

EXISTENCE AND MODALITY IN KANT

LESSONS FROM BARCAN

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Abstract: This essay considers Kant's theory of modality in light of a debate in contemporary modal metaphysics and modal logic concerning the Barcan formulas. The comparison provides a new and fruitful perspective on Kant's complex and sometimes confusing claims about possibility and necessity. Two central Kantian principles provide the starting point for the comparison: that the possible must be grounded in the actual and that existence is not a real predicate. Both are shown to be intimately connected to the Barcan formulas, and Kant's views on what he distinguishes as three different kinds of modality are then considered in light of this connection.

Keywords: Kant, existence, modality, Barcan, de re / de dicto, grounding, logical possibility, formal possibility, real possibility, substance, essence, laws, a priori

1. Introduction

The Barcan formulas have played a central role in some of the most important modern-day developments in modal logic and metaphysics.¹ They were introduced and endorsed in the earliest fully formal systems of quantified modal logic, syntactically in Barcan (1946; 1947) and semantically in Carnap (1946; 1947), and they were the first examples chosen by Kripke (1963, 87–90) to illustrate his ‘possible worlds’ model theory, showing how to invalidate the formulas by varying the domain of individuals between worlds.² Questions concerning their interpretation, validity, and significance touch on many of the topics that have shaped the contemporary debate, not least concerning essentialism, *de re* modality, the distinction between logical and metaphysical conceptions of modality, the nature of possibilities, and the relation between being, existence, quantification, and predication. So what are the Barcan formulas and how might they be relevant for our understanding of Kant?

Together they encode the view that it is possible that there is something that satisfies a condition just in case there is something that possibly satisfies the condition; or equivalently, that it is necessary that everything satisfies a condition just in case everything necessarily satisfies the condition. Remarkably, Kant appears to reject exactly this view in the course of his seminal attack on the ontological argument.

He says: “The unconditioned necessity of judgements, however, is not an absolute necessity of things” (A593/B621).³ If we take this to be a general claim, then it can seem to involve a rejection of the view encoded in the Barcan formulas. Consider an instance where the condition in question is that of being *F*. The Barcan formulas say that it is necessary that everything is *F* just in case

¹ See Williamson (2013) for a comprehensive study and Janssen-Lauret (forthcoming) for a recent historical account.

² This result, which will be of special significance here, is already set as a task in Kripke (1959a, 13) and foreshadowed in Kripke (1959b, 324). In these early papers Kripke always mentions the Barcan formulas alongside the closely related topic of the necessity of identity (cf. Barcan 1947; 1961), which would go on to be so important in his own engagement with Kant (Kripke 1971; 1980).

³ References to the *Critique of Pure Reason* take the standard A/B format. References to Kant’s other works are by volume and page number of the Academy edition with abbreviations listed at the end. Translations are from the Cambridge Edition.

everything is necessarily *F*. The claim that it is necessary that everything is *F* is a *de dicto* modal claim. In Kant's terms, it attributes necessity to a *judgement*, namely the judgement that everything is *F*. The claim that everything is necessarily *F* is a *de re* modal claim. In Kant's terms, it attributes a necessity to *things*. It says of all of them that they are necessarily *F*. According to the Barcan formulas, the two claims are equivalent. According to Kant, it would seem, they are not. Whereas the Barcan formulas equate certain *de dicto* modal claims with certain *de re* modal claims, Kant insists on maintaining a sharp distinction between the two.

Is there a genuine connection here, or is the appearance of Kant's rejection of the view encoded in the Barcan formulas merely superficial and misleading? One would be forgiven for presuming the latter. After all, histories of the *de re / de dicto* distinction tend to jump from the fourteenth century to the twentieth, as though it slipped from view in the meantime.⁴ And of course the technical machinery involved in modern logic would be unknown to Kant. In general, there is no prior guarantee that Kant even determinately has views or commitments on such things as the Barcan formulas. In particular, there are certainly other things he might mean by insisting on a distinction between the unconditioned necessity of judgments and the absolute necessity of things.⁵

Yet there are grounds for further investigation. For instance, there is increasing appreciation of the debt Kant's philosophy owes to certain broadly Aristotelian strands of thought in the Scholastic tradition. Perhaps some appreciation of a modal *de re / de dicto* distinction is another example of this. More generally, there is a clear overlap between the issues that arise in relation to the Barcan formulas, some of which were listed in the first paragraph above, and those that are of central importance to Kant. And crucially, some of the central questions relating to the Barcan formulas do not require for their sheer formulation technical means that Kant did not possess. On the contrary, we will see that Kant could make good sense of them.

In this essay I will argue that there is indeed a genuine and deep connection between the issues surrounding the Barcan formulas, which have been so

⁴ See e.g. Kneale (1962), Dutilh Novaes (2004), and Keshet & Schwarz (2019).

⁵ One obvious alternative is that Kant is referring to his distinction between logical and real modalities (A596/B924). As we will see in §4 and §6, the two distinctions are closely related.

important for the contemporary debate, and Kant's theories of existence and modality, in which there has been a recent surge in interest. My aim is to show that this connection provides a new and fruitful perspective on Kant's complex and sometimes confusing claims about possibility and necessity. Exploring it will enable us to draw together several key Kantian doctrines, it will show us how unified and sophisticated Kant's account of modality is, both internally and by the standards of contemporary formal work, and it will lead us to the heart of Kant's system and the Critical method.

§2 provides a more formal introduction to the Barcan formulas. I raise a set of informal questions which will serve as a litmus test for whether Kant would accept the view they encode and I connect the formulas to a position I call *Modal Particularism*, which is related to Kant's claim that the possible must be grounded in the actual. In §3 I connect the formulas to Kant's famous dictum that existence is not a real predicate and in §§4–6 I explore how these connections play out with regard to three Kantian species of modality.

In §4 I consider a broadly logical conception of modality and argue that Kant would reject the Barcan formulas for this case because of his views about analyticity and existence. In §5 I consider an empirical or causal conception of modality and argue that Kant would accept the Barcan formulas for this case because of the principle he lays out in the First Analogy. In §6 I consider a conception of modality that concerns agreement with our forms of experience and argue that Kant would reject the Barcan formulas for this case because of his views about existence and the *a priori*. For each case there are both philosophical and technical corollaries. I argue that whether or not Kant accepts the Barcan formulas tracks whether or not he accepts Modal Particularism and I show that Kant is implicitly committed to formalisms that likewise cohere with whether or not he accepts the Barcan formulas and Modal Particularism. I explain how a single fundamental commitment underlies all of these results.

2. The Barcan Formulas, Domain Variance, and Modal Particularism

The Barcan 'formulas' are really formula schemas governing the interaction of quantifiers and modal operators. Together they form an equivalence schema, the Barcan Equivalence (BE). Where v is any individual variable and ϕ any formula:

$$\text{BE} \quad \Diamond \exists v \phi \leftrightarrow \exists v \Diamond \phi$$

The left-to-right direction is the *Barcan Formula*; the right-to-left direction is the *Converse Barcan Formula*. Different interpretations of the formulas raise different issues. For the purposes of this section we interpret the quantifier as unrestricted and the modal operator with some as yet unspecified conception of alethic possibility. The non-vacuous cases will be those where v is free in ϕ , so that we can think of ϕ as stating a condition on v . The evaluable cases (i.e. the closed or semantically complete cases) will be those where only v is free in ϕ , so that instances of the left-hand side will be *de dicto* formulas and instances of the right-hand side will be *de re* formulas (where a formula is *de dicto* if no variable occurs free within the scope of a modal operator and *de re* otherwise). The Barcan Equivalence then says that it is possible that there is something that satisfies a condition if and only if there is something that possibly satisfies the condition.

By the duality of the quantifiers ($\forall v =_{\text{df.}} \neg \exists v \neg$) and the operators ($\Box \phi =_{\text{df.}} \neg \Diamond \neg \phi$), which we can assume for the moment are stipulative definitions, BE is equivalent to:

$$\text{BE}^* \quad \Box \forall v \phi \leftrightarrow \forall v \Box \phi$$

In BE*, the left-to-right direction is the Converse Barcan Formula, the right-to-left direction the Barcan Formula. Where v and only v is free in ϕ , BE* says that it is necessary that everything satisfies a condition if and only if everything necessarily satisfies the condition, and again, instances of the left-hand side will be *de dicto* while instances of the right-hand side will be *de re*.

The formalism here is modern but many of the issues it raises are not. The Barcan formulas turn out to be intimately connected to the question of whether it is necessary which things there are, or whether what there is could have been different. In particular, is there something that exists but which could have failed to do so, and could there have existed something that in fact failed to do so?

