**The Divine Fractal:**

**1st Order Extensional Theology**

**Abstract**: In this paper, I present what I call the *symmetry conception of God* within 1st order, extensional, non-well-founded set theory. The symmetry conception comes in two versions. According to the first, God is that unique being that is universally symmetrical with respect to set membership. According to the second, God is the universally symmetrical set of all sets that are universally symmetrical with respect to set membership. I present a number of theorems, most importantly that any universally symmetrical set is identical to its essence, that show that the two symmetry conceptions intersect with some dominant theological conceptions of God. The theorems also show that both of the symmetry conceptions of God entail that God has a fractal like structure.

Formal methods have advanced considerably investigations into the nature and existence of God. Modal semantics and modal logic facilitated analyses of the concept of the greatest possible being and Anselm’s ontological argument. (Adams 1971, Plantinga 1974, Tooley 1981, Lewis 1983, van Inwagen 1987, Chandler 1993, Oppy 1995) In addition, Gödel (1995) formalized Leibniz’ understanding of and argument for God within the context of second order modal logic; and subsequently several philosophers and logicians have presented analyses of it.

(Sobel 1987, Anderson 1990, Oppy 1996, Hazen 1999, Kovacs 2003, Maydole 2009, Pruss 2009) Both Anselm’s and Leibniz’ ontological arguments have also been subject to computational analysis. Using theorem prover, Prover 9, Oppenheimer and Zalta (2011) reduced Anselm’s ontological argument to its simplest form. And using the Coq and Isabelle proof assistants, Benzmüller and Woltzenlogel-Paleo (2014) provided a computational analysis of Gödel’s reconstruction of Leibniz’ argument.

Despite the conceptual clarity of the formal treatments of Anselm’s and Leibinz’ arguments, the fact that they appeal in both cases to intensional notions and in Gödel’s case to second-order logic raises various philosophical concerns. This is not the place to discuss the many debates concerning modality and second-order logic that have occurred, but the controversies surrounding them may naturally make one wonder whether it is possible to pursue the nature and existence of God within an extensional first-order theory. Indeed, even those who are comfortable with the appeal to intensional concepts and second-order logic might wonder, if for no other reason than philosophical curiosity, whether some extensional first-order understanding of God is possible. For, if the nature of God can be expressed within an extensional 1st order mathematical framework, even someone as doggedly opposed to intensions and 2nd order logic as Quine could not object to the pursuit of theology. Quine might of course object to the details of the theology that is offered, but he could not dismiss the pursuit of theology on the general grounds that it requires intensional concepts and 2nd order logic.

It is the purpose of this paper to present a 1st order extensional analysis of the concept of God. In addition to differing in terms of the formal context within which the analysis is presented, the treatment of God in this paper differs from the Anselmian and Leibnizian treatments in the conception of God to which it appeals. Both Anselm and Leibniz accepted what can be called a maximalist conception of God: Anselm thought that God was the greatest possible, i.e. maximally great, being; and Leibniz thought that God was that being that instantiates all and only positive properties, i.e. is the maximally positive being. In this paper, I articulate what can be called the *symmetry conception* of God. The symmetry conception comes in two varieties. The first is that God is that unique being that is universally symmetrical. The second is that God is the universally symmetrical property of being universally symmetrical.

Although the symmetry conception differs in intension from better known conceptions of God, one of the goals of this paper is to show that it intersects directly with the Thomistic theological tradition. As I discuss in section III it can be proven that a set is universally symmetrical if and only if it is identical to its essence. Because Thomists hold that God is identical to her essence, they are committed to the symmetry conception of God. In addition to intersecting with the Thomistic theological tradition, the symmetry conception also potentially intersects with the Leibnizian theological tradition. Although Leibniz and Gödel understand God as instantiating all the positive properties, they leave the concept of a positive property undefined. At the end of section III, I suggest a definition of a positive property that stems naturally from the discussion up to that point. With such a conception of a positive property in place, it follows that God instantiates all and only the positive properties.

In addition to showing that the symmetry conception of God intersects with the Thomistic and Leibnizian traditions, this paper has a second goal, which is to show that the symmetry conception of God entails that God has an infinitely repeating membership relation and hence has a fractal like structure. The work of various mathematicians and physicists have shown not only that fractals are an exceptionally beautiful type of mathematical object but that they are abundantly present in the physical world. Coastlines, clouds, leaves, proteins, heart sounds, lightning bolts, snowflakes, mountain ranges, rings of Saturn are just a few of the many physical phenomena that have a fractal structure. So pervasive are fractals that some physicists have proposed that spacetime itself is a fractal. (Nottale 1993, Benedetti 2009, Ho, 2014) Two of the defining features of fractals are that they are self-similar and hence infinitely repeating, and that they have a Hausdorff dimension. As I shall discuss, the two different conceptions of God entail both that she has an infinitely repeating membership relation and that God’s Hausdorff dimension equals 0. The best-known fractals have a positive Hausdorff dimension and so occupy space. Unlike the best-known fractals, God as I define her is point-like.

In what follows I present six axioms within 1st-order set theory that allow one to demonstrate the existence of objects that conform to the first and second symmetry conceptions of God. Two of the axioms – Extensionality and the Empty Set axiom -- are familiar parts of well-founded set theory. The other four axioms are inconsistent with the axiom of regularity, which is a defining axiom within well-founded set theory. Hence, the mathematical theory that results from the axioms is part of non-well-founded set theory. Although the theory in this paper requires countenancing mathematical objects that diverge from the standard, iterative, conception of a set, the mathematical development of non-well-founded set theories should lay to rest any contention that the concept of a non-well-founded set resists extensional mathematical treatment. (Aczel 1988, Forster 1994, Esser 1999)

Before proceeding to the substance of the paper, I want to address an objection that one might raise at this point. Addressing the objection should make clear the methodological stance I am taking. Using non-well-founded set theory to analyze the structure and assert the existence of God may seem to entail the view that God is a set. But here, someone might sensibly retort that whatever God is, she cannot be a set. How, after all, could the living God be a mathematical object? A similar sort of objection has been raised to certain theories of properties. (Bealer 1980) According to the objection, a property could not be a function from possible worlds to sets, since that would entail that when I savor the taste of pineapple, I am savoring a function. But certainly, one might object, I cannot savor a function.

