Iter Italicum and Leibniz/Giordano correspondence

Francesco Tampoia

Abstract

Letters exchanged by scientists are a crucial source by which to trace the process that accompanies their scientific evolution. In this paper -accomplished through a historical approach- I aim to throw new light on Leibniz's continuing interest in classical geometry and to stress the significance of his correspondence with the Italian mathematician Vitale Giordano.

 Kew words: G.W. Leibniz; V. Giordano; the straight line.

Percurri nonnulla *Euclidis* Tui *restituti*, et magna cum voluptate vidi multa a Te feliciter suppleri[.] Nec cum iis facio qui rigorosas demonstrationes contemnunt Etsi libens agnoscam viris magnis qui quaedam notiora tanquam concessa admisere ut ad majora progrederentur esse ignoscendum[.] Interim laudandam est posterorum praetermissa supplentium industria” G. W. Leibniz.

We cannot think of a line without *drawing* it in thought, we cannot think of a circle without *describing* it, we cannot represent the three dimensions of space at all without *placing* three lines perpendicular to each other at the same point, and we cannot even represent time without ... *drawing* a straight line. Kant, *Critique of Pure Reason.*

## *1)Introduction*

## Towards the end of the 18th, dealing with Leibniz in his youthful book *Ideas for a Philosophy of Nature* F. W. J. Shelling wrote the following judgement: “His mind despised the fetters of the schools; small wonder that he survived among us only in a few kindred spirits and among the rest has long become a stranger. He belonged to the few who treat science as a free activity. He had in himself the universal spirit of the world, which reveals itself in the most manifold forms, and, where it enters, life expands”[Wilson,1995, 470]. A century later echoing the same words, W. Dilthey defined G. Leibniz as the most universal spirit modernity had produced up to Goethe. And more than two decades ago, in the intriguing book *Le pli* (1988), the philosopher G. Deleuze called G. Leibniz a baroque spirit for his personality, his manifold mindsets and his way of thinking, in his dual role as philosopher and scientist. Perhaps the best way to understand these judgements and to offer an idea of the multifaceted scientific and philosophic learning of Leibniz, as well as his intense activity, is that of reviewing his book of *monad-letters*, the rich correspondence he enjoyed throughout his life with scholars, scientists, politicians and sovereigns. It has been calculated that, from the age of twenty onwards, Leibniz wrote almost a letter a day, of which we have nearly fifteen thousands.

During the 1680s Leibniz has reached the pinnacle of European science and philosophy. In collaboration with Otto Mencke, in 1682, he founded the *Acta Eruditorum*, where in the year 1684 he published the famous *Memory* on infinitesimal calculus. Meanwhile he corresponded with the most important philosophers and scientists on the Continent. Between ‘1672 and ’76, he journeyed through Europe via France, and in 1687, he begins a second journey through South Germany, Austria and Italy.

 The famous *Iter italicum* brought him to Rome on 14th of April 1689. He remained there for more than six-seven months, where he had the opportunity to frequent the *Academia fisicomatematica* of D. Ciampini. Furthermore, he developed close relationships with academics. In his well-known book, *Iter Italicum (Mars 1686- Mars 1690) La dynamique de la République des Lettres Nombreux textes inédites*, A. Robinet writes that these encounters, although short, were nevertheless fruitful as were the Parisian ones Leibniz established with the members of Académie des Sciences. Ciampini, the founder of Academia fisico-matematica, would be no less than the Roman Huygens. With some reservations I agree with him, although I do not see Leibniz as a tourist, I am sure Leibniz admired Rome, particularly Piazza Navona and the Vatican, where he had frequent contacts with scientists, intellectuals, philosophers and clergymen. He discussed with them on humanistic learning and scholarship: humanities, history, science, and religion. Some of them became his new correspondents [Robinet, 1988, Introduction]. Yet, within the voluminous Leibnitian correspondence only a few letters about mathematics are with Italian correspondents. Among these exist three letters with Vitale Giordano.

In those years Giordano is at the zenith of Italian mathematics. Owing to his academic position and prestige, he is regarded, along with Vivian and Guglielmini, as one of the best-known Italian mathematicians. The encounter between Giordano and Leibniz happened within the environs of the Academia Ciampiniana: on that occasion Giordano offered Leibniz a copy of his *Euclide restituto*. To achieve a precise chronological reconstruction of their relation, during Leibniz’s Roman sojourn, we must begin from October 1689. Leibniz met Giordano on 25 of October and asked him for a letter of introduction to Vivian, which Giordano wrote in Italian as reported in full below (English):

*‘Letter of Mr. Vitale Giordani to Mr. Viviani.* Here is in your presence German Mr. Leibniz, a fully literate person, who as Your Lordship will find out, before returning to his native land, is very eager to know Italian men of letters and particularly the person of Your Lordship. I cannot find wanting both his noble desire, and the singular merit of Your Lordship. I am introducing him to you fully aware that two great men will enjoy such a pleasant occasion. While I am pleased to accept your commands, I always confirm myself obedient servant of Your Lordship. Vitale Giordani Rome on 25 October 1689.To Most Illustrious and very learned Lord Mr. Vincenzio Viviani in Florence’ [ Robinet, 1988, 228, my translation].

On his way back to Germany, Leibniz visited Vivian in Florence on 27 of November 1690.

*2)The correspondence Leibniz/Giordano*

From the very first exchanges of letters between the two scholars, a contemporary reader is confronted with two personalities, shaped by training and culture of different shades: on the one hand Leibniz, born in Leipzig, a singularly attractive and well-mannered man, a refined universal spirit, forged in the academic environment of German high society. On the other hand Giordano, born in Bitonto (Italy), to poor parents, lived a very difficult, albeit adventurous and eventful life. What draws them together is a deep interest in geometry, in the very foundations of elementary geometry and in the growth of mathematics. (\*)

 The first letter from Leibniz is datable between the end of October and the beginning of November 1689. The second, Giordano’s reply, dates to 11th November, and Leibniz’ answer in middle of that same month.