This is crucial because, *prima facie*, these are questions that Kant could make sense of. Indeed, *prima facie*, they are questions that Kant would think it important to settle. Questions of existence and modality are central to his

philosophy and questions of the modal status of what exists are an important aspect of this. As we will see, just what Kant's answers to such questions would be, and how he would reach them, will depend on what exactly they are asking—they will depend first and foremost on the kind of modality at issue. But one thing formal techniques provide is a way to abstract from specifics and see, quite generally, that the Barcan Formula will fail if there could have been something that in fact there isn't, while the Converse Barcan Formula will fail if there is something that there might not have been. This result will provide us with a litmus test for whether Kant would accept the view encoded in the Barcan formulas for each of the cases we consider.⁶

Now, it might seem obvious that it *is* possible for what there is to vary in this way. Couldn't Kant have had a child even though he in fact died childless? If so, then it can seem as though there could have been something that actually there isn't, namely a child of Kant. And couldn't Kant's parents have not had children? If so, then it can seem as though there is something that there might not have been, namely Kant himself. There are several moves that defenders of the Barcan formulas can make in the face of such putative counterexamples. Generally speaking, they can either take a narrow view of what is possible or a broad view of what there is. If nothing is possible that is not also actual, then Kant could not have had a child when in fact he died childless. Or perhaps Kant would still have existed had he never been born, albeit without then being the child of his parents or possessing many other of his actual properties.⁷ Both kinds of approach will be relevant in what follows and I will expand on what they amount to in due course. For now I want to articulate another way of thinking about the Barcan formulas which will also be important for what follows.

⁶ The general result first became visible with technical advances in possible worlds model theory culminating in Kripke (1963)—see Copeland (2002) and Goldblatt (2003) for informative histories. But we needn't think of it in such terms. As we will see from §4 onwards, our questions can be replicated at the level of instances of the Barcan formulas, which for present purposes can in turn be treated as translations of natural language sentences that Kant would be in a position to assess.

⁷ See Linksy and Zalta (1994; 1996) and Williamson (2013) for general versions of this second kind of approach. There will be any number of ways to response to specific examples, since these will often involve specific assumptions. Those I just gave involving Kant, for instance, require the necessity of origins (Kripke 1980, 110–15) and that constitution is not identity (Johnston 1992).

Suppose that general modal truths are grounded in particular modal truths. Just as general existential truths and general truths involving extensional relations are grounded in particular existential truths and particular truths involving extensional relations, so general modal truths are grounded in truths about particular things and their modal properties. More specifically, and taking possibility as our primitive modal notion, let *Modal Particularism* be the following view. If it is possible *that* something ϕ 's, this is grounded in the fact that ϕ 'ing is a possibility *for* some particular thing (or things). As with existence and extensional relations, it might not matter which particular thing (or things) can ϕ , but there must at least be something that can ϕ if it is to be possible that something ϕ 's. Such a view gives a certain primacy to the *de re* over the *de dicto* and is one way to elaborate on the idea that modality is fundamentally about how *things* could be. It also looks like one way to implement Kant's principle that "*All possibility is given in something actual*" (OPA 2:79).⁸

Modal Particularism naturally validates the Barcan formulas and thus the Barcan Equivalence, BE ($\Diamond\exists v \phi \leftrightarrow \exists v \Diamond\phi$).⁹ It says that truths of the kind that form instances of the left-hand side of BE—general or *de dicto* possibility truths—are grounded in truths of the kind that form (or are themselves instances of) the corresponding instances of the right-hand side—particular or *de re* possibility truths. On such a view, the right-to-left direction of BE—the Converse Barcan Formula—follows from the widely accepted principle that, roughly and informally, if A is grounded in B then B entails A.¹⁰ The left-to-right direction of BE—the Barcan Formula—will not follow from any such *general* principle of grounding. It is not always the case that if A is grounded in B then A entails B. Nevertheless, the entailment will hold for the case at hand, as it does for many other cases. This was part of the point of the analogy with existence claims and claims involving extensional relations. If it is true that humans exist then it is true of some particular human that they exist; if Sally met someone then there is some particular person whom Sally met. And similarly, if the general possibility that something ϕ 's is grounded in there being something that can ϕ , in the way

⁸ This principle survives in some form into the Critical period (PR 28:1036), though there is widespread disagreement about just what that form is and part of the aim in what follows is to provide a new perspective on the issue—see especially §6. For discussion see Chignell (2009; 2012), Stang (2016), Abaci (2019), and Barker and Marshall (forthcoming).

⁹ Cf. Barcan (1985) and Simchen (2013).

¹⁰ See e.g. Rosen (2010, sec. 7).

envisioned by Modal Particularism, then it will only be true that it is possible that something ϕ 's if it is also true that there is something that can ϕ .¹¹

The relationship between Modal Particularism and the Barcan formulas is not strict. There are ways to hold either without the other. For my purposes here, however, it suffices that they form a natural pair. Modal Particularism will provide a general perspective on how the Barcan formulas relate to Kant's views about modality. For each of the kinds of modality Kant distinguishes and which are to be discussed here, we can ask: does Kant hold, for the modality in question and in the manner envisioned by Modal Particularism, that general modal truths are grounded in truths about particular things and their properties? We will see that the answer tracks whether or not he would accept the view encoded in the Barcan formulas for the modality in question. And it will do so in a way that sheds light on Kant's principle about possibility being grounded in actuality.

There is a lot more that could be said here and more will come out in what follows. My aim in this section has been to introduce the Barcan formulas and surrounding issues in enough detail to facilitate an initial comparison with Kant.

3. Existence is Not a Real Predicate

In this section I connect the Barcan formulas to Kant's views about existence. Most commentators agree that, for Kant, existence is not a discriminating property of things, where a discriminating property of things is a property such that, in a sense to be specified below, it is possible that some things have it while other things don't. Thus being human is a discriminating property of things because it is possible that some things are human while other things are not. Discriminating properties are able to divide the domain of things. Existence is not a discriminating property, then, because it cannot divide the domain of things in this way. It is not possible that some things exist while other things don't. And since it is clearly possible that some things exist, that must be because

¹¹ If the grounds are instances of the existentially quantified facts rather than the existentially quantified facts themselves—see §5—then we will need a rule of existential instantiation to get from Modal Particularism to the Converse Barcan Formula and a rule of existential generalization to get from Modal Particularism to the Barcan Formula. This is related to the failure of the Barcan formulas in free modal logics—see the technical corollaries to §4 and §6.

it is not possible that some things do not exist. While there can be non-human things, there cannot be non-existent things.

This is the view that I want to connect to the Barcan formulas. It will not be uncontroversial but it suffices for present purposes that the view is textually well-supported and widely shared. It follows from two highly plausible readings of Kant's famous dictum:

Being is obviously not a real predicate, i.e. a concept of something that could add to the concept of a thing. (A598/B626)

On one reading, Kantian real predicates just are predicates that express discriminating properties of things, so what Kant is saying here just is that existence is not a discriminating property of things.¹² On another reading, Kantian real predicates are simply predicates that express properties of things, whether or not they discriminate among them, so Kant is saying something stronger, namely that existence is not a genuine property of things at all, thus a fortiori not a discriminating property of things.¹³ I present the former reading and connect it to the Barcan formulas before explaining how we get the same result from the latter reading.

We start with Kant's positive gloss on real predication and work backwards from there. The second clause of the line quoted above implies that, for Kant, a real predicate is "a concept of something that could add to the concept of a thing". What does it take to be a concept *P* of something that could add to the concept *S* of a thing? On the reading under consideration, there are two conditions. First, *S* and *P* must be consistent. Otherwise *P* would not *be able* to add to *S*, in the sense that the result of trying to add *P* to *S* would be a contradictory concept which "cancels itself out" (A292/B348). Second, *P* must not already be analytically contained in *S*. Otherwise *P* would not be able to *add to* *S*, in the sense that the result of trying to add *P* to *S* would just be *S* and so no addition would really have been made (A7/B11).

¹² See Stang (2016, ch.2; 2015), Van Cleve (1999, 188–90), and Plantinga (1974, 97). For critical discussion see Bader (2018), Abaci (2019, chs.3, 8), and Rosefeldt (2020).

¹³ See Bennett (1974, 228–32), Wiggins (1994), Wolff (1995), Forgie (2009), and Cuffaro (2012). For critical discussion see Rosenkoetter (2010), Vanzo (2014), Kannisto (2017), and Rosefeldt (2020). The differences between these two readings should not be overstated. As presented here they disagree about real predication, but they can agree on much else.

Consistency and analyticity are broadly logical notions, so we can express these conditions in terms of a corresponding notion of logical possibility, broadly construed.¹⁴ Moreover, as Kant is clear, we are talking about concepts *of things*. A real predicate is “a concept *of something* that could add to the concept *of a thing*”. Or as he puts it in the previous paragraph, a real predicate is “the determination *of a thing*” (A598/B626, emphasis added; cf. OPA 2:72). Thus we can say that two concepts S and P are consistent just in case it is logically possible that there is something that falls under both S and P ; and P is not analytically contained in S just in case it is logically possible that there is something that falls under S but not under P . Finally, as Kant also makes clear in the previous paragraph, “Anything one likes can serve as a **logical predicate**” (A598/B626). I take this to imply that existence, even though it is not a real predicate, can nevertheless function grammatically as a predicate.

