. Then, in the *Principia*, Newton offered a mechanics of bodily motion that theorized gravitational behaviors of bodies by appeal to attraction. Understood as a physics, such an attraction implies action-at-a-distance between bodies, and many at the time thought this violated the precepts of "mechanical philosophy." However, by the end of the 18th century, any successful treatment of bodily motion by rational mechanics brings those motions within the remit of "mechanical philosophy," and so Newtonian gravitation now falls under the umbrella of mechanical philosophy: contact action is no longer a precept of "mechanical philosophy."

[5] Newton 1999, 382.

[6] Guicciardini 2009.

other hand, fell under the authority of mathematicians. While some people at the time, including some of the most influential figures of the period, were both philosophers and mathematicians, the two disciplines were distinct. They had distinct methods, distinct goals, and domains of authority.

With this in mind, we see that early 18th-century physics is importantly different from physics today, in its goal (giving a causal account of the nature, properties, and behaviors of bodies in general), methods (which were qualitative), and disciplinary relations (within philosophy, and distinct from mechanics). Physics thus understood is key to the arguments of our book, and it is this 18th-century conception that the term "physics" denotes. Sometimes, we will use the term "philosophical physics" as a reminder that physics in the 1700s was a non-mathematical branch of philosophy.

Descartes' account of bodies falls within his philosophical physics. His rules of collision, insofar as they are mathematical and exact, fall under the remit of rational mechanics. According to the analysis that we offer, Descartes' theory of collisions integrates resources from physics and rational mechanics in order to provide a philosophical mechanics of collisions. By itself, this makes overly hard work of Descartes on collisions, with a superfluity of terminology for little philosophical gain. The payoff comes from applying the same analytical tools over the next 150 years of developments.

Newton's Principia *as a project in philosophical mechanics*

Newton's treatise contains a rational mechanics, but it is not *merely* a work in rational mechanics. His choice of title, *Mathematical Principles of Natural Philosophy*, is revealing. Newton declares that the forces to be treated mathematically include natural forces, such as gravity. In this way, rational mechanics becomes a tool for the pursuit of natural philosophy. In Book III, Newton applies the results of his rational mechanics from Books I and II to the particular case of gravity, and there he gives a causal account of the motions of material bodies under gravity: he offers a contribution to philosophical physics.

The *Principia* thus contains both rational mechanics and physics. Newton explicitly set out the relationship between the two, as he understood it. Together, the overall project forms a framework for pursuing the science of bodies in motion, in which a rational mechanics (providing an exact mathematical treatment of bodies and the forces that act upon them) is to be integrated with a physics (providing a treatment of the causes of the motions of bodies). Newton's physics is incomplete, but his intention to contribute to both rational mechanics and physics is clear.[7] Indeed, Newton's "Axioms, or Laws of Motion" belong to both. The 18th century, prior to Newton, had seen discussions over whether the laws of nature (such as Descartes') might also serve as axioms of mechanics. Up until Newton, books of physics and books of mechanics were distinct, and the principles of each differed. The *Principia*, in attempting to combine rational mechanics with physics, is an important example of a philosophical mechanics.[8]

4 Constructive and principle approaches

We have seen the centrality of collisions in Cartesian natural philosophy. Yet, as is well known, his own rules of collision were rejected for their inadequacy with respect to observable collisions.[9] The ensuing 17th century discussions are an important background for our book. First, they reveal hints of two distinct heuristics for tackling *Body*—constructive and principle—and we discuss these here. Second, they preview the problems with collision theory that 18th century natural philosophy was to inherit: see section 4, below.

In October 1668, Henry Oldenburg, secretary of the Royal Society, wrote to Huygens and Wren asking for collision theories. Soon, Huygens, Wren, and Wallis submitted their proposed rules of impact. Huygens' and Wren's cover the case of perfectly elastic bodies (as we would call them today), while Wallis' rules cover perfectly inelastic collisions.[10] One might think that this is where the story should end: we have the correct rules of collision, so what else is there?

As Jalobeanu explains, the issues with collisions were far from resolved by the arrival of the rules from Huygens, Wren, and Wallis. In 1668, Oldenburg wrote to Wallis (and others) asking about the physical causes of rebound, about

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[7] For explanation and argument, see see Brading, forthcoming.
[8] See Brading, forthcoming.
[9] Descartes himself maintained that his rules applied only to microscopic (and therefore unobservable) collisions, not to the bodies of our experience. The rejection of this defense of his rules speaks to the question of epistemology: of the means by which we are to determine whether or not the proposed rules are to be accepted.
[10] The problem of how to categorize bodies—as hard, soft, elastic, inelastic, rigid, malleable, unbreakable, infinitely divisible, etc.—and how to correlate these terms with the various behaviors of bodies (when pressed upon, during impacts, etc.) persisted into the 18th century, as we shall see. For discussion of the17th century struggles with "hardness" in the context of collisions, and in relation to the Royal Society debates, see Scott (1970, 12ff.)

whether resting matter resists motion, whether motion is conserved, whether motion is transferred from one body to another when they collide, and so forth. The questions concerned the material nature of bodies, and the physical causes of their behaviors during impact.

