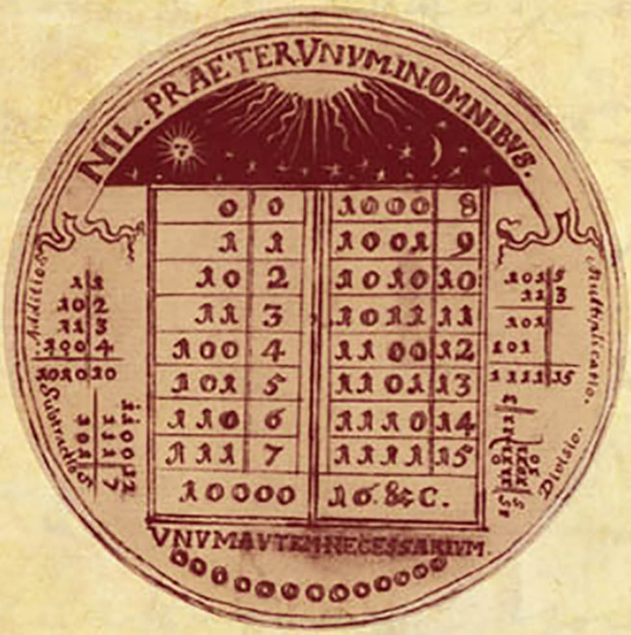
Leibniz on Binary

The Invention of Computer Arithmetic



Lloyd Strickland and Harry Lewis

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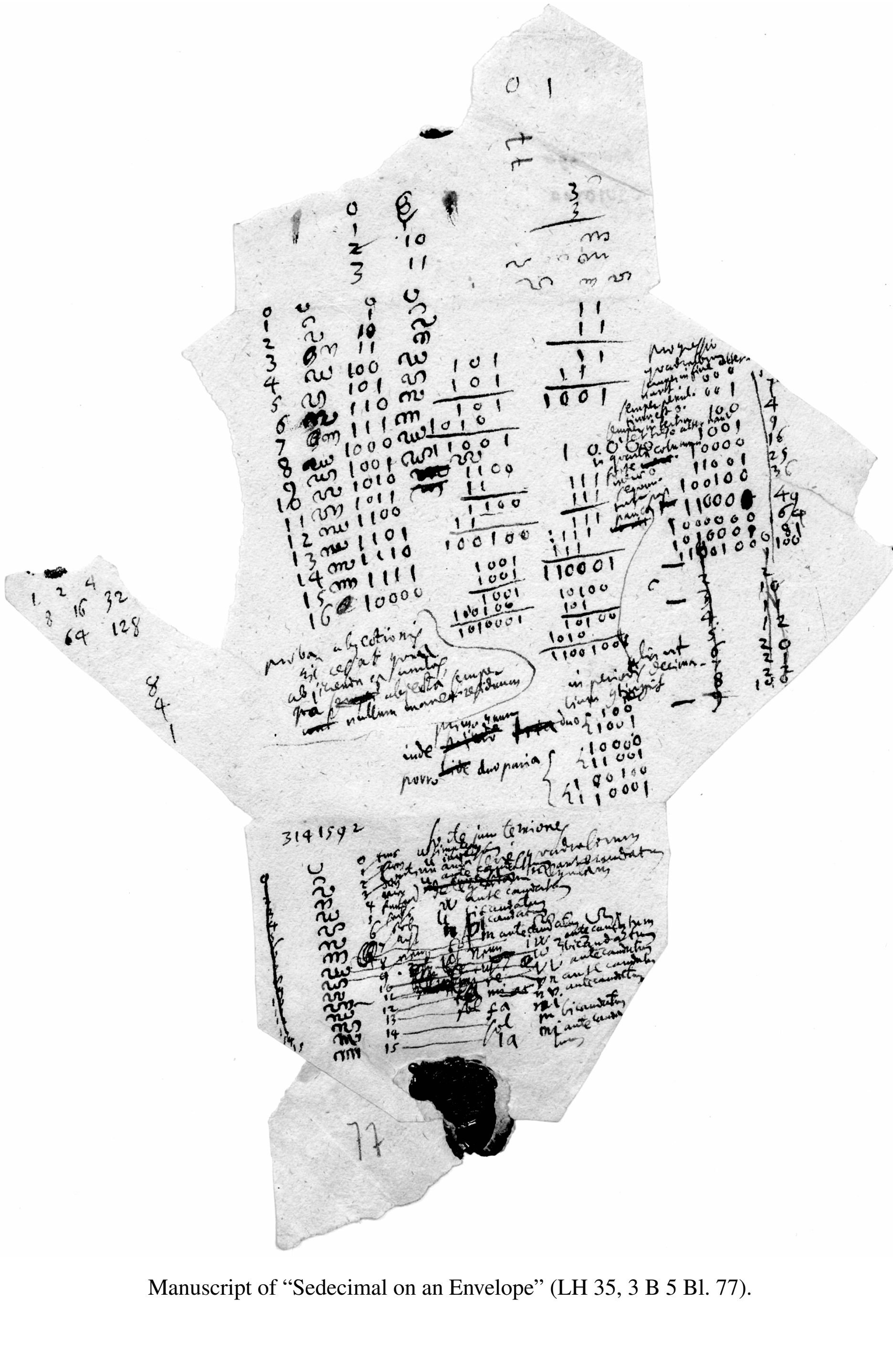