There are two plausible responses to this objection. (Oddie 2001) First, one might insist that the identification of properties with functions from possible worlds to sets is a proper reduction and hence that, appearances notwithstanding, one does savor a function when savoring the taste of pineapple. Although such a response is certainly possible with respect to the understanding of God in this paper – I could claim to have reduced God to a set – there is a second, less extreme, response to the objection that is, I believe, more plausible in the case of God. (Whether the second response is more plausible in the case of properties is not something I take a stand on.) Instead of claiming that one can reduce properties to functions, one might instead claim that appealing to such functions is a way of understanding the logical structure of properties, whatever properties end up being. So, for instance, one might claim that such an appeal can explain why extensions do not fix intensions. Analogous to such a response, one can use the axioms in this paper to illuminate the structure of God, whatever God ends up being. As in the case of properties, the appeal to a mathematical depiction of God can help illuminate God’s structure and thereby resolve some long-standing theological questions. By the end of this paper, I will have shown that some traditional conceptions of God, which have both been proposed and objected to on the grounds of conceptual incoherence, pass at least a minimal test of coherence and that they all, surprisingly, revolve around the concept of symmetry. Moreover, in the conclusion of this paper I discuss the fact that certain mathematical facts can illuminate the question as to whether the definitions go beyond being minimally coherent and are instead what one might call deeply consistent, that is consistent with some fully developed axiomatic treatment of mathematics.

The remainder of this paper is structured as follows. In sections I and II, I discuss two of the three axioms needed for the first symmetry conception of God. The first axiom corresponds to the concept of Being and the second axiom corresponds to the concept of Essence. I go on in section II to discuss six theorems that can be proven from Extensionality, the Empty set axiom and what I call the *Being* and *Essence* axioms as well as the structure that is described by those six theorems. In section III, I articulate what I call the *First God Axiom* and then motivate and propose the first symmetry conception of God. I go on in section III to discuss three theorems that can be proven from the five axioms that have been proposed. As shall become apparent, one of the theorems establishes the first main claim in this paper, which is that an object conforms to the symmetry conception of God if and only if it is identical to its essence. I then go on to discuss the fact that God is a fractal like object with a Hausdorff dimension equal to 0.

In section IV, I discuss what I take to be shortcomings of the first symmetry conception of God. I go on to propose the second symmetry conception of God. The second symmetry conception of God leads to a revision of the Being axiom and one final axiom, what I call the *Second God Axiom*. I show how one can combine both the first and second symmetry conceptions of God so as to characterize a structure in which there are three Gods within Being – what I call *the Mother*, *the Daughter*, and the *Holy Spirit* – along with one God beyond Being. As is the case with the first symmetry conception of God, the second symmetry conception of God entails that God is fractal like, has a Hausdorff dimension equal to 0, and is identical to her essence.

I want to stress that my aim in this paper is not to argue for the existence of God. As will become apparent, two of the axioms that I discuss assert the existence of God. In a debate against a theist or an agnostic, such assertions would clearly beg the question. My goal is instead to articulate a set of simple first order axioms that together with standard axioms in first order set theory entail the existence of an object that not only has a God-like structure but that also can be investigated within first order extensional mathematics. Even the most committed atheist, I assume, could find some interest in pursuing the nature of God in this way.

Section I – Being

(∃x)(∀y)(y ∈ x & x = Being)

The first metaphysical axiom, the *Being axiom*, asserts the existence of what is commonly called ‘the universal set’ but what I will call *Being.* As is well-known, the non-existence of Being can be demonstrated from the axiom of regularity, which is part of well-founded set theories, as well as from the comprehension principles of ZF or NBG set theories. It is difficult to present a non-question begging argument for or against the existence of Being. Its rejection is sometimes motivated by the fact that ZF and NBG are well-founded set theories and that they provide the foundations for all of mathematics. The force of such a consideration is blunted, however, by other set theories, for instance positive set theory and Quine’s NF, that not only entail the existence of a universal set but also can provide mathematically serviceable alternative non-well-founded set-theoretic foundations to mathematics. (Holmes 2017)

In addition to considerations concerning the foundations of mathematics, one might raise various philosophical objections to a first-order property of Being. There is a long and distinguished line of philosophers, including Aristotle, Hume, Kant, Rusell, and Frege who denied that existence is a property of individuals. Much of the debate concerning this issue in the 20th century stems from Russell’s arguments about negative existentials. (Russell 1905) This is not the place to advance a full response to the considerations raised by these various philosophers. I will only note that the results of this paper should go some way toward showing that the Being axiom as articulated above can be incorporated into a fully extensional axiomatized understanding of God. Moreover, God as understood in this paper has a fractal like structure and so is similar to structures that are known to play a significant role in the physical world. To the extent that metaphysical theses in particular and philosophical theses in general are to be judged by the extent to which they can be incorporated into contemporary mathematical and physical theories, the view of Being expressed by the Being axiom should be given some weight. Whether such a view can be given an entirely satisfactory defense is a topic for another occasion.

Despite my desire to avoid a full response to the considerations against viewing Being as a first-order property of individuals, there is one issue that should be addressed. Were one to add the axioms of classical logic to the Being axiom, the Being axiom would entail that everything, even Superman, is a being. This would either require Being as expressed by the above axiom to be interpreted as something other than existence; it would require one to claim that Superman, despite appearances exists; or it would require a restriction of the quantifier to objects that exist. Although notable philosophers have defended each of these possibilities, I would rather avoid them. For the purposes of what follows, therefore, I will suppose that the axioms are to be added to the axioms of positive free logic. Because the axioms of positive free logic do not allow one to instantiate the above axiom to Superman unless one has already asserted the existence of Superman, the problem is avoided. (Lambert, 1960, Nolt, 2020)

Section II – Essence

(∀x)(∃y)(∀z)(z ∈ y ≡ x ∈ z)

An essence is typically tied to the identity of an object across modal space. (Ishii, *et al.* 2018) Modality, however, is not at play within the context of extensional set theory. There is, however, a concept that is closely tied to the concept of an individual, as opposed to a kind, essence, which is present in extensional set theory, namely the concept of all of an object’s properties. Were one to make the modal assumption that a mathematical object has all of its properties necessarily, then the non-modal concept of all of an object’s properties, when restricted as it is in this paper to mathematical entities, is entailed by a modal conception of an essence according to which an object has all of its essential properties necessarily. Within the framework of sets, the concept of all of X’s properties can be understood as the set of all sets that contain X. For the purposes of this paper, I shall understand an essence in such a way; and I shall suppose that every entity has an essence so understood. The above axiom, the *Essence axiom*, asserts that every object, X, has an essence.

