The RL+ Cosmological Model

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Abstract

We present a cosmological model (RL+) that offers exact predictions for the Hubble constant, the cosmological constant, the total energy density of the universe, and a curvature that matches current observational constraints. The model predicts a cosmological constant energy density that constitutes approximately 64% of the total energy budget, in agreement with current estimates from the standard ΛCDM model. Furthermore, the model addresses several longstanding cosmological problems—namely, the problem of infinite initial density, the coincidence problem, and the flatness problem—all with the introduction of a single temporal parameter.

Keywords: Hubble constant, cosmological constant, total energy density, curvature, phenomenological model

1. The Model

The standard model of cosmology, known as the ΛCDM model, has been remarkably successful in explaining a wide range of observational data, from the cosmic microwave background to the large-scale structure of the universe. However, certain fundamental questions remain unresolved, particularly those related to the nature of the cosmological constant, the initial conditions of the universe, and the apparent flatness of space. In this paper, we introduce a new cosmological model, referred to as the RL+ (Root-Log+) model, which addresses these issues with a novel approach.

1.1. Cosmological Constant

We begin by introducing the cosmological constant $\Lambda(t)$, which evolves with time as:

$$\Lambda[t] = \frac{\left(\frac{\ln(t^4)+\gamma}{t^4}\right)^{1/2}}{4\pi}.$$

In this equation, γ is the Euler-Mascheroni constant. Several studies have suggested a time-dependent cosmological constant. (Cervero, 1979; Lopez and Nanopoulos, 1996; Kantha, 2016; Oztas et al., 2018) The approach in this paper differs from these studies in two ways. First, it is phenomenological and so does not offer a physical mechanism underlying the equation. Second, it yields an exact match. At $t = 8.08 \times 10^{60}$, the currently observed age of the universe in Planck units, the cosmological constant has been measured at 2.888×10^{-122}, and the RL+ model predicts 2.888×10^{-122}, an exact match. Although the function is introduced phenomenologically, we note both its mathematical simplicity as well as the appearance of the natural logarithm, the square root function, and 4π. Each of these has known connections to the physical world, which suggests that this particular form likely has a natural physical interpretation. We also note that the occurrence of t within the logarithm function needs to be divided by one Planck moment, so that the expression is unitless. We have omitted this for ease of expression.

1.2. Energy Densities

Einstein's Field Equations require that the cosmological constant energy density, $\rho_\Lambda[t]$, is related to the cosmological constant by:

$$\rho_\Lambda[t] = \frac{\Lambda[t]}{8\pi}.$$

This leads to the expression:

$$\rho_\Lambda[t] = \frac{\left(\frac{\ln(t^4)+\gamma}{t^4}\right)^{1/2}}{32\pi^2}.$$

On phenomenological grounds, we assume the total energy density $\rho_{\text{total}}[t]$ is proportional to the cosmological constant energy density by a factor of $\frac{\pi}{2}$, leading to:

$$\rho_{\text{total}}[t] = \frac{\left(\frac{\ln(t^4)+\gamma}{t^4}\right)^{1/2}}{64\pi}.$$

Although this function, like the previous one, is introduced phenomenologically, we propose that the factor of $\pi/2$ may naturally arise from quantum fluctuations induced by the cosmological constant energy density. The observed value of the total energy density is 1.8×10^{-123}, and the RL+ model predicts the same value, 1.8×10^{-123}, indicating exact agreement with observational data. This precise correspondence between the model and observations underscores the potential physical significance of the introduced factor, suggesting that the RL+ model accurately captures the dynamics of the universe's energy content.

1.3. Curvature Term and Additional Structure

To derive the curvature term, we make the assumption that the sum of the energy density term and the cosmological constant term in the Friedmann equation equals $\pi\times$ the curvature term:

$$\frac{8\pi G}{3}\rho_{\text{total}}[t] + \frac{\Lambda[t]}{3} = \pi K[t],$$

from which the curvature term is derived.

$$K[t] = \frac{(2+\pi)\left(\frac{\ln(t^4)+\gamma}{t^4}\right)^{1/2}}{24\pi^2}.$$

Ideally, some explanation of the additional structure required for this derivation would be given. Once again, we justify it on phenomenological grounds. This equation predicts a curvature equal to 7.879×10^{-123}, consistent with current observations indicating that the universe is nearly flat.

1.4. Hubble Parameter

Finally, the Hubble parameter is derived using the first Friedmann Equation along with the expressions for the total energy density, cosmological constant, and curvature:

$$H[t] = \frac{\sqrt{-2+\pi+\pi^2}\left(\frac{\ln(t^4)+\gamma}{t^4}\right)^{1/4}}{2\sqrt{6}\pi}.$$

When evaluated at the current age of the universe, the model predicts the Hubble constant as $H_0 \approx 1.29 \times 10^{-61}$ in Planck units. This corresponds to $H_0 \approx 73$ km/s/Mpc, which matches the value reported from the distance ladder measurements.

2. Results and Predictions

The RL+ model makes several exact predictions:

- **Cosmological constant:** The model predicts 2.888×10^{-122}, identical to the observed value.
- **Total energy density:** The model predicts 1.8×10^{-123}, identical to the observed value.
- **Hubble constant:** The model predicts $H_0 \approx 1.29 \times 10^{-61}$ in Planck units, which corresponds to $H_0 \approx 73$ km/s/Mpc in commonly used units.
- **Curvature:** The model predicts a curvature equal to 7.879×10^{-123}, consistent with observations indicating a nearly flat universe.
- **Dark energy fraction:** The model predicts that the cosmological constant energy density is $2/\pi$ times the total energy density, which is approximately 64%, in line with ΛCDM estimates.

3. Theoretical Implications

The RL+ model addresses several fundamental problems in cosmology:

- **Infinite Initial Density Problem:** The model avoids the problem of infinite initial density because its functions are undefined for times $t < 1$, and at $t = 1$, each function has a finite value. This implies that time begins at $t = 1$, and at that moment, the energy density is finite, eliminating the need for an infinitely dense initial state.
- **Coincidence Problem:** The model explains the current coincidence between the cosmological constant and the total energy density, providing a natural explanation for why these quantities are of the same order of magnitude at the present epoch.
- **Flatness Problem:** The model predicts a nearly flat universe, with a curvature on the order of 10^{-123}, consistent with current observations, and does so without requiring fine-tuning.

4. Conclusion

The RL+ model offers a phenomenologically grounded framework that provides exact predictions for key cosmological parameters. While two functions are introduced phenomenologically, their mathematical forms suggest the potential for natural physical interpretations. Further investigation is required to clarify the precise physical mechanisms underlying these functions. Likewise, some explanation for the additional structure required to derive the curvature term should be found. Nonetheless, even without a full physical interpretation, the model's ability to resolve major cosmological challenges—such as infinite initial density, the coincidence problem, and the flatness problem—using only a single temporal parameter, while remaining consistent with observed values for the Hubble constant, total energy density, cosmological constant, the shape of the universe, and the fraction of the universe's energy budget attributed to the cosmological constant, establishes it as a promising candidate for further theoretical and observational investigation.

References

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