*Identity to Essence*

**Abstract**

The idea that a being can be identical to its own essence has a long and venerable history in theological speculation. As with many ideas in theology, however, such an idea has never been given an adequate mathematical formulation. The key to such a formulation, I argue, is introducing an essence axiom into non-well-founded set theory. According to such an axiom, for every set x there is a set that contains all and only those sets that contain x. With such an axiom, it becomes possible to introduce an essence function that takes each set to its essence. An object that is identical to its essence, then, is simply a fixed point of the essence function. In this paper, I first discuss a theorem according to which any fixed point of the essence function is universally symmetrical with respect to the set membership relation. After proving the theorem, I discuss its theological implications. I go on to argue that the process of counting the members of a set that is identical to its essence is a Sisyphusian process. Such a fact I suggest shows that the absurdity of human existence is not alleviated by the existence of God but rather is entailed by it.

Therefore, the essence of a composite thing is not predicated in every way of the composite thing itself, for we cannot say that a man is his own quiddity. But the essence of a simple thing, which is its form, can only be signified as a whole, for there is nothing there apart from the form, so there is nothing that could receive the form. Therefore, no matter how we consider the essence of a simple substance, it is predicated of the simple substance. This is why Avicenna says that the quiddity of a simple substance is the simple substance itself, for there is nothing else [in the simple substance] to receive it. (Aquinas, *On Being and Essence*)

The idea of identity to essence is a central part of the history of theological speculation. Despite the idea’s centrality, however, it has not been given an adequate formal articulation. Because it involves the concept of essence, one might naturally think that a formal presentation of the idea would invoke modal concepts. Typically, an essential property as opposed to an accidental property is understood as a property that an object must have in all possible worlds in which it exists. The appearance of modality in such a description is obvious.

There is, however, another understanding of essence in the philosophical tradition that can be given an extensional treatment, namely Leibniz’s idea of a *complete individual concept* (CIC).

The nature of an individual substance or of a complete being is to have a notion so complete that it is sufficient to contain and to allow us to deduce from it all the predicates of the subject to which this notion is attributed. (A VI iv 1540/AG 41)

Were one to use the language of sets, one could say that a CIC of x is the set that contains all the sets that contain x. Because it is possible to pursue set theory within an extensional framework, it is possible to formulate Leibniz’s idea extensionally. Moreover, because extensional set-theory is undeniably a part of mathematics, it is thereby possible to provide a *mathematical* analysis of the concept of a CIC, or what I will henceforth generally call an essence, and in turn to provide a mathematical analysis of the concept of identity to essence.

 One might object at this point that an extensional treatment of Leibniz’s idea runs afoul of his claim that every *predicate* of a subject must be able to be deduced from its individual concept. If a predicate is a linguistic item, predicates are far more fine-grained than extensions. In response to such an objection, I reiterate Zalta (2010) who argues that such an approach yields too many individual concepts. Here is Zalta’s argument in his own words.

So suppose that Adam’s complete individual concept is the concept (i.e., property) P. Then pick your favorite proposition, say q, and consider the distinct property [λy P y & (q ∨ ¬q)]. Call this property ‘Q’. P and Q are exactly the kind of necessarily equivalent
but distinct properties that are the subject of property theory. But if P is a complete individual concept of Adam, so is Q. If Adam is the unique thing exemplifying P, he is the unique thing exemplifying Q. So, we have distinct complete individual concepts that are both complete individual concepts of Adam, contrary to the assumption that there is a
unique individual concept of Adam.

It turns out, then, not only *can* the concept of an individual concept be treated within an extensional framework, but such a concept *is best* treated within an extensional framework.

 An analysis of identity to essence is made possible by introducing the following axiom, what I call the *Essence Axiom*, according to which every set has an essence.

1. .[[1]](#footnote-1)

It follows trivially from extensionality that no set has two essences: for suppose that x contains all and only those sets that contain z and so too does y. Then, x and y contain all the same sets and so are identical. This trivial result is a formal demonstration of Zalta’s insight about the appropriateness of an extensional framework for treating individual concepts. Because every set has an essence and no set has two essences, we can talk about *the* *essence* of some set x. So, let E(x) stand for the essence of x. It follows a little less trivially but still easily that the essence function is 1 to 1:

To see this. suppose x y. Then, there is a set, w, such that either w is a member of x and is not a member of y, or w is a member of y and is not a member of x. Suppose w is a member of x and not of y. Then, by the Essence Axiom, E(w) contains x and not y. Hence, by the Essence Axiom, E(x) contains E(w) and E(y) does not. Hence, by Extensionality, E(x ). Similar reasoning shows that if w is a member of y and not x then E(x). Either way, , and so by contraposition and generalization (2) follows.

 The fact that the essence function is 1-1 shows that it satisfies a criterion for any understanding of a CIC -- no two non-identical beings have the same CIC. But here someone might object on the grounds that a CIC is the wrong type of essence to consider when considering whether some being is identical to its essence. So, for instance, according to Aquinas, when he says that simple substances are identical to their essences, the kind of essence in question is a species, not an individual essence.

