

Final author version. Please see
The 2020 Logica Yearbook
edited by Martin Blichá and Igor
Sedlár for the published version
with correct page numbers.

Imperative Bilateralism

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Abstract: This paper provides a proof-theoretic account of imperative logical consequence by generalising Greg Restall’s multiple conclusion bilateralism for declarative logic. According to imperative bilateralism, a sequent $\Gamma \vdash \Delta$ is valid iff jointly commanding all the imperatives $\Phi \in \Gamma$ and prohibiting all the imperatives $\Psi \in \Delta$ clashes. This account has three main virtues: (1) it provides a proof-theoretic account of imperatives; (2) it does not rely on the controversial notion of imperative inference; and (3) it is neutral regarding cognitivism about imperatives.

Keywords: imperatives, bilateralism, logical consequence, inferentialism

1 Introduction

This paper provides a proof-theoretic account of imperative logical consequence by generalising Greg Restall’s multiple conclusion bilateralism for declarative logic. According to imperative bilateralism, a sequent $\Gamma \vdash \Delta$ is valid iff jointly commanding all the imperatives $\Phi \in \Gamma$ and prohibiting all the imperatives $\Psi \in \Delta$ clashes. In the following Section 2, a trilemma for imperative logic is introduced, along with the contemporary debate between cognitivists and non-cognitivists about imperatives. A virtue of imperative bilateralism is that it allows one to remain neutral about this debate. Next, in Section 3, bilateralism is introduced, first for declaratives, second for imperatives, and third for a mixed language containing both imperatives and declaratives. A second virtue of this theory is that, in spite of its proof-theoretic orientation, it does not rely on the controversial notion of imperative inferences. A third, is that it provides a general definition of logical consequence and a sequent calculus in which the meaning of logical connectives is neutral regarding sentence type.

¹I would like to thank David Ripley and Lloyd Humberstone for their comments on this paper, as well as Greg Restall and Shawn Standefer for their help and comments on earlier versions of this work. This work is funded by an Australian Government Research Training Program Scholarship.

2 Introducing imperatives

Imperative logic faces the following trilemma:

Claim 1 (Relations) *Imperatives stand in logical relations.*

Claim 2 (Truth Apt) *Imperatives are not truth-apt.*

Claim 3 (Relata) *The relata of logical relations must be truth apt.*²

One cannot consistently affirm all three claims, yet each seems well motivated. Consider Claim 1 (Relations). Imperatives stand in logical relations such as incompatibility (1a and 1b) and equivalence (2a and 2b).

Example 1 (Incompatibilities and Equivalences)

1. (a) Both buy jam and don't buy marmalade [$\phi \wedge \neg\psi$]
(b) Either don't buy jam or do buy marmalade [$\neg\phi \vee \psi$]
2. (a) Neither buy quinoa nor soy falafel mix [$\neg(\phi \vee \psi)$]
(b) Don't buy quinoa falafel mix and also not the soy [$\neg\phi \wedge \neg\psi$]

There also appear to be valid arguments involving imperatives. Consider the following example from Peter Vranas (2011). Say someone is sitting an exam. They read instructions 1, 2, and 3 below. They then notice that the third follows from the first two.

Example 2 (Exam)

1. Answer exactly three out of the six questions;
2. Do not answer both questions 3 and 5;
3. Answer at least one even-numbered question (Vranas, 2011, p.369).

For a mixed declarative and imperative argument, consider:

Example 3 (Umbrella)

1. If it's raining, then bring your umbrella
2. It's raining.

²I have taken and slightly modified this presentation from Hannah Clark-Younger (2012).

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3. Therefore: bring your umbrella

Claim 2 (Truth Apt) also has intuitive motivations, as well as critics. For the motivation, one appears to make a mistake in calling imperative sentences true or false. Consider:

Example 4 ('True' and 'False')

3. (a) Wipe the bench
(b) #That's true.
4. (a) Do your homework
(b) #That's false.