 Let us read the letters.

 216. Leibniz an Vitale Giordani

[ Rom. Ende Oktober-Anfang November 1689]

English translation:

Gottfried Wilhelm Leibniz to Vitale Giordano

I reviewed much of your *Euclide restituto* and with pleasure I see that you have carefully restored it in many points. I would not say that with those who despise the rigorous demonstrations, though we have to forgive the great men who admitted to having lost a few well known things to better develop. On the other hand, we have to praise posterity’s commitments to complete what is lacking.

Firstly, I see you are well learned regarding the parallels and the composition of proportions. In Belgium a certain Nonancourt wrote a booklet on the proportions, I remember that I have seen, of which Arnauld-celebrated nearby Theologians, but also excellent in all kinds of learning- praises and follows the method in the second edition of *Liber Gallicus*, entitled *Nova Geometria Elementa*. Both develop the proportions by means of the fraction in which the numerator is the antecedent, the denominator the consequent, but it seemed to me there was something lacking in the utmost rigor of the demonstration. Working on the proportions, truly, the fraction appears a more suitable means than the same ratio. I would like to know your opinion about, as it comes out in the *V Book of Elements* by Galileo, posthumous published by Viviani.

In reference to the demonstrations taken from others in your Euclide, I do not find any difficulty. In fact, in Thales’ demonstration at p. 21, that the straight line passing by the centre bisects the circle in two parts, only one case allows an exception: if one said that a segment ABC shifted to another place, part in and part out, intersects another segment ADC. The same happens in the demonstration of axioms at p. 22 and p. 23, where two straight lines do not have any common part, nor contain any space. It follows that two points G, F at p. 22 and two points E, F at p. 23, in which two straight lines, not coincident between them, are bisected by the circle. What needed to demonstrate. And, supposed that in the demonstration of posterior axioms Clavius would replace this principle, he fell in the same error, imagining a new circle drawn from the centre D on the straight line ACO, secant the straight line, and not coincident on the points E and F. But, in the universe of their axioms on the demonstration of the straight line I find the same difficulty, because the definition of the straight line is not present; nor any property of the straight line is contained in an axiomatic way. The definition of the straight line, accepted by you, you correctly use for the property of the parallels, is that the straight line is the shortest between two points; but here you neither point it out, nor do you suppose another axiom of the straight line. In all of these demonstrations any line could be considered a straight line, and this would be wrong. Indeed, it seems to me something lacks in these demonstrations. And the demonstration will be hardly solved, unless we do not assume the notion of a straight line to which I am accustomed; that is, given any body rolled through two motionless points, the locus of all fixed points is a straight line, or at least the straight line is the line that cuts the endless plane in two equal parts, in which the plane is the surface secant the endless solid in two equal parts.

To Vitale Giordano

 217. Vitale Giordani AN Leibniz

[ Rom. Ende Oktober-Anfang November 1689]

English translation:

V. Giordano to Gottfried Wilhelm Leibniz

I decided to come and see you when a new commitment prevented my advice. I thank you very much for your honourable judgement on my *de momentis Dissertatione* and *Euclide restituto*. There is only one point that must make in regard to your very learned remarks, not to defend myself, but to explain to you the reasons for which I thought there were well-grounded motives for my engagement to restore Euclid.

First, I want to recall that my chief purpose has been to give Elements the best clarity by means of some adjustments in favour of Tyron who- if wrong figures get entangled in the hall- at once come confused and loose the spirit. In reference to Thales’ demonstration I exposed at p. 21, I left as his Author made it, without adding the third case, because the third case is similar. And also because in Theoremata it is the first negative demonstration, nor is the Tyron used to conceiving of the semicircle as a very different figure, as AFC, that could be easily cause of confusion. What happens not at all after, and much less in what follows. He [ Tyron] is used to conceive the negative demonstration in an easily constructed figure, and does not create any difficulty in more complex others, as at p. 123, where nothing forbids to add any case.

With regard to Proclus’ demonstration at p. 22, I do not see clearly where there is difficulty. If we suppose that the straight lines AD, CD distinct each other and only concurrent in D, I do not know how we may conceive of them as concurrent in G and F; at most, as continued to A and C, they may concur with the part AC. If we put DB minor than DA and DC, the circumference will intersects the straight lines DA, DC in G and F; if we put in a minor point of the straight lines DA, DC any point G or F, with centre in D, and interval DG or DF, we will draw the circle EGH, whose circumference will intersects DB, continued in the oblique point B; what is the same as before.

Nor less unknown is for me the difficulty at p. 23, where the straight lines BAD and BCD continued, or concur with the circumference in a point K or cut the circumference in two points. In fact, if they concurred before arriving at the circumference, they would go on, either they concur with the circumference in a point or they cut the circumference in two points. If they cut the circumference in the two points then the Proclus’ demonstration is excellent: if they concur with circumference in a point, as K, then they put any point D on the straight line BC, so that BD is major than DO with a centre in D and interval DB. In such a way, we will construct the circle BGE whose circumference will cut the straight lines OHK, OFK in E and F. And, this way the Clavius’ demonstration correctly concludes. But, all these complications are not timely; indeed, they would cause much confusion in Thyrons’ minds, thanks whom it seems to me not proceeding to other cases.

Now I proceed to the definition of a straight line. I judge excellent that one of Euclid – the straight line is the very that lies equally as to its points- whose sense seems to me that the straight line is the one that extends between its extremes. But, I am used to the Heron’ definition for the simple reason that it seems to me more suitable to the Thyrons’ understanding; and well distinct from the other lines when he says, -The line that is not the shortest between two points, I call curve. Certainly, any line we take for a straight line, or it will be the shortest interval between the extremes, or will not: if will be the shortest interval then it will be a straight line, if it will not be the shortest interval, then it will be curve.

