ON THE COMPATIBILITY BETWEEN QUANTUM THEORY AND GENERAL RELATIVITY

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Abstract. I propose a gentle reconciliation of Quantum Theory and General Relativity. It is possible to add small, but unshackling constraints to the quantum fields, making them compatible with General Relativity. Not all solutions of the Schrödinger's equation are needed. I show that the continuous and spatially separable solutions are sufficient for the nonlocal manifestations associated with entanglement and wavefunction collapse. After extending this idea to quantum fields, I show that Quantum Field Theory can be defined in terms of partitioned classical fields. One key element is the idea of integral interactions, which also helps clarifying the quantum measurement and classical level problems. The unity of Quantum Theory and General Relativity can now be gained with the help of the partitioned fields' energy-momentum. A brief image of a General Relativistic Quantum Standard Model is outlined.

Contents

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1. **Introduction**

In this article I propose a reconciliation of the two main branches of fundamental physics: General Relativity (GR) and Quantum Theory (QT) (including the Standard Model). These two fields together contain almost entirely the fundamental physics, but when we try to combine them, we encounter serious difficulties. The program proposed here implies minimal changes in both theories: the spacetime will remain four-dimensional, in fact, GR will remain untouched. Just one extra restriction will be proposed for the relativistic fields, in the form of a principle. Nonrelativistic Quantum Mechanics (QM) and Quantum Field Theory (QFT) will suffer some changes, which, we will see, are minor, and don't contradict the experimental data.

There are several difficulties we encounter when we want to unite GR with QT. One of them is that, in QM, it seems that the Schrödinger equation is from time to time (when we observe the system) violated by the so-called wavefunction collapse. In GR fields should be continuous, so that their energy-momentum tensor is well defined and locally conserved (this also required the fields to be differentiable). Another difficulty comes from the *entanglement*, which is a manifestation of the idea that a joint system is described by the tensor product of the Hilbert space of its subsystems. Most solutions of the Schrödinger equation governing the joint system cannot be separated in solutions describing the subsystems. Consequently, the fields have a nonlocal character, which makes impossible to define an appropriate energy-momentum tensor.

I show that we can obtain the appearance of wavefunction collapse without violating the unitary evolution governed by Schrödinger's equation. Moreover, the combined

systems can be always spatially separable, but still presenting the appearance of nonlocality associated with entanglement. Therefore, there is no need to keep the spatially nonseparable state vectors of a composite system's state space.

To generalize this solution of the QM problems to quantum fields, I propose the idea of field partition. The quantum states arise, in this formalism, as partitioned classical fields, and the field operators are constructed as operators on the state space. Related to the field partition, I propose the Principle of Integral Interactions. This principle may arise as a consequence of a deeper phenomenon, probably of the topology of the bundles describing the fields. It is responsible for the quantum manifestations in classical fields, by providing a support for the field partitioning. It is also an important ingredient in solving two main problems of QM – that of measurement, and the one of the relation between the classical and quantum levels.

When trying to relate the quantum fields to gravity, normally we encounter the problem that gravity is nonrenormalizable. A more modest approach is to take the expectation value of the energy-momentum quantum operator as a source for gravity, but this solution is considered provisory, and requires a renormalization procedure for the energy-momentum expectation value. The partitionable fields on the other hand, have associated the classical energy-momentum, which can now be naturally taken as the source of gravity and curvature.

I will now detail the program, and the steps done so far.

2. **Preliminaries**

2.1. **The General Relativistic World**

According to General Relativity, the space and time are warped together in a continuum, a four dimensional differentiable manifold M, having a metric g_{ij} of signature $(+---)$ on its tangent bundle. The matter is represented by vector fields on M, *i.e.* sections of vector bundles over M (spinor, tensor, complex vector fields, and various combined constructions).

The matter fields are solutions of some partial differential equations (*e.g.* the "classical" – *i.e.* not quantized – Maxwell, Dirac, or Yang-Mills equations), which in turn can be derived from a Lagrangian, from which we can construct an energy-momentum tensor T_{ij}. The fields bend the spacetime, according to Einstein's equation:

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij} \tag{1}$$

where $R_{ij} = R^k_{ijk}$ is the Ricci curvature tensor, $R = R^i{}_i$ the scalar curvature, G, c, and Λ are respectively the gravitational constant, the speed of light, and the cosmological constant.

GR describes very well the way matter generates gravity, and how is affected by it. Yet, there still are important cosmological mysteries, like those related to the Big Bang, dark matter, and dark energy.

2.2. The Quantum World

2.2.1. Quantum Mechanics

In Quantum Mechanics, the systems are represented as rays in a Hilbert space, and the time evolution of systems is represented as solutions of Schrödinger's equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(x,t)\rangle = \hat{H}|\psi(x,t)\rangle \tag{2}$$

The observables are represented by Hermitian operators on the Hilbert space. After a measurement, the system is found in a state described by an eigenvector of the observable. In this standard view, developed by von Neumann, there is a contradiction between the unitary evolution described by Schrödinger's equation, and the condition of finding the system in an eigenstate of the observable. In general, two successive measurements of the same system impose conditions that cannot be satisfied by the same solution to the Schrödinger equation. The standard approach is to consider that, between the two measurements, a discontinuous jump takes place: the rotating vector disappears here and appears there in the Hilbert space. The particle jumps discontinuously from the state $|\psi\rangle$ to $|\psi'\rangle$. For example, by determining the position of a particle, its wavefunction collapses to the point where the particle is found. The probability to find the particle at x is given by $|\psi(x,t)|^2 = \langle\psi(x,t)|\psi(x,t)\rangle = |\langle\psi(t)|x\rangle|^2$, according to Born's rule. More general, the probability to find that the particle $|\psi\rangle$ is in the eigenstate $|\psi'\rangle$ of the observable is given by $|\langle\psi|\psi'\rangle|^2$.

The Hamiltonian of a many-particle system acts on a state space which is a tensor products between the particles' Hilbert spaces. Since the corresponding Schrödinger equation is linear in this case too, there are solutions which cannot be considered simple products of wavefunctions for single particles, but rather superpositions of them. These solutions are named *entangled states*.

2.2.2. Quantum Field Theory

Klein-Gordon and Dirac equations are Lorentz-invariant versions of the Schrödinger equation. To overcome the problems of QM when we go to the special relativistic regime, we need to quantize classical fields instead of particles. This led to the QFT which is able to treat very well the particles and their interactions. All the known forces (apart from gravity) and particles are described by quantizing fields of the gauge theory based on the group $\mathrm{U}(1)_Y \times \mathrm{SU}(2) \times \mathrm{SU}(3)_C$. This description forms the Standard Model of particle physics, which is satisfactory for the particle regime, although there still are important unanswered questions.

In QFT we can use the occupation number representation to describe the states of more particles of the same type. Let $\mathcal{H}$ be a separable Hilbert space of one particle, and let $(|e_i\rangle)_{i\in\mathbb{N}}$ be an orthonormal basis of $\mathcal{H}$. We represent the system consisting of n_i particles in the state e_i by $|n_1, n_2, \ldots, n_k, \ldots\rangle$, where $n_i \in \mathbb{N}$ for bosons, and $n_i \in \{0,1\}$ for fermions. The Hilbert space $\mathcal{H}_N$, generated by these vectors and the scalar product

$$\langle n_1, \ldots, n_k, \ldots | n'_1, \ldots, n'_k, \ldots\rangle = \delta_{n_1 n'1} \ldots \delta_{n_k n'k} \ldots \tag{3}$$

represents the many particle systems. The vacuum state is represented by the vector $|0\rangle = |0, \ldots, 0, \ldots\rangle$.

The creation operators $\hat{a}^\dagger(|e_k\rangle)$ for bosons create a particle in the state $|e_k\rangle$, and $\hat{a}(|e_k\rangle)$ annihilate a particle from the state $|e_k\rangle$:

$$\hat{a}^\dagger(|e_k\rangle)|\ldots, n_k, \ldots\rangle = \sqrt{n_k+1}|\ldots, n_k+1, \ldots\rangle, \tag{4a}$$

$$\hat{a}(|e_k\rangle)|\ldots, n_k, \ldots\rangle = \sqrt{n_k}|\ldots, n_k-1, \ldots\rangle, \tag{4b}$$

$$\hat{a}(|e_k\rangle)|\ldots, 0, \ldots\rangle = 0. \tag{4c}$$

They obey the commutation relations

$$[\hat{a}^\dagger(|e_i\rangle), \hat{a}^\dagger(|e_j\rangle)] = [\hat{a}(|e_i\rangle), \hat{a}(|e_j\rangle)] = 0, \tag{5a}$$

$$[\hat{a}(|e_i\rangle), \hat{a}^\dagger(|e_j\rangle)] = \delta_{ij}. \tag{5b}$$

In general,

$$[\hat{a}(|u\rangle), \hat{a}^\dagger(|v\rangle)] = \langle u|v\rangle. \tag{6}$$

