Entanglement of two Josephson junctions: Current Locking revisited.

G. Stephens[1]

Abstract.

In this essay we take the view that too much reality has been afforded to the notion of 'particles' and to 'flow of supercurrent,' in the superconducting state. Instead we take the original point of view of Josephson that " It is clear that intuition is of no great help in understanding the supercurrent as a flow of Cooper pairs " which is more akin to, and in line with, a "telegraphing of amplitudes" approach. With this conception in mind, we examine the results of Jillie et al and Smith et al. of two Josephson junctions connected in series by a superconducting join. We argue that

their results can best be understood in terms of the entanglement of current elements via the interfering of amplitudes. We sketch an approach to calculating the current spanning two entangled Josephson junctions, which reduces to the relation for a single junction when the current is set zero in either of the pair, or the entanglement ceases.

We speculate that if this interfering of amplitudes was found to persist, after the separation of the junctions in space, there still remaining a connection in their common past, then this would furnish, at least the possibility, of a new means of signalling without wires. Experiments are suggested.

1. g_stephens_@hotmail.com

Part I.

I. Introduction.

Since antiquity Mankind has wondered about the matter perceived to be around him. For a long time it was thought to be composed of particles,-- that is was particulate. The first signs that this might not be so came at around the time of the rays of light of Newton, in 1671. Newton was aware of Hooke's wavelike viewpoint for light matter, and even partook in demonstrations upon this hypothesis, in letters to Hooke.

But it was not until de Broglie in 1924 guessed that all matter might be wavelike--- in his so called 'matter wave' hypothesis, that the wavelike view became more well known, if not entirely understood.

Now the essential characteristic of a particle is that it is at one point. The essential characteristics of a wave is that it is at more than one point. So the clue to the real nature of matter might lie more in the notion of it being at more than one point, than in other ways of looking at it. This has come into focus more recently--over the last half century, that is, in the interest that has been given to EPR-type entanglement for quantum particles. That is the innate ability for ALL matter (or its properties,) in the right circumstance, to be in two places at once, that is: to be entangled.

Following along this line of reasoning, if we could continually observe this wavelike aspect of matter, ---then we might, instead, in reality, be continually observing the matter being in two places at once. This is the subject of this essay.

Before we proceed to this subject proper, (in a more detailed way,) let us first say a few words on superconductivity—ie. about the matter in which we envision the above proposition might be true.

Probably the most accurate summary of superconductivity has been given by Josephson[1] "The characteristic features of superconducting systems, the Meissner effect, zero resistivity, quantised persistent currents in macroscopic rings and quantised flux lines, are a consequence of a long-range ordering process of an essentially quantum nature."

The first two, the Meissner effect (B=0) and zero resistivity (E=0) can be thought of as one and the same thing, when viewed through the prism of Relativity, insofar as the electric field transforms partly into the magnetic field and vice versa, in different frames, moving uniformly with respect to each other.

The latter two, persistent currents in rings and quantised flux lines, are essentially wave aspects, or perhaps, as we ventured above, aspects of matter being extended in space-being in more than one place at once.

Excepting quantised flux lines, it is worth noting that none of the above effects, listed by Josephson, were predicted or guessed at in advance, by the ingenuity of Mankind; they were discovered only by chance.

Ia. The Orthodox view of Superconductivity.

There is a dogma that has grown up over the years, since the discovery of superconductivity by Kamerlingh Onnes in 1911, that is best characterized as 'the particulate school of reality'.

Its professors and promulgators are long and distinguished. One of the earliest was

Einstein[2], who in 1922, envisioned a chain of molecules, each passing an electron from left to right along the chain: "It seems unavoidable that superconducting currents are carried by closed chains of molecules (conduction chains) whose electrons endure ongoing cyclic exchanges".

Then there was the frozen crystal of electrons idea of Lindemann (1915) and J J Thomson (1922)[3] "the electron space-lattice can move unimpeded through the atom space-lattice"

The first real theory of superconductivity per se, was given in the 1930s and 40s by Fritz and Heinz London[4]. In the first place they devised, by heuristic means, an electro-dynamics, - they guessed at these electrodynamic relations which seemed to fit the main features of the recently discovered electrodynamic behaviour. It is known today as a 'phenomenological' model. In the second place they introduced the crucial idea of a 'wavefunction' for the superconducting condensate, albeit in a vague way.

There then emerged, shortly thereafter, another theory called the Ginzburg Landau [5] 'theory of superconductivity,' which was, in truth, an extension of the Londons' theory. They took F London's idea of a wavefunction and turned it into an 'order parameter' in such a way that no one could fathom if they were talking about a quantum mechanical wavefunction or a classical order parameter

It focused on the tapering-off regions of superconductivity,-- those regions in which the magnetic field, and normal-type or quasi normal-type current can exist.

---In our view these tapering off regions are places where the electric matter is continuously being subsumed into, and manifested out from, the superconducting matter. The superconducting matter itself being only interfering occurrences.---

The first faint signs that the superconducting matter might not be particulate came in the BCS and Gor'kov theories[6,7], of the late 1950s, early 1960s. Moreover, entanglement was implicit in Josephson's analysis of the supercurrent through barriers in his generalized BCS-Bogoliubov formalism.

The main idea of Bardeen, Cooper, Schrieffer, was the construction of a wavefunction out of quasi-particles, which were in turn linear combinations of creation and annihilation operators. To handle these operators, Gor'kov invented 'normal' and 'anomalous' Green's functions[8]:

$$G_{\alpha\beta}(x-x') = -i \langle N | T\ \psi_\alpha(x)\psi^+_\beta(x') | N \rangle$$

$$F_{\alpha\beta}(x-x') = -i \langle N | T\ \psi_\alpha(x)\psi_\beta(x') | N+2 \rangle$$

$$F^+_{\alpha\beta}(x-x') = -i \langle N+2 | T\ \psi^+_\alpha(x)\ \psi^+_\beta(x') | N \rangle$$

where $|N\rangle$ and $|N+2\rangle$ are the ground states of the system with numbers of particles N and N+2. T is the time-ordering operator and $\psi^{+}_{\beta}, \psi_{\alpha}$ the creation and annihilation operators.

The power of the Gor'kov method came from the ability to use the wavelike properties of Green's functions (ie. Fourier Transforms) in the discrete Heisenberg like picture of creation and annihilation operators.

By using the Gor'kov formalism, Josephson was able to discover, in a very ingenious way, purely superconducting processes, which look very much like, or are akin to, 'interfering occurrences'. These original conceptions of Josephson have, on account of their rather abstruse mathematical nature, been left somewhat obscured.

And so the GL 'order parameter' has won out in the minds of most researchers and experimentalists. They imagine a kind of super laminar ‘flow’ of particles, on account of them being in the same ‘state,’ a quantum state, in the lexicon.

Thus, on the whole, the orthodoxy so developed has not really changed from the earliest attempts to understand superconductivity. This orthodoxy is of ‘particles’ that ‘flow’, unimpeded.

Pre-amble:

Out of the Matrix mechanics of Heisenberg, and the Wave Mechanics of Schrodinger, inspired by de Broglie’s ‘matter waves,’ emerged a new type of mechanics, for the very small. Based on the old, but with new and baffling qualities and features. These new features of micro-mechanics so clearly and concisely set down in the brilliant exposition ‘Principles of Quantum Mechanics’ by Dirac, are still the subject of interpretation and fascination today.

One of the features that emerged was the abstract notion of a ‘state’ or ‘wavefunction’ which was supposed to represent a micro-particle---- an electron or atom of light.

Of the most interesting, yet most strange rules that emerged from the new micro-mechanics was the rule regarding how two or more particles, or particle ‘states’ were to be treated, in particular when they were proximate to each other, such that one particle or state might be able to interfere with the other particle or state.

Imagine we have an observable A taken over from the old mechanics, -- say position or momentum, or more generally any observable. To bring this observable over into the new micro-mechanics we say it operates on the particle state $|a\rangle$ and alters it in some way. A way to handle two particles, or two states $|a\rangle$ and $|b\rangle$ might be: that an observable A operating on the product $|a\rangle|b\rangle$ operates only on the $|a\rangle$ factor and commutes with the $|b\rangle$ factor. In which case we could simply write the combined state, for the two particles, as

$$|a\rangle|b\rangle = |b\rangle|a\rangle = |ab\rangle \qquad (1)$$

since the operation $A|b\rangle = |b\rangle A$ leaves $|b\rangle$ unaltered. This is not the general case though. If the operator A does not leave the $|b\rangle$ state unaltered, we have to add this contribution to the alteration, so for the combined state we write $|a\rangle|b\rangle + |b\rangle|a\rangle$. And so it was noticed that the general state for an assembly was given by the sum of the permutations for the separate particle states $|a\rangle, |b\rangle$.

$$\Sigma\ P\ |ab\rangle$$

$$|ab\rangle = |a\rangle|b\rangle + |b\rangle|a\rangle \qquad (2)$$

[There was found, by comparing results of calculation with experiment, to be two types of particle, one in which the permutation left the state unaltered $P\ |ab\rangle = |ba\rangle$ and one in which the permutation changed the sign $P\,|ab\rangle = -|ba\rangle$. In the case that we will be sketching shortly, we have the former case that leaves the sign unaltered.]

Another way of looking at this was discovered by Feynmann:

Say we have identified two different occurrences 1, 2 that might occur in a system of two particles a and b. Then these occurrences, whatever they were, could each be represented by a path, and the whole a diagram. These occurrences or alternatives as Feynman calls them, can interfere with each other.

The probability of these occurrences happening is given by:

$$|\varphi(1,2;a,b) + \varphi(2,1;a,b)|^2 = 4p \qquad (3)$$

Where $\varphi(1,2;a,b)$, $\varphi(2,1;a,b)$ are amplitudes for different occurrences. Cf. Ref (9).

We will see in the next section, that the occurrence of the current element being across junction 1 will interfere with the alternative occurrence of the current element being across junction 2 and vice versa. So we will see there is an interference of these two alternatives.

Feynman was able to take this idea of interfering occurrences, or alternatives, and go through the whole of quantum mechanics and quantum electrodynamics applying it to this or that case. This led to the 'Feynman diagram' approach.

II. Demonstration.

Here we will sketch a method to calculate the entangled supercurrent spanning two junctions, depicted below, together with the connecting voltage and current leads.

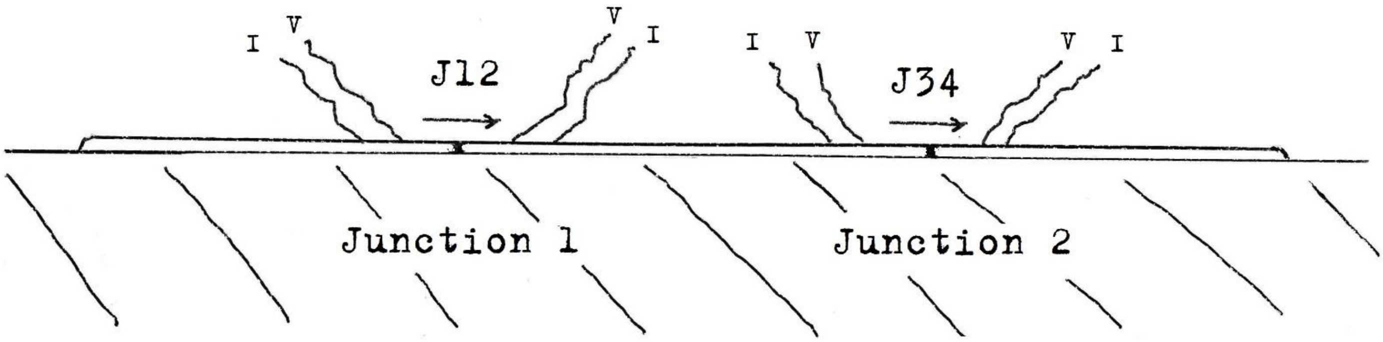


Fig. 1

Consider the three expressions below. The first is the standard equation in GL theory for the supercurrent. It is also exactly the same as the equation for current in the Schrodinger picture for a single particle. We will take the Ψ, below, to mean a 'wavefunction' proper —in the sense of the Schrodinger picture, and not an 'order parameter' as in a GL picture.