Your twofold definition certainly is very ingenious, but also subject to the same exceptions, the biggest of which seems to me that it supposes as known either body or plane, what is (as people say) to put the cart before the horse. In the same way D. Borrelli was wrong in his Euclide restituto, in which imagining the body as know, from this notion inferred the notion of surface, of line and of point, and then in *V Book* he went on to define the body. That is unlike a person of Geometry. Another exception is that the line that cuts the plane in two equal points may could be curved and tortuous.

The one and the other definitions, nevertheless, seem so hard that they can be scarcely understood by the most experts, even less by candidates to Geometry. In my *Archimede* I defines the straight line so -*the line rolled round to its motionless extremes, whose parts always maintain the same place of before, I call straight line*- but I acknowledge that I will give up that definition, I will cross it out and replace it with the Euclidean one that I think is the best (this explains it the precision).

I send you my little book entitled *Fundamentum doctrinae motus Gravium*. It is lacking the reply to some objections because the text was in print. I will give it to D. Ciampini who will hand it over to you. If possible, I would like to wait for your judgement on this paper, what has much value to me.

Finally, I beg that you will please me (with more) of your letters and will continue to love me.

Rome Third Ides of November 1689

218. Leibniz an Vitale Giordani

[ Rom. Ende Oktober-Anfang November 1689]

English: translation

Leibniz to Vitale Giordano

According to a certain way of thinking: the body is before the surface, and the surface before the line. In fact, the extremes of the body are not per se; they are certainly derived from the section of the body. There is no easier method to prove it than to cut an endless body- whose extremes are not considered so that we have a common extreme- cutting at the same way either the one or the other, or the equal convergent parts. And then we have the plane. Again, if a plane is cut in the same way we have the straight line. Mark that seems to merit a certain praise by ingenuous persons for its novelty and simplicity.

I do not believe that something about a straight line can be proved without using the definition of a straight line or other property. It is impossible to know what belongs to the line described by the demonstrations taken from Proclus or Clavius- i.e. the shortest between the extremes-, or to suppose another notion of a straight line. Suppose my reply is that the lines AD, CD (or AO, CO) in D (or in O) only concur. But, that was not to suppose, rather to demonstrate and to show, that is: given points E, F not coincident, the drawn circle with centre in D goes through them. I would believe it is possible to eliminate the fault in Thales’ demonstration without any bias for Tyron’s.

I make these remarks not to be contradictory, but rather- because I know they are very helpful for the perfection of Geometry to ensure that Euclidean axioms on the type of the straight line are precisely demonstrated.

I thank you very much, most learned man, for the new gift, which will be a pleasant reading during my journey. I should have liked to do personally what I am doing by sheet of paper if I were no busier than to come easily to you. Moreover, I am not a man who would his demonstrations are collected as tripod, and overall I love the simplicity of those who acknowledge to have usefully been corrected.

And, while you object to my definition of the straight line, I recognize something worthy to be considered~~:~~ if someone is wrong because he supposes the notion of a plane and a solid, or more if he merits praise for this. I will let you examine that more carefully before we together say to put the cart before the horse. In fact, there will be some who will hold the notion of a body prior to the notion of surface and a line, in the same way of extremes of a body, not subsistent per se. You know this by a section of a body. Since the beginning I take as endless whether the extremes are not considered or the same section gives the extremes. But, the first and very simple section of a body lies in the congruent parts per se, cutting the one and the other part of that section in the same way; and this is the plane. According to this view, we have a very simple origin of the plane and of the straight line that surely would encourage some praise of ingenuous men. Nor do I censure other ways (any and mine) in order to have equal clarity that until this point we have not known in the works of Euclid. Meanwhile in any notion of a straight line that we use, it is necessary to go into theorems that we have to demonstrate about the straight line. Otherwise, it is not clear if those theorems are up to that object, whose definition has been given.

In those demonstrations of Euclidean axioms you take from Proclus and Clavius- I gave a nod- I also desire the same, though you omitted it in your reply. From them we know what is up to straight line, the very short between its extreme points. Moreover, as suggested in your Euclid that, where allowed, all things must be exactly demonstrated. May be you do not deny that it is possible to restore more precisely the cases called to the perfection of demonstration I believe are not of prejudice to Tyrons. For the aforementioned reasons no one ever wrote a book so well- without any matter of reproach and without losing praise. And, supposed that at p. 23 two straight lines BA, BC concur in D or O, this does not prevent, often, from concurring and that E and F are coincident. One cannot suppose, therefore, as you say, that straight lines concur only in D.

But, I do not wish to keep you for more time, (willing) only to reply that you do not believe I hasten in these rash objections for contradiction. For a long time I desired to know the exact demonstrations of these axioms, because I well know they lead to the perfection of Geometry and so I would put my doubts to you in order to push you, as he who is able to overtake difficulties, to replace those are lacking.

Vale

 *3. The significance of this correspondence to historians and mathematicians.*

Written between decade 1682-1692, these letters ought to be considered alongside the articles published in Acta Eruditorum which had wide circulation in continent. They provide, in my view, additional knowledge of Leibniz, the geometrician, and their discovery encourages further studies on Leibniz’s attitude towards classical geometry.