For the fermions, the creation and annihilation operators act by:

$$\hat{a}^\dagger(|e_k\rangle)|\ldots, n_k = 1, \ldots\rangle = \hat{a}(|e_k\rangle)|\ldots, n_k = 0, \ldots\rangle = 0, \tag{7a}$$

$$\hat{a}^\dagger(|e_k\rangle)|\ldots, n_k = 0, \ldots\rangle = (-1)^{n_1+\ldots+n_{k-1}}|\ldots, n_k = 1, \ldots\rangle, \tag{7b}$$

$$\hat{a}(|e_k\rangle)|\ldots, n_k = 1, \ldots\rangle = (-1)^{n_1+\ldots+n_{k-1}}|\ldots, n_k = 0, \ldots\rangle, \tag{7c}$$

and obey the anticommutation relations

$$\{\hat{a}^\dagger(|e_i\rangle), \hat{a}^\dagger(|e_j\rangle)\} = \{\hat{a}(|e_i\rangle), \hat{a}(|e_j\rangle)\} = 0, \tag{8a}$$

$$\{\hat{a}(|e_i\rangle), \hat{a}^\dagger(|e_j\rangle)\} = \delta_{ij}. \tag{8b}$$

All the states can be obtained from the vacuum state by repeatedly applying such operators, and by linear combinations. A different basis $(|e'_i\rangle)$ satisfies $|e'_i\rangle = \sum_j \langle e_j|e'_i\rangle |e_j\rangle$. Therefore, $a^\dagger(|e'_i\rangle) = \sum_j \langle e_j|e'_i\rangle a^\dagger(|e_j\rangle)$ and $a(|e'_i\rangle) = \sum_j \langle e'_i|e_j\rangle a(|e_j\rangle)$.

If the fields are complex, there are also antiparticles. We denote by $\hat{a}^\dagger_+$ and $\hat{a}_+$ the creation/annihilation operators for particles, and by $a^\dagger_-$ and $\hat{a}_-$ the creation/annihilation operators for antiparticles. They too satisfy the relations (5) and (8).

Note that, if we want to construct the many-particle states as tensor products of the one-particle Hilbert space with itself, because the identical particles are indistinguishable, we have to take the symmetric tensor products for bosons, and antisymmetric ones for fermions. We obtain an equivalent representation, but I will prefer in the following the occupation number one, because it emphasizes better some locality related aspects and avoids some confusions with the entangled states.

In the canonical quantization, we start with the field variable φ and its canonical conjugate $\pi_i = \delta\mathcal{L}/\delta\varphi_{,i}$. We replace the fields by operators $\hat{\varphi}$ and $\hat{\pi}_i$ which satisfy the commutator relations corresponding to the Poisson brackets. The field operators $\hat{\varphi}(x)$ are in general obtained by expressing the classical field in the basis $(|e_i\rangle)_{i\in\mathbb{N}}$, and then replacing the expansion coefficients with the creation/annihilation operators. In particular, we can use the Fourier expansion:

$$\hat{\varphi}(\mathbf{x}) = \int \left(e^{-i\mathbf{k}\cdot\mathbf{x}}\hat{a}(\mathbf{k}) + e^{i\mathbf{k}\cdot\mathbf{x}}\hat{a}^\dagger(\mathbf{k})\right) d^3\mathbf{k}. \tag{9}$$

We can obtain the operator corresponding to the energy-momentum tensor by inserting the field operators in its definition (or its symmetric version)

$$\hat{T}_{ij} = \hat{\mathcal{L}} g_{ij} - \hat{\pi}_i \hat{\varphi}_{;j}. \tag{10}$$

3. Wavefunction Collapse, Continuity, and Global Consistency

3.1. Conservation Laws Require Continuity

The energy-momentum operator (10) at each point is calculated in terms of the field operator (9) at that point. Its mean value is conserved:

$$\langle T_{ij} \rangle_{;k} = 0. \tag{11}$$

For the Schrödinger quantized field for a constant number of particles we obtain the QM. The momentum operator in QFT gives, for monochromatic plane waves, the same results as the momentum operator in QM.

A discontinuity in the wavefunction's evolution will correspond to a discontinuity in the quantum field. This leads to a violation of the mean energy-momentum conservation (11), therefore of the momentum and energy, and also of the electric charge, since its density current also depends locally on the field.

In fact, at the beginnings of QM, Bohr proposed that the momentum conservation is broken, being valid only in average [1, 2, 3]. Of course, an experiment proposed [4] by Bothe and Geiger confirmed [5] the conservation.

Let's consider an isolated system made of two electrically charged particles, which initially have definite momenta. During the interaction between them, their momenta can become undefined, but the total system still is in an eigenstate of the momentum operator. This shows that the momentum is conserved even if the systems are not all the time in a momentum eigenstate. How an undefined quantity can be conserved? Well, the mean value of momentum (and the energy-momentum density) still can be defined, and they are conserved. When one subsystem becomes again a momentum eigenstate, the momentum eigenvalues of the composite system "reassemble themselves from thin air", and the mean (or expectation) value coincides with the eigenvalue.

Let us suppose that we measured a particle's momentum, then it's position, then again it's momentum. Clearly the measurement of the position disturbed indefinitely its momentum, so the initial and final momenta are different. But the momentum is conserved; it simply was transferred between the measurement devices and the particle. And when the particle had an undefined momentum, the measurement device's momentum differed with the same undefined quantity as the particle (in "opposite undefined direction"). But their momenta's mean values were well defined all the time, and were conserved.

This discussion of the momentum conservation also works for electric current and all other locally conserved quantities. The conserved observables are those commuting with the Hamiltonian, and a discontinuous evolution can guarantee neither theirs nor their mean value conservation. The only way to force the conservation laws during a

discontinuous collapse would be to admit global conservation, but local violation. But the relativity of simultaneity makes this impossible, since a global conservation law which is not local will appear, from another reference frame, as being violated at global level too.

To resume: a discontinuous collapse would imply that the mean value of the energy-momentum is not conserved, which would lead to violations of the momentum conservation, which were not detected. Therefore, the discontinuous collapse doesn't exist. Instead, the collapse can happen in a continuous manner, as we shall see.

3.2. **Principle of Continuity**

Principle 1 (of Continuity)**.** The only admitted wavefunctions (in QM) or fields (in QFT) are continuous, and have continuous time evolution. Their partial derivatives are also required to exist and to be continuous, to the degree required by the PDE defining the fields.

If the wavefunction $|\psi(t_0)\rangle$ is smooth, then the solution of Schrödinger equation does not allow discontinuity in time, as the collapse seems to imply. The discontinuous collapse contradicts clearly the Schrödinger equation. We should never need a principle to restore something which was already contained in the Schrödinger equation.

Other fields are obtained from partial derivatives of the wavefunctions, they also require the continuity of derivatives of the needed order. For example, (11) requires the energy-momentum to be differentiable, therefore in general $|\psi\rangle$ should be two times differentiable. Nicely behaved energy-momentum is required by the GR.

This principle also rules out the observables having discontinuous eigenfunctions or distributions, such as the position operator.

It seems to be a problem with this principle. How can we say that the measurements have definite results, if we don't allow the state vector to be projected on an eigenspace of the observable? Well, it is possible to allow this projection, but still require it to be **continuous**.

3.3. **Delayed Initial Conditions**

My proposed solution [6] to the apparent contradiction between the Schrödinger equation and the conditions imposed by the measurements is to admit that the solution of the Schrödinger equation is not determined completely by the initial conditions, but also by future conditions consisting in the choice observables to be measured. This idea may look crazy, but it is already contained in the idea of delayed-choice experiment. We can just extrapolate it to the entire past, completely removing the discontinuous collapse.

Einstein hoped that, in the two-slit experiment (figure 1), we can detect through which slit the photon passed by measuring the recoil of the wall containing the slits. But making this measurement will prevent the interference to happen. Consider that we choose whether to detect *which-way*, or the interference (*both-ways*), after the photon passes through the slit, according to Wheeler's delayed-choice version of this experiment. We conclude that our choice selected which version of the past already happened: the

one in which the two-slit wall had a definite recoil, therefore the photon passes through one slit, and the one in which the recoil is undefined, witnessing a photon which passed through both slits. In both cases, the momentum conservation, and the continuity of the wavefunction evolution, are respected, if we admit that the photon was, even before going through the two-slits wall, in a state anticipating our choice. We can interpret this as thinking that the photon was entangled with the two-slit wall, in a state

$$1/\sqrt{2}(|\text{photon up}\rangle \otimes |\text{recoil up}\rangle + |\text{photon down}\rangle \otimes |\text{recoil down}\rangle). \tag{12}$$

If we choose to measure the *which-way*, the entanglement resolves into one of the two states in the superposition, while choosing to measure interference let the system in superposition.

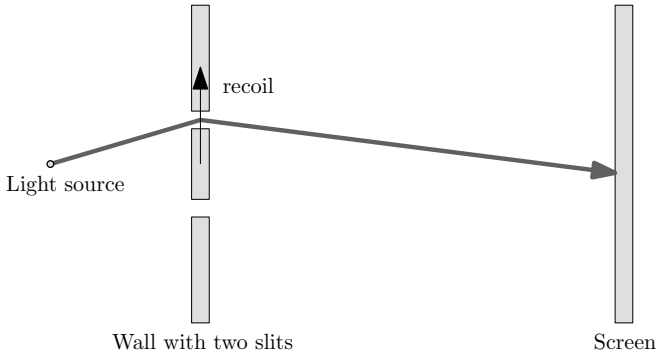

FIGURE 1. If the photon travels through the upper slit, the two-slits wall recoils upwards.