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The Invention of Computer Arithmetic

Lloyd Strickland and Harry Lewis

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This book is dedicated
by Lloyd to Vernon Pratt, mentor, friend, and *vir eximie*, as Leibniz would say.
As an indefatigable source of wisdom, inspiration,
and knowledge of where to find good desserts,
you are too good for this world.
by Harry to the memory of his father, Emil Harold Lewis,
who with the 16th General Hospital, from 1943 to 1945,
helped save his father's birthland, free his grandfather's,
and create the enduring Anglo-German-American friendship
of which this book is fruit.
Medio tutissimus ibis.

All programmers are optimists.
—Frederick P. Brooks Jr., *The Mythical Man-Month*

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Abbreviations

A	Leibniz (1923–). Cited by series, volume, and page or item number (N).
DSR	Leibniz (1992).
Dutens	Leibniz (1768).
GM	Leibniz (1849–1863). Reprinted as Leibniz (1977).
Gotha	Manuscript held by Forschungsbibliothek Gotha, Erfurt, Germany.
GP	Leibniz (1875–1890).
LBr	Manuscript held by Gottfried Wilhelm Leibniz Bibliothek, Niedersächsische Landesbibliothek, Hanover, Germany. Cited by shelfmark and Blatt [sheet].
LGR	Leibniz (2016a).
LH	Manuscript held by Gottfried Wilhelm Leibniz Bibliothek, Niedersächsische Landesbibliothek, Hanover, Germany. Cited by shelfmark and Blatt [sheet].
LTS	Leibniz (2011b).
PPL	Leibniz (1969).
SLT	Leibniz (2006b).
W	Leibniz (2006a).
WOC	Leibniz (1994).
Z	Zacher (1973).

Preface

> It would be pointless to say that Mr. Leibniz was a mathematician of the first rank, [since] it is through mathematics that he is most generally known.
> —Bernard le Bovier de Fontenelle (1718, 108–109)

Despite his original contributions to a host of disciplines, such as law, philosophy, politics, languages, and many areas of science, the fame and renown of the polymath Gottfried Wilhelm Leibniz (1646–1716) has always rested principally on his pioneering mathematical work, in particular his independent invention of the calculus in 1675. Another of his enduring mathematical contributions was his invention of binary arithmetic, though the utility of binary went unrecognized until it became the basis for today's world of digital computing and communications. And in a second flash of foresight, Leibniz also invented the other number system commonly used in computing, namely, base 16, or hexadecimal in modern parlance (Leibniz's own term for it was sedecimal). These two inventions are the focus of this book.

Leibniz's groundbreaking work in mathematics is all the more remarkable given that he did not begin serious study of the subject until he was in his mid-twenties. Born in Leipzig in 1646 to a professor of moral philosophy, Leibniz obtained his bachelor's and master's degrees in philosophy in 1663 and 1664, respectively, before turning to the law, in which he obtained a bachelor's degree in 1665 and a doctorate a year later. By the end of the 1660s, he was at the court of Mainz working alongside the diplomat Johann Christian von Boineburg (1622–1672) on reforming the legal code. In February 1672, Boineburg dispatched him on a diplomatic mission to Paris, where he met some of the foremost mathematicians of the day, most notably Christiaan Huygens (1629–1695). Leibniz later recalled that he had arrived in Paris with "a superb ignorance of mathematics" (GM III, 71), but this was soon rectified by Huygens's tutelage and his own insatiable thirst for inquiry. He initially focused on summing infinite series and the classical Greek problem of squaring the circle, and from his intensive work on both, he eventually arrived at the calculus.[1] In October 1675, he devised the symbols d and $\int$, which are still used today.