Truth and falsity simply don't seem applicable to imperatives (Charlow, 2018; Parsons, 2013; Portner, 2004). Cognitivists about imperatives argue, however, that this move is too hasty, partly because truth is central to both standard theories of semantics and logical consequence. In response, various cognitivist theories of imperatives have been proposed in which the proponents attempt to explain away this apparent non-truth aptness (Kaufmann, 2012; Lewis, 1970, 1979).

Before continuing, let us explicitly distinguish between sentences, speech acts and contents, as it allows us to be clearer about neutrality regarding cognitivism. Declarative sentences are paradigmatically used in speech acts of asserting and denying and are normally taken to express truth apt propositions. Similarly, imperative sentences are paradigmatically used to command and prohibit and express some kind of content, truth apt or not. For example, if imperatives express sets of actions that comply with the imperative, as in Fine (2018), then we can naturally say that one commands and prohibits actions, analogous to asserting and denying propositions. In contrast, if the content of imperatives is propositional (Kaufmann, 2012), some rephrasing would be required to avoid the unidiomatic "commanding and prohibiting propositions". All that's intended is that imperatives express some kind of content and that the following is neutral regarding this content. A further terminological note is that one often speaks of asserting or denying some declarative *A*, even though, perhaps, strictly speaking one only uses the sentence to assert or deny a proposition. In the same way, we will speak of commanding or prohibiting some imperative Φ , even though, strictly speaking, one commands or prohibits the content expressed by using the sentence.

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Most theories of imperative logical consequence generalise standard truth-conditional approaches to content and logical relations formalised using model theory. Such theories either treat imperatives as truth apt or apply a different predicate analogous to truth. For example, Josh Parsons (2013) and Hannah Clark-Younger (2014) apply the notion of *compliance* to imperatives, with valid pure imperative arguments preserving compliance. In contrast, proof-theoretic approaches remain unexplored.³ Much proof-theory for declarative logics is motivated by inferentialism, according to which meaning is understood in terms of inferential relations rather than truth and reference (Peregrin, 2014; Steinberger & Murzi, 2017). Adopting inferentialism allows one to remain neutral about Claim 3 (Relata). For if inference is taken as basic, truth need not be built into the notion of logical relations. Neutrality regarding Claim 3 (Relata) also allows the inferentialist to remain neutral regarding Claim 2 (Truth Apt), because unlike someone who already endorses Claim 1 and Claim 3, the inferentialist's hand is not forced on Claim 2. They are open to either endorse Claim 2 and reject Claim 3 or vice versa.

This line of reasoning, however, runs into a problem. This is because there is an ongoing debate about whether there are genuine imperative inferences, in the sense of 'inference' as an activity (Clark-Younger, 2012; Hansen, 2008; Vranas, 2010; Williams, 1963). Inferentialists would seem to only avoid one debate at the cost of having to stake a substantial position in the other. A way to avoid both debates is to adopt an interpretation of proof theory that takes some notion other than inference *per se* as basic. An example is Greg Restall's bilateralist interpretation of the classical multiple conclusion sequent calculus (2005). Restall's bilateralism is outlined below in Section 3.1 before showing how it can be generalised to imperatives in Section 3.2.

3.1 Declarative bilateralism

We briefly summarise Restall's bilateralism (2005), in order to ground the approach taken to imperatives in the next section. To represent declaratives in our metalanguage lower case Latin p and q are used for atomics, upper case,

³See Fox (2012) for one of the few proof-theoretic approaches to imperatives, and Vranas (2019) and Fine (2018) for proof systems for their respective model-theoretic definitions of imperative consequence.

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A and B , for arbitrary declaratives, and upper case X and Y for multisets of declaratives. X, A is shorthand for $X + [A]$.⁴

Declarative logical consequence will be defined for the standard (declarative) propositional language \mathcal{L}_D .