There is no special reason why we should stop with inferring the past at the moment the photon was emitted. The emission should have been caused by the source's past, and so on, and the wall's state also was caused by something in its past. The very causality principle seems to require that there is no discontinuous collapse. But isn't the selection of the past to fit future choices a violation of causality?

3.4. **Principle of Global Consistency**

What is causality principle? Is it the idea that the present state should be univocally caused by its past, in a deterministic manner? If so, then maintaining the continuity in the evolution of the wavefunction obeys to this principle, while the discontinuous collapse breaks it. The wavefunction's unitary evolution is deterministic, and the choice of what observable to measure contributes only to selecting a specific, deterministic solution. This may look strange, but it explains what happens during the apparent collapse without violating the conservation laws. If we admit that the collapse happens continuously, there is no way but to admit the idea of delayed initial conditions.

I will show now that this idea can be viewed in a more acceptable way, from mathematical viewpoint. As a parallel, consider a power series as in complex analysis $\sum_{k=0}^{\infty} a_k(z - z_0)^k$, convergent in a disc $D_r(z_0) \subset \mathbb{C}$. It defines a holomorphic function $f : D_r(z_0) \to \mathbb{C}$. We can enlarge the domain $D_r(z_0)$ of f by *analytical continuation.*

If we develop in power series around a point $z_1 \neq z_0$, $z_1 \in D_r(z_0)$, we obtain another convergence disk, which may go outside $D_r(z_0)$. We can go from z_0 to a point $z_f \notin D_r(z_0)$, by following two different paths. If the contour enclosed within the two paths doesn't contain singularities, then the value of f at z_f is independent of the path. Going back to the Schrödinger equation (2), let $|\psi(t_0)\rangle$ be the state describing the world at t_0 (including therefore both the measurement device and the observed system), and let's assume that at the instant t_1 and place x_1 the two-slit experiment is performed. We can develop $|\psi\rangle$ locally to obtain its state at (t_1, x_1). Let's assume that we are following, on the one hand, the evolution of the measurement device (including the observer), and on the other hand, that of the source emitting the photon. They both are considered local subsystems of $|\psi\rangle$. It is analogous to the analytical continuation along two paths: that of the measurement device, and that of the photon. When the two paths met one another, they should have consistent states. Otherwise, we would violate the mathematical consistency of the solution $|\psi\rangle$. Hence, the same state vector $|\psi(t_0)\rangle$ is responsible for the choice of the observable, and of the initial condition of the observed particle. A similar reasoning shows us that, in an EPR type experiment, the entangled particles should be found in states which are mutually compatible, states following from the same initial state vector.

Think at a four dimensional spacetime, as in Relativity, and $|\psi\rangle$ the wavefunction describing the world. It is a block world, containing equally the past, present and future. In fact, there is no special present marked on it. Why should there be an absolute initial condition for $|\psi\rangle$? Why the conditions selecting $|\psi\rangle$ among other solutions of the equation (2) be all at an initial time (whatever "initial" may mean), and not spread at various points in spacetime? In fact, why even having such points, when in fact we only need the solution, and the initial conditions, delayed or not, are used only to determine it among all possible solutions? There is certain relativity in choosing what the points in spacetime are where the initial conditions should be imposed. Well, if there is an initial singularity, the Big Bang, then it seems that this point is special. It is also a point of very low entropy, explaining why the entropy seems to increase with time. But even if we define the initial conditions of the universe at that point, they still should be mathematically consistent, such as the time development doesn't lead to contradictions. The initial conditions should always be well posed, so that the future development is consistent. It is not important when the initial data of a field is defined, important is that the solution is globally self-consistent [7].

Principle 2 (of Global Consistency)**.** The fields describing the matter should be globally consistent.

It is difficult to find a reason to reject this principle. So why did I even mentioned it? Simply because it contains the main intuition which led us thinking in terms of causality. The idea of whatever we understand by causality is that the phenomena should be consistent. We feel uncomfortable about delayed initial conditions, because we worry that a cause in the present may change the past. But what we really want is that the phenomena are consistent, and there is no reason to worry about this.

The Principle of Global Consistency allows the world to look like most of the initial conditions are defined in the past, and that our choice of what to measure only refines

them. Thinking from the atemporal "God's view", the initial conditions may very well be delayed. The real reason why we see causes in the past and effects in the present is that the time is revealing to ourselves from past towards the future.[1] In fact, there only exist fields, which are correlated and consistent with one another and with themselves. I would mention a phenomenon which seems to reveal a different situation where delayed initial conditions occur, in some experiments with photons [8]. They seem to go back in time, because of a very special experimental arrangement where they have negative group velocity in a medium.

Applying the Principle of Global Consistency, we can see that there are consistent solutions of the equation (2) which appear like a wavefunction collapse, but which are not. The idea is that the observed particle interacted with the preparation and measurement devices, and is entangled with them, and the experiment simply "chooses" from the entanglement (which is a superposition) a solution that fits an eigenstate of the observable [6]. A more temporal way to view this is that the initial conditions and the previous interactions of the observed system, for example the interaction with the preparation device, are precisely such that the measurement finds the system in the eigenstate of the observable.

The idea of global consistency is fundamental and elementary, and it is enough to explain the correlations which occur in experiments such as EPR, two-slit, Mach-Zehnder, and quantum eraser, with their delayed-choice versions. The systems are correlated because they are parts of the same wavefunction. There is no reason to think that an instantaneous message, or one traveling backwards in time, is sent. There is no need to send back in time another wavefunction which changes the already happened one. The spins in Bohm's version of the EPR experiment [9] are correlated among them and with the direction chosen by the experimenter, not because they are signaling, but simply because they are parts of the same solution of equation (2). There is no need to conceive additional mechanism, like faster-than-light signaling, waves traveling back in time to erase the past and create a new one, or esoteric laws ensuring spooky correlation and momentum conservation at a distance.

4. **Principle of Spatial Separability**

4.1. **Entanglement without Entanglement**

We have seen that it is possible to have, to paraphrase Wheeler, "collapse without collapse" – the appearance of wavefunction discontinuous collapse, which in fact is unitary (and of course continuous) evolution. What about entanglement?

Two systems are entangled, if the composed system cannot be expressed as a simple tensor product of states $|\psi_1\rangle \otimes |\psi_2\rangle$, but only as a superposition of at least two such products:

$$|\psi_1^1\rangle \otimes |\psi_2^1\rangle + \ldots + |\psi_1^n\rangle \otimes |\psi_2^n\rangle. \tag{13}$$

[1] Principle 2 is also compatible with the closed timelike curves, but there is absolutely no relation between them and the delayed initial conditions.

The entangled (or nonseparable) states were predicted by Schrödinger [10] and Einstein, Podolsky and Rosen [11], although they considered them "bugs" of QM. The theoretical origin of the entanglement is the principle of superposition: if ψ_1 and ψ_2 are solutions of the Schrödinger equation (2), then $\alpha_1\psi_1 + \alpha_2\psi_2$ is also a solution. Since a system composed of two systems is represented on the Hilbert space given by the tensor product of the two Hilbert spaces, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, it follows that the superpositions of the form (13) are also solutions. The experimental tests of the violation of Bell's inequality [12, 13, 14, 15, 16] are taken as confirming the reality of entanglement.

When a two-particles system is in an entangled state like in the EPR experiment, determining the state of the first particle as being $|\psi_1^k\rangle$ results in determining the state of the second one to be $|\psi_2^k\rangle$ [2]. It is therefore possible that the two states were $|\psi_1^k\rangle$ and $|\psi_2^k\rangle$ even before the measurement, and the particles were not in fact entangled! What I want to say is that we don't really need the particles to be in the entangled state (13), in order to obtain the correlations specific to entanglement, and to violate Bell's inequality[17, 18]. We can have "entanglement without entanglement", since the correlations are the same. In the case of the EPR experiment, the correlation are dictated by the momentum conservation, and not by the particles really being in an entanglement of all possible outcomes with the total momentum equal to the initial one.

Of course, it is possible, in theory, that the systems really are in an entangled state, but in the case when we disentangle them by measurement, it turns out that they were not entangled, just correlated. What if we perform an experiment to measure an observable with all the eigenstates being nonseparable? Of course, if we would have such a measurement device, we may prove that the states were really entangled. This would show that there really are states which aren't separable. But can such a measurement device be constructed?

4.2. **Principle of Spatial Separability**

But since we can detect the separated states, this implying that they were separated all the time, why would we assume that there really are nonseparable states? The reason is that there are some states which are indeed not separable. For example, the singlet state $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$, which is entangled in the spin space, or the meson $\pi^0 = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$, which is entangled in the quark space. These examples have in common the fact that they are entangled in some internal spaces, although they can still be separated in space. So, I propose a "weak" principle of separability:

Principle 3 (of Spatial Separability)**.** All admitted quantum states of composite systems are separable in space. The time evolution preserves the separability.