The second expression is the same as the first, but in zero applied magnetic field (B=0, A=0) and with $e\hbar/2m$ set equal to 1, to strip out clutter; here $|ab\rangle$ is taken the represent the wavefunction for the assembly of two junctions, a and b, in the sense discussed above. The third is merely a concise form of the second, with cc. denoting the complex conjugate.

$$j = -ie\hbar/2m \{ \Psi V \Psi^* - \Psi^* V \Psi \} - 2e^2|\Psi|^2/mc\ A$$

$$i \langle ab\ U|\ ab\rangle - i \langle ab\ |U\ ab\rangle$$

$$i \langle ab\ U|\ ab\rangle - i\ cc. \tag{4}$$

U is supposed to represent the disturbance to the superconducting wavefunction across a junction, (to the superconducting matter) initially undisturbed, brought about by the connecting leads supplying the current. The letter U was deliberately used instead of p 'momentum' or v 'velocity' to prevent the mind from thinking of 'flow' of particles,-- the p being strongly related to the classical notion of the motion of particles in classical mechanics, ie. possessing a definite position and inertia. Moreover, in what follows, we want the reader to perceive of a 'telegraphing of amplitudes,' rather than the 'flow' of physical matter. The action of U on ‹a| is to produce a wavefunction ‹a U| or amplitude ‹a U|a› for junction 1 (assumed to be of the form const. $e^{i\theta a}$) that represents that junction *being in a state of carrying a supercurrent*. And similarly the action of U on ‹b| is to produce a wavefunction ‹b U| or amplitude ‹b U|b› for junction 2 (assumed to be of the form const. $e^{i\theta b}$) that represents that junction *being in a state of carrying a supercurrent*. These supercurrents can now interfere with each other in the following manner.

$$j = \text{amp (J12 or J34)} = i \langle ab\, U|\, ab\rangle - i\ \text{cc.} \tag{5}$$

Using

$$\langle a\ U|a\rangle = \text{const.}\ e^{i\theta a} \quad \text{where} \quad \theta_a = \chi_2 - \chi_1$$

$$\langle b\ U|b\rangle = \text{const.}\ e^{i\theta b} \quad \& \quad \theta_b = \chi_4 - \chi_3 \tag{6}$$

We then have, up to a multiplying constant,

$$i \{ e^{i\theta a} e^{i\theta b} + e^{i\theta b} e^{i\theta a} \}$$
$$- i \{ e^{-i\theta a} e^{-i\theta b} + e^{-i\theta b} e^{-i\theta a} \} \qquad (7)$$

The 'minus the complex conjugate' just has the effect of multiplying the result (the imaginary part) by a factor 2. We will take the current to be real as other authors have done cf. ($j = e/m \, Re\{\phi^*(p\phi)\}$ cf. ref (10)) so that

by taking the Img. Part:

$= -i4 \, (e\hbar/2m)$

$\{ i \, Sin(\theta_a)Cos(\theta_b) + i \, Sin(\theta_b)Cos(\theta_a)\}$

(8)

We find, with $e\hbar/m = \varkappa$

$$j(\text{entangled}) = \varkappa \; 2/\sqrt{2} \; Sin(\theta_a + \theta_b \}$$

(9)

(With $1/\sqrt{2}$ inserted for normalisation)

Hereafter we will put $\varkappa$ as unity for the following reason. We do not necessarily believe that: while the telegraphing of amplitudes and their interference occurs—while in the pure superconducting state, that the matter in this state can be thought of as being composed of myriads of particles of mass m and a charge e. In the above, it was the quantum mechanical structure that we were most concerned with. That structure furnished the essential apparatus in the calculation of the entanglement current.

This quantum mechanical entanglement <u>acts</u> on the current fed into the system at the connecting leads, $|j1|$,$|j2|$, therefore the expression for the resultant entanglement current should be of the form:

$$j(J12 \text{ or } J34) = (|j1|+|j2|)2/\sqrt{2}\ \mathrm{Sin}(\ \theta_a + \theta_b\ ) \qquad (10)$$

(With $1/\sqrt{2}$ inserted for normalisation) where the phases θ_a, θ_b, handle the polarity of the applied current/supercurrent.

In the above we did not specify $\langle a|$, at the outset, but we assumed that when it was acted upon by the operator U, it produced a wavefunction $\langle a\ U|$; its amplitude was assumed to be of the form $\langle a\ U|a\rangle$ = const. $e^{i\theta a}$. This then represents the wavefunction carrying a current across the barrier, with phase difference $\chi_2 - \chi_1$. And similarly for the other barrier.

With this assumed wavefunction we then proceeded to calculate the supercurrent spanning both junctions, taking into account the interfering of amplitudes, which in this case amounts to the interfering of current elements occurring across junction 1 with the current elements occurring across junction 2. Once we have the expression for the resultant entanglement current, we can check to see if our choice of wavefunction for the current carrying junctions singly were good ones, by setting the current across one or other of the pair to zero, which is the same as setting the phase difference across one or other of the pair to zero (θ_a or θ_b = 0) and seeing if the expression reduces to the standard expression for the supercurrent across one junction on its own, ie. j = j1 sin (θ_a), which, (with the entanglement factor $2/\sqrt{2}$ set to unity,) is the case. This gives us a reasonable expectation that the assumed current carrying wavefunction used earlier, was a good one.

Remarks

(i)

What is interesting here, is that without knowing the expression for the supercurrent across one junction, j = j1 sin (θ), we obtain it in any case, by the circuitous route of <u>at first</u> finding the expression for two junctions which can interfere with each other in the entanglement sense, and then reducing that expression to the single junction case, by setting the phase difference, and thus the supercurrent, through one or other of the entangled junctions to zero, and taking out the entanglement factor $2/\sqrt{2}$; ie. setting this to unity.

The interpretation of the entanglement current, sketched above, is not fully understood, and needs further contemplation, and so <u>the following Remarks are uncertain.</u>

(ii)

In a situation in which a current jo is supplied through connecting leads at one end of a two junction system, and is collected by the connecting leads at the other end of a two junction system, it could be the case that the phases do not come into play when considering the resultant entanglement current through the junctions. This is because we are measuring the voltage across both junctions, and as the current is fed in at one end, the phases of both junctions change in unison, and so any effects of differences between these phase differences, (one junction to another,) is nullified. So that the equation for the entanglement current in this case

$$j(\text{entangle}) = jo\ 2/\sqrt{2}\ \text{Sin}(\ \theta_a + \theta_b\ ) \tag{12}$$

might better be expressed as

$$j(\text{entangle}) = jo\ 2/\sqrt{2}\ \text{Sin}(\varphi) \tag{13}$$

That is to say, the junctions can be looked upon as behaving like a single junction with phase difference φ.

Even though the phases do not come into play in this configuration, we still find the resultant supercurrent at each junction is at a higher value than would normally be the case without entanglement, by a factor $2/\sqrt{2}$.

It is because the current elements are partly in two places at once, and shows up in the experimental results for this configuration. Cf. Figures 5,5a Part II.

(iii) The "critical current" of a junction is that threshold current that may be passed in at the connecting leads, above which a voltage begins to develop between the junction terminals. This critical current can vary from junction to junction, depending on fabrication characteristics.

Suppose we measure the critical current of junction 1 on its own. We find it is 13½µA.

Suppose we then measure the critical current of junction 2 on its own. We find it is 16½µA.

Taking the sum of these two and dividing by 2 we get the average critical current for the two junctions, 15µA.

Suppose, now, we have the configuration described in Remark (ii) in which the current is passed in (by the connecting leads) at the left of the two junction system, and collected (by connecting leads) on the right of the two junction system. The voltage measured spans both junctions.

One might think that if the junctions are entangled then the maximum supercurrent for the two junction system, as described above, would be the average for the two junctions, ie. 15µA. However, since the entangled current is augmented by a factor $2/\sqrt{2}$, on account of the current at each junction being partly at the place of the other junction, then the actual maximum supercurrent, that may be passed in and out at the connecting leads, in this system, is 10.6 µA. So 10.6 µA should be the critical current of this two junction system.

This is in (what appears to be exact) agreement with the results in Figure 5, Part II.

Likewise, if for example, the critical current for Junction 1 was 20µA and the critical current for junction 2 was 20µA, then the maximum voltage free supercurrent one could pass from left to right through this two junction system, in the configuration described, would be 20µA x $\sqrt{2}/2$ = 14.1 µA.

(iv) In situations in which the connecting leads are placed across each junction separately, so that one can bias a supercurrent through one junction, and look at the effect (V,I) at the other junction, then the interference term Sin($\theta_a + \theta_b$) of the respective phase differences $\theta_a = \Delta\chi_a$, $\theta_b = \Delta\chi_b$ between the junctions comes into play.

From Sin $(\Delta\chi_a - \Delta\chi_b)$ or Sin $(\Delta\chi_a + \Delta\chi_b)$ or Sin $(-\Delta\chi_a - \Delta\chi_b)$ or Sin $(-\Delta\chi_a + \Delta\chi_b)$ we see that the overall entanglement current, j(entangle)=j(J12 or J34) spanning both junctions is either augmented or diminished depending upon the relative signs and magnitudes, -- the respective polarities, of the applied current across each junction. The net current—the resultant entanglement current j(J12 or J34) at each junction, is not that which is supplied by the connecting leads, but instead a single value, which is proportional to the Sine of the resultant phase difference of the entangled pair.

This circumstance is reflected in the Figures (6a) (6b) (6c) (6d) and 7(a) of Part II. Results.

(v)

Smith et al[11] studied the results of Jillie et al[12], and expanded upon them with their own findings. They concluded that the experimental results could not be explained without the inclusion of a 'coupling' term, Sin($\theta_a + \theta_b$) between the junctions, into their model.

We will discuss the results of Smith et al together with the results of Jillie et al in an intuitive way in Part II. Results.

(vi) Since the Josephson frequency relation is independent of the critical current of a junction, Ic, & moreover other fabrication characteristics, and since the effect of entanglement of two junctions is essentially to produce another junction with a different critical current, then it is difficult to see how the Josephson frequency relation would be affected. The only difference being, is that it is the entanglement current that oscillates, and we have $d(\theta_a + \theta_b)/dt = 2e/\hbar\ V$ or $d(\varphi')/dt = 2e/\hbar\ V$ instead of $d(\varphi)/dt = 2e/\hbar\ V$ for a single junction. (Cf. Remark ii, above)

III. Summary.

In concluding let us summarize our viewpoint in the follow way:

Electric matter can be thought of as dissolving into the superconducting condensate at the connecting leads, being transmitted through the condensate via the telegraphing of amplitudes, and then re-emerging or manifesting at the other end, before flowing outward, in the usual sense, along the exit leads, to complete the circuit. In the case of partitioned systems, like two junctions connected in series by a superconducting join, then interference between these telegraphing amplitudes, or possibility of occurrences may occur.

Once this conception is borne in mind, which is not new --- it can be found in the interpretations of a supercurrent by Josephson, then the possibility of interference between physically separated junctions, becomes real. Experiments along these lines are given at the end of Part II.

PART II. Experiments.

I. Introduction.

In 1976 DW Jillie, during the course of his PhD. investigations at Stony Brook, New York, discovered some very interesting yet difficult to explain interactions between two microbridge Josephson junctions connected in series by a superconducting strip.

Later on in 1990 Smith et al produced similar reports and expanded upon the results of Jillie et al.

We have seen in Part I. that if we follow the original conception of Josephson, ---which amounts to an interfering of amplitudes rather than a flow of physical matter,--- then there is a possibility that the supercurrent across one junction will interfere with the supercurrent across another junction connected in series, in such a way that a portion of the supercurrent elements across one junction are also present across the other junction due to quantum mechanical type entanglement: That is to say the supercurrent elements cannot be separated out, one junction from the other.

This possibility is summarized in the following expression:

j(entangled) =

$$j(J12 \text{ or } J34) = (|j1|+|j2|)2/\sqrt{2}\ \mathrm{Sin}(\theta_a + \theta_b) \quad (1)$$

which relates to the following arrangement.