Putting these points together, our consistency and non-analyticity conditions on existence being a real predicate can be given the following formalization (which will be further explained and justified below). Where \diamond_L is our logical possibility operator, x an individual variable ranging unrestrictedly over ‘things’, and E our first-order, ‘logical predicate’ for existence, then E is a real predicate just in case, for some ‘concept of a thing’, S , both of the following conditions are satisfied:

$$\begin{array}{ll} (\textit{Consistency}) & \diamond_L \exists x (Sx \ \& \ Ex) \\ (\textit{Non-Analyticity}) & \diamond_L \exists x (Sx \ \& \ \neg Ex) \end{array}$$

¹⁴ We might prefer to call this ‘conceptual’ possibility, insofar as it concerns consistency in conceptual content and the corresponding conception of necessity is analyticity. But my usage here is consonant with Kant’s, whose conception of ‘logical’ possibility also concerns consistency in conceptual content (Bxxvi, A244/B302, A596/B624, A610/B638; Prog. 20:325; PR 28:1016, 1033). Kant’s primary use of the category of logical possibility is in contrast to real possibility, and in this broad contrastive sense, matters of conceptual consistency and analyticity are non-real, hence logical. Note moreover that Kant thinks of the law of non-contradiction as belonging to pure general logic, while tending to formulate it in terms of conceptual content and calling it the supreme principle of all analytic judgments (A150–53/B189–93). In line with Paton (1936a, 214) and MacFarlane (2002, 25) but contra Vanzo (2014, 228), then, I do not think that Kant’s pure general logic abstracts from all matters of content whatever, including conceptual content, so that it abstracts even from the analytic/synthetic distinction. As general logic it abstracts from matters intuitional and as pure logic from matters of empirical psychology, but freedom from contradiction in conceptual content is still, in the relevant sense, a matter for the ‘mere form of thinking’. See §4 and §6 for further discussion.

If existence is *not* a real predicate, that must be because it fails to satisfy one or both of these conditions. Clearly existence satisfies *Consistency*. It is logically possible that there is something that both exists and falls under S , for some S which is a concept of a thing. After all, humans exist. The culprit must be *Non-Analyticity*—existence must not be a real predicate, on this reading, because it is *not* logically possible for something to fall under a concept of a thing and yet not exist.¹⁵ So far this would allow for the logical possibility of there being things that do not exist, so long as they also do not fall under any concept of a thing. But if we assume that it is not logically possible for there to be things that do not fall under any concept of a thing,¹⁶ then it follows that existence is not a real predicate because it is not logically possible for there to be things that do not exist. And we have arrived at the key claim of the present reading. Kant’s doctrine that existence is *not* a real predicate (KE) can be captured in the formula:

$$\text{KE} \quad \neg \diamond_L \exists x \neg Ex$$

It is *not* logically possible that there is something that does not exist. According to the present reading, this is why Kant thinks existence is not a discriminating property of things, the determination of a thing, a real predicate.

As in §2, assume the standard duality of the quantifiers and the operators. We can treat the duality of the quantifiers as a stipulation—I say more about their meaning and scope in a moment—and Kant is frequently reported as affirming the duality of possibility and necessity.¹⁷ KE is then equivalent to:

$$\text{KE}^* \quad \Box_L \forall x Ex$$

¹⁵ This is akin to what Plantinga (1983, 11) calls *serious actualism* and Williamson (2013, ch.4) calls the *being constraint*. See fn.27 for related discussion.

¹⁶ This assumption is shared by both of the readings considered here and I return to it below. It is akin to the converse of serious actualism and the being constraint. See fn.27 for related discussion.

¹⁷ E.g. “Necessary is that of which the opposite is impossible” (ML₂ 28:557). I take duals to be definable in this way for each of the kinds of modality I consider. For an opposing view see Bader (forthcoming).

It is logically necessary that everything exists. According to the present reading, this is why Kant thinks existence is not a discriminating property of things, the determination of a thing, a real predicate.

It will already be clear where I want to draw the initial point of connection between this central doctrine of Kant's and the Barcan formulas. KE is the negation of an instance of the left-hand side of the Barcan Equivalence, BE ($\Diamond \exists v \phi \leftrightarrow \exists v \Diamond \phi$), where the modality is logical and the condition is that of not existing. Equivalently, KE* is an instance of the left-hand-side of BE* ($\Box \forall v \phi \leftrightarrow \forall v \Box \phi$), where the modality is logical and the condition is that of existing. Thus we can already start to see the significance of the *de re / de dicto* distinction in the question of whether and in what sense it might be necessary not only *that* everything exists but *which* things exist. These are the issues I will explore in detail in the remainder of the essay and readers who are satisfied with the preceding should feel free to skip to the next section. But before moving on I want to explain how we get the same result from an alternative reading of Kant's claim that existence is not a real predicate, according to which he means to say not only that existence is not a discriminating property of things but that it is not a genuine property of things at all.

The basic idea here is straightforward. If existence is not a property of things, then nor is it a discriminating property of things and hence there cannot be things that do not exist. But we need to say a little more to show that KE and KE* can be read in a way that suitably captures such a view. For don't they treat existence as a property of things, albeit a universal rather than a discriminating one, simply by virtue of employing a first-order existence predicate? In fact my use of a first-order existence predicate here and throughout is dispensable and nothing in what follows requires that the 'Ex' in KE and KE* be taken at face value. It *can* be read as representing existence as a genuine first-order property of things, but it can also be read as shorthand for a formula that does not represent existence in this way. Explaining this point will serve to further explain and justify my formalism.

On the kind of reading I have in mind, Kant's denial that existence is a real predicate embodies a proto-Fregean account of existence according to which first-order existence claims are analysed in terms of the second-order property of being instantiated. To say that humans exist, for example, is really to say, of the

property of being human, that it is instantiated. We express this second-order property by means of our first-order quantifier, which we now treat as a second-order predicate. Thus “ $\exists xHx$ ” is read as saying, of the property denoted by “ H ”, that it is instantiated. We can still think of this quantifier as ranging unrestrictedly over ‘things’, as we did above, but we can now elaborate a little on what we mean—it ranges over all and only the instantiators of first-order properties, or as Kant might put it, over that which “must always be considered as subject, never as mere predicate” (B129). That is, we can specify the domain of our first-order quantification by means of Kant’s pure concept of substance.¹⁸

So this kind of view takes the fundamental logical form of first-order existence claims to be given by quantification rather than first-order predication. Crucially, however, it may still allow us to employ a first-order existence predicate—again, “Anything one likes can serve as a **logical predicate**” (A598/B626). It requires only that we treat formulas involving such a predicate as shorthand for something of proper quantificational form. What, then, is the proper quantificational form of a formula like Ex , on such a view? Consider two natural candidates:

$$X_1 \quad Ex =_{\text{df.}} \exists y y=x$$

$$X_2 \quad Ex =_{\text{df.}} \exists X Xx$$

We can think of X_1 as encoding the view that, to say of some individual thing x that it exists is just to identify it as one of the things over which our first-order quantifiers range. And we can think of X_2 as encoding the view that, to say of

¹⁸ Cf. A147/B186, B225, A242–43/B300–301, A348; B411n; MFNS 4:503; R 5295–97, 18:145. For related discussion see Thompson (1972, 334), who relates Kant’s pure concept of substance to the “ x ” in “ Fx ”, and Stang (2021), who marks an important distinction between a notion closely akin to the present one, which he calls the “quantificational concept of an object”, and Kant’s “concept of an object in general” (A290/B346), which Stang argues is a “representational concept of an object”, i.e. the concept of that which a representation represents. The latter concept is absolutely central to Kant’s transcendental philosophy but it does *not* correspond to the concept of something that instantiates first-order properties. For Kant, “All representations [*Vorstellungen*], as representations, have their object [*Gegenstand*]” (A108)—for every ‘putting before’ there is a ‘standing against’. But it is not the case that every representation represents something that instantiates a first-order property. After all, we can represent first-order properties. I have tried to mark this point by using the term “thing” instead of “object” in the main text. This was occasioned by Kant’s usage in my central quotation and related passages, but I make no claim to find any such stable terminological distinction in Kant.

some individual thing x that it exists is just to say that it instantiates some first-order property. These views are unified by our identification of the things over which our first-order quantifiers range with the instantiators of first-order properties. Thus in effect both X_1 and X_2 say that what it is for an individual thing to exist is for it to be one of the instantiators. The *definientia* in X_1 and X_2 remain importantly different.¹⁹ But what is crucial for my purposes here is that they can both very plausibly be taken to entail KE ($\neg \diamond_L \exists x \neg Ex$) and KE* ($\Box_L \forall x Ex$).

X_1 entails KE and KE* on the assumption that it is logically necessary that everything is self-identical. That is, given X_1 , KE would say that it is not logically possible that there is something that is not identical to anything (even itself): $\neg \diamond_L \exists x \neg \exists y y=x$. While KE* would say that it is logically necessary that everything is identical to something (for instance itself): $\Box_L \forall x \exists y y=x$. Similarly, X_2 entails KE and KE* on the assumption that it is logically necessary that everything instantiates at least one first-order property (or ‘concept of a thing’—see above). That is, given X_2 , KE would say that it is not logically possible that there is something that instantiates no first-order property: $\neg \diamond_L \exists x \neg \exists X Xx$. While KE* would say that it is logically necessary that everything instantiates at least one first-order property: $\Box_L \forall x \exists X Xx$. In other words, and as already suggested above, the ‘instantiators’ (of first-order properties, i.e. the instantiators that cannot also be instantiated) are *just what we mean* by the ‘things’.