Along with Extensionality and the Empty set axiom, the Being and Essence axioms entail a rich structure. It will be worthwhile to examine that structure before turning to the First God axiom. The first thing to note is that the Essence axiom and the Empty set axiom entail an infinite progression of sets. Let ‘E(x)’ denote the essence of x and ‘∅’ denote the empty set. Then, the two axioms entail the existence of the empty set, ∅, the existence of the essence of the empty set, E(∅), the existence of the essence of the essence of the empty set, E(E(∅)), and so on. Likewise, the Essence axiom and the Being axiom entail an infinite progression of sets: Being, E(Being), E(E(Being), and so on. For the ease of expression, I will call any set that is part of the progression of essences stemming from the empty set an ‘empty-set essence’ and any set that is part of the progression of essences stemming from Being a ‘Being essence’. I will also employ the following notation – En(x) – to stand for the essence function applied n times repeatedly beginning with x. So, for instance, E3(∅) = E(E(E(∅))). In the limit when n=0, En(x)=x.

There are six theorems that express the structure entailed by the four axioms so far discussed. Before looking at the theorems, it may be helpful to see a visual representation of it. Because the theory is first-order, structures that are not isomorphic to the structure to be described also make the axioms true. This follows from the Löwenheim-Skolem theorem. What is described can be considered the intended structure -- it stands to the axioms in this paper as the intended structure of arithmetic stands to the axioms of Peano Arithmetic.

In the intended structure the empty set essences are all finite sets whose members are Being essences. Each empty set essence, En(∅) contains all the Being essences Em(Being) such that m < n. So, for instance, E1(∅) contains E0(Being), E2(∅) contains E0(Being) and E1(Being), and so on. The Being essences are all infinite sets. Each Being essence contains every Being essence. In addition, each being essence Em(Being) contains every empty set essence En(∅) such that n ≥ m. The following is a visual representation of the first several Being and empty set essences.

E0(Being) = {Em(Being), En(∅)}for all m≥0 and all n≥0.

E1(Being) = {Em(Being), En(∅)}for all m≥0 and all n≥1.

E2(Being) = {Em(Being), En(∅)}for all m≥0 and all n≥2.

E3(Being) = {Em(Being), En(∅)}for all m≥0 and all n≥3.

E4(Being) = {Em(Being), En(∅)}for all m≥0 and all n≥4.

E0(∅) = {}

E1(∅) = {E0(Being)}

E2(∅) = {E0(Being), E1(Being)}

E3(∅) = {E0(Being), E1(Being), E2(Being)}

E4(∅) = {E0(Being), E1(Being), E2(Being), E3(Being)}

In this structure, the Being Essences progressively lose more and more of the empty set essences. So, for instance, E0(Being) contains everything, both all the Being essences and all the empty set essences. E1(Being) contains all but one thing: It contains all the Being essences as well as all the empty set essence except E0(∅). E2(Being) contains everything but two things. And so on. It is as if the progression of Being essences is progressively drained of the empty set essences. Were one to take such a progression out to infinity, one would reach a set that contains all of the Being essences and none of the empty set essences. In other words, Eω(Being) = {Em(Being)}for all m≥0. The progression of empty set essences, on the other hand, does not consist in a successive loss of sets but rather a successive gaining of sets. E0(∅), i.e. the empty set, contains nothing, E1(∅) contains one set, namely E0(Being). E2(∅) contains two sets, namely E0(Being) and E1(Being). And so on. Were one to take such a progression out to infinity, one would again reach the set that contains all of the Being essences: Eω(∅) = {Em(Being)}for all m≥0. Hence, Eω(Being) = Eω(∅). What can be called ‘the point at infinity’ for both the Being and empty set essences is the set that contains all the Being essences.

The following four theorems describe the intended structure. (Proofs of the theorems appear in the appendix.) Let A be the set of axioms so far asserted.

*Theorem 1*: For all m≥0, all n≥0, A |- Em(Being) ∈ En(Being)

Theorem 1 shows that in the intended structure every Being essence is in every Being essence. Were one to interpret the membership relation as the accessibility relation within modal logic, theorem 1 shows that the Being essences have the same structure as possible worlds within S5 – every world accesses every world.

*Theorem 2*: For all m≥0, all n≥0, A |- Em(∅) ∉ En(∅)

Theorem 2 shows that unlike the Being essences the empty set essences are disconnected from each other – no empty set essence is in any other empty set essence.

*Theorem 3*: For all m, all n such that 0≤m<n, A |- Em(Being) ∈ En(∅);

*Theorem 4*: For all m, all n such that 0≤n≤m A |- Em(Being) ∉ En(∅)

Theorems 3 and 4 show that each empty set essence contains a finite number of Being essences. The number of Being essences that an empty set essence contains depends on the number, n, of the empty set essence. So, for instance, E0(∅) = {}, E1(∅) = {E0(Being)}, E2(∅) = {E0(Being), E1(Being)}, and so on.

*Theorem 5*: For all m, all n such that 0≤m≤n, A |- En(∅) ∈ Em(Being);

*Theorem* 6: For all m, all n such that 0≤n<m, A |- En(∅) ∉ Em(Being).

Unlike the empty set essences, the Being essences are all infinite. This follows from theorem 1, which shows that every Being essence contains every Being essence. In addition, theorems 5 and 6 show that each Being essence also contains all the empty set essences whose number is greater than or equal to the number of the Being essence. So, for instance, E1(Being) contains E1(∅), E2(∅), E3(∅), and so on. E2(Being) contains E2(∅), E3(∅), E4(∅), and so on. And so on.

One theological conclusion can be drawn from these mathematical theorems. Because no two Being essences contain exactly the same sets, no two Being essences are identical. Hence, Being is not identical to its essence, nor is the essence of Being identical to its essence, nor is the essence of the essence of Being identical to its essence, and so on. To the extent that identity to one’s essence is a necessary condition for being God, contra Aquinas God is not Being. This raises two questions: could there be a Being that is identical to its essence? And if so what would such a being be like? In the next section, I answer these questions.

Section III – God Within Being

(∃x)((∀y)(y ∈ x ≡ x ∈ y) & (∀z)((∀y)(y ∈ z ≡ z ∈ y) ⊃ z = x) & x=God)

The *First God axiom*, asserts the existence of one and only one object, x, such that x is in some set, y, if and only if y is in x, and it identifies that object with God. Because such an understanding of God differs from more well-known conceptions such as Anselm’s and Leibniz’, some explanation as to the motivation for it is in order.

The motivation begins at one level of abstraction below metaphysics, with physics. The idea that physics would provide insights for the correct formulation of metaphysics can be found in Aristotle who thought that first philosophy was to be pursued upon the completion of physics. Although Aristotle was no doubt naïve in what he thought would be required to complete physics, and although physics is not yet complete, 20th century physics has already provided a conceptual lesson for any correct metaphysics, namely the importance of symmetry in the physical world.

