Beyond Proportional Analogy
A Structural Model of Analogical Mapping

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ABSTRACT: A model of analogical mapping is proposed that uses five principles to construct an analogical map from a source to a target domain. The principles follow the fine conceptual structure of the domains. They are: (1) the principle of proportional analogy; (2) the principle of mereological analogy, (3) the principle of chain reinforcement; (4) the principle of transitive reinforcement; and (5) the principle of mutual inconsistency. Each principle generates hypotheses regarding assignments of elements of the source domain to analogous elements of the target domain. A constraint-satisfaction network is used to find the set of hypotheses that preserves the greatest relational structure of the source. In contrast to the model proposed here, most models of analogical mapping use only the principle of proportional analogy. The use of many principles is shown to be superior in that it permits smoother integration of pragmatic factors and results in a more efficient mapping process.

1. Introduction

1.1 The Process of Analogical Cognition

Analogical cognition is a mental process that exploits parallelisms within existing knowledge to generate new knowledge. Here we are concerned with analogies involving propositional knowledge.[1] Propositional knowledge consists of terms organized into propositions. A proposition is a structure containing a name, a predicate, and a set of arguments. The name, predicate, and arguments are all terms. We allow terms to correspond to concepts in long-term memory, and therefore hold that propositions are conceptual structures in long-term memory.[2] Analogical cognition working with propositional knowledge is generally thought to have three phases (Hall, 1989): (1) an access phase; (2) a mapping phase; (3) a transfer phase. The access and mapping phases find parallelisms between conceptual structures already existing in long-term memory; the transfer phase uses these parallelisms to extend conceptual structures, thereby creating new knowledge.

Analogical cognition begins with a particular system of terms about which the mind desires to know more. This initial system is traditionally called the *target*. Given the target, *the access phase* finds another system of terms which shares some relational structure with the target system. This other system is traditionally called the *source*. The two systems involved in analogical cognition are usually called *domains*. Hence we

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[1]Analogies involving propositional knowledge can be contrasted with perceptual and motor analogies.

[2]We include both semantic and episodic memory in long-term memory; that is, we include both semantic and world knowledge in long-term memory. We hold that the same principles organize both semantic and world knowledge. Long-term memory contains both propositional and non-propositional knowledge: images (from all sensory modalities) are one kind of non-propositional knowledge; motor-programs are another.

speak of the *target domain* and the *source domain*. We conceive of a domain as a set of propositions. A set of propositions is a complex conceptual structure. Domains can be thought of in terms of the scripts proposed by Schank and Abelson (1977), the experiential gestalts proposed by Lakoff and Johnson (1980), or the semantic fields proposed by Kittay (1987).

Once the target and source have been isolated, *the mapping phase* specifies a precise set of correspondences between source and target terms. Formally, the task of the mapping phase is to produce an *analogical mapping relation* f_M that pairs source terms with target terms. The relation f_M is a set of ordered pairs (s, t) where s is in the source and t is in the target. Such ordered pairs are sometimes called *matches*, *correspondences*, or simply *analogies*. The relation f_M must preserve as much of the relational structure of the source as possible. A function f_M preserves the relational structure of the source to the extent that for each relation R(x,y) in the source there is a corresponding relation $R(f_M(x), f_M(y))$ in the target. According to Gentner (1982, 1983), Gentner and Gentner (1983), and Falkenhainer, Forbus, and Gentner (1989), the structure-preserving relation f_M must be a one-to-one function. That is, f_M must be an isomorphism. As Holyoak & Thagard (1989) point out, the constraint that f_M be one-to-one is too strong; they require only that f_M be a function, thus allowing it to be many-to-one. However the analogical mapping relation is conceived, analogical mapping generally is divided into two subprocesses: (1) generating candidate analogical matches (match hypotheses); (2) finding the "best" set of analogical matches within the set of match hypotheses. This best set is the analogical mapping relation f_M.

Whenever the knowledge associated with a source term is more extensive than the knowledge associated with a corresponding target term, *the transference phase* copies the source knowledge and associates it with the target term. New knowledge about the target is thereby created.

1.2 Approaches to Analogical Mapping

Recently several highly sophisticated computational approaches to analogical mapping have been developed. Foremost among these recent approaches are Gentner's *structure-mapping approach* (Gentner, 1982, 1983, 1988, 1989; Falkenhainer, Forbus & Gentner,1989) and Holyoak & Thagard's constraint-satisfaction approach (Holyoak & Thagard, 1989, 1990). Each of these approaches is embodied in a computer program. Gentner's approach is realized in the Structure-Mapping Engine (SME), while Holyoak & Thagard's approach is embodied in the Analogical Constraint Mapping Engine (ACME).

The structure-mapping approach represents propositions as predicate calculus expressions, and domains as lists of such expressions. It uses a set of match rules to generate match hypotheses. The match rules used to compute match hypotheses implement a single principle, the classical principle of proportional analogy. According to the principle of proportional analogy, if $R(S_1, S_2, \ldots S_n)$ is a proposition in the source, and $R(T_1, T_2, \ldots T_n)$ is a proposition in the target, then (S_i, T_i) is a match hypothesis for $i = 1, 2, \ldots n$. Once match hypotheses have been computed, a variety of combinatorial techniques are used to combine them into maximally consistent collections called global matches. Each global match is given a structural evaluation score. The analogical mapping relation is the global match with the highest score. The structure mapping approach is primarily syntactic, concerned only with "the syntactic properties of the knowledge representations, without regard to either the specific content of the domains or the goals to be accomplished" during analogical cognition (Novick, 1988: 126). The structure-mapping approach incorporates neither semantic nor pragmatic considerations.

The ACME approach to analogical mapping (Holyoak & Thagard, 1989a, 1989b, 1990) incorporates syntactic, semantic, and pragmatic considerations. ACME represents propositions as predicate calculus expressions, and domains as lists of such expressions. The ACME approach use a single principle, again the principle of proportional analogy, to generate match hypotheses. According to Holyoak & Thagard, the analogical map is the set of match hypotheses that best satisfies a number competing syntactic, semantic, and pragmatic constraints. This set is easily found using standard connectionist constraint-satisfaction techniques. We agree with Holyoak & Thagard that constraint-satisfaction techniques are most appropriate for finding the analogical map once the set of possible matches has been established. Because ACME incorporates semantic and pragmatic considerations, we regard ACME as a mapping engine superior to SME.

While both SME and ACME are powerful approaches to analogical mapping, we believe neither is adequate. Both SME reduce relations among terms to participation in predicate calculus expressions. However, we believe there are many different kinds of relations among terms besides mere participation in predicate calculus expressions. Both SME and ACME utilize a single principle, namely, the principle of proportional analogy, to generate match hypotheses. The principle is applied universally to all terms in all propositions in the domains. In other words, the generation of match hypotheses in SME and ACME is a brute force combinatorial process. In contrast to both SME and ACME, we believe there are many principles for generating match hypotheses. Reducing all relations among terms to the single relation of participation in a predicate calculus expression, and then applying a single matching principle universally, these mapping engines cannot be sensitive to the fine conceptual structure of the source and target domains.

In contrast to SME and ACME, we develop an approach to analogical mapping that is sensitive to the fine conceptual structure of the domains involved. The approach developed here is realized in a computer program called NETMET.[3] We argue that an approach to mapping that is sensitive to the fine conceptual structure of the domains makes for a more efficient computation of the analogical mapping relation and allows for a better integration of pragmatic factors into analogical cognition. NETMET refines ACME's approach to generating match hypotheses, and uses ACME's constraint-satisfaction approach to finding the analogical mapping relation. Because both ACME and NETMET involve constraint-satisfaction, we develop NETMET in contrast to ACME. First, we present ACME. Second, we discuss the difficulties that emerge from using only one principle to generate match hypotheses. Third, we present NETMET. Fourth, we compare the performance of ACME and NETMET on several examples. Our comparisons reveal the value of an approach to mapping that is sensitive to the fine conceptual structure of the mapped domains.

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[3]NETMET is written in C and runs on IBM PCs. A copy of NETMET can be obtained by writing to the author.

2. Analogical Mapping by Constraint Satisfaction

Holyoak and Thagard's ACME model of analogical mapping is a powerful and general engine for producing analogical maps by constraint satisfaction. ACME represents a proposition as a predicate calculus expression; target and source domains are lists of such expressions. A proposition has the form N: $P(A_1, A_2, \ldots A_n)$ where N is the name of the proposition, P is its predicate, and A_i for i = 1 to n are the arguments of the proposition.

2.1 Generating Match Hypotheses

ACME uses only one principle to generate match hypotheses. This is the principle of proportional analogy. ACME applies the principle of proportional analogy to every pair of propositions with the same number of arguments in the target and source, regardless of the semantic similarity of the predicate. ACME generates match hypotheses whose elements are arguments of propositions, predicates of propositions, and names of propositions. If P is a proposition, let NameOf(P) be the name of the proposition, PredOf(P) be the predicate of the proposition, and Arg(P,i) be the i-th argument of the proposition. For each target proposition T, and for each source proposition S, ACME generates: (1) a match hypothesis (NameOf(S), NameOf(T)) for the proposition names, (2) a match hypotheses (PredOf(S), PredOf(T)) for the proposition predicates, (3) a match hypothesis (Arg(S, i), Arg(T,i)) for each argument in the propositions. It should be noted that the generation of match hypotheses by ACME is a brute force combinatorial process.