The second difference is that the essences of composite things, since they are received in designated matter, are multiplied by the division of designated matter, whence in their case it happens that there are numerically distinct things in the same species. However, since the essence of a simple thing is not received in matter, in their case there cannot be this kind of multiplication; therefore, in the case of these substances, there cannot be several individuals in the same species, but there are as many species as there are individuals, as Avicenna expressly claims.

Hence, the objection continues, a mathematical examination of a being that is identical to its CIC is not thereby an investigation into the kind of being that simple substances are, at least not according to Aquinas.

This objection, however, has an answer, one whose basis appears already in the above quotation. Aquinas claims that a simple substance, though a species, necessarily has only one instance. It follows, therefore that simple substances satisfy the 1-1 condition on the essence function: For any two simple substances, x and y, if the essence of x is identical to the essence of y, then x is identical to y. Hence, the essence axiom characterizes Aquinas’s simple substances; and so, a mathematical investigation of essence as understood in this paper is indeed an investigation into the kind of being that according to Aquinas the simple substances are.

Not only does this objection have an answer, but there is a very good reason to think that for the purposes of understanding the nature of God it is precisely the Leibnizian sense of essence rather than the species sense of essence that is important. Unlike a species, A CIC represents its object. Hence, a CIC that is identical to its essence is a CIC that represents itself. In different terminology, a CIC that is identical to its essence is thought thinking itself. So understood, a CIC that is identical to its essence conforms exactly to Aristotle’s conception of God: ‘Therefore it must be of itself that the divine thought thinks (since it is the most excellent of things), and its thinking is a thinking on thinking.’ (Aristotle, *Metaphysics, XII, 9)*

Although identity to essence can be given a formal treatment within set-theory, it is not hard to see that the set theory in question must be non-well-founded. If a set is identical to its essence, then it contains all the sets that contain it and so violates the axiom of foundation. In this paper, I won’t take a stand as to which non-well-founded set-theory provides the proper set theoretic framework for a theory of identity to essence, since the axioms I employ are part of the two best known and mathematically serviceable non-well-founded set theories, namely Quine’s NF and Positive Set Theory. In fact, for the purposes of presenting the theory we will not need any comprehension schema. Rather, in addition to the Essence Axiom, we need only the existence of the universal set, or what I call *Being*, the existence of the empty set, what I call *non-Being*, and Extensionality.

 In what follows I prove one theorem and discuss the structure entailed by several others that serve to characterize a being that is identical to its essence and go on to comment on the theological significance of the mathematical results. The theorem can be proven from the essence axiom and extensionality alone. It establishes an equivalence between the concept of identity to essence and the concept of a perfect Being. Some may find jarring the claim that a mathematical theory can talk about a perfect Being. But, as shall become apparent, the sense of perfection in question can be given a mathematical formulation in terms of the concept of symmetry. The other theorems require in addition the existence of Being and non-Being. They show that the process of counting the members of a set that is identical to its essence is a Sisyphusian process. Once again, the idea that something like Camus’ Myth of Sisyphus could be represented mathematically may seem initially jarring. But, as in the case of perfection, there is a mathematically precise way of characterizing such a process by way of the concept of symmetry. Although my rendering of the concepts of a perfect Being and a Sisyphusian process may not conform to every philosopher’s understanding of those concepts, as far as I know, no philosopher’s understanding of perfection or a Sisyphusian process has ever been given a mathematical articulation.

 In section I of this paper, I argue that identity to essence entails perfection. In section II, I argue that counting the members of a being that is identical to its essence is a Sisyphusian process. Each section is divided into two parts, a first that contains mathematical preliminaries and a second that contains a discussion of the philosophical implications of those preliminaries.

**Section I – Identity to Essence and Universal Symmetry**

*Part I – Mathematical Preliminaries*

The claim that identity to essence entails perfection depends on the following theorem, which expresses an equivalence between identity to essence and universal symmetry with respect to the set-membership relation.

1. (

The set-membership relation is the only non-logical relation within set theory and, because of set theory’s foundationalist credentials, arguably the only non-logical relation needed for all of mathematics and the physical sciences. (3) thus expresses the fact that identity to essence is equivalent to being universally symmetrical with respect to what is arguably the most fundamental non-logical relation, one that is standardly taken to represent the is of predication. Let us suppose that to be a symmetry of being is to be related symmetrically by the set membership relation to every other set. (3) expresses the fact that being identical to one’s essence is equivalent to being a symmetry of being.

 It has long been recognized, as the following quotation from Aristotle attests, that symmetry is both a mark of beauty and a formal cause in mathematics.

