Questions about Proof Theory vis-à-vis Natural Language Semantics

ESSLLI 2007 CD Version, interspersed with some of the lecture slides[1]

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Semantics plays a role in grammar in at least three guises. (A) Linguists seek to account for speakers' knowledge of what linguistic expressions mean. This goal is typically achieved by assigning a model theoretic interpretation[2] in a compositional fashion. For example, *No whale flies* is true if and only if the intersection of the sets of whales and fliers is empty in the model. (B) Linguists seek to account for the ability of speakers to make various inferences based on semantic knowledge. For example, *No whale flies* entails *No blue whale flies* and *No whale flies high*. (C) The well-formedness of a variety of syntactic constructions depends on morpho-syntactic features with a semantic flavor. For example, *Under no circumstances would a whale fly* is grammatical, whereas *Under some circumstances would a whale fly* is not, corresponding to the downward vs. upward monotonic features of the preposed phrases.

It is usually assumed that once a compositional model theoretic interpretation is assigned to all expressions, its fruits can be freely enjoyed by inferencing and syntax. What place might proof theory have in this picture? This paper attempts to raise questions rather than offer a thesis.

1. Model theory and proof theory

Two approaches to semantics are the model theoretic and the proof theoretic ones. Using a familiar example, consider the model theoretic and the proof theoretic

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[1] An earlier version was presented at the 2005 Semantics Workshop at Rutgers University. I thank the commentators, Matthew Stone and Ken Shan, as well as the participants for criticism. Mark Steedman, Johan van Benthem, Chris Barker, Ed Stabler, Zoltan Szabo, Jason Stanley, and Barry Schein have kindly discussed these matters with me; this text does not yet do justice to all their suggestions.

[2] There may be a discrepancy in the use of the term "semantics" between formal semanticists and philosophers. The former do not concern themselves with questions of reference and, in general, the relationship between expressions and the world out there; the entities in the models are abstract and linguistically/mathematically motivated.

faces of propositional logic. The interpretation of the connectives $\wedge$,$\vee$, and $\neg$ in terms of truth tables is the simplest kind of model theoretic semantics. It also determines relations between formulae. For example, $\neg p$ is a logical consequence of $\neg(p\vee q)$ because all the ways of assigning values to p and q that make $\neg(p\vee q)$ true also make $\neg p$ true. Thus the inference is said to be semantically valid (notated as $|=$). Compare this with how the propositional calculus approaches the same inference. It offers a set of transition steps, the de Morgan law $\neg(p\vee q)= \neg p\wedge\neg q$ among them, with which the string of symbols $\neg p$ can be derived from the string $\neg(p\vee q)$. This demonstrates the so-called syntactic validity of the inference (notated as $|-$). A calculus is sound if whatever it derives is true in the intended models (ϕ |- ψ only if ϕ |= ψ); complete if it can derive whatever is true in the models (ϕ |- ψ whenever ϕ |= ψ); and decidable if an algorithm can effectively determine whether ϕ |- ψ holds. The propositional calculus is sound, complete, and decidable; the first order predicate calculus is sound and complete. More complex calculi have at most generalized completeness.

As the term `syntactic validity' indicates, proof theory involves symbol manipulation. Nevertheless, given soundness and (some interesting degree of) completeness, a calculus deserves the name `proof theoretic semantics' in that it cashes out model theoretic semantic relations in its own syntactic terms, rather than concerning itself with the plain well-formedness of expressions, e.g., whether ($p\wedge$) is well-formed.

On this view, model theory has primacy over proof theory. A language may be defined or described perfectly well without providing a calculus and thus, a logic for it, but (on this view) a calculus is of distinctly limited interest without a class of models with respect to which it is known to be sound and (to some interesting degree) complete.

It seems fair to say that mainstream formal semantics as practiced by linguists is exclusively model theoretic.[3] As I understand it, the main goal is to elucidate the meanings of expressions in a compositional fashion, and to do that in a way that offers an insight into natural language metaphysics (Bach 1989) and uncovers universals of the syntax/semantics interface.[4] Non-linguists sometimes regard the compositional interpretation of natural language expressions either as impossible or just a simple exercise. In contrast, linguists have come to think of it as a huge but rewarding empirical enterprise. The fact that the insights accumulated over the past

[3] I am not quite sure of the position of the formal pragmatics work on implicatures, inspired by Grice. As far as I can see, the techniques are clearly model theoretic but the goals may or may not be.