For example, consider the target and source domains for the THEAETETUS IS A MOTHER analogy shown in Table 1. The analogy is taken from Plato's *Theaetetus*, in which the production of ideas by the young man Theaetetus is compared to the production of a baby by a mother. Notice that most of the conceptual structure of these domains is provided by the relation "contains". We use the relation "contains" to express a generalized part-whole relation; to say that X contains Y is equivalent to saying that X is a whole that contains or includes Y in some way. Part-whole relations are also called *mereological* relations, and we will refer to them as such. Given the domains in Table 1, ACME generates the match hypotheses shown in Table 2. In Table 2, each target proposition is shown paired with all source propositions to generate match hypotheses.

Source (MOTHER)	Target (THEAETETUS)
S1:produce(mother,baby)	T1:produce(Theaetetus, idea)
S2:contains(mother,womb)	T2:contains(Theaetetus, mind)
S3:contains(womb,baby)	T3:contains(mind,idea)

Table 1. Source and target for analysis of ACME.

T1:produce(Theaetetus, idea)
    S1:produce(mother,baby)
        (T1,S1), (produce,produce), (Theaetetus,mother), (idea,baby)
    S2:contains(mother,womb)
        (T1,S2), (produce,contains), (idea,womb)
    S3:contains(womb,baby)
        (T1,S3), (Theaetetus,womb)
T2:contains(Theaetetus, mind)
    S1:produce(mother,baby)
        (T2,S1), (contains,produce), (mind,baby)
    S2:contains(mother,womb)
        (T2,S2), (contains,contains), (mind,womb)
    S3:contains(womb,baby)
        (T2,S3)
T3:contains(mind, idea)
    S1:produce(mother,baby)
        (T3,S1), (mind,mother)
    S2:contains(mother,womb)
        (T3,S2)
    S3:contains(womb,baby)
        (T3,S3)

Table 2. Generation of match hypotheses by ACME.

2.2 Relations between Match Hypotheses

Match hypotheses in a set of match hypotheses are not isolated; they bear two important relations to one another. These relations are (1)*consistency* and (2) *inconsistency*.

All the match hypotheses produced by a single application of the principle of proportional analogy to a pair of propositions are consistent. For each target proposition T, and for each source proposition S, if T and S both have n arguments, ACME generates relations of consistency as follows. Relations of consistency are generated between (S, T) and (PredOf(S), PredOf(T)). For i = 1 to n, relations of consistency are generated between (S, T) and (Arg(S,i), Arg(T,i)). For i = 1 to n, relations of consistency are generated between (PredOf(S), PredOf(T)) and (Arg(S,i), Arg(T,i)). Finally, for i = 1 to n and for j = 1 to n, ACME generates a relation of consistency between (Arg(S, i), Arg(T, i)) and (Arg(S,j), Arg(T,j)). The generation of some of the relations of consistency for the domains in Table 1 is illustrated in Table 3. In Table 3, the two-headed arrow "↔" between match hypotheses indicates two relations of consistency, one from each match to the other. It should be noted that in ACME the generation of relations of consistency is a brute force combinatorial process. The relation of consistency functions as a structural constraint on an analogical map. Consistency is a *positive constraint* on an analogical map; if two matches are consistent, an analogical map that contains the one is constrained to contain the other. Consistency is a symmetrical relation: if a match X is consistent with Y, then Y is consistent with X. ACME generates the relations of consistency as it generates match hypotheses.

Inconsistency occurs when a single source concept is matched with many target concepts, or when a single target concept is matched with many source concepts. The relation of inconsistency functions as a structural constraint on an analogical map. Since ACME prefers to maximize isomorphism, inconsistency is a *negative constraint* on an analogical map; if two matches are inconsistent, an analogical map that contains one is constrained not to contain the other. Importantly, inconsistency functions as a soft constraint; it is a pressure that discourages many-many mappings, but it does not forbid them. Like consistency, inconsistency is a symmetric relation. ACME generates the relations of inconsistency after all match hypotheses have been generated.

T1:produce(Theaetetus, idea)
- S1:produce(mother,baby)
  - Matches:
    - (T1,S1), (produce,produce), (Theaetetus,mother), (idea,baby)
  - Relations of Consistency:
    - (T1, S1) ↔ (produce, produce)
    - (T1, S1) ↔ (Theaetetus, mother)
    - (T1, S1) ↔ (idea, baby)
    - (produce, produce) ↔ (Theaetetus, mother)
    - (produce, produce) ↔ (idea, baby)
    - (Theaetetus, mother) ↔ (idea, baby)
- S2:contains(mother,womb)
  - Matches:
    - (T1,S2), (produce,contains), (Theaetetus,mother), (idea,womb)
  - Relations of Consistency:
    - (T1, S2) ↔ (produce, contains)
    - (T1, S2) ↔ (Theaetetus, mother)
    - (T1, S2) ↔ (idea, womb)
    - (produce, contains) ↔ (Theaetetus, mother)
    - (produce, contains) ↔ (idea, womb)
    - (Theaetetus, mother) ↔ (idea, womb)
- S3:contains(womb,baby)
  - Matches:
    - (T1,S3), (produce,contains), (Theaetetus,womb), (idea,baby)
  - Relations of Consistency:
    - (T1, S3) ↔ (produce, contains)
    - (T1, S3) ↔ (Theaetetus, womb)
    - (T1, S3) ↔ (idea, baby)
    - (produce, contains) ↔ (Theaetetus, womb)
    - (produce, contains) ↔ (idea, baby)
    - (Theaetetus, womb) ↔ (idea, baby)

Table 3. Relations of consistency among some match hypotheses.

2.3 Finding the Analogical Mapping Relation

The task of finding the maximal coherent one-to-one map in a set of match hypotheses is a combinatorially complex task. Thagard and Holyoak (1989, 1990) showed that this task can be thought of as a *constraint satisfaction problem*. They identified three kinds of constraints on finding an analogical map: (1) structural constraints, (2) semantic constraints, and (3) pragmatic constraints.

Structural constraints favor the emergence of a maximally consistent map. The maximally consistent map between two domains is the largest possible isomorphism between them. Structural constraints encourage the analogical mapping relation to be the closest possible approximation to such an isomorphism. Semantic constraints favor matches that are supported by semantic similarity. The match hypotheses associated with propositions S and T are favored to the degree that the predicate of S is semantically similar to the predicate of T. Pragmatic constraints favor match hypotheses that contribute to the goal of the analogy.

Having shown how the problem of finding the analogical mappng relation can be formulated as a constraint satisfaction problem, Thagard and Holyoak then showed how the problem could be solved in parallel by a *constraint-satisfaction network* (a CS network). A CS network is a connectionist network. CS networks were first used by McClelland and Rumelhart (1981) to model visual word recognition. McClelland (1981) and McClelland and Rumelhart (1985) used CS networks for the storage and retrieval of general and specific information.

A CS network consists of nodes linked by weighted connections. In the case of analogical mapping, the *nodes* in the CS network are match hypotheses. The CS network is therefore called a *hypothesis network*. *Connections* between match hypotheses embody the relations of consistency and inconsistency between match hypotheses, thus capturing the structural constraints on the analogical mapping function. If two match hypotheses are consistent, there is an *excitatory connection* from each to the other; if two match hypotheses are inconsistent, there is an *inhibitory connection* from each to the other. Excitatory and inhibitory connections are given *weights* corresponding to the strength of the excitatory or inhibitory relations they represent. Excitatory connections have a weight proportional to the number of proposition pairs that induce those connections. For example, both (T2, S3) and (T3, S3) induce the excitatory connection (contains, contains) × (idea, baby). The weight of an excitatory connection is a constant +0.1 multiplied by the number of proposition pairs inducing the connection. Inhibitory connections have a weight of -0.2. Figure 1 shows the hypothesis network constructed from the domains in Table 1. In Figure 1, "+" indicates an excitatory connection, "-" an inhibitory connection. The network contains 20 nodes, 106 excitatory connections, and 64 inhibitory connections for a total of 170 connections.

Every node in a CS network is associated with a parameter called its *activation*. In the case of analogical mapping, the activation of a match hypothesis is its *degree of membership* in the analogical mapping relation. Semantic and pragmatic constraints are used in ACME to supply activations to the hypotheses in the network. Thus the semantic and pragmatic constraints used by ACME are posterior to the principle of proportional analogy, since activations are assigned to match hypotheses already constructed on the basis of the principle of proportional analogy alone. The semantic constraint is implemented by giving all predicate-predicate match hypotheses excitatory connections to a node called the Semantic Unit. If S is a source proposition and T a target proposition, the weight of the excitatory connection between (PredOf(S), PredOf(T)) and the semantic unit is proportional to the semantic similarity of PredOf(S) and PredOf(T).

