Not Every Truth Could Have a Truthmaker

Abstract: Mark Jago argues for truthmaker maximalism in some recent papers based on a key premise: that every truth could have a truthmaker. Jago contends that many would pretheoretically accept this principle and that counterexamples to it would be difficult to find. In this note, I show how truthmaker non-maximalists can use a modified version of Milne’s argument against maximalism to provide a counterexample to this key premise.

1. Introduction

Truthmaker maximalists claim that all truths have truthmakers. But consider the claim that for each truth there is a logically possible truthmaker, stated formally below:

(Key Premise) A → ◇TMA.¹

Jago (2020; 2021) has recently argued that this claim, when combined with some additional assumptions, leads to truthmaker maximalism. Jago’s main argument is that Key Premise combined with the following principles:²

(Fact) TMA → A,

(Dist) TM(A & B) → (TMA & TMB),

plus the following inference rule:

(N) From ⊢ ~A derive ⊢ ~◇A,

¹ See Jago (2020, 2021). In Jago (2021), TMA’ is revised to ‘NecA,’ where ‘NecA’ is ‘there is some entity which necessitates A’s existence. Formally:

∃x□(∃y y = x → A).

I will use TMA in this paper because I take my main argument to carry over to Jago’s revised version, but also see note 7 for a ‘NecA’ formulation of the main argument.

² Jago (2020: 40) says that logically possible situations respect Fact and Dist, which is equivalent to taking them to be logically necessary. The idea being that if Fact and Dist are not logically necessary, the necessitation rule N would be unusable. After all, the proof works by showing that there is no logically possible situation where TM(A & ~TMA) has a truthmaker. Trueman (2020) gives reasons to doubt that Fact and Dist are logically necessary, but I do not raise the issue here.
leads to truthmaker maximalism (where Fact says that truthmaking is factive, Dist says that truthmaking distributes over conjunctions, and N is a necessitation rule). Jago’s proof (2020: 43-44) goes as follows:

1. TM(A & ~TMA) Assumption
2. TMA & TM~TMA 1, (Dist)
3. TMA & ~TMA 2, (Fact)
4. ~TM(A & ~TMA) 1-3, Reductio
5. ~◇TM(A & ~TMA) 4, (N)³
6. (A & ~TMA) → ◇TM(A & ~TMA) (Key Premise)
7. ~(A & ~TMA) 5, 6, MT
8. A → TMA 7, logic.

Informally the reasoning is as follows:

If we assume for the sake of argument that the following conjunction:

(Conj) A & A does not have a truthmaker

*itself* has a truthmaker, a contradiction follows. For if Conj has a truthmaker, it’s conjuncts both have truthmakers. If the conjuncts both have truthmakers, then the left conjunct has a truthmaker while the right conjunct is true, which is a contradiction. Thus, Conj could not have a truthmaker. But there is no truth which could not have a truthmaker. So, Conj is not true.

Jago’s argument uses elements of the Church-Fitch paradox of knowability (Church 2009, Salerno 2009) where truthmaking is substituted for knowledge.

³ This step in the proof is only usable if Fact and Dist are taken to be logically necessary.
In addition to claiming that many would accept Key Premise pretheoretically,\(^4\) Jago contends that we should accept this premise for two reasons: first, that maximalist friendly ontologies are logically consistent, and second, that we can infer Key Premise from an inductive step that appeals to several examples of candidate truths such as “wombats are marsupials” and “1+1 = 2” (2020: 44). Jago suggests that a successful counterexample to Key Premise would need to be “an uncontested truth, for which there is no logically possible truthmaker” (2020: 43). The significance of a counterexample to Key Premise is clear: if there is a counterexample to Key Premise, then a reductio similar in style to Jago’s original argument would undermine unrestricted truthmaker maximalism itself. If some truths could not have truthmakers, then some truths do not have truthmakers. In this note I propose such a counterexample.

2. A Truth Without a Possible Truthmaker

How might a non-maximalist question Key Premise? Jago offers some constraints for a successful counterexample:

…your best bet is to deny the induction step to [Key Premise], on the grounds that the base isn’t sufficiently broad. If our examples include truthmaking talk, then we’ll find a false instance: A & ~TMA. This move is dialectically ineffective, however, for it assumes the falsity of maximalism in taking A & ~TMA as an example truth. Whether there is such a truthmakerless truth A is precisely what is in question. An effective counterexample to [Key Premise] requires an uncontested truth, for which there is no logically possible truthmaker. But such truths, I suggest, are hard to come by (2020: 43).

This seems partially correct. The following strategy, for example, would not be effective: It’s true that for every successful counterexample to maximalism, there will be a corresponding true

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\(^4\) See Jago (2020: 40).
conjunction which could not have a truthmaker. For example, if some negative existential such as “there are no arctic penguins” is a truth without a truthmaker, then its corresponding conjunction: “there are no arctic penguins & ‘there are no arctic penguins’ doesn’t have a truthmaker” could not have a truthmaker. Thus, counterexamples to maximalism could be a springboard to formulate counterexamples to Key Premise. Such a strategy is unlikely to work because one would be using the invalidation of maximalism to invalidate Key Premise. Such an approach would lead to a dialectical stalemate.\(^5\)

However, I contend that one need not use this “corresponding conjunction” strategy to propose a counterexample to Key Premise. Consider Milne-style arguments against maximalism that appeal to the following:

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\text{(Milne sentence M) This sentence has no truthmaker.}\(^6\)
\]

Although Milne sentence M (or simply M for short) was originally intended as a counterexample to truthmaker maximalism, a modified argument involving M can be used to formulate a counterexample to Key Premise. Informally, the reasoning would go as follows:

If M is not true, then M has a truthmaker and is thus true. Contradiction. If M has a truthmaker, then M is true and thus does not have a truthmaker. Contradiction. Since supposing M not to be true leads to a contradiction and supposing M to be true does not, M is true. Since supposing M to have a truthmaker leads to a contradiction, M could not possibly have a truthmaker.\(^7\)

Since M says of itself that it has no truthmaker, we can obtain the following biconditional:

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\(^5\) For more on this dialectical stalemate, see Trueman (2021: 272).
\(^7\) In fact, in Milne’s (2005: 223) original article, he suggests that a way out for truthmaker theorists is to take M to be a necessary truth and to restrict the truthmaker principle to contingent truths. Of course, this route would be problematic for an upholder of Key Premise.
If we assume that M-Equiv is logically necessary, a proof can continue as follows:

1. \( \neg M \)  
2. \( \neg \neg M \)  
3. \( M \)  
4. \( M \land \neg M \)  
5. \( M \)  
6. \( \neg \neg M \)  
7. \( M \)  
8. \( \neg \neg \neg M \)  
9. \( \neg \neg \neg \neg M \)  

\( \neg \neg \neg \neg \neg M \) can be simplified to \( \neg \neg \neg M \)  
10. \( \neg \neg \neg \neg \neg \neg M \)  
11. \( \neg \neg \neg \neg \neg \neg \neg M \)  
12. \( \neg \neg \neg \neg \neg \neg \neg \neg M \)  

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8 Barrio and Rodriguez-Pereyra (2015) have suggested that there is no reason that maximalists should accept these sorts of biconditionals. However, Brendel (2020: 1654) has recently shown how biconditionals of this form can be obtained with minimal means via diagonalization when a truthmaker predicate is added to Robinson arithmetic. Using this method, one can obtain 
\[ \Gamma = \neg \neg \neg M \]  
I do not replicate Brendel’s treatment here, but the proof can be appropriately modified to do so. I should note that Brendel’s own proof includes an important assumption which I contend non-maximalists should not grant (see note 9).

9 Similarly to Fact and Dist, M-Equiv needs to be logically necessary in order for the necessitation rule N to be usable in the proof. M-Equiv is logically necessary if M is logically equivalent to the claim that M has no truthmaker.

10 Note that when TMA is replaced with NecA, one can modify Milne’s (2013) revised argument in a similar way:
One might suspect that this argument resembles the knower paradox because M seems to have an epistemic counterpart in the following formulation:

(K) K is not known to be true.  

But this would be a mistake. Although one can argue that M is a truth which could not have a truthmaker, K is not clearly an unknowable truth. Supposing K to be known to be true leads to a contradiction, which amounts to a proof that K is not known to be true. But, if proof entails knowledge, proving K not to be known true would amount to a proof that K is known to be true after all. In other words, step 10 in the proof above would lead to a paradox. On the other hand, supposing M to have a truthmaker leads to a contradiction, which amounts to a proof of M. However, the idea that proof entails truthmaking is not as plausible as its epistemic counterpart and the non-maximalist has no motivation to grant it and they may even have motivation to reject it outright. Some philosophers, intuitionists for example, have proposed that proofs can function as truthmakers for mathematical and logical truths. Even so, such a view comes with its share of troubles. The first two are concerns for taking proofs to be truthmakers on a general level and the third is a concern for non-maximalists specifically. First, there is a general concern that it mistakes the epistemology of provable truths from their metaphysics (of course intuitionists would be suspicious of this distinction). Second, for any consistent and finite axiomatization of arithmetic, some mathematical truths will not be provable, and it would be

10. M
11. ¬◇∃x□(∃y y = x → M)
12. M & ¬◇∃x□(∃y y = x → M)

The proof diverges from Milne’s at step 11.

11 Kaplan and Montague (1960)
12 Brendel, for example, contends that M does not undermine truthmaker maximalism and is instead genuinely paradoxical. However, as Brendel points out, this only follows given two assumptions: (I) If a sentence σ has a truthmaker, then σ is true.
(II) If a sentence σ is provable in Robinson arithmetic, then σ has a truthmaker. (Brendel 2020: 1654).
13 See Dummett (2000: 4) for example. Thanks to an anonymous referee for raising this point.
strange if some mathematical truths had truthmakers and others are true for some other reason (Read 2001). Third, provable truths are necessary, and non-maximalists have long suspected that necessary truths are true irrespective of what exists. Beebee and Dodd put the point nicely:

…one might wonder whether necessary— and, in particular, analytic— truths need truthmakers; an analytic truth, it could be said, is true however the world is, and so is not made true by anything (Beebee and Dodd 2005: 2).

Since provable truths are necessary, non-maximalists will tend towards the view that such truths are true irrespective of whether a proof exists. These kinds of concerns do not arise when considering the claim that proof entails knowledge. There are key differences between M and K.

3. Possible Responses

I have claimed that the Milne-style argument against Key Premise does not beg the question against maximalism because it does not appeal to the “corresponding conjunction” strategy which uses counterexamples to maximalism to facilitate counterexamples to Key Premise. However, Rodriguez-Pereyra, (2006) Barrio and Rodriguez-Pereyra, (2015) have suggested that Milne’s argument is question-begging for a different reason. The thought is that M can be assimilated to the liar sentence because maximalism is committed to the following thesis:

(Maximalist Thesis) Every sentence is such that it is true if and only if it has a truthmaker (2006: 261).

Because Maximalist-Thesis equates truth and having a truthmaker, maximalists could respond to M similarly to the liar paradox. Rodriguez-Pereyra puts the point this way:

Of course, M gives rise to no inconsistency unless one assumes [Maximalist-Thesis] or some principle to the same effect. But this shows that in order to be justified in refusing
to assimilate M to the Liar one must have prior reasons to reject or at least not accept [Maximalist-Thesis]. But if so, discussion of M as a counterexample to Truthmaker Maximalism is of little interest, since one has already rejected or refused to accept Truthmaker Maximalism (2006: 262).

There are two issues with this response when applied in the current context. First, since M is being put forward as a counterexample to Key Premise, the dialectic is shifted. Whereas Milne’s argument is intended as an outright refutation of truthmaker maximalism, the argument presented here merely defends non-maximalism against arguments based on Key Premise. Non-maximalists are justified in rejecting Key Premise even if maximalists would be inclined to reject their justification. Second, even if we grant that rejecting Maximalist Thesis outright is question-begging against the maximalist in this context, the same does not apply to the mere non-acceptance of Maximalist Thesis. Otherwise, one would have to accept Maximalist Thesis in order to avoid begging the question against it, which is implausible. After all, part of Key Premise’s appeal is supposed to be that it is pretheoretically acceptable. Finally, refusing to accept TM might be a closed-minded stance for an arguer to take, but the status of the argument itself need not be impacted even if the arguer takes such a stance. The two issues seem entirely independent: a closed-minded arguer could make an argument that is not question-begging, and an open-minded arguer could make an argument that is question-begging. An argument does not become question-begging simply because an arguer refuses to accept an opposing view.

Jago’s response to M in What Truth Is is mired with similar dialectical concerns. Jago’s approach to addressing M appeals to what he calls the “no-proposition approach.” This approach is to take truthbearers to be propositions, develop an account of the nature of propositions, and to classify certain kinds of sentences as failing to express propositions based upon this account.
Jago puts this approach to work in addressing the liar sentence and other kinds of unusual expressions. With respect to M, Jago (2018: 311) puts it this way:

The same can be said of the No-truthmaker sentence [M]. If false, then it has no truthmaker; but if it has no truthmaker, it appears to be true. If so, it is a truthmakerless truth, contrary to maximalism (Milne 2005). The response is as before: our theory of propositions implies that [M] expresses no proposition. But that doesn’t make [M] true, for being truth (sic) requires a proposition to be expressed.

What is the theory of propositions that the no-proposition approach appeals to? As Jago puts it:

A given proposition’s nature is not merely to be true or false at a given world, but rather to be made true or made false by specific ways things could be. Propositions are not truth-conditions: they are truthmaker conditions.

Elsewhere Jago characterizes truthmaker conditions as “sets of possible truthmakers” (2020, 255). In other words, on Jago’s account it is a proposition’s nature to possibly have a truthmaker. Consequently, for a proposition to be true is for it to have a truthmaker (or an entity in virtue of which the proposition is true) and for a proposition to be false is for it have a falsemaker (or an entity in virtue of which the proposition is false).14 This response is problematic in the current context for two reasons: First, given Jago’s argument about M, there are two possibilities:

(i) that M lacks truthmaker conditions and therefore does not express a proposition,

(ii) that truthmaker maximalism is inconsistent because it implies that M both has a truthmaker and a falsemaker

Although Jago takes M’s behavior to be evidence of i, one could easily take it to be evidence of ii. On Jago’s account, if M does express a proposition, then M either has a truthmaker or a falsemaker.

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14 See (Jago 2018, 125-126) and (Jago 2018, 356).
falsemaker. As a result, proving that M cannot have a truthmaker amounts to simultaneously showing that M is true (since M says that M has no truthmaker) and showing that M is false (because M has no truthmaker and thus has a falsemaker). Second, once again, Key Premise is supposed to be pretheoretically acceptable. Appealing to a theory of propositions to address an objection based on M immediately eliminates that benefit. Although Jago’s theory of propositions may be a reasonable defense of maximalism against arguments based on M, the non-maximalist need not adopt a theory of propositions which entails truthmaker maximalism.

4. Conclusion

M is arguably a truth which could not possibly have a truthmaker. We have also seen that arguments based on M are not analogous with the knower paradox. Whereas the knower paradox requires the assumption that proof entails knowledge, the truthmaker non-maximalist need not grant that proof entails truthmaking. Additionally, because of a shift in the dialectic, the extant replies to Milne’s argument by truthmaker maximalists cannot be straightforwardly repurposed in order to defend Key Premise. It might be the case that truths without possible truthmakers are “hard to come by” (Jago 2020: 43) but they are not impossible to come by.

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References


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