Since the good (ἀγαθόν) and the beautiful (καλόν) are different (since the good always occurs in action and the beautiful occurs also in immobile things), those who say that the mathematical sciences say nothing about the beautiful or the good are wrong. For these speak and offer proofs about these things most of all; for even if they do not name them, they prove things about their results (ἔργά) and their formuals (λόγους). For the greatest forms of beauty are order and symmetry and the determinate, which are the very things the mathematical sciences prove most of all. And since, to be sure, these (I mean, for instance order and the determinate) appear as causes of many things, it is clear that they would speak of this sort of cause as well, and that the beautiful is also a certain sort of cause. (Metaphysics,)

As became clear in the 20th century due to the work of Noether, Einstein and other mathematicians and physicists, symmetry is also a formal cause of the physical laws. For those familiar with 20th century physics, this is a familiar claim. But for those not familiar, it may be worthwhile to quote Richard Feynman whose interest and indeed awe concerning the matter should show just important symmetry is.

“The symmetries of the physical laws are very interesting at this level, but they turn out, in the end, to be even more interesting and exciting when we come to quantum mechanics. For a reason which we cannot make clear at the level for the present discussion – a fact that most physicists still find somewhat staggering, a most profound and beautiful thing, is that in quantum mechanics, *for each of the rules of symmetry there is a corresponding conservation law*; there is a definite connection between the laws of conservation and the symmetries of physical laws.” (Feynman Lectures, Vol. 1, Ch. 52)

Near the end of his discussion of symmetry, Feynman asks: “So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why.”

 Although Feynman understandably does not suggest a metaphysical answer to his question, it is worth considering whether some metaphysical answer could be proposed. Presumably, a metaphysical answer would involve something analogous to the symmetry principles in physics. (3) shows that it is indeed possible to make sense of a symmetry of being and moreover that any such object must be identical to its essence. Thus, symmetry, in addition to being a formal cause in mathematics and physics can also be a formal cause in metaphysics. Although I will not try to argue that a symmetry of being is the correct answer to Feynman’s question, it is certainly noteworthy that because it is identical to its essence, a symmetry of being would halt an explanatory regress.

Someone might object at this point that there is a disanalogy between the type of symmetries in other branches of mathematics and physics and the type considered here. Symmetry in mathematics and physics is typically understood in terms of an object’s remaining the same under some operation, whereas the symmetry of being in (3) is a logical notion defined in terms of the material biconditional. In response to this objection, one can point out that the axioms needed in order to prove (3) are so minimal and hence that the proof is of such a fundamental nature that the symmetry in question must be of a logical nature. The standard cases of symmetry in geometry presuppose that the objects in question have a considerable amount of structure that allow for some type of operation to be performed on the elements of the object. At this point in the development of the present theory, however, we have only assumed extensionality and the essence axiom. We thus don’t have any robust structure to work with. To the extent that some notion of symmetry can be employed, it must be the logical notion. In a logical sense, a relation, R, is symmetrical if and only if . An object, G, exhibits such a symmetry if and only if . And because there is only one non-logical predicate in the theory, namely the set-membership relation, the symmetry in question must therefore be: . Moreover, (3), already goes some way to integrating such a logical sense of symmetry into a more familiar geometrical sense in that it shows the equivalence between the logical sense of symmetry and being a fixed point of a function. And not just any function but arguably the most important function in metaphysics, the essence function.

The proof of (3) is straightforward.

First, assume . Suppose . Then, by the symmetry of , . Therefore, by the definition of an essence, . Suppose . Then, by the definition of an essence, . Therefore, by the symmetry of G, . Therefore, for any set, , . Therefore, by extensionality, .

Second, assume that . Suppose that . Then, by the identity of G to its essence, . Therefore, by the definition of an essence, . Suppose that . Then by the definition of an essence, . Therefore, by the identity of and , . Therefore, for any set, , .

*Part II – Philosophical Implications*

 Theologians within the tradition of *Perfect Being Theology* (PBT) claim that God is not just good but perfectly good. Theologians within the same tradition have differed, however, in their understanding of what that means. Although I won’t attempt an exhaustive comparison between the concept of perfection in this paper and the concept as it has appeared in the philosophical tradition, I will situate the account relative to a recent trend within PBT. Recently, philosophers have argued that perfect goodness entails perfect moral agency. (Morris 1989b, p. 26; Wierenga 1989, p. 202) A perfectly good being has morally unsurpassable agency where this is thought to entail constraints on the desires, beliefs, and actions of such a being. Though widely accepted, the plausibility of such a claim rests on the plausibility of the claim that a perfect being is a person. For if the perfect being is not a person, then there is no reason to suppose that it must possess perfect moral agency, which is a defining feature of a person, not a defining condition of a being. And there are historical and contemporary examples of religions and philosophers who have advanced conceptions of an ultimate metaphysical principle that is not a person. Brahman in the Hindu tradition is one obvious example. And Schellenberg has recently argued very persuasively that a general conception of God need not be personal but can nonetheless exhibit what he calls metaphysical and axiological ultimacy. (Schellenberg, 2005, 2007, 2009)

 Schellenberg does not say precisely what metaphysical ultimacy is, arguing instead that it can be filled out in various ways. But, he claims, that it involves some notion of explanatory fundamentality. Historically, identity to essence was considered one way of expressing explanatory fundamentality. If something’s features are explained by its essence, then to be identical to one’s essence entails being the explanation of one’s own features. If one of those features is existence, then a being identical to its essence explains its own existence. We can plausibly maintain, therefore, that the concept being analyzed in this paper is one way of filling out the notion of metaphysical ultimacy.