The pragmatic constraint is implemented by giving each match hypotheses that is important for pragmatic reasons an excitatory connection to a node called the Pragmatic Unit that supplies it with activation. Connections to the Semantic and Pragmatic Units are not shown in Figure 1.

	(T1,S1)	(T1,S2)	(T1,S3)	(T2,S1)	(T2,S2)	(T2,S3)	(T3,S1)	(T3,S2)	(T3,S3)	(pro,pro)	(pro,con)	(con,pro)	(con,con)	(Th,mom)	(Th,womb)	(mind,mom)	(mind,womb)	(mind,baby)	(idea,womb)	(idea,baby)
(T1,S1)		-	-	-			-			+				+						+
(T1,S2)	-		-		-			-			+			+					+	
(T1,S3)	-	-				-			-		+				+					+
(T2,S1)	-				-	-	-					+		+				+		
(T2,S2)		-		-		-		-					+	+			+			
(T2,S3)			-	-	-				-				+		+					+
(T3,S1)	-			-				-	-			+				+				+
(T3,S2)		-			-		-		-				+			+			+	
(T3,S3)			-			-	-	-					+				+			+
(pro,pro)	+										-			+						+
(pro,con)		+	+							-			-	+	+				+	+
(con,pro)				+			+							+		+		+		+
(con,con)					+	+		+	+		-			+	+	+	+		+	+
(Th,mom)	+	+		+	+					+	+	+	+		-	-	+	+	+	+
(Th,womb)			+			+					+		+	-			-		-	+
(mind,mom)							+	+				+	+	-			-	-	+	+
(mind,womb)					+				+				+	+	-	-		-	-	+
(mind,baby)				+								+		+		-	-			-
(idea,womb)		+						+			+		+	+	-	+	-			-
(idea,baby)	+		+			+	+		+	+	+	+	+	+	+	+	+	-	-	

Figure 1. Network for domains in Table 1.

2.4 Running the Hypotheses Network

A CS network solves a constraint satisfaction problem by running through a number of *cycles*. On each cycle, each node interacts directly or indirectly through its connections with all the other nodes in the network. How a node X influences another node Y is a function of X's activation and its connection to Y. If X has an excitatory connection to Y, then the activation of X tends to *increase* the activation of Y. In other words, if X and Y are consistent match hypotheses, the degree of membership of X in the maximally consistent map tends to increase Y's degree of membership in that map. If X has an inhibitory connection to Y, then the activation of X tends to *decrease* the activation of Y. In other words, if X and Y are inconsistent match hypotheses, the degree of membership of X in the maximally consistent map tends to decrease Y's degree of membership in that map.

On each cycle, each node in the network computes a new activation. To see how this is done, let $a_j(t)$ be the activation of node j on cycle t. Let $out_i(t)$ be the output of node i on cycle t; $out_i(t)$ is $\max(a_i(t), 0)$. Let w_{ij} be the weight of the connection from unit i to unit j; note that $w_{ij} > 0$ if the connection is excitatory, $w_{ij} < 0$ if the connection is inhibitory. The net excitatory input to a node j is $enet_j = \Sigma_i w_{ij} out_i(t)$ for $w_{ij} > 0$ and the net inhibitory input to a node j is is $inet_j = \Sigma_i w_{ij} out_i(t)$ for $w_{ij} < 0$. The new activation $a_j(t+1)$ of node j is based on the rule given in Formula 1.

$$[1]\ a_j(t+1) = a_j(t)(1-d) + enet_j(max-a_j(t)) + inet_j(a_j(t) - min)$$

where d is is the rate at which activation decays, max is the maximum activation of a node, min is the minimum activation of a node. In ACME, $decay = 0.1$, $max = 1.0$, $min = -1.0$.

The nodes in an CS network interact with one another, cycle after cycle, until their activations stabilize. This is called running the network to convergence. The network is run to convergence by updating the activations of all nodes until the change in the activation of each node from cycle to cycle is less than some specified constant. Once the network has converged, activations of match hypotheses can be used to rank-order them. Among competing match hypotheses, the one with the greatest activation is the best match and is referred to as the winner. A one-to-one map can be constructed by accepting for each source term only the winning match hypothesis for that term.

3. Difficulties with Proportional Analogy

3.1 Sensitivity to Conceptual Structure in Analogical Mapping

Both SME and ACME utilize a single principle, the principle of proportional analogy, to generate match hypotheses and their relations. If a single principle is used for generating match hypotheses, that principle is applied universally to all concepts in the source and target domains, regardless of their structural positions in those domains. Consequently, if a single principle is used for generating match hypotheses, the fine conceptual structure of the source and target domains cannot be taken into consideration.

A pragmatic analysis argues against the universal application of a single principle for generating match hypotheses and their interconnections. The purpose of analogical mapping is to discover the most coherent map between two existing domains. A process that ignores the fine conceptual structures of the two domains discourages that goal by creating match hypotheses and connections that can only frustrate the emergence of a coherent global map. For instance, in domains structured by mereological relations, the brute force application of the principle of proportional analogy produces match hypotheses that both violate and respect the mereological structures of the domains. Match hypotheses that violate the mereological structure of the domains compete with match hypotheses that respect those structures; such competition degrades the coherence of all the match hypotheses that respect the conceptual structures of the domains. The result is larger constraint-satisfaction networks that converge to less coherent maps.

In the case of the THEAETETUS IS A MOTHER analogy, ACME applies the principle of proportional analogy to the propositions whose predicate is "contains". Such application creates the seven match hypotheses: (mother, Theaetetus), (womb, Theaetetus), (womb, mind), (baby, mind), (mother, mind), (womb, idea). But this proliferation is absurd. Attending to the mereological relations that hold between Theaetetus, the mind in Theaetetus, and the idea in the mind in Theaetetus, and also to those that hold between the mother, the womb in the mother, and the baby in the womb in the mother, it is apparent that "contains" should only induce the match hypotheses: (mother, Theaetetus), (womb, mind), and (baby, idea). Consequently, ACME generates 4 invalid match hypotheses; these invalid match hypotheses are involved in 24 invalid excitatory connections.

Application of the principle of proportional analogy to propositions whose predicate is "contains" creates many invalid match hypotheses and connections. This is because "contains" creates a hierarchical structure in each domain; matching concepts at different levels in these mereological hierarchies violates the mereological order of the domains. Hence such match hypotheses are mereologically invalid. Figure 2 shows both mereologically valid and invalid match hypotheses. In Figure 2, the arrow "∅" denotes the relation "contains". Note that ACME generates both mereologically valid and invalid match hypotheses. The mereological relation "contains" is not the only relation that does not induce analogies according to the principle of proportional analogy. Other pervasive and transitive relations, such as spatial, temporal, and causal relations, fail to induce proportional analogies and instead induce analogies according to other principles. In what follows, only the mereological relation "contains" is analyzed; similar analyses can be performed for spatial, temporal, and causal relations.

We do not believe that the mind generates match hypotheses and their interconnections by the brute force application of a single principle. We believe that the mind is sensitive to the fine conceptual structure of the source and target domains. The structure of the knowledge in the source and target domains guides the production of match hypotheses and their interconnections. We believe that the mind generates match hypotheses by tracing out parallel patterns through the structure of the source and target domains. For each move that is made in the source, a parallel move is made in the target. Such moves should not be determined by a single rule. Different rules are more appropriate for moving along the different relations that structure the knowledge in the two domains. For example, the rule of proportional analogy is acceptable for generating match hypotheses from propositions. However, several other rules are used to generate match hypotheses from mereological relations.

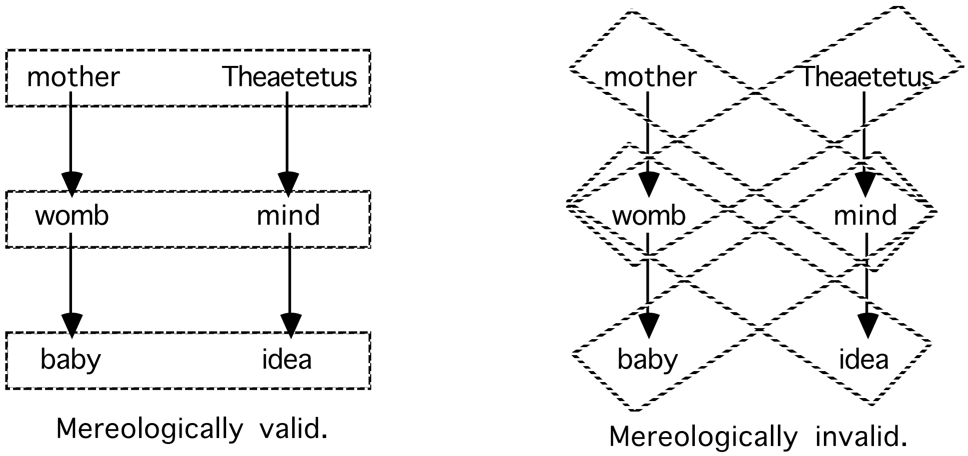

Figure 2. Mereologically valid and invalid match hypotheses.