Definition 1 (\mathcal{L}_D) \mathcal{L}_D is the language whose vocabulary is made up of denumerably many atomic declaratives p_1, p_2, p_3, \dots ; the one-place connective \neg and the two-place connectives \wedge, \vee and \supset ; and whose sentences are all and only those generated recursively from the following rule: all atomic declaratives p are sentences and if A and B are sentences then so are $\neg A, A \wedge B, A \vee B$ and $A \supset B$.

Instead of interpreting proofs in terms of inference, according to Restall's declarative bilateralism, proofs show that there is a clash or incoherence in a position that both asserts the premises and denies the conclusions.

Definition 2 (Declarative Positions and Clash) *If X and Y are both multisets of declaratives, then $X : Y$ is the declarative position that asserts all $A \in X$ and denies all $B \in Y$. $X \vdash Y$ expresses the claim that the position $X : Y$ clashes.*

For example, $A \vee B \vdash A, B$ records that the position which both asserts the disjunction $A \vee B$ and denies both disjuncts A and B clashes.

Clash is used to define declarative logical consequence.

Definition 3 (Declarative Consequence) *Y is a declarative consequence of X iff the declarative position $X : Y$ clashes.*

This provides a natural reading of the following classical multiple conclusion sequent calculus. Each rule says that if each position above the line clashes then so does the position below the line. The rules can also be read contrapositively, saying that if the position below the line does not clash, then neither does at least one of the positions above the line.

$$\frac{}{A \vdash A} \text{Id} \quad \frac{X \vdash A, Y \quad X, A \vdash Y}{X \vdash Y} \text{Cut}$$

$$\frac{X \vdash Y}{X, A \vdash Y} \text{KL} \quad \frac{X \vdash Y}{X \vdash A, Y} \text{KR} \quad \frac{X, A, A \vdash Y}{X, A \vdash Y} \text{WL} \quad \frac{X \vdash A, A, Y}{X \vdash A, Y} \text{WR}$$

⁴ $[A]$ is a singleton multiset, whose only member is A . $X + Y$ is the sum of X and Y , where for any x that occurs n times in X and m times in Y , x occurs $n + m$ times in $X + Y$.

$$\begin{array}{c}
 \frac{X, A \vdash Y}{X, A \wedge B \vdash Y} \wedge_{L1} \quad \frac{X, B \vdash Y}{X, A \wedge B \vdash Y} \wedge_{L2} \quad \frac{X \vdash A, Y \quad X \vdash B, Y}{X \vdash A \wedge B, Y} \wedge_R \\
 \\
 \frac{X, A \vdash Y \quad X, B \vdash Y}{X, A \vee B \vdash Y} \vee_L \quad \frac{X \vdash A, Y}{X \vdash A \vee B, Y} \vee_{R1} \quad \frac{X \vdash B, Y}{X \vdash A \vee B, Y} \vee_{R2} \\
 \\
 \frac{X \vdash A, Y \quad X, B \vdash Y}{X, A \supset B \vdash Y} \supset_L \quad \frac{X, A \vdash B, Y}{X \vdash A \supset B, Y} \supset_R \\
 \\
 \frac{X \vdash A, Y}{X, \neg A \vdash Y} \neg_L \quad \frac{X, A \vdash Y}{X \vdash \neg A, Y} \neg_R
 \end{array}$$

As examples of how to read the rules: Id records that assertion and denial are incompatible in the sense that asserting and denying the same thing always clashes. Id is unrestricted, in the sense that it applies to all sentences rather than just atomics. However, if it were restricted to atomics, then identity sequents could be derived for sentences of arbitrary logical complexity; $\neg L$ says that if a position that denies A clashes, then so does one that swaps A 's denial for the assertion of its negation $\neg A$. Read contrapositively, $\vee L$ says that if it is coherent to assert a disjunction, then, in the same context, it is coherent to assert at least one of the disjuncts. The same rules will be applied to imperatives in the following section where they will be given a more detailed interpretation.