The fundamental role of symmetry in physics is not a claim from a philosophical theory but rather something insisted upon by physicists. Nobel prize winner P.W. Anderson (1972), for instance, asserted, ‘It is only slightly overstating the case to say that physics is the study of symmetry.’ Other physicists corroborate Anderson’s contention. David Gross (1996) describes the role of symmetry in Einstein’s work as follows.

“Einstein’s great advance in 1905 was to put symmetry first, to regard the symmetry principle as the primary feature of nature that constrains the allowable dynamical laws. Thus, the transformation properties of the electromagnetic field were not to be derived from Maxwell’s equations, as Lorentz did, but rather were consequences of relativistic invariance, and indeed largely dictate the form of Maxwell’s equations.”

And Richard Feynman (1963) describes the relation of symmetry to conservation laws as follows.

“The symmetries of the physical laws are very interesting at this level, but they turn out, in the end, to be even more interesting and exciting when we come to quantum mechanics. For a reason which we cannot make clear at the level for the present discussion – a fact that most physicists still find somewhat staggering, a most profound and beautiful thing, is that in quantum mechanics, *for each of the rules of symmetry there is a corresponding conservation law*; there is a definite connection between the laws of conservation and the symmetries of physical laws.”

The physical world owes a great deal of its structure to symmetries, so much so that Feynman says near the end of his discussion: “So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why.”

Feynman, not surprisingly, does not suggest theism as an answer to his question. But the very fundamentality of the phenomenon he is discussing, namely the nature of the physical laws, would require an answer, if indeed an answer is possible, that appeals to something that is more fundamental than the physical laws. And what could that be? One answer to that question is: God. Not just any God, however, but a universally symmetrical God. In that way, the symmetries in the laws of nature could be seen as a reflection of the perfect symmetry of God. If there is to be an answer to Feynman’s question, the answer, I contend, must appeal to a God that is universally symmetrical. But universally symmetrical in what way? The answer to that question lies in the subject matter of metaphysics. Whereas the symmetries involved in the conservation laws are continuous symmetries of time, space or spatial orientation, which are symmetries that are appropriate to the subject matter of physics, the symmetries in God would have to be more fundamental than space and time and would have to be appropriate to the subject matter of metaphysics. Because metaphysics is the study of Being, a universally symmetrical God would have to be universally symmetrical with respect to being.

But what does it mean to say that an entity exhibits symmetries of being? If one supposes that there are three main senses of being – the being of existence, the being of identity, and the being of predication – the symmetry would have to involve one of those three. Because we are dealing with types of being and not geometrical concepts, the symmetry cannot be classified as either continuous or discrete – those are concepts appropriate to space and time. At the level of fundamentality involved, the symmetry would have to be a logical symmetry: The symmetry would have to be such that for any object, x, God R x if and only if x R God, where R is a relation of being. Existence, however, is not a relation. So, the R in question must either be the is of identity or the is of predication. One might think that R could be identity, since identity is by nature symmetrical. But, therein lies the difficulty in using identity as the basis for the symmetry principle: If R were replaced by the identity relation, the resulting principle would be a logical principle applicable to all things: for all objects, y, it is the case that for any object, x, x is identical to y if and only if y is identical to x. Because such a principle applies to all objects, it would not uniquely characterize God. To have a substantive symmetry principle that characterizes only God, the being in question must be the being of predication. To say that God is universally symmetrical with respect to the being of predication is to say that for any object x, God instantiates x if and only if x instantiates God. Or to use set-theoretic language it is to say that for any set, x, God is in x if and only if x is in God. The First God axiom asserts the existence of one and only one object that satisfies this condition.

With the God axiom in place, it is possible to prove three theorems about God. The first theorem demonstrates that being universally symmetrical with respect to set membership is equivalent to being identical to one’s essence. Hence, the first theorem shows that God according to the first version of the symmetry conception of God is identical to her essence. Moreover, it shows that anything that is identical to its essence is universally symmetrical with respect to the being of predication. The second two theorems show that God is the point at infinity for the progressions of the Being and empty set essences. Let the set A of axioms be augmented with the First God axiom.

*Theorem 7*: For any, x, x is universally symmetrical with respect to the being of predication if and only if x = E(x).

Theorem 7 shows that God as defined is identical to her essence. Such a fact lends considerable credence to the definition of God as that unique being that is universally symmetrical with respect to the is of predication. Unlike the meta-theorems, it is provable directly from the axioms in A.

*Theorem 8*: For all n≥0 A |- En(Being) ∈ God

Theorem 8 shows that in the intended structure every Being essence is in God. It is an easy corollary from the God axiom that God is in every Being essence.

*Theorem 9*: For all n≥0, A |- En(∅) ∉ God.

Theorem 9 shows that in the intended structure no empty set essence is in God. It is an easy corollary from the God axiom that God is not in any empty-set essence. Because all the Being essences are in God and none of the empty set essences are, God as defined is the point at infinity for both the Being and empty set essences.

With theorems 7-9 in place, one can augment the above visualization of the intended structure of the axioms as follows.

God = {Em(Being)} for all m≥0.

E0(Being) = {God, Em(Being), En(∅)}for all m≥0 and all n≥0.

E1(Being) = {God, Em(Being), En(∅)}for all m≥0 and all n≥1.

E2(Being) = {God, Em(Being), En(∅)}for all m≥0 and all n≥2.

E3(Being) = {God, Em(Being), En(∅)}for all m≥0 and all n≥3.

E4(Being) = {God, Em(Being), En(∅)}for all m≥0 and all n≥4.

E0(∅) = {}

E1(∅) = {E0(Being)}

E2(∅) = {E0(Being), E1(Being)}

E3(∅) = {E0(Being), E1(Being), E2(Being)}

E4(∅) = {E0(Being), E1(Being), E2(Being), E3(Being)}

In this structure, God has a fractal like structure. A defining condition of a fractal is that it is self-similar. The whole of a self-similar object repeats in every one of its parts. Peitgen *et al (1991).* Hence, a fractal is infinitely repeating at smaller and smaller scales. Because God contains all and only the Being essences and all the Being essences contain God, the membership relation in God is infinitely repeating. Because the Being essences are countable, God is a countable set. Hence, because the Hausdorff dimension of a countable set is 0 (Schleicher, 2007), God’s Hausdorff dimension equals 0. God is thus a point-like fractal-like object.