In this way, the present reading of Kant’s claim that existence is not a real predicate will also have him committed to KE and KE*. It will understand these

¹⁹ They raise distinct issues that I cannot address here. For instance, contra Stang (2015; 2016), it is not immediately clear whether X_1 is compatible with the second-order-predicate understanding of our first-order quantifiers given Kant’s distinctly un-Fregean views about the essential generality of concepts (A68/B93, A320/B377; JL 9:91). For X_1 would then seem to treat being identical to something (“...=x”) as a first-order property, saying of such a property that it is instantiated. Yet isn’t such a property essentially singular? One option in the face of this worry would be to adopt X_2 , but we would then owe an account of its second-order quantifier—is it, for Kant as for Frege, a third-order predicate with equal existential import? A better option I suspect might be to stick with X_1 but loosen the analogy between Kant’s and Frege’s accounts of existence. Perhaps, for Kant, to say that humans exist is not *quite* to say, of the property of being human, that it is instantiated. The idea of ‘absolutely positing’ something that instantiates that property is not very far away, and perhaps we can think of our first-order quantifiers directly in terms of this central Kantian notion, rather than in terms of second-order predication. This proposal is related to that of treating our quantifiers as modal operators—see fn.20.

formulas slightly differently to the previous reading, but it can allow them to be glossed in just the same way. Whether Kant thinks that existence is not a real predicate because he thinks it is not a discriminating property of things or because he thinks it is not a genuine property of things at all, he is thereby committed to the claim, suitably captured by KE and KE*, that it is not logically possible that there is something that does not exist, i.e. that it is logically necessary that everything exists. As in §2, there is much more that could be said here and more will come out in what follows. But let us turn to our case studies.

4. Logical Modality

In the previous section we have seen an initial point of connection between Kant's claim that existence is not a real predicate and the Barcan formulas. In the rest of this essay I expand on this connection and explore its consequences with a view to shedding light on Kant's views about the logic, metaphysics, and epistemology of modality. Kant distinguishes a number of different kinds of modality and I focus on three of them, that is, I consider what Kant would make of three different interpretations of the Barcan formulas. For each kind of modality I present two test cases, which is to say two formulas that raise questions that Kant could make sense of and which bring out the fact that the Barcan formulas will fail if what there is could have been different, if the domain of what exists can vary. I then draw out technical and philosophical corollaries to these test cases. I begin with the most straightforward case, that of logical modality.

Test Cases

Take the Converse Barcan Formula first, specifically the formulation that is the left-to-right direction of BE* ($\Box \forall v \phi \leftrightarrow \forall v \Box \phi$). We have seen that KE* ($\Box \forall x Ex$) is an instance of the left-hand-side of BE* where the modality is logical and the condition that of existing, so our first and simplest test case is as follows:

$$\text{LI} \quad \Box \forall x Ex \rightarrow \forall x \Box Ex$$

The antecedent is KE*, i.e. Kant's claim that existence is not a real predicate. So LI says that, if existence is not a real predicate, then everything is such that it is logically necessary that it exists. Would Kant accept this consequence of the Converse Barcan Formula? Clearly not. Kant famously thinks that nothing exists

with *logical* necessity, not even God. This is why he believes that an ontological proof of God’s existence cannot succeed (A592–602/B620–30). Thus it seems that Kant would reject the consequent of LI for reasons that are intimately related to his acceptance of its antecedent. I will elaborate below but first let us turn to the Barcan Formula.

This time I take as our test case a somewhat more complicated instance of the Barcan Formula—the contrapositive of the left-to-right direction of BE ($\Diamond\exists v \phi \leftrightarrow \exists v \Diamond\phi$), where the modality is logical and the condition is that of actually not existing, “@ $\neg Ex$ ”:

$$L2 \quad \neg\exists x \Diamond_L @ \neg Ex \rightarrow \neg\Diamond_L \exists x @ \neg Ex$$

If it is not the case that there is something such that it is logically possible that actually it does not exist, then it is not logically possible that there is something that actually does not exist. The antecedent follows from KE, but again, it is clear that Kant would reject the consequent. He thinks it *is* logically possible for something to exist that does not actually exist; he thinks it *is* logically possible to add to the domain of what there is. Let me unpack this.

First a note on my use of the actuality operator, @. This is merely an artifact of the formalism. It is required to display, at the level of an object-language formula and without having to make further assumptions, the general meta-linguistic point that the Barcan Formula will fail if there could have been something that there in fact is not, if we can add to the domain. I do *not* mean our actuality operator to align to Kant’s own modal category of actuality, or existence.²⁰ All I

²⁰ On Fregean readings of Kant the role of his modal category of actuality or existence is played by the existential quantifier (§3). Opponents of such readings sometimes object that Kant clearly separates his quantifiers from his modalities and treats existence as a modality (Rosefeldt 2020; Bader 2020; Kannisto 2017). This objection is mistaken in two important respects and it is worth briefly explaining why. First, Fregean readings need only claim that Kant’s *account of existence* prefigures Frege’s existential quantifier. They need *not* claim that Kant’s *own quantifiers* prefigure Frege’s. Kant’s quantifiers are the traditional binary quantifiers operating on two predicates, not Frege’s unary quantifiers taking a single argument, and Frege’s translation of the former into the latter should be viewed as entirely his own innovation. This will be important below and in §6. Second, the distinction between our modern quantifiers and modal operators is not as sharp as the objection supposes. Possible world semantics shows us how to think of modality in terms of quantification and it should not be surprising that we can do the reverse. Indeed doing so—

require here is that Kant could make sense of, and would reject, the claim that, if existence is a not real predicate, then it is not logically possible for there to have been something that in fact there is not, which in effect is what L2 says. I take the antecedent and consequent in turn.

How does the antecedent of L2 ($\neg\exists x \diamond_L @ \neg Ex$) follow from KE ($\neg \diamond_L \exists x \neg Ex$), i.e. from Kant's claim that existence is not a real predicate? We treat @ in the normal way, which is to say as a rigidifying operator that always 'returns' us to the actual world. In the antecedent of L2, this has the effect of making $\diamond_L @$ redundant. With no wider scope modal operators we start off in the actual world, the diamond takes us to a logically possible world, but then the actuality operator brings us straight back to the actual world again, so the detour was redundant. But if the possibility operator is redundant then so is the actuality operator. That is, $\neg\exists x \diamond_L @ \neg Ex$ is equivalent to $\neg\exists x @ \neg Ex$ which is equivalent to $\neg\exists x \neg Ex$. And KE entails $\neg\exists x \neg Ex$ given that logical impossibility entails falsity (A59/B84, A151/B190).²¹ Thus KE, Kant's claim that existence is not a real predicate, entails $\neg\exists x \diamond_L @ \neg Ex$, the antecedent of L2.²²

Why, then, would Kant reject the consequent of L2 ($\neg \diamond_L \exists x @ \neg Ex$)? It says that it is not logically possible for there to be something that actually does not exist. Kant would reject this because he thinks it is logically possible for something to exist that does not actually exist; he thinks it is logically possible to add to the domain of what there is. To see this, consider the following (and note that here, in particular, the generality of L2 will be crucial).

treating quantifiers as modal operators—turns out to be particularly straightforward for the monadic fragment of first-order quantificational logic (Montague 1960; Prior 1968; Lewis 1973, 112; Kuhn 1980; Koslow 2019). This is important because that is plausibly the fragment of modern logic that Kant anticipates (Thompson 1972, 334; Friedman 1992, 63n.9). Kant can think of existence along the lines of a modern existential quantifier while also thinking of it as a (unary, sentential) modal operator that is fundamentally distinct from his (binary, predicate) quantificational operators.

²¹ Equivalently, logical necessity entails truth—the T-axiom holds for logical modality. All of the modalities I discuss here are alethic in this sense.

²² In a way that this shows is that the antecedent of L2 is not really *de re*—of course it does have a free variable in the scope of a modal operator, but that very operator is redundant. More generally, the upshot of this section will be very congenial to the idea that, for Kant, there can be no genuinely *de re* logical modalities. After all, logical modality is exclusively conceptual and he thinks that concepts are essentially general (A68/B93; JL 9:91). See Heide (2021, 601–602) for relevant discussion.

Something is logically possible, in the relevant sense, just in case its concept is consistent. And there are lots of consistent concepts under which nothing happens to fall. There is no inconsistency in the concept of a pink elephant, for instance, which is just to say that pink elephants are logically possible. We are not yet finished, however. Just because pink elephants are logically possible even though none exist, it does *not* immediately follow that it is logically possible for something to exist that does not actually exist. For it might be logically possible for something that *does* actually exist to be a pink elephant.

This looks plausible for the case at hand, but what if we generalize? To salvage the consequent of L2 in its full generality, the claim here would have to be that, for *any* consistent concept (which is a first-order concept ‘of a thing’), there is something that actually exists such that it is logically possible that the thing fall under the concept. And this would mean that showing the conceptual consistency of a concept would suffice to show that something actually exists (since there would then have to be something actually existing that might have fallen under the concept). But for Kant, showing conceptual consistency can never suffice for showing that something actually exists. For Kant, existence claims like this are always synthetic, whereas conceptual consistency is an analytic matter (A151/B191, A598/B626). *This* is why he would reject the consequent of L2 and thus the Barcan Formula for logical modality.

Kant’s doctrine that existence is not a real predicate, coupled with his rejection of the ontological argument and his claim that existence claims are synthetic, means he would reject both directions of the Barcan formulas for logical modality. This supports my suggestion at the outset of our investigation, that one of the things that is going on in Kant’s discussion of the ontological argument is a rejection of the view encoded in the Barcan formulas alongside some at least implicit appreciation of the *de re / de dicto* distinction. For Kant, it is logically necessary *that* everything exists but it is logically contingent *which* things exist—logic alone does not determine what there is. There are both technical and philosophical corollaries of this result.