[4] Conservativity, a property of determiners or, more generally, of expressions denoting relations between sets, may be the best studied universal. Det is conservative iff Det(A)(B) = Det(A)(A$\cap$B).

decades have been obtained by investigating denotation conditions[5] plays an immense role in linguists' acceptance of and adherence to the model theoretic approach.

The view that model theory is not only necessary but also the primary source of insights is not the linguist's invention. For example, in the Gamut textbook's chapter on "Arguments and Inferences", the authors--here, probably, Johan van Benthem and/or Dick de Jongh—put forward that "[S]emantic methods tend to give one a better understanding" but, they go on, "[they are] based on universal quantification over that mysterious totality, the class of all models (there are infinitely many models, and models themselves can be infinitely large). The notion of meaning that we use in the syntactic approach is more instrumental: the meaning of some part of the sentence lies in the conclusions which, because precisely that part appears at precisely that place, can be drawn from the sentence […]."

This observation is nothing new, of course, but the idea that our way of doing semantics is both insightful and computationally (psychologically) unrealistic has failed to intrigue formal semanticists into action.[6] Why? There are various, to my mind respectable, possibilities. (i) Given that the field is young and still in the process of identifying the main facts it should account for, we are going for the insight as opposed to the potential of computational/psychological reality. (ii) We don't care about psychological reality and only study language in the abstract. (iii) We do care about potential psychological reality but are content to separate the elucidation of meaning (model theory) from the account of inferencing (proof theory). – But if the machineries of model theory and proof theory are sufficiently different, option (iii) may end up with a picture where speakers cannot know what sentences mean, so to speak, only how to draw inferences from them. Is that the correct picture? Perhaps we could have our cake and eat it too. Or have an altogether better cake if we cared to modify the recipe. It seems that a better understanding of the choice we made and of other choices that we might make would be useful. The present paper wishes to highlight this need and to elicit comments from linguists and from the neighboring fields.

In my initial attempt to reduce my ignorance, I seem to have identified three

[5] I am carefully avoiding the term `truth (conditions)', because only a fraction of natural language sentences and subsentential expressions can be said to be true or false to begin with, and also because it is immaterial from this perspective whether sentences are associated with truth values, situations, events, or something else.

[6] Clearly, we are talking about proof theory offering the abstract possibility of psychological reality; there is no claim to the effect that people have, say, Natural Deduction machines in their heads.

interestingly different proof theoretic perspectives on semantics.[7]

2. Severing denotation conditions from infinity

Infinity is one typical complaint against model theoretic semantics. Indeed, finite systems may not literally house and manipulate an infinite set of infinite models, but this need not be a knockdown argument against the model theoretic approach. Infinity is necessary to capture the uncertainty as to what model, and what part of that model, we are talking about, but capturing this uncertainty need not belong to the object language. At the object language level infinity might be traded for partiality, retaining the strategy of assigning denotation conditions to expressions. Barwise and Perry 1983, Muskens 1995, and Kamp and Reyle 1993, 1996 come to mind. Denotation conditions, not infinity, are at the heart of the linguist's attraction to model theory; the two could be disentangled.[8]

A very interesting proposal of this sort is van Lambalgen and Hamm 2005. Here an event calculus is combined with minimal models in which events that the scenario of the given activity does not require to occur are assumed not to occur and enlargement of the model leads to nonmonotonic progression. What makes the proposal especially interesting is the fact that it puts to linguistic use a program for semantics where the sense of an expression is the algorithm that allows one to compute the denotation of the expression (Moschovakis 1994, 2006). Van Lambalgen and Hamm submit that only a computational notion of meaning is compatible with the results of psycholinguistics, but (drawing from Kamp's and Steedman's work on tense) the representations their theory computes are not alien to the denotational semantic intuition linguists have found insightful to work with.

