Endogenous Ambiguity and Rational Miscommunication

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May 31, 2023

Abstract

This paper studies a sender-receiver game in which both players want the receiver to choose the state-optimal action. Before observing the state, the sender observes a "contextual signal," a payoff-irrelevant signal that correlates with states and is imperfectly shared with the receiver. Once the sender observes the state, the sender sends a message to the receiver, incurring a small messaging cost. It is shown that there is no miscommunication in any efficient equilibrium if the messaging cost is uniform or contextual information is poorly shared between players. However, if the messaging costs are different between some messages, and contextual information can affect the probability ranking of states and is shared reasonably well, any efficient equilibrium that favors the sender exhibits miscommunication. Furthermore, the messages that cause miscommunication can be coarse or ambiguous, depending on how well players share contextual information.

Keywords: Miscommunication, linguistic ambiguity, language, context, message cost

JEL Codes: D83

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[†]I am grateful to the anonymous referees and the editor for their thoughtful suggestions. I appreciate the seminar participants at the University of Arizona and Keio University for their valuable comments. Finally, I appreciate Andreas Blume and Bart Lipman for their helpful comments and encouragement.

1 Introduction

Many economic activities are cooperative, and communication is essential for efficient operations. At first glance, when agents share an objective and have no incentive to lie, communication seems to be a trivial task. However, in reality, we occasionally fail to understand each other regardless of our intention to communicate. The purpose of this paper is to provide a model of equilibrium miscommunication to apprehend the subtlety of communication between players who share an objective.

Miscommunication can be caused in various ways. To illustrate the type of miscommunication this paper studies, consider a situation where a teacher wants to describe a complicated idea to a student. Suppose that even though it is taxing for the teacher to describe the idea precisely, she still prefers to make the student understand the idea by describing it precisely rather than leaving him uninformed. However, if she believes that the student has been listening to the earlier part of her lecture, she would simplify her description, expecting that the student would interpret it correctly with the help of the context the earlier part of her lecture would give. Then, miscommunication occurs when the student has missed the earlier part of her lecture and fails to interpret her imprecise description correctly. This paper provides a formal model to analyze this type of miscommunication. It is shown that there is no miscommunication in any efficient equilibrium if the messaging cost is uniform or the players share contextual information too poorly. However, if the messaging costs differ between some messages, and the contextual information can affect the probability ranking of states and is shared reasonably well, any efficient equilibrium that favors the sender exhibits miscommunication. Moreover, the messages that cause miscommunication can be either coarse or ambiguous, depending on the distribution of states and how well players share contextual information.

Section 2 introduces the model. There is a sender (she) and a receiver (he). At each state, there is a unique state-optimal action: both players want the receiver to choose the optimal action at each state. First, the sender observes a noisy signal about the state, which is called contextual information and is imperfectly shared with the receiver. Specifically, the sender does not know how much the receiver knows about the contextual information. The sender then observes the state and sends a message to the receiver, who does not observe the state. Sending a message is costly, and the cost can vary across messages, reflecting that speaking or writing is a taxing activity whose cost depends on the length of speech or text. The receiver then chooses an action given the message and his knowledge of the contextual information. Both players get rewarded only if the receiver chooses the state-optimal action. If the message cost is higher than the value of the optimal action, imperfect communication is inevitable. Thus, we consider the setting where the message cost is smaller than the reward from the state-optimal action. This paper then analyzes the perfect Bayesian equilibria of the game.

In Section 3, we analyze the model. To begin with, we introduce basic concepts. First, a message is coarse if it is used at more than one state given a contextual signal. Thus, even if the receiver observes a contextual signal, a coarse message does not reveal the state. Second, a message is ambiguous if it is used at a different state across some contextual signals. Thus, unless the receiver observes a contextual signal, he cannot identify the state an ambiguous message refers to. Third, a message is precise if it is used at only one state across contextual signals, and the receiver can pin down the state even without contextual information. Finally, we say an equilibrium exhibits miscommunication if the receiver can fail to choose the state-optimal action with a positive probability in the equilibrium.

Before stating the main results, this paper provides some preliminary analysis. The first lemma states that any miscommunication in this model is caused by either a coarse message or an ambiguous message. Thus, we can focus on the equilibrium use of those messages to study miscommunication. In the current model, there is always an equilibrium with miscommunication as well as one without miscommunication, suggesting that the model does not preclude perfect communication by design. Thus, our focus is on when and how miscommunication occurs in Pareto-efficient equilibria that favor the sender, i.e., "sender-optimal equilibria." The second lemma shows that if a sender-optimal equilibrium does not exhibit miscommunication, the equilibrium strategy can take a simple form. However, finding a sender-optimal equilibrium strategy becomes much harder when we do not know whether a sender-optimal equilibrium exhibits miscommunication. For example, the most economical strategy that fully separates states seems to be a good candidate for a sender optimal equilibrium strategy. Nevertheless, such a strategy can fail to be an equilibrium. That is, a sender-optimal equilibrium strategy needs to solve the trade-off between the informativeness and economy of communication under equilibrium constraints.

The first result of this paper provides some necessary conditions for miscommunication in a sender-optimal equilibrium. The first condition is that the messaging cost must not be constant. The second condition is that contextual information must be shared reasonably well. Specifically, we provide an upper bound for the probability that the receiver misses a contextual signal under which a sender-optimal equilibrium can exhibit miscommunication. Thus, any sender-optimal equilibrium only uses precise messages and exhibits no miscommunication if the messaging cost is uniform or/and contextual information is shared too poorly. It is also shown that whenever a coarse message causes miscommunication in a sender-optimal equilibrium, players do not share contextual information very well. Specifically, this paper provides a lower bound for the probability that the receiver observes a contextual signal under which a sender-optimal equilibrium can use a coarse message. The result suggests that any miscommunication in a sender-optimal equilibrium is caused by an ambiguous message if players share contextual information sufficiently well. It is also shown that whenever miscommunication in a sender-optimal equilibrium is caused by an ambiguous message, the probability ranking of states can be changed by contextual information. That is, a sender-optimal equilibrium never uses an ambiguous message if the distribution of states is too "stable" regardless of contextual information.

The next result provides a sufficient condition for miscommunication in a sender-optimal equilibrium. It is shown that a sender-optimal equilibrium exhibits miscommunication if the messaging costs vary across some messages, and the contextual information can affect the probability ranking of states and is shared reasonably well. The result also gives the formula that quantifies how well contextual information needs to be shared in order to have miscommunication in a sender-optimal equilibrium. The proof of the result is by construction. First, we construct a communication strategy with ambiguous messages by modifying the strategy in the best equilibrium without miscommunication. While the ambiguous messages in the modified strategy cause miscommunication, it allows the sender to save her communication cost when the messaging costs vary across some messages, and the contextual information can affect the probability ranking of states. Then, it can be shown that the strategy with ambiguous messages can be supported in equilibrium, and the expected gain from the ambiguous communication exceeds the expected loss from the ambiguity when the probability that the receiver observes the contextual signal satisfies the provided condition. That is, the condition guarantees the existence of an equilibrium with miscommunication that gives the sender a higher expected payoff than the best equilibrium without miscommunication. It is also demonstrated that the degree of ambiguity in a sender-optimal equilibrium can be substantial; an ambiguous message can be used in a sender-optimal equilibrium even if the receiver can miss contextual information with a probability close to 0.5, and the ambiguous message can mislead the receiver with a probability close to 0.5 conditional on the state and the contextual signal.

In order to obtain insight into the efficient use of coarse messages, this paper also gives another sufficient condition for miscommunication in a sender-optimal equilibrium. Suppose contextual signals can be categorized into either "usual" or "unusual," where one state is more likely than another conditional on a usual contextual signal, whereas it is reversed conditional on an unusual contextual signal. This paper then provides a sufficient condition for miscommunication in a sender-optimal equilibrium on the probability that the receiver observes the contextual signal. The condition guarantees that a strategy with a coarse message is supported in equilibrium, and the sender strictly prefers the equilibrium to the best equilibrium without miscommunication. The coarse message in the equilibrium strategy exhibits the defining property of vagueness; there is a borderline state that is referred to by one meaning but not by another meaning of the coarse message. It is also shown that a sender-optimal equilibrium can use such a vague message rather than an ambiguous message if contextual information is reasonably but rather poorly shared.

Section 4 provides discussions. First, if the value of the correct decision is much higher for the receiver than the sender, miscommunication in a sender-optimal equilibrium can be very costly for the receiver. In such a case, the receiver might pre-announce how he responds to each message, i.e., "an interpretation rule," before communication. This paper then provides the property of interpretation rules that maximize the receiver's expected payoff when the sender responds optimally. It is shown that such a rule possesses a property commonly observed in professional languages and organizational codes. Second, it is illustrated how the basic insights of this paper can be preserved in a more general setting. Third, it is argued that the current model suggests "context" does not exist independently by itself but is merged together with the equilibrium use of a message. Finally, we discuss how different types of equilibrium messages can be interpreted with various linguistic concepts.

The paper is concluded in Section 5.

Related literature: There is a vast literature on imperfect communication in economics; particularly, the role of a conflict of interests in imperfect communication has been studied extensively since Crawford and Sobel [1982]. The current paper contributes to a growing literature that studies imperfect communication that is caused by the presence of communication friction rather than a conflict of interests. For example, Cremer et al. [2007] and Jäger et al. [2011] consider the model where the set of messages is smaller than the set of states and analyze the optimal use of coarse messages. Blume and Board [2013] study the model where the set of available messages, i.e., "vocabulary," is private information. In their model, since the sender with a rich and a poor vocabulary use the same message differently, the message cannot reveal the exact state in the efficient equilibrium. Blume [2018] analyzes the role of higher-order uncertainty about language availability in imperfect communication.

The current paper provides another framework in this literature. There are several differences between the existing papers and the current paper. First, unlike in the existing literature, the communication friction of this paper is small enough to have an equilibrium that perfectly reveals the state. That is, this paper considers the model where the sender has a rich set of messages with small messaging costs. Our question is then when and how an efficient equilibrium can exhibit miscommunication. Second, in this paper, imperfect communication is not caused by single friction but by a combination of frictions. In fact, no efficient equilibrium exhibits miscommunication if there is no communication cost or imperfectly shared contextual information. Third, unlike in the existing models where larger communication friction can make the equilibrium communication noisier, larger friction can make an efficient equilibrium more informative in this paper. Specifically, even though the probability of miscommunication in the sender-optimal equilibrium can increase when contextual information is shared less accurately, the equilibrium miscommunication can disappear when contextual information is shared too poorly. Finally, in the existing literature, imperfect communication is caused by coarseness, i.e., the message that refers to a set of states. By contrast, imperfect communication in this paper is caused not only by coarseness but also ambiguity, i.e., the message refers to a specific but different state across contextual signals. This paper shows when contextual information is shared poorly between players, imperfect communication can be caused by coarseness. However, ambiguity plays the dominant role in imperfect communication when contextual information is shared sufficiently well.

2 Model

There are two players; a sender (she) and a receiver (he). Let Ω be a finite set of payoff-relevant states, and let A be a finite set of actions for the receiver. For each ω, there is a unique state-optimal action $a_\omega \in A$, which is different for each state. Both parties wish the receiver to choose the state-optimal action.

Before the sender observes ω, which is private information, the sender observes contextual information θ, which can be any information that can affect the probability distribution of ω, e.g., locations, surroundings, earlier statements, past relevant events, etc.[1] Let Θ be a finite set of θ, and let $\pi(\omega, \theta)$ be a joint distribution of (ω, θ) where $supp(\pi) = \Omega \times \Theta$. The receiver may or may not observe contextual information perfectly. Specifically, he observes a private signal $s \subset \Theta$, which indicates that the contextual signal the sender observed is in s. One way to interpret this setting is that θ is a history of relevant events, and the receiver can miss or forget some part of the history with some probability. Let $S = P(\Theta)\backslash\{\emptyset\}$ where $P(\Theta)$ is the power set of Θ. Then, let $g(s, \theta)$ be a joint distribution such that $supp(g(.|\theta)) = \{s \in S : \theta \in s\}$ and $supp(g(.|s)) = \{\theta \in \Theta : \theta \in s\}$. Assume that s and ω are independent given θ.

Given (ω, θ), the sender chooses a message $m \in M$, where M is finite and $|M| \geq |\Omega \times \Theta|$. That is, M is rich enough to refer to every (ω, θ). His communication strategy is then $\sigma(\omega, \theta)$ where $\sigma : \Omega \times \Theta \to M$. Given a message m and a private signal s, the receiver chooses an action $a \in A$. Formally, his decision strategy is $f(m, s)$, where $f : M \times S \to A$.

Sending a message is assumed to be costly, and the cost can vary across messages. Formally, let $c(m)$ be the cost of sending m where $c : M \to [0, \infty)$. This setting abstractly reflects that players use a language to communicate; in any language, some expressions are longer and more taxing to speak or write than others. Moreover, since a short message has smaller variations than longer messages, shorter messages tend to be more scarce in the set of available messages.[2]

Both players get rewarded only if the receiver chooses the state-optimal action a_ω

[1] This paper calls θ "contextual information" rather than "context" since whether θ actually gives context to a message, i.e., whether θ affects the meaning of a message, is determined in equilibrium.

[2] For example, if a message is a binary string, there are only two messages with length one and four messages with length two. Similarly, in natural language, since one can create longer expressions by adding words to a short expression, longer expressions have more variations. Moreover, if a feasible message is limited to a string of symbols that satisfies the syntax of a language and is relevant to the communication, it makes cheap (short) messages in M even more scarce.

at ω. Specifically, if the sender uses m at ω, and the receiver chooses an action a, the sender's payoff is $u(a,\omega) - c(m)$ where $u(a,\omega) = v > 0$ if $a = a_\omega$ and $v(a,\omega) = 0$ if $a \neq a_\omega$. The receiver's payoff from a at ω is $w(a,\omega)$ where $w(a,\omega) = V > 0$ if $a = a_\omega$ and $w(a,\omega) = 0$ if $a \neq a_\omega$. Note that even though both players wish the receiver to choose the state-optimal action, the current model allows the importance of the decision to be different between players. For example, if the receiver actually faces the consequence of his decision, whereas the sender wishes him to choose the right action based on her sense of responsibility, V can be much larger than v.[3]

This paper is interested in a communication problem where the value of the optimal action is higher than the cost of sending a message.[4] Thus, assume that $c(m) \in [0, v)$ for all $m \in M$.

This paper uses the solution concept of *perfect Bayesian equilibrium* to analyze the game. Formally, we say (σ^*, f^*, μ^*) is an equilibrium if

1. The receiver's belief μ^* is consistent. That is, given σ^*, the receiver updates his belief according to Bayes' rule whenever possible;

$$\mu^*(\omega|m,s) = \frac{\sum_{\theta\in\{\tilde{\theta}:\sigma^*(\omega,\tilde{\theta})=m,\tilde{\theta}\in s\}} g(s|\theta)\pi(\omega,\theta)}{\sum_{(\omega',\theta)\in\{(\tilde{\omega},\tilde{\theta}):\sigma^*(\tilde{\omega},\tilde{\theta})=m,\tilde{\theta}\in s\}} g(s|\theta)\pi(\omega',\theta)}.$$

2. The receiver's decision strategy $f^*(m,s)$ is optimal given μ^*;

$$f^*(m,s) \in \arg\max_{a\in A} \sum_{\omega} w(a,\omega)\mu^*(\omega|m,s).$$

3. The sender's communication strategy $\sigma^*(\omega,\theta)$ is optimal given f^*;

$$\sigma^*(\omega,\theta) \in \arg\max_{m\in M} \sum_{s} u(f^*(m,s),\omega)g(s|\theta) - c(m).$$

[3]This paper considers the simple payoff function to avoid unnecessarily complex notations and computations. The basic insight of this paper can be preserved under a more general payoff function. For a detailed discussion, see Section 4.2.

[4]If the cost of sending a message is higher than the value of the optimal action, imperfect communication is an immediate outcome. The current paper is interested in how players who share a common objective can fail to communicate perfectly even if a rich and inexpensive language is available.

	θ_1	θ_2
ω'	m'	m'''
ω''	m'	m''
ω'''	m''	m''''

Figure 1: Coarse, ambiguous, and precise messages

3 Analysis

3.1 Basic concepts

The main interest of this paper is in how the sender and the receiver can fail to communicate even though they share a common objective and a rich set of inexpensive messages. In this paper, we say an equilibrium exhibits **miscommunication** if there exists (ω, θ) such that $f^*(\sigma^*(\omega, \theta), s) \neq a_\omega$ for some s. That is, an equilibrium exhibits miscommunication if an equilibrium message can induce a wrong action with positive probability.

In order to introduce the next concepts, let

$$\Omega_\sigma(m, \theta) = \{\omega : \sigma(\omega, \theta) = m\}.$$

That is, $\Omega_\sigma(m, \theta)$ is the set of states at which σ uses a message m given θ. Put differently, if the sender uses m at θ according to σ, a message m refers to $\Omega_\sigma(m, \theta)$. It is also convenient to define

$$\Omega_\sigma(m) = \bigcup_\theta \Omega_\sigma(m, \theta).$$

Definition 1. A message m in σ is
(i) **coarse** if $|\Omega_\sigma(m, \theta)| > 1$ for some θ;
(ii) **ambiguous** if $|\Omega_\sigma(m, \theta)| \leq 1$ for all θ and $|\Omega_\sigma(m)| > 1$;
(iii) **precise** if $|\Omega_\sigma(m)| = 1$.

First, a message is coarse if it refers to a *set* of states rather than a particular state given some θ. In Figure 1, m' is a coarse message since it refers to $\{\omega', \omega''\}$ if $\theta = \theta_1$. Thus, the receiver cannot tell whether the state is ω' or ω'' from m' given $s = \{\theta_1\}$.

Second, a message is ambiguous if the message refers to a different specific state, depending on θ. For example, in Figure 1, m'' is ambiguous since m'' refers to ω''' given θ_1 whereas it refers to ω'' given θ_2. Thus, unless the receiver knows θ, he cannot tell whether m'' refers to ω''' or ω''.[5]

Finally, a message is precise if it always refers to the same state. In Figure 1, m''' and m'''' are precise; m''' refers to ω' given θ_2, and m'''' refers to ω''' given θ_2. If the receiver gets a precise message, he knows the exact state the message refers to regardless of s.

The following examples illustrate coarse, ambiguous, and precise messages in ordinary communication. Suppose a payoff-relevant state is determined by (a) whether Ken speaks English or not and (b) whether Ken has an MD or not. First, when the expression "Ken has an MD" is used if and only if Ken has an MD, it only conveys the literal meaning. Then, the use of the expression, which does not reveal whether Ken speaks English or not, makes the expression a coarse message. Second, suppose that Ken's country of residence is contextual information. Then, if the contextual information is that Ken lives in the US, and the state is that Ken speaks English and has an MD, a speaker would simply describe the state as "Ken has an MD", expecting that the listener would infer that Ken, who lives in the US, would speak English.[6] Similarly, if the contextual information is that Ken lives in Japan, and the state is that Ken has an MD and does not speak English, a speaker would describe the state also as "Ken has an MD," expecting that the listener would infer that Ken, who lives in Japan, would not speak English; in fact, if Ken could speak, the speaker would mention the special skill. The above use of "Ken has an MD" makes the expression an ambiguous message since it refers to a different state, depending on Ken's country

[5] In linguistics, ambiguity is defined as "a word or expression that can be understood in two or more possible ways." There are at least three kinds of linguistic ambiguities; lexical, syntactic, and pragmatic. The first one is caused by a *word* that has two different meanings, e.g., "Ken is near the *bank*," whereas the second one is caused by an ambiguous *sentential structure*, e.g., "The chicken is ready to eat." The third one takes various forms, but one example is "Ken has an MD," illustrated in this section. One common feature of linguistic ambiguities is that while it is ambiguous without any context, it becomes clear once a proper context is given. Since the current paper considers a message without preexisting meaning, it is not intended to study a particular type of linguistic ambiguity but the use of a message that exhibits the defining property of linguistic ambiguity. Finally, it might be worth noting that ambiguity is different from vagueness as a formal concept; the former is caused by having more than one specific and distinct interpretation, whereas the latter is caused by a lack of specificity characterized by borderline cases. Thus, in the current paper, vagueness is a type of coarseness. In Proposition 4, we provide some result that is related to the use of vagueness.

[6] In pragmatics, the branch of linguistics that studies meaning by virtue of use, this is called "conversational implicature," introduced by Grice [1975].

of residence. Finally, if a speaker uses the expression "Ken has an MD and speaks English" only at the state where Ken has an MD and speaks English, the expression always refers to the specific state. The expression is then a precise message.

3.2 Preliminary analysis

We start our analysis with the following lemma.

Lemma 1. *If an equilibrium message in σ causes miscommunication, the message is either coarse or ambiguous.*

Proof. If an equilibrium message m' is not coarse, then $|\Omega_\sigma(m', \theta)| \leq 1$ for all θ. If m' is also not ambiguous, then $|\Omega_\sigma(m')| \leq 1$. Thus, there exists ω' such that $\Omega_\sigma(m', \theta) = \{\omega'\}$ for all θ where $\Omega_\sigma(m', \theta) \neq \emptyset$. Then, by definition, m' is precise. If m' is precise, clearly $\mu(\omega'|m', s) = 1$ for all s and thus $f_\sigma(m', s) = a_{\omega'}$ under any s. Hence, m' cannot cause miscommunication. $\square$

Lemma 1 suggests that we can focus on the equilibrium use of coarse and ambiguous messages to analyze miscommunication in the current model.

As in most communication games, this game has various equilibria.

Observation 1: There exists an equilibrium with miscommunication as well as an equilibrium without miscommunication.

Let $n(\omega)$ be the probability ranking of ω based on $\pi(\omega)$, i.e., ω is the $n(\omega)$-th most likely state. Moreover, let m_k be the k-th cheapest message in M given a cost ranking of m.[7] The simplest equilibrium with miscommunication might be the one with $\sigma(\omega, \theta) = m_1$ for all (ω, θ). The coarse message m_1 then causes miscommunication. For an equilibrium without miscommunication, consider the strategy $\sigma(\omega, \theta) = m_{n(\omega)}$. That is, the sender uses the $n(\omega)$-th cheapest message at ω regardless of θ. Since every $m_{n(\omega)}$ is precise and any off-path message is at least as costly as any on-path message, this is an equilibrium strategy.

Since there is always an equilibrium with and without miscommunication, this paper focuses on how and when a Pareto-efficient equilibrium, particularly one that favors the sender, *"a sender-optimal equilibrium,"* exhibits miscommunication.[8]

[7] If there is more than one ranking due to ties, choose any of them.

[8] Note that if the sender can pre-commit to an equilibrium strategy, she chooses a sender-optimal equilibrium strategy. A sender-optimal equilibrium has also been shown to be evolutionarily stable by Blume et al. [1993].

The following lemma states that if a sender-optimal equilibrium does not exhibit miscommunication, a sender-optimal equilibrium strategy takes a simple form.

Lemma 2. *If a sender-optimal equilibrium does not exhibit miscommunication, then* $\sigma(\omega,\theta) = m_{n(\omega)}$ *is a sender-optimal equilibrium strategy.*

Proof. See appendix. □

From Lemma 1, any equilibrium strategy that does not cause miscommunication only uses precise messages. Then, if a sender-optimal equilibrium does not exhibit miscommunication, the equilibrium strategy has the lowest expected messaging cost among those that only use precise messages. It can be shown that even though there can be more than one cost-minimizing strategy among those that only use precise messages, σ in Lemma 2 is always one of them.

Finding a sender-optimal equilibrium can be challenging when we do not know whether it exhibits miscommunication. To illustrate this, let Σ^* be the set of strategies that are separating in ω given any θ, i.e., σ such that given any θ, $\sigma(\omega',\theta) \neq \sigma(\omega'',\theta)$ if $\omega' \neq \omega''$. Thus, the strategy in Lemma 2 is in Σ^*, and Σ^* also includes strategies that use ambiguous messages. Then, let

$$\sigma_{\min} \in \arg\min_{\sigma\in\Sigma^*} \sum_{\omega}\sum_{\theta} c(\sigma(\omega,\theta))\pi(\omega,\theta).$$

That is, this is the most economical strategy that is separating in ω given any θ.

Observation 2: $\sigma_{\min}$ can fail to be a sender-optimal equilibrium strategy.

Observation 2 follows from the fact that $\sigma_{\min}$ can fail to be an equilibrium strategy, i.e., it is not always incentive compatible. If $c(\sigma_{\min}(\omega,\theta')) > c(\sigma_{\min}(\omega,\theta''))$ for some ω, then the sender can have the incentive to use the cheaper message $\sigma_{\min}(\omega,\theta'')$ at (ω,θ') if players do not share contextual information well enough. Observation 2 illustrates the subtly of a sender-optimal equilibrium; it needs to solve the trade-off between the informativeness and economy of communication under equilibrium constraints.

3.3 Main results

In order to state Proposition 1, let $m_{|\Omega\times\Theta|}$ be the $|\Omega\times\Theta|$-th cheapest message in M.[9]

[9] If there is more than one ranking due to ties, choose any of them.

Proposition 1. *If a sender-optimal equilibrium exhibits miscommunication, then (i) $c(m') \neq c(m'')$ for some $m', m'' \in M$, and (ii) $g(s = \Theta|\theta) \leq \frac{c(m_{|\Omega \times \Theta|})}{v}$ for some θ. Moreover, if the miscommunication is caused by a coarse message, then $g(s = \{\theta\}|\theta) \leq \frac{v + c(m_{|\Omega \times \Theta|})}{2v}$ for some θ.*

Proof. See appendix. $\square$

Condition (i) states that a sender-optimal equilibrium exhibits miscommunication only if the messaging cost is not homogeneous. This condition can naturally be satisfied if we consider communication with a language; since most languages use a string of symbols as a message, some message is longer and more costly to speak or write than others. The idea behind the result is simple; if the cost of messaging is constant, the sender cannot save her communication cost by using a message across states and contextual signals. Then, any sender-optimal equilibrium only uses precise messages to avoid miscommunication.

Condition (ii) states that if the receiver completely misses contextual information with a probability that is higher than $\frac{c(m_{|\Omega \times \Theta|})}{v}$, there is no miscommunication in any sender-optimal equilibrium. Put differently, miscommunication in a sender-optimal equilibrium is not caused by poorly shared contextual information but reasonably shared contextual information. The basic idea of the proof is as follows. Suppose a sender-optimal equilibrium with σ uses some imprecise messages.[10] Then, for each imprecise message m, there exists $\omega_m \in \Omega_\sigma(m)$ such that m induces a_{ω_m} when the receiver completely misses θ, i.e., $s = \Theta$. Let $\Lambda_\sigma(m) = \{(\omega, \theta) : \sigma(\omega, \theta) = m\}$, and construct the alternative strategy σ' such that each imprecise message m in σ is used only at $(\omega, \theta) \in \Lambda_\sigma(m)$ where $\omega = \omega_m$, whereas, for $(\omega, \theta) \in \Lambda_\sigma(m)$ where $\omega \neq \omega_m$, each state uses a unique off-path message in σ. It can be shown that if the receiver responds to σ' optimally, σ' can weakly improve the sender's expected payoff at any (ω, θ) unless $g(\Theta|\theta)$ is low enough to satisfy condition (ii). Moreover, if σ' is not an equilibrium strategy, we can always construct an equilibrium strategy from σ' that weakly improves the expected payoff at each (ω, θ) under σ'.

The last part of Proposition 1 suggests that whenever a coarse message causes miscommunication in a sender-optimal equilibrium, the probability that the sender and the receiver share contextual information cannot be too high. In other words, when-

[10] Imprecise messages are messages that are not precise. That is, they are either ambiguous or coarse messages.

ever the receiver observes contextual information with a reasonably high probability, any miscommunication in a sender-optimal equilibrium is caused by an ambiguous message. To see the idea of the proof, suppose a sender-optimal equilibrium with σ uses coarse messages. If m is coarse in σ, each θ such that $\sigma(\omega, \theta) = m$ for some ω has $\omega_m^\theta \in \Omega_\sigma(m, \theta)$ such that m induces $a_{\omega_m^\theta}$ if the receiver observes $s = \{\theta\}$. Then, construct the alternative strategy σ' where each coarse message m in σ is used only at $(\omega_m^\theta, \theta) \in \Lambda_\sigma(m)$, whereas, for $(\omega, \theta) \in \Lambda_\sigma(m)$ where $\omega \neq \omega_m^\theta$, each state uses a unique off-path message in σ. By construction, σ' uses only precise and ambiguous messages. It can be shown that if $g(\{\theta\}|\theta)$ is high enough to satisfy the condition in Proposition 1, σ' is an equilibrium strategy, and the sender strictly prefers the equilibrium to the equilibrium with σ.

For the next result, let $n_\theta(\omega)$ be the probability ranking of ω based on $\pi(\omega|\theta)$. If there are ties, there can be more than one ranking. For consistency, consider the same tie breaking rule between θ.

Definition 2. A probability distribution $\pi(\omega, \theta)$ is **order sensitive** to θ if $n_{\theta'}(\omega') \neq n_{\theta''}(\omega')$ for some $(\theta', \theta'', \omega')$.

In short, π is order sensitive to θ if some state can be less or more likely than another state, depending on θ. For example, the probability distribution of one's academic degree is order sensitive to the contextual information of occupation; the probability distribution of one's nationality is order sensitive to the contextual information of location.

Proposition 2. *If* $\min_\theta g(\{\theta\}|\theta) > \frac{v + c(m_{|\Omega \times \Theta|})}{2v}$ *and a sender-optimal equilibrium exhibits miscommunication, then* π *is order sensitive to* θ*.*

Proof. See appendix. □

From Proposition 1, if the condition on $g(\{\theta\}|\theta)$ in Proposition 2 is satisfied, any miscommunication in a sender-optimal equilibrium must be caused by an ambiguous message. Thus, Proposition 2 suggests that miscommunication is caused by an ambiguous message in a sender-optimal equilibrium, π must be order sensitive to θ. To see the idea, note that the benefit of using an ambiguous message is to save the messaging cost by changing the use of a cheap message across states so that it is used for a state that occurs with a relatively high probability conditional on θ. Thus, if θ

does not affect the probability ranking of ω, there is no gain from using an ambiguous message.

The next result provides a sufficient condition for miscommunication in a sender-optimal equilibrium. As mentioned earlier, a sender-optimal equilibrium solves a delicate trade-off between the informativeness and economy of communication under equilibrium constraints. Since this is a combinatorial optimization problem that cannot be approached analytically, a challenge is to find a condition that is concise enough to provide insight into the efficient use of imprecise messages.

To state the condition, let $n(\omega)$ be the probability ranking of ω in $\pi(\omega)$. Moreover, let m_k be the k-th cheapest message in M.[11] Then, define

$$\xi(\omega', \omega'') = \frac{v - (c(m_{n(\omega')}) - c(m_{n(\omega'')}))}{2v}$$

$$\zeta(\theta', \omega', \omega'') = 1 - \frac{(\pi(\omega'', \theta') - \pi(\omega', \theta'))(c(m_{n(\omega'')}) - c(m_{n(\omega')}))}{\sum_\theta (\pi(\omega', \theta) + \pi(\omega'', \theta))v}.$$

Proposition 3. *If* $\min_\theta g(\{\theta\}|\theta) > \max\{\xi(\omega', \omega''), \zeta(\theta', \omega', \omega'')\}$ *for some* $(\theta', \omega', \omega'')$, *then there exists an equilibrium with miscommunication such that the sender strictly prefers the equilibrium to any equilibrium without miscommunication. That is, any sender-optimal equilibrium exhibits miscommunication.*

Proof. See appendix. □

The proof of Proposition 3 is by construction. Let σ_0 be the strategy in the sender's most preferred equilibrium without miscommunication, i.e., $\sigma_0(\omega, \theta) = m_{n(\omega)}$ from Lemma 2. Since $\min_\theta g(\{\theta\}|\theta) \in (0, 1)$, if the condition in Proposition 3 is satisfied, $\pi(\omega') - \pi(\omega'')$ and $\pi(\omega'|\theta') - \pi(\omega''|\theta')$ must have different signs, i.e., π is order sensitive to θ, and $c(m_{n(\omega'')}) - c(m_{n(\omega')}) \neq 0$. Without loss of generality, suppose $\pi(\omega') > \pi(\omega'')$ and $\pi(\omega''|\theta') > \pi(\omega'|\theta')$. Then, construct the strategy σ' from σ_0 by exchanging the use of $\sigma_0(\omega', \theta')$ at (ω', θ') and $\sigma_0(\omega'', \theta')$ at (ω'', θ'); specifically, $\sigma'(\omega', \theta') = \sigma_0(\omega'', \theta') = m_{n(\omega'')}$, $\sigma'(\omega'', \theta') = \sigma_0(\omega', \theta') = m_{n(\omega')}$, and $\sigma'(\omega, \theta) = \sigma_0(\omega, \theta) = m_{n(\omega)}$ for the rest. Figure 2 illustrates σ_0 and σ' when $|\Omega| = 3$, $\omega_1 = \omega'$, $\omega_2 = \omega''$, and $\theta_2 = \theta'$. Since ambiguous messages $m_{n(\omega')}$ and $m_{n(\omega'')}$ in σ' can cause miscommunication, it is not always an equilibrium strategy. However, it can be shown that if $g(\{\theta'\}|\theta') > \xi(\omega', \omega'')$, the sender has no incentive to deviate from the use of

[11] If there is more than one ranking due to ties, consider any of them.

σ_0	θ_1	θ_2
ω_1	m_1	m_1
ω_2	m_2	m_2
ω_3	m_3	m_3

σ'	θ_1	θ_2
ω_1	m_1	m_2
ω_2	m_2	m_1
ω_3	m_3	m_3

Figure 2: Illustration of σ_0 and σ' for the proof of Proposition 3

ambiguous messages in σ'. Note that the strategy σ' saves the expected messaging cost since the cheaper message $\sigma(\omega', \theta')$ is used at ω'' that has a higher probability than ω' conditional on θ'. It can be shown that if $\min_\theta g(\{\theta\}|\theta) > \zeta(\theta', \omega', \omega'')$ and the receiver optimally responds to σ', the saved messaging cost from the use of ambiguous messages is higher than the loss from miscommunication caused by those messages. Depending on the setting, $\zeta(\theta', \omega', \omega'')$ can be larger or smaller than $\xi(\omega', \omega'')$. If $\min_\theta g(\{\theta\}|\theta) > \max\{\xi(\omega', \omega''), \zeta(\theta', \omega', \omega'')\}$, σ' is an equilibrium strategy, and the sender's ex-ante expected payoff in the equilibrium is strictly higher than that in the best equilibrium without miscommunication.

It is worth noting that even if $\min_\theta g(\{\theta\}|\theta)$ violates the condition in Proposition 3, a sender-optimal equilibrium can still exhibit miscommunication. In fact, as we will see in Example 1 and Proposition 4, the condition on $\min_\theta g(\{\theta\}|\theta)$ in Proposition 3 can be weakened when π or c satisfies an additional condition.

Proposition 3 offers more than a sufficient condition for miscommunication in a sender-optimal equilibrium. Note that the strategy σ', which is obtained simply by exchanging the use of $m_{n(\omega')}$ and $m_{n(\omega'')}$ at θ' in σ, preserves the meaning of every on-path message in σ under some contextual signal.[12] Thus, the condition in Proposition 3 guarantees the existence of an equilibrium with miscommunication that uses almost the same language as the best equilibrium without miscommunication and gives the sender a strictly higher expected payoff than any equilibrium without miscommunication.

Corollary 1. *Suppose* $\pi(\omega') > \pi(\omega'')$, $\pi(\omega'', \theta') > \pi(\omega', \theta')$, *and* $c(m_{n(\omega'')}) \neq c(m_{n(\omega')})$ *for some* $(\omega', \omega'', \theta')$. *Any sender-optimal equilibrium exhibits miscommunication if* $\min_\theta g(\{\theta\}|\theta) \in (0, 1)$ *is sufficiently high.*

The following example shows that "sufficiently high $g(\{\theta\}|\theta)$" in Corollary 1 can be just around 0.5, and an ambiguous message in a sender-optimal equilibrium can mislead the receiver with a substantial probability.

[12] Formally, for any (ω, θ), there exists θ' such that $\Omega_{\sigma'}(\sigma(\omega, \theta), \theta') = \Omega_\sigma(\sigma(\omega, \theta), \theta)$.

Example 1. Suppose $\Omega = \{\omega_1, \omega_2\}$, $\Theta = \{\theta_1, \theta_2\}$, $M = \{m_1, m_2, m_3, m_4\}$, and $g(\{\theta\}|\theta) = \rho \in (0,1)$ for all θ. Moreover, suppose that $\pi(\theta_1) = \pi(\theta_2) = 0.5$ and $\pi(\omega_1|\theta_1) = \pi(\omega_2|\theta_2) = \lambda > 0.5$, i.e., two states are ex-ante equally likely $\pi(\omega_1) = \pi(\omega_2) = 0.5$. Furthermore, assume that $c(m_1) = 0 < c(m_k) = \beta v$ for all $k \neq 1$, where $\beta \in (0,1)$. Then, if $n(\omega') = 1$, $n(\omega'') = 2$, and $\theta' = \theta_2$, then

$$\xi(\omega', \omega'') = \frac{1+\beta}{2},$$

$$\zeta(\theta', \omega', \omega'') = 1 - (\lambda - \frac{1}{2})\beta.$$

Thus, if $\beta = 0.5$, $\max\{\xi(\omega', \omega''), \zeta(\theta', \omega', \omega'')\}$ in Proposition 3 can take any value between $(0.75, 1)$, depending on $\lambda \in (0.5, 1)$.

Note that the strategy that derives the condition in Proposition 3, i.e., $\sigma(\omega_1, \theta_1) = \sigma(\omega_2, \theta_2) = m_1$ and $\sigma(\omega_1, \theta_2) = \sigma(\omega_2, \theta_1) = m_2$, is not always a sender-optimal equilibrium. In this particular setting, the best alternative candidate for the sender-optimal equilibrium strategy is obtained by modifying σ by replacing m_2 with m_3 at (ω_1, θ_2); $\sigma'(\omega_1, \theta_1) = \sigma'(\omega_2, \theta_2) = m_1$, $\sigma'(\omega_1, \theta_2) = m_3$ and $\sigma'(\omega_2, \theta_1) = m_2$. That is, m_1 is the only ambiguous message in σ'. It can be shown that if $\rho > \max\{\beta, 1-\beta\}$, σ' is the sender-optimal equilibrium strategy. Then, if $\beta = 0.5$, the sender-optimal equilibrium can use the ambiguous message as long as $\rho \in (0.5, 1)$. That is, the sender-optimal equilibrium can use the ambiguous message m_1 even if the receiver misses θ with a probability close to 0.5, and m_1 at (ω_2, θ_2) can mislead the receiver with a probability close to 0.5.

The condition in Proposition 3, which is derived from a strategy with ambiguous messages, is concise but does not provide much insight into the efficient use of a coarse message. Since the receiver's optimal response to a coarse message can vary, depending on the setting, it is hard to obtain a concise condition for miscommunication in a sender-optimal equilibrium from a strategy with a coarse message. However, if we consider π that satisfies a certain property, we can still obtain a relatively concise condition. Let

$$\Theta_0(\omega', \omega'') = \left\{\theta : \sum_{\theta' \in s} \pi(\omega', \theta') > \sum_{\theta' \in s} \pi(\omega'', \theta') \text{ for all } s \text{ such that } \theta \in s\right\}$$

That is, this is the set of θ such that whenever s includes θ, ω' is more likely than ω'' conditional on s. Intuitively, $\Theta_0(\omega', \omega'')$ is the set of "typical contextual signals" under which ω' is more likely than ω''. Moreover, let

$$\Theta_1(\omega', \omega'') = \{\theta : \pi(\omega', \theta) < \pi(\omega'', \theta)\}.$$

That is, this is the set of θ such that ω'' is more likely than ω' conditional on θ. Then, define the following.

$$\hat{\xi}(\omega', \omega'') = \frac{c(m_{n(\omega'')}) - c(m_{n(\omega')})}{v}$$

$$\hat{\zeta}(\omega', \omega'') = \frac{\sum_{\theta \in \Theta_1(\omega', \omega'')} \pi(\omega'', \theta)(v - (c(m_{n(\omega'')}) - c(m_{n(\omega')})))}{\sum_{\theta \in \Theta_1(\omega', \omega'')} [\pi(\omega'', \theta) - \pi(\omega', \theta)]v}$$

Proposition 4. *If there exist ω', ω'' and θ' such that (i) $\Theta \backslash \Theta_0(\omega', \omega'') = \Theta_1(\omega', \omega'') \neq \Theta$ and (ii) $\min_\theta g(\{\theta\}|\theta) > \max\{\hat{\xi}(\omega', \omega''), \hat{\zeta}(\omega', \omega'')\}$, then there exists an equilibrium that uses a coarse message, and the sender strictly prefers the equilibrium to any equilibrium without miscommunication.*

To illustrate the idea of Proposition 4, suppose $\Omega = \{\omega', \omega''\}$, $\Theta = \{\theta', \theta''\}$, and $\pi(\omega') > \pi(\omega'')$. Then, if $\pi(\omega', \theta') > \pi(\omega'', \theta')$ and $\pi(\omega'', \theta'') > \pi(\omega', \theta'')$, we have $\Theta_0(\omega', \omega'') = \{\theta'\}$ and $\Theta_1(\omega', \omega'') = \{\theta''\}$, satisfying condition (i) in Proposition 4. For condition (ii), consider the communication strategy in Figure 3. In this strategy, m_1 is a coarse message; it refers to $\{\omega', \omega''\}$ if $\theta = \theta''$, whereas it refers to ω' if $\theta = \theta'$. One way to interpret such a coarse message in ordinary communication is a general expression whose meaning depends on the context, such as "interesting"; it is an informative message in the context θ', whereas it is just a "polite" uninformative message in the context θ''. It can be shown that if $\min_\theta g(\{\theta\}|\theta) > \hat{\xi}(\omega', \omega'')$, this is an equilibrium strategy. Moreover, if $\min_\theta g(\{\theta\}|\theta) > \max\{\hat{\xi}(\omega', \omega''), \hat{\zeta}(\omega', \omega'')\}$, the sender strictly prefers the equilibrium with the coarse message to the best equilibrium without miscommunication.

To see how the coarse message works, consider the case where the receiver observes contextual information. If $s = \{\theta'\}$, the receiver understands that m_1 means ω' and chooses $a_{\omega'}$. By contrast, if $s = \{\theta''\}$, he understands that m_1 means $\{\omega', \omega''\}$ and chooses $a_{\omega''}$ since ω'' is more likely than ω'. Turning to the case where the receiver misses contextual information, if $s = \{\theta', \theta''\}$, m_1 means either $\{\omega', \omega''\}$ or $\{\omega'\}$,

σ'	θ'	θ''
ω'	m_1	m_1
ω''	m_2	m_1

Figure 3: Illustration of the strategy with coarse message m_1

exhibiting vagueness.[13] Then, since $g(\theta') > g(\theta'')$, the receiver infers ω' is more likely than ω'' and chooses $a_{\omega'}$. Hence, in this case, the coarse message induces the optimal action at ω' whereas it induces a suboptimal action at ω''. However, if $\min_\theta g(\{\theta\}|\theta) > \hat{\zeta}(\omega', \omega'')$, the strategy with the coarse message saves the communication cost large enough to compensate for the probability of inducing the suboptimal action.

The strategy with the coarse message may or may not be used in a sender-optimal equilibrium. In fact, from Proposition 1, we know that if the receiver shares contextual information well enough, there is no sender-optimal equilibrium with a coarse message. However, if the receiver shares contextual information not so well but not too poorly, the strategy with the coarse message can be a sender-optimal equilibrium as in the following example.

Example 2. Consider the setting in Example 1 with two modifications: $\pi(\theta_1) = 0.6$, and $c(m_3) = \beta' v$ where $\beta' \in (\beta, 1)$. Then, if $n(\omega') = 1$ and $n(\omega'') = 2$, π satisfies condition (i) in Proposition 4. Note that

$$\hat{\xi}(\omega', \omega'') = \beta$$

$$\hat{\zeta}(\omega', \omega'') = \frac{\lambda(1-\beta)}{2\lambda - 1}$$

From Proposition 4, if $\rho > \max\{\beta, \frac{\lambda(1-\beta)}{2\lambda-1}\}$, the strategy that is used to derive the condition in Proposition 4, i.e., $\sigma'(\omega_1, \theta_1) = \sigma'(\omega_1, \theta_2) = \sigma'(\omega_2, \theta_2) = m_1$ and $\sigma'(\omega_2, \theta_1) = m_2$, is an equilibrium strategy, and the sender prefers this equilibrium to any equilibrium without miscommunication.

There are two plausible alternatives for the sender-optimal equilibrium strategy. The first strategy replaces m_1 at (ω_1, θ_2) in σ' with m_2, i.e., $\sigma''(\omega_1, \theta_1) = \sigma''(\omega_2, \theta_2) = m_1$ and $\sigma''(\omega_1, \theta_2) = \sigma''(\omega_2, \theta_1) = m_2$. It can be shown that if $\frac{1+\beta}{2} > \rho$, the sender prefers the equilibrium with σ' than that with σ''. The second alternative replaces m_2 at (ω_1, θ_2) in σ' with the off-path message m_3, i.e., $\sigma'''(\omega_1, \theta_1) = \sigma'''(\omega_2, \theta_2) = m_1$,

[13]In linguistics and semantics, an expression is *vague* if it has a borderline case. In this example, ω'' is a borderline case since the meaning of m' may or may not include ω''.

σ'	θ_1	θ_2
ω_1	m_1	m_1
ω_2	m_2	m_1

σ''	θ_1	θ_2
ω_1	m_1	m_2
ω_2	m_2	m_1

σ'''	θ_1	θ_2
ω_1	m_1	m_3
ω_2	m_2	m_1

Figure 4: Strategies in Example 3

$\sigma'''(\omega_1, \theta_2) = m_3$ and $\sigma'''(\omega_2, \theta_1) = m_2$. We can show that σ''' is an equilibrium strategy only if $\rho \geq \beta'$. Thus, σ' is the sender-optimal equilibrium if

$$\min\left\{\beta', \frac{1+\beta}{2}\right\} > \rho > \max\left\{\beta, \frac{\lambda(1-\beta)}{2\lambda - 1}\right\}.$$

For instance, if $\lambda = 0.9$, $\beta = 0.5$, and $\beta' = 0.8$, the sender-optimal equilibrium uses a coarse message under any $\rho \in (0.5625, 0.75)$.

Even though a coarse message with vagueness can play an important role in a sender-optimal equilibrium, vagueness is not an essential property of a coarse message in a sender-optimal equilibrium; in fact, we can construct an example in which a sender-optimal equilibrium uses a coarse message without vagueness. Whether a coarse message in a sender-optimal equilibrium exhibits vagueness or not is mainly a quantitative question; even if a coarse message without vagueness is used in a sender-optimal equilibrium, a coarse message with vagueness can become optimal once the cost of the cheapest off-path message is slightly reduced, preserving the cost ranking, or the probability distribution is slightly shifted, preserving the probability ranking. Due to its quantitative nature, there is no intuitive explanation for when a coarse message in a sender-optimal equilibrium exhibits vagueness.

Nevertheless, we can consider that a coarse message in a sender-optimal equilibrium is typically vague. The reason is that when a sender-optimal equilibrium uses a coarse message, the message needs to induce the state-optimal action with positive probability at every (ω, θ) where the message is used. This condition is more stringent for a coarse message without vagueness, in which the meaning is always "broad" given any θ; in fact, the condition is never satisfied if $|\Theta| = 2$. Moreover, even though we can construct an example that satisfies the condition, the construction is intricate and artificial, lacking a natural interpretation.[14]

[14]Lipman [2009] argues that it is hard to explain vague languages in the standard cheap talk game. This example shows vagueness can be the outcome of an efficient equilibrium communication once we take into account message costs and imperfectly shared contextual information.

4 Discussions

4.1 Prevention of miscommunication

Consider a situation where the receiver faces the direct consequence of his decision, whereas the sender only wishes the receiver to make the right decision as an advisor. In such a situation, whether an optimal action is chosen or not is often much more important for the receiver than the sender, i.e., $V - v$ is large. As a result, even when the probability of miscommunication is small enough to be optimal for the sender, the expected loss from miscommunication can be significant for the receiver.

When the receiver regularly gets advice from the sender, the receiver might announce how he interprets and responds to each message beforehand. Formally, define an *interpretation rule* $r(m, s)$ where $r : M \times S \to A$. That is, an interpretation rule is the specification of an action given a message and the receiver's signal about contextual information. We say an interpretation rule is *optimal* if it maximizes the receiver's ex-ante expected payoff when the sender responds to the rule optimally. Note that an interpretation rule may or may not be consistent with an equilibrium of our basic game. Thus, call an interpretation rule *credible* if it can be supported as the receiver's equilibrium decision strategy in our basic game.

Proposition 5. *In any optimal interpretation rule, if the sender chooses m' at (ω', θ) as the optimal response to an optimal interpretation rule r, then $r(m', s) = a_{\omega'}$ for all s. Moreover, any interpretation rule such that $r(m_{n(\omega)}, s) = a_\omega$ for all s and ω is optimal and credible.*

The first part of Proposition 5 states that if a message is the optimal response to an optimal interpretation rule, the optimal interpretation rule always responds to the message with the same action regardless of s. For the second part, there can be an optimal interpretation rule that is not credible. However, any interpretation rule that responds to the $n(\omega)$-th cheapest message by a_ω regardless of s is optimal and credible.

Proposition 5 suggests that optimal interpretation rules make the sender use a message so that each message has a fixed meaning independently of contextual information, precluding pragmatic inference. Such a communication rule can be found in professional languages such as aviation English and organizational codes.[15] In the ex-

[15] For example, according to Estival et al. [2016], an emergency needs to be explicitly stated rather

isting literature, such as Arrow [1974] and Cremer et al. [2007], organizational codes are considered as the way to economize communication. This paper provides another rationale for using professional language; it is an interpretation rule that prevents miscommunication.

4.2 General payoff function

In order to focus on the basic idea, this paper considered the setting where both players are rewarded only if the receiver chooses the state-optimal action. However, even if we consider a more general setting where the payoff from a suboptimal action can be positive at some ω, the basic insight can be preserved. To see this claim, suppose that, as in the current model, the cost of sending each message is smaller than the benefit of distinguishing each state, i.e., $u(a_\omega, \omega) - u(a, \omega) > c(m)$ for all $a \neq a_\omega$ and $m \in M$. Note that if the payoff from a suboptimal action can be positive rather than zero for some ω, it becomes less important for the sender to induce the state-optimal action, making the use of imprecise messages more attractive. Then, since the condition under which the sender can save her communication cost with imprecise messages remains the same, the sender-optimal equilibrium would exhibit miscommunication under a condition analogous to but less demanding than that in the basic setting.

4.3 Contextual information and context

In this paper, contextual information is defined as any information that can affect the probability distribution of states, e.g., locations, surroundings, earlier statements, past events, etc. It is important to note that whether specific contextual information provides context to a message, refining the meaning, is determined in equilibrium. In fact, contextual information may or may not provide context to a message depending on the communication environment. Proposition 1 shows that contextual information never provides context to any message if contextual information is shared poorly between the sender and the receiver. Proposition 2 suggests that even if contextual information is shared well, the information can provide context to a message only if

than implicated in aviation English. Thus, even if a pilot's report "fuel is running out" is enough to convey the sense of emergency as a natural English expression, it does not indicate the state of emergency in aviation English.

it affects the probability ranking of states. Finally, whether contextual information θ' provides context to message m' can depend not only on $\pi_{\theta'}(\omega)$ and $c(m')$ but also on $\pi_\theta(\omega)$ for $\theta \neq \theta'$ and $c(m)$ for $m \neq m'$. These observations might explain why the concept of context is so elusive; without knowing the specific communication environment, we often cannot tell which information provides context to an expression.[16]

4.4 Relationship to linguistics

This paper considers the model in which each message has no preexisting meaning and can be used at any state. On the one hand, this abstraction allows us to obtain clear insights into how the defining property of linguistic ambiguity emerges and causes miscommunication in a sender-optimal equilibrium without being bothered by linguistic details. On the other hand, the abstraction limits the formal linguistic implications we can obtain from the model. Specifically, since the sender can use each message across contexts without any constraint, a sender-optimal equilibrium uses messages more flexibly across contexts than in reality. In order to obtain linguistically more realistic results, we need to impose linguistic restrictions on the sender's strategy. For example, if we incorporate literal meaning, imposing the verifiability assumption, we can obtain linguistically more realistic outcomes while preserving the basic insight of this paper.[17]

Even though the current model does not provide a formal linguistic implication, the abstraction allows us to interpret equilibrium messages with various linguistic concepts.[18] As mentioned in Section 3-1, one linguistic interpretation of an ambiguous message is conversational implicature. If ambiguous and precise messages are used at the same state where Ken has an MD and speaks English, the ambiguous

[16] For some types of words, context can be formalized systematically. In semantics and logic, Kaplan [1989] introduced the formal notion of context to analyze demonstratives. In economics, Suzuki [2020] shows that the finest mutually self-evident event always gives context to indexical silence in efficient communication.

[17] To see how the basic insight is preserved, note that the condition in Proposition 3 is based on the construction of the payoff-improving equilibrium strategy with an ambiguous message. Even if the use of messages is restricted, as long as the analogous condition is satisfied between states that share some feasible messages, we can still construct a similar payoff-improving equilibrium strategy with an ambiguous message. The current paper did not follow such an approach since the linguistic constraints on the sender's strategy make the model much less tractable, limiting the insight we can formally obtain from the model.

[18] This is contrary to the literature that derives a specific pragmatics concept such as implicature with game theory, e.g., Benz et al. [2005].

message can be the expression "Ken has an MD" used in the context where Ken lives in the US, whereas the precise message can be the expression "Ken has an MD and speaks English" used in the context where Ken lives in Japan. When the receiver misses the contextual information, but the ex-ante probability that Ken lives in Japan is high, "Ken has an MD" uttered in the context where Ken lives in the US can be misinterpreted as Ken has an MD and does not speak English, causing miscommunication.

Another linguistic interpretation of an ambiguous message is indexical. In linguistics and semiotics, an indexical is an expression whose reference can shift from context to context, e.g., "today," "she," "that," etc. To illustrate the idea, suppose there is a group of females, and the state is determined by the one the sender wishes to indicate, and the contextual information is the last one the sender mentioned in the conversation. Moreover, suppose the set of messages consists of "she" and their names, and the cost of using a specific name is constant, whereas using "she" is cheaper. Then, if it is more likely to continue to talk about the last one the sender mentioned than talk about someone else, there is a sender-optimal equilibrium in which the last one is called "she," whereas the rest are called by their names. If the receiver forgets the last one the sender mentioned, he can misinterpret "she" as a different person, causing miscommunication.

In this paper, a cheap (or short) message can be coarse, ambiguous, or precise in a sender-optimal equilibrium. As mentioned earlier, if a short message is ambiguous, a plausible linguistic interpretation is implicature or indexical. By contrast, if a short message is precise and used at the state regardless of contextual information, a natural interpretation of such a message is the name of the state.[19]

5 Conclusion

This paper introduced a model of equilibrium miscommunication to understand the subtlety of communication between players who share an objective. We incorporated two common communication frictions in ordinary communication: (i) uttering or writing messages is costly, and the cost can vary across messages, reflecting that

[19] Kripke [1980] defines a proper name as a "rigid designator," i.e., the term that refers to the same thing under all "possible worlds." Thus, if "possible worlds" are interpreted as "possible contextual signals," a message that refers to the same state regardless of contextual signals can be considered a rigid designator.

linguistic expressions have various lengths; (ii) there is some probability that the sender fails to share contextual information with the receiver. This paper then shows that a message in a sender-optimal equilibrium can display ambiguity or coarseness and causes miscommunication, depending on the environment. More specifically, a sender-optimal equilibrium never exhibits miscommunication if the messaging cost is uniform or contextual information is shared too poorly. However, any sender-optimal equilibrium exhibits miscommunication if the message cost can vary across some messages, and contextual information can change the probability ranking of some states and is shared sufficiently well. Furthermore, a coarse message plays the main role in miscommunication when contextual information is shared rather poorly, whereas an ambiguous message plays the dominant role when contextual information is shared reasonably well.

One potential application of the current model can be found in network formation. Consider a network formation model where the benefit of forming a link depends not only on what the new link can provide but also on how efficiently a player can communicate by using a costly language. The current paper suggests that a sender prefers to communicate with a receiver who shares contextual information well. Since individuals with more similar backgrounds tend to share contextual information better, a network formation model that incorporates the use of a costly language may produce clusters among those who share similar backgrounds without the assumption of homophily or preferential attachment.

6 Appendix

This section provides the omitted proofs.

6.1 Proof of Lemma 2

Suppose σ is not a sender optimal equilibrium. From Lemma 1, if an equilibrium does not exhibit miscommunication, the equilibrium strategy only uses precise messages. Thus, there exists $\sigma' \neq \sigma$ such that it only uses precise messages, and the expected messaging cost is the lowest among those that only use precise messages and strictly lower than that of σ.

If there is no (ω', θ') and (ω'', θ') such that $c(\sigma'(\omega', \theta')) < c(\sigma'(\omega'', \theta'))$, σ' uses messages with the same cost. Then, since σ' only uses precise messages, the expected cost of σ' should be at least as high as that of σ. Thus, we must have (ω', θ') and (ω'', θ') such that $c(\sigma(\omega', \theta')) < c(\sigma(\omega'', \theta'))$. First, if $n(\omega'') > n(\omega')$ for all such (ω', θ') and (ω'', θ'), then clearly the expected cost of σ is at least as low as that of σ'. Second, if $n(\omega'') < n(\omega')$ and $\pi(\omega'') = \pi(\omega')$ for all such (ω', θ') and (ω'', θ'), we can exchange the use of $\sigma'(\omega', \theta')$ and $\sigma'(\omega'', \theta')$ without affecting the expected cost. Then, again, the expected cost of σ is at least as low as that of σ'. Finally, if $n(\omega'') < n(\omega')$ and $\pi(\omega'') \neq \pi(\omega')$, then $\pi(\omega'') > \pi(\omega')$. I claim that, in this case, σ' cannot be sender-optimal. To see the claim, let $\Lambda_\sigma(m) = \{(\omega, \theta) : \sigma(\omega, \theta) = m\}$. Consider the alternative strategy σ'' such that (i) $\sigma''(\omega, \theta) = \sigma'(\omega', \theta')$ for $(\omega, \theta) \in \Lambda_{\sigma'}(\sigma'(\omega'', \theta'))$; (ii) $\sigma''(\omega, \theta) = \sigma'(\omega'', \theta')$ for $(\omega, \theta) \in \Lambda_{\sigma'}(\sigma'(\omega', \theta'))$; (iii) $\sigma''(\omega, \theta) = \sigma'(\omega, \theta)$ for the rest. The expected cost of σ' is

$$\sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega',\theta'))} \pi(\omega, \theta) c(\sigma'(\omega', \theta')) + \sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega'',\theta'))} \pi(\omega, \theta) c(\sigma'(\omega'', \theta'))$$
$$+ \sum_{(\omega,\theta)\notin\cup_{\omega',\omega''}\Lambda_{\sigma'}(\sigma'(\omega,\theta'))} \pi(\omega, \theta) c(\sigma'(\omega, \theta)),$$

whereas the expected messaging cost of σ'' is

$$
\sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega',\theta'))} \pi(\omega,\theta)c(\sigma'(\omega'',\theta')) + \sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega'',\theta'))} \pi(\omega,\theta)c(\sigma'(\omega',\theta'))
$$

$$
+ \sum_{(\omega,\theta)\notin\cup_{\omega',\omega''}\Lambda_{\sigma'}(\sigma'(\omega,\theta'))} \pi(\omega,\theta)c(\sigma'(\omega,\theta)).
$$

The expected messaging cost saved by σ'' is then

$$
\begin{aligned}
&\sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega',\theta'))} \pi(\omega,\theta)c(\sigma'(\omega',\theta'))(c(\sigma'(\omega',\theta')) - c(\sigma'(\omega'',\theta'))) \\
&+ \sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega'',\theta'))} \pi(\omega,\theta)(c(\sigma(\omega'',\theta')) - c(\sigma(\omega',\theta'))) \\
&=(\sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega'',\theta'))} \pi(\omega,\theta) - \sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega',\theta'))} \pi(\omega,\theta))(c(\sigma(\omega'',\theta')) - c(\sigma(\omega',\theta')))
\end{aligned}
$$

Note that since σ' only uses precise messages, if $\pi(\omega'') > \pi(\omega')$, then

$$
\sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega'',\theta'))} \pi(\omega,\theta) - \sum_{(\omega,\theta)\in\Lambda_{\sigma'}(\sigma'(\omega',\theta'))} \pi(\omega,\theta) > 0.
$$

That is, the expected cost of σ'' is lower than that of σ', a contradiction.

6.2 Proof of Proposition 1

For (i), suppose $c(m)$ is constant in m but a sender-optimal equilibrium with σ exhibits miscommunication. From Lemma 1, σ uses a coarse or ambiguous message. Then, consider any strategy σ' that only uses precise messages. Since $c(m)$ is constant in m, the expected messaging cost of σ' and that of σ are the same. Then, since σ' always induces the state-optimal action, the sender strictly prefers the equilibrium with σ' to that with σ, a contradiction.

For (ii), suppose a sender-optimal equilibrium with σ uses an imprecise message, i.e., a message that is ambiguous or coarse. Let $\Lambda_\sigma(m) = \{(\omega,\theta) : \sigma(\omega,\theta) = m\}$. Note that if the receiver gets an imprecise message m and chooses a_ω where $\omega \notin \Omega_\sigma(m)$, his payoff is 0. Thus, each imprecise message m has $\omega_m \in \Omega_\sigma(m)$ such that $f(m, s = \Theta) = a_{\omega_m}$. Then, construct the strategy σ' from σ as follows. For each imprecise message m in σ, (i) if $(\omega, \theta) \in \Lambda_\sigma(m)$ and $\omega = \omega_m$, σ' uses m as in σ; (ii) if $(\omega, \theta) \in \Lambda_\sigma(m)$ and $\omega \neq \omega_m$, σ' uses a unique off-path message in σ that is equal to or cheaper than $m_{|\Omega\times\Theta|}$ for each state. Moreover, keep the use of precise messages as in σ. Once we obtained σ', construct σ'' from σ' such that $\sigma''(\omega, \theta) = \sigma'(\omega, \theta_\omega^{\min})$ where $\theta_\omega^{\min} \in \arg\min_\theta c(\sigma'(\omega, \theta))$. By construction, σ'' only uses precise messages. Moreover, since each state uses only one precise message across contextual signals, it is clearly an equilibrium strategy.

Now, I claim that the sender strictly prefers the equilibrium with σ'' to that with σ if $g(\Theta|\theta) > \frac{c(m_{|\Omega\times\Theta|})}{v}$ for all θ. To see the claim, first, note that, for $(\omega, \theta) \in \Lambda_\sigma(m)$ such that $\omega = \omega_m$, the sender's expected payoff from the precise m in σ'' is equal to or higher than that from the imprecise message m in σ'. Second, since $f(m, s = \Theta) = a_{\omega_m}$, the sender's expected payoff from m in σ at any $(\omega, \theta) \in \Lambda_\sigma(m)$ such that $\omega \neq \omega_m$ is at most

$$(1 - g(\Theta|\theta))v - c(\sigma(\omega, \theta)),$$

whereas the sender's expected payoff from σ'' at $(\omega, \theta) \in \Lambda_\sigma(m)$ is $v - c(\sigma''(\omega, \theta))$. Thus, if σ is a sender-optimal equilibrium strategy, we must have

$$(1 - g(\Theta|\theta))v - c(\sigma(\omega, \theta)) \geq v - c(\sigma'(\omega, \theta))$$

or

$$g(\Theta|\theta) \leq \frac{c(\sigma'(\omega, \theta)) - c(\sigma(\omega, \theta))}{v}$$

for some θ. However, since $c(\sigma''(\omega, \theta)) \leq c(m_{|\Omega\times\Theta|})$ by construction, if $g(\Theta|\theta) > \frac{c(m_{|\Omega\times\Theta|})}{v}$ for all θ, the above inequality cannot be satisfied.

For the last part, consider a sender-optimal equilibrium with σ that uses a coarse message. If m is a coarse message in σ, there exists $\omega_m^\theta \in \Omega_\sigma(m, \theta)$ such that $f_\sigma(m, s = \{\theta\}) = a_{\omega_m^\theta}$. Then, construct the alternative strategy σ' from σ as follows: (i) if $(\omega, \theta) \in \Lambda_\sigma(m)$ and $\omega = \omega_m^\theta$, σ' uses m as in σ; (ii) if $(\omega, \theta) \in \Lambda_\sigma(m)$ and $\omega \neq \omega_m^\theta$, use a unique off-path message in σ that is equal to or cheaper than $m_{|\Omega\times\Theta|}$ for each state. If this modification of σ makes a precise message in σ cheaper than other precise messages for the same state, replace the latter with the former.

Claim 1. If $g(s = \{\theta\}|\theta) > \frac{v + c(m_{|\Omega\times\Theta|})}{2v}$ for all θ, σ' is an equilibrium strategy.

To see the claim, note that, by construction, σ' uses either ambiguous or precise messages. First, suppose $\sigma'(\omega, \theta)$ is precise. By construction, if $\sigma'(\omega, \theta')$ where $\theta' \neq \theta$

is precise, there is no incentive to use other precise messages at (ω, θ). If $\sigma'(\omega, \theta')$ where $\theta' \neq \theta$ is an ambiguous message, the sender's expected payoff from the ambiguous message at (ω, θ) is at most $(1 - g(\{\theta\}|\theta))v$. Then, she has no incentive to use it at (ω, θ) if

$$v - c(\sigma'(\omega, \theta)) \geq (1 - g(\{\theta\}|\theta))v$$

or $g(\{\theta\}|\theta) \geq \frac{c(\sigma'(\omega,\theta))}{v}$. Since $\frac{c(\sigma'(\omega,\theta))}{v} \leq \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$, the condition is satisfied if $g(\{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$.

Second, suppose $\sigma'(\omega, \theta)$ is ambiguous. Then, since $f_{\sigma'}(\sigma'(\omega, \theta), s = \{\theta\}) = a_\omega$, the expected payoff from the ambiguous message at (ω, θ) is at least $g(\{\theta\}|\theta)v - c(\sigma'(\omega, \theta))$. If $\sigma'(\omega, \theta')$ where $\theta' \neq \theta$ is ambiguous, the expected payoff from $\sigma'(\omega, \theta')$ at (ω, θ) is at most $(1 - g(\{\theta\}|\theta))v$. If $\sigma'(\omega, \theta')$ where $\theta' \neq \theta$ is precise, and the sender uses $\sigma'(\omega, \theta')$ at (ω, θ), the receiver cannot update his belief according Bayes' rule whenever $s = \{\theta\}$. Then, if we choose an off-path belief that induces a state-suboptimal action, her expected payoff from the deviation is at most $(1 - g(\{\theta\}|\theta))v$. Thus, at (ω, θ), the sender prefers $\sigma'(\omega, \theta)$ to any message if

$$g(\{\theta\}|\theta)v - c(\sigma'(\omega, \theta)) \geq (1 - g(\{\theta\}|\theta))v$$

or $g(\{\theta\}|\theta) \geq \frac{v+c(\sigma(\omega,\theta))}{2v}$. Then, since $c(\sigma'(\omega, \theta)) \leq c(m_{|\Omega\times\Theta|})$, if $g(\{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$, the above condition is satisfied.

Claim 2. If $g(s = \{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$ for all θ, the sender's expected payoff at each (ω, θ) in the equilibrium with σ' is equal to or higher than that with σ.

In the equilibrium with σ, the sender's expected payoff from a coarse message m for $(\omega, \theta) \in \Lambda_\sigma(m)$ such that $\omega \neq \omega_m^\theta$ is at most $(1 - g(\{\theta\}|\theta))v$, whereas the payoff from $\sigma'(\omega, \theta)$ is at least $v - c(m_{|\Omega\times\Theta|})$. Thus, if $g(\{\theta\}|\theta) > \frac{c(m_{|\Omega\times\Theta|})}{v}$,

$$v - c(m_{|\Omega\times\Theta|}) > (1 - g(\{\theta\}|\theta))v.$$

Then, since $\frac{c(m_{|\Omega\times\Theta|})}{v} < \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$, the above inequality holds if $g(\{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$. For other (ω, θ), by construction, the sender's expected payoff at (ω, θ) under σ' is equal or strictly higher than that under σ. Thus, the sender's expected payoff at each (ω, θ) under σ' is equal to or higher than that under σ.

6.3 Proof of Proposition 2

Suppose a sender-optimal equilibrium with σ^* exhibits miscommunication when π is not order sensitive to θ and $\min_\theta g(\{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$.

Claim 1: If $\min_\theta g(\{\theta\}|\theta) > \frac{v+c(m_{|\Omega\times\Theta|})}{2v}$, σ^* is separating in ω given any θ.

From Proposition 1, if the condition on $\min_\theta g(\{\theta\}|\theta)$ is satisfied, any miscommunication is caused by an ambiguous message. Then, since every message in σ^* is either precise or ambiguous, σ^* is separating in ω given any θ.

Let Σ^* be the set of strategies that are separating in ω given any θ, and let

$$\sigma_{\min} \in \arg\min_{\sigma\in\Sigma^*} \sum_\omega \sum_\theta c(\sigma(\omega,\theta))\pi(\omega,\theta).$$

Claim 2: If $\sigma \in \Sigma^*$ uses an ambiguous message and π is not order sensitive to θ, then $\sigma \neq \sigma_{\min}$.

Suppose $\sigma \in \Sigma^*$ uses an ambiguous message but $\sigma = \sigma_{\min}$. Let ω_n be the state whose probability ranking in $\pi(\omega)$ is n. Then, since σ uses an ambiguous message, there exists $(\omega_{n'}, \omega_{n''}, \theta')$ such that $m_{k'} = \sigma(\omega_{n'}, \theta')$ and $m_{k''} = \sigma(\omega_{n''}, \theta')$ where $k' < k''$ and $n' > n''$. Then, the expected messaging cost of σ is

$$\begin{aligned}&\pi(\omega_{n'},\theta')c(m_{k'}) + \pi(\omega_{n''},\theta')c(m_{k''})\\ &+ \sum_{\omega\neq\omega_{n'},\omega_{n''}} \pi(\omega,\theta')c(\sigma(\omega,\theta')) + \sum_\omega \sum_{\theta\neq\theta'} \pi(\omega,\theta)c(\sigma(\omega,\theta))\end{aligned}$$

Consider the alternative strategy σ' such that $m_{k''} = \sigma'(\omega_{n'}, \theta')$ and $m_{k'} = \sigma'(\omega_{n''}, \theta')$ but the rest is the same as σ. Then, the expected messaging cost of σ' is

$$\begin{aligned}&\pi(\omega_{n'},\theta')c(m_{k''}) + \pi(\omega_{n''},\theta')c(m_{k'})\\ &+ \sum_{\omega\neq\omega_{n'},\omega_{n''}} \pi(\omega,\theta')c(\sigma(\omega,\theta')) + \sum_\omega \sum_{\theta\neq\theta'} \pi(\omega,\theta)c(\sigma(\omega,\theta))\end{aligned}$$

The expected messaging cost saved by σ' is then

$$
\begin{aligned}
&\pi(\omega_{n'},\theta')(c(m_{k'})-c(m_{k''}))+\pi(\omega_{n''},\theta')(c(m_{k''})-c(m_{k'})) \\
=&(\pi(\omega_{n''},\theta')-\pi(\omega_{n'},\theta'))(c(m_{k''})-c(m_{k'}))
\end{aligned}
$$

Note that if π is not sensitive to θ, then $\pi(\omega_{n''},\theta') > \pi(\omega_{n'},\theta')$. Thus,

$$
(\pi(\omega_{n''},\theta')-\pi(\omega_{n'},\theta'))(c(m_{k''})-c(m_{k'})) > 0
$$

The inequality contradicts $\sigma = \sigma_{\min}$.

Claim 3: $\sigma_{\min}$ is an equilibrium strategy.

From Claim 2, $\sigma_{\min}$ does not use any ambiguous message. Then, since $\sigma_{\min} \in \Sigma^*$, $\sigma_{\min}$ only uses precise messages, and the sender has no incentive to use an on-path message for a different state. Moreover, since all off-path messages are at least as costly as any on-path message in $\sigma_{\min}$, there is no incentive to use any off-path message under any off-path belief.

Claim 4: The sender strictly prefers the equilibrium with $\sigma_{\min}$ to that with σ^*.

Since the equilibrium with $\sigma_{\min}$ exhibits no miscommunication, whereas the expected messaging cost of $\sigma_{\min}$ is strictly lower than that of any $\sigma \in \Sigma^*$ with an ambiguous message, the sender prefers the equilibrium with $\sigma_{\min}$ to the equilibrium with σ^*.

6.4 Proof of Proposition 3

Let $\sigma_0(\omega,\theta) = m_{n(\omega)}$ for all ω. From Lemma 2, this is an equilibrium strategy, and the sender prefers this equilibrium to any equilibrium that exhibits no miscommunication. I claim that if the condition in Proposition 3 is satisfied, there is an equilibrium with an ambiguous message in which the sender's expected payoff is higher than that in the equilibrium with σ_0.

First, choose any ω', ω'' such that $n(\omega') < n(\omega'')$. Then, consider the strategy σ' where the use of $m_{n(\omega')}$ and $m_{n(\omega'')}$ in σ_0 are exchanged at θ'. That is, $\sigma'(\omega',\theta') = m_{n(\omega'')}$ and $\sigma'(\omega',\theta) = m_{n(\omega')}$ for all $\theta \neq \theta'$; $\sigma'(\omega'',\theta') = m_{n(\omega')}$ and $\sigma'(\omega'',\theta) = m_{n(\omega'')}$ for all $\theta \neq \theta'$. Let $\rho = \min_{\theta\in\Theta} g(\{\theta\}|\theta)$.

Claim 1. If $\rho \geq \xi(\omega',\omega'')$, then σ' is an equilibrium strategy.

First, if the sender is at (ω, θ) where $\omega \neq \omega', \omega''$, $\sigma'(\omega, \theta)$ is precise and constant in θ. Thus, clearly, the sender has no incentive to deviate if she is at $\omega \neq \omega', \omega''$.

Second, suppose the sender is at $(\omega', \theta \neq \theta')$ or (ω'', θ'). Then, the probability that the ambiguous message $m_{n(\omega')}$ induces $a_{\omega'}$ is at least ρ. Thus, the sender's expected payoff from $m_{n(\omega')}$ at $(\omega', \theta \neq \theta')$ or (ω'', θ') is at least $\rho v - c(m_{n(\omega')})$, whereas her expected payoff from $m_{n(\omega'')}$ at $(\omega', \theta \neq \theta')$ or (ω'', θ') is at most $(1-\rho)v - c(m_{n(\omega'')})$. Thus, the sender has no incentive to use $m_{n(\omega'')}$ at $(\omega', \theta \neq \theta')$ or (ω'', θ') if

$$\rho v - c(m_{n(\omega')}) \geq (1-\rho)v - c(m_{n(\omega'')}).$$

or

$$\rho \geq \frac{v - (c(m_{n(\omega'')}) - c(m_{n(\omega')}))}{2v}$$

Note that since $n(\omega') < n(\omega'')$, $c(m_{n(\omega')}) \leq c(m_{n(\omega'')})$. Thus, $\xi(\omega', \omega'') > \frac{v-(c(m_{n(\omega'')})-c(m_{n(\omega')}))}{2v}$. Hence, if $\rho \geq \xi(\omega', \omega'')$, the above condition is satisfied.

Third, suppose the sender is at (ω', θ') or $(\omega'', \theta \neq \theta')$. Then, the probability that $m_{n(\omega'')}$ induces the state optimal action is at least ρ. Thus, the sender's expected payoff from $m_{n(\omega'')}$ at (ω', θ') or $(\omega'', \theta \neq \theta')$ is at least $\rho v - c(m'')$, whereas the payoff from m' is at most $(1-\rho)v - c(m_{n(\omega')})$. Thus, the sender has no incentive to use $m_{n(\omega')}$ at (ω', θ') or $(\omega'', \theta \neq \theta')$ if

$$\rho \geq \frac{v - (c(m_{n(\omega')}) - c(m_{n(\omega'')}))}{2v}$$

The above condition is satisfied if $\rho \geq \xi(\omega', \omega'')$.

Claim 2. The sender prefers the equilibrium with σ' to that with σ_0 if $\rho \geq \zeta(\theta', \omega', \omega'')$.