3.2 Pragmatics and Sensitivity to Conceptual Structure

An approach to analogical mapping that follows the fine conceptual structure of the domains by using several rules is more open to pragmatic factors than an approach that universally applies a single rule. Pragmatic factors can influence constraint-satisfaction models of analogical mapping in two ways: (1) by encouraging or discouraging individual match hypotheses; (2) by guiding the construction of the hypothesis network itself.

In constraint-satisfaction approaches to analogical mapping that use only one rule for creating their match hypotheses and interconnections, pragmatic factors can encourage or discourage individual match hypotheses, but cannot guide the construction of the hypothesis network. No guidance is possible, since the application of the rule to the source and target domains is purely combinatorial. For example, in ACME pragmatic factors influence individual match hypotheses through connections to the pragmatic unit; however, pragmatic factors do not guide the construction of the hypothesis network.

In constraint-satisfaction models of mapping that use more than one rule for creating match hypotheses and interconnections, pragmatic factors can both encourage or discourage individual match hypotheses and can also guide the construction of the hypothesis network. We believe that match hypotheses and their interconnections are created by making parallel moves through the source and target systems. Structured systems of concepts can be traversed in many ways. At each point in such a system, it is possible to branch in several directions via different conceptual relations. Pragmatic factors determine which moves are made (i.e. which rules are executed and which relations are followed). The context and purpose of analogical cognition determine which rule is applied at each point. Pragmatic factors also determine how far moves are made in the domains. For example, given a match hypothesis (s,t), pragmatic factors determine whether or not we explore the mereological relations in which s and t participate. If it is decided to explore these relations, pragmatic factors determine how far they are explored.

4. A Structural Model of Analogical Mapping

4.1 Representing Propositional Knowledge in Semantic Networks

While SME and ACME represent propositions as predicate calculus expressions, we represent propositions as semantic frames. A *semantic frame* consists of a set of interconnected terms. We use semantic field theory to provide principles for the organization of terms into semantic frames. According to semantic field theory (de Saussure, 1966; Lyons, 1977; Cruse, 1986; Kittay, 1987), terms are organized by sense relations. If two terms are brought together by a sense relation, they are linked in a semantic frame by a connection that is labelled with that sense relation.

Sense relations determine semantic frames. We classify semantic frames according to the sense relations that structure them. Sense relations are divided into paradigmatic and syntagmatic relations. *Paradigmatic* relations involve terms from the same conceptual category. Here we allow conceptual categories to be grammatical categories, such as nouns, verbs, adjectives, and adverbs. Paradigmatic relations are divided into contrastive and affinitive relations. *Contrastive* relations include binary and n-ary oppositions among terms. Here we consider only binary opposition. A *contrastive frame* consists of two terms with approximately opposite senses symmetrically linked by connections labelled ANTONYM. *Affinitive* relations are generally divided into synonymic relations, taxonomic relations and mereological relations. A *synonymic frame* consists of two terms with approximately the same sense symmetrically linked by connections labelled SYNONYM. *Taxonomic* relations are relations between a term and its subordinates (for example, "dog" is a subordinate of "animal"; "walks" is a subordinate of "moves"). A *taxonomic frame* consists of a term and a subordinate term linked both by a connection labelled SPECIES directed from the term to the subordinate and a connection labelled GENUS directed from the subordinate to the term. *Mereological* relations are relations between a term and its parts. A mereological frame consists of a term and a part linked both by a connection labelled CONTAINS directed from the term to its part and by a connection labelled WHOLE directed from the part to the term. *Syntagmatic* relations group terms from different conceptual categories. Syntagmatic relations include relations between a proposition and its predicate (either verb or adjective) as well as relations between that proposition and its arguments (nouns). Such relations are often called thematic relations or *thematic roles*. In case the predicate of a proposition is a verb, the *syntagmatic frame* representation of that proposition contains the name of the proposition, its verb predicate, and its noun arguments. Connections labelled VERB symmetrically link the verb predicate to the proposition name. Connections labelled with thematic roles, such as AGENT, PATIENT, and INSTRUMENT, among others, symmetrically link each noun argument to the proposition name. In case the predicate of a proposition is an adjective, the syntagmatic frame representation of that proposition consists of a connection labelled FEATURE that links the adjective predicate to the noun of which it is predicated. Semantic frames form complex networks in long-memory. Such networks are called *semantic networks*. Figure 3 shows the knowledge from Table 1 encoded in a semantic network. Connections are labelled with thematic roles; an arrow X∅Y means "X contains Y".

We hold that different principles of analogical mapping are used for different sense relations, hence for different kinds of semantic frames. NETMET incorporates principles for all the types of semantic frames discussed above. In what follows, however, we outline principles of analogical mapping only for syntagmatic and mereological frames. First, we describe two analogies for the illustration of semantic networks and the principles of analogical mapping. These analogies are the ATOM IS A SOLAR-SYSTEM analogy and the COULOMB'S LAW IS NEWTON'S LAW. The mereological relation "contains" occurs pervasively in these analogies. Second, we describe the principles of analogical mapping themselves. Third, we discuss the hypothesis networks built by NETMET.

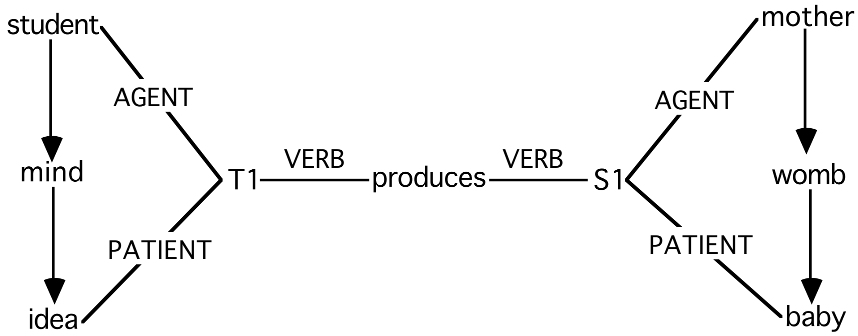

Figure 3. Semantic network version of knowledge in Table 1.

4.2 The ATOM IS A SOLAR-SYSTEM Analogy

We use the analogy between the ATOM and the SOLAR-SYSTEM as an example of a complex analogy for illustrating the structural model of analogical mapping. Both the ATOM and the SOLAR-SYSTEM are structured wholes.[4] The propositional knowledge in these domains is represented as a list of predicate calculus expressions in Table 4.

Source (SOLAR-SYSTEM)	Target (ATOM)
contains(solar-system, sun)	contains(atom, nucleus)
contains(solar-system,asteroid-belt)	contains(atom, electron-cloud)
contains(solar-system, planetary-system)	contains(electron-cloud, electron-shell)
contains(asteroid-belt, asteroid)	contains(electron-shell, electron)
contains(planetary-system, planet)	orbits(electron, nucleus)
contains(planetary-system, moon)	surrounds(electron-cloud, nucleus)
contains(planetary-system, ring)	
contains(ring, subring)	
contains(subring, debris)	
orbits(asteroid, sun)	
orbits(planetary-system, sun)	
orbits(moon, planet)	
orbits(debris, planet)	
surrounds(asteroid-belt, sun)	
surrounds(ring, planet)	

Table 4. The SOLAR-SYSTEM and the ATOM.

[4]Note that very different analogies involving the ATOM arise if the ATOM is thought of, not as a structured whole, but as unities involving attraction, repulsion, and union. In such analogies, the atom is personified. For instance, flourine is hungry for electrons, or fluorine lusts after electrons, or fluorine is greedy for electrons. The "noble" gasses are not reactive; they don't share their electrons, they don't lust after the electrons of other atoms, they're self-sufficient. Mr. Oxygen lusts after Miss Hydrogen's electrons, and grab's them; when they get married, they form water.

A semantic network representation of the source domain SOLAR-SYSTEM and target domain ATOM is shown in Figure 4. An arrow X→Y means "X contains Y". Proposition names have the form "Fi", where i is some number. Due to lack of space, connections are not labelled; however, the thematic roles can be easily inferred (e.g. an asteroid orbits the sun, so the asteroid is the AGENT and the sun is the PATIENT).

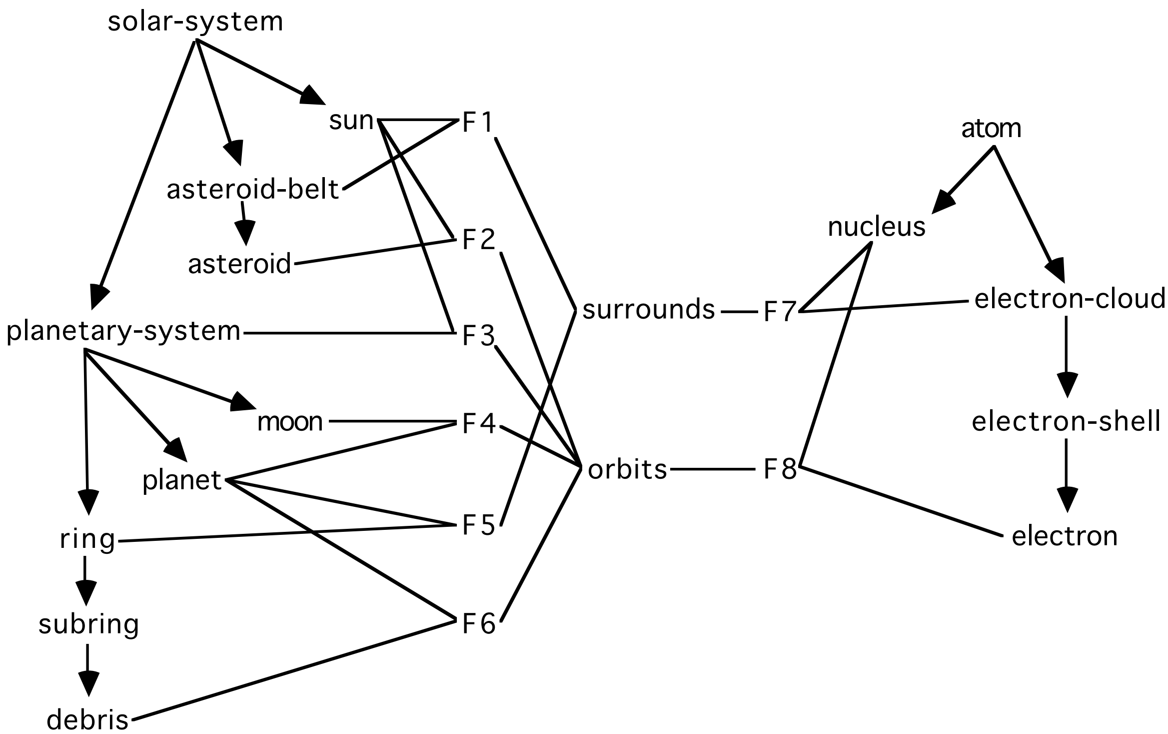

Figure 4. Semantic net representation of SOLAR-SYSTEM and ATOM.

4.3 The COULOMB'S LAW IS NEWTON'S LAW Analogy

Mathematical formulae are often analogous. Newton's law of gravitational attraction is analogous to Coulomb's law of electrostatic attraction (Halliday & Resnick,1974: 424-5). Another example of such a mathematical analogy is the analogy between a mass-spring system performing simple harmonic motion and an oscillating LC circuit. (Halliday & Resnick,1974: 628-9). We use the analogy between Coulomb's law and Newton's law as another example to illustrate the structural model of analogical mapping. Newton's law is shown in Formula 2.

[2] Newton's law: A = G*((m1*m2)/pow(d,2))

where A is the attraction between masses m1 and m2, d is the distance between the masses, G is a constant, and pow(x,y) is x raised to the y-th power. Coulomb's law is shown in Formula 3.

[3] Coulomb's law: F = k*((q1*q2)/pow(r,2))

where F is the attraction between charges q1 and q2, r is the distance between the charges, and k is a constant.

Each of these equations is a structured whole in which one mathematical expression contains another. The containment relations are given by the parentheses (except for functions, such as pow()). Formulae 4 and 5 show these functions written in LISP notation. LISP notation reveals the structure of the analogy between the equations more clearly.

```
[4] (= A (* G (/ (* m1 m2) (pow d 2))))
[5] (= F (* k (/ (* q1 q2) (pow r 2))))
```

If Newton's law is taken as the target domain and Coulomb's law is the source domain, these domains can be written as the lists of predicate calculus expressions shown in Table 5. Notice that most of the information in the domains is mereological. The semantic network representations of the two equations are shown in Figure 5.

Source (COULOMB'S LAW)	Target (NEWTON'S LAW)
contains(E1, F)	contains(Z1, A)
contains(E1, E2)	contains(Z1, Z2)
contains(E2, k)	contains(Z2, G)
contains(E2, E3)	contains(Z2, Z3)
contains(E3, E4)	contains(Z3, Z4)
contains(E3, E5)	contains(Z3, Z5)
contains(E4, r)	contains(Z4, d)
contains(E4, 2)	contains(Z4, 2)
contains(E5, q1)	contains(Z5, m1)
contains(E5, q2)	contains(Z5, m2)
equals(F, E2)	equals(A, Z2)
multiplication(k, E3)	multiplication(G, Z3)
division(E5, E4)	division(Z5, Z4)
pow(r, 2)	pow(d, 2)
multiplication(q1, q2)	multiplication(m1, m2)

Table 5. Coulomb's and Newton's laws.

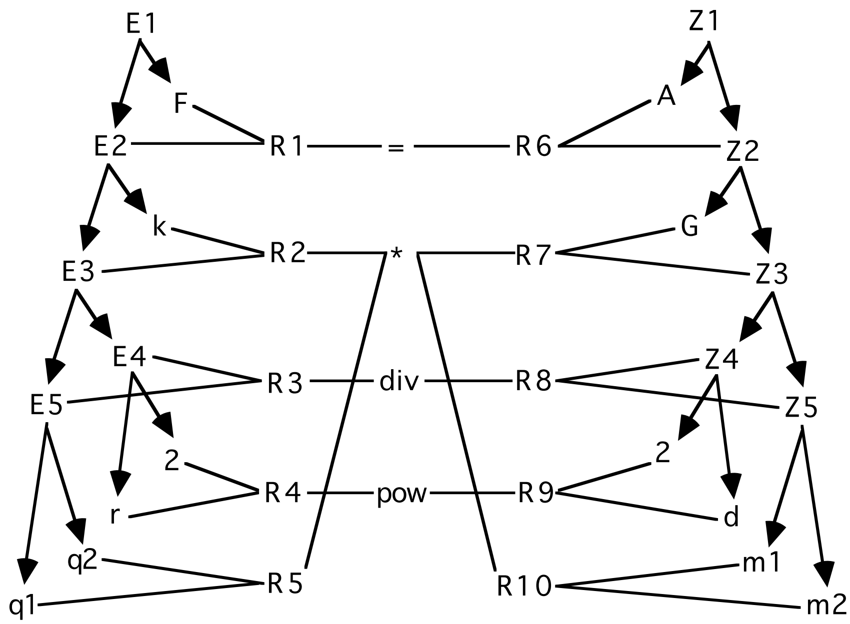

Figure 5. Semantic net representation of Coulomb's and Newton's laws.

4.4 Principles for Generating a Hypothesis Network

While SME and ACME apply one principle of analogical mapping to every predicate calculus expression, NETMET applies a different principle to each different kind of semantic frame. Here we discuss only the principles for syntagmatic and mereological frames. These are: (1) the principle of proportional analogy, (2) the principle of mereological analogy, (3) the principle of chain reinforcement, and (4) the principle of transitive reinforcement. The four principles for generating match hypotheses listed above generate relations of consistency between matches. To generate relations of inconsistency between matches, we use a fifth principle. This principle is (5) *the principle of mutual inconsistency*. Each of these principles is discussed below.

4.4.1. *The principle of proportional analogy*. If SF:P(S_1, S_2, . . . S_n) is a syntagmatic frame in the source and TF:Q(T_1, T_2, . . . T_n) is a syntagmatic frame in the target, and if the similarity SIM(P,Q) of source predicate P to target predicate Q exceeds some threshold, then (SF, TF) is a match and (S_i, T_i) are matches for i = 1, 2, . . . n. The match hypotheses {(SF, TF), (S_1, T_1), . . . (S_n, T_n)} are all consistent. We stipulate that SIM(P,Q) is 0 for no similarity and 1 for identity. The threshold is determined by pragmatic considerations. We use the pair (P,Q) to form the semantic input table, discussed below. Note we do *not* treat (P,Q) as a match hypothesis. The matches induced by the principle of proportional analogy are called *proportional matches*. We speak of the pair of predicates (P,Q) as inducing the proportional matches. If P = Q, we simply speak of the predicate P as inducing the proportional matches.

For example, the predicate "orbits" in the propositions "SF:orbits(debris, planet)" and "TF:orbits(electron, nucleus)" induces the proportional matches {(SF, TF), (debris, electron), (planet, nucleus)}; all these matches are consistent with one another. These proportional matches, and their generation in the semantic network, are illustrated in Figure 6.

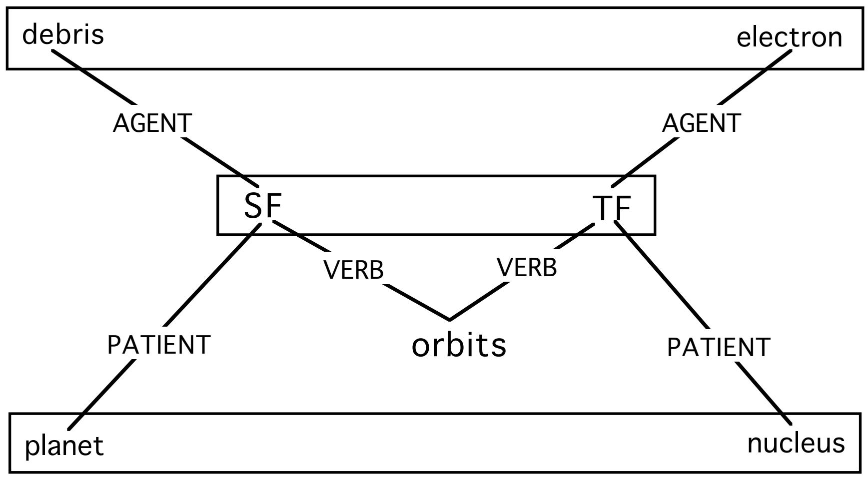

Figure 6. Proportional matches induced by "orbits".