In this connection we may also mention that the extensive literature on the

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[7] Proof theoretic approaches to natural language using categorial type logics have a rich tradition (Moortgat 1997, 2002; Oehrle 2003). But as far as I can see, the main focus has been on the syntax of natural languages. The central interpretive concern is limited to the Curry-Howard correspondence between formulae and lambda terms, i.e. to the interpretive effects of syntactic assembly. Aspects of meaning that go beyond type specification are not studied with any systematicity. Thus the actual results so far bear only on some of the issues formal semanticists tend to be interested in. Bernardi 2002, Bernardi and Szabolcsi 2005 are among the exceptions.

[8] Chomsky has often declared that semantics has no place within grammar. One might read this as a rejection of infinite models and as a commitment to proof theory, but most likely what Chomsky strives to reject is incorporating (links to) the real world, i.e. semantics in the philosopher's sense. If so, then what formal semanticists do with abstract models may be fine with him and he would regard it as part of syntax. So, the mentalistic view of language may entail an answer to the model theory vs. proof theory dilemma, but it is not my impression that Chomsky's actual statements are about this issue.

psychology of reasoning features two main approaches: mental models and mental logic. This may sound like a distinction corresponding to model theoretic versus proof theoretic semantics, but Johnson-Laird’s mental models are equally about building representations. Bonatti 1994 offers a good comparative evaluation.

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Approach 1
Keep denotation conditions, which we love, and get rid of (too much) infinity.

Message: It's not all or none. It is possible

- to cut down on infinity even while using model theoretic (MT) methods, and
- to construct "snapshot" representations even with proof theoretic (PT) methods.

MT and partiality:

Barwise and Perry 1983

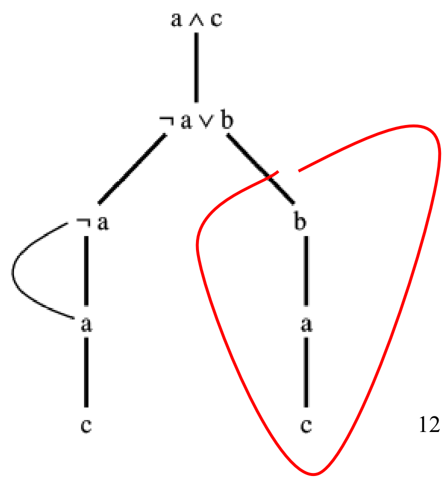
Situations are partial worlds (e.g. that part of the world that you are perceiving)

Muskens 1995

From Montague's IL to Gallin's TY_2 to Muskens's TT_2: type theory to capture the intuition that partial models are situation-like entities that can be ordered by growth of information

PT and “snapshots”:

A calculus can show how to construct crucial (counter-) examples for satisfiability. E.g. the **tableau calculus**: {a∧c, ¬a∨b} is satisfiable iff each of a, b, and c is true.



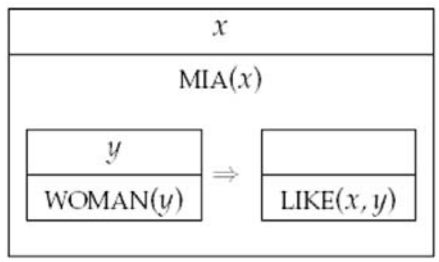
(If you hear “calculus”,
don’t immediately think “sequent”…)

PT and partiality:
Kamp & Reyle 1993, 1996

Discourse representation structures between syntax proper and model theory.

A DRS is true if it can be embedded in the real world:

Mia likes every woman.



PT and partiality:
van Lambalgen & Hamm 2005

An event calculus combined with minimal models;

events that the scenario of the given activity does not require to occur are assumed not to occur;

enlargement of the model leads to non-monotonic progression (unlike in DRT).

Theories of reasoning in psychology

recall guise (B): inferences

Attempt to explain human performance;
Hypothesize that humans
use mental logical schemas or
build mental models;
Assume that difficulty is measured by the number of steps involved in the deduction.

Is mental logic to mental models as proof theory is to model theory?