*Technical Corollary*²³

Kant would reject classical logic in favor of a universal (or inclusive) free logic for logical modality, and the Barcan formulas are invalid in systems of quantified modal logic that take a universal free logic as their base predicate logic.²⁴

KE*, Kant's claim that existence is not a real predicate, says that it is logically necessary that everything exists. By classical modal reasoning, it follows that it is logically necessary that something exists. But Kant would deny that it is logically necessary that something exists. The culprit here is that classical logic requires the domain of quantification to be non-empty—this is what validates the inference from “everything exists” to “something exists”. So on pain of inconsistency, Kant needs a logic in which the domain of quantification can be empty and there is no such entailment.²⁵ That is to say, he needs a universal free logic.

Why think Kant would deny that it is logically necessary that something exists? It does not suffice that Kant thinks nothing is such that it exists with logical necessity. This is compatible with it nevertheless being logically necessary that *something or other* exists. Instead what is crucial for the present point is that Kant thinks it is logically possible that nothing whatsoever exists. He says, for instance, “There is no inner contradiction in the negation of all existence” (OPA 2:78; cf. A595/B623).²⁶ In fact we can think of both of these claims as following from

²³ When I say in this and the corresponding parts of later sections that Kant would need, or is implicitly committed to, this or that formal system, I of course do not mean that he had any such system in mind. What I mean is that this or that system would be the appropriate one for modelling Kant's views for the discourse in question. Reconstructing Kant's commitments in this way will shed further light on his views, not least by helping to bring out how unified and sophisticated they are. It will also allow us to pinpoint and clarify some disagreements in the literature where similar reconstructions have been offered—see fn.27.

²⁴ See e.g. Garson (2006, 250) and Williamson (2013, 39–44).

²⁵ Recall that our quantifiers are not Kant's (fn.20), so I am *not* here denying that Kant allows universal-particular inferences for his own quantifiers (JL 9:116; A608/B636).

²⁶ I should note that Kant does say that “*it is absolutely impossible that nothing at all should exist*” (OPA 2:79). But he is here referring to what he would later call “real” as opposed to “logical” impossibility. The claim follows from Kant's principle about possibility being grounded in actuality, which also needs to be so specified (see §1 and §6). Very roughly, the absolutely really impossible is that which would cancel all real possibility, and that nothing at all should exist fits the bill, because all real possibility must be grounded in something actual. See Stang (2016, chs. 5 and 9) for extensive discussion.

Kant's view that existence claims are synthetic and explain the point in these terms.

KE*, the claim that it is logically necessary that everything exists, can, on the operative, broad conception of what is logical, be rephrased as the claim that it is analytic that everything exists. But as we have seen, Kant says that “every existential proposition is synthetic” (A598/B626). Is there a tension here? There is not. “Everything exists” does not express an existential proposition, in Kant's sense. Kant makes clear earlier in the same paragraph that, when he says every existential proposition is synthetic, he is talking about propositions of the form “*This or that thing... exists*” (A597/B625). These are propositions to the effect that *something exists* or *something in particular exists*—which accordingly Kant would deny can be logically necessary—and they have quite a different form to the claim that everything exists. That isn't quite the end of the matter, however. Plausibly, Kant would take analyticity to be closed under entailment: if A is analytic and A entails B, then is B analytic. If so, then if “everything exists” is analytic and entails “something exists”, then “something exists” would be analytic, too. And this *would* be in tension with Kant's claim that existential propositions are synthetic. Kant's commitment to KE* and the synthetic nature of existence claims means he must deny that “everything exists” entails “something exists”. In contemporary terms, he must adopt a universal free logic, the modal extension of which invalidates the Barcan formulas.²⁷

²⁷ This is in line with Stang (2017, 75), Stephenson (2017), and Vanzo (2014, 228–29) but contra Van Cleve (1999, 197). Van Cleve's objection to attributing free logic to Kant is especially significant here because of what I claimed in §3, in effect that Kant equates being with being propertied. Van Cleve says “if we adopt a free logic we depart at least from the spirit of the Kant-Frege view [of existence]” because “free logic places a companion restriction on the rule of existential generalization, requiring ‘*Ea*’ as an auxiliary premise before one can get from ‘*Fa*’ to ‘ $\exists xFx$ ’, and that requirement has point only on the assumption that *a* can instantiate the property of being *F* even if *a* does not exist”. But this is incorrect. Free logics *per se* make no such assumption. Only positive free logics allow atomic formulae to come out true that involve singular terms that do not denote existing things. Negative free logic require all such formulae to be false while neutral free logics require them to be truth-valueless. Informally, both negative and neutral free logics can be thought of as allowing only existing things to instantiate properties. Instead then, and consonant with the discussion in the main text, the ‘point’ of free logic restrictions on existential generalization can be seen as a purely formal requirement on quantifier duality *given* the companion restriction on universal instantiation. This latter restriction is more basic to free logic. That is, all free logics require an auxiliary premise to the effect that *a* exists in order to validate moves from “Everything is *F*” to “*a* is *F*”. This requirement ‘has point’ because, without it, “*a*” might fail to denote something, and so it might not be the case that *a* is *F* even

Philosophical Corollary

The philosophical corollary of all this is that Kant rejects Modal Particularism for logical modality. He does *not* think that general truths about what is logically possible are grounded in particular truths about things and their properties, in the way required for Modal Particularism. Indeed, in a sense, he thinks logical modality has nothing fundamentally to do with *things*—it is, as he often puts it, a modality for concepts (A244/B302).²⁸ What is or is not logically possible or necessary is determined by the laws of pure general logic and concerns concepts and their relations of containment, exclusion, compatibility, and so forth. But neither the laws of logic nor concepts, for Kant, are things—they are not substances in which accidents inhere.

Thus what we have here is a remarkable degree of unity and sophistication, both internally and by the standards of contemporary formal work on modality. Kant's claim that existence is not a real predicate; his rejection of the ontological argument and his view that nothing exists with logical necessity; his conception of analyticity and the synthetic nature of existence claims; and his view that it is logically possible that nothing whatsoever exists. These intimately related views together imply Kant's rejection of the view encoded in the Barcan formulas for logical modality, his commitment to a universal free logic on which the Barcan formulas fail, and his rejection of the Modal Particularism that goes naturally with the Barcan formulas.

though everything is *F*. This is a requirement that holds across positive, negative, and neutral free logics and it involves no suggestion that non-existents can instantiate properties, merely that singular terms might fail to denote. And if we restrict ourselves to universal free logics the same reasoning holds for inferences from "Everything is *F*" to "Something is *F*". This time the auxiliary premise "Something exists" is required because "Everything is *F*" will be vacuously true when there is nothing. Again, there is no suggestion here that non-existents can instantiate properties. See Stephenson (2017) for further discussion. (Vanzo (2014, 229) makes a related error in suggesting that universal free logics reject the inference from "*Fa*" to " $\exists xFx$ " while licensing that from " $\forall xFx$ " to " $\exists xFx$ "—negative universal free logics, for instance, do neither.)

²⁸ I do not here mean to deny that analytic judgments, as truth-evaluable judgments, must have "relation to an object"—I am using "thing" in a much more specific sense than Kant uses "object" (fn.18). See A58/B82, A68/B93, A151/B190 and for discussion Paton (1936a, 214), MacFarlane (2002, 51), Tolley (2012), and Lu-Adler (2018, ch.5).

It is tempting to think of all of these commitments as aspects of the Critical doctrine of the formality of pure general logic (A52–56/B76–80), but we will see later that they follow from a more general commitment of Kant's concerning pure logic as such, be it general or transcendental.

5. Empirical Modality

In this section I turn to what I will call *empirical* modality, by which I mean a kind of modality that concerns what is possible and what is necessary given the actual laws of nature and some actual state of the phenomenal world. It will be significant later that Kant is a determinist about the phenomenal world in the sense that he thinks nothing is empirically possible that is not also actual and therefore empirically necessary. But let us first turn to the Barcan formulas.

Recall that the Barcan formulas are valid just in case it is impossible for what there is to vary. If we suppose that the appropriate domain of individuals for empirical modality is the domain of phenomenal substances, then the Barcan formulas will be valid for empirical modality just in case it is empirically impossible for what phenomenal substances there are to vary. And now we can see that the validity of the Barcan formulas for empirical modality, so understood, is a consequence of what Kant aims to establish in the Analogies of Experience, in particular the First Analogy.