 Schellenberg understands axiological ultimacy as follows: “To say that something is axiologically transcendent is to say that its intrinsic value – its splendour, its excellence – exceeds that of anything found in mundane reality alone.” (Schellenberg, 2016) Importantly, he does not think that the value in question must provide the foundations of ethics, which presumably explains his appeal to the concepts of splendour and excellence rather than the concept of unsurpassable moral agency. He says: “Some might be tempted to assume that I have in mind, when speaking of axiological ultimacy, some claim to the effect that the existence of the Ultimate is the foundation for ethics or value theory or some such thing…But no, this is not the case. Of course, there are conceivable elaborations of axiological ultimacy that run in this direction, but nothing of the sort is entailed by it.” If a Being is splendid or excellent in an unsurpassable way, according to Schellenberg, it can be considered axiologically ultimate.

The value that (3) involves is symmetry with respect to the most fundamental non-logical relation. The fact that it is unsurpassably symmetrical follows from the fact that it is universally symmetrical. For, it is not possible to be more symmetrical than universally symmetrical. So, does that make such a being unsurpassably splendid or excellent? Yes. Or so it seems to me at any rate. A thorough discussion of the various ways that symmetry enters moral and aesthetic judgements as well as the way that it is involved in mathematical and physical theories is well beyond the scope of this paper. But it is difficult to think of some other concept that could simultaneously link together the mathematical, the physical, the aesthetic, and the moral domains. To be sure, the concepts of symmetry in these areas differ. And to be sure in each domain one must take account of asymmetries in order to achieve a thorough understanding of the phenomena. Nonetheless, it is difficult to deny that symmetry, though a strict mathematical concept, can plausibly be understood as a feature that confers value, either moral, aesthetic, or explanatory, on an object. Exhibiting logical symmetry with respect to the *most fundame*ntal non-logical relation thus plausibly increases the value of that value, so to speak, to a level at which no greater possible value of that sort exists. Having that value universally would then quite plausibly make the object in question unsurpassably valuable, or as I have said previously, perfect. The ultimate philosophical implication of (3), therefore, is that it shows the equivalence of axiological and metaphysical ultimacy, at least as those concepts can plausibly be filled out.

 Although the perfection involved in unsurpassable symmetry is more general than the concept of perfect moral agency, there is a relation between the two. Suppose we follow Aristotle as he was quoted above and distinguish between the beautiful and the good -- the good is properly speaking a predicate of actions, whereas beauty, which Aristotle correctly notes is linked to symmetry, is a predicate of mathematical objects. So too, one might argue, that the universal symmetry that identity to essence entails is a value that is logically prior to the domain of action and so does not, until further specified, entail perfect moral agency. But, if appropriately specified by reference to the actions that God might take, (assuming that God is the sort of being that can act), the universal symmetry entailed by identity to essence entails perfect moral agency.

 One might of course wonder at this point what connection there is between symmetry and perfect moral agency. A rather quick answer, one that must be filled out considerably if it is to be adequately defended, can be given that should indicate the direction a more thorough discussion would take. In short, symmetry is partly constitutive of love. When two agents love each other, they become one. Their becoming one is in some sense metaphorical. But it is also literally true insofar as agents who love each other begin to have identical interests. The identity of interests can be understood as entailing a symmetry in their relationship. A husband who loves his wife, for instance, will tend to have desires for their future if and only if the wife has those same desires. Hence, a being that is identical to its essence has a perfected form of a relation that is partly constitutive of love. Hence, universal symmetry is a general and perfect form of a fundamental structure of love. And because God is identical to her essence, she just is perfect love. Because a being that is perfect love would exhibit perfect moral agency, we have a conceptual route from universal symmetry to perfect moral agency. By locating an unsurpassable value that God is independent of his being an agent, one can provide a non-circular and informative account of why God is a perfect moral agent: God acts in a morally unsurpassable way because she is the form of perfect love

 Once again, I reiterate that a philosopher may have a different conception of perfection than universal symmetry. But once again, I reiterate that Perfect Being Theologians have not articulated any such conception within a mathematical framework. Indeed, the standard practice within PBT is to take the concept of perfection as undefined. Consider, for instance, what Brian Leftow (2021) says about Jeff Speaks’ book (2018), a book that Leftow claims is the most thorough and extended critique of PBT in the history of philosophy. Notice the parenthetical remark.