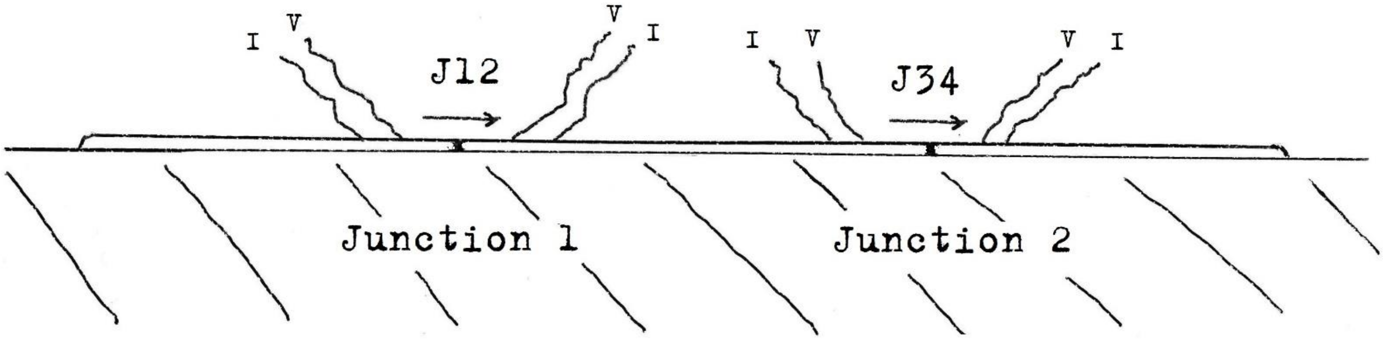


Fig.1

Where the polarity of the current supplied at each junction, |j1|, |j2|, is reflected in the sign of the phase difference, $+\theta_a$ or $-\theta_a$, $+\theta_b$ or $-\theta_b$, across each junction.

By the above expression (1) it is seen that, in the entangled state, the supercurrent occurring across junction 1, (J12) and the supercurrent occurring across junction 2, (J34) cannot be separated out, but are one and the same thing, j(entangled): they have the same value.

This value is proportional to Sin($\theta_a + \theta_b$), meaning, they are proportional to the sine of the combined sum of the phase differences, θ_a, θ_b across each junction.

This means this that, if one were to supply a fix current across junction 2, j2, and then continually alter the current and accompanying phase difference at the other junction,-- junction 1, j1, then, the resultant supercurrent, the actual or net supercurrent across junction 2 would not be that supplied by the connecting leads, but would be, instead, j(entangled), <u>the same for both junctions</u>.

Exactly such an experimental configuration is described in Fig 6c and 6d, below. These results were so baffling to the experimenters, that they attempted no explanation, other than to say there was a severe distortion of the current-voltage characteristic in this arrangement. But we will look at this in more detail, in turn, as we come to it.

These and results of this kind, for different configurations of the applied currents, were so extraordinary and incomprehensible, that Jillie et al invented a number of classical and quasi classical hypotheses, in an effort to make sense of their findings. They ended up with 5 or 6 separate hypotheses divided into two classes, depending on whether the currents supplied were in the same or opposite directions through the junctions.

Before we come to these results, and later the results of Smith et al, let us first say a few words about the experimental arrangement.

II. Jillie et al's Experimental arrangement

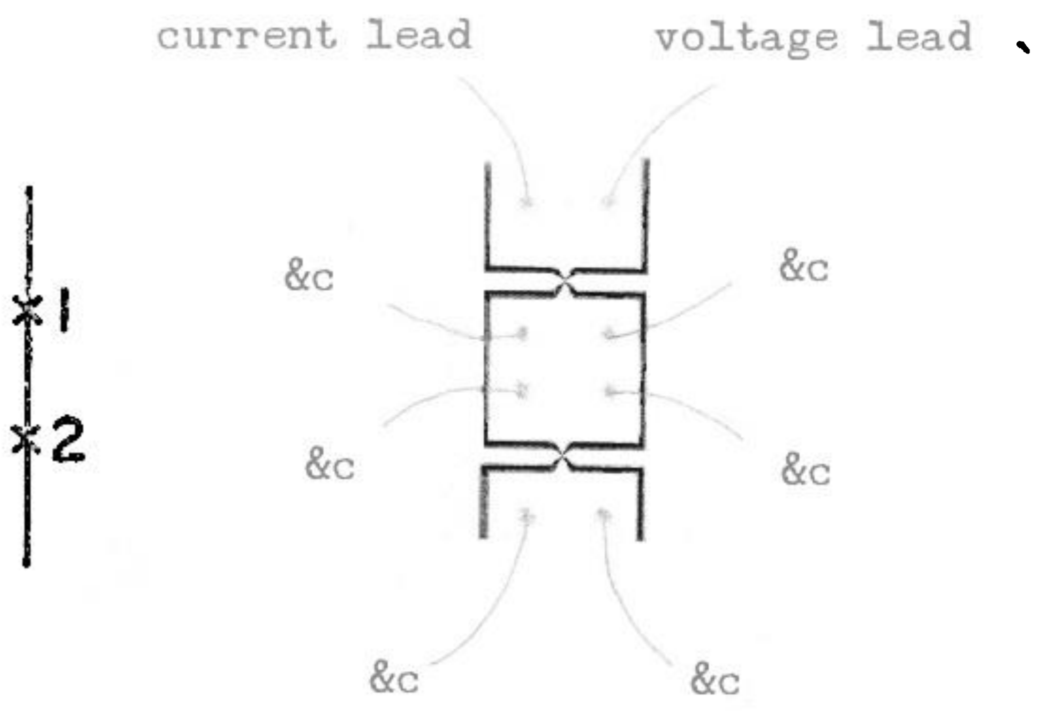


Figure 2. (Experimental arrangement of Jillie et al.)

Two microbridge Josephson junctions, denoted by 1 and 2 respectively are fabricated on a strip of Indium (In) or Tin (Sn). The distance between junction 1 and junction 2 in this example is 2µm. For details of the fabrication process please refer to Jillie et al (dissertation 1976). A 'four terminal' technique was used for connecting the voltage and current leads to the sample, as indicated in the diagram above.

The simplest question one can ask in the above arrangement is: what happens to the voltage-current characteristics or the supercurrent window when you bias a current through a. junction 1 only, b. through junction 2 only, or and lastly c. through junction 1 and junction 2 combined, the voltage in the latter case being taken across both junctions together.

II.a Results for junction 1 only.

Jillie et al obtained the following result[1] for case a. junction 1 only.

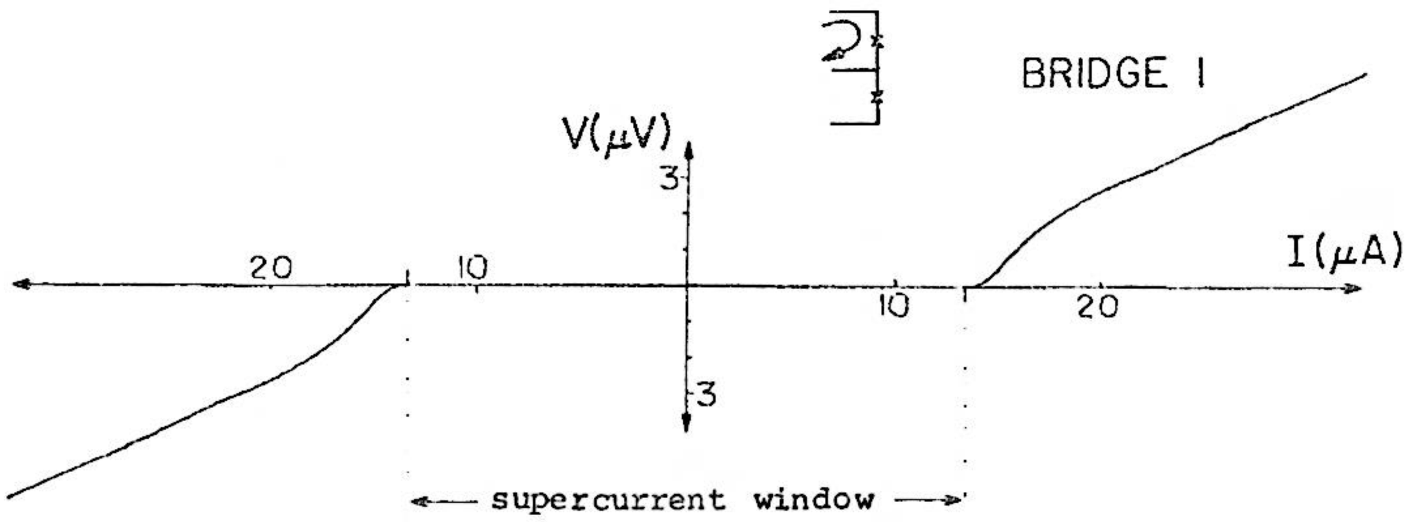


Figure 3a. (Result for case a. junction 1 only)

1. See APPENDIX 1.

It is this supercurrent window that varies in an oscillatory manner with respect to an applied magnetic field (or vector potential); the supercurrent window being dependent on the relative quantum mechanical phases across the divide[13]:

$$j = j1 \sin (\chi_2 - \chi_1 - 2e/m \int A_i \, da^i) \qquad (2)$$

But we will not consider the effect of an applied magnetic field in this essay. For simplicity we consider only the supercurrent through a junction or junctions with B=0. In this case the supercurrent is given simply by the famous Josephson relation:

$$j = j1 \sin (\chi_2 - \chi_1)$$

or $$j = j1 \sin (\theta) \qquad (3)$$

where θ is the phase difference $\theta = \chi_2 - \chi_1$, across the divide.

A supercurrent can increase through a junction until such time that a voltage begins to develop between its terminals. This threshold or 'critical current' can vary from junction to junction and depends on fabrication characteristics.

II.b Results for junction 2 only.

Jillie et al obtained the following result[1] for case b. junction 2 only

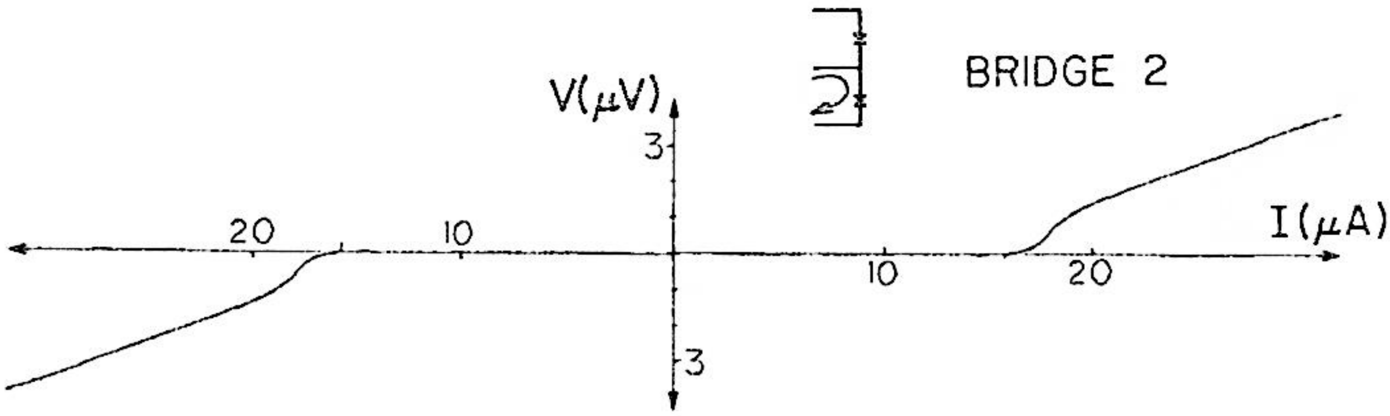


Figure 3b. (Result for case b. junction 2 only)

2. See APPENDIX 1.

We notice here the supercurrent window or the "critical current," of junction 2 is slightly larger than that pertaining to junction 1. In other respects these junctions appear broadly similar.

II.c Hypothetical results for junction 1 and 2 combined.

Let us pretend, for the time being, that there is no interaction between the junctions. In which case we might expect the following result for case c. junction 1 and junction 2 combined:

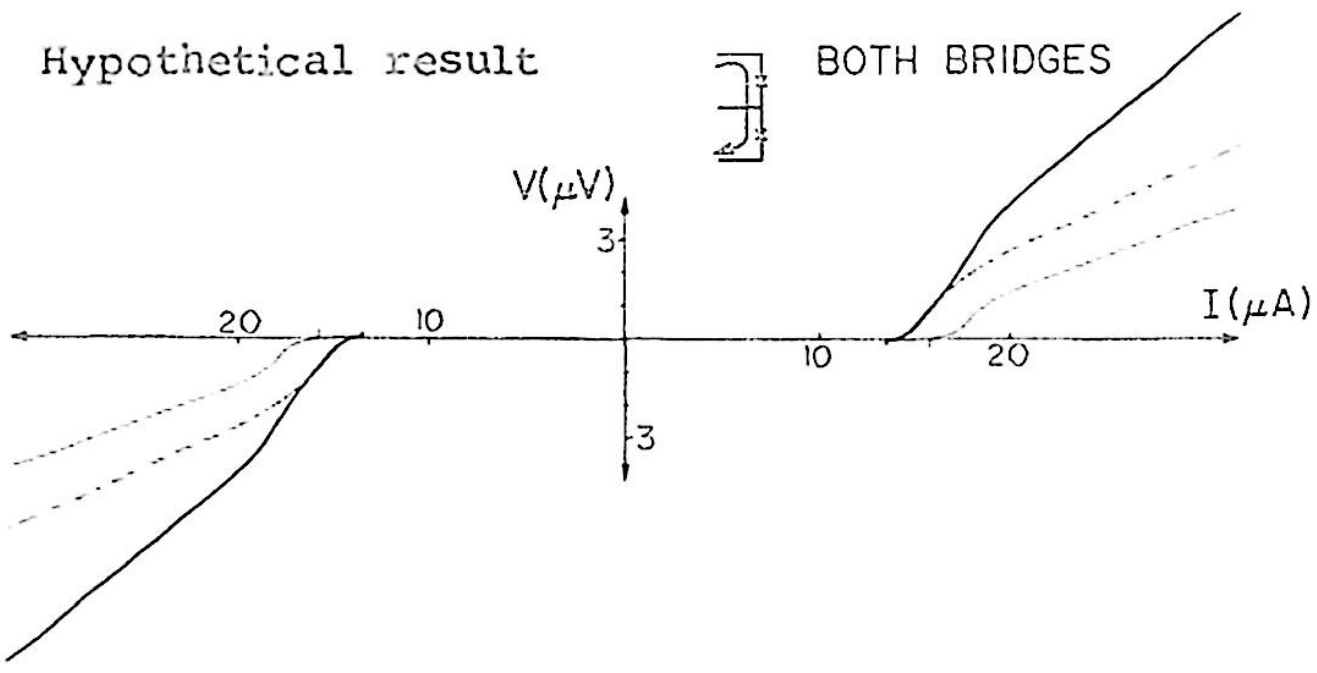


Figure 4. (Hypothetical result for case c. junction 1 and junction 2 combined)

Supercurrent is biased through junctions 1 and 2, in series, until such point that junction 1 just begins to develop a voltage--the voltage across junction 2 remaining at zero.

As the current increases further, the critical-current of junction 2 is reached, and this then also begins to develop a voltage, the total now being the sum of the two: the voltage across junctions 1 is augmented by voltage across junction 2, at a particular bias current. Figure 4 shows this hypothetical result.

II.d Actual results for Junction 1 and 2 combined.

The *actual result*[1], obtained by Jillie et al, for case c. through junction 1 and junction 2 combined, is shown in the lower trace of Figure 5 below, the voltage being taken across both junctions together.

1.See APPENDIX 1

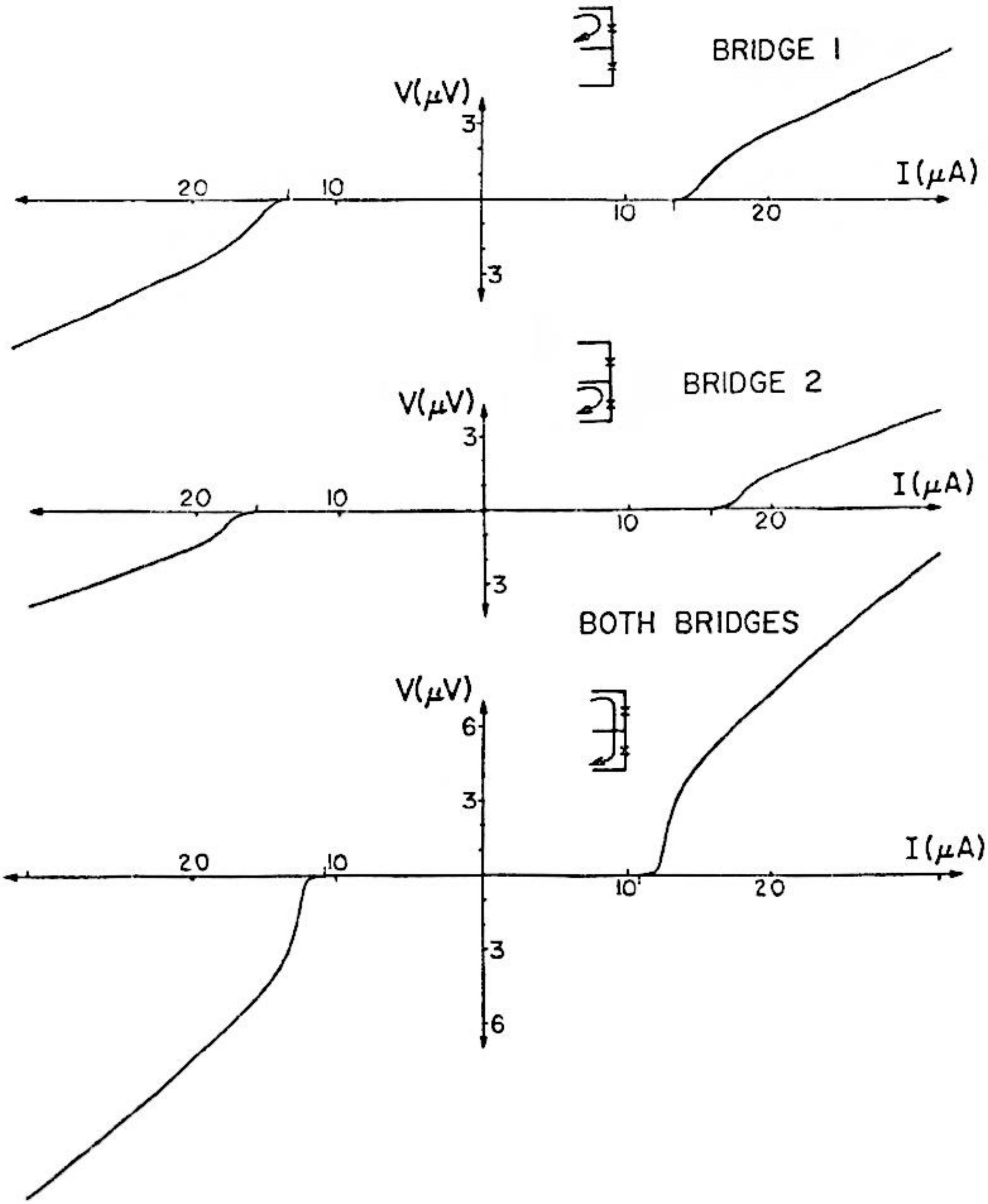


Figure 5. (Actual result for case c. junctions 1 and junction 2 combined)

Let us take a moment to describe the essential features of this lower trace. They are:

i. What Jillie et al called a "locking" of the critical currents of the two bridges to the same lower value, different from either of the bridges: that somehow the junctions act in unison or 'lock' together in such a way that not only did they manifest a voltage at the same, lower, current, but they also appeared locked together above this nascent voltage state, into the VI- trace.

ii. The "reduction of Ic" i.e. the lowering of the critical current above which a voltage begins to develop, producing a reduced supercurrent-window.

iii. A sharpening of the VI- curve, or as Jillie et al put it, the "sharpening of the resistive transition."

Excluding the concept of entanglement, these results on their own are difficult to explain. Combining them with the results that we will present in the next section, in which the currents are biased in opposite directions through the bridges, an inordinately complex picture emerges; one in which the results become next to impossible to explain without recourse to <u>multiple causes or mushrooming hypotheses</u>.

We should remember that at the time of Jillie et al investigations the concept of entanglement was not available to them. They were not aware of aspects of Josephson's original analysis and thought of a Josephson junction in terms of 'tunnelling' only. They did, however, have the concept of 'quasiparticle' flow-- a type of normal flow of current that is supposed to exist in the presence of a supercurrent flow. They also had 'heating effects' -- vaguely that a current flow may produce heating and this heating may affect the critical current at another place not far away. Certainly, a classical current flow can produce heating effects, the idea presumably being that the quasiparticle or normal type flow may also produce heating effects. Thirdly a supposed suppression of the order parameter by the presence of a supercurrent. Fourthly the idea of a 'phase slip,' and associated dissipation.

Jillie et al attribute the decrease in critical current in the lower trace to 'at least 3 separate processes.'[12]

i. Depression of the order parameter due to increasing supercurrent density in the neighbourhood.
ii. Effects of heating from a 'phase slip' process
iii. Effects of 'nonequilibrium quasiparticles' from a 'phase slip' process.

The above processes are characterised as 'symmetric interactions' as contra-distinct to other processes which are supposed to occur when the supercurrents flow in opposite directions through the bridges. Jillie et al term them 'asymmetric interactions'. We will come to those configurations later on.

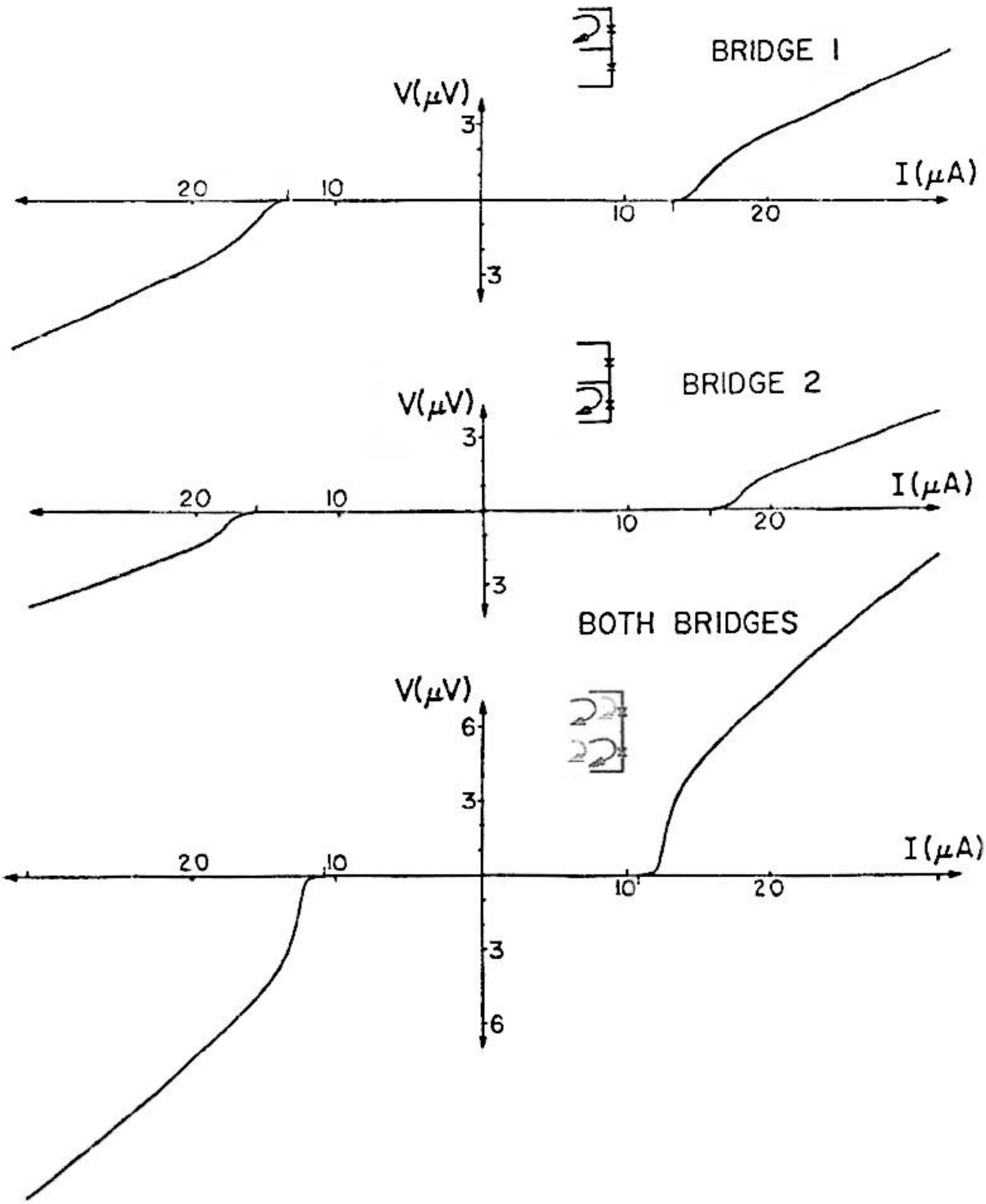


Figure 5a. (See text, case c. junctions 1 and junction 2 combined)

Let us now see, in the following, how we can account for the results of Figure 5. in terms of the expression for the entanglement current.

II.e Discussion Junction 1 and 2 combined.

We believe in this configuration, as discussed in Remark ii, Part I., that, owing to the respective phase-differences across each junction transforming in unison, as the current jo is fed in on the left of the two junction system, then the equation for the entanglement current:

$$j(\text{entangle}) = jo\ 2/\sqrt{2}\ \text{Sin}(\ \theta_a + \theta_b\ ) \tag{4}$$

might be better be expressed as

$$j(\text{entangle}) = jo\ 2/\sqrt{2}\ \text{Sin}(\varphi) \tag{5}$$

That is to say, the junctions can be looked upon as behaving like a single junction with phase difference φ.

This appears to be the case: the current voltage characteristics are smooth and with no kinks, unlike those plotted out in the hypothetical case, above, in which the junctions behaved as if separate entities.

Even though the phases do not come into play in this configuration, we still find the resultant supercurrent at each junction is at a higher value than would normally be the case without entanglement, by a factor $2/\sqrt{2}$.

This, in turn, means the critical current will be diminished by the same factor from what it would otherwise have been without entanglement.

Taking the critical currents for junctions 1 and 2 on their own to be $13\frac{1}{2}\mu A$, and $16\frac{1}{2}\mu A$ respectively, we can calculate the expected critical current for the entangled pair by taking the average, $15\mu A$, and then finding that value which must be multiplied by the entanglement factor $2/\sqrt{2}$ to give this average, which is 10.6µA. This appears to be in close if not exact agreement with the results, cf. Figure 5 or 5a.

In what follows we will sometimes talk of a supercurrent 'emanating' from, or being 'native' to, a place. This is to help us picture the entanglement current j(entangled) in a less abstract way, and is to be looked upon as a mental aid.

The contribution from the 'entangled' supercurrent element emanating from junction 1 will act to augment the 'native' supercurrent of junction 2, *and vice versa*. That is to say, in effect, a higher 'net supercurrent' is produced due to these 'other junction' contributions. It follows that the critical-current for the pair will be lower, and the supercurrent- window narrowed. Figure 5a. lower trace, illustrates the case.

The sharpening of transition may be understood as follows. At the critical-current of the entangled pair, a nascent voltage appears across junction 1 arising from the supercurrent in junction 1. Secondly, at the same time a nascent voltage appears across junction 2 arising from the supercurrent in junction 2. Thirdly a nascent voltage appears across junction 1 arising from the entangled supercurrent contribution emanating from junction 2. Fourthly a nascent voltage appears across junction 2 arising from the entangled supercurrent contribution emanating from junction 1. Thus the contributions combine to produce an apparent sharpening of the transition.

In this picture, these supercurrent elements only become real or actual when they are manifested out of the condensate at the connecting leads. Within the superconducting matter itself, they remain only 'telegraphing amplitudes' or 'interfering occurrences'.

Let us now turn our attention to the case in which the currents are biased not in the same direction, as was the case in the above configuration, but rather, more generally *in opposition directions*, --at least for a half portion of the trace, as shown below in figure 6.

II.f Actual results for Junction 1 and 2 combined. A fixed current is supplied to Junction 2. The current across junction 1 is swept from negative to positive values.

(Junction 1 is considered)

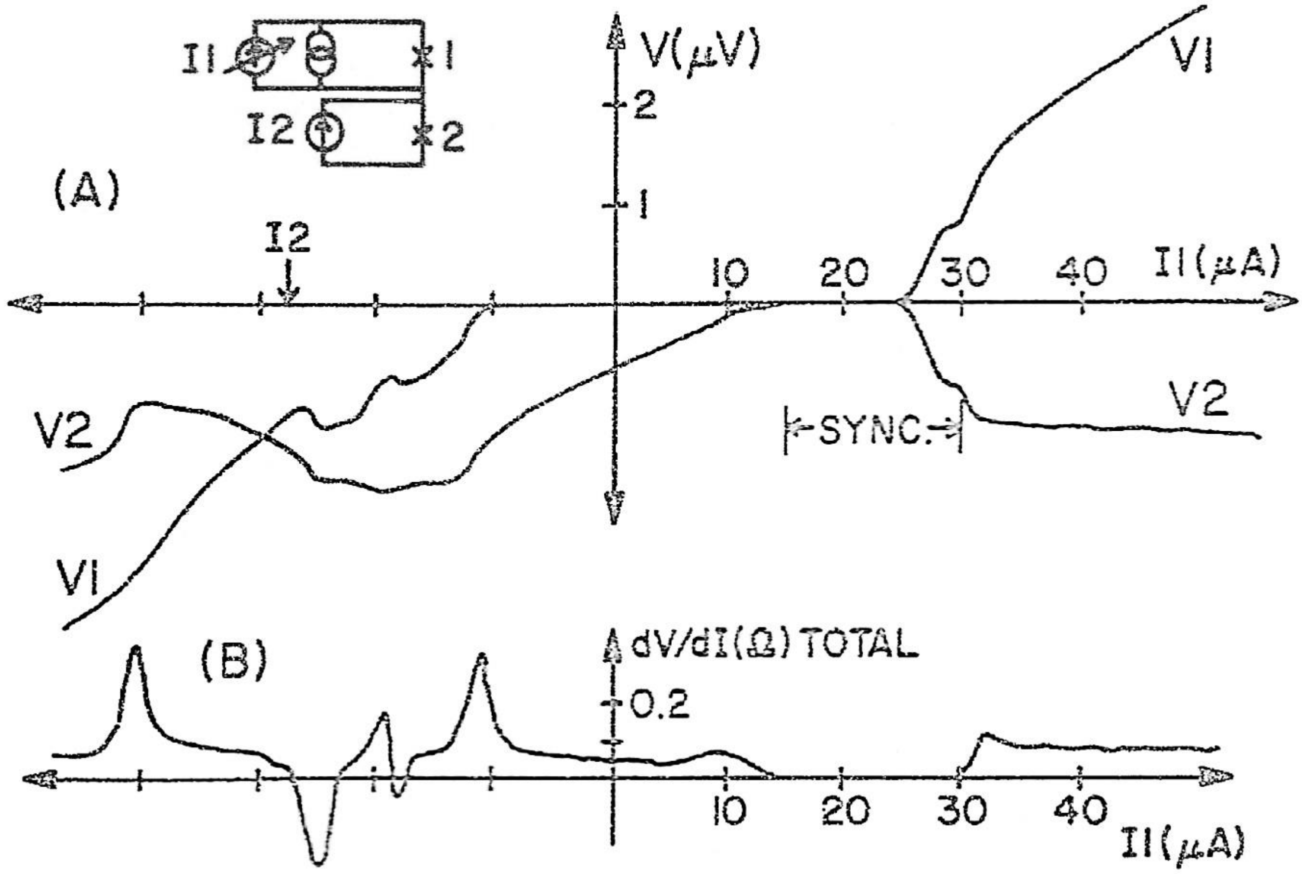


Figure 6. (From Fig.2 of reference 12. " (A) Variation of the voltages across both bridges separately as the current through 1 is varied. I2 is held constant at the value shown. The intrinsic critical currents are 21μA and 26μA for bridges 1 and 2, respectively. (B) Slope of V1 + V2 in (A). Note the area of voltage synchronization from 14 to 30μA. There is no evidence of synchronization for negative current.")

Here a current is biased through junction 2. at the fixed value shown by the arrow, I2, in the above figure. To form the trace, a current I1 is biased through junction 1, from negative to positive values, while V1 and V2 are traced out. The applied current I1 is in the same direction as I2 for a half portion of the trace (the left half,) and in an opposing direction for the remainder (the right portion of the trace).

Jillie et al characterise the main features of their results as:

i. "A large distortion of the IV curves of the bridges"
ii. That somehow the voltages of the junctions are 'pulled' together at zero volts, or 'synchronised' or 'locked' over a region of I1, in the right hand portion of the trace. There is also smaller region of I1 in which $|V1|=|V2|$ appear 'synchronised' together.

iii. There is no locking or synchronization in the left hand portion of the trace, at least for the case given.

Jillie et al attempt to explain this 'voltage locking' by a number of complicated interactions involving quasiparticle diffusion, phase slips, and a conjecture that quasiparticle currents induce compensating supercurrents. These additional processes were termed 'asymmetric interactions'.

Not mentioned explictly, is the shift of the supercurrent window, for junction 1, to the right along the current axis, shown more clearly in figure 6a below.

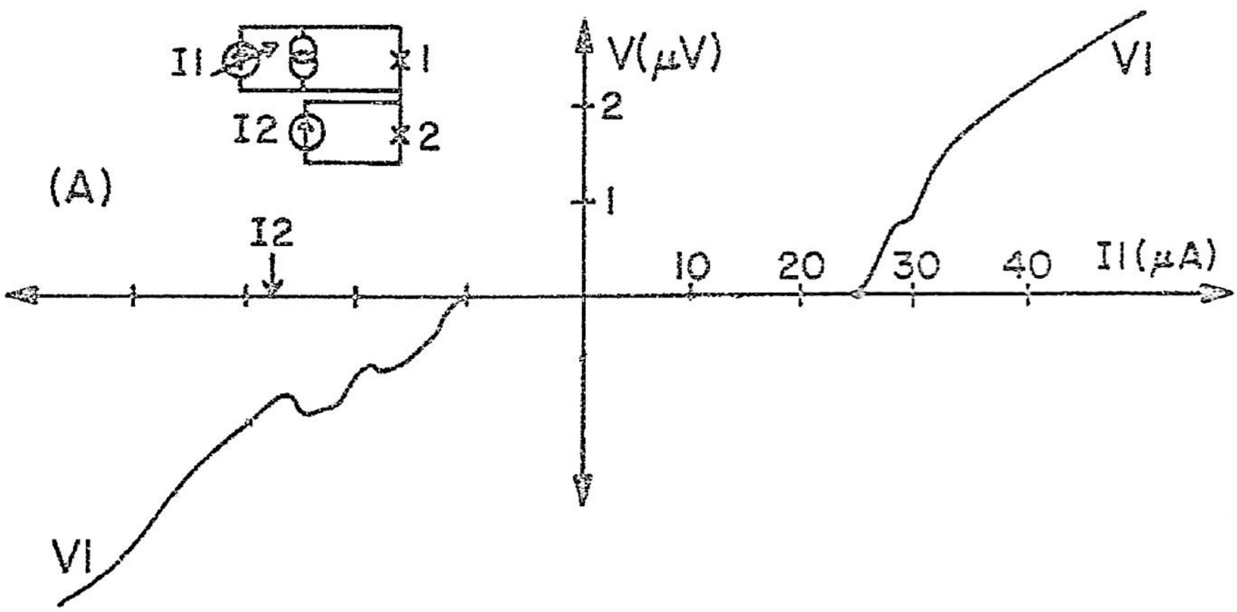


Figure 6a. (Shift of the supercurrent window, for junction 1, to the right along the current axis)

This shift, together with the shift of the supercurrent-window for junction 2, shown in figure 6c, are, in truth, *the main phenomenon* of these results. And as we will see shortly, the 'voltage locking,' or 'pulling' & and the distortion of the VI curves are, moreover, apparent or incidental, and in this respect illusory: they are bought about by the main phenomenon.

II.g Discussion for Junction 1.

There is no voltage "pulling" or "synchronization" so to speak,-- (there is no need for this additional hypothesis,) the results are simply a reflection of the combined or net supercurrent passing through each junction. By 'net supercurrent' we really mean resultant phase-difference from the entanglement of one junction with the other.

The first thing we should note is that the current I2 is biased through junction 2 at a fixed value (26µA or 27µA), and that this value is at or slightly above the critical current for this junction, --- or at least it seems so from figure 6. Why Jillie et al chose this ambiguous value for I2 is somewhat a mystery. We should remember, however, that even above the critical current at these nascent voltage values, there must still be supercurrent elements present, albeit diminished: if there were not, then we would flick to a normal state resistivity, in a binary on off sense, which clearly is not the case. Conversely just below the critical current we have supercurrent elements at full strength, as it were.

Considering first, the left half of the trace in figure 6a, where both bridges are biased in the same direction, the contribution from the 'entangled' supercurrent element emanating from junction 2 will act to 'augment' the native supercurrent in junction 1. And so the supercurrent window for the left half will be contracted or narrowed from what it would otherwise have been without these contributions, ie. from what it would have been with I2=0.

Turning now to the right half of the trace of figure 6a, in which the bridges are biased in opposite directions, the contribution from the 'entangled' supercurrent element emanating from junction 2 will act to 'diminish' the native supercurrent in junction 1, and so a higher value of I1 will be required in order to reach the threshold of the critical current: The supercurrent window on the right-hand side will be extended due to the contribution from junction 2.

Said differently, the phase-difference of the entangled supercurrent element, emanating from junction 2, is of *opposite sign* to the phase-difference of the native supercurrent in junction 1, and this causes the supercurrent-window on this side to be extended.

Figure 6b illustrates the case.

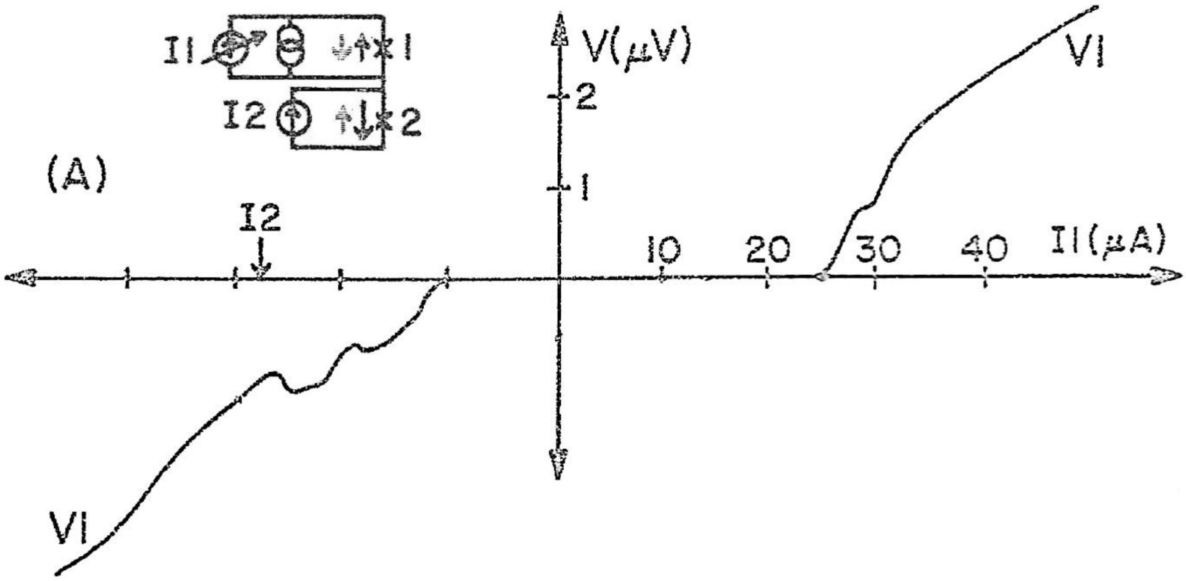


Figure 6b. (See text, shift of the supercurrent window, for junction 1, to the right along the current axis)

If the above explanation for the shift be true, we would expect a whole spectrum of supercurrent-window shifts to the right or to the left depending upon the polarity and strength of the fixed applied current at the other junction. Jillie et al did not perform such measurements, but fortunately Smith et al did, as we will see in the next section.

II.h Discussion for Junction 2

Let us first bring our attention back to the results of Jillie et al and consider the peculiar IV-trace for junction 2, shown below in figure 6c.

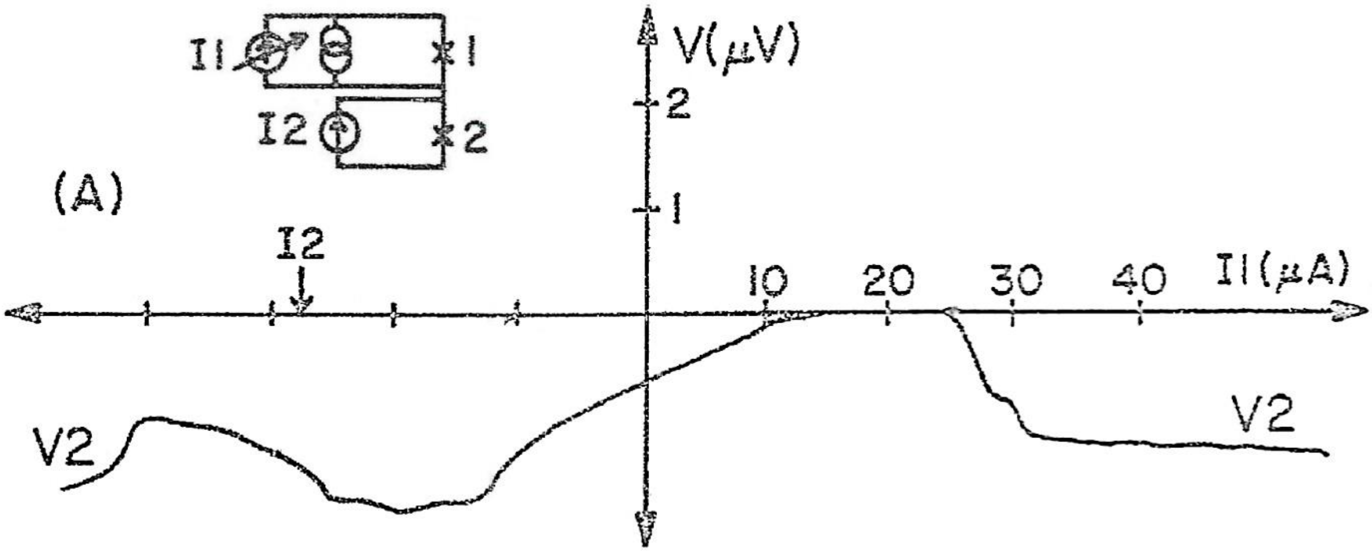


Figure 6c. (Peculiar IV-trace for junction 2)

As with all the VI-traces in this report the results are simply a reflection of the net-supercurrent passing through the bridge.

For the left hand portion of the trace, the fluctuation in V2 is caused by the interference of the supercurrent elements in junction 1 with those in junction 2, with the added complication that junction 2 is at or slightly above its critical current.

As we move into to the right hand side of the trace, we see $|V2|$ decrease, as the supercurrent elements of junction 1 act to diminish the net supercurrent through junction 2, --(the phase-diffences contributions of junction 1 now being of *opposite sign* to those of junction 2). At around 14-25µA the supercurrent elements in junction 2 are diminished enough to fall below the threhold of the critical current for this junction. And so the V2=0 section of the trace corresponds to the supercurrent-window of junction 2, but from the perspective of the current biased through junction 1. At values of I1 above 21µA, ie. the critical current of junction 1, we see a falling off of interaction, so that the more or less flat region of the trace above 30µA corresponds simply to the nascent voltage of junction 2 on its own, set slightly above its critical current, with no further contributions emanating from junction 1. Figure 6d illustrates the case.

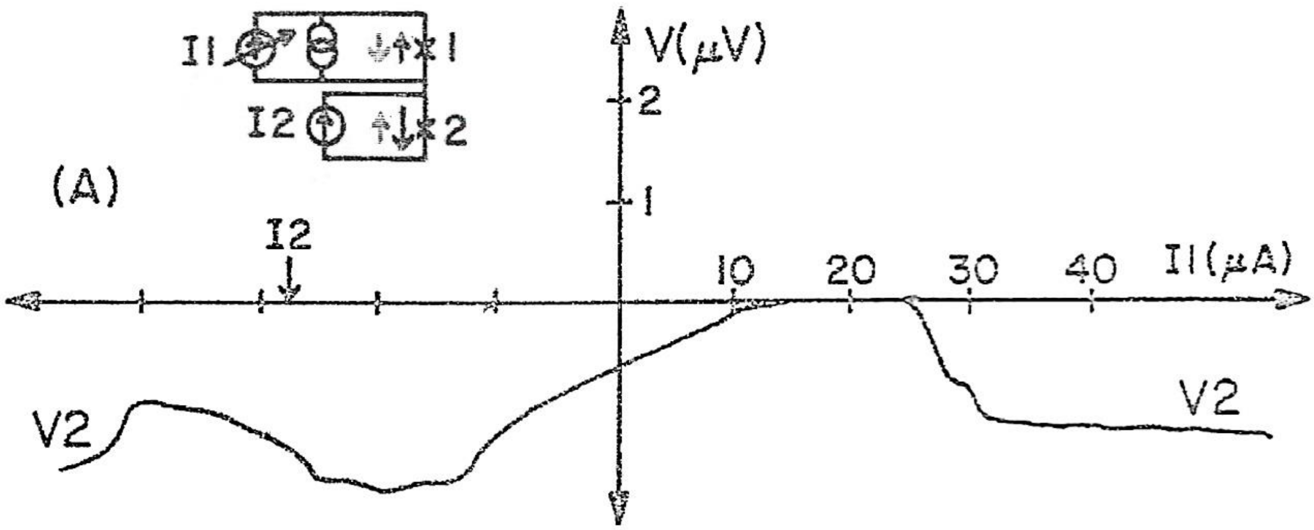


Figure 6d. (See text, peculiar IV-trace for junction 2)

*   *

III. Results of Smith et al.

In this section we will consider the results of Smith et al. They had a similar experimental setup to Jillie et al, with figure 2 sufficing to describe their arrangement. In their case two Tin (Sn) microbridge-junctions were fabricated with a separation of 0.2μm. "The current-voltage characteristics of one microbridge was monitored while a fixed bias was maintained on the other."[11] A number of VI-traces were made for junction 1, each for a different magnitude and polarity of the applied current I2, at junctions 2. They found a way to display all these traces in a single figure by displacing them, according to their fixed applied current magnitude I2, along the voltage axis. Figure 7 shows these results.

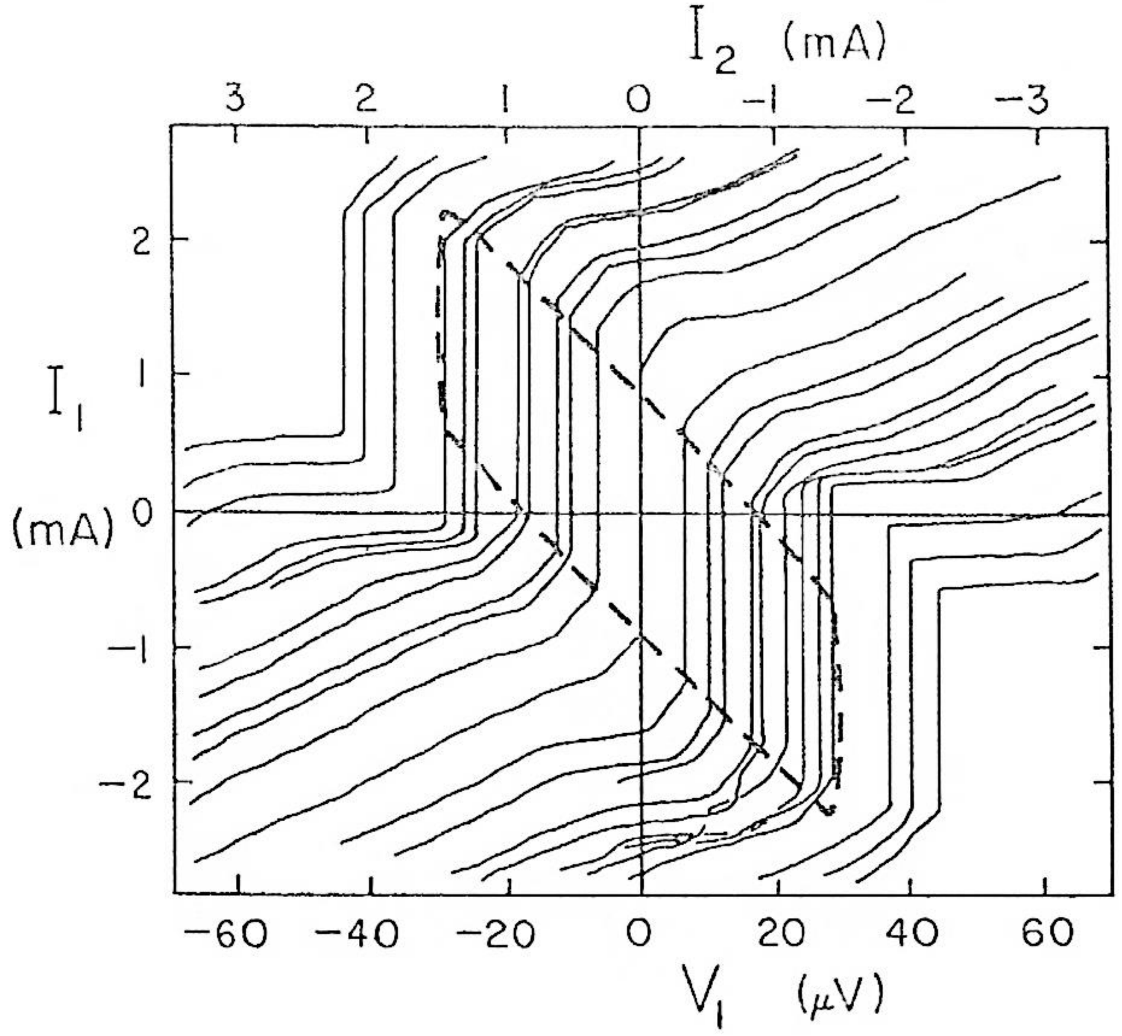


Figure 7. (From Fig.3 of reference 11. " Current-voltage (I_1,V_1) characteristics of microbridge 1 at T/Tc=0.978 for different values of I_2. The curves have been shifted horizontally by the magnitude of I_2. The dashed line is a map of the critical current of microbridge 1 as a function of I_2, for I_{c1}=0.81mA, I_{c2}=0.80mA and zero potential across

microbridge 2". (NB. There is a typo in the value for I_{c2} in the original figure, corrected above.))

Let us flip this figure about the long axis of the rhomboid shape (the dotted line shape) lest we miss an opportunity to present these results in their clearest possible manner. Cf. figure 7a, below.

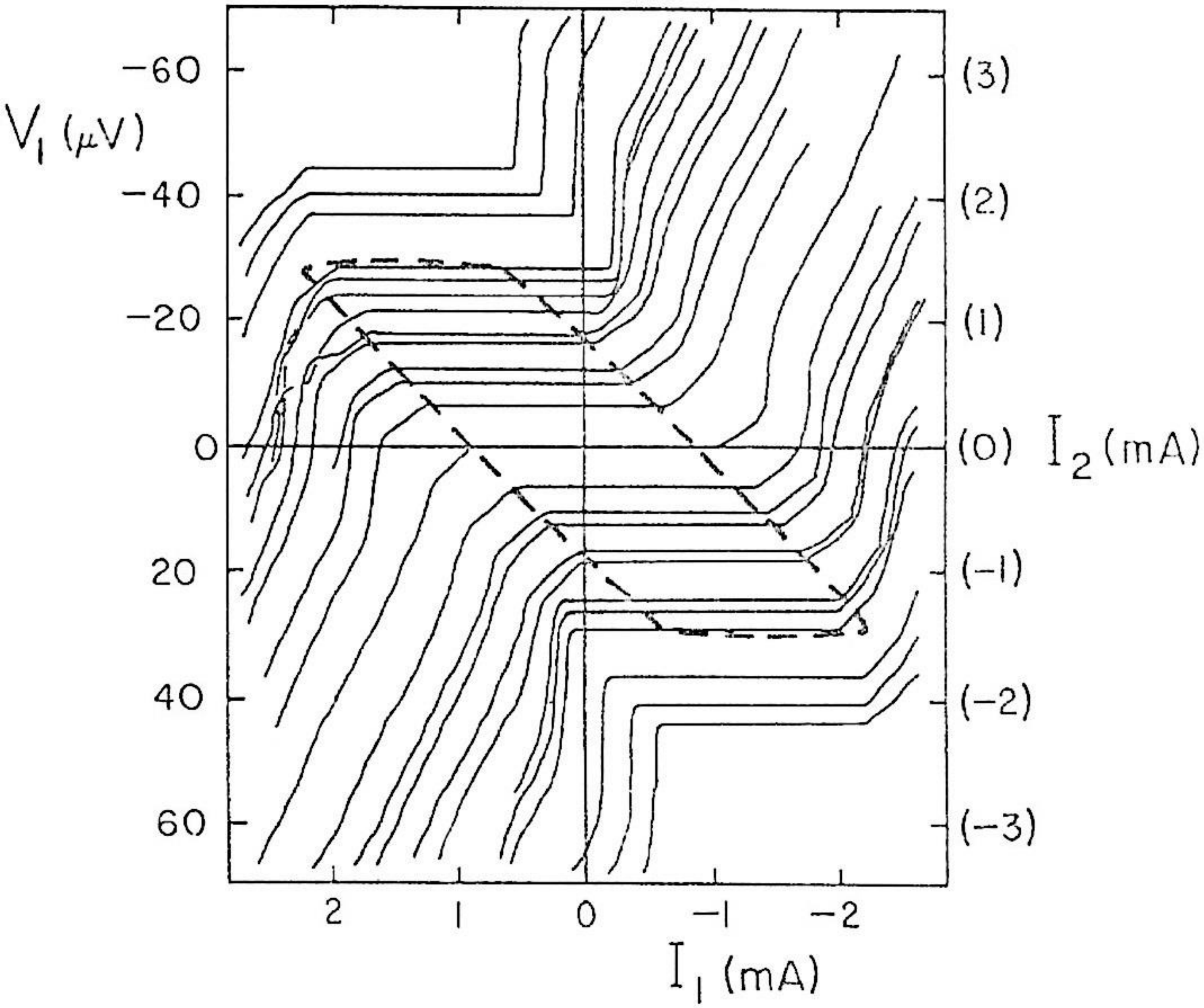


Figure 7a. (Figure 7 re-arranged)

These VI-traces constitute a the whole spectrum of supercurrent-window shifts to the right or to the left depending upon the polarity and strength of the applied current at the other junction.

The above figure may be understood in terms of the expression for the entanglement current, by noticing:

j(entangled) is proportional to Sin $(\theta_a + \theta_b)$,

that is to say it is proportional to the Sine of the phase difference $\theta_a = \Delta\chi_a$, $\theta_b = \Delta\chi_b$, at each junction, or

$$j(\text{entangled}) \text{ is prop. to } \mathrm{Sin}\,(\Delta\chi_a + \Delta\chi_b)$$

In the configuration relating to the traces of Figure 7a, let us assume θ_a and θ_b are each respectively, not too far from zero. In this approximation, we can then replace Sin $(\theta_a + \theta_b)$ with $\theta_a + \theta_b$, or $\Delta\chi_a + \delta\chi_b$; $\delta\chi_b$ denoting the phase difference brought about by the fixed current at junction 2, so that

$$j(\text{entangled}) \text{ is prop. to } \Delta\chi_a + \delta\chi_b$$

As the applied current at junction 1, I1 is swept from negative to positive values, then so $\Delta\chi_a$ will go from $-\Delta\chi_a$ to $+\Delta\chi_a$, while $\delta\chi_b$, which is brought about by I2 is fixed at discrete values. When I2 = 0, (the middle trace), we have $\delta\chi_b = 0$, and the resultant entanglement current j(entangled) is symmetric about the I1=0 point.

This means that the supercurrent window is symmetrical about I1=0: j(entangle) reaches the critical current at the same value, positive or negative, for I1, either side of the I1=0 point.

This symmetrical state of affairs is broken when $\delta\chi_b$ either adds to the resultant phase difference of the entangled pair; $\Delta\chi_a + \delta\chi_b$, or detracts from the resultant phase difference of the entangled pair $\Delta\chi_a - \delta\chi_b$, depending on whether the current applied at junction 2, I2 is positive (traces above the I2=0 line) or negative (traces below the I2=0 line).

If the $\delta\chi_b$ adds to the result phase difference of the entangled pair $\Delta\chi_a + \delta\chi_b$, then the j(entangled) is augmented on the right hand side of the trace and diminished on the left hand side of the same trace. This in turn means that the supercurrent window is shifted to the left.

ie. the critical current on the rhs is diminished, and the critical current on the lhs is augmented, for that trace.

The situation is the reversed if $\delta\chi_b$ detracts from the resultant phase difference of the entangled pair, $\Delta\chi_a - \delta\chi_b$. In this case the supercurrent window shifts to the right.

ie. The critical current on the lhs is diminished, and the critical current on the rhs is augmented, for that trace.

This accounts for the skewed-rectangle shape in Figure 7a for the supercurrent windows of junction 1, at various applied I2 values for junction 2.

Smith et al note the following main features of their results:

"The increase or decrease of the critical current of one microbridge depending on the current through the other microbridge even with no potential across the second microbridge. "[11]

"This is an effect that non-equilibrium quasiparticle coupling cannot predict."[11]

And conclude that an "inclusion of a phase coupling term," Sin($\theta_a + \theta_b$), is "necessary to explain qualitatively the experimental data."[11]

It is this phase coupling or interference between the supercurrent elements in one junction with the supercurrent elements in the other junction, that furnishes the most straightforward and simple explanation of all the above results.

IV Suggested Experiments

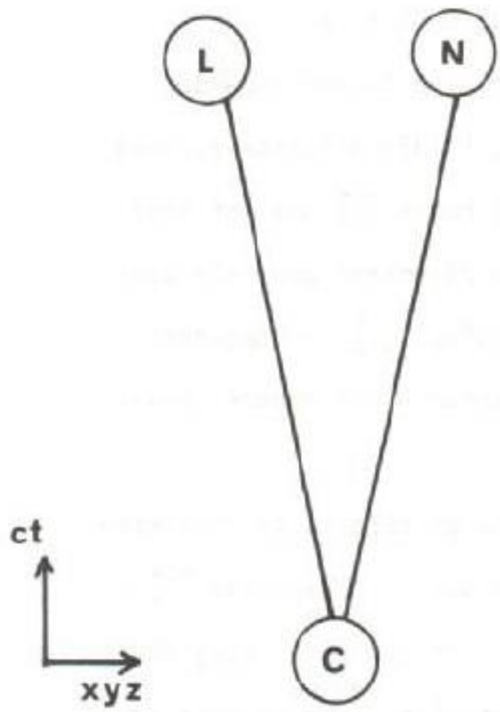


Figure 8.

Costa de Beauregard made a study of EPR-type correlations--- entanglement phenomena. His conclusion is that the separated matter is tied together either in its past, or in its future (so called 'echelon absorption,') the correlations he argues, being, in fact, time symmetric at the quantum level.

In our case, the junctions are tied together in their common past.

Once separated in space, lets suppose these correlations persist: they would be transmitted along L C N in the junctions' mutual common past. (Cf. Fig 8 from Ref.(14))

There is, in truth, nothing to stop the occurrences that might occur when the junctions are together, continuing to occur, when the junctions are separated. This is because, these interfering occurrences or alternatives do not occur at any one time, but throughout the whole extent of the 'body's time,' just like interfering vibrations, with nodes and resonances, occur throughout the whole extent of a 'body's space,'--- as we commonly think of a material extended in space.

Consider figure 9. The purpose of this arrangement is to find out at which point, if at all, do the junctions behave independently of one another. And moreover, if they remain 'in contact,' so to speak, in any mode or fashion what-so-ever, after the joining strip goes normal.

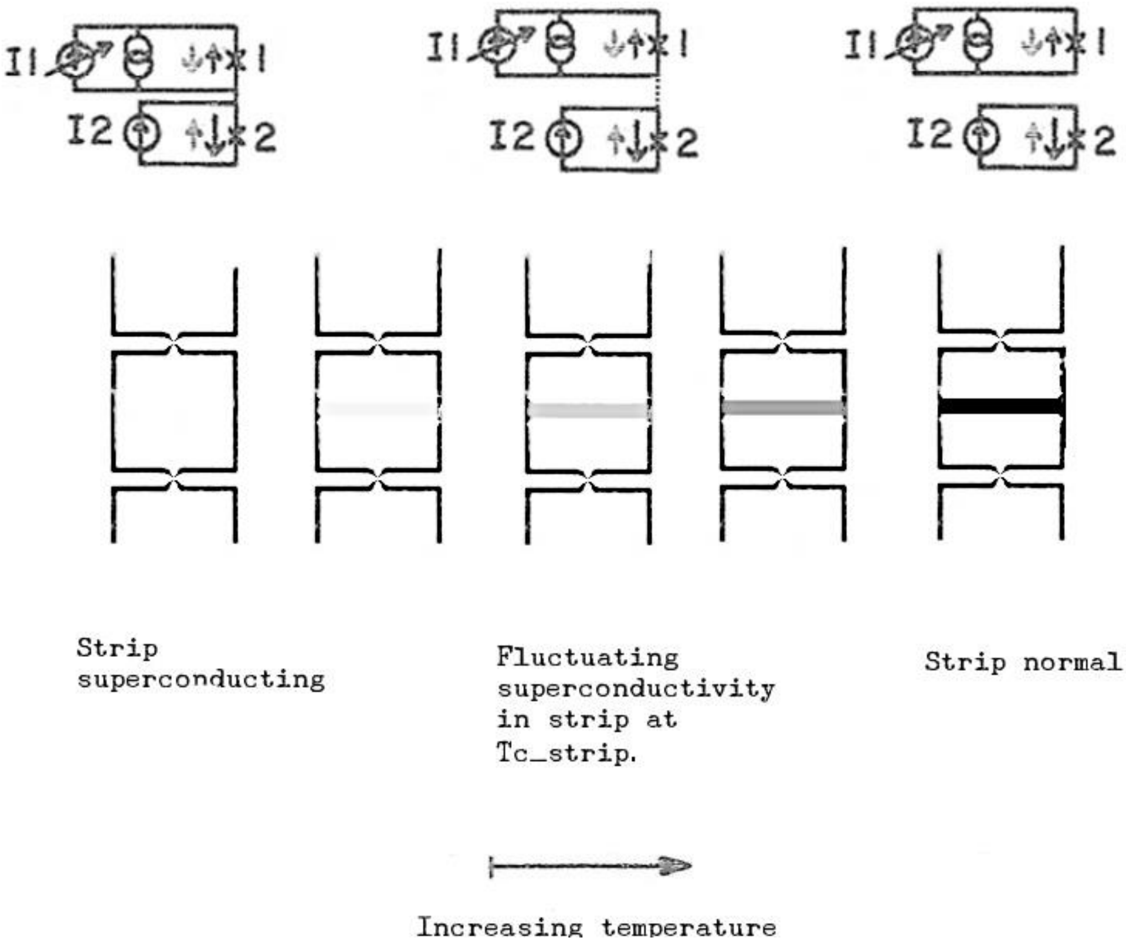


Figure 9. (Separation of junctions via transformation of strip)

The advantage of the above scheme is that it separates the junctions in the most delicate way imaginable, employing no moving parts. The microbridge junctions in this figure being at all times fully superconducting. Only the strip transforms from fully superconducting to fully normal over the temperature range used.

Lastly, consider the below scheme depicted in figure 10. Here the entangled junction-bridges are separated by mechanical means, the junctions being joined initially by a portion of 'superconducting paste'. If there were found to be any interference between the separated junctions either as a one-off, or continously, in any mode or fashion what-so-ever, then there is the possibility, albeit speculative, of a new means of signalling without wires. The separated junctions could be decanted off into separate dewars of liquid helium, to investigate if the entanglement-type inteference was dependent upon the distance between them.

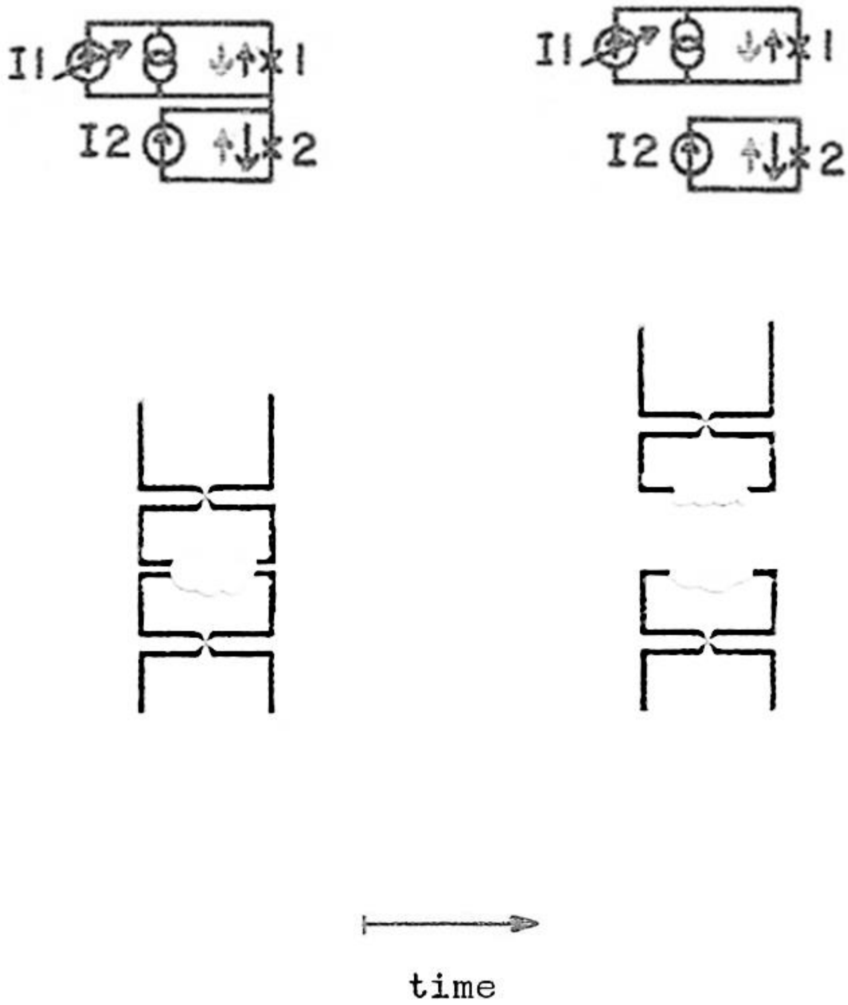
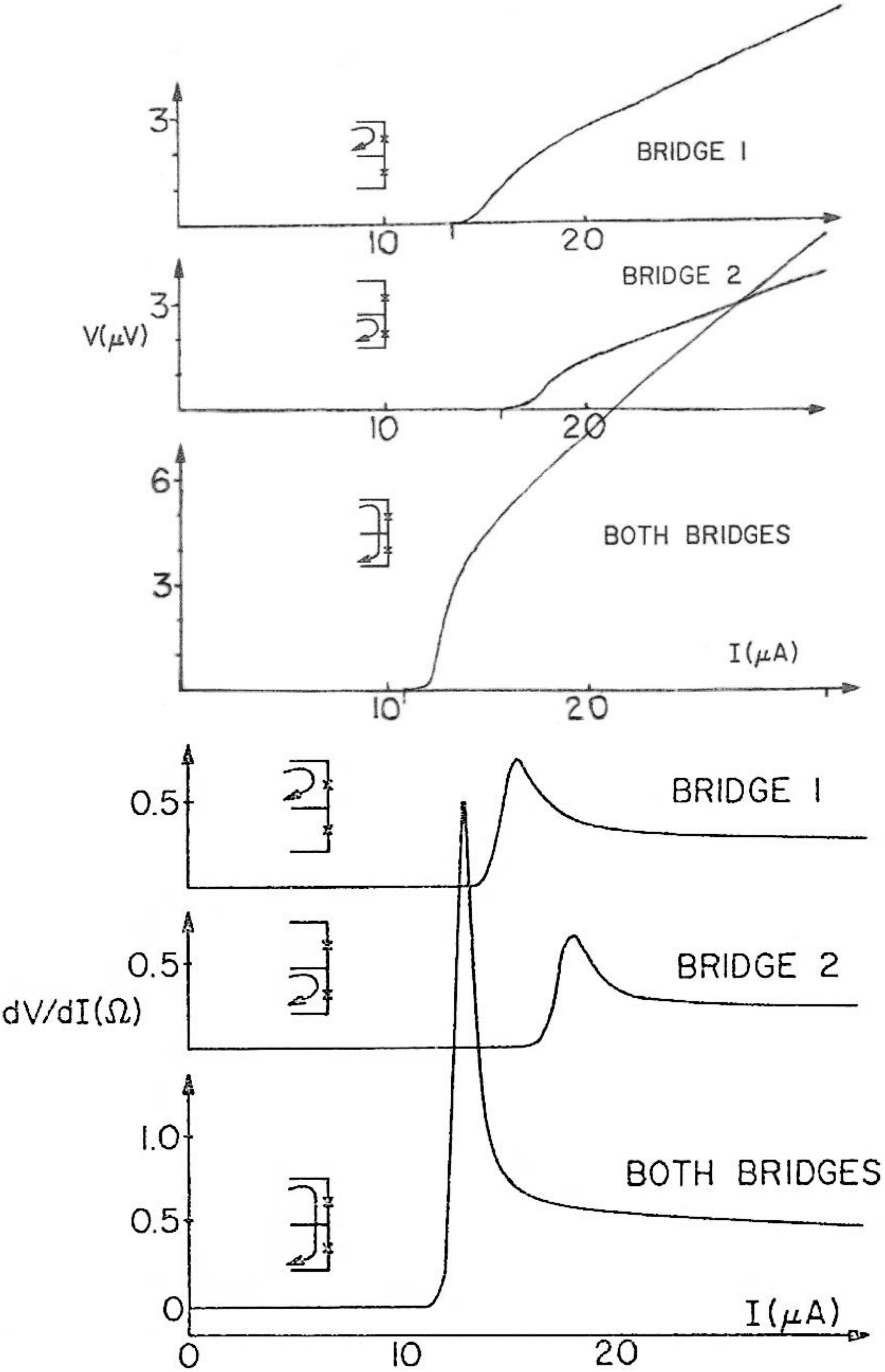


Figure 10. (Mechanical separation of junctions)

APPENDIX 1.



(The V-I plots, above, were obtained from the Jillie et al plots of dV/dI - I, also above, from ref. 10. by integration. Cf. APPENDIX 2.)

APPENDIX 2.

The curves in APPENDIX 1. were integrated by a method original with our self, albeit, arguably self-evident, once the idea of Galileo (1599,) is known. He found the quadrature of the Cycloid by weighing a cut-out of the cycloid against a cut-out of the generating circle, both being fabricated from the same sheet of metal. The ratio of their areas was discovered to be about 3.

To find the area-curve under any crooked line:

Cut the crooked line figure into segments or strips of white-card, 2½mm wide, then add them to a balance, piece by piece, against the weight of the whole crooked line figure. The tipping of the scales will be a measure of the area under the curve, & can be marked off on a board: the measure will gradually increase as more pieces are added, & will be commensurate with the area under the curve at that segment (x-value).

The x-axis scale was taken from the printout from which the figure was cut. That scale gave the current in µA from 0 up to a maximum of 30µA.

The y-axis scale may be found, again with help of a balance.

Place a rectangular piece of white-card of reckoned slightly greater proportion (and weight), to the cut-out crooked line figure, on the balance. Strips of lesser and lesser extent are cut from the rectangular card, by trial and error, until the balance is made.

This rectangular piece of card was then placed on the printout from which the original crooked line figure was cut, and aligned with the axes. The area of the rectangular figure could then be determined using the scales of the printout. This value must be the maximum area of the crooked line figure and was found to be approximately 4µV, 6µV and 13µV respectively. The y-axis scale was thus determined to go from 0 to around 13µV.

In practice a professional digital mini scale TL-series was used to weigh the pieces of white-card, accurate to 0.001g.

References.

(1) Josephson, B.D. (1962) The Relativistic Shift in the Mossbauer Effect and Coupled Superconductors. College Fellowship Dissertation. & Josephson, B. D. (1964) Ph.D. thesis, University of Cambridge, England

(2) Einstein, A. (1922), Gedenkboek aangeb. Aan H. Kamerlingh Onnes, eaz. Leiden, E. pp435

(3) L. Hoddeson, G. Baym, and M. Eckert, (1987), Rev. Mod. Phys, 56, no 1, p287.

(4) London, F Superfuids (Dover)

(5) V. I. Ginzburg and L.D. Landau (1950), J Exptl. Theoret. Phys. (USSR). 20, 1064

(6) J. Bardeen, L.N.Cooper, and J. R.Schrieffer (1957), Phys Rev. 106, 162 & 168, 1175

(7) L. P. Gor'kov (1958), Soviet Phys. JETP 7, p505; (1959) Soviet Phys. JETP 9, p1364. See

also Lifshitz, E.N and Pitaevskiĭ, L.P (1981) Statistical Physics, Part 2. Chapter 5.

(8) B. D. Josephson, private communication.

(9) R. P. Feynman and A. R. Hibbs (1965) Quantum Mechanics and Path Integrals. Dover.

(10) Wysin, G.M. (2011). Probability Current and Current Operators in Quantum Mechanics.

(11) Smith, H.J.T. and Dion, M. (1990) Physical Review B, 42, number 1, 206

(12) Jillie, D.W., Lukens, J.E., Kao, Y.H., & Dolan, G.J. (1976) Physics Letters 55A, number 6. 381.; Jillie, D.W. (1976) Ph.D dissertation, State University of New York at Stony Brook.

(13) Mercereau, J. E. (1969) Chapter 8 in Superconductivity, vol.1, ed. R. D.Parks

(14) O. Costa de Beauregard (1979) « Einstein, 1879-1955 », colloque du centenaire au Collège de France, les 6-9 juin, pp. 299-305, éd. du Centre National de la Recherche Scientifique, 1980. See also. http://www.costa-de-beauregard.com

O. Costa de Beauregard, private communication.