It may be helpful to step back from a mathematical characterization of the structure and offer a more descriptive characterization. In this structure, the Being essences, finding their origin in Being, are positive, infinite, universally connected with each other, and symmetrically predicated of God. They are aptly described as ideas in the mind of God who both unifies the being essences and, being identical to her own essence, unifies herself. On the other hand, nothing unifies all the empty-set essences. Finding their origin in something that is defined negatively – the empty set is the set that does *not* contain any other set – they are finite, disconnected from each other, and yet are integrated into the Being essences by both containing and being contained by them.

The fact that the essences in the structure bifurcate into positive and negative essences makes it natural to identify the class of positive properties with the Being essences. If one accepts such an identification, then the symmetry conception of God intersects with the Leibnizian theological tradition. According to the Leibnizian conception of God, God instantiates all and only the positive properties. To the extent that all and only the Being essences are positive properties, God instantiates all and only the positive properties. What the Leibnizian conception does not obviously entail is that all the positive properties instantiate God, i.e. that God contains all the positive properties. In this structure, however, not only do all and only the positive properties contain God but God contains all and only the positive properties. God is thus universally symmetrical with respect to the is of predication and so, by theorem 7 above, is, as Aquinas holds, identical to her essence.

Section IV – God Beyond Being

I have so far articulated five axioms from which one can prove the existence of a unique being that is identical to her essence. Because being identical to one’s essence is a central feature of God in the Thomistic tradition, I have called such a being ‘God’. One might not be too concerned with the question as to whether the label ‘God’ is appropriate for the entity in question. For, call it what you will, the concept of an entity that is identical to its essence is a fundamental part of the philosophical theological tradition. Yet, such a concept is paradoxical enough that one might be excused for thinking that it could not be analyzed within the type of mathematical framework that is now widely accepted as the standard for mathematics, namely an axiomatized first-order extensional theory, and so must, like many of the accoutrements of ordinary religious belief be relegated to the domain of mere belief. By showing that that such a concept can be so analyzed, the concern over the legitimacy of the concept of an entity that is identical to her essence vanishes. And that fact alone is quite striking even if one does not call such an entity ‘God’. Yet, despite a possible lack of concern over whether such an entity is correctly called ‘God’, there are three reasons for denying that it is correctly so called. Addressing those reasons will not only show the weaknesses of the symmetry conception of God as it has so far been articulated but will also allow for the articulation of a definition of God that is continuous with the symmetry conception of God, that avoids the problems to be discussed, and so, I contend, really is a plausible candidate for the label ‘God’.

The first objection one might make to the symmetry conception of God is that it simply asserts the uniqueness of God by way of the God axiom. There are two reasons to be dissatisfied with such an assertion. First, in the Thomistic tradition, identity to essence is necessary for being God. But, Aquinas thought, there are a plentitude of entities that are not identical to their essences, with God occupying a unique position among them. (Aquinas 1983) Were one to approach the issue with the concept of a point at infinity in mind, a similar objection can be raised. For, there is nothing mathematically incoherent in supposing that there is more than one point at infinity. By asserting the uniqueness of God in the God axiom, I have disallowed the possibility of a multiplicity of entities that are identical to their essences. Alternatively, I have disallowed the possibility of a multiplicity of points at infinity. Moreover, it would be ideal were a definition of God to express a property that entails the uniqueness of God. Ideally, whatever property God instantiates is such that it could only be instantiated by one individual. Of course, it is possible to guarantee God’s uniqueness by explicitly including a uniqueness clause in the definition of the property. But it would be preferable were one to be able to derive the uniqueness from the property without its being explicitly stated. So, one might ask, is there some way to provide a conception of God that provably makes God unique among a multitude of entities that are identical to their essences?

Second, although the symmetry conception of God endows God with a crucial feature, namely symmetry, it leaves out another crucial feature, namely reflexivity. As early as Aristotle, God has been characterized as thought thinking itself: ‘Therefore it must be of itself that the divine thought thinks (since it is the most excellent of things), and its thinking is a thinking on thinking.’ (Aristotle, *Metaphysics, XII, 9).* Within non-well-founded set theory, such a feature would naturally be modeled by making God reflexive. Although one could simply add an assertion that the God set so far described is reflexive, such an assertion would face a complaint similar to the complaint that can be raised against the assertion that there is a unique entity that is universally symmetrical: It would be a mere assertion. What would be more satisfying would be a demonstration of God’s reflexivity from the concept of God.

There is one final aspect of God that the symmetry conception does not capture. As far back as Plato, philosophers have speculated that the original first principle is somehow set apart from other beings. Plato expresses this idea when he says in the *Republic* that the Good outranks Being in dignity in power. (Plato, *Republic* 509b) More recently, the twentieth century has seen a number of theologians who have not only portrayed God as beyond Being (Tillich 1951, Robbins and Rodkey 2010), but have also accused conceptions of God as within Being of being idolatrous. (Stenmark 2015) Despite the prevalence of such views in the twentieth century, they face a quick, and dismissive, rejection from those who think that such a view makes no sense. Van Inwagen, for instance, claims that such ascriptions, if they mean anything at all, mean that there is no God. (Van Inwagen 2006) One might of course reject, as Van Inwagen does, such an understanding of God as being overly mystical or obscurantist. Nonetheless, the thought that God is somehow utterly distinct from other Beings is such a long-standing part of various theological traditions that it would be ideal were a conception of God to be able to capture such a thought. The symmetry conception, however, does not, since, as described by the symmetry conception, God is contained by Being.

It turns out that all three of these objections can be met by moving one level of abstraction above the universally symmetrical sets to the set that contains all and only universally symmetrical sets. By itself, the assertion that a set that contains all and only the universally symmetrical sets exists does not yield a set that avoids the three objections just raised. However, if one in addition asserts that such a set is symmetrical, then one arrives at a set that does avoid the above objections.

To make such a structure vivid, let us first begin with three universally symmetrical sets -- call them ‘The Mother’, ‘The Daughter’, and ‘The Holy Spirit’ – that contain each other but not themselves. Such beings would be symmetrical but not reflexive. The fact that there are three such sets is not crucial. In principle, any number of such sets is possible. On the assumption that there are only three such sets, one would have a structure that looked like the following.

Mother = {Daughter, Holy Spirit, Em(Being)} for all m≥0.

Son = {Mother, Holy Spirit, Em(Being)} for all m≥0.

Holy Spirit = {Mother, Daughter, Em(Being)} for all m≥0.

E0(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥0.

E1(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥1.

E2(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥2.