4.4.2. *The principle of mereological analogy*. In a mereological frame, if X contains Y, then X is the WholeOf(Y). If (A,B) is a match and if both WholeOf(A) and WholeOf(B) are non-nil, then (WholeOf(A), WholeOf(B)) is a match, and (A,B) and (WholeOf(A), WholeOf(B)) are consistent. Matches generated by application of this principle are called *mereological matches*. This principle can be applied recursively to generate a *containment chain*. A containment chain is a list of matches: ((A, B), (WholeOf(A), WholeOf(B)), (WholeOf(WholeOf(A)), WholeOf(WholeOf(B))). All the matches in a containment chain are consistent with one another.

The principle of mereological analogy dictates that mereological matches are generated in a bottom-up fashion in a mereological hierarchy. Matches at the bottom level are typically not mereological, but are induced by some other principle, such as the principle of proportional analogy. The classical analogy between an animal and a watch supports the view that proportional matches induce mereological matches. This analogy was proposed by Hobbes (1651/1962) and later by de la Mettrie (1748/1912). Here is Hobbes's statement of the analogy between an animal and a watch:

> For seeing life is but a motion of limbs, the beginning whereof is some principal part within; why may we not say, that all automata (engines that move themselves by springs and wheels as doth a watch) have an artificial life? For what is the heart but a spring; and the nerves, but so many strings; and the joints, but so many wheels, giving motion to the whole body, such as was intended by the artificier? (Hobbes, 1651, Introduction).

Here it is apparent that the animal and watch are analogous because their parts are analogous. That is, because proportional analogies hold among the parts, there is a mereological analogy among the wholes containing those parts.

Figure 7 illustrates how mereological matches are generated from a proportional match by making parallel moves up the mereological hierarchies in the source and target. Pragmatic factors determine whether and how far such moves are made. In Figure 7, as in all figures involving matches, an arrow X∅Y indicates the *inverse* of "contains" and means "X is contained by Y". In Figure 7, as in all figures involving matches, a box drawn with a solid line indicates a match induced by the principle of proportional analogy; a match drawn with a broken line indicates a match induced by some other principle. The containment chain generated in Figure 7 is ((debris, electron), (subring, electron-shell), (ring, electron-cloud), (planetary-system, atom)).

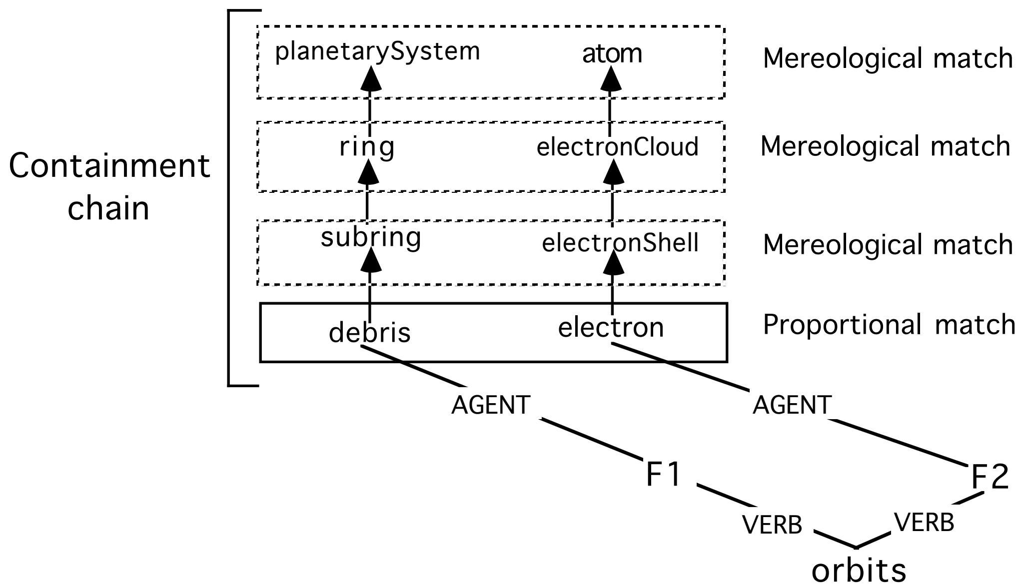

Figure 7. Mereological matches induced by a proportional match.

4.4.3. *The principle of chain reinforcement*. If two containment chains generated by the same pair of predicates (or verb) have identical last members, then these two containment chains converge on the same wholes, and the chains themselves are *mereologically consistent*. All the matches in both chains are consistent.

To see how chain reinforcement works, consider the enlarged versions of the SOLAR-SYSTEM and ATOM domains shown in Table 6. In the domains in Table 6, the verb "orbits" induces the following proportional matches: (debris, electron) and (planet, nucleus). These matches represent the proportional analogy that debris is to planet as electron is to nucleus. The containment chain generated by (debris, electron) is ((debris, electron), (subring, electron-shell), (ring, electron-cloud), (planetary-system, atom)). The containment chain generated by (planet, nucleus) is ((planet, nucleus), (planetary-system, atom)). These containment chains are shown in Figure 8.

Source Domain	Target Domain
contains(planetary-system, planet)	contains(atom, nucleus)
contains(planetary-system, ring)	contains(atom, electron-cloud)
contains(ring, subring)	contains(electron-cloud, electron-shell)
contains(subring, debris)	contains(electron-shell, electron)
orbits(debris, planet)	orbits(electron, nucleus)

Table 6. Sample source and target domains.

Since both containment chains converge on the same matches, the chains are mereologically consistent and the all matches in each are consistent with one another. This means that the union of the matches in both containment chains is a set whose members are all consistent.

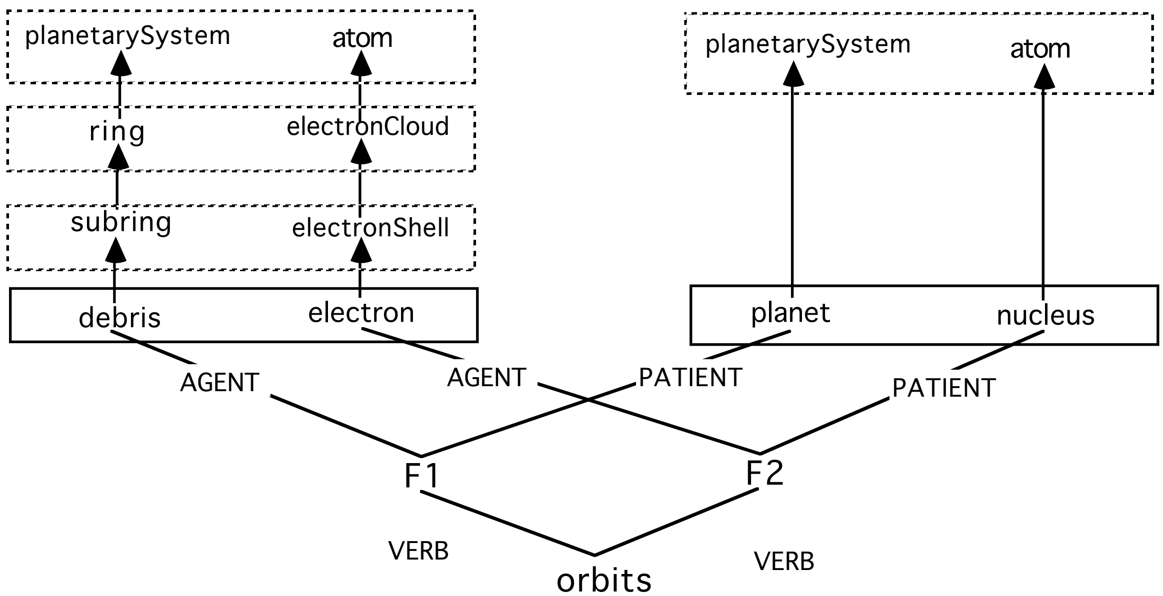

Figure 8. Mereologically *consistent* containment chains.

The validity of the chain reinforcement principle can be seen by considering what happens when two containment chains generated by the same verb *fail* to converge. The failure of two chains to converge is a *mereological inconsistency*. To see the failure of convergence, consider the versions of the SOLAR-SYSTEM and ATOM domains shown in Table 7.

Source Domain	Target Domain
contains(planetary-system, planet)	contains(atom, nucleus)
contains(planetary-system, moon)	
contains(planetary-system, ring)	contains(atom, electron-cloud)
contains(ring, subring)	contains(electron-cloud, electron-shell)
contains(subring, debris)	contains(electron-shell, electron)
orbits(moon, planet)	orbits(electron, nucleus)

Table 7. Sample source and target domains.