 During the Early Modern period, the earlier tradition of mathematics was often treated as a burden, but also as an unavoidable point of departure for the study of new runs. Often a classic difficult problem might easily be solved by changing the technique. It is sufficient, for example, to keep in mind that at the beginning of the famous *Nova Methodus* Leibniz gives us a geometrical interpretation of the basis of calculus. Afterwards he changes his approach and translates the spatial relations into algebraic terms. For a while he follows the Cartesian program of analytical geometry, and then (he) deviates to interpret calculus through a different method. Leibniz was always advocating the idea that he was pioneering a new movement for development by his successors. He tenaciously defends his principles; yet he does not give up classical geometry as we see at the close of his third letter to Giordano: ‘For a long time I desired to know the exact demonstrations of these axioms, because I know well they lead to the perfection of Geometry and so I would put my doubts to you in order to push you, as he who is able to overtaken the difficulties, to replace those are lacking.’

 The correspondence with Giordano, in detail and for some aspects, helps us to understand the course of Leibniz’s scientific position/evolution at a precise moment of his life, as he himself significantly voices in the letter to Magliabechi dated 18th August 1692.

*The issue of geometrical exactitude*

Let us focus on some insights, which are useful for understanding the foundations of geometry in accordance with Leibniz and Giordano’s practice.

 The first letter applies to the book by Giordano *Euclide restituto Libri XV*, Roma 1686, and claims to concern itself with defining the concept of straight line. A careful reading, however, between the lines, so to speak, allows us to discover attitudes, opinions, ideological and doctrinal beliefs. Leibniz praises the book *Euclide restituto,* and (he) writes that, although he had not entirely read it, as a scientist he approves of this aim, one worthy of a mathematician, and praises *Elements* for its precision and perfection. In polemic with Descartes, he recognizes his debt toward classical and modern geometry; he is conscious, too, as he states in a letter to Gallois at the end of 1678, that the application of algebra to geometry is not the natural way to solve geometrical problems. He planned to go beyond Descartes, in the direction of a *geometry of situation*, i.e. a linear characteristic [ Knobloch E., 2006, outlines 3-4].

 Given that among Euclid’s definitions, axioms, and other basic objects of geometry there are some ideas which need still to be better defined *extremity, length*, the *exactitude* to which Leibniz refers when he writes in agreement to Giordano is, I think, the cleaning of all psychological and existential passages. Dealing with the straight line, which Euclid defines as a straight line is a line, which lies evenly with respect to its points, a classic definition in Euclidean literature, Leibniz critically comments upon Giordano’s definition, and proposes two others.

 Let us remember that of this argument, Leibniz is a leading mathematician in the world. Among his numerous mathematical writings, one thinks of the essay *Characteristica geometrica* ( 1679) or of the numerous pages of *Mathematische Schriften*. In *Mathematische Schriften*- part IV- Euclidis *ΠPΩTA*, he defines elements, such as point ”Punctum est cujus pars nulla est”, line “Linea est longitudo latitudinis expers”, extremes of line “Lineae Termini sunt puncta”, straight line “Recta Linea est quae ex aequo sua interjacet puncta”, surface “Superficies est quae longitudinem latitudinemque tantum habet” and so on [ Leibniz, 1971,V].

In reply to Leibniz, Giordano apologizes for not having personally thanked him for his qualified judgment on both the paper *de momentis dissertatione* and the *Euclide restituto*. Giordano’s gratitude for the judgment on *de momentis dissertatione* can be better understood if keeping in mind that Leibniz closely followed the controversy started by Jesuit G. F. Vanni against Galileo for the *momenta* of heavy bodies on an inclined plane. In November 1684, on Acta Eruditorum, G. F. Vanni published a brief paper *Specimen libri de momentis gravium Autore I. F. V. Lucensi,* censured by Leibniz as well as by Giordano for its fallacious doctrine [Torrini, 1979, 105-237]

 Back to geometry, it is well known Giordano’s disagreement with his friend A. Borrelli. Giordano appeal to the greatness of Euclidean system, to the very meaning of Elements, *στοιχεĩα*, as collections of propositions upon which the treatise is founded. The Euclidean method functions according to the mathematical reasoning that develops from proposition to proposition, from concept to concept, and from construction to construction when given the first two or three terms. Giordano is conscious of this, tanks his close reading of numerous commentaries (Clavio and others) on the Elements, Proclus’ *Commentary* *on the First Book of Elements of Euclid*, who clearly states “We call ‘elements’ those theorems whose understanding leads to the knowledge of the rest and by which the difficulties in them are resolved. As in written language there are certain primal elements, simple and indivisible, to which we give the name *στοιχεĩα*” [ Proclus, 1992, 72 ].

*The concepts of space and body*

 Leibniz’s third letter, divided into two parts, concludes the exchange of views on the definition of a straight line. The first part, the shortest, composed of five enumerated periods, appears as a letter per se, or better the original draft of the letter, whose concepts are repeated in the second part.

 In the beginning Leibniz focuses on the concepts of surface, body and the extremes of surface in order to deepen some aspects related to geometry “According to a certain way of thinking: a body is before a surface, and a surface before a line. In fact, the extremes of a body are not per se, they are certainly derived from the section of a body. There is no easier method to prove it than to cut an endless body whose extremes are not considered so that we have a common extreme cutting at same way either the one and the other, or the equal convergent parts, and then we have the plane. And ahead, if the plane is cut at the same way we have the straight line”. For historians familiar with Leibniz’s correspondence the passage bear similarity to passages of letters written to Arnauld, letters weighed at length. According to Leibniz such principles must be referred to within a special *dynamical geometry* and obviously to a new physics, very different from Descartes’, in which bodies are just extensions, completely inert and lacking any kind of motion. For Leibniz, of course, body and motion are concepts which allude to the problem of infinitesimals and indicate a sort of activity, i.e. “ the principle of continuity”. The essential elements of this view may be found, among other texts, one in the *Discours de Métafisique* of 1686, as well as the massive *Dynamica de Potentia et legibus Naturae corporeae*, works Leibniz wrote during his journey in Italy.

Let us, here, pay more attention to the concepts of space and body.

 It is well known the Leibnizian attack on the doctrine of absolute and uniform space and time. For Leibniz space is not ‘real’ but ideal: it is an entity of reason, an ideal system of relations between bodies. Put roughly, the "relationist" Leibniz maintains that space and time are simply a space-temporal relationship among existing material bodies; they are orders of existence and, as such, abstract, and they occupy an ontological realm distinct from that of phenomena. On this matter Daniel Garber remands us that Leibniz defends the view that space is not ‘real’ but ideal; it is an entity of reason, an ideal system of relations holding between bodies. Put roughly, the "relationist" Leibniz holds that space and time are simply space-temporal relationship among existing material bodies; they are orders of existence and as such abstract, and occupy an ontological realm distinct from that of phenomena. On this Daniel Garber reminds us that “Leibniz was interested in the notion of space from his earliest writings. Though there is a strong suggestion in some of his earlier writings that he thought of space as something distinct from body, in his mature writings he quite clearly denies the independent reality of space, particularly in opposition to the specific form that that doctrine was given in Newton’s writings” [ Garber, 1995, 302]. I quote, in addition, from the same Garber the Leibniz’s response to Samuel Clarke:

 “The author [i.e., Clarke] contends that space does not depend upon the situation of bodies. I answer: It is true, it does not depend upon such or such a situation of bodies, but it is that order which renders bodies capable of being situated, and by which they have a situation among themselves when they exist together, as time is that order with respect to their successive position. But, if there were no creatures, space and time would only be in the idea of God (Fourth letter to Clarke, par. 41, G VII 376-77: AG 331) [ Garber, 1995, 303 ].

 The extension as space, discussed in section 3.1, just implicates the order of things, and presupposes the existence of things to be ordered, bodies, ultimately substances, which are the only things that are real in the proper sense for Leibniz” [ Garber, 1995,. 303]. More clearly, when he addresses the concept of space Leibniz gives us his definition of space and time writing “space is nothing but the order of the existence of things possible at the same time, while time is the order of existence of things possible successively”. That is, extension and surfaces, lines, and points are only relations of order or orders of coexistence, thus they are derived from co-existent and successive states of substance. Leibniz, in sum, opposes the reduction of extension to something primary or metaphysic, and at the same way he opposes the reduction of quality-senses to quantity, figure and motion in the manner of the mechanicists (see in particular the Letter to Arnauld, 30 April 1687). He maintains that extensionality of bodies is phenomenal; that it is the result of our imperfect senses, which impose geometrical concepts onto bodies which are, in their real nature, quite different and which do not fit them exactly. He marks, put differently, a sharp distinction between the world of mathematical entities (lines, surfaces and numbers)and the world of concrete things. Referring to Leibnizian mathematics for its objects, namely to discuss imaginable things, Robert Mcrae remarks, “There are no actual determinate figures, or circles, or ellipses in nature, but they can be drawn in the imagination if one has their concepts or definitions. To have a reality as opposed to a merely nominal definition is to know the possibility of the thing. In geometry, this possibility can be known in either of two ways: by an analytical definition showing that the concept contains no contradiction or logical incompatibility of terms or by a casual definition showing the method by which the thing can be produced” [ Garber, 1995, 186]. The Leibnizian point of view is that geometrical extension is something that exists outside the world of concrete things.

 Then, what does it mean for a body to be extended?

 In reality, bodies do not have and cannot have the geometrical shapes that we attribute to them; their boundaries are infinitely complex, and cannot be captured by geometry. In opposition to Descartes on the one hand and against Newton on the other, the idea of a thing as a simple geometrical extension and nothing else is unintelligible to Leibniz (absolute or metaphysical space). Leibniz’s view on the nature of body as an extended thing is like diffuse impenetrability, to which geometrical notions apply, at least approximately. Its extension is imaginary and phenomenal to the extent that bodies are made up of infinity of smaller corporeal substances furnished of boundaries of infinite complexity.

 Indeed, they (bodies) do not have the exact geometrical shapes they seem to have.

 With his usual fair play, towards the end of the third letter, following other remarks on the definition of the straight line, Leibniz acknowledges the validity of Giordano’s definition in contrast his second definition, and reminds Giordano of his suggestion and that “whenever possible, all things must be exactly demonstrated”.

The correspondence with Giordano comes to the end with the third letter. The Iter Italicum finishes on Mars 1690, and Leibniz returns to Hanover in mid-July 1690.

#### Latin text of letters

 *Letter 216-* *Gerhardt enumeration*

216. Leibniz an Vitale Giordani

[Rom, Mitte November 1689.]

Gerhardt: G. W. Leibniz, Dritte Reihe- Vierte Band July 1683-Dezember 1690.

Latin text:

 Percurri nonnulla Euclidis Tui restituti, et magna cum voluptate vidi multa a Te feliciter suppleri[.] Nec cum iis facio qui rigorosas demonstrationes contemnunt Etsi libens agnoscam viris magnis qui quaedam notiora tanquam concessa admisere ut ad majora progrederentur esse ignoscendum[.] Interim laudandam est posterorum praetermissa supplentium industria.

 In primis circa parallelas et rationum compositiones video te profunde meditatum. Quidam Nonancurtius in Belgio libellum de rationibus scripsit, quem me videre memini, hujus methodum laudat et secutus est Arnaldus(celebris apud Theologos, sed idem in omni doctrinarum genere excellens) in secunda editione libri Gallici, quem inscripsit nova Geometriae Elementa. Ambo rationem exprimunt per fractionem, cujus numerator sit antecedens, denominator consequens, sed videbatur mihi deesse aliquid ad summum demonstrandi rigorem. Et fractio ista potius est aliquid rationem determinans quam ipsa ratio. Velim nosse quae tua sit circa hoc argumentum sententia de posthumis Galilaei a Cl. Viviano editis.

 Circa demonstrationes quasdam quas ab aliis in tuum Euclidem transsumsisti, nonnihil difficultatis reperio. Nam in demonstratione Thaletis et pag. 21 quod recta per centrum ducta circulum bisecet, unus casus negligitur, si scilicet diceret aliquis unum segmentum ABC in alteram partem translatum, partim intra partim extra alterum segmentum ADC cadere. Item in demonstratione axiomatum pag. 22, 23, quod duae rectae non habeant partem communem, nec spatium includant; Supponitur duo puncta G, F, pag. 22, et duo puncta E, F pag. 23, quibus duae rectae a circolo secantur non coincidere inter se, quod tamen adhuc demonstrandum erat. Et licet in axiomatis posterioris demonstratione Clavius hanc instantiam removere voluerit, attamen ipsemet in eandem denuo incidit, supponendo novum circulum quem describit ex centro D sumto in recta ACO, secare rectas in punctis E et F non coincidentibus. Sed in universum in horum axiomatum de recta demonstrationibus illam difficultate reperio, quod in eas nullo modo ingreditur definitio rectae nec ulla rectae proprietas axiomate aliquo praemittendo contenta. Definitio enim rectae a te assumpta est quod sit brevissima inter duo puncta, quae puchre uteris pro parallelarum proprietate, sed hic eam non adhibes nec aliud de recta axioma praemittis. Itaque in omnibus istis demonstrationibus posset alia quaecunque linea pro recta assumi quod tamen male fieret. Itaque videtur aliquid his demonstrationibus desse. Et difficulter assolvi poterit demonstratio nisi quis assumat notione[m] rectae, qualis est qua ego uti soleo, quod corpore aliquo duobus punctis immotis revoluto locus omnium punctorum quiescientium sit recta vel saltem quod recta sit linea secans planum inteminatum in duas partes congruas; et planum sit superficies secans solidum interminatum in duas partes congruas.

Ad Dn. Vitalem Jordanum

 *Letter 217*

Gerhardt: G. W. Leibniz, Dritte Reihe- Vierte Band July 1683-Dezember 1690.

Latin text:

217. VITALE GIORDANI AN LEIBNIZ

Rom, II. Novembre 1689

Latin text:

Clarissimo et Doctismo Viro D. Gotfrido Guilielmo Leibnitio Vitalis Jordanus S. P. D.

Statueram ad te venire; cum nova occupatio fregit consilium meum. De honorifico Judicio tuo super mea de momentis Dissertatione, atque Euclide restituto, mirificas tibi gratis ago. Hoc unum superest, ut aliquid ipse dicam de doctissimis tuis Animadversionibus, in elementa factis ; non quo mea sim defensurus : sed, ut rationes aperiam tibi, quibus adductus, putavi, ea, quae conatus sum, satis esse posse ad Euclidis restitutionem, quam mihi proposueram. Primum itaque monitum te volo, praecipuum meum institutum fuisse, ut iisdem Elementis eam conciliarem claritatem, quae esset quammaxime accommodata captui Tyronum, qui si ipso in vestibulo intricatas figuras offendant, statim confunduntur, atque animo cadunt. Hoc factum est, ut Thaletis demonstrationem, quam pag. 21 exposui, talem reliquerim, qualem suus fecit Auctor, sine tertii casus addizione; tum quia tertius ille casus non dissimili ratione demonstratur; tum etiam, quia cum hoc in Theoremate sit prima demonstratio negativa, neque adhuc Tyro assuetus sit concipere pro semicircolo figuram longe diversam, qualis est notata AFC, facili negozio confundi is potuisset: id quo minime fit in sequente, multoque minus in ea,

 

quae sequenti succedit; quia assuetus jam concipere demonstrationem negativam in figura facilis constructionis, nullam deinde difficultatem experitur in aliis implicatioribus, ut in pag. 123 ubi nihil obstitit, quominus eundem casum adderem.



Quod ad Procli demonstrationem attinet in pag. 22, non plane video, ubi sit difficultas: Quoniam, cum rectae AD, CD supponantur una extra alteram, et in D tantum concurrentes, equidem ignoro, quonam modo concipi possint, ut concurrentes in G et F; ad summum enim contendi posset, ut continuatae versus A et C possint tandem concurrere ad partes AC: quare si fiat DB minor, quam DA et DC, circumferentia secabit rectas DA, DC, ut in G et F; vel, si sumatur in minore rectarum DA, DC punctum quodcumque G vel F, facto centro in D, intervalloque DG, vel DF, describatur circulus EGH, ejus circumferentia secabit DB continuatam in puncto aliquo B; quod item est, ac prius.

  

Neque minus ignota mihi est difficultas ad pag. 23 ubi rectae BAD, BCD productae aut concurrunt cum circumferentia in uno puncto K, aut secant circumferentia in duobus punctis; si enim concurrerent prius, quam pervenirent ad circumferentiam, pergamus eas producere, quousque aut concurrant cum circumferentia in uno puncto aut secent cincumferentiam in duobus punctis: si eam secent in duobus punctis, optima est Procli demonstratio: si cum circumferentia concurrant in uno puncto, ut in K, tunc sumpto in recta BCI puncto aliquo D ita, ut BD sit maior quam DO, et centro in D, intervalloque DB describatur circulus BGE, ejus peripheria secabit rectas OHK, OFK, ut in E et F; et hoc modo Clavii demonstratio recte concludit. At tot hae complicationes non sunt opportunae; imo immane quantum confusionis ingererent mentibus Tyronum, in quorum gratiam mihi visum est ad alios casus non procedere.

 Jam ad rectae lineae definitionem accedo. Ipse equidem optimum puto Euclideam - recta linea est, quae ex aequo sua interjacet puncta: cujus sensus mihi videtur esse, quod recta linea sit illa, quae aequaliter inter sua estrema extenditur. At Heronis definitionem sum usus, non alia de causa, nisi quia visa mihi est accommodatior Tyronum intellectui; et ab aliis lineis tunc optime distincta est, cum dixi: lineam, quae non brevissima est inter duo puncta, vocavi curvam. Certe quaecumque linea sumatur pro linea recta proposita, aut erit brevissimum intervallum inter estrema rectae propositae, aut non erit: si erit brevissimum intervallum, ea erit recta linea; si non erit brevissimum intervallum, ea erit curva.

 Duplex tua definitio, satis ea quidam ingeniosa est, sed suis etiam exceptionibus obnoxia: quarum maxima videtur esse, quod supponit cognitum, tum corpus, tum planum; quod est ponere (ut ajunt) currum ante boves. Idem peccavit D. Borellius in suo Euclide restituto, qui supponens cognitum corpus, ex ea cognitione deduxit notitiam superficiei, lineae, et puncti; deinde in 6° libro ei definiendum fuit, quid esset corpus. Hoc sane alienum est a persona Geometrae: Alia exceptio est, quod linea, secans planum in duas partes congruas, esset potest curva, imo etiam tortuosa. Utraque tandem definitio tam oscura videtur, ut vix concipi possit a peritioribus, nedum a Candidatis Geometriae. In meo Archimede sic rectam lineam definivi- la linea revoluta intorno ai suoi estremi immoti, le di cui parti ritengono sempre il medesimo sito di prima, la chiamo linea recta- sed fateor: ea in definitione non acquiesco: expungam ipsam, et Euclideam, quam optimam duco (atque rectitudinem explicat) reponam.

 Mitto tibi opusculum meum, inscriptum: Fundamentum doctrinae motus Gravium: Deest responsio ad non nullas objectiones, quae non dum est impressa; eam tamen tradam Illmo D. Ciampino, qui curabit ad te perferendam. Si per tempus licet, exopto, atque expecto tuum de hoc opuscolo Judicium, quod plurimum apud me valet. Ceterum te rogo, ut tuis mandatis me velis exornatum, et me amare perge.

Romae Tertio Idus Novembris 1689

 Letter 218 *Gerhardt enumeration*

218. Leibniz an Vitale Giordani

[Rom, Mitte November 1689.]

Latin text:

Uberliefering:

L1

L2

(L1)

1. Secundum certum considerandi modus corpus est prius superficie, et superficies prior linea. Sunt enim termini corporis nec per se subsistunt. Unde bene ex corporis sectione derivantur. Nec simplicior derivandi ratio est quam secando corpus indeterminatum vel cujus termini non considerantur et ita quidam ut terminus communis seu secans utrinque se habeat eodem modo seu secet in partes respective congruentes et tunc fit planum. Et si rursus planum tale eodem modo secetur prodit recta. Quae consideratio novitate et simplicitate sua applausum aliquem ab ingenuis mereri posse videbatur.
2. Non video quomodo aliquid de recta demonstrari possit, non utendo rectae definitione aut proprietate ulla. Sane ex demonstrationibus a Proclo aut Clavio mutuatis conosci non potest, utrum pertineant ad lineam antea definitam quae scilicet brevissima est inter extrema, nec ullam aliam rectae notionem praesupponunt.
3. Lineas AD, CD (vel AO, CO) in D (vel in O) tantum concurrere, supponi ait responsio. At hoc ipsum non supponendum erat sed demonstrandum, adeoque ostendendum est dari puncta E et F non coincidentia per quae circulus centro D descriptus transeat.
4. Crediterim defectum in demonstratione Thaletis sine ullo tironum praejudicio emendari posse.
5. Annotavi ista non libidine contradicendi, sed quia scio magni referre ad perfectionem Geometriae, ut axiomata Euclidea de natura Rectae perfecte demonstrentur.

(L2)

Gratias Tibi maximas Clarissime Domine pro novo munere ago, quod in itinere jucundam lectu materiam suppeditabit. Facerem coram quod per schedam nunc exequor, nisi essem occupatior quam ut revisere ad Te facile possim.

Caeterum non is ego sum qui mea velut ex tripode dicta statim recipi velim: et ingenuitatem eorum inprimis amo, qui non diffitentur se utiliter admonitos. Itaque quod contra meam rectae definitionem objicis dignum consideratu agnosco: utrum scilicet in eo peccet, quod plani et solidi notiones supponit, an potius vel ideo laudem mereatur. Quod tibi porro examinandum relinquo exactius, antequam dicamus tecum, currum esse positum ante boves. Erit enim qui arbitretur corporis notionem priorem esse nozione superficiei et lineae, tanquam corporis terminorum, nec per se subsistentium; et has corporis sectione cognosci. Quod inizio assumo interminatum vel ita ut termini ejus non considerentur; ita ut ipsa sectio det terminos. Prima autem et simplicissima corporis sectio est in partes sibi respondentes congruas, seu ita ut secans ad utramque secti partem se habeat eodem modo ; et haec fit per planum. Et prima rursus plani sectio eodem modo fit per rectam, nec (quantum ego video) nisi per rectam. Habemus ergo plani et rectae originem simplicissimam secundum hunc considerandi modum qui sane novus apud ingenuos aliquem applausum sperare poterat. Nec ideo alios considerandi modos improbo(quales et ipse habeo), dummodo par claritas obtineatur, quam in Euclidea nondum hactenus agnoveramus. Interim quacunque demum utamur notione rectae, eam influere ut ita dicam, oportet, in theoremata quae de recta demonstrare volumus, alioqui ignotum est, utrum ea quae demonstramus ad eam rem pertineant, cujus data est definitio. Idque in illis demonstrationibus Euclideorum Axiomatum quas a Proclo et Clavio mutuatus es desiderare me jam innui, etsi hoc in responsione tua praeterieris. Quomodo enim ex iis sciemus pertinere ad lineam brevissimam inter sua puncta extrema. Caeterum cum propositum esset in Euclide tuo omnia qua licet exacte demonstrare, fortasse non diffiteberis rectius suppleri casus qui ad perfectionem demonstrationis desiderantur quod tironibus opinor praejudicium facere non poterat. Neque quisquam unquam tam bene subductis rationibus librum scripsit quin aliqua hujusmodi admonitionum materia supersit, quas sine detrimento existimationis agnoscere possumus. Et licet pag. 23 duae rectae BA, BC concurrant in puncto D vel O, hoc nihil prohibet quin adhuc saepius concurrant atque adeo coincidant E et F. Non igitur supponitur (quod ais) esse tantum concurrentes in D. Sed nolo te his tenere diutius, voluique tantum respondere, ne me putes quadam contradicendi libidine temerarias objectiones festinasse. Nam diu desideravi exactas videre axiomatum istorum demonstrationes quoniam sciebam magni referre ad perfectionem Geometriae, itaque dubitationes meas vel ideo tibi proponete volebam, ut Te quem parem superandae difficultati putabam, ad supplenda quae desunt, excitarem. Vale et me ama.