Perfect being theologies are machines to crank out divine attributes. As Speaks sees it, each has two moving parts. One is a claim that God is the greatest being in some range—actual, possible, or conceivable beings. “Pure” PBT adds a greatness condition. “Impure” adds a bridge principle. Attributes that meet a greatness condition add to a thing’s greatness. (Speaks does not worry about what greatness is. He in effect spots PBT what it needs to get going, and argues that even so, it doesn’t work*.*)

Speaks’ strategy here is not in question. If he can find insuperable problems with PBT independent of any view about greatness, then he has successfully criticized it. But his omission is notable and in line with many perfect being theologians who take the concept of greatness as not needing to be defined in order to determine the characteristics that such a being must have. (Leftow, 2010) One need not be overly positivistic to find such an approach to be less than ideal.

It is not my intention, however, to engage with alternative approaches to the concept of greatness in PBT. Rather, I simply note that understanding perfection as universal symmetry yields an interesting result, namely a proof that perfection is equivalent to identity to essence. The concept used in the proof is of the same general type as a concept that plays an ineliminable role in mathematical theories, in the foundations of physics, and in people’s judgements about beauty and goodness. And as a result of the account, those theologians who think that God is a person and hence has unsurpassable moral agency can now provide a non-circular informative explanation as to why God acts in a morally unsurpassable way. Such an explanation goes well beyond simply insisting that God must act in such a way because she is, well, perfect. A being that is identical to its essence must have in an unsurpassable way a kind of value that is plausibly construed as being partly constitutive of love and hence must lead an agent who has such a value to act, if such an agent can act, in a morally unsurpassable way.

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***Section II – A Sisyphusian Process***

*Part I – Mathematical Preliminaries*

 The claim that counting the members of a being that is identical to its essence is a Sisyphusian process depends on the internal structure of the members of such a being. To describe what can be considered the minimal structure of those members requires two additional axioms that assert what can be considered the extremes of both the metaphysical world and the world of sets. One of the axioms is ruled out in well-founded theories but is part of both Quine’s NF and Positive Set Theory, while the other is a standard part of both well-founded and non-well-founded set theories. In stating the axioms, I have used ‘Being’ instead of ‘the universal set’ and ‘non-Being’ instead of the empty set. Although a mere terminological difference, the terminology I employ tracks much more closely historical metaphysical debates than the usual set-theoretic terminology.

1.

With just (1), (4), (5), and extensionality, a structure reminiscent of an Escher drawing, is entailed. Studtmann (2021) has presented elsewhere proofs of several meta-theorems that serve to characterize the structure. For the purposes of this paper, I omit the proofs and instead present the structure informally.

The first thing to note is that the Essence Axiom and the non-Being Axiom jointly entail an infinite progression of sets. Let ‘E(x)’ denote the essence of x and ‘∅’ denote non-Being. Then, the two axioms entail the existence of non-Being, ∅, the existence of the essence of non-Being, E(∅), the existence of the essence of the essence of non-Being, E(E(∅)), and so on. Likewise, the Essence axiom and the Being axiom entail an infinite progression of sets: Being, E(Being), E(E(Being), and so on. For the ease of expression, I will call any set that is part of the progression of essences stemming from non-Being a ‘non-Being essence’ and any set that is part of the progression of essences stemming from Being a ‘Being essence’. I will also employ the following notation – En(x) – to stand for the essence function applied n times repeatedly beginning with x. So, for instance, E3(∅) = E(E(E(∅))). In the limit when n=0, En(x)=x.

With this notation, it is possible to state one of the fundamental facts about God. But first, let us make explicit by way of an axiom that God is identical to her essence.

1. God = E(God).

(4)-(6) entail that God contains En(Being) for all natural numbers n. This can be seen by first noting that because everything is in E0(Being), God E0(Being). Because God is universally symmetrical with respect to the set membership relation it follows that E0(Being) God. Because E1(Being) contains all and only those sets that contain E0(Being), God E1(Being). By the symmetry of God, E1(Being) God. And so on. A second easily provable fact is that none of the non-Being essences is in God. Consider first that if E0(∅) God, then by the symmetry of God, God E0(∅), which is impossible. If E1(∅) God, then by the symmetry of God, God E1(∅). By the definition of an essence, E0(∅) God, which has just been proven to be impossible. Such a proof can be carried out for any En(∅).