The principle of the First Analogy, Kant says, deserves to be “at the head of the pure and completely *a priori* laws of nature” (A184/B227). It states, in the A-edition, that “All appearances contain that which persists (**substance**) as the object itself, and that which can change as its mere determination, i.e., a way in which the object exists” (A182); and in the B-edition that “In all change of appearances substance persists, and its quantum is neither increased nor diminished in nature” (B224). I take it that at least *part* of what Kant means to establish here is that individual substances are permanent in the sense that, if a substance exists at all, then it exists at all times and does not come into or go out of existence in time.²⁹ As he elaborates:

²⁹ I do not claim that this is all Kant means to establish in the First Analogy. I think he also wants to argue that the total *amount* of substance is conserved—as it were the substantial stuff that composes individual substances. Indeed, as I understand Kant, he thinks that individual

all change in time can only be regarded as a modus of the existence of that which lasts and persists. Therefore in all appearances that which persists is the object itself, i.e., the substance (*phaenomenon*), but everything that changes or that can change belongs only to the way in which this substance or substances exist, thus to their determinations... in all alterations in the world the substance remains and only the accidents change. (A183-4/B227)

This is enough to establish the validity of the Barcan formulas for empirical modality when our domain is that of phenomenal substances. The reason is the way in which empirical possibility is indexed to the actual world. Say that something is empirically possible just in case it is compatible³⁰ with the actual laws of nature and some actual state of the world, where a state of the world is a complete description of the phenomenal world at a time. Any actual state of the world takes you to a point of time in the actual world and tells you what phenomenal substances exist at that time. The law of nature that is the principle of the First Analogy then tells you that, if those substances exist at that time, then those and no others exist at all times. It would thus be incompatible with the actual laws of nature and some actual state of the world that any of those substances do not exist or that any other substances do exist. More generally and in model-theoretic terms, any state of the world of evaluation together with the principle of the First Analogy fixes the domain of phenomenal substances for all worlds empirically accessible from that world. It is thus empirically impossible for what phenomenal substances there are to vary across empirically accessible worlds. The Barcan formulas are valid for empirical modality. We can confirm this result by looking at our test cases.

Test Cases

substances persist and the total amount of substance in the world is conserved *because* the total amount of substance in any individual substance is conserved. My point here is more generic: I can allow that Kant often uses “substance” as a mass-noun to refer to the substance of substances, and that he means to establish the persistence of what is thereby named, so long as he also thinks that the kind of things that are the values of our individual count-noun variables, i.e. individual substances, persist. For discussion see Paton (1936b, ch.42), Bennett (1974, ch.3, ch.9 sec.56), Guyer (1987, 230–35), Langton (1998, ch.3), Friedman (2013, chs.2-3), Watkins (2019, ch.3), and Messina (2021).

³⁰ I leave the notion of compatibility unanalysed for now but see fn.34 below.

\diamond_E is our empirical possibility operator and our quantifiers are restricted to phenomenal substances. This time we can consider instances of the Converse Barcan Formula and the Barcan Formula together:

$$\begin{aligned} \text{E1} \quad & \neg\diamond_E \exists x \neg Ex \rightarrow \neg\exists x \diamond_E \neg Ex \\ \text{E2} \quad & \neg\exists x \diamond_E @ \neg Ex \rightarrow \neg\diamond_E \exists x @ \neg Ex \end{aligned}$$

The antecedents of E1 and E2 follow from KE ($\neg\diamond_L \exists x \neg Ex$), Kant's claim that existence is not a real predicate. Recall from §4 that the operators in the antecedent of E2 are redundant, so it is equivalent to $\neg\exists x \neg Ex$. Thus it follows from KE on the assumption that logical impossibility entails falsity. The antecedent of E1 likewise follows from KE given two very plausible assumptions. First we assume that logical impossibility entails empirical impossibility—contradictions cannot be realized in nature. Second we assume that phenomenal substances are things—they are things in space and time. If existence is not a real predicate so that it is *logically* impossible that there is *something* that does not exist, which is what KE says, then it must also be *empirically* impossible that there is *a phenomenal substance* that does not exist, which is what the antecedent of E1 says.

Would Kant accept the consequents of E1 and E2? He would. They say: there is no phenomenal substance for which it is empirically possible that it not exist; and, it is not empirically possible for there to be a phenomenal substance that actually does not exist. That is, together they say that it is not empirically possible for the domain of phenomenal substances to vary from the phenomenal substances there in fact are. And we have seen that this is part of what Kant aims to establish in the First Analogy. Whatever is actually in the domain of phenomenal substances, it is fixed relative to empirical possibility so that it is not empirically possible to either take such a substance away from the domain or add such a substance to it. Kant would therefore accept these instances of the Barcan formulas for empirical modality. Of course, accepting instances of a schema does not require accepting the schema in the way that rejecting instances of a schema requires rejecting the schema. That is why we had to go over the general result above. But these cases nevertheless confirm what we saw there.

There are technical and philosophical corollaries to Kant's acceptance of the Barcan formulas for empirical modality.

Technical Corollary

First of all note that Kant would have to reject as the base predicate logic for a quantified modal logic of empirical modality the universal free logic that would invalidate the Barcan formulas. For he would accept that it is empirically necessary that some phenomenal substance exists—the domain of individuals cannot be empty when what is at issue concerns what is possible and what is necessary given the actual laws of nature *and some actual state of the phenomenal world*. But a more interesting and far stronger technical corollary to Kant's acceptance of the Barcan formulas for empirical modality stems from his determinism about the phenomenal world. There are many textual resources to draw on with regard to this point. I focus on the most salient.

Empirical modality is what Kant is concerned with in the third of the Postulates of Empirical Thinking: “That whose connection with the actual is determined in accordance with general conditions of experience is (exists) **necessarily**” (A218/B266). He says in elucidating this postulate that “It certainly looks as if one could increase the number of that which is possible beyond that of the actual, since something must be added to the former to constitute the latter. But I do not acknowledge this addition to the possible” (A231/B284). This looks like a commitment to the determinist view that whatever is empirically possible is also actual. We can formalize this view with a version of the TRIV-axiom: $\Diamond_E \phi \rightarrow \phi$. And if we add such an axiom to a system of alethic modality (in which the T-axiom holds), then the possible, the actual (or true), and the necessary all become equivalent and modal operators become redundant.³¹ The Barcan formulas are trivially valid on such a system, for without their modal operators, they simply state the equivalence of $\exists v \phi$ with itself and $\forall v \phi$ with itself. Kant's determinism about the phenomenal world requires his acceptance of the Barcan formulas for empirical modality.

Philosophical Corollary

³¹ See e.g. Hughes & Cresswell (1996, 65).

The philosophical corollary of Kant's acceptance of the Barcan formulas for empirical modality is that Kant accepts a form of Modal Particularism for this kind of modality. That is, in the way required for Modal Particularism, he holds that general truths about what is empirically possible are grounded in particular truths about things and their properties. Specifically, he holds that general truths about what is empirically possible are suitably grounded in truths about phenomenal substances and their real essences. I cannot give a full account of this complex issue here and I will just assume the following basic picture. As with my account of Kant's doctrine that existence is not a real predicate, it will not be uncontroversial, but it suffices for present purposes that the account is textually well-supported and widely shared.³²

The Second Analogy (A189–211/B232–56) seeks to establish that every change in the accidents of a phenomenal substance, i.e. every alteration, is the effect of a determining cause. Causation is not a relation between alterations but between (the activation of) a causal power in one substance and an alteration in another substance. Causal powers are not themselves accidents of substances. Rather they are part of the essential natures of substances, their real essences. The Third Analogy (A211–18/B256–65) seeks to establish that all spatial substances stand in thoroughgoing mutual interaction insofar as they are simultaneous, i.e. exist at the same time. A set of substances stand in thoroughgoing mutual interaction just in case each member of the set causally affects every other member, or equivalently, just in case each member of the set is causally affected by every other member. Thus if substances persist, as per the First Analogy, it follows that all spatial substances stand in thoroughgoing mutual interaction at all times in and throughout one time.

Laws of nature describe the relations between real essences and accidents. They are either causal and describe the relations between causal powers in one phenomenal substance and alterations in another, or they are non-causal and describe the relations between the accidents of one phenomenal substance and its own real essence. For Kant, then, it is particular truths about the real essences of phenomenal substances that ground both the laws of nature, which describe how essences determine accidents, as well as the states of the world, which describe how things are with essences and the accidents they determine at a time.

³² The account is drawn primarily from Watkins (2005; 2019) and Stang (2016; 2019).

And since grounding is transitive and general truths about what is empirically possible are in turn grounded in the actual laws of nature and some actual state of the world, they will in turn be grounded in particular truths about the real essences of actually existing phenomenal substances.

There is an important complication here. Clearly Kant thinks that truths about our forms of experience also play some role in grounding the laws of nature and the states of the world. Thus it would seem that particular truths about the real essences of phenomenal substances can only be *partial* grounds of the laws of nature, the states of the world, and thereby of the general truths about what is empirically possible.³³ And this can seem to undermine my claim that what we have here is a philosophical corollary to Kant's acceptance of the Barcan formulas for empirical modality. The problem is that the argument from Modal Particularism to the Barcan formulas that was presented in §2 can fail for merely partial grounding. Specifically, there is no guaranteed grounding-entailment link for partial grounding. If A is only partially grounded in B then it will not *in general* be true that B entails A. For B might obtain without A obtaining, *if* B can obtain while the other partial grounds of A do not. Is the form of Modal Particularism that we have here too weak to be suitably connected to the Barcan formulas?