E3(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥3.

E4(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥4.

E0(∅) = {}

E1(∅) = {E0(Being)}

E2(∅) = {E0(Being), E1(Being)}

E3(∅) = {E0(Being), E1(Being), E2(Being)}

E4(∅) = {E0(Being), E1(Being), E2(Being), E3(Being)}

Now, moving up a level in abstraction, consider the set that contains all and only those sets that are universally symmetrical. Such a set can be considered the property of being universally symmetrical, what Plato might have called ‘Symmetry Itself’. Such a set would contain the Mother, The Daughter, and The Holy Spirit, since they are all universally symmetrical. But would it contain itself? Not necessarily. In fact, without modifying the Being axiom, there is an easy proof that such a set does not contain itself. For, suppose that Being contains everything and so contains the set of all the universally symmetrical sets. If the set of all universally symmetrical sets contains itself, it would be universally symmetrical and so, because it is contained by Being, would have to contain Being. But Being is not symmetrical – it contains a set, namely the empty set, that does not contain it. So, one would have a contradiction. Hence, in order to allow for the possibility that the set of all universally symmetrical sets is universally symmetrical, one must modify the Being axiom.

Let us call the set of all universally symmetrical sets ‘God\*’. And let us write the Being axiom as follows.

Being\* Axiom: (∃x)(∀y)((y ≠ God\* ⊃ y ∈ x) & x = Being))

By itself, the Being\* Axiom does not entail that God\* is not in Being. It only entails that everything that is not identical to God\* is in Being. The Being\* axiom thus leaves open the possibility that God\* is not in Being without by itself entailing that to be the case.

We can then define God\* as follows.

(x)(x is God\* ≡ [(y)(y ∈ x ≡ (z)(y ∈ z ≡ z ∈ y)) & (z)(z ∈ x ≡ x ∈ z)])

According to this definition, God\* contains all and only the universally symmetrical sets and

is herself universally symmetrical. Notice that the definition of God\* does not include an assertion of uniqueness. The Second God Axiom can now be asserted.

God\*: (∃x)(x is God\*)

With the Second God axiom in place, it is possible to prove that God\* is unique, reflexive and beyond Being. Like theorem 5, the following three theorems can be carried out at the object level.

*Theorem 10*: (x)(x is God\* ⊃ (y)(y is God\* ⊃ x=y))

Theorem 10 shows that the property expressed by the definition of God\* is uniquely satisfied. The proof of 10 is straightforward. Suppose that there are two sets x and y each of which contains all and only those sets that are universally symmetrical. Because two such sets contain the same sets, by extensionality they are identical.

*Theorem* 11: God\* ∈ God\*.

Theorem 11 shows that God\* is reflexive. The proof of theorem 9 is straightforward. God\* contains all and only the universally symmetrical sets. Because God\* is universally symmetrical, God\* contains God\*.

*Theorem* 12: God\* ∉ Being

Theorem 12 shows that God\* is beyond Being. The proof of theorem 12 is straightforward. God\* contains all and only the universally symmetrical sets and is universally symmetrical herself. If God\* is in Being, by the symmetry of God\*, Being is in God\*. If Being is in God\*, then by the definition of God\* Being is symmetrical. But Being is not symmetrical. Hence, God\* is not in Being.

Were one to assert the existence of God\* as well as the existence of the Mother, the Daughter, the Holy Spirit, and the Being\*, empty set, and Essence axioms, one would arrive at the following structure.

God\* = {God\*, Mother, Daughter, Holy Spirit}

Mother = {God\*, Daughter, Holy Spirit, Em(Being)} for all m≥0.

Son = {God\*, Mother, Holy Spirit, Em(Being)} for all m≥0.

Holy Spirit = {God\*, Mother, Daughter, Em(Being)} for all m≥0.

E0(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥0.

E1(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥1.

E2(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥2.

E3(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥3.

E4(Being) = {Mother, Daughter, Holy Spirit, Em(Being), En(∅)}for all m≥0 and all n≥4.

E0(∅) = {}

E1(∅) = {E0(Being)}

E2(∅) = {E0(Being), E1(Being)}

E3(∅) = {E0(Being), E1(Being), E2(Being)}

E4(∅) = {E0(Being), E1(Being), E2(Being), E3(Being)}

In this structure, a single reflexive entity, God\*, stands at the head, containing and contained by all and only those sets that are universally symmetrical with respect to predication. As is the case with the God within Being, it should be clear that the membership relation in God\* is infinitely repeating -- it is contained by every set that contains it. Indeed, not only is God\*’s membership relation infinitely repeating, but all of its members, namely the Mother, the Daughter, and the Holy Spirit. have infinitely repeating membership relations as well. It should also be clear that God\* is a countable set and hence that its Hausdorff dimension, like the Hausdorff dimension of God within Being, equals 0.

It is instructive to consider the extent to which the above structure is characterized by universal symmetry. God\* and its elements -- the Mother, the Daughter, and the Holy Sprit -- are all universally symmetrical. The Being essences and the empty set essences, on the other hand, are nearly universally symmetrical. Their universal symmetry is broken only by a single instance of asymmetry. Each Being essence contains an empty set essence that does not contain it. For instance, E0(Being) contains E0(∅), which does not contain E0(Being). And each empty set essence is contained by a Being essence that it does not contain. To use the same example again, E0(Being) contains E0(∅), which does not contain E0(Being). Otherwise, each Being and empty set essence is symmetrical – with one exception in each case, each Being and empty set essence is contained by a set if and only that set contains it. So, the top two levels in the above structure contain universally symmetrical sets, while the bottom two levels contain sets that are nearly universally symmetrical. It would be hard indeed, then, to find a metaphysical structure more suited to provide an answer to Feynman’s question – Why is nature so nearly symmetrical? – than the one described by the axioms in this paper.

Conclusion

In this paper I have presented an axiomatic basis for two different but related definitions of God. According to the first definition, God is that unique Being that is universally symmetrical with respect to the Being of predication; and according to the second definition God is a universally symmetrical object that contains all and only universally symmetrical objects. I have also described structures that make all the axioms for each of the two different conceptions true. Hence, each of the sets of axioms is consistent and therefore so too is each of the definitions. This, one might reasonably contend, is a non-trivial result. Although the two definitions are novel, they both entail conceptions of God that not only are deeply embedded in the history of theology but have been charged with vacuity or incoherence.

Despite their being consistent, however, one might nonetheless wonder whether the two sets of axioms are not just consistent but consistent with the axioms of some centrally important type of non-well-founded set theory. For, without this latter consistency, one might doubt that the two sets of axioms could be fully integrated into mathematics and might thereby doubt that the definitions suffice to describe an object that is genuinely possible in some robust sense of possibility that goes beyond mere consistency. Consistency, after all, is a rather low bar. Genuine possibility, or what can be called ‘deep consistency’, is the real hurdle for any adequate metaphysics. (Studtmann 2010)

The response to this challenge, I contend, is instructive. For, it shows not only a substantive difference between the two definitions of God, but it also makes the conditions for the consistency of the definitions mathematically precise. The non-well-founded set theory that I will use in answering the challenge is positive set theory, which, along with Quine’s NF, is considered by set theorists to be one of the two mathematically serviceable positive set theories. (Holmes 2017)

Positive set theory restricts the comprehension schema to positive formulas. A positive formula is a formula which belongs to the smallest class of formulas containing a false statement ⊥, all atomic membership and equality formulas and closed under the formation of conjunctions, disjunctions, universal and existential quantifications. Positive set theory also includes the empty set axiom and the axiom of extensionality. (Holmes 2017) It is clear from the definition of a positive formula that ‘x=x’ and ‘x ∈ z’ are both positive formulas. The first of these formulas yields an axiom asserting the existence of all sets that are self-identical. Because (x)(x=x) is a logical axiom, the Being axiom is contained within positive set theory. The second formula yields a formula that with one instance of universal generalization yields the Essence axiom. Hence, four of the five axioms needed for the first symmetry definition of God are parts of basic positive set theory.

The God axiom, however, is not an instance of the comprehension schema. The formula needed to assert the existence of God contains a biconditional, and formulas that contain biconditionals are not positive. Because the God axiom entails the existence of an infinite set, albeit one that contains all and only sets whose existence is entailed by the basic axioms of positive set theory, in order to prove the existence of the God set, positive set theory would need to be augmented with an axiom of infinity. It is here, then, that one arrives at a mathematically and metaphysically interesting juncture. For, positive set theory with an axiom of infinity is equiconsistent with Morse Kelley set theory, which is a very strong set theory. As is well known, Morse Kelley can interpret ZFC. That makes both Morse Kelley and positive set theory with an axiom of infinity capable of providing the foundations of mathematics. But it also makes them so strong that one could reasonably doubt their consistency. Here is a succinct description of this issue as it relates to positive set theory by Holmes (2017).

One obvious criticism is that this theory is *extremely* strong, compared with the other systems given here. This could be a good thing or a bad thing, depending on one’s attitude. If one is worried about the consistency of a weakly compact, the level of consistency strength here is certainly a problem… On the other hand, the fact that the topological motivation for set theory seems to work and yields a higher level of consistency strength than one might expect…might be taken as evidence that these are very powerful ideas.

What, then, can be said about the deep consistency of the first symmetry conception of God? Well, until someone derives a contradiction from Morse Kelley set theory, it will remain not obviously inconsistent. But, because of the limitations on consistency proofs stemming from Gödel’s second incompleteness theorem, it will remain forever not provably consistent.

The best one could hope for would be some kind of relative consistency proof. But such a relative consistency proof would at best, because of Gödel’s theorem, be merely relative. Hence, we arrive at a situation that is familiar from traditional formulations of Anselm’s ontological argument. Although perhaps sound, it is doubtful that the crucial premise asserting God’s possible existence can be defended *a priori* without presupposing its truth. (Rowe 1976) The argument in other words fails to be cogent. Analogously, it is hard to see how one could present a cogent ontological argument stemming from the possibility of the symmetry conception of God, since it is hard to see how one could demonstrate the deep consistency of the definition without presupposing the consistency of an even stronger theory within which the proof could be carried out. Indeed, because of Gödel’s second theorem, any purported demonstration of the deep consistency of the definition within a weaker theory than positive set theory would ipso fact entail the definition’s inconsistency. It is worth emphasizing that such an epistemic situation characterizes not just the first symmetry conception of God but also any conception of God, for instance the Thomistic conception, according to which God is identical to her essence. For, as I demonstrated above, identity to essence entails universal symmetry.

The second symmetry definition yields a much different epistemic situation for the simple reason that it entails that there is something that is not in Being. Within positive set theory it is possible to establish both the existence of Being and that everything belongs to Being. Hence, the second symmetry definition of God is inconsistent with positive set theory. One might take such a result to show that those who have objected to the idea that God is beyond Being are correct – such a view leads to a deep inconsistency. But the idea that something lies beyond Being is, even according to those who accept it, already so peculiar and potentially counterintuitive, that the inability to integrate such an object into standard mathematics may serve simply to reinforce such an understanding of God. For, as I proved above, the second symmetry definition, which entails that God is beyond Being, is minimally consistent. So, one cannot reasonably contend that the idea of a God beyond Being is wholly incoherent or devoid of sense. Moreover, the fact that the second symmetry definition cannot be integrated into mathematics, which is perhaps the paradigmatic repository of Being, may just go to show that God is, well, beyond Being.

**Appendix**

Theorems 1-6, 8 and 9 are meta-theorems. They all stem from the fact that some set, x, is in a set, y, if and only if y is in the essence of x. With an appropriate definition of the essence-of function, such a fact can be proven from the Essence axiom and extensionality. Call that fact ‘E’. Formally stated,

E: (∀x)(∀y)(x ∈ y ≡ y ∈ E(x)).

Completely rigorous proofs of the meta-theorems would require using induction on the syntax. Instead of complicating the presentation by formulating inductive proofs, I exhibit several iterations of the patterns involved and then use the informal phrase, ‘and so on’. It should be obvious in each case that an inductive proof could be supplied.

Proof of Theorem 1.

For n≥0, A |- En(Being) ∈ Being. By instantiation of E, for all n≥0, A |- En(Being) ∈ Being ≡ Being ∈ En+1(Being). Hence, for n≥0, A |- Being ∈ En+1(Being). By the Being axiom, A |- Being ∈ Being. So, for n≥0 A |- Being ∈ En(Being). By instantiation of E, for all n≥0, A |- Being ∈ En(Being) ≡ En(Being) ∈ E1(Being). So, for n≥0 A |- En(Being) ∈ E1(Being). By instantiation of E, for all n≥0, A |- En(Being) ∈ E1(Being) ≡ E1(Being) ∈ En+1(Being). Hence, for n≥0, A |- E1(Being) ∈ En+1(Being). By the Being axiom, A |- E1(Being) ∈ E0 (Being). Hence, for n≥0 A |- E1(Being) ∈ En(Being). By instantiation of E, for all n≥0, A |- E1(Being) ∈ En(Being) ≡ En(Being) ∈ E2(Being). So, for n≥0, A |- En(Being) ∈ E2(Being). By instantiation of E, for all n≥0, A |- En(Being) ∈ E2(Being) ≡ E2(Being) ∈ En+1(Being). Hence, for n≥0, A |- E2(Being) ∈ En+1(Being). By the Being axiom, A |- E2(Being) ∈ E0(Being). Hence, for n≥0, A |- E2(Being) ∈ En(Being). And so on. Hence, for all m≥0 and all n≥0, A |- Em(Being) ∈ En(Being).