Note that $f_{\sigma'}(\sigma'(\omega, \theta), s = \{\theta\}) = a_\omega$ for all (ω, θ). Thus, the sender's ex-ante expected payoff from σ' is at least

$$\sum_{\omega \neq \omega', \omega''} \sum_{\theta} \pi(\omega, \theta)[v - c(\sigma_0(\omega, \theta))] + \sum_{\omega = \omega', \omega''} \sum_{\theta} \pi(\omega, \theta)[g(\{\theta\}|\theta)v - c(\sigma'(\omega, \theta))],$$

whereas the sender's expected payoff in the equilibrium with σ_0 is

$$v - \sum_{\omega} \sum_{\theta} \pi(\omega, \theta) c(\sigma_0(\omega, \theta)).$$

Thus, the sender strictly prefers the equilibrium with σ' to that with σ_0 if

$$\sum_{\omega=\omega',\omega''}\sum_{\theta}\pi(\omega,\theta)[g(\{\theta\}|\theta)v - c(\sigma'(\omega,\theta))] - \sum_{\omega=\omega',\omega''}\sum_{\theta}\pi(\omega,\theta)[v - c(\sigma_0(\omega,\theta))] > 0$$

Since $\rho = \min_{\theta\in\Theta} g(\{\theta\}|\theta)$, we can rewrite the above inequality as follows.

$$(\rho-1)\sum_{\theta}(\pi(\omega',\theta)+\pi(\omega'',\theta))v + [(\pi(\omega'',\theta')-\pi(\omega',\theta'))(c(m_{n(\omega'')})-c(m_{n(\omega')}))] > 0$$

By rearranging for ρ,

$$\begin{aligned}\rho &> \frac{\sum_{\theta}(\pi(\omega',\theta)+\pi(\omega'',\theta))v - [(\pi(\omega'',\theta')-\pi(\omega',\theta'))(c(m_{n(\omega'')})-c(m_{n(\omega')}))]}{\sum_{\theta}(\pi(\omega',\theta)+\pi(\omega'',\theta))v}\\ &= 1 - \frac{(\pi(\omega'',\theta')-\pi(\omega',\theta'))(c(m_{n(\omega'')})-c(m_{n(\omega')}))}{\sum_{\theta}(\pi(\omega',\theta)+\pi(\omega'',\theta))v}.\end{aligned}$$

That is, $\rho > \zeta(\theta',\omega',\omega'')$.

From Claim 1 and 2, if $\rho > \max\{\xi(\omega',\omega''),\zeta(\theta',\omega',\omega'')\}$, then σ', which uses ambiguous messages, is an equilibrium strategy, and the sender strictly prefers the equilibrium with σ' to any equilibrium without miscommunication.

6.5 Proof of Proposition 4

As in the proof of Proposition 3, let $\sigma_0(\omega,\theta) = m_{n(\omega)}$. Note that if $\Theta_0(\omega',\omega'') \neq \emptyset$, by definition, $\sum_{\theta}\pi(\omega',\theta) > \sum_{\theta}\pi(\omega'',\theta)$ and thus $n(\omega') < n(\omega'')$. Then, consider the following strategy σ' where $\sigma'(\omega'',\theta) = m_{n(\omega')}$ for $\theta \in \Theta_1(\omega',\omega'')$ and $\sigma'(\omega,\theta) = \sigma_0(\omega,\theta)$ for the rest. That is, $m_{n(\omega')}$ in σ' is a coarse message.

Claim 1. σ' is an equilibrium strategy if

$$\min_{\theta} g(\{\theta\}|\theta) \geq \max\left\{\frac{v + c(m_{n(\omega')}) - c(m_{n(\omega'')})}{v}, \frac{c(m_{n(\omega'')}) - c(m_{n(\omega')})}{v}\right\}.$$

Clearly, the sender has no incentive to deviate if she is at $\omega \neq \omega''$. Thus, consider the case where the sender is at ω''.

First, if the sender is at (ω'',θ) with $\theta \in \Theta_1(\omega',\omega'')$, her expected payoff from $m_{n(\omega')}$ is at least $g(\{\theta\}|\theta)v - c(m_{n(\omega')})$, whereas her expected payoff from $m_{n(\omega'')}$ is

$v - c(m_{n(\omega'')})$. Thus, she prefers $m_{n(\omega')}$ to $m_{n(\omega'')}$ if

$$g(\{\theta\}|\theta)v - c(m_{n(\omega')}) \geq v - c(m_{n(\omega'')})$$

or

$$g(\{\theta\}|\theta) \geq \frac{v + c(m_{n(\omega')}) - c(m_{n(\omega'')})}{v}.$$

Second, if the sender is at (ω'', θ) with $\theta \in \Theta_0(\omega', \omega'')$, her expected payoff from $m_{n(\omega'')}$ is $v - c(m_{n(\omega'')})$, whereas her expected payoff from $m_{n(\omega')}$ is at most $(1 - g(\{\theta\}|\theta))v - c(m_{n(\omega')})$. Thus, she prefers $m_{n(\omega'')}$ to $m_{n(\omega')}$ if

$$v - c(m_{n(\omega'')}) \geq (1 - g(\{\theta\}|\theta))v - c(m_{n(\omega')})$$

or

$$g(\{\theta\}|\theta) \geq \frac{c(m_{n(\omega'')}) - c(m_{n(\omega')})}{v}$$

Thus, if the condition in Claim 1 is satisfied, the sender at (ω'', θ) has no incentive to deviate for any θ.

Claim 2. If σ' is an equilibrium strategy, the sender prefers the equilibrium with σ' to that with σ_0.

Note that if condition (i) is satisfied, $f_{\sigma'}(m_{n(\omega')}, s) = a_{\omega''}$ if $s \subset \Theta_1(\omega', \omega'')$, whereas $f_{\sigma'}(m_{n(\omega')}, s) = a_{\omega'}$ if $s \not\subset \Theta_1(\omega', \omega'')$. Then, let

$$\eta_\theta = \sum_{s \subset \Theta_1(\omega', \omega'')} g(s|\theta).$$

The sender's ex-ante expected payoff in the equilibrium with σ' is

$$\sum_{\omega \neq \omega', \omega''} \sum_{\theta} \pi(\omega, \theta)(v - c(m_{n(\omega)})) + \sum_{\theta \in \Theta_0(\omega', \omega'')} \pi(\omega', \theta')(v - c(m_{n(\omega')}))$$
$$+ \sum_{\theta \in \Theta_1(\omega', \omega'')} [\pi(\omega'', \theta)(\eta_\theta v - c(m_{n(\omega')})) + \pi(\omega', \theta)((1 - \eta_\theta)v - c(m_{n(\omega')}))]$$

The sender's ex-ante expected payoff in the equilibrium with σ' is higher than that in the equilibrium with σ if

$$\sum_{\theta\in\Theta_1(\omega',\omega'')} [\pi(\omega'',\theta)(\eta_\theta v - c(m_{n(\omega')})) + \pi(\omega',\theta)((1-\eta_\theta)v - c(m_{n(\omega')}))]$$
$$> \sum_{\theta\in\Theta_1(\omega',\omega'')} [\pi(\omega'',\theta)(v - c(m_{n(\omega'')})) + \pi(\omega',\theta)(v - c(m_{n(\omega')}))]$$

or

$$\sum_{\theta\in\Theta_1(\omega',\omega'')} \pi(\omega'',\theta)((\eta_\theta - 1)v + c(m_{n(\omega'')}) - c(m_{n(\omega')}))$$
$$> \sum_{\theta\in\Theta_1(\omega',\omega'')} \pi(\omega',\theta)\eta_\theta v$$

By rearranging for η_θ,

$$\sum_{\theta\in\Theta_1(\omega',\omega'')} [\pi(\omega'',\theta) - \pi(\omega',\theta)]\eta_\theta v$$
$$> \sum_{\theta\in\Theta_1(\omega',\omega'')} \pi(\omega'',\theta)v - \sum_{\theta\in\Theta_1(\omega',\omega'')} \pi(\omega'',\theta)(c(m_{n(\omega'')}) - c(m_{n(\omega')}))$$

Then, the above condition can be satisfied if

$$\min_{\theta\in\Theta_1(\omega',\omega'')} \eta_\theta > \frac{\sum_{\theta\in\Theta_1(\omega',\omega'')} \pi(\omega'',\theta)(v - (c(m_{n(\omega'')}) - c(m_{n(\omega')}))}{\sum_{\theta\in\Theta_1(\omega',\omega'')}[\pi(\omega'',\theta) - \pi(\omega',\theta)]v}$$

or

$$\min_{\theta\in\Theta_1(\omega',\omega'')} \eta_\theta > \hat{\zeta}(\omega',\omega'')$$

Note that $g(\{\theta\}|\theta) \leq \min_{\theta'\in\Theta_1(\omega',\omega'')} \eta_{\theta'}$ for all $\theta \in \Theta_1(\omega',\omega'')$. Thus, the above condition is satisfied if $\min_\theta g(\{\theta\}|\theta) > \hat{\zeta}(\omega',\omega'')$. Moreover, since $\hat{\zeta}(\omega',\omega'') > \frac{v+c(m_{n(\omega')})-c(m_{n(\omega'')})}{v}$, if $\min_\theta g(\{\theta\}|\theta) > \hat{\zeta}(\omega',\omega'')$, then $\min_\theta g(\{\theta\}|\theta) > \frac{v+c(m_{n(\omega')})-c(m_{n(\omega'')})}{v}$. Hence, if $\min_\theta g(\{\theta\}|\theta) > \max\{\hat{\xi}(\omega',\omega''), \hat{\zeta}(\omega',\omega'')\}$, σ' is an equilibrium strategy, and the sender prefers the equilibrium with σ' to that with σ_0.

6.6 Proof of Proposition 5

Given an interpretation rule r, the sender at (ω, θ) chooses m that solves

$$\max_{m \in M} \sum_{s \in \{s' : \theta \in s', r(m,s') = a_\omega\}} g(s|\theta)v - c(m).$$

Let $m_r(\omega, \theta)$ be the sender's optimal response to the interpretation rule r at (ω, θ). Suppose r is optimal but $r(m_r(\omega, \theta), s') \neq a_\omega$ for s' such that $\theta \in s'$. If the receiver changes r to r' so that $r'(m_r(\omega, \theta), s') = a_\omega$, the sender's expected payoff from $m_r(\omega, \theta)$ becomes higher whereas her expected payoff from other messages are equal or lower under r'. Then, since the sender's optimal response to r' at (ω, θ) is still $m_r(\omega, \theta)$, the receiver can improve his expected payoff by using r', a contradiction.

For the second part, consider the interpretation rule $r(m_{n(\omega)}, s) = a_\omega$ for all s. That is, the receiver chooses a_ω regardless of s if the sender uses the $n(\omega)$-th cheapest message. Since $m_{n(\omega)}$ is the cheapest message that induces the state-optimal action, the sender's optimal response to r at (ω, θ) is $m_{n(\omega)}$ for all θ. Then, since the sender's message induces the state-optimal action regardless of s, r is an optimal interpretation rule. To see the rule is credible, suppose the sender uses the strategy $\sigma(\omega, \theta) = m_{n(\omega)}$. Clearly, this strategy is consistent with the sender's optimal response to the interpretation rule. Moreover, since $m_{n(\omega)}$ is precise, the receiver chooses a_ω regardless of s, which is also consistent with the interpretation rule. Furthermore, by construction, any off-path message in σ is at least as costly as any on-path message. Thus, the sender has no incentive to use any off-path message under any off-path belief.

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