The verb "orbits" induces the matches (moon, electron) and (planet, nucleus). The match (moon, electron) generates the containment chain ((moon, electron), (planetary-system, electron-shell)). The match (planet, nucleus) generates the containment chain ((planet, nucleus), (planetary-system, atom)). Comparing the last members of these chains, it is easy to see that the analogy "moon is to planet as electron is to nucleus" is *not* mereologically consistent, since it implies two *inconsistent* matches for "planetary-system".[5] Figure 9 shows these inconsistent containment chains.

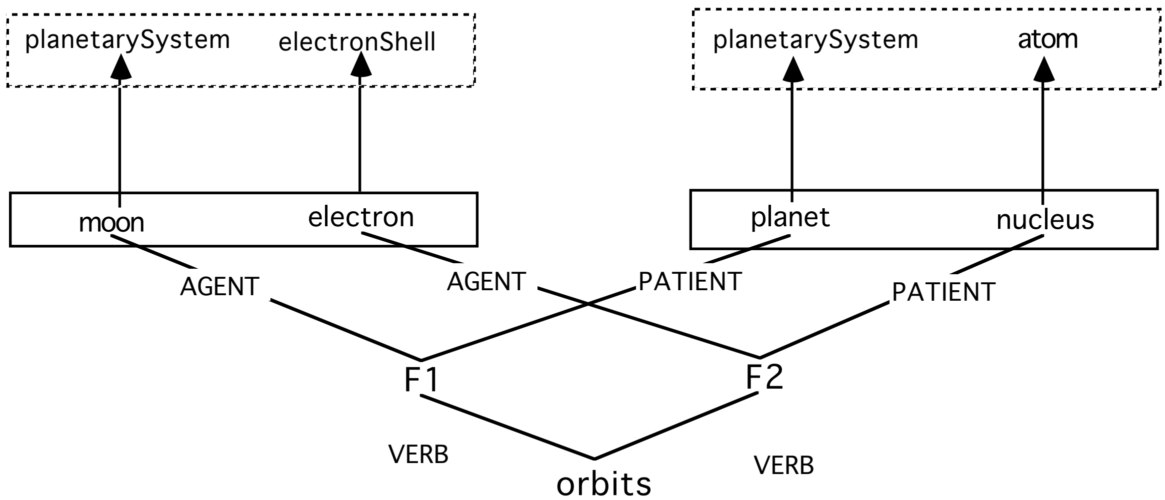

Figure 9. Mereologically *inconsistent* containment chains.

4.4.4. *The principle of transitive reinforcement*. If (x,y) and (a,b) are proportional matches, and if x directly or indirectly contains a and y directly or indirectly contains b, then (x,y) and (a,b) are consistent. Notice that this principle enables a proportional match to supersede the mereological orderings of the domains. To see how the principle of transitive reinforcement works, consider the analogies indicated in Figure 10.

[5]The failure of structural consistency leads to the idea that if the ends of two containment chains induced by the same semantic relation are not equal, the resulting structural inconsistency should be represented by making the elements of these chains mutually inconsistent, and by weakening the support of the match hypotheses originally induced by the pair of predicates. Here again, pragmatic factors determine whether or not this is to be done.

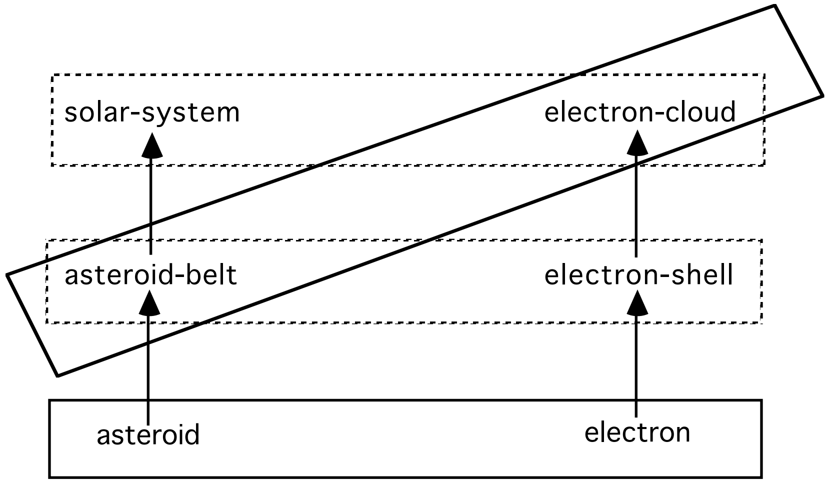

Figure 10. Some matches.

Figure 10 contains the proportional matches (asteroid, electron) and (asteroid-belt, electron-cloud); it contains the mereological matches (asteroid-belt, electron-shell) and (solar-system, electron-cloud). According to the principle of mereological analogy, (asteroid, electron), (asteroid-belt, electron-cloud), and (solar-system, electron-cloud) are all consistent. But, due to the transitivity of CONTAINS, "electron-cloud" contains "electron", so (asteroid-belt, electron-cloud) and (asteroid, electron) should be consistent.

4.4.5. *The principle of mutual inconsistency*. A match is an ordered-pair. We denote the first member of a match X as FIRST(X) and the second member of a match X as SECOND(X). Two matches X and Y (i.e. two ordered pairs) are mutually inconsistent if FIRST(X) is equal to FIRST(Y) but SECOND(X) is not equal to SECOND(Y), or if SECOND(X) is equal to SECOND(Y) but FIRST(X) is not equal to FIRST(Y). For instance, (solar-system, electron-cloud) is mutually inconsistent with (solar-system, atom) and is also mutually inconsistent with (asteroid-belt, electron-cloud). The rule of mutual inconsistency is applied to all the pairs of matches generated by the other rules.

4.5 Hypotheses Networks Constructed by NETMET

Hypotheses networks constructed by NETMET are very similar to those generated by ACME. Like ACME, NETMET encodes relations of consistency in excitatory connections and relations of inconsistency in inhibitory connections. Like ACME, NETMET allows semantic and pragmatic factors to supply activations to match hypotheses. However, NETMET does not supply activations to match hypotheses in the same way that ACME does. ACME supplies such activations indirectly, through the Semantic and Pragmatic units. NETMET supplies activations directly to the match hypotheses themselves. Activation supplied directly to the match hypotheses is called external activation, and is either pragmatic or semantic. Pragmatic activation supplied to a match hypothesis is proportional to the importance of that match. We now discuss how NETMET supplies external semantic activations to its match hypotheses.

Since the match hypotheses generated by the principle of proportional analogy are the basis for all other analogical match hypotheses, proportional matches receive external semantic activation. Each pair of predicates (P,Q) that induces a proportional match supplies that proportional match with external activation equal to the quantity SIM(P,Q). Minimally, SIM(P,Q) is the threshold for inducing proporitional matches. Maximally, in case P and Q are identical, SIM(P,Q) is 1. If a pair of predicates (P,Q) induces the proportional matches (A,C) and (B,D), then (P,Q) gives external activation equal to SIM(P,Q) to both of the match hypotheses (A,C) and (B,D). Since (P,Q) may induce many analogies, (P,Q) may supply external activation to many match hypotheses. Since (P,Q) may induce many analogies in which the same match hypothesis occurs, (P,Q) may supply external activation equal to some integral multiple of SIM(P,Q) to a single match hypothesis.

To see how pairs of predicates supply external activation to match hypotheses, consider the verbs "orbits" and "surrounds" in the target domain of the ATOM and the source domain of the SOLAR-SYSTEM, as shown in Table 8. To save space in the discussion, matches of proposition names are not shown in Table 8.

Source Domain	Target Domain
orbits(asteroid, sun)	orbits(electron, nucleus)
orbits(planetary-system, sun)	
orbits(moon, planet)	
orbits(debris, planet)	
surrounds(asteroid-belt, sun)	surrounds(electron-cloud, atom)
surrounds(ring, planet)	

Table 8. Sample source and target domains.

The occurence of "orbits(planetary-system, sun)" and "orbits(asteroid, sun)" means that "orbits" induces (sun, nucleus) twice. Likewise, "orbits" induces (planet, nucleus) twice. In each of these cases, "orbits" supplies these two match hypotheses with two units of external activation, since SIM(orbits, orbits) is 1. Similar remarks apply to the match hypotheses induced by "surrounds". The quantity of external activation supplied by each of these pairs of predicates is shown in Table 9.

Match Hypothesis	"orbits"	"surrounds"	Total
(asteroid,electron)	1	0	1
(sun,nucleus)	2	1	3
(planetary-system,electron)	1	0	1
(debris,electron)	1	0	1
(planet,nucleus)	2	1	3
(moon,electron)	1	0	1
(asteroid-belt,electron-cloud)	0	1	1
(ring,electron-cloud)	0	1	1

Table 9. Units of external activation for match hypotheses.

After the external activations have been assigned to all match hypotheses, they are normalized so that they vary between 0 and 1. The normalized external activation assigned to a match hypothesis is the total quantity of external activation for that match hypothesis divided by the maximum number of times that pairs of predicates that induce matches, where this maximum is taken over the whole set of match hypotheses. In the example, the maximum number of times that pairs of predicates induce match hypotheses is 3. The normalized external activations are shown in Table 10. This table is called the *semantic input table*. If a match hypothesis is not in the semantic input table, its external activation is set to zero.

Semantic Input Table

Match Hypothesis	External Activation
(asteroid, electron)	1/3
(sun, nucleus)	1
(planetary-system, electron)	1/3
(debris, electron)	1/3
(planet, nucleus)	1
(moon, electron)	1/3
(asteroid-belt, electron-cloud)	1/3
(ring, electron-cloud)	1/3

Table 10. The semantic input table.

Like ACME, NETMET runs the hypothesis network as a constraint-satisfaction network. However, NETMET uses updating rules proposed by McClelland & Rumelhart (1986) rather than those proposed by Grossberg.[6] On each cycle, each node in the network computes its net input in accordance with the rule in Formula 6.

$$[6]\ net_i = \Sigma_j w_{ij}\, output_j + extinput_i$$

where net_i is the net input to unit i, w_{ij} is the weight of the connection from unit j to unit i, $output_j$ is the output of unit j, and $extinput_i$ is the external input to unit i. Once the net input has been computed for every unit in the network, the change in the activation of each unit is given by the rules in Formulae 7 and 8,

$$[7]\ \Delta a_i = (max - a_i)net_i - decay(a_i - rest) \text{ if } net_i > 0$$

$$[8]\ \Delta a_i = (a_i - min)net_i - decay(a_i - rest) \text{ if } net_i <= 0$$

where a_i is the activation of unit i, *decay* is a parameter indicating the rate at which activation decays, *max* is the maximum activation of a unit, *min* is the minimum activation of a unit, and *rest* is the resting activation of a unit. For NETMET, *decay* = 0.1, *max* = 1.0, *min* = -0.2, and *rest* = -0.1.

[6]ACME also can use rules like those of McClelland & Rumelhart. Except for some complex cases (Holyoak & Thagard, 1989, p. 315), there is little difference between these updating rules and those of Grossberg.

5. Comparison of NETMET and ACME

In order to compare NETMET and ACME, we applied each program to the THEAETETUS IS A MOTHER analogy, the ATOM IS A SOLAR-SYSTEM analogy, and to the COULOMB'S LAW IS NEWTON'S LAW analogy. In each case, we compare the hypothesis networks constructed by NETMET to those constructed by ACME. Since NETMET does not use a semantic unit as ACME does, we do not consider the connections between the semantic unit and other units in ACME's hypothesis networks. Since connections between the pratmatic unit and other units are the same for NETMET and ACME, we do not consider those connections in our comparisons.

5.1 Performance on the THEAETETUS IS A MOTHER Analogy

NETMET was applied to the source domain of the MOTHER and target domain of the STUDENT from Table 1. The resulting hypothesis network is shown in Figure 11. As before, a plus sign "+" indicates an excitatory connection and a minus sign "-" indicates an inhibitory connection. The network constructed by NETMET contains 4 match hypotheses, 10 excitatory connections, and 0 inhibitory connections for a total of 10 connections. The network converged after 35 cycles. By comparison, the network constructed by ACME contains 20 match hypotheses, 106 excitatory connections, and 64 inhibitory connections for a total of 170 connections. The NETMET network in Figure 11 should be compared with the ACME network in Figure 1.

	(T1, S1)	(Th, mom)	(idea,baby)	(mind,womb)
(T1, S1)		+	+	
(Th, mom)	+		+	+
(idea,baby)	+	+		+
(mind,womb)		+	+	

Figure 11. NETMET network for THEAETETUS IS A MOTHER.

5.2 Performance on the ATOM IS A SOLAR-SYSTEM Analogy

NETMET was applied to the source domain of the SOLAR-SYSTEM and target domain of the ATOM from Table 4. The resulting hypothesis network is shown in Figure 12. In Figure 12, a plus sign "+" indicates an excitatory connection and a minus sign "-" indicates an inhibitory connection. The network constructed by NETMET contains 21 match hypotheses, 72 excitatory connections, and 62 inhibitory connections for a total of 134 connections. By comparison, the network constructed by ACME contains 153 match hypotheses, 970 excitatory connections, and 2428 inhibitory connections for a total of 3398 connections.

	(F1, F7)	(F5, F7)	(F2, F8)	(F3, F8)	(F4, F8)	(F6, F8)	(moon,electron)	(planet,nucleus)	(plansys,eshell)	(solsys,ecloud)	(plansys,atom)	(aster,elect)	(sun,nucleus)	(astbelt,eshell)	(solsys,atom)	(plansys,elect)	(solsys,eshell)	(debris,elect)	(subring,eshell)	(ring,ecloud)	(astbelt,ecloud)
(F1, F7)		-											+								+
(F5, F7)	-							+												+	
(F2, F8)				-	-	-						+	+								
(F3, F8)			-		-	-							+			+					
(F4, F8)			-	-		-	+	+													
(F6, F8)			-	-	-			+										+			
(moon,electron)					+			+	+	+		-				-		-			
(planet,nucleus)		+			+	+	+				+		-					+	+	+	
(plansys,eshell)							+			+	-			-		-	-		-		
(solsys,ecloud)							+		+			+		+	-		-			-	-
(plansys,atom)								+	-						-	-		+	+	+	
(aster,elect)			+				-			+			+	+		-		-			+
(sun,nucleus)	+		+	+				-				+			+	+					+
(astbelt,eshell)									-	+		+					-		-		-
(solsys,atom)										-	-		+				-				+
(plansys,elect)				+			-		-		-	-	+				+	-			
(solsys,eshell)									-	-				-	-	+			-		
(debris,elect)						+	-	+			+	-				-			+	+	
(subring,eshell)								+	-		+			-			-	+		+	
(ring,ecloud)		+						+		-	+							+	+		-
(astbelt,ecloud)	+									-		+	+	-	+					-	

Figure 12. NETMET network for the ATOM IS A SOLAR-SYSTEM.

In the NETMET simulation, match hypotheses received external activations shown in the semantic input table in Table 10. The network converged after 77 cycles. The activations of the match hypotheses on convergence are shown in Table 11. The winning match hypothesis for each member of the Target is shown in boldface.

Match Hypotheses	Activations
(moon, electron)	0.52
(planet, nucleus)	**0.93**
(planetary-system, electron-shell)	-0.17
(solar-system, electron-cloud)	-0.17
(planetary-system, atom)	**0.84**
(asteroid, electron)	0.75
(sun, nucleus)	0.90
(asteroid-belt, electron-shell)	-0.17
(solar-system, atom)	0.61
(planetary-system, electron)	-0.16
(solar-system, electron-shell)	-0.17
(debris, electron)	**0.86**
(subring, electron-shell)	**0.86**
(ring, electron-cloud)	**0.88**
(asteroid-belt, electron-cloud)	0.83

Table 11. Map for the ATOM IS A SOLARSYSTEM analogy.

5.3 Performance on the COULOMB'S LAW IS NEWTON'S LAW Analogy

NETMET was applied to the COULOMB'S LAW IS NEWTON'S LAW analogy from Table 5. The network constructed by NETMET contains 26 match hypotheses, 126 excitatory connections, and 44 inhibitory connections. By comparison, the network constructed by ACME contains 367 match hypotheses, 2418 excitatory connections, and 8764 inhibitory connections for a total of 11182 connections. The NETMET network converged after 20 cycles. The activations of the match hypotheses on convergence are shown in Table 12. The winning match hypothesis for each member of the target is shown in boldface.

Match Hypotheses	Activations
(q1, G)	0.88
(q2, Z3)	0.88
(E5, Z2)	-0.10
(E3, Z1)	-0.16
(K, G)	**0.91**
(E3, Z3	**0.95**
(E2, Z2)	**0.97**
(E1, Z1)	**0.94**
(K, m1)	0.88
(E3, m2)	0.88
(E2, Z5)	-0.10
(E1, Z3)	-0.16
(q1, m1)	**0.93**
(q2, m2)	**0.93**
(E5, Z5)	**0.95**
(E4, Z4)	**0.95**
(r, d)	**0.94**
(2, two)	**0.94**
(F, A)	**0.91**

Table 12. Map for the COULOMB'S LAW IS NEWTON'S LAW analogy.

6. Conclusion

Most theories of analogical mapping generate analogical match hypotheses by applying a single principle, the classical principle of proportional analogy, to all the propositions in the source and target domains in the analogy. When only a single principle is applied to all the propositions in the domains, the generation of match hypotheses cannot be sensitive to the fine conceptual structure of the domains.

In contrast, the theory proposed here uses several principles generate analogical match hypotheses and to integrate these hypotheses into a constraint-satisfaction network for producing an analogical map. The principles are designed to attend to the fine conceptual structures of the domains involved in the analogies. The use of several principles enables pragmatic factors to be smoothly incorporated into the construction of the constraint-satisfaction networks for analogical mapping. Moreover, the networks constructed using several principles are smaller and more coherent than those constructed using only the principle of proportional analogy. Consequently, we believe that the mind uses many principles to construct analogical maps, and that its principles are designed to trace out the parallelisms between the fine conceptual structures of the source and target domains.

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