Proof of Theorem 2.

Suppose that for arbitrary m≥0 and n≥0, A |- Em(∅) ∈ En(∅). By instantiation of E, A |- En-1(∅) ∈ Em(∅) ≡ Em(∅) ∈ En(∅), A |- Em-1(∅) ∈ En-1(∅) ≡ En-1(∅) ∈ Em(∅), A |- En-2(∅) ∈ Em-1(∅) ≡ Em-1(∅) ∈ En-1(∅), and so on. Hence, A |- Em-n(∅) ∈ En-n(∅) or A |- En-m(∅) ∈ Em-m(∅). Hence, A |- (∃x)(x ∈ ∅), which contradicts the empty set axiom. Hence for all m≥0, all n≥0, Em(∅) ∉ En(∅).

Proof of Theorem 3.

For all n≥0, A |- En(∅) ∈ Being. Hence, by instantiation of E, A |- En(∅) ∈ Being ≡ Being ∈ En+1(∅), A |- Being ∈ En+1(∅) ≡ En+1(∅) ∈ E(Being), A |- En+1(∅) ∈ E(Being) ≡ E(Being) ∈ En+2(∅), and so on. So, for all n≥0, all m ≥1, A |- En(∅) ∈ Being ≡ Em(Being) ∈ En+m(∅). Hence, For all m, all n such that 0≤m<n, A |- Em(Being) ∈ En(∅)

Proof of Theorem 4.

Assume that 0≤n≤m. By instantiation of E, A |- En-1(∅) ∈ Em(Being) ≡ Em(Being) ∈ En(∅), A |- Em-1(Being) ∈ En-1(∅) ≡ En-1(∅) ∈ Em(Being)…. A |-Em-n(Being) ∈ En-n(∅) ≡ En-1(∅) ∈ Em(Being). Hence, A |- Em(Being) ∈ En(∅) ⊃ Em-n(Being) ∈ En-n(∅). Hence, A |- Em(Being) ∈ En(∅) ⊃ (∃x)(x ∈ ∅). Hence, A |- Em(Being) ∉ En(∅).

Proof of Theorem 5.

For all n≥0, A |- En(∅) ∈ Being. By instantiation of E, A |- En(∅) ∈ Being ≡ Being ∈ En+1(∅), A |- Being ∈ En+1(∅) ≡ En+1(∅) ∈ E(Being), A |- En+1(∅) ∈ E(Being) ≡ E(Being) ∈ En+2(∅), A |- E(Being) ∈ En+2(∅) ≡ En+2(∅) ∈ E2 (Being), and so on. So, for all m, all n such that 0≤m≤n A |- En(∅) ∈ Being ≡ En+m (∅) ∈ Em(Being). Hence, for all m, all n such that 0≤m≤n A |- En+m (∅) ∈ Em(Being).

Proof of Theorem 6.

Assume that 0≤m<n. By instantiation of E, A |- Em-1(Being) ∈ En(∅) ≡ En(∅) ∈ Em(Being), A |- En-1(∅) ∈ Em-1(Being) ≡ Em-1(Being) ∈ En(∅)… A |- Em-n(Being) ∈ En-n(∅) ≡ En-n(∅) ∈ Em-n+1(Being). Hence, A |- En(∅) ∈ Em(Being) ⊃ Em-n(Being) ∈ En-n(∅). Hence, A |- Em(Being) ∈ En(∅) ⊃ (∃x)(x ∈ ∅). Hence, A |- Em(Being) ∉ En(∅).

Proof of Theorem 7.

First, for a proof of the sufficient condition, assume that God is universally symmetrical with respect to the is of predication, i.e. that for any set, x, x ∈ God ≡ God ∈ x. Suppose x ∈ God. Then, by the symmetry of God, God ∈ x. Therefore, by the definition of an essence, x ∈ E(God). Suppose x ∈ E(God). Then, by the definition of an essence, God ∈ x. Therefore, by the symmetry of God, x ∈ God. Therefore, for any set, x, x ∈ God ≡ x ∈ E(God). Therefore, by extensionality, God = E(God). Second, for a proof of the necessary condition, suppose that God is identical to her essence, i.e. that God = E(God). Suppose that x ∈ God. Then, by the identity of God to her essence, x ∈ E(God). Therefore, by the definition of an essence, God ∈ x. Suppose that God ∈ x. Then by the definition of an essence, x ∈ E(God). Therefore, by the identity of God and E(God), x ∈ God. Therefore, for any set, x, x ∈ God ≡ God ∈ x.

Proof of Theorem 8.

By the Being axiom, A |- God ∈ Being. Hence, by the God axiom, A |- Being ∈ God. By instantiation of E, A |- Being ∈ God ≡ God ∈ E(Being). Hence, A |- God ∈ E(Being). By the God axiom, therefore, A |- E(Being) ∈ God. By instantiation of E, A |- E(Being) ∈ God ≡ God ∈ E2(Being). Hence, A |- God ∈ E2 (Being). By the God axiom, therefore, A |- E2(Being) ∈ God. And so on.

Proof of Theorem 9.

By the God axiom, for all n, A |- En(∅) ∈ God ⊃ God ∈ En(∅). By the fact E, A |- En-1(∅) ∈ God ≡ God ∈ En(∅). Hence, A |- En(∅) ∈ God ⊃ En-1(∅) ∈ God. Hence, by the God axiom, A |- En(∅) ∈ God ⊃ God ∈ Em-1(∅). By the fact E, A |- En-2(∅) ∈ God ≡ God ∈ En-1(∅). Hence, A |- En(∅) ∈ God ⊃ En-2(∅) ∈ God. Hence, by the God axiom, A |- En(∅) ∈ God ⊃ God ∈ En-2(∅). And so on, until A |- En(∅) ∈ God ⊃ God ∈ Em-m(∅). Hence, A |- En(∅) ∈ God ⊃ (∃x)(x ∈ ∅). Hence, A |- En(∅) ∉ God.

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