God, then, contains an infinite number of individual concepts, namely all the Being essences, and does not contain any non-Being essences. Moreover, God as ordered by the essence function is isomorphic to the set of natural numbers. This follows from the fact that the four first order Peano axioms are entailed by defining the successor function as the essence function, 0 as Being, and being a natural number as membership in God. Those proofs very quickly stated are as follows. As noted above, God E0(Being) and so E0(Being) God. Hence, zero (Being) is a natural number (is in God). Moreover, E0(Being) is not the essence of any set. This follows from the fact that E0(Being) contains the empty set. If E0(Being) were the essence of some set, x, then x would be in the empty set, which is impossible. Hence, 0 (Being) has no predecessor (is not the essence of any set). Third, every natural number, (set in God), has a successor, (essence) that is a natural number (is in God). For, suppose that some set, x, is in God. Then, by the essence axiom, such a set has an essence that contains God. By the symmetry of God, God contains the essence of x. Hence, any set in God has an essence that is in God. Finally, as noted above, the essence function is 1-1. Hence, the essence function is a successor function defined on the Being essences with membership in God being equivalent to being a natural number. To paraphrase Augustine, the natural numbers are ideas in the mind of God.

In addition to being isomorphic to the natural numbers, the Being essences are structured like possible worlds within S5 modal logic. This structure comes not from the essence function but from the membership relation. It is possible to prove that for any m and n, the axioms so far introduced entail that Em(Being) En(Being). Hence, every Being-essence is in every Being-essence. If one were to define accessibility in terms of set membership, then God as so far portrayed contains objects that have the structure of possible worlds in S5 modal logic. God is thus at one and the same time structured like the natural numbers and S5 modal space.

I will return to the Being essences in a moment. Their structure will be more easily seen, however, if we first turn to the non-Being essences. From the axioms so far discussed, one can prove that for any m and n, En(∅) contains all the Being essences Em(Being) such that . So, for instance, E1(∅) contains E0(Being), E2(∅) contains E0(Being) and E1(Being), and so on. So, the non-Being essences have the following structure:

E0(∅) = {}

E1(∅) = {E0(Being)}

E2(∅) = {E0(Being), E1(Being)}

E3(∅) = {E0(Being), E1(Being), E2(Being)}

E4(∅) = {E0(Being), E1(Being), E2(Being), E3(Being)}

If one were to use the language of representation, one would say that the non-Being essences represent the possible initial complete finite sequences of Being essences. If a ‘path’ through the Being essences always begins with E0(Being), is finite, and complete, then the non-Being essences represent all the paths through the Being-essences. It is not hard to see that were one to take this process out to infinity, one would approach a set that contains all and only the Being essences. In other words: Eω(∅)= {Em(Being) | m is a natural number}.

 Now, let us return to Being. If we restrict ourselves to just the Being-essences, that is if we don’t yet include (6) in our set of axioms, then Being looks like the following:

E0(Being) = {E0(Being), E1(Being), E2(Being)… E0(∅), E1(∅), E2(∅), E3(∅)…}

Consider, then, E1(Being), the essence of Being. It contains every set that contains Being. But notice that every set in Being with one exception, namely the empty set, contains Being. Hence, the essence of Being must contain all the sets in Being minus the empty set.

E1(Being) = {E0(Being), E1(Being), E2(Being)… E1(∅), E2(∅), E3(∅)…}

Notice that every set in E1(Being) contains E1(Being) except E1(∅), which contains only E0(Being). Hence, E2(Being) contains every set in E1(Being) except for E1(∅). Hence,

E2(Being) = {E0(Being), E1(Being), E2(Being)… E2(∅), E3(∅)…}

And so on. Each Being essence contains one fewer non-Being essences than its predecessor. Taking this out to infinity, one approaches a set that contains no non-Being essences and all and only Being essences. The point at Infinity for the progression of the Being essences is the same as the point at infinity for the progression of non-Being essences. The two polar extremes of the metaphysical world – Being and non-Being – converge through repeated application of the essence function to the same set. As I have already mentioned, God contains all and only the Being essences. So, rather remarkably, the two chains of Leibnizian essences that begin with Being and non-Being both converge to God.

 If one adds (6) to the set of axioms, there is one change to the Being essences. As proven above, God contains En(Being) for each of the natural numbers, n. By the symmetry of God, each En(Being) contains God. And so, we arrive at the following structure.

 God = {

E0(Being) = {God, E0(Being), E1(Being), E2(Being)… E0(∅), E1(∅), E2(∅), E3(∅)…}

E1(Being) = {God, E0(Being), E1(Being), E2(Being)… E1(∅), E2(∅), E3(∅)…}

E2(Being) = {God, E0(Being), E1(Being), E2(Being)… E2(∅), E3(∅)…}

E3(Being) = {God, E0(Being), E1(Being), E2(Being)… E3(∅)…}

…}

The structure so far can be considered the minimal structure in God. Even this minimal structure demonstrates several of God’s features. It shows that God, what we can along with Aristotle call thought thinking itself, is universally symmetrical with respect to set-membership, contains an infinite number of ideas structured both like the natural numbers and like possible worlds within S5 modal logic, and stands as a point at infinity for two essence chains, one stemming from non-Being and the other stemming from Being.