Mental Logic: Braine, O'Brian, Rips,…

Basic inference schemas, with experimentally estimated difficulty rating and % of errors in one-step problems involving just this schema [Braine et al.]

$$\frac{p_1 \text{ OR } \ldots \text{ OR } p_n \,;\ \sim p_i}{p_1 \text{ OR } \ldots \text{ OR } p_{i-1} \text{ OR } p_{i+1} \ldots \text{ OR } p_n} \qquad (1.38;\ 2.5\%)$$

Mental models: Johnson-Laird, Byrne, …

Reasoners construct a set of models of the premises that make explicit the minimum amount of information; they formulate a parsimonious conclusion based on this set; to test for validity, they search for counterexamples, perhaps fleshing out the initial models.

There is a circle and there is a triangle:

O Δ (one model, implicit)

There is a circle or there is a triangle:

O (two

Δ models, implicit)

There is a circle or there is a triangle, but not both.

[O] (exhaustivized

[Δ] models)

First model explicitly fleshed out: [O] [$\neg\Delta$]

Mental logic is clearly proof theoretic, but so are mental models (compare the tableau calculus and DRT).

Caution against over-optimism:

What if a sentence does NOT have finite models?

$\forall x\, \exists y\, Rxy$ & $\forall x \forall y\, \neg(Rxy\ \&\ Ryx)$ & $\forall x \forall y \forall z\, ((Rxy\ \&\ Ryz) \rightarrow Rxz)$

(Boolos—Burgess—Jeffrey 2002)

Every number has a predecessor.

There are infinitely many stars in at least one circle. (Altmann—Peterzil—Winter 2005)

3. Meaning via proofs

A second approach rejects the notion that inferences should play second fiddle to denotations. As Kahle and Schröder-Heister (2006) put it, "Proof-theoretic semantics proceeds the other way round, assigning proofs or deductions an autonomous semantic role from the very onset, rather than explaining this role in terms of truth transmission. In proof-theoretic semantics, proofs are not merely treated as syntactic objects as in Hilbert's formalist philosophy of mathematics, but as entities in terms of which meaning and logical consequence can be explained." See Prawitz (2006) on Gentzen, Dummett, and his own views. In a similar spirit, Moss (2005) wonders, "If one is seriously interested in entailment, why not study it axiomatically instead of building models? In particular, if one has a complete proof system, why not declare it to *be* the semantics? Indeed, why should semantics be founded on model theory rather than proof theory?"

Given the absence of pertinent literature, I am not in a position to judge how a semantics founded on proof theory would fare for natural language. In addition to a possibly major conceptual shift, I suspect that it may involve shifts in the detailed intuitions captured. To use a simple example, consider the model theoretic and the natural deduction treatments of the propositional connectives. The two ways of explicating conjunction and disjunction amount to the same thing indeed: if you know the one you can immediately guess the other. Not so with classical negation. The model theoretic definition is in one step: $\neg p$ is true if and only if p is not true. In contrast, natural deduction obtains the same result in three steps. First, elimination and introduction rules for $\neg$ yield a notion of negation as in minimal logic. Then the rule Ex Falso Sequitur Quodlibet is added to obtain intuitionistic negation, and finally Double Negation Cancellation to obtain classical negation. While it may be a matter of debate which explication is more insightful, it seems clear that the two are intuitively not the same, even though eventually they deliver the same result. See Hintikka 2002 for the possible linguistic relevance of this.

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Gentzen 1934

“The introductions constitute, as it were, the `definitions’ of the symbols concerned, and the eliminations are, in the final analysis, only consequences of this, which may be expressed something like this: At the elimination of a symbol, the formula with whose outermost symbol we are dealing may be used only `in respect of what it means according to the introduction of that symbol’.”

Prawitz 2006 on Gentzen, Dummett, own work:

Introductions represent canonical ways of inferring the sentence. Other ways have to be justified (shown to be valid) by reduction to canonical ones.

The task is to develop an appropriate notion of validity and show that certain legitimate forms of reasoning are valid in the sense defined. Not just for Gentzen’s eliminations but for any non-canonical inference.

Prawitz, cont'd

An argument whose skeleton (tree of formulas) is closed (all occurrences of assumptions and of variables are bound) and is in the canonical form (its last step is an introduction) is valid provided its immediate subarguments are valid.

A closed argument not in the canonical form is valid iff it reduces to a valid argument in the canonical form.

An open argument is valid iff all its appropriate instances are valid.

Prawitz, cont'd

Gentzen's suggestion that the meanings of logical constants are given by introductions is a special case of the idea that the meaning of a sentence is given by what counts as a direct verification of it.

Most serious counterexamples to the verificationist project concern universal sentences in empirical discourse. Distinguish direct vs indirect verification, assertion vs inferential use.

Moschovakis 1994, 2006: the sense of an expression is the algorithm that allows one to compute the denotation of the expression

Muskens 2005: three levels:
propositions (algorithms) $\Rightarrow$
sets of possible worlds $\Rightarrow$
denotations (truth values)

Applications (Muskens 2005):

Propositional attitudes: identifying "sense" with algorithm makes more distinctions than intensions, but possibly fewer than intensional isomorphism

Potentially captures the Liar paradox (non-terminating program)

4. Natural Logic and semantically flavored syntactic features

A third relevant approach, Natural Logic, bears out the slogan that proof theory "syntacticizes semantics", not only in the sense that it manipulates representations, but also in the sense that it lives off of the actual syntactic representations of expressions. It uses linguistic structures,[9] as opposed to models or an auxiliary logical language, as the vehicle of inference. The literature contains a collection of small subsystems that are individually sound and complete in terms of the standard models. The techniques are fairly diverse. Johan van Benthem's Monotonicity Calculus, explored further by Victor Sánchez-Valencia, tags all items for monotonicity and for polarity position, and computes the increasing/decreasing inferential status of any expression in tandem with the categorial grammatical derivation. Larry Moss presents a syllogistic logic with quantifiers, notably including *most*, which is not first order definable. Yoad Winter handles inferences with restrictive modification, monotonicity, and quantifier scope, exploiting insights from generalized quantifier theory.

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[9] That is, strings together with their syntactic and possibly intonational analyses. This is a straightforward response to the "misleading form" objection to Natural Logic.

Monotonicity calculus (quotes from Bernardi 2002)

Monotonicity is a semantic property of functions, passed to argument positions while building a formula. Polarity is a syntactic notion that can be computed for all positions in a formula. The Curry-Howard correspondence makes it possible to associate CTL derivations with polarity on lambda terms and thus their monotone positions.

DEFINITION 4.9. [Polarity of Occurrences] Given a lambda term N and a subterm M of N. A specified occurrence of M in N, is called *positive* (*negative*) according to the following clauses:

i. M is positive in M.
ii. M is positive (negative) in PQ if M is positive (negative) in P.
iii. M is positive (negative) in PQ if M is positive (negative) in Q, and P denotes an upward monotone function.
iv. M is negative (positive) in PQ if M is positive (negative) in Q, and P denotes a downward monotone function.
v. M is positive (negative) in $\lambda X.P$ if M is positive (negative) in P and $X \notin FV(M)$.

PROPOSITION 4.10. If M_b is positive (resp. negative) in N_a, then N_a is upward (resp. downward) monotone in M_b.

example

(a) Monotonicity markers

$$\dfrac{\underset{+}{s/s^-} \qquad \dfrac{\dfrac{\underset{+}{(s/(np\backslash s)^+)/n^-} \qquad \underset{-}{n}}{\underset{+}{s/(np\backslash s)^+}} \qquad \underset{+}{np^+\backslash s}}{\underset{-}{s}}}{\underset{+}{s}}$$

(b) Polarity markers

$$\dfrac{\underset{+}{s/s} \qquad \dfrac{\dfrac{\underset{-}{(s/(np\backslash s))/n} \qquad \underset{+}{n}}{\underset{-}{s/(np\backslash s)}} \qquad \underset{-}{np\backslash s}}{\underset{-}{s}}}{\underset{+}{s}}$$

inferences. For instance, the inference (A3) in Example 4.5 can be derived by replacing 'good_logician' with the more general term 'logician'. We use the thick inference line to distinguish this new inference from the logical and structural rules of the logical system.

$$\dfrac{(\text{not}^+((\text{every}^-\text{good\_logician}^+)^-\text{wanders}^-)^-)^+ \vdash s}{(\text{not}^+((\text{every}^-\text{logician}^+)^-\text{wanders}^-)^-)^+ \vdash s}$$

Note that the information regarding the partial order holding among the expression involved in the substitution is still based on formal semantics and it is computed on the corresponding lambda terms. See [FWF00] for a natural logic where the order relation is computed within the system.

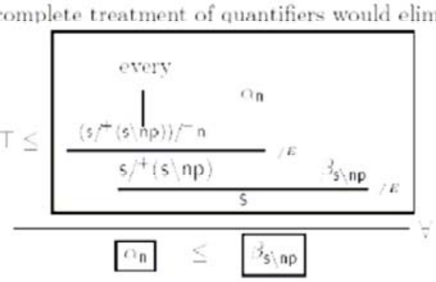
Fyodorov—Winter—Francez 2003

4.3 Lexical/extra-logical order statements

We postulate the following order statements between derivation trees of simple expressions (for simplicity, only the expressions are given, without the corresponding derivation trees in the grammar):

- *exactly* = *at least and at most*
- *two* $\leq$ *three* $\leq$ *four*
- *student* $\leq$ *person*, *teacher* $\leq$ *person*, *boys* $\leq$ *people*, *girls* $\leq$ *people*
- *ran* $\leq$ *moved*, *walked* $\leq$ *moved*
- *hugged* $\leq$ *touched*, *kissed* $\leq$ *touched*

Another *ad hoc* order relation is derived by the presence of the determiner *every* and is given in Figure 1. This rule is used to enrich the system with more order relations that are derived from assumptions in natural language (in the form '*every x y*'). Of course, a more complete treatment of quantifiers would eliminate this rule.



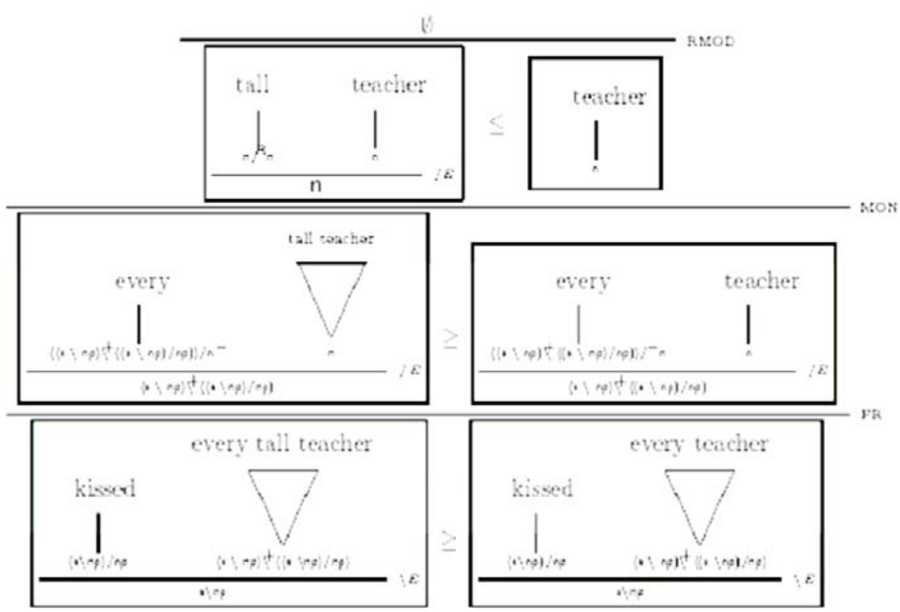


FIG. 2. kissed every teacher $\leq$ kissed every tall teacher

Moss, Syllogistic Logic (ESSLLI 2007)

Syntax: We start with *variables* X, Y, …, representing plural common nouns of English. We also also *names* J, M, …. Then we consider sentences S of the following very restricted forms:
All X *are* X, *Some* X *are* X, *No* X *are* X, J *is an* X, J *is* M.

Semantics: One starts with a set $\mathcal{M}$, a subset $[\![X]\!] \subseteq \mathcal{M}$ for each variable X, and an element $[\![J]\!] \in \mathcal{M}$ for each name J. This gives a *model* $\mathcal{M} = (\mathcal{M}, [\![\ ]\!])$.
We then assign a semantics $[\![S]\!] \in \{\mathsf{true}, \mathsf{false}\}$ to the sentence S in a model $\mathcal{M}$, as follows:

$$\begin{array}{lll}
[\![\text{All } X \text{ are } Y]\!] = \mathsf{true} & \text{iff} & [\![X]\!] \subseteq [\![Y]\!] \\
[\![\text{Some } X \text{ are } Y]\!] = \mathsf{true} & \text{iff} & [\![X]\!] \cap [\![Y]\!] \neq \emptyset \\
[\![\text{No } X \text{ are } Y]\!] = \mathsf{true} & \text{iff} & [\![X]\!] \cap [\![Y]\!] = \emptyset \\
[\![J \text{ is an } X]\!] = \mathsf{true} & \text{iff} & [\![J]\!] \in [\![X]\!] \\
[\![J \text{ is } M]\!] = \mathsf{true} & \text{iff} & [\![J]\!] = [\![M]\!]
\end{array}$$

We write $\mathcal{M} \models S$ if $[\![S]\!] = \mathsf{true}$. And if Γ is a set of sentences, then we write $\mathcal{M} \models \Gamma$ to mean that $\mathcal{M} \models S$ for all $S \in \Gamma$.

Main semantic definition: $\Gamma \models S$ means that every model which makes all sentences in the set Γ true also makes S true. We say Γ *semantically implies* S.

Moss, cont'd.

$$\frac{}{\textit{All } X \textit{ are } X} \qquad \frac{\textit{All } X \textit{ are } Z \quad \textit{All } Z \textit{ are } Y}{\textit{All } X \textit{ are } Y}$$

$$\frac{\textit{Some } X \textit{ are } Y}{\textit{Some } X \textit{ are } X} \qquad \frac{\textit{All } Y \textit{ are } Z \quad \textit{Some } X \textit{ are } Y}{\textit{Some } Z \textit{ are } X}$$

$$\frac{}{J \textit{ is } J} \qquad \frac{J \textit{ is } M \quad M \textit{ is } F}{F \textit{ is } J}$$

$$\frac{J \textit{ is an } X \quad J \textit{ is a } Y}{\textit{Some } X \textit{ are } Y} \qquad \frac{\textit{All } X \textit{ are } Y \quad J \textit{ is an } X}{J \textit{ is a } Y}$$

$$\frac{M \textit{ is an } X \quad J \textit{ is } M}{J \textit{ is an } X} \qquad \frac{\textit{All } X \textit{ are } Z \quad \textit{No } Z \textit{ are } Y}{\textit{No } Y \textit{ are } X}$$

$$\frac{\textit{No } X \textit{ are } X}{\textit{No } X \textit{ are } Y} \qquad \frac{\textit{No } X \textit{ are } X}{\textit{All } X \textit{ are } Y}$$

$$\frac{\textit{Some } X \textit{ are } Y \quad \textit{No } X \textit{ are } Y}{S}$$

Figure 7: A complete set of rules for $\mathcal{L}(\textit{all}, \textit{some}, \textit{no}, \text{names})$.

If proof theory syntacticizes semantics, it may be of particular interest to pay attention to semantic properties that natural language already singles out as syntactically relevant. I will dub these semantically flavored syntactic features. Some fairly standard examples are [wh] (i.e. interrogative), [topic], [focus], [negative], [agent], [number], [telic], [evidential], and so on. It turns out that such features are quite pervasive, and generative syntax uses them as conditions for syntactic operations ("merge" and "move"). The so-called negative inversion construction of English is an example.

(1) *Under no/few circumstances would a whale fly.*
(2) **Under some/most circumstances would a whale fly.*

(1) is acceptable, (2) is not, although it does not seem incoherent. The generalization is that the initial position accepts an adjunct only if it is (roughly) decreasing.[10] One way to implement this is to assume that whenever the decision to fill this position arises in a syntactic derivation, the compositional model theoretic interpretation of the adjunct is inspected for decreasingness. This is what semanticists would do by default. Another implementation is to assume that certain adjuncts have a purely syntactic feature [de]; the set of expressions with [de] may substantially overlap or even coincide with those whose denotations are decreasing, but this fact has no place in the theory. This is what syntacticians would do by default. In contrast, Stabler (1997) proposes that [de] is a properly syntactic feature but, in addition to licensing syntactic operations like negative inversion, it features in the proof theoretic component and speakers use it to draw inferences. Natural language syntax can be sensitive to semantic properties precisely because its semantics is proof theoretical in nature, and those particular properties play a role in this proof theory. If this view is correct, we may say that syntax is a "window on the format of semantics". (This formulation is somewhat stronger than Stabler's but in keeping with Stabler's views and intentions.)

A more conservative view maybe a hybrid one. Here natural language would have, in addition to a full model theoretic semantics, a partial proof theoretic one, which provides shortcuts in the cases of some shallow semantic features. For example, Geurts and van der Slik (2005) observe that monotonicity properties are shallow. Even though speakers often disagree about the precise truth conditions of donkey-sentences (is (3) true or false if farmers have more than one donkey each and do not treat all their donkeys alike?), they are quick to recognize that (3) entails

[10] This seems to be the same property as the one involved in the licensing of negative polarity items like *ever*. Given $A \leq B$, a function f is monotonically increasing (upward entailing) iff $f(A) \leq f(B)$, and decreasing (downward entailing) iff $f(A) \geq f(B)$. See Ladusaw 1980, von Fintel 1999.

(4) and is entailed by (5):

(3) *Every farmer who owns a donkey beats it.*
(4) *Every farmer who owns a male donkey beats it.*
(5) *Every farmer who owns a donkey beats it with a stick.*

This indicates that monotonicity inferences do not mobilize the whole model theoretic semantics of the sentences. We may now hypothesize that semantically flavored syntactic features are shallow ones. This would preserve Stabler's insight without committing us to handle all complexities of linguistic meaning in a proof theoretic fashion.

5. Some general questions

So, some general questions arise, for the global approaches as well as for the particular variants.

Proponents of proof theoretic methods seem confident that only their approach, not the model theoretic one, can be integrated with the rest of cognitive science. Is that correct? If yes, what is the crux of the matter -- finite representations or inferential character?[11] Is cognitive science possible without relating to the world outside?

Do model theoretic and proof theoretic semantics differ as to what general conception of language they fit with? Would there be gains or losses in domains not considered above?

What are the prospects of extending the proof theoretic approach to intensional phenomena, presupposition, and implicatures?

What kind of compositionality would proof theoretic approaches afford? Although it has sometimes been suggested that any effective procedure that computes meanings will do, I believe that there is an important consideration that suggests that we must be more particular. Whatever one might think of the specific theories generative grammar has come up with over the past decades, I believe it has been demonstrated beyond any reasonable doubt that natural languages, while superficially wildly different, exhibit very detailed and thoroughgoing structural similarities; in other words, that "universal grammar" is not merely a wishful thought. Therefore no theory incapable of accounting for the unity behind the superficial variation stands a chance to be an even remotely valid theory of natural language. Now, cross-linguistic variation in syntax is to some extent parallelled by cross-linguistic variation in interpretation. Here are two simple examples.

[11] Referring to mathematical results that the set of first order quantifiers is not identifiable in the limit from examples, Stabler 2005 points out that they might be learnable given inferential evidence.

(i) Given the right predicate, bare plurals in English and German may have an existential or a generic reading:

(6) *Professors are sick*.
`There are professors stricken with illness'
`Professors in general are disgusting'

But it is well-known that in many other languages, Romance languages among them, one or both interpretations may be unavailable.

(ii) The interaction of negation with disjunction and conjunction in English and German straightforwardly bears out the de Morgan laws:

(7) *John didn't study flute or accounting.*
primary reading: `neither'
(8) *John didn't study flute and accounting.*
primary reading: `not both'

In many other languages, Russian, Italian, Japanese, and Hungarian among them, the above interpretations are missing. The literal counterparts of (7) mean exclusively `One or the other he didn't study' and the literal counterparts of (8) mean exclusively `He studied neither one'.

Given such variation, it does not suffice to provide some effective procedure that delivers the correct interpretations for the constructions of the individual languages; what is needed is a compositional analysis that accounts for exactly how languages differ.[12] Without that, human languages will appear to be incommensurable.

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[12] Chierchia 1998 and Szabolcsi and Haddican 2004 are such attempts.

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