This principle simply takes back what was lost in accepting the nonseparability, because the entanglement in space is no longer needed. And the "entanglement in internal spaces" is simply a way to speak about the tensor products of the vector bundles in which the fields are sections. Instead of tensoring Hilbert spaces of fields, we tensor vector bundles in which the fields are sections.

[2] If we also measure the second one, the probabilities obey Born's rule relatively to $|\psi_2^k\rangle$.

I mentioned in the discussion of the two-slit experiment (see figure 1) that the interference is usually considered to be $1/\sqrt{2}(|\text{photon up}\rangle \otimes |\text{recoil up}\rangle + |\text{photon down}\rangle \otimes |\text{recoil down}\rangle)$, which is a nonseparable state. In fact, the interference is as well a classical wave phenomenon. The photon's wavefunction splits like a classical wave, and the state is in fact

$$\left(\frac{1}{\sqrt{2}}|\text{photon up}\rangle + \frac{1}{\sqrt{2}}|\text{photon down}\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|\text{recoil up}\rangle + \frac{1}{\sqrt{2}}|\text{recoil down}\rangle\right), \tag{14}$$

which is separable. Let us name this combination *pseudo-superposition*, as opposed to the genuine superposition, which allows composite systems to be nonseparable. The two states, (12) and (14), differ (except a multiplicative constant) by two crossed terms that appear when we multiply the two parentheses:

$$\frac{1}{2}|\text{photon up}\rangle \otimes |\text{recoil down}\rangle + \frac{1}{2}|\text{photon down}\rangle \otimes |\text{recoil up}\rangle.$$

The part of the wave $|\text{photon up}\rangle$, and the point of the lower slit where $|\text{recoil down}\rangle$ is applied, are spatially separated enough so that their mutual interaction can be neglected. Recall that, in general, the Hamiltonian contains a potential which expresses the interaction between each pair of particles. The interaction is small for large enough distances, as in the case of our crossed terms. Therefore, the entangled state (12) evolves approximatively like (14). I claim that the correct description is given by (14). In some experimental arrangements, we may compare the two predictions.

Even in the two-slit experiment with buckyballs (molecules of C60 fullerene), we can consider each of the elementary particles wavefunction split in two parts, one going through the upper slit, another through the lower slit. Yet, there are differences between the entangled and the separable state vectors, and the differences are expected to increase with the complexity of the systems, so that we cannot obtain interference phenomena for any system.

4.2.1. **Hamiltonians and Separability**

A separable state $|\psi_1\rangle \otimes |\psi_2\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$ can be transformed by a unitary operator in a nonseparable one. This means that not all possible Hamiltonians preserve separability. In the case of a free particle, the unitary evolution preserves the purity of the state. If we have two interacting particles $|\psi_1(t_0)\rangle \in \mathcal{H}_1$ and $|\psi_2(t_0)\rangle \in \mathcal{H}_2$, will they evolve from $|\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle$ into a nonseparable state?

The total Hamiltonian is $\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{\text{int}}$. $\hat{H}_1$ and $\hat{H}_2$ are the free Hamiltonians of the particles, acting separately on $\mathcal{H}_1$ and $\mathcal{H}_2$. The interaction Hamiltonian $\hat{H}_{\text{int}}$ acts on the product $\mathcal{H}_1 \otimes \mathcal{H}_2$. But it has two parts, each acting on one particle:

$$\hat{H}_{\text{int}}(|\psi_1\rangle \otimes |\psi_2\rangle) = \hat{H}_{21}(|\psi_2\rangle)|\psi_1\rangle + \hat{H}_{12}(|\psi_1\rangle)|\psi_2\rangle,$$

where $\hat{H}_{21}(|\psi_2\rangle)$ acts on $\mathcal{H}_1$ and $\hat{H}_{12}(|\psi_1\rangle)$ on $\mathcal{H}_2$. Therefore, we have the following coupled system

$$\begin{cases} i\hbar\frac{\partial}{\partial t}|\psi_1(t)\rangle = \left(\hat{H}_1 + \hat{H}_{21}(|\psi_2(t)\rangle)\right)|\psi_1(t)\rangle \\ i\hbar\frac{\partial}{\partial t}|\psi_2(t)\rangle = \left(\hat{H}_2 + \hat{H}_{12}(|\psi_1(t)\rangle)\right)|\psi_2(t)\rangle \end{cases} \tag{15}$$

which preserves separability.

Like the Hamiltonian, the observables too preserve the separability. The known observables have physical meaning for one particle, for example position, momentum, charge, *etc.*. These observables are always reducible to the corresponding observables of the elementary particles, and their eigenvalues are function of the subsystems' eigenvalues, *e.g.* sums or products.

4.2.2. **Tensor Product *vs.* Direct Sum**

The representation of composite systems by tensor products of the individual Hilbert spaces originated in the quantization of classical systems. The classical systems of n particles are described by $3n$ positions and $3n$ momenta. By quantizing a one-particle system, we obtain a wavefunction, a complex function on $\mathbb{R}^3$, from the space $L^2(\mathbb{R}^3)$. It is natural to represent a system of n particles as a wavefunction from $L^2(\mathbb{R}^{3n})$. But since $L^2(A\times B) = L^2(A)\otimes L^2(B)$, it follows that the natural way to represent composite quantum systems is by tensor products of their individual Hilbert spaces.

The tensor product is disadvantageous if we want to keep track of the individual phase or amplitude of a particle in the system. If we represent the states as tensor products, the notions of individual phase or individual amplitude are meaningless. But in interference processes, both individual amplitude and phase play a key role. Moreover, Aharonov and Bohm's effect [19], confirmed experimentally by S. Olariu and Ioan-Iovit̨ Popescu in 1985 [20], shows that the phase of a field is real. Instead, the direct sum can track both phase and amplitude separately for each particle.

On the other hand, the linear combinations of separated states, even if are not separable, are useful, when we don't know the real state, and we want to obtain statistical estimations. Therefore, the tensor product is useful, and I don't suggest to replace it completely with direct sums.

If the states of a system composed of two systems represented by $\mathcal{H}_1$ and $\mathcal{H}_2$ belong to the direct sum $\dim(\mathcal{H}_1)\oplus\dim(\mathcal{H}_2)$, what about the Hamiltonian and other operators? One reason why it should be preferred the tensor products to the direct sum is that the composed system should have $\dim(\mathcal{H}_1)\cdot\dim(\mathcal{H}_2)$ (not necessarily distinct) eigenvalues, and not $\dim(\mathcal{H}_1)+\dim(\mathcal{H}_2)$ (see *e.g.* [21]). Moreover, in general, the scalar product of the total Hilbert space obtained as tensor product is more appropriate, allowing us to define unitary and hermitian operators.

A compromise solution can be to use the Segre embedding[3] of the direct sums into tensor products of the state spaces. This way, we can keep as states the separable ones, and use the hermitian scalar product and the unitary and hermitian operators in the same way.

[3]Usually, the Segre embedding refers to the embedding of projective spaces $\mathbb{P}^m\times\mathbb{P}^m\mapsto\mathbb{P}^{m+n+mn}$, but it can be used also in the sense of embedding the underlying vector spaces.

5. Consequences of the Continuity and Spatial Separability Principles

5.1. Back to the Classical Values?

We had to pay a price for explaining away the discontinuous collapse – we had to understand that we cannot say at an instant of time that the fields are determined, since the initial data needs to be collected by further experiments. And choosing what to observe determines what solution already happened. But we also brought back into Physics the possibility that the fields have ontological reality, and are spatially separated. The vision which emerges from here is closer to the classical physics than we expected. The fields are again local, and there is again something that happens between the measurements: we regained the *local realism*. We can now see that the wavefunction provides a complete description of reality.

Moreover, we can see that the *determinism* is back too, since the unitary evolution is deterministic. But we still seem to be able to choose what to measure, and therefore what happened, so that we can obtain a particular set of possible states and not another. So, if the standard interpretation of QM allowed the *free will*, ours allows it too. The collapse takes place smoothly, deterministically, and we can't predict the outcomes because we don't know the initial conditions, which are the truly undetermined part.

Heisenberg's principle is about the size of the wave packet in position *vs.* frequency representations. It has nothing to do with indeterminism, because the particles are in fact waves, and it is not appropriate to speak about position and momentum.

5.2. Actual and Potential

Not all solutions of the Schrödinger equation respect the Principle of Spatial Separability. Potentiality is described by the full set of solutions subject to the constraints accumulated so far, including therefore the mixed states. In order to describe potentiality, we need to employ density matrices. They are used to predict the possible actual solutions, which are spatially separable and continuous. Although the actual solutions can be thought as superpositions, or they even split in smaller parts which follow different paths, they remain local, and cannot jump discontinuously. Only the potential solutions can be projected, only they can collapse, since they are waves of information, describing our momentary knowledge. Any result of a measurement collapses them. But the actual solutions behave locally and respects the Hamiltonian evolution (hence the Principle of Continuity) and the Principle of Spatial Separability.

6. Quantum Fields, Field Partitions, and Spatial Separability

I will develop in this section a way to represent the quantum fields by using partitioned classical fields. This representation can be used for quantization, but it will also help

extending the Principle of Spatial Separability to QFT. In this way, we can establish the link between the quantum and classical levels, including General Relativity.

6.1. **Field Partitions**

Let us start with a relativistic equivalent of the Schrödinger equation (2), such as Klein-Gordon or Dirac equation. The classical solutions will be sections in vector bundles, for example, in the case of the Dirac field, in the bundle of Dirac spinors. We can expand the classical solutions in Fourier series, having thus negative and positive-frequency eigenstates of momenta. This allows us to construct two spaces of classical solutions, those of positive and negative energies. We interpret these solutions as the particle and antiparticle states, which are wavefunctions like those in QM. We use them to construct the state space for the occupation number representation, as usual. We define the creation/annihilation operators, by equations (4) and (7).

In general, the emphasis is put on the field operators. They are required to obey Heisenberg's equation, which captures the dynamics, while the state is present only as the vacuum state, on which the field operators act. On the other hand, to represent the quantum state, we can use the states composing the occupation number as well. They can be viewed as wavefunctions which are solutions of the relativistic correspondent of the Schrödinger equation.

For example, $|n_1, n_2, \ldots, n_k, \ldots\rangle$ can be viewed as a classical field Ψ partitioned in the eigenfunctions $|e_i\rangle$

$$\Psi = \underbrace{|e_1\rangle \odot \ldots \odot |e_1\rangle}_{n_1 \text{ times}} \odot \ldots \odot \underbrace{|e_k\rangle \odot \ldots \odot |e_k\rangle}_{n_k \text{ times}} \odot \ldots$$

In general, a partition is specified by

$$\Psi = \underbrace{|\psi_1\rangle \odot \ldots \odot |\psi_1\rangle}_{n_1 \text{ times}} \odot \ldots \odot \underbrace{|\psi_k\rangle \odot \ldots \odot |\psi_k\rangle}_{n_k \text{ times}} \odot \ldots, \tag{16}$$

where $|\psi_k\rangle$ are state vectors of distinct directions from the Hilbert space of one particle $\mathcal{H}$. The symbol $\odot$ means here that the particle currents (*e.g.* for electron is $\bar{\psi}\gamma^k\psi$) are related by

$$j(\Psi) = n_1 j(|\psi_1\rangle) + \ldots + n_k j(|\psi_k\rangle) + \ldots$$

We do not require $|\psi_k\rangle$ to be linearly independent, only to have distinct directions in $\mathcal{H}$. A state vector $|\psi\rangle$ with $\int_\Sigma \rho(|\psi\rangle)\mathrm{d}\sigma = 1$ represents an *integral* particle. If all $|\psi_k\rangle$ are integral, we say that the partition is *integral*. Fields admitting an (integral) partition are named *(integrally) partitionable*. The partition is not unique, so the particles are not uniquely defined. A partition of a classical field is not implicit in the field, nor in the equation, for the moment it can be considered as an extra structure. A partition of a classical field can be translated in time, by evolving the parts independently.

The field operators are simply operators acting on the space linearly generated by the partitioned solutions of the classical field equation. They can be used to express any state in terms of the vacuum state. Therefore, the field operators are totally definable in terms of partitioned fields.

Because the parts of the field are not necessarily orthogonal, the partitioned field may appear as a superposition of separable states in the occupation number representation corresponding to a particular orthogonal basis. For example, the partitioned state $|1,0\rangle\odot(\alpha|1,0\rangle+\beta|0,1\rangle)$, with $\alpha^2+\beta^2=1$, when expressed as $\alpha|2,0\rangle+\beta|1,0\rangle\odot|0,1\rangle$, appears to be a superposition. But it is separable into the one-particle pure states $|1,0\rangle$ and $\alpha|1,0\rangle+\beta|0,1\rangle$, only that this is not obvious in this particular basis, which is a tensor product of the elements of the one-particle Hilbert space's basis.

Any state in $\mathcal{H}_N$ can be expressed as linear combination of classical field partitions. Moreover, we can even think even at the superposition of separable states as representing a double-layered partition: the first layer is given by the linear combination, and the second layer by the partition itself. This makes QFT look incredibly classical. Admitting only spatially separable fields makes it even more classical-like. The partition representation shows that the classical and quantum fields are the same, except that the quantum one is partitionable. Quantization is just partitioning of classical fields.

6.2. **Commutation, Anticommutation, Uncertainty**

The operators $\hat{a}^\dagger_\pm(|e_k\rangle)$, $\hat{a}_\pm(|e_k\rangle)$, $\hat{\varphi}(\mathbf{x})$, $\hat{\pi}_i(\mathbf{x})$, defined by (4), (7), (9), can be used to construct states, starting with the vacuum state $|0\rangle$. In terms of the classical field partition representation, $|0\rangle$ corresponds to a constant solution of the relativistic version of Schrödinger's equation. The operators $\hat{a}^\dagger(|\psi\rangle)$ can be interpreted as just adding a particle in the state $|\psi\rangle$.

One would expect that creating and then annihilating the same quanta will leave the state identical, which is not true, because of the way the coefficients in (4) were chosen. These coefficients are the only cause of the commutation relation (5), and similar is for the anticommutation of fermions (8). They lead to the commutation relations for the field operators $|\varphi\rangle$ and $|\pi_0\rangle$

$$[\hat{\varphi}(x,t),\hat{\varphi}(y,t)]=[\hat{\pi}_0(x,t),\hat{\pi}_0(y,t)] = 0 \tag{17a}$$

$$[\hat{\varphi}(x,t),\hat{\pi}_0(y,t)] = i\hbar\delta(x-y). \tag{17b}$$

For fermions we replace the commutator $[,]$ with the anticommutator $\{,\}$.

These relations express the principle that the field observables, which are bilinear forms in the field operators, are independent at distinct points separated by spacelike intervals. From their independence and bilinearity it follows that the field variables should obey either commutation, or anticommutation relations. On the other hand, the reason why at the same spacetime point, or for timelike separated events, do not (anti)commute, is in the coefficients in the definition of $\hat{a}^\dagger(|e_k\rangle)$ and $\hat{a}(|e_k\rangle)$. These coefficients has the origin in the study of the harmonic oscillator.

Any state $|\psi(x,t)\rangle$, which is a solution of the field equation, can be constructed from the vacuum with the help of the field operators $\hat{\varphi}(x,t)$ – which are used to create a point particle at the point x.

The first commutation relation (17a) saids that the field's values at any points x and y are independent. The same holds for its canonical conjugate. These conditions are natural if $|\psi(t)\rangle$ are bosonic fields, because the values of $|\psi(x,t)\rangle$ and $|\psi(y,t)\rangle$ are observable and should be independent. But the relation (17b) saids that the field's and

its canonical conjugate values are independent only at spacelike separated points. The fields $|\psi(x,t)\rangle$ and $|\pi(x,t)\rangle$ cannot commute, since $|\pi_i\rangle = \delta\mathcal{L}/\delta|\varphi_{;i}\rangle$, and in fact $|\pi_i(x,t)\rangle$ depends locally on $|\psi(x,t)\rangle$.

Such commutation conditions hold for each observable, as I mentioned. In the case of fermionic fields, the anticommutation rules for $|\psi(x,t)\rangle$ express commutation rules for an observable – which is a bilinear form in $|\psi(x,t)\rangle$. For example, in the case of a Dirac field, we can take the charge current $j_k = ie\overline{\psi}\gamma_k\psi$.

Applying the commutation rule (6) to eigenstates $u(x') = \delta_x(x')$ and $v(x') = e^{\frac{i}{\hbar}px'}$ of the quantum operators $\hat{x}$ and $\hat{p}_x$ leads to $[\hat{a}(u), \hat{a}^\dagger(v)] = \langle u|v\rangle = \int \delta_x(x')e^{\frac{i}{\hbar}px'}\mathrm{d}x' = e^{\frac{i}{\hbar}px}$. The two operators $\hat{x}$ and $\hat{p}_x$ cannot have common eigenstates, and this leads to Heisenberg's relations. Please recall Principle 1, which does not allow the discontinuous collapse. The interpretation of Heisenberg's relations is not about uncertainty, but simply a statement of the fact that a wavefunction cannot be eigenfunction of both $\hat{x}$ and $\hat{p}_x$, which originates in Fourier analysis. The probabilistic interpretation of Heisenberg's relations occurs only in conjunction with Born's rule, which expresses the probability to obtain a particular eigenstate of the observable. But this projection on the eigenspace is taking place smoothly, not discontinuously.

6.3. **Principle of Spatial Separability for Quantum Fields**

As we have seen, any state in $\mathcal{H}_N$ can be expressed as linear combination of classical field partitions. But we will want even more, so we will extend the Spatial Separability Principle to quantum fields.

Principle 3' (of Spatial Separability for Quantum Fields)**.** The admitted quantum states are the ones that can be represented as integral classical field partitions, and the unitary evolution of these quantum states maintains the partition.

I want to emphasize again that we refer to separability in space only, and not in internal spaces.

We are now in possession of a bridge between the quantum and classical worlds. Of course, even if the Principle of Spatial Separability for Quantum Fields will be rejected by experiment, we can still use the partition representation, which emphasizes the relation between the quantum and the classical levels. But I think that the double-layered partition, although can express the QFT as we know it, allows too many possible states. It allows all possible classical fields, but with a more complicated mechanism, for expressing them as superpositions, for partitioning each element of the superposition, and for keeping them independent. On the other hand, there may be a simple reason for the single-layered partition, *i.e.* for states satisfying the spatial separability principle.

7. Principle of Integral Interactions

7.1. Interactions

For the interactions of the classical fields, we use coupled equations. They are described by each field's Lagrangians, plus the interaction Lagrangians, *e.g.*:

$$\mathcal{L} = \underbrace{\underbrace{\bar{\psi}(i\gamma^k\partial_k - m)\psi}_{\text{Dirac}} \; \underbrace{-\frac{1}{4}F^{jk}F_{jk}}_{\text{Maxwell}}}_{\text{free}} \; \underbrace{+e\bar{\psi}\gamma^k A_k\psi}_{\text{interaction}}, \tag{18}$$

to which we add a gauge fixing for A_k, as is required by quantization (it is needed to partition the potential A_k in photons).

If the fields are partitioned, we expect the partition to "survive" during the interaction. During the interaction it is possible for electron/positron Dirac field to lose quanta from its partition, and the Maxwell field to gain, for example in the electron–positron annihilation $|e^+\rangle \odot |e^-\rangle \to |\gamma\rangle \odot |\gamma'\rangle$. What happens is that the state $|e^+\rangle \odot |e^-\rangle \odot |\text{other particles}\rangle$ unitarily evolved into $|\gamma\rangle \odot |\gamma'\rangle \odot |\text{other particles}\rangle$. If we account for the Principle of Spatial Separability, during the evolution the states must remain separable. This means that the wavefunction of the electron and positron should attenuate continuously, as they met, and in the meantime the two photons must be created continuously. Both the Dirac and Maxwell fields' norm is changed during this process. But they evolve unitarily, according to the scalar product (3) on the occupation number representation.

The field partition of electron/positron field lost two particles, and the one of the electromagnetic field gained two. This process took place gradually, but the idea is that we can relate the partition of the electron/positron field to that of the photon field. During the interactions, different fields correlate their partitions. This gives us a relative measure of the part of the field that should be a quantum.

7.2. Point Particles *vs.* Waves

The solutions of the Schrödinger equation are waves, or wavefunctions. According to the Principle of Continuity, they cannot be point particles. It is difficult to explain the quantum phenomena with point particles, and usually we have to appeal to waves anyway (*e.g.* pilot wave, probability wave *etc.*). And if we consider that a particle is point-like, since when we measure its position we obtain the particle concentrated at a point, then if we would measure its momentum, we would have to admit that it suddenly became a plane wave. If we consider it to be a wave, which can have various shapes according to its state, we can explain the quantum phenomena better, as we have seen.

In QFT, when we calculate the Wick's expansion of the S-matrix, we use Feynman diagrams, which depict interacting point particles. But this is only a formal way to reduce an impossible calculation to something easier to do. Like expanding a function in power series, or in Fourier series, expressing the S-matrix in terms of Feynman diagrams is an approximation, which is supposed to converge to the desired result. In fact, this expansion diverges, and to make it convergent we have to apply renormalization. These

perturbative techniques are very useful, since we don't know other way to solve this kind of problems. But we should remember that the point particles, like pure plane waves, are pure abstractions. The image of particles interacting by exchanging virtual photons or other bosons is interesting, if we want to think the interaction fields in terms of point particles, but it is less realistic than an approximation. This doesn't mean that all problems vanish if we consider the particles as parts of a classical field. We still need to use use perturbation theory, to get concrete results out of the formulas.

7.3. **Principle of Integral Interactions**

When we detect a photon, we detect the entire photon, and not only a part of it. If we pass the photon through a beam splitter, then we try to detect the "half-photon" which was transmitted, we find that the entire photon was either transmitted, or reflected. The photon is detected by processes which change the number of particles: it is absorbed in an atom which gets excited, or it is used to start the emission of an avalanche of electrons, or cause ionizations, or a chemical reaction in a photographic plate, but any way, the number and types of particles is changed during these processes. This may suggest that, in general, in order to detect a particle, or some properties of that particle, we need processes that change the number or types of particles. Let us call this type of interaction *particle-changing* interaction or process.

I do not suggest that the "half-photon" cannot interact. Obviously, it can be reflected or transmitted by a mirror, but this kind of interaction is not *particle-changing*. Well, in terms of Feynman's theory, a photon which is reflected or transmitted is absorbed and emitted many times (in fact a new photon is emitted). But as a wave, from classical viewpoint, it can simply be reflected or transmitted. The emissions/absorptions sum-up to the same result as the classical transmissions and reflections. These processes are Feynman diagrams from a development of the interaction in terms of idealized interactions between point particles. So, I wouldn't worry about this procedure which allows fractions of the photon participating to distinct particle-changing processes. They are perturbative approximations of classical wave behaviors. In general, if we are able to observe a particle-changing process, then the particles involved change integrally, and not only partially.

Particles can decay, can be annihilated, absorbed, emitted. These processes are observable, when they have enough amplitude to be real, and not simply virtual (*i.e.* terms of a perturbation expansion), nor "fractions" of particles (obtained *e.g.* by splitting them with a beam splitter). What I want to say is that the particle-changing interactions should be *integral*, and not change only fractions of them.

It seems natural to state that the only possible processes that change the numbers and types of particles are those being integral. But since this is not obvious, and does not seem to result from other principles, I will state it as an independent principle:

Principle 4 (of Integral Interactions)**.** Particle-changing processes can annihilate or create only integral particles.

A partitionable field can be partitioned in many ways. A particle changing interaction determines parts of a partition, but the parts determined by other particle changing

interactions are not necessarily parts of the same partition. Remembering this may help when discussing interference.

Some objections may be raised, though. Particle-changing processes are, of course, governed by the PDE, so they should be continuous. As I explained in §7.1, parts of one field are "transformed" in parts of other fields. This kind of process needs space, and more important, time. How long do they take? What I can say is that they have to be done before the particles being involved in the next particle-changing process. If they are observed, then the particle-changing interaction is already finished.

7.4. **Particle-Changing Interactions and Field Partitions**

It is known that Dirac said [22]:

> each photon interferes only with itself. Interference between different photons never occurs.

His statement was disproved by interference of distinct maser light beams [23]. The classical field partitioning, in addition to expressing the spatial separability of states (and separability of phase and amplitude) for quantum fields, also allows the interference between distinct particles of the same type. This is because the field can interfere with itself.

It should be added that the field partitions should be considered locally, and not necessarily globally. The partitions are induced by interactions changing the numbers or types of particles, which are postulated to be integral. Of course, they can be extended globally, but the idea is that two particle-changing interactions may induce distinct field partitions. Distinct parts of a partition may very well interfere. Two "halves" of two identical particles combine in another particle of the same type, as in the interference between different photons.

Because the wavefunctions are not concentrated around a point, but rather spread in space, in every given point of the space the field is the sum of amplitudes of many individual wavefunctions. The interference phenomena manifest as fluctuations, and when the amplitude reaches a given value, particle-antiparticle pairs may be created. The Principle of Integral Interactions works in this situation as well, although it seems that the pairs are created out of vacuum. The reason is the constructive interference of distinct photons for instance, which locally created enough amplitude to obtain photons, which then combined in the particle-antiparticle pair. These fluctuations are smooth. There is an impression that the fluctuations are infinite at small scales, but they cancel out at larger scales. This impression comes from conferring ontological reality to the perturbative expansions, which are nothing but approximations which in some situations converge towards the result, but in important situations in QFT, like this one, are divergent. Another reason for thinking the fluctuations as infinite is the improper usage of the time-energy Heisenberg relation, $\Delta E \Delta t \geq \hbar$. This relation is true as an expression of the Fourier analysis relation $\Delta\omega\Delta t \geq 1$, but its meaning is that, if a process takes a too short period of time, the frequencies composing it occupy a wide spectrum. This means that, expressed as linear combination of energy eigenstates, it contains a wide spectrum of energy. If this state collapses in a high energy eigenstate, then indeed we

can have a large fluctuation of energy. If we make $\Delta t \to 0$, we expect that there are infinite fluctuations in energy. But because of Principle of Continuity, we can't have such discontinuous collapse, although we can still detect the particle at high energy, as a consequence of delayed initial conditions. Infinite fluctuations are not observed, what is observed are the detected particles, and their energy is always finite.

7.5. **Remarks on the Principle of Integral Interactions**

Principle 4 introduces a condition to the field, which breaks the superposition at a certain level. Principles 1 and 3 are much close to the classical view, and we no longer need to reject them in order to embrace QM. Unlike them, Principle 4 seems more non-classical.

This principle, by itself, is responsible for building the Quantum World out of classical field equations, to structure the classical fields in quanta, which are integral parts of the field. In addition, it offers stability to a world constructed from waves, as we shall see in §9.3, and offers a reason for the problem of outcomes of measurements (§9.1). These tasks can be done in conjunction with the Principle of Global Consistency.

Maybe the Principle of Integral Interactions can be explained as a consequence of a nonlinearity in the field equations, maybe it arise from topological conditions, I cannot say yet.

A topological explanation seems to naturally support the delayed initial conditions, when combined with the Principle of Global Consistency. In general, the global properties of manifolds and bundles are topological in nature. Topological solutions, like solitons, monopoles, instantons, may be the parts of the field partition. Then, also the topology will not allow any type of transformations. For example, since the solitons and instantons are classified, according to their behavior at infinity, by the homotopy groups, when they combine, they have to maintain the topological constraints at infinity. This may allow only processes which change an integral number of particles, and which obey both the conservation laws following from Noether's theorem, and the ones resulting from topology. Another interesting topological object is the magnetic monopole. The presence of a magnetic monopole leads naturally to charge quantization. Another possibility to be considered is the Misner and Wheeler's wormhole model [24] of the electron/positron pair. But some of these solutions may be too localized to allow quantum effects. It is hard to imagine how we can split with a beam splitter a wormhole, and make its Poincaré dual[4] interfere with itself (maybe a kind of double-solution explanation, on the lines of Bohm and de Broglie?). Perhaps the most plausible direction to explore are the topological properties of the gauge fields.

If the field partitioning and the Principle of Integral Interactions will turn out to be consequences of topology, the fields will result to be "already quantized", to paraphrase Wheeler.

But because the mechanism behind this is not yet clear, I prefer, for the moment, and for the purpose of the present research, to formulate the Principle of Integral Interactions in a generic way.

[4]Wormholes are objects in the homology of spacetime, and the corresponding source-free electromagnetic field is an element of the cohomology group.

8. **Classical Fields and Quantum Fields**

A classical particle is point-like, and have at any given instant a position and a momentum. When QM described the particles as wavefunctions, strange things seemed to happened, such as wave-particle duality, uncertainty, *etc.*. But there is no wave-particle duality, only waves. Finally, we can consider them spacetime waves, and not only abstract phase space waves. There is no reason to try to find simultaneously the exact position and the precise momentum, since a wave doesn't have such properties – it is just a field. The complete state, and the complete information, is given by the wavefunction. Of course, there is a problem if we try to measure the state of a particle, since we will find the wavefunction as an eigenstate of the observable. This means that our observation contributed to what we observed. But not only because it disturbed the position or the momentum, rather because it imposed a delayed initial condition to the wavefunction.

In the relativistic case, Schrödinger's equation was replaced by the Klein-Gordon and Dirac equations. Some nice properties of the Schrödinger's wavefunctions – positive energy and positive probability density – had to be revised. It occurred that the fields which are the solutions of the relativistic equations contained more particles, which are not very distinguishable.

The canonical solution of QFT was to take the quantization of classical point-particles and to apply it for fields, relying on the idea that what we quantize are in fact the degrees of freedom. So, when a point-particle leads, by quantization, to a wavefunction, a field leads to something corresponding to an infinite number of degrees of freedom. In Heisenberg's picture, the fields are operators, in fact operator valued distributions [25].

Since we first go from point-particle to field (wavefunction) by quantization, then we quantize the field, the field quantization was often referred as "second quantization". Not only is the name tributary to history, but the procedure of quantization itself. Remember that initially Einstein proposed that the electromagnetic field is made of particles. After a while, de Broglie observed that what we initially thought to be particles have wave behavior, being thus fields. Schrödinger expressed de Broglie's intuition mathematically. At that moment, what we had were only fields. Both electrons, as wavefunctions, and electromagnetic field were fields. The main characteristics of the Quantum World, the wave-function duality and the uncertainty principles, were already presented in the first quantization. Why then introducing a second quantization, and why relying so much on operator valued distribution? Because the canonical quantization was useful to replace point particles with fields, and then it seemed a good idea to replace the fields with even more, something corresponding to the fields' degrees of freedom. And then, because QFT was the only way to accommodate some observed quantities which classical mechanics or nonrelativistic QM were not able to accommodate, this approach was considered validated.

As we have seen, we can take the classical solutions of Dirac and Klein-Gordon equations, and then partition them. Partitioning is the only extra information which structures the field. Then we naturally obtain the multiparticle Hilbert space in occupation number representation, and of course, the operators acting on this space, which can be combined to construct systems of particles out of the zero-particles (vacuum) state

$|0\rangle$. So, the operator-valued distributions are intrinsic constructs of the partitioned field. We can use them, but why not use directly the state vectors, which are real, being the partitioned fields? By taking instead the operator valued distributions as real, we run into serious problems, having to deal with divergences (infinities). If we work carefully, they eventually cancel out. But the partitioned fields are already free of divergences. Wouldn't they make the things simpler? I think that here is a lot work to be done, to see whether we can obtain with field partitions the same results as with standard QFT, or even simplify it. I do not expect the perturbative techniques to disappear completely, since they are very useful in obtaining honorable results for otherwise unsolvable equations. But they should not lie at the basis of the conceptual framework of QFT.

9. Interface between Classical and Quantum Worlds

We can say that there are four levels of conditions. First, the bundles over the spacetime determine what kind of fields are allowed. Second, the PDE impose the possible classical solutions. Third, the Principle of Integral Interactions keeps only some of these solutions, by imposing conditions at various moments of time. These conditions need to be fulfilled by the wavefunctions, in addition to being solutions of the Schrödinger equation. Fourth, from all these solutions we have to choose one, which represent the actual fields in our world, and here the initial conditions come in place, to fix the remaining free variables. The Principle of Global Consistency is a key principle here, because it selects the possible initial conditions, to make them compatible with the Principle of Integral Interactions.

9.1. Measurements

A measurement device is an apparatus which allows us to extract information about a quantum system. Obviously, this should be done by interaction. We can "read" the effects of the interaction, therefore, the effects have to be stable enough. The way to acquire stability is to use systems which have a set of possible states we can observe, all stable, and which can switch among these states. They should be arranged so that they can change their states in function of the state of the observed system. This kind of device involves particle-changing processes. If we want to observe a particle, we need to make it participate to such interactions.

Suppose now that our device detects the particle in the state corresponding to one of its stable states. It cannot detect it in another state simultaneously, because the particle can participate only integrally to particle-changing interactions. To detect a particle, it should impress a photographic plate, ionize some atoms, initiate a photomultiplication process, *etc.*.

Measurements are in fact conditions imposed to particles by the Principle of Integral Interactions. In conjunction with the Principle of Global Consistency, this principle constrains the initial conditions of the particle. The measurement device's final state cannot be a "superposition" of detecting the particle in two different states, because this would involve fractions of the observed particles participating in distinct particle-changing processes, therefore it would violate the Principle of Integral Interactions.

So, Schrödinger's cat needs not to worry, because the Geiger counter cannot simultaneously detect and not detect radiation.

9.2. **Born Rule**

Imagine now a source of light, sending light through a slit in a wall, towards a screen parallel to the wall. From classical viewpoint, light passing through the slit is a wave described by the Huygens Principle. But at very small level, the pattern is, in fact, composed of tiny spots – the impact points of the photons. The photons were forced by the atoms in the screen and the Principle of Integral Interactions to be detected as being localized. The light wave turn out (because of the way it was detected) to be in fact made of these localized waves. When a large number of photons reach the screen, the pattern converges towards the classical pattern. This means that the well localized photons are distributed to approximate the classical wave. The average density of incident photons on the screen is proportional to the squared amplitude of the wave predicted by Huygens Principle. This led Born to his rule.

More generally, the wave of potentialities (see §5.2) is an average of all possibilities. It equals in fact the wavefunction of the observed system, had it evolve undisturbed (also unobserved), since all the possible disturbances cancel out. The observable imposes a condition which filters only the possibilities corresponding to its set of eigenstates. The amplitude to obtain each eigenstate is such that, in average, the wave of potentialities is obtained. Therefore, it is the length (*i.e.* norm) of the projection of the state vector representing the potentialities. Hence, the probability is the squared amplitude of the projection, $|\langle\psi|\psi'\rangle|^2$. The Born rule appears as a consequence of the fact that the undisturbed wave is the average of all possible outcomes. It is this "coincidence" which created confusion between the wave of possibilities and the actual wave, and this resulted in the false idea of discontinuous collapse.

9.3. **The Classical World**

The Quantum World seems full of superpositions and collapses, while in the Classical World the things look different. We don't see, in a normal state of mind, a person in two places, nor do we see this person being everywhere, and then suddenly collapsing at one place. Not even smaller objects, like a dust speck.

The Principle of Continuity forbids discontinuous collapse even at the quantum level, and the Principle of Spatial Separability forbids nonlocal fields. Isn't this enough to obtain the Classical World? Maybe these principles also forbid Schrödinger's cats at macroscopic level. But just invoking them does not solve the problem, since, as we have seen, the appearance of collapse and entanglement still are possible at microscopic level, due to the Principle of Global Consistency.

The appearance of collapse is explained by allowing the initial conditions to be delayed. We can still see the initial conditions as initial, but chosen such that they anticipate the future conditions. The Principle of Global Consistency forces the initial condition to satisfy the future constraints. But what constrain can be imposed to the fields, other

than the equations they obey? The answer is the Principle of Integral Interactions. This principle is the one responsible for the partitioning of the classical field, and for selecting, among the possible solutions of a PDE, only some. This principle, with the Principle of Spatial Separability, breaks the principle of superposition for composite systems.

From this image, we see that we can have macroscopic waves, and interference. Even we can have macroscopic Schrödinger's "kittens", as long as they does not really forbid the Principle of Spatial Separability.

What about Schrödinger's cat? Can we split its particles' wavefunctions and combine them in two half-cats, like we did for the C60 molecule in §4.2? The usual superposition employed in QM sees here no problem, since we just superpose the two possible states of the Schrödinger's cat. This process is linear. On the other hand, the particle-changing interactions that can be observed are those involving integral particles. We can think the C60 molecule as being able to survive only by field interactions in the classical way, without changing the particles' numbers and types. But when it comes to cats, the things are more complicated. The superposition is not genuine (since this is allowed only for one-particle systems), but rather the pseudo-superposition described in §4.2. We have a difference, that between genuine superposition and pseudo-superposition, which may result in distinct predictions for more complex objects.

The principles stated so far strengthened the relation between classical and quantum worlds. The Principle of Continuity makes the wavefunctions more classical-like, by removing the discontinuity. The Principle of Spatial Separability allows the fields to be local, although the appearance of entangled phenomena is maintained by correlations resulting from the Principle of Global Consistency. We have seen that we can maintain both continuity and separability and still accounting for the known phenomena. For QFT is shown to be possible to construct the field operators as based on classical fields, which are somehow partitioned. The partition results from the interactions, via the Principle of Integral Interactions. We see that the classical world is no longer that far from the quantum one.

The objects are, according to these principles, networks of waves, linked by the Principle of Integral Interactions. This image looks like a Feynman graph, except that the edges are all kind of possible states of single particles (which are fields), and the vertices are governed by the Principle of Integral Interactions. These fields interact not only at vertices, since they are fields and they usually overlap, being parts of field partitions. But we don't have superpositions of composite states, we only have classical fields, partitioned in a specific way.

10. **Quantum Theory and General Relativity**

10.1. **Field Partitions in Curved Spacetime**

For Special Relativistic fields, the vacuum solution is Poincaré invariant, and so is the Fourier decomposition of fields. But in noninertial frames, the things are different.

In [26] there is a general discussion of how we construct the positive frequency one-particle Hilbert space, as a space of classical solutions, and how this construction leads to unitarily inequivalent representations.

Let us fix a coordinate system, and obtain an equivalence class of coordinate systems by applying to the fixed coordinate system local Poincaré transformations. The vacuum and the decomposition of the field in two such bases are related by unitary transformations (like they are in an inertial frame, in conformity with Wigner's theorem [27, 28]). But the passing to a coordinate system which does not belong to the equivalence class changes nonunitarily both the vacuum and the decomposition. The numbers and types of particles depend therefore on the coordinate system, if we admit noninertial coordinates. This may lead to Unruh effect and Hawking radiation.

10.2. **Quantum Field Theory in Curved Spacetime**

The field equations (Dirac, Maxwell, Yang-Mills) are replaced on curved spacetime by their covariant versions. The Principles 1-4 can be extended without problems to curved spacetime. Continuity and consistency are obviously the same as in flat spacetime. A field partition still remains a partition, and the particles still remain separated. The Principle of Spatial Separability still can be required without problems. The Principle 4, of Integral Interactions, still holds, because the property of a field to be integral is generally covariant.

10.3. **The Energy-Momentum Tensor**

One difficulty in QFT in curved spacetime is the problem of the backreaction. The spacetime geometry is related to the matter fields by Einstein equation (1). Because in QFT the fields are replaced by operator-valued distributions, in Einstein's equation the energy-momentum tensor is thought to be replaced by the expectation value of the corresponding operator, $\langle \hat{T}_{ij} \rangle$. But $\hat{T}_{ij}$ is expressed in terms of $\hat{\varphi}(x)$, and we have to multiply distributions, and we run into problems that can be solved only by renormalization.

Instead, if we use field partitions, what we have is the classical field. It is exactly like a classical field, with the additional constrain that it is partitionable. We can use its classical energy-momentum. Einstein's equation will relate the energy-momentum of the classical fields to the Ricci curvature.

10.4. **Quantizing Gravity**

The following remark of Dirac [29] contains the main reason for searching a quantization of gravity:

> There is no experimental evidence for the quantization of the gravitational field, but we believe quantization should apply to all the fields of physics. They all interact with each other, and it is difficult to see how some could be quantized and others not.

On the other hand, the source of gravity is the energy-momentum of partitioned fields. This means that any property the quantum fields may have, there is a corresponding property of the gravity.

Moreover, since Einstein equation (1) relates the energy-momentum of the field to the spacetime geometry, it follows that any property of the field is somehow reflected in the very spacetime geometry.

The field partition approach suggests that the quantum fields are in fact just like the classical ones, except that they are partitionable in a specific way. Therefore, the solutions of the field equations are subject to some constraints. These constraints are reflected in the curvature of the spacetime. Constraining an expression on which the curvature depends, we constrain the curvature, which in turn constrains the metric tensor, hence the geometry. In other words, the metric should be such that its curvature corresponds, via Einstein equation, to partitionable fields.

11. **The Unity of Nature**

The spacetime is a four-dimensional semiriemannian spin manifold M with Lorentz metric g. Over the spacetime, it is defined a G_{SM}-bundle $G_{SM}(M) \to M$, where $G_{SM} = \mathrm{U}(1)_Y \times \mathrm{SU}(2) \times \mathrm{SU}(3)_C$ is the standard model group. We also need a spinor bundle over M, corresponding to the tangent bundle and the metric g, with the structure group $\mathrm{SL}(2,\mathbb{C})$. The product of the two bundles is a $\mathrm{SL}(2,\mathbb{C}) \times G_{SM}$-bundle over M. The fundamental fermions (leptons and quarks) are sections in associated vector bundles. The forces come from potentials A^μ defined as connections on the bundle $G_{SM}(M)$. The curvatures of these connections define the Yang-Mills and Maxwell fields $F^\mu = \mathrm{d}A^\mu$, *i.e.* the electroweak and strong forces. The Bianchi identity provides half of the Yang-Mills and Maxwell equations, $\mathrm{d}F^\mu = 0$, the other half coupling the force fields with the Dirac fields $\mathrm{d}^* F^\mu = e\bar{\psi}^\mu \gamma^k \psi^\mu$.

The fields involved in these equations are partitionable. The partitions are defined locally by the Principle of Integral Interactions. Globally, the partition may be broken, *e.g.* in interference. The Principle of Integral Interactions is likely to have the origin in topological conditions imposed to the fields. A partitionable field can be partitioned in many ways, important is that it is partitionable. Starting from the field partitions, we can construct the occupation number representations and the operators needed in QFT. Only the spatially separable states are admitted, because we started from a partitionable classical field. The Principle of Spatial Separability is a direct consequence of using classical field partitions. We don't need to worry about the fact that separability seems to prevent entanglement effects, which apparently were confirmed experimentally. Entanglement-like effects still are allowed, but the system turns eventually that it was all the time in the separable state in which it is find by measurements. The continuity is also a necessary property of dealing with classical fields. The wavefunction collapse does not need to happen discontinuously, because the Principle of Integral Interactions together with the Principle of Global Consistency act like global conditions (delayed initial conditions). From a temporal viewpoint, the wavefunction evolves continuously

toward the eigenstate in which is detected. The measurement problem, and the relation between the quantum and classical level, becomes more transparent.

The metric is determined, according to Einstein's equation (1), from the energy-momentum of the matter fields. There is no need for special quantization of gravity, since the energy-momentum tensor already contains the quantization of matter fields, in the form of field partitionability.

12. About the Principles

Using the word "principle" may make the reader think that I propose some radically new ideas in the form of the principles. The Principle of Continuity and the Principle of Spatial Separability are natural requirements for classical fields, as well as for GR. I had to state them just to change the paradigm of discontinuous collapse, and that of nonlocal fields, back to the general relativistic paradigm. The essential idea resides not only in the principles, but mainly in the fact that there is no need to break them.

It is self-evident that a theory should be consistent, that the fields should be consistent with the principles. Then, why the need to state the Principle of Global Consistency? The main reason is that this is the real source of our intuition about causality. Causality may seem to be broken by the delayed choice initial conditions, which seem to act backwards in time, but what in fact matters is consistency, whose temporal manifestation is the causality. Another reason to emphasize the consistency is the wide acceptance of the idea that observations break the law expressed quantitatively by Schrödinger equation.

The Principle of Integral Interactions is required to induce locally the field partitions, so needed for passing from classical to quantum fields. It is also needed as an underlying mechanism for the quantum measurement, which can be reduced to integral interactions. Integral interactions impose global constraints on the initial conditions, which appear as delayed.

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