The structure can be extended by introducing various sets. For instance, because the Being-essences are structured like the natural numbers, one can sensibly assert the existence of the prime Being-essences. Such a set, like all sets, is a member of E0(Being) By the essence axiom, there is an essence of the set of prime Being-essences, which is contained in E1(Being). And so on. The entire chain of essences stemming from the set of prime Being essences is contained in the corresponding Being-essences, which are in God. And the same is true for any set. In this way, God, one can maintain, knows all things. At least she knows all things about sets and their essence chains. Which sets the Being essences contain depends upon one’s theory. Studtmann (forthcoming) has discussed the theory that results from adding a comprehension schema that comprehends all the arithmetically definable sets of Being-essences and a first-order set-theoretic version of the induction schema. Any Being-essence would then contain all and only those arithmetically definable properties that its predecessor instantiates. Within such a theory, each Being-essence becomes infinitely complex. And yet the essence function continues to structure the Being essences like the natural numbers.

With the added complexity, God becomes a rather remarkable object, which is what one would expect of God. She is thought thinking itself. She contains an infinite number of ideas (Leibnizian essences). her ideas are structured in the simplest possible way, i.e., like the naturals, but are themselves infinitely complex, each one representing its predecessor. This latter fact makes God ineffable, where an effable set can be considered one all of whose members have an effectively enumerable membership relation. She stands as a point at infinity for the progression of ideas that she contains as well as for a progression of ideas that those ideas contain. And she is contained by all the ideas that she contains. This latter fact results from God’s universal symmetry, which, as proven above, is equivalent to identity to essence and which, as argued previously, makes God the form of pure love.

For the sake of the present discussion, however, I will ignore the extra complexity that arises from introducing a comprehension schema and focus instead on the structure so far presented. For, it is possible to notice an intriguing fact about the Being essences as so far presented – each Being essence is universally symmetrical with respect to the set membership relation with only one exception. In the case of Being, every set, x, but non-Being is such that Being is in x if and only if x is in Being. In the case of the essence of Being, every set, x, but the essence of non-Being is such that the essence of Being is in x if and only if x is in the essence of Being. And so on. Suppose, then, that one construes the process of counting the Being essences in terms of the attempt to restore perfect symmetry to each Being essence by removing the one source of asymmetry in it. Such a process is equivalent to repeatedly applying the essence function to the Being essences. Begin with Being and remove the one source of asymmetry in it, namely the empty set. The result is the essence of Being, which has a single source of asymmetry, namely the essence of non-Being. Suppose again that one tries to restore perfect symmetry by removing that one source of asymmetry. The result is the essence of the essence of Being, which has its own single source of asymmetry. And so on.

*Part II – Philosophical Implications*

It is not through lack of love that Don Juan goes from woman to woman. It is ridiculous to represent him as a mystic in quest of total love. But it is indeed because he loves them with the same passion and each time with his whole self that he must repeat his gift and his profound quest. (Camus, *The Myth of Sisyphus*)

 Let us call a process Sisyphusian if it involves an attempt to accomplish a goal by performing an action only for the result of that action to yield a situation in which the structure that led to the original goal is again present. Famously, Camus considered human existence to be Sisyphusian. Although readers of Camus typically focus on Camus’s discussion of Sisyphus, Camus’s description of Don Juan provides the clearest description of a Sisyphusian process. Don Juan has a fundamental passion that leads him from woman to woman. Each time his passion is sated, it rises again to lead him to the next woman. There is no escape from the process for Don Juan. Exactly in line with Camus’s view of human existence, Sartre views consciousness as Sisyphusian. The fundamental goal of consciousness, a goal that arises because of consciousness’ nature as a lack, is to become God, or what Sartre calls an In-Itself For-Itself. (*Being and Nothingness* ) But, because God is an impossible synthesis of those two opposing types of Being according to Sartre, any success that consciousness might have in achieving its goal leads to a desire being sated, not satisfied. Once sated, consciousness seeks out another object to fill itself with. And so on. Consciousness, according to Sartre, is a useless passion.

 Because both Camus and Sartre were atheists, it is natural to suppose that the Sisyphusian nature of human existence is tied to God’s non-existence. And theists have generally wanted to claim that the existence of God suffices to endow humans and the universe with a telos. Although they typically don’t use such language, such a telos one might naturally suppose, makes both human existence and the universe non-Sisyphusian. But despite these loose historical and conceptual connections, the analysis in this paper shows that the existence of God, assuming she is identical to her essence, entails the existence of a Sisyphusian process, one that occurs as a result of the members of God. At each step in the process of counting the Being essences the goal is to achieve perfect symmetry, i.e., to become like God. In this way, such a counting process mirrors exactly Sartre’s understanding of consciousness. The means for achieving that goal is by removing the asymmetry that keeps a Being essence from universal symmetry. And the result is a set that has a different source of asymmetry. And so on, *ad infinitum*. Though the goal of the process is to become like God, God is forever out of reach. Far from being opposed to a Sisyphusian universe, God entails the existence of a Sisyphusian process.