Fortunately, the present case is one in which the grounding-entailment link required to pair Modal Particularism with the Barcan formulas holds even though the relation is only one of partial grounding. For the particular truths about phenomenal substances and their real essences cannot obtain unless the relevant truths about our forms of experience also obtain. That is, the former partial grounds are not suitably independent of the latter partial grounds for their status as such—as merely partial grounds—to generate failures of the required

³³ I should mention two alternative reactions to this observation. One would be to take our forms of experience to be themselves part of the real essences of phenomenal substances, in which case there is no issue for my account. Another would be to take their grounding role to play out at one level down, i.e. to combine with some feature of noumenal reality in grounding the real essences of phenomenal substances, in which case the relevant response would be to point out that nothing in Modal Particularism or its relation to the Barcan formulas requires that the grounding it posits be terminal. My approach in the main text has been to argue that my claim holds even for the hardest case. (I take it that structurally similar responses would be available to the objection that the relevant ground in the *Metaphysical Foundations of Natural Science* is the generic real essence of a *kind* of phenomenal substance—matter—rather than that of individual phenomenal substances.)

grounding-entailment link. So it does not here matter that the particular truths about phenomenal substances and their real essences are merely partial grounds of the general truths about what is empirically possible—Kant’s weak form of Modal Particularism about empirical modality will still validate the Barcan formulas by the argument of §2. And we saw above that accepting the Barcan formulas for empirical modality was required by his determinism about the phenomenal world. So once again, we see that Kant has a unified and sophisticated system of views.

Here is an equivalent way to state the view I have just sketched. Particular truths about phenomenal substances and their real essences fully ground the general truths *about the formally contingent aspects* of empirical modality, i.e. those aspects of empirical modality that are not fully grounded in, and thereby fully determined by, our forms of experience alone. It is to this formal conception of modality that I now turn.

6. Formal Modality

In the first of the Postulates of Empirical Thought, Kant characterizes a kind of modality that is absolutely central to his Critical philosophy: “Whatever agrees with the formal conditions of experience (in accordance with intuition and concepts) is **possible**” (A218/B265). Call this *formal* possibility. The formal conditions of experience are the forms of sensibility and intuition—space and time—and the forms of understanding and thought—the categories, the logical functions of judgment, and the laws of logic. The appropriate domain of individuals for formal modality will again be that of phenomenal substances, or things in space and time. But note that the notion of formal possibility defined in the first Postulate is not the dual of the notion of empirical necessity defined in the third Postulate. This is to be expected. The first and third Postulates do not look at all like they define duals and Kant suggests that the third category under each moment is somehow supposed to arise from the synthesis of the first two (B110–11), rather than arising definitionally via negation from only the first, in the way of duals (i.e. $\Box\phi =_{\text{df.}} \neg\Diamond\neg\phi$). Thus, very roughly, the empirical necessity defined in the third Postulate arises from constraining the formal possibility defined in the first Postulate by way of a connection to the actual defined in the second Postulate. This picture will be confirmed in what follows. Let us turn to our test cases.

Test Cases

\diamond_F is our formal possibility operator, with \square_F defined in the usual way, and our quantifiers are restricted to phenomenal substances, or things in space and time. Consider the following instance of the Converse Barcan Formula, which by now will be familiar:

$$\text{FI} \quad \square_F \forall x Ex \rightarrow \forall x \square_F Ex$$

Since spatiotemporal things are things, the antecedent follows from KE^* ($\square_L \forall x Ex$), i.e. Kant's claim that existence is not a predicate, if we assume that logical necessity entails formal necessity. This assumption is warranted because, for Kant, the law of non-contradiction, which defines logical possibility, is an aspect of the forms of the understanding and thought, and thus of our forms of experience (A150–53/B189–93, A596/B624). If nothing can agree with the forms of experience unless it is free from contradiction and hence logically possible, then logical impossibility entails formal impossibility, or equivalently logical necessity entails formal necessity.³⁴ If existence is not a real predicate so

³⁴ This is clear on e.g. model-theoretic ways of thinking about modality. Our forms of experience add constraints to those of mere thought and conceptual consistency so the set of formally possible worlds is a proper subset of the set of logically possible worlds. But it also holds if we think of modality in terms of grounding. Roughly and in terms of propositions (modified from Stang 2016, ch.7): p is formally possible iff it is not the case that some fact or facts about our forms of experience wholly ground $\neg p$. If the law of non-contradiction is an aspect of our forms of experience, then for any p that is a contradiction and hence is logically impossible, it will be the case that some fact or facts about our forms of experience wholly ground $\neg p$, which is to say that logical impossibility entails formal impossibility, or equivalently that logical necessity entails formal necessity. Note that, by duality, this kind of view will have it that logical necessities are wholly grounded in some fact or facts about our forms of experience. But this does *not* entail that logical necessities are wholly grounded in *the fact that we have the particular forms of experience that we do*, or anything else that sounds suspiciously logically contingent. For grounding is nonmonotonic: if A is grounded in B it does not follow that A is grounded in any fact that includes B . Moreover, grounding can be complete and multiple, as when existential generalizations are wholly and multiply grounded in their true instances. Thus while logical necessities will be wholly grounded in some fact or facts about our forms of experience, on this view, they will also be wholly grounded in some fact or facts about every other form of cognition, insofar as such forms also include those of thought and hence the law of non-contradiction, and thus regardless of whether or not they are specifically spatiotemporal or even sensible. Thinking of logical necessities as wholly grounded in some fact or facts about our forms of experience does not preclude thinking of them, and with them the laws of logic, as more fundamental and more

that it is *logically* necessary that *everything* exists, then it will be *formally* necessary that every *spatiotemporal* thing exists.

Would Kant accept the consequent of F1? He would not. It says that every spatiotemporal thing is such that it is formally necessary that it exists. If this were true, then we could know *a priori* through reflection on the forms of experience that something in space and time exists. To be formally necessary is to be required by, or wholly grounded in, the forms of experience. It is, for instance, formally necessary that everything we experience is spatiotemporal and causally integrated. We can know this *a priori* through reflection on the forms of experience. But we cannot know existence claims about things in space and time in this way. This time the problem is not the synthetic nature of such claims, as it was in §4. We can know synthetic claims through reflection on the forms of experience, if not through reflection on the mere forms of thought. This time the problem is the *a posteriori* nature of existence claims, at least about spatiotemporal things. Kant says “the existence of appearances cannot be cognized *a priori*” (A178/B211) and “no existence of objects of the senses can be cognized fully *a priori*” (A226/B279). For nothing about the forms of experience requires that spatiotemporal things exist. In other words, for us to know that a spatiotemporal thing is actual, it has to agree not only with the forms of experience but also be “connected with the *material* conditions of experience (of sensation)” (A218/B266, emphasis added). Kant would accept the antecedent of F1 but reject its antecedent—he would reject the Converse Barcan Formula for formal modality.

Similar resources can be used to show that Kant would also reject the other direction of the Barcan formulas for formal modality, the Barcan Formula itself. Consider the following instance, which again will be familiar:

$$F2 \quad \neg\exists x \diamond_F @ \neg Ex \rightarrow \neg\diamond_F \exists x @ \neg Ex$$

As with previous cases, the antecedent of F2 follows from KE ($\neg\diamond_L \exists x \neg Ex$), Kant’s claim that existence is not a real predicate. The operators in the antecedent of F2 are redundant for the reasons given in §4, so it is equivalent to $\neg\exists x \neg Ex$,

general than our particular forms of experience. (For discussion of this issue see Gomes, Moore, and Stephenson (2022).) Parallel points hold for the assumption I made in §5, that logical impossibility entails empirical impossibility.

which follows from KE on the assumption that logical impossibility entails falsity (A59/B84, A151/B190). What about the consequent of F2? It says that it is not formally possible for there to be a phenomenal substance that actually there is not. Kant would reject this because he thinks it is formally possible for a phenomenal substance to exist that actually does not exist; he thinks it is formally possible to add to the domain of phenomenal substances. The argument for this parallels that against the Barcan Formula for logical modality.

Recall that in §4 I argued that, for Kant, it must be logically possible to add to the domain of things because otherwise every logical possibility would have to be a logical possibility for something that actually exists, and that in turn would mean that we can always prove that something actually exists merely by proving a logical possibility, i.e. merely by showing a concept to be consistent. This argument had two components. First, a conditional: if it is not logically possible to add to the domain of things, then any method for proving logical possibility must also be a method for proving that something actually exists. Second, a counterexample to the consequent of this conditional that allowed us to reach our conclusion by *modus tollens*. Since proving conceptual consistency is a method for proving logical possibility that is *not* also a method for proving that something actually exists, we could infer that it *is* logically possible to add to the domain of things. A parallel argument can be made for formal modality.

First note that the above conditional generalizes, for it is simply a manifestation of the way in which the Barcan Formula allows us to move between *de dicto* and *de re* modalities. Thus we have a corresponding conditional for the present case: if it is not formally possible to add to the domain of phenomenal substances, then any method for proving formal possibility must also be a method for proving that some phenomenal substance actually exists. To be more specific, we can reason as we did in §4. Proving the formal possibility of pink elephants, even though none exist, would *not* be enough to falsify the consequent of F2, for pink elephants might be formally possible only because there is some phenomenal substance that actually exists that could, in the relevant sense, be a pink elephant. But to salvage the consequent of F2 in its full generality in this way, it would have to be the case that every formal possibility is merely a formal possibility for some phenomenal substance that actually exists. And this in turn would mean that any method for proving formal possibility must also be a method for proving that some phenomenal substance actually exists. We can now draw on the

resources above, with one additional assumption, to show that this is not the case.

We saw above that Kant thinks that the formally necessary is knowable *a priori* and that existence claims about things in space and time are not knowable *a priori*. Now assume further that Kant would accept the characteristic axiom of S₅ for formal modality, so that truths about what is formally possible are themselves formally necessary ($\diamond_F \phi \rightarrow \Box_F \diamond_F \phi$).³⁵ It follows that formal possibilities are knowable *a priori* and hence in a way that existence claims about things in space and time are not.³⁶ Thus it is not the case that every method for proving formal possibility is also a method for proving that some phenomenal substance actually exists. By *modus tollens* on our conditional we can thus conclude that it *is* formally possible to add to the domain of phenomenal substances—Kant would reject the Barcan Formula for formal modality.