Restall's bilateralism has two main advantages as an account for generalising to imperatives. First, the central notion of clash makes no reference to truth and instead takes incompatibilities between assertions and denials as basic. Thus, in generalising to imperatives the question of truth doesn't come up. Second, although it is nominally an inferentialist theory, it is the norms governing clash, rather than inference as a cognitive process, that are front and centre. This suits imperatives, because incompatibilities between imperatives are uncontroversial regardless of whether there are genuine imperative inferences. Whether Restall's theory is *really* a form of inferentialism is open to debate and largely hinges on how liberal we're being about what counts as inferentialism. *If* for a theory to count as inferentialist it needs to treat the activity of inference as central in explaining meaning, then the clash-based interpretation of the sequent calculus in this paper is certainly not inferentialist. However, many inferentialists take the relevant notion of inference to be inferential rules, instead of the activity. This is the approach of Jaroslav Peregrin (2014) who, like Restall, also focuses on the constraining nature of rules of language use—a rule of inference from 'A' to 'B' does not require

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that we infer ‘B’ whenever we know ‘A’ and instead prohibits rejecting ‘B’ when we affirm ‘A’. If the rules of the multiple conclusion sequent calculus count as inference rules, then their clash-based reading could count as a form of inferentialism like Peregrin’s. I will not argue for the antecedent here. Even if this is not accepted, Restall’s theory is an adjacent form of normative pragmatism, in the sense that meaning is explained in terms of norms of language use.

3.2 Imperative bilateralism

Restall’s bilateralism is now generalised to imperatives, drawing on the virtues for doing so identified above. This section only discusses imperatives, rather than declaratives also, and uses lower case Greek ξ and ρ for atomic imperatives, upper case Φ and Ψ for arbitrary imperatives, and upper case Γ and Δ for multisets of imperatives. In the following section, the system will be extended to accommodate both imperatives and declaratives together.

We define an imperative language \mathcal{L}_I , analogous to the declarative language \mathcal{L}_D .

Definition 4 *\mathcal{L}_I is the language whose vocabulary is made up of denumerably many atomic imperatives $\xi_1, \xi_2, \xi_3, \dots$; the one-place connective \neg and the two-place connectives \wedge, \vee and \supset ; and whose sentences are all and only those generated recursively from the following rule: all atomic imperatives ξ are sentences, and if Φ and Ψ are sentences then so are $\neg\Phi, \Phi \wedge \Psi, \Phi \vee \Psi$ and $\Phi \supset \Psi$.*

In the declarative case, assertions “rule in” propositions or ways the world could be, whereas denials rule these out. In contrast, uses of imperatives shift the practical commitments of agents, an idea shared by cognitivists and non-cognitivists alike (Charlow, 2014; Kaufmann, 2012; Lewis, 1979; Mastop, 2011; Parsons, 2013; Portner, 2004). What’s needed for an imperative bilateralism are speech acts that rule in and out actions for agents, shifting their practical commitments. Commands rule in actions for someone by requiring their performance, whereas prohibitions rule out actions for someone by forbidding their performance. Commands and prohibitions also clash just as assertions and denials do. For example, the respective command and prohibition of 2a and 2b from Section 2. Note that, if one thinks that imperatives express propositions, then commands and prohibitions will rule actions in and out by, e.g., making true deontic modals or requiring that

propositions expressing the fulfilment conditions of the imperative are made true.

We first define the notion of an *imperative position*.

Definition 5 (Imperative Positions and Clash) *If Γ and Δ are both multisets of imperatives, then $\Gamma : \Delta$ is the imperative position that commands all $\Phi \in \Gamma$ and prohibits all $\Psi \in \Delta$. $\Gamma \vdash \Delta$ expresses the claim that $\Gamma : \Delta$ clashes.*

Think of these positions as representing a simple ‘‘Overseer-Underling’’ type situation, where Overseer gives Underling commands and prohibitions but not the reverse. This gives us a natural notion of imperative logical consequence.