In their submissions, none of Huygens, Wren or Wallis had discussed the material constitution of the bodies, let alone used it to set up or constrain the problem. In fact, Huygens is explicit about setting aside causes altogether:[11]

> Whatever may be the cause of hard bodies rebounding from mutual contact when they collide with one another, let us suppose that when two bodies, equal to each other and having equal speed, directly collide with one another, each rebounds with the same speed which it had before the collision. (Huygens 1977, 574)

Wallis, in contrast, responded by claiming that the rules themselves provide an account of the physical causes:

> I have this to adde ... you tell mee yt *ye Society in their present disquisitions have rather an Eye to the Physical causes of Motion, & the Principles thereof, than ye Mathematical Rules of it*. It is this, That ye Hypothesis I sent, is indeed of ye *Physical* Laws of Motion, but *Mathematically* demonstrated. (Oldenburg 1968, 220-2)

But, their quietism about material properties and causes met with resistance at the Royal Society. Another member, William Neile, objected that the collision rules needed to be supplemented by an underlying matter theory that would provide an account of the "physical causes" of the observed phenomena, such as rebound.[12]

The problem arises thus. If the properties of bodies (such as hardness and "springyness") and the principles concerning the behavior of bodies (such as conservation of quantity of motion) appealed to in the rules of collision arise from the nature of matter (as they do in Cartesian physics), then a problem in mechanics (finding the correct rules of collision) is inevitably entangled with

[11] This excerpt is from a paper published posthumously in 1703. See also Murray, Harper and Wilson (2011, 189, n. 8) who note that this phrase does not appear in the original letter sent to Oldenburg but was added prior to publication.
[12] For context and discussion, see Stan 2009.

matter theory. More generally, Neile's objection signals a theme that persists late into the 18th century: the search for a causal-explanatory account of the properties of bodies and of the collision process that integrates the rules of impact into a theory of matter: a philosophical mechanics of collisions.

The dispute above can be analyzed as offering two general approaches for tackling problems within philosophical mechanics. Following Neile, we may decide to begin with a theory of matter, and develop our collision theory from there. We call this the *constructive* approach. Following Wallis, we may decide to begin with the rules, and seek to build our theory of matter from there. We call this the *principle* approach. By "approach" we here mean a general strategy consisting of a broad heuristic along with a reservoir of initial evidence and explanatory premises. We will argue that 18th-century attempts to solve *Body* are best understood as pursuing a philosophical mechanics of bodies by means of these two general approaches. We further specify them as follows.

> **Constructive approach (Bodies):** The qualities and properties of matter are the primary resource for solving *Body*.

From the properties and powers of matter, we build concepts of body (Nature) and bodily action (Action) consistent with Principle and Evidence (see 1, above) to arrive at a philosophical mechanics in which the resulting account of bodies yields, or is at least consistent with, the notion of body that rational mechanics presupposes.

This approach comes in two varieties, a stronger and a weaker. The stronger begins with an explicit theory of matter, and constructs bodies from it. The weaker eschews a foundation in matter theory, and works directly with bodies instead. It presumes that the methods and resources of philosophical physics itself are sufficient for determining their qualities and properties.

> **Principle approach (Bodies):** Theoretical principles, such as the laws of motion, are the primary resource for solving *Body*.

In the case of laws of motion, the principle approach means drawing the concept of body, and of bodily action, from the laws themselves, without appeal to any prior theory of material constitution. From there, we arrive at a philosophical mechanics by showing that the resulting account coheres with a philosophically viable theory of matter in meeting the demands of NAEP. By "coheres with," we again denote two possibilities, a stronger and a weaker. The weaker is that the laws are deemed a *necessary* component in constructing a concept of body, but additional, extra-legal ingredients are required, drawn perhaps from an independent matter theory. The second, stronger, position is that the laws are both necessary *and* sufficient for the construction of an adequate body concept.

Insofar as matter theory and physics fall under the authority of philosophy, and laws of motion under rational mechanics, the constructive and principle approaches align with two distinct routes to a philosophical mechanics: one which prioritizes philosophy—including matter theory and physics—and the other which puts rational mechanics first. To see this play out requires the rest of our book.

The constructive and principle approaches generalize beyond bodies. At the beginning of the 18th century, bodies were presumed to be the objects of study in both physics and rational mechanics. But as the century wore on, this presumption came under increasing pressure. From the perspective of mechanics, candidates for the object of study in the 1700s included point particles, flexible and elastic solids, inviscid fluids, and mass volumes in equilibrium configurations.[13] If we relax the assumption that the objects of physical theorizing are bodies, then it becomes an open question what those objects might be, and *Body* becomes a more general problem, viz. the *Problem of Objects*: What are the objects of physical theorizing? And how can we know? As our analysis unfolds in later chapters, we see that the constructive and principle approaches track this generalization.

> **Constructive approach (Object):** Matter theory is the primary resource for constituting the objects of physical theorizing.
>
> **Principle approach (Object):** Theoretical principles, such as laws, are the primary resource for constituting the objects of physical theorizing.

As with Bodies above, these approaches come in weaker and stronger forms.

[13] Once we move beyond the 18th century, we soon add classical fields, quantum particles, quantum fields, and so forth to the possibilities for the objects of theorizing. The problem persists of how best to specify these objects.

5 The unity of physical theory

As the 18th century began, the role of physics was to provide a general account of bodies. Other areas of inquiry (in philosophy and beyond) took such bodies for granted, as a given in theorizing. Special areas of natural philosophy, e.g. botany, took the general account of body and then studied the additional specifics appropriate to plants. The general concept of body provided by physics thus played a unifying role as the common object of philosophy, mechanics, and so forth.

In order for bodies (more generally, for objects of any kind) to play such a unifying role, solutions to *Body* must presuppose that there is some single kind of object that serves as the subject-matter of our theorizing: some single ontology that unifies the various theories and areas of inquiry. As we have seen, one may adopt either a constructive or a principle approach to constituting the bodies (or objects) that play this unifying role.

However, having distinguished between the constructive and principle approaches, an alternative source of unity presents itself. Rather than unifying our theorizing through a shared *object* (such as a shared account of body), we locate unity directly in the *principles* (such as the laws). To see how this new possibility emerges, we make the contrast with the constructive approach explicit.

> **Ontic Unity:** a single type of object unifies physical theory.

For example, unity may be achieved via a single account of the bodies common to all areas of theorizing, and in such a case this unity may be achieved either by the constructive or by the principle approach to ontology. However, even where the objects of physical theory are varied, a constructive approach might yield a single matter theory consistent with them all, thereby providing the underlying ontology that unifies the diverse objects of different physical theories.

There is an alternative to ontological unity, one that arises from the principle approach:

> **Nomic Unity:** a single set of principles, such as a set of dynamical laws, unifies physical theory.

That is, unity comes from the (small) set of principles that entail the properties and behaviors of all physical systems, such as when the laws entail equations of motion for such systems—or at least all those regarded as tractable at the time. Such an approach makes no explicit commitments about the ontology associated with the principles: it allows for a diverse ontology, for diverse objects, and for cases where there is no explicit specification of objects at all; it locates the unity in the laws.

As we will see in the second half of our book, this latter conception of unity emerged in the context of rational mechanics as a consequence of the persistent failure to solve *Body*. But it too faced a challenge, viz. to ensure that mechanics is *one* theory, not a patchwork of local accounts joined arbitrarily by blunt juxtaposition in a textbook. Facing up to this challenge is crucial—for if mechanics lacks even this unity then it is unclear whether it has a subject-matter at all. Towards the end of this book, we see what this approach to unity in fact amounts to.

6 Collisions and constraints: PCOL and PCON

We cannot hope to cover all the many aspects of *Body* in one book. However, when viewing the 18th century through the lens of philosophical mechanics, we see that two somewhat better defined problems were the main loci of investigation: the problem of collisions (PCOL) and the problem of constrained motion (PCON). We explain these at length in our book, because they are the focus of the first and second halves, respectively. Here, we state them for future reference, and attempt to give the gist of their significance.

We have already noted that collisions became central to natural philosophy after Descartes. As a result, the question became pressing:

PCOL: What is the nature of bodies such that they can undergo collisions?

We argue that solving PCOL became a necessary—but not sufficient—condition on solving *Body*. The task was to give a causal-explanatory account of collisions by integrating the rules of impact with a theory of matter. In other words, to provide a philosophical mechanics of collisions.

There are two routes to that: one prioritizes philosophical physics, the other puts rational mechanics first. Within the former, we find two versions of a constructive approach, consistent with the stronger and weaker versions described above. The stronger is matter-theoretic: it starts with an overt, philosophical theory of matter. From its resources, this approach articulates a physics of bodies undergoing impact. That physics will include their properties relevant to the collision process, and their behaviors during and after impact. The weaker presumes that physics itself has methods and resources sufficient for determining and justifying the qualities and properties of material bodies, without the need to appeal to any explicit theory of matter. For ease of reference we will call these the matter-theoretic and physical versions of the material approach. The constructive approach to PCOL is the subject of the first half of our book.

Despite concerted efforts by a wide range of philosophers, as of the mid-1700s the constructive approach had yet to succeed. Meanwhile, developments in rational mechanics began to change the philosophical space in important ways, so that a new problem supplanted PCOL as the most important locus of research relevant to *Body*. This is the focus of the second half of our book, and it is here that the principle approach comes to the fore.

A key assumption in attempts to solve PCOL was that bodies are extended and mobile. Such are the bodies treated in rational mechanics, and following Euler's *Mechanica* in the 1730s, collisions fall within a projected general theory of the motions of extended bodies. The 1600s had tackled the motion of an extended body by tracking a representative point, which proved hard enough for many systems. This approach has two serious limitations that yielded two corresponding challenges for 18th century rational mechanics, and both proved highly consequential for attempts to solve *Body*.

First, treating a representative point yields the overall trajectory of a body (e.g. the path of an asteroid), but it does not determine the motions of the parts of the body as it executes that trajectory (e.g. the tumbling of the asteroid as it careens towards the Earth). The challenge was to construct a rational mechanics that goes beyond the representative point when treating the motion of the whole. Within mechanics, it falls within the theory of constrained motions, and we call it MCON1.

> **MCON1: Given an extended body subject to *internal* constraints, how does it move?** More specifically, given an extended body whose parts are mutually constrained among themselves (i.e. held together to form *one* body), what is the motion of each of the parts?

A solution to MCON1 would enable us to determine the motion of *every* part of an extended body, as the whole moves. The simplest case is the hard (or rigid) body, in which there are no relative motions among the parts of the body. To achieve rigidity, the presumptions are that (i) forces acting on the body produce no change in shape (no compression forces, no torsion); (ii) forces acting on the body through a point other than the centroid (representative point) produce only rotational motion, no torsion; and (iii) any rotational motion has no effect on the shape of the whole. As soon as we relax these assumptions, relative motions among the parts of the body (and their effects on the motion of the whole) must be addressed. As you might imagine, MCON1 is horribly difficult.

The second limitation concerns the motions of bodies that are impeded by other bodies, such as when a ball is prevented from falling by the presence of an inclined plane. One might hope to treat such obstructions in terms of Newton's laws of motion, via the forces at work as the obstacle acts on the moving body. However, such hopes are often ill-founded, especially when the forces are many or when they change at every moment of the motion. As a classic example, consider a bead constrained to move along an arbitrarily curved wire. As the bead moves, the direction and magnitude of the impressed force changes at every instant. To overcome this complexity, and the resulting intractability of the problem,[14] we consider the bead as subject to *kinematic constraints*: we treat the wire as restricting the motion of the bead to a particular spatial region, without concern for the forces that bring this about. More generally, we theorize the obstructed motion of the target body as encountering obstacles that render certain regions of space inaccessible. By this means, we can seek to determine the motions of bodies when subject to a variety of external obstacles. We call this MCON2.

> **MCON2: Given an extended body subject to *external* constraints, how does it move?** More specifically, when the motion of a body is impeded by an obstacle, what is its resulting motion?

[14] It can be tempting to think that intractability concerns the limits of what is practical for us and is therefore unimportant for the claim that "in principle" everything moves in accordance with Newton's Second Law. This would be a mistake, because of the relation between Evidence and Nature: our evidence for our claims about Nature depend on showing that bodies do indeed move in accordance with theories of motion that are at least consistent with Nature, so if we cannot solve problems of motion using those theories, then we break the link between Evidence and Nature, and our claims about Nature lose their justification. For allied discussion, see also Wilson 2009.

A solution to MCON2 would enable us to determine the motion of a body when subject to any kinematic constraints whatsoever.

Both MCON1 and MCON2 belong to the theory of constrained motions within rational mechanics. As this theory developed in the 18th century, it provided a new locus of investigation into *Body* that we call "the problem of constrained motion," PCON:

> **PCON:** What is the nature of bodies such that they can be the object of a general mechanics?

The parallels with PCOL will be helpful. In tackling PCOL, the rules of impact place important demands on the nature of bodies undergoing such collisions—and thereby play a significant role in determining conditions of adequacy for any solution to *Body*. In moving beyond collisions to consider a more general mechanics, we turn our attention to the more general rules for the behavior of bodies, as formulated in equations of motion, equilibrium conditions, and the like. In particular, we seek a rational mechanics that provides solutions to MCON1 and MCON2. The resulting theory in relation to PCON is analogous to the rules of collision in relation to PCOL. The crucial difference is that, unlike with PCOL where the rules of impact had been found by 1668, as of the early 18th century mechanics did not yet have the "rules of motion" for a general mechanics. Rather, it was only in the 1700s that a general treatment of the possible motions of extended bodies became an explicit task of rational mechanics (MCON1 and MCON2), and thereby PCON came to the fore as a critical locus of investigation for *Body*. This is the subject of the second half of our book.

We argue that addressing PCON becomes a necessary condition on any adequate solution to *Body*. The nature of bodies must be such that they cohere so as to move as MCON1 demands, and of undergoing constrained motion in accordance with the demands of MCON2. As the century progressed, the relationship of *Body*—and the vulnerability of proposed solutions thereof—to developments in PCON became increasingly fraught with philosophical and conceptual difficulties. For philosophers, the lesson is this: any attempted solution to *Body* must keep up with developments in rational mechanics, especially MCON1 and MCON2.

Historically, we can map the relationships between *Body*, PCOL, and PCON in the 18th century, as follows. From the 1600s philosophers inherited collisions as fundamental for natural philosophy, along with an unsolved problem, PCOL. The failure of philosophical physics to solve PCOL during the early 18th century (see chapters 2-6) coincided with the independent rise of rational mechanics (see chapter 7). Following this, PCON emerged as an alternative route to solving PCOL, in which PCOL is a problem within PCON. However, as it turns out, PCOL is a very complex problem within PCON, lying far downstream of the foundational problems in the newly developing generalized theories of rational mechanics (see chapters 8-11). What then of *Body* come the end of the 18th century (see chapter 12)?

7 Methods

Body belongs to philosophy, and remains a problem in contemporary philosophy today.[15] Yet we approach it from an historical vantage point, and limit our attention to the 18th century. Why?

Three reasons lead us to take an historical approach. First, *Body* is more than 300 years old, as are attempts to solve it. If we study just its current version, we risk working in an impoverished problem-space, bound by a thin, narrow slice of a philosophical picture that is bigger and richer than the present. In consequence, even those with strictly contemporary interests stand to benefit from taking a longer temporal view. We are familiar with this from work by philosophers of physics on space, time and motion, where not just early 20th century, but also 17th, 18th, and 19th century considerations deepen our understanding of the philosophical issues at stake. The same is true for *Body*.

Second, the contours of *Body* are historically sensitive: how *Body* is formulated, its place in the system of knowledge, the preferred heuristics for solving it, and—most importantly—the criteria for an acceptable solution, vary with time as the philosophical context for addressing it shifts and changes.

[15] There are two main strands of *Body* in contemporary philosophy. The first concerns macroscopic bodies, their metaphysical status, and their relationship to "fundamental" objects. The second is the generalization of *Body* to the *Problem of Object*: that is, the problem of specifying the object of a given theory, whether that be a body, particle, field, gene, or whatever it may be. See van Inwagen 1990, and the subsequent literature in metaphysics; the vast literature on reductionism in philosophy of science; discussions of the appropriate ontology for quantum mechanics (see, for example, Ney 2020); quantum field theory (see, for example, Fraser 2008); and so forth.

Therefore, *Body* raises different philosophical challenges and questions at different moments in the history of philosophy. These are of philosophical interest in their own right.

Finally, this diachronic dimension is developmental and interactive. Philosophers' understanding of *Body* changed and developed over time not only in response to, but also—and crucially—*contributing to* the evolving philosophical context. To study the unfolding of *Body* in the history of philosophy is, in our view, the best path to understanding both *Body* itself and its significance for philosophy.

A contrastive characterization of our project may be helpful, for our book is situated between two alternative historical approaches. For one, it is not a work in intellectual history: we do not set out to track the "emergence" of a concept or idea, or the semantic shifts undergone by a word or concept. For another, neither is it a work in the history of material culture. We do not seek to map chains of belief transmission through networks of patronage, mentorship, correspondence, and the like; or to study how such transmission takes place. Rather, our book studies a philosophical problem as a historically situated object whose characteristics are: *determined by* the historical figures who formulated it and struggled with it; *revealed by* the argumentative and evidential resources those figures employed; and *presented in* the material books, papers, letters, manuscripts and notes those figures left behind. As a result, conceptual developments and material circumstances have an important role to play, but only when and where they make a *philosophical* difference to the argumentative, explanatory, or evidential elements of *Body*.

To pursue our goal, we employ three methodological heuristics. First, we seek to recover meaning from use, where by "use" we mean: in philosophical argumentation and in theoretical problem-solving. We do not confine ourselves to prefaces, manifestos, and programmatic declarations. Rather, we give greater evidential weight to the details that come later: the places where the opening declarations are tamed and re-shaped by the argumentative and evidential constraints of the problems at hand. It is here that most of the philosophical action takes place, in our view.

Second, we use anachronism judiciously. Situated in the 21st century as we are, post-1700 developments (in philosophy and classical mechanics) provide us with resources unavailable to our protagonists. We use the resulting insights only where they can be translated without remainder into the concepts of the historical period at issue. We aim to state and shape our explanations or objections just as a premier authority, fully *au courant* with the state of the art then, could have done, with the proviso that some of the words we use have changed their meaning since that time (and where this is of philosophical import for our project, we say so). We neither state nor assess any historically given answers to *Body* in terms or by standards that greatly post-date the context of the answer at issue. In this way, we seek to preserve the diachronic dimensions of *Body* described above.

Finally, we explicitly recognize our own authorship. *Body* is both *our* problem and *their* problem, and this book is the product of an engagement between the two, of course. We chose *Body* as our central theme because it interests us. As philosophers in the 21st century, our philosophical backgrounds, sensibilities, and motivations for embarking on this project frame and guide the work that we do in this book. We do not offer a history that pretends to wash out our own presence: rather, we offer a philosophy of a problem that has a long history, and we seek to explain what interests us about it and why.

8 Audience

We offer three strands of argument, of interest to the three groups of people for whom we wrote this book: philosophers of physics; historians of modern philosophy; and philosophers of science and metaphysicians interested in the epistemology and metaphysics of science.

Philosophy of physics

From Aristotle to Newton, physics was the study of bodies. If we turn our attention to modern physics, however, we find that bodies are no longer its principal object, and indeed "body" is not even among its central concepts. This observation, mundane though it may seem, turns out to hide an abundance of interesting philosophical problems. When we ask: "Why did bodies get displaced from their privileged position, how did that come to be, and with what consequences?," the answers that we demand—and that we offer in this book—are metaphysical, epistemological and conceptual. One upshot is this: the 18th century becomes a period of focal interest for philosophers of physics, equal to the 17th and early 20th centuries in its import. This is because physics in the 1700s proved unable to articulate a satisfactory account of body, and rational mechanics (then a separate discipline) attempted to fill the void—also unsuccessfully, it turns out. In the process, the conceptual foundations of physics and mechanics received profound scrutiny and reformation. The consequences shape contemporary physics today, as we will see.

Most philosophers of physics will be familiar with 18th century debates over space, time, motion, and gravity. This area of philosophical mechanics (celestial mechanics and gravitation theory) has received widespread attention, and so it will not be our focus in this book. Instead, our goal is to open up a new area of inquiry for philosophers of physics, one that has yet to receive detailed scrutiny. If we consider the *Principia* as a work in philosophical mechanics, we see that the scope of his rational mechanics contrasts sharply with that of his physics: while Books 1 and 2 are intended to be general, Book 3 concerns one force only, gravitation. Against this foil, we restrict our attention to philosophical mechanics where the gravitational behavior of bodies is *not* at issue: non-gravitational mechanics and terrestrial physics. It is here that many foundational issues in classical mechanics were worked out, for it is here that the pressing need to treat constrained systems, and the limitations of Newton's laws for this purpose become clear.

The comparison with gravitation is useful in a further respect. Recent work has done much to elucidate the evidential support for Newton's theory of universal gravitation, as it was developed and accrued during the 18th century. Unlike celestial mechanics, in the 1700s non-gravitational mechanics did not have centuries of observational data and mathematical theorizing to work with. The only potential analogue of positional astronomy for terrestrial mechanics is the set of terrestrial machines studied in ancient mechanics, but Enlightenment mechanics explicitly sought to move beyond this limited set. The contrast with gravitation brings a new question into focus: what counts as evidence in the parts of philosophical mechanics *not* concerned with gravitation?

History of modern philosophy

Philosophers have long read the 18th century as grappling with problems inherited from Descartes: Cartesian skepticism, Cartesian dualism, and the Cartesian circle, to name but a few. Often, these are cast in an epistemological vein. But there is another problem, also originating with Descartes and equally evident in the collective philosophical struggles of the 1700s: *Body*.

Body directly confronts core topics of the period: substance and causation, along with the associated issue of how, if at all, we can arrive at knowledge of either. All created substances were presumed (by almost everyone at the time) to depend on God, the primary substance and primary cause of all things. As a result, discussions of substance and causation divide into two: primary and secondary. Our concern is exclusively with secondary substances (bodies) and secondary causation (agency among bodies). When approached with primary substance and causation as the entry point, we find the familiar range of opinions on secondary substance and causation, from Leibniz's pre-established harmony, to Malebranche's occasionalism, to physical influx; re-hashing these debates is not our goal. But if we focus exclusively on secondary substances and causation a different picture emerges—in which there is (surprisingly) wide, broad-brush agreement on what counts as an adequate theory of secondary substance and causation. The disagreements are over how, and whether, any such theory can be developed. Detailing *this* debate and its consequences is our concern.

Tackling the problems of 18th-century natural philosophers required developments not just in the relevant technologies (experimental, mathematical, conceptual), but also in the appropriate epistemologies and accompanying methodologies. Canonical figures such as Locke, Berkeley, Hume and Kant, like Descartes before them, presume a starting point for epistemology in our ideas: they assumed an individual to have ideas whose contents she may inspect; and that some of these contents are both determinate enough and sufficiently accessible to base a viable epistemology on them. We are familiar with reading Hume's views as the terminus of this line of inquiry into causation, and conceding the point lies at the heart of Kant's Critical turn.[16] However, not all who contributed to *BODY* take ideas as their epistemological fountainhead, hence *BODY* requires us to widen our epistemological purview. For example, many of the figures we study approached questions of justification, certainty and truth through criteria of success (theoretical and empirical) in solving problems.

Whereas Enlightenment natural philosophy has predominantly been cast as grappling with the world according to Newton, a different picture emerges when *BODY* provides our lens. Though he engaged with *BODY*, Newton was neither the first nor the last to do so. For the most part, 18th-century philosophers attacked *BODY* within parameters set largely by Descartes; or they recast it in new terms that had no precedent in Newton. As a result, our book yields a new perspective on the relation between philosophy and the exact

[16] See Clatterbaugh 1999.

science of nature in the 1700s. More generally, it is an invitation to historians of philosophy to revisit the epistemological and metaphysical assumptions, arguments and methodologies of Enlightenment philosophers grappling with the material world, and the philosophical consequences of these that we inherit today.

Philosophy of science and metaphysics of science

Body lies at the intersection of metaphysics, physics and mechanics during the 18th century. Attempts to address it involve inferences from one domain to another, disputes over the authority of one domain with respect to another, and indeed they problematize where the boundaries between domains might be drawn, and on what basis. And, these attempts lead directly to questions of the appropriate epistemology and methodology for solving *Body*, including questions of what principles might be used to guide, constrain, or evaluate a solution. Enlightenment attempts to grapple with these issues are interesting in themselves, and also for the light they shed on their contemporary counterparts. We offer our book as an invitation to philosophers of science, metaphysicians of science, and anyone interested in "scientific metaphysics," to engage with either or both.

9 Overview

In this introductory chapter we have presented several technical terms (*Body;* philosophical mechanics; Goal; Nature, Action, Evidence, and Principle; the constructive and principle approaches; PCOL and PCON). In the chapters that follow, these terms will be re-introduced slowly as the need for each arises naturally in the argument of the book. We have collected them together here as a guide to the overall structure of our analysis, and for ease of reference going forward. Whether all are necessary will be clear only by examining the work that they do in the remainder of this book. We proceed as follows.

In the book's first half, our primary focus is impact. Accordingly, Chapter 2 is an account of collision theory in France after Descartes. It documents and explains protracted efforts, by Malebranche and his posterity, to integrate coherently a broadly Cartesian matter theory and the rules of impact.

Chapter 3 uncovers analogous efforts—to build a philosophical mechanics of collision—in Germany after Leibniz and in Newton's Britain, from resources they had respectively bequeathed. The joint upshot of these two chapters is that, by 1730, natural philosophy regarded collision theory as the main locus for solving *Body*, and no satisfactory solution was available.

In Chapter 4, we broaden our scope to *Body*. We introduce physics as a sub-discipline of philosophy, and show that *Body* was its central problem We articulate NAEP within this context, and illustrate some of the difficulties facing philosophers then in their attempts to find appropriate resources—metaphysical and epistemological—whereby to tackle *Body*. We show the relationship of *Body* to familiar debates over substance and causation in the period. Finally, we show that PCOL arises naturally within the context of *Body*, and that its philosophical significance is best understood against this backdrop. We conclude that *Body* is a problem to be solved not just within philosophy, but within philosophical mechanics.

This sets the scene for Chapter 5, which examines mid-century attempts to address *Body*. We argue that the two most promising proposals, by du Châtelet and Euler respectively, faced insurmountable obstacles. Both begin with physics, and seek to integrate relevant resources from the mechanics of collision. We claim that success demands meeting Goal (see above): providing a single, well-defined concept of body that is consistent with an intelligible theory of matter; adequate for a causal-explanatory account of the motion behaviors of bodies; and sufficient for the purposes of mechanics. And we argue that neither succeeds.

Chapter 6 studies two radically new ways of constructing extended, impenetrable, mobile and interacting bodies, found in Kant and Boscovich. We explain how their theories transform the goals of physics while falling short when it comes to a philosophical mechanics of collision. And, these attempts need to be assessed in the context of concurrent developments in mechanics. For, during the period covered in Chapters 2-6, rational mechanics had been undergoing rapid changes of direct relevance to *Body*.

In Chapter 7, we argue that around 1750, the locus for grappling with *Body* moves from philosophy to rational mechanics. We explain the reasons for this watershed transition: conceptual difficulties with three notions that philosophers had relied on, viz. mass, contact action, and extended-body motion. We argue that professional philosophers then failed to incorporate pertinent advances in mechanics into their accounts of *Body*, thus opening a rift between philosophical physics and rational mechanics. And, we uncover the key challenge in mechanics going forward; namely, the problem of constrained motion (PCON). This problem is our lens, throughout the subsequent chapters, for analyzing the developments in rational mechanics most relevant to *Body*.

In Chapter 8, we revert to the early 1700s, and review developments in the theory of constrained motion then, as they pertain to the goals of this book. We survey a wealth of work on the vibrating string and the compound pendulum. Theorists at this time sought general principles and uniform methods for treating constraints. But, they generally fell short of these desiderata. That insufficient outcome would shape the agenda for rational mechanics through the latter half of the century.

Against this background, in Chapter 9 we turn to d'Alembert's *Treatise on Dynamics*. He made the first systematic attempt at a general treatment of the mechanics of constrained motions. We show that his *Treatise* exemplifies the enormous difficulties involved in PCON, and argue that it is pivotal for the growth of philosophical mechanics in the latter half of the 18th century.

Chapters 10 and 11 assess two different strategies for solving MCON. In Chapter 10, we examine how the 18th century dealt with MCON1. After 1730, rational mechanics learned how to tackle the motion of extended bodies with internal constraints, e.g. rigidity and incompressibility. Here the greatest advances were due to the Bernoullis, d'Alembert, and especially Euler. Accordingly, we focus on their key breakthroughs in pursuit of a rational mechanics of extended bodies.

In Chapter 11 our focus is on Lagrange. The problem of external constraints found a general solution in his analytic mechanics of 1788. Two ingredients were key to his solution: a dynamical law, viz. the Principle of Virtual Velocities; and the method of Lagrange multipliers. This combination allowed him to unify all rational mechanics then available. We assess Lagrange's achievement, via a constructive- and then principle reading of his theory.

By now, we have reached the end of the 18th century. What, then, is the state of philosophical mechanics? To see this, we follow two strands of development, one seeking nomic unity (following Lagrange), and the other ontic unity (in the physics of the Laplacian School). Work by Cauchy spawned alternatives to both: a new approach to nomic unity, by means of balance laws of force and torque; and a new approach to ontic unity, based in deformable-continuous matter. The upshot is pluralism in philosophical mechanics, with consequences for *Body* and for the relationships among philosophy, physics and mechanics. Chapter 12 ends with a brief review of the main conclusions of our book.

10 Conclusions

The 18th century was a golden age for philosophical mechanics. As the century began, physics was a subdiscipline of philosophy, and its primary task was *Body*. By 1800, this was no longer the case. Physics had become an independent discipline, and *Body* was not its driving concern anymore. In this book, we argue that the philosophical reasons for this transformation, and thus its consequences, come into view if we analyze the 18th century as an era of philosophical mechanics. That is, as an age of widespread, long-lasting and concerted efforts to address *Body* by integrating rational mechanics with philosophical physics.

This is an entirely new way of thinking about philosophy, physics and mechanics in the 18th century, and it diverges sharply from prior accounts. According to Mach, once we have Newton's *Principia* then classical mechanics is complete as regards its principles; all that remains is the technical challenge of *using* these principles to treat ever more complex and difficult phenomena. This view of post-Newtonian mechanics is perpetuated in Kuhn. For him, the *Principia* is the culmination of a scientific revolution, after which all "classical mechanics" becomes normal science within the Newtonian paradigm. The principles, methods, and basic ontological commitments are secure; all we need to do now is solve puzzles.[17]

But it is simply not true that 18th century physics is "normal science" or that Enlightenment mechanics has settled foundations and is philosophically uninteresting. This is not a new point to make, yet for its significance to shine through, for it to be something we can use in our research and teach in our classrooms, we need an alternative way of framing the history, one that is different from Mach's or Kuhn's.

Our proposal, philosophical mechanics, is built on evidence from the books and papers of the time, and it enables us to do all sorts of new and interesting things. Works whose philosophical importance is largely invisible under old framings (such as du Châtelet's *Foundations of Physics* and d'Alembert's *Treatise on Dynamics*) become highly visible and prominent. New questions arise about more familiar works (such as Boscovich's *Theory of Natural Philosophy* and its relation to constrained mechanics). Under the Mach-Kuhn framing of history, the work of some physicists, such as Newton

[17] See, respectively, Mach (1883, 239) and Kuhn 1962.

and Einstein, is philosophically important. But the work of others, such as Euler and Lagrange, is philosophically inert—after all, all they did was use Newton's principles to solve problems within the already-existing Newtonian paradigm. Our framing, in contrast, enables the recovery of a broad range of first-rank physicists as doing work with *philosophical* import. The work on *BODY* that we discuss makes little sense from a Mach-Kuhn perspective. Yet, it was important. From our vantage point, it appears as a widely shared and long lasting project—of integrating philosophical physics with rational mechanics.

We intend our book to be an example of the importance of telling and re-telling our history, keeping it alive over and again with every new generation of students and scholars. We hope you will find it worthwhile.