Paul Tillich famously claimed that God is the ground of Being. (Tillich 1997) To the extent that God’s containing Being makes her the ground of Being, Tillich was correct. But God grounds far more than just Being. She grounds *all* the Being essences. Because the process of counting the Being essences is a Sisyphusian process, and because a process that is Sisyphusian is absurd, we can synthesize Tillich’s view and the French existentialists by saying that God is the ground of the absurd. One can infer from her grounding the absurd a connection between God and human consciousness. For it is human conscious experience that is Sisyphusian. The nature of consciousness is of course one of the most hotly debated issues in contemporary philosophy. And I don’t want to enter the seemingly endless debates involved in it. I will finish by simply reiterating the fact that a being that is identical to its Leibnizian essence is the very form of representational self-awareness. Humans are self-aware. Hence, humans instantiate, albeit to a limited degree, the structure of God’s consciousness. Because the process of counting God’s members is Sisyphusian, one would expect human consciousness to inherit the Sisyphusian character of God’s consciousness. Human consciousness may thus be a useless passion not because consciousness faces the absurdity that results from God’s non-existence but rather because consciousness inherits a structure that, as Sartre might have said, lies coiled within the heart of God, like a worm.

**Conclusion**

 In this paper, I have presented a theory of identity to essence within extensional first-order non-well-founded set theory and argued that the theory characterizes a being that Aristotle calls ‘thought thinking itself’. I argued that according to such a conception, God is perfect thought thinking itself, identical to her essence, contains ideas that are structured like the numbers and possible worlds, and is the form of pure love. Moreover, it contains a structure that both Sartre and Camus argued lies at the core of human conscious experience.

 I want to emphasize just how atypical such a mathematical object is. It is about as far from a well-founded set as one can get. No well-founded set could be contained by all its members. No well-founded set could be identical to its essence. No well-founded set could be a point at infinity for a chain of concepts stemming from Being and non-being. No well-founded set could be considered thought thinking itself or the most general form of pure love.

I emphasize these points in order to respond to an objection that I have heard both informally and from journal referees. Consider, for instance, the following objection raised by a referee.

We should ask ourselves: why should this be of interest to philosophers of religion? The only suggested connection is the speculative identification of the set S with God. But the idea that God is a set is one that is rejected out of hand by almost everyone who currently works in philosophy of religion, theists, atheists and agnostics alike… it is hard to see why this should be of any particular interest to those working in philosophy of religion.

It is not my place to comment on the interests of contemporary philosophers of religion. But I can speak to their supposed rejection out of hand of the identification of God with a set. Such a rejection is inevitably informed by their conception of a set. And for most, perhaps all, philosophers of religion, their conception of a set is tied to the well-founded conception, since that is almost certainly the only conception with which they have any significant acquaintance. To this extent I am sympathetic with the objection. If the only conception of a set that I had was the well-founded conception, I too would reject out of hand the identification of God with a set. Indeed, I reject that God could be any mathematical object that one typically runs across within standard mathematics. But were someone to convince me that some object, one whose intrinsic nature was for the moment left unspecified, was (i) identical to its essence; (ii) perfect; (iii) thought thinking itself; (iv) a point at infinity; (v) constituted by an infinite number of ideas that are structured like the natural numbers and possible worlds; (vi) the form of pure love; and (vii) ineffable in its totality; I couldn’t help but think that such an object is God, at least as God has been portrayed by many major theologians in the philosophical tradition. It bears repeating Schellenberg’s correct claim that a general conception of God need not be personal. Perhaps Aristotle’s God was a person, perhaps not. But only an excessive attachment to a personal God would lead someone to think that Aristotle’s view about God was not in fact a view about God. If I then learned that the object in question is a set, I would have to conclude as a result of basic logical inference that God is a set. I would thus be forced to conclude that sets are more interesting than mathematicians have led people to believe. Indeed, I might even be inclined to conclude that perhaps everything, dogs, people, atoms, and everything else, is a set and hence that of course God, if she exists, is a set too. Set theory, after all, provides the foundations for all of mathematics. Why might it not also provide the foundations for a theory of Being itself? If, as I suspect is the case, most philosophers of religion would agree that satisfying (i)-(vii) suffices for an object to satisfy at least a general conception of God and are in addition wedded to the most basic forms of logical inference, then they are committed to the possibility that God is a set. Although such a conclusion may jar one’s philosophical intuitions, it nonetheless is in line with a thought familiar to most schoolchildren, namely Galileo’s claim that math is the language in which God wrote the universe. If the view in this paper is correct, one can extend Galileo’s thought by including God in the universe that her language so accurately describes.

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1. As stated, the Essence Axiom is restricted to sets. One might object that this deviates from Leibniz’s understanding of a CIC, since he clearly intended the notion to apply to individuals. Such an objection, however, presupposes individuals are not sets but should be treated as ur-elements within set theory. Although I won’t pursue the matter here, it is possible to develop a theory of individuals without ur-elements by treating individuals as essence chains that stem from Quine atoms. [↑](#footnote-ref-1)