Al Signor Vitale Giordano

#### References

\*Vitale Giordano was born in Bitonto (Italy) probably on 15 October 1633. As an adolescent, he left his city and, after an eventful youth, became a soldier in the Pontifical Army. During these adventures, he read his first book of mathematics, the Aritmetica by P. Clavius. At twenty-eight, living in Rome, he decided to devote himself to mathematics. The most important book he studied was Euclid’s Elements translated by Commandino in Italian. In Rome, he made acquaintance with the renowned mathematicians G. A. Borelli and M. Ricci, who became his friends. Christina of Sweden employed him for a year as a mathematician in 1667; he became a teacher at the Academy of France founded by Louis XIV in Rome, and in 1685 he gained the lectureship of mathematics at the University La Sapienza in Rome. Vitale Giordano met Leibniz in Rome when Leibniz stayed there during his journey through Italy in the years 1689-90. Giordano died on November 3, 1711, and was buried in the [San Lorenzo in Damaso](http://en.wikipedia.org/wiki/San_Lorenzo_in_Damaso) basilica Garber D., 1995 Leibniz: Physics and philosophy. The Cambridge Companion to Leibniz, Cambridge University Press, p. 302.

 Knobloch E., 2006 *Beyond Cartesian limits: Leibniz's passage from algebraic to “transcendental” mathematics,* Historia Mathematica Volume 33, Issue 1, February 2006 p. 113-131

Leibniz G. W.1971, Mathematische Schriften, 5, in Samtlinke Schriften und Briefe Akademie Verlag. Hildesheim- New York 1971 herausgegeben von C. I. Gerhardt Band V.

 Mcrae R., in Garber D. 1995, 186. For the absolute space in Leibniz see directly The Leibniz-Clarke Correspondence, ed. H.G. Alexander (New York: Barnes & Noble 1956).

Proclus, 1992, A commentary on the First Book of Euclid’s Elements, Prologue II, p.72. Princeton University Press.

Robinet A.1988, G. W. Leibniz, Iter Italicum (Mars 1686- Mars 1690) La dynamique de la République des Lettres Nombreux textes inédites- Leo S. Olschki Editore.

 Robinet, 1988. Original Italian, p. 228: Vitale Giordani a Vincenzo Viviani, Rome, 25 oct. 1689 (copie de la main de Leibniz sur papier italien, LHs XLI, 7, b, f. 5)

Lettera originale del Sig. Vitale Giordani al Sig. Viviani.

Viene costà, il Sigr Leobiniz Germanico persona di quella piena letteratura che V. S. Ill. ma sperimenterà, e molto desideroso di conoscere prima del suo ritorno in patria, i letterati d’Italia e particolarmente la persona di V. S. Ill. ma, io non devo mancare né al nobile desiderio di lui né al merito singolare di V. S. Ill. ma, lo indirizzo a lei con ferma credenza che due grand’huomini goderanno di sì bella occasione ch’io le presento: mentre pregandola dei suoi riveritissimi comandamenti mi confermo per sempre di V. S. Ill. ma dev.mo servitore obl mo

 Vitale Giordani

 Roma lì 25 8bre 1689

All’Ill mo Signor mio padrone Col mo

il Signor Vincenzio Viviani in Firenze

Torrini M,1979 Dopo Galileo. Una polemica scientifica- Firenze S. Olshki.

See also the monography: Francesco Tampoia, Vitale Giordano, un matematico bitontino nella Roma barocca, Armando Editore Roma 2005. in Rome.

For the works by Vitale Giordano, see the brief scientific communication: M. Teresa Borgato, Unpublished manuscripts of Vitale Giordani, correspondent of Leibniz. Leibniz Tradition und Aktualitat V. Internationaler Leibniz-Kongress, unter der Schirmherrschaft des Niedersachsischen Ministerprasidenten Dr. Ernst Albrecht, Vortrage Hannover 14-19 November 1988.

 Giordano gave Leibniz a copy of the second edition of his book: Vitale Giordano, Euclide restituto Libri XV, Roma 1686. My translations from Italian and Latin.

Wilson Catherine, 1995, The reception of Leibniz in the eighteenth century, The Cambridge Companion to Leibniz, Cambridge University Press, p. 470, quoted from Schelling F. W. J., Ideas for a Philosophy of Nature, Translated by E. E. Harris and P. Health. Cambridge: Cambridge University Press 1988.