There are technical and philosophical corollaries to Kant’s rejection of the Barcan formulas for formal modality. As we may now expect, they closely parallel those for logical modality.

Technical Corollary

We saw above that Kant thinks we cannot have *a priori* knowledge of the existence of phenomenal substances, or things in space and time. Yet he would

³⁵ He says e.g. that “possibility in general is certainly necessary” (R 3712, 17:252; cf. OPA 2:83–84). To deny this would be to hold that what is possible could have been impossible; for alethic modalities, like formal modality, it amounts to the view that modal status in general is necessary. In terms of grounding and agreement, then, the claim here is that whether or not some putative thing agrees with or is wholly grounded in facts about our forms of experience will itself be wholly grounded in facts about our forms of experience. See Chignell (2009, 167) and Stang (2016, 127, 134).

³⁶ What, specifically, is Kant’s *a priori* epistemology for formal modality? Considerations of conceptual consistency and containment, which yield only analytic knowledge of logical modality, cannot suffice. Knowledge of formal modality must be synthetic—it must involve intuition. I suspect Kant offers an imagination-based epistemology for formal modality. He thinks that whether or not a concept represents a formal possibility tracks whether or not an object corresponding to that concept can be ‘constructed’ or ‘exhibited’ in intuition, where this process takes place in the pure or productive imagination and the imagination can play this role because it has the very same forms as those of experience, i.e. those that define formal possibility. For some especially relevant texts see A94–102, B151–52, A140–46/B179–85; Disc. 8: 191–92; Prog. 20:325–26; Anthr. 7:153, 167–68; MFNS 4:480–508. For relevant discussion see Stephenson (2015), Matherne (2016), Grüne (2017), and especially Rosefeldt (2021).

allow that we can have *a priori* knowledge that it is formally necessary that every such thing exists (i.e. the antecedent of F1). For as we also saw above this follows from the doctrine that existence is not a real predicate on the assumption that logical possibility is a condition on formal possibility, and Kant would presumably think that both of these are *a priori* knowable and that *a priori* knowability is closed under (*a priori* knowable) entailment: if we can know *a priori* that *p* and that *p* entails *q* then we can know *a priori* that *q*. Thus on pain of inconsistency, Kant would have to deny that “every phenomenal substance exists” entails “some phenomenal substance exists”. That is, he would have to reject classical logic in favor of a universal free logic for formal modality, and the Barcan formulas are invalid in systems of quantified modal logic that take a universal free logic as their base logic.

Philosophical Corollary

The philosophical corollary of Kant’s rejection of the Barcan formulas for formal modality is that he would reject Modal Particularism for formal modality. Kant does not think, in the way required for Modal Particularism, that general truths about what is formally possible are grounded in particular truths about things and their properties. Rather, he thinks that general truths about what is formally possible are grounded in particular truths about our forms of experience, that they are space and time and the categories etc. But our forms of experience are not *things*—they are not substances in which accidents inhere. So Kant rejects Modal Particularism for this case.

There is an instructive complication here. Are not our forms of experience properties of ourselves and therefore of things? Are they not part of our real essences? It is a complicated question in what sense Kant might think we are or can be known to be substances (A341–405/B399–432). He does not think we are phenomenal substances in space and time. But he does connect our cognitive faculties and thus their forms to the notion of power and the notion of power to that of substance (A204/B249; MMr 29:771; R 5650, 18:298–300).³⁷ So might there be a sense in which even formal modality is grounded in things and their properties?

³⁷ For discussion see Rosefeldt (2000, ch.3), Kitcher (2011, ch.10), Warren (2015), Longuenesse (2017, chs.4-5), Boyle (2020), and McLear (2020).

Suppose there is. This would not undermine my claim that Kant rejects Modal Particularism for formal modality. For the general possibilities that are grounded in the forms of our cognitive faculties and thus in ourselves and our properties need not be possibilities *for us*, as would be required for Modal Particularism. In the technical terms of the Barcan formulas, our assignments of values to variables must be *uniform* across the *de dicto* and *de re* sides of the conditional. That was the case for empirical modality but not here: it is empirically possible that there are phenomenal substances that ϕ because there are phenomenal substances for which it is an empirical possibility that they ϕ , while it is formally possible that there are phenomenal substances that ϕ because there are cognizers with certain forms of experience that ground this fact even when they themselves cannot ϕ , for instance because ϕ -ing requires being something substantial in space and time. The point is instructive, however, because it allows us to elaborate on the relation between Modal Particularism and Kant’s principle about possibility being grounded in actuality (§2).

Kant says in the pre-Critical *Beweisgrund* essay that “All possibility is given in something actual, *either as a determination existing within it or as a consequence arising from it*” (OPA 2:79, emphasis modified). This is supposed to hold not of logical modality but of what Kant would later call “real” modalities, that is, of modalities that are not merely for concepts but for things (A244/B302; A596/B624). It is widely held that some such principle survives into the Critical-era and the present picture supports this.³⁸ Both empirical and formal modality, as modalities for things, are real modalities, and both satisfy the principle, albeit in different ways that correspond to their contrasting relations to Modal Particularism and the Barcan formulas. The difference is not simply that in the empirical case the actual grounds are things while in the formal case they are

³⁸ See fn.8 for references. It might be objected that the points made here about formal modality could also be made about logical modality, insofar as we think of the law of non-contradiction as an aspect of our forms of experience (fn.34). By these lights, won’t even logical modality satisfy Kant’s actualist principle? It won’t. At most, logical modality is *in fact* grounded in our forms of experience (albeit non-fundamentally and while also being grounded in every other form of cognition—see fn.34). But Kant means for his principle to posit a *conceptual*, logically *necessary* connection between real modality and being grounded in something actual. He says “we have no *concept* of real possibility except through existence” (PR 28:1036). This is not the case for logical modality, for it is logically contingent that anything exists, and so logically contingent that logical modality is grounded in anything actual, even God.

forms. It is, moreover, that in the empirical case the grounded possibilities are “determinations existing within” the actual grounds, and thus are possibilities for those grounds, validating Modal Particularism and the Barcan formulas, while in the formal case they are “consequences arising from” the actual grounds, and thus need not be possibilities for those grounds, invalidating Modal Particularism and the Barcan formulas.

We are now in a position to generalize, and thereby further unify and explain, the results of the last three sections. Kant insists on the complete separation of matters of existence in general, of what there is or which things there are, from matters of logical possibility and necessity, of conceptual consistency, analyticity, and the laws of pure general logic. He would therefore reject the Barcan formulas and Modal Particularism and adopt a universal free logic for logical modality. The above suggests that the same is not quite true of formal modality, which must be grounded in something actual, namely ourselves and our cognitive forms. But for Kant such forms “lie ready...in the mind *a priori*” (A20/B34) and thus formal modality is still a pure science, corresponding not to pure general logic but to a pure transcendental logic (A55–57/B79–82). As such it remains entirely independent of what phenomenal substances there are, of which things there are in space and time, which is to say of the existence of the things for which it is a modality. So Kant would likewise reject the Barcan formulas and Modal Particularism and adopt a universal free logic for formal modality. And the converse holds for empirical modality, which is grounded in phenomenal substances and their properties in the way required for Modal Particularism, is therefore not *a priori* but rather conditioned by “the material conditions of experience” (A218/B266), and for which Kant would reject universal free logic but accept the Barcan formulas.

My aim in this essay has been to shed light on Kant’s theory of modality by means of a comparison with the Barcan formulas. I have argued that this comparison provides a new and fruitful perspective from which to appreciate the ways and extent to which Kant’s complex and sometimes confusing claims about possibility and necessity form a unified and sophisticated system of views. These views, as well as the kind of interplay we have seen emerge between matters logical, metaphysical, and epistemological, are by no means peripheral to Kant’s

philosophy, and they are intimately connected to the issues surrounding the Barcan formulas.³⁹

Abbreviations

I have used the following abbreviations when referencing works in *Kants Gesammelte Schriften* (Berlin: De Gruyter and predecessors, 1900–).

Anthr. = *Anthropology from a Pragmatic Point of View*

Disc. = *On a Discovery Whereby Any New Critique of Pure Reason is to be Made Superfluous by an Older One*

JL = *Jäsche Logic*

MFNS = *Metaphysical Foundations of Natural Science*

ML₂ = *Metaphysics Anon-L₂*

MMr = *Metaphysics Mrongovius*

OPA = *The Only Possible Argument in Support of a Demonstration of the Existence of God*

PR = *Pölitiz Religion / Lectures on the Philosophical Doctrine of Religion*

Prog. = *What Real Progress has Metaphysics Made in Germany Since the Time of Leibniz and Wolff?*

Prol. = *Prolegomena to Any Future Metaphysics that will be able to Come Forward as a Science*

R = *Reflections*

Translations have been from the following works in the *Cambridge Edition of the Works of Immanuel Kant* (Cambridge: Cambridge University Press, 1992–):

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Critique of Pure Reason, edited and translated by Paul Guyer and Allen W. Wood. 1997.

³⁹ Acknowledgements

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