Despite his growing reputation as a mathematician, Leibniz was unable to secure a post in Paris. One year after his breakthrough with the calculus, and a little over four and a half years since his arrival in Paris, Leibniz departed for the northern German city of Hanover to take up the post of court counselor, a role he held for the remaining forty years of his life (though in the intervening years, his facility at moving within courtly circles enabled him to add roles in Wolfenbüttel, Berlin, and Vienna to his portfolio). Leibniz later explained that he stayed in Paris as long as he did in order "to stand out a little bit in mathematics" (A II 1, 753).

It was during his Paris years that Leibniz developed his lifelong interest in the automation of calculation. In the 1640s Blaise Pascal (1623–1662) had invented a machine capable of adding and subtracting, but Leibniz wanted to go further, proposing a machine capable of multiplying, dividing, and even (in his initial plans, in 1670, for a "new arithmetic instrument") extracting roots (LH 42, 5 Bl. 16v). His designs were refined over the next three years, but putting into practice his belief that

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1. See Probst (2018).

wheels and cranks could perform the labors of the mind proved a challenge. A machine he demonstrated at the Royal Society in London in 1673 was, by all accounts, imperfect.[2] Further refinements were communicated to the craftsman tasked with building the device, but progress was slow. Leibniz several times reported that the machine had been completed and tested,[3] but these successes appear to have been short-lived at best, and it is doubtful that the machine ever worked reliably. Even in the last year of his life, a frustrated Leibniz was firing off instructions for fixing various faults, which he blamed on a craftsman's lack of diligence and precision.[4] Of the three versions of Leibniz's machine built in his lifetime, only the last survives. When it was rediscovered in 1879, Arthur Burkhardt (1857–1918), a leading mechanical engineer, was tasked with making it functional, but he was unsuccessful and concluded that it had probably never worked.[5] Nonetheless Burkhardt soon began manufacturing calculating machines incorporating some of Leibniz's ideas, and in the twentieth century, engineers were able to construct fully-functioning "replicas" of Leibniz's machine,[6] one of which is displayed at the G. W. Leibniz Bibliothek in Hanover.

The G. W. Leibniz Bibliotek also holds the vast majority of Leibniz's surviving writings, which total some 200,000 manuscript pages, a sizable proportion of which have yet to be published. These writings cover a wide variety of topics and disciplines, and range from completed books and journal articles to draft essays, rough jottings, and personal reading notes, as well as letters to and from more than a thousand correspondents. This material offers a window into Leibniz's prodigiously wide-ranging interests, ideas, and inventions, not to mention his various projects. Between 1680 and 1686, he sought to improve the productivity of the Harz silver mines by designing pumps and windmills to drain the floodwaters that made mining impossible after heavy rains.[7] In 1686, he took on the task of writing a history of the House of Guelph (or Welf) in order to enhance his employer's dynastic ambitions (the history was still not complete at the time of his death thirty years later, due to Leibniz's excessive thoroughness and many distractions).[8] From the early 1680s onward, he sought to facilitate a reunion between Catholics and Lutherans and, in the late 1690s, a reunion between Lutherans and Calvinists.[9] In 1700, he lobbied for the formation of the Kurfürstlich Brandenburgische Societät der Wissenschaften [Electoral Brandenburg Society of Sciences], and was immediately installed as its first president.[10] He drew up plans for a universal encyclopedia that would contain everything known, wrote Latin poetry, built a full philosophical system that would have great influence for centuries to come, and undertook pioneering studies on the origin of languages.[11]

Leibniz's scope, reach, and genius were such that he was described as "the ornament of Germany" by the Dutch scientist Herman Boerhaave (1715, 13). The last years of his life were overshadowed by the priority dispute with Sir Isaac Newton (1643–1727) over the invention of the calculus and his frantic but unsuccessful efforts to complete the history of the Guelph house. On 6 November 1716, he became bedridden by an attack of gout and arthritis, and he died on 14 November.

2. For further details, see Beeley (2020).

3. For example, he claimed in July 1674 that his arithmetic machine had been "successfully completed" (A III 1, 119) and in April 1695 that he had "finally had [it] completed" (LH 42, 5 Bl. 63r; English translation: Leibniz 1695a).

4. See Leibniz (1845).

5. See Morar (2015).

6. However, the "replicas" required a little technological license and modern engineering techniques to function. See Stein et al. (2006) and Kopp and Stein (2008).

7. For further information, see Wakefield (2010).

8. For further information, see Antognazza (2018).

9. For further information, see Jordan (1927).

10. For further information, see Rudolph (2018). The institution survives today as the Berlin-Brandenburg Academy of Sciences and Humanities.

11. For additional information on Leibniz's life and work, see Aiton (1985), Belaval (2005), and Antognazza (2009).