Definition 6 (Imperative Consequence) *Σ is an imperative consequence of Γ iff the imperative position $\Gamma : \Sigma$ clashes.*

We can apply this definition to give an intuitive interpretation of the same classical multiple conclusion sequent calculus rules as for declaratives. As with assertions and denials in the declarative case, commands and prohibitions are both, in a sense, mutually exclusive and exhaustive. The sense in which they are mutually exclusive is represented by the identity axiom Id. Id says that a position that commands and prohibits the same thing clashes. It is of course possible to command and prohibit the same thing, but not coherently.

$$\frac{}{\Phi \vdash \Phi} \text{Id} \quad \frac{\Gamma \vdash \Phi, \Sigma \quad \Gamma, \Phi \vdash \Sigma}{\Gamma \vdash \Sigma} \text{Cut}$$

The sense in which commands and prohibitions are exhaustive is represented by the Cut rule. Cut, read contrapositively, tells us that commanding and prohibiting are exhaustive in the sense that whenever an imperative position does not clash, then neither does its extension with either commanding or prohibiting any arbitrary Φ . This does not mean that every position in fact either commands or prohibits Φ .

$$\frac{\Gamma \vdash \Sigma}{\Gamma, \Phi \vdash \Sigma} \text{KL} \quad \frac{\Gamma \vdash \Sigma}{\Gamma \vdash \Phi, \Sigma} \text{KR} \quad \frac{\Gamma, \Phi, \Phi \vdash \Sigma}{\Gamma, \Phi \vdash \Sigma} \text{WL} \quad \frac{\Gamma \vdash \Phi, \Phi, \Sigma}{\Gamma \vdash \Phi, \Sigma} \text{WR}$$

The other structural rules of weakening K and contraction W fall out of the intended reading of clash. Weakening holds because once a position clashes then adding further commands or prohibitions to it will not remove the clash. Contraction records that commanding or prohibiting once is equivalent to doing so multiple times. If a position that commands or prohibits Φ, Φ clashes,

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then it will still clash if it commands or prohibits Φ just once. This makes sense of readings according to which someone commanding ‘Buy a bottle of milk’ and then saying so again later is reinforcing the first command with the second rather than telling them to buy two bottles of milk.

The operational rules receive readings very similar to in the declarative case, but replacing assertions and denials with commands and prohibitions. Conjunction and disjunction are primarily connected respectively to commands and prohibitions.

$$\frac{\Gamma, \Phi \vdash \Sigma}{\Gamma, \Phi \wedge \Psi \vdash \Sigma} \wedge L_1 \quad \frac{\Gamma, \Psi \vdash \Sigma}{\Gamma, \Phi \wedge \Psi \vdash \Sigma} \wedge L_2 \quad \frac{\Gamma \vdash \Phi, \Sigma \quad \Gamma \vdash \Psi, \Sigma}{\Gamma \vdash \Phi \wedge \Psi, \Sigma} \wedge R$$

$$\frac{\Gamma, \Phi \vdash \Sigma \quad \Gamma, \Psi \vdash \Sigma}{\Gamma, \Phi \vee \Psi \vdash \Sigma} \vee L \quad \frac{\Gamma \vdash \Phi, \Sigma}{\Gamma \vdash \Phi \vee \Psi, \Sigma} \vee R_1 \quad \frac{\Gamma \vdash \Psi, \Sigma}{\Gamma \vdash \Phi \vee \Psi, \Sigma} \vee R_2$$

Read contrapositively, the $\wedge L$ rules say that if there is no clash in commanding $\Phi \wedge \Psi$, then neither is there in commanding both Φ and also Ψ . Hence, commanding $\Phi \wedge \Psi$ rules out prohibiting either conjunct. However, prohibiting $\Phi \wedge \Psi$ does not require prohibiting both of Φ and Ψ . Rather, what $\wedge R$ records, read contrapositively, is that if there is no clash in prohibiting $\Phi \wedge \Psi$, then it does not clash to prohibit at least one of Φ or Ψ in the same context. Prohibiting $\Phi \wedge \Psi$ only clashes with commanding both conjuncts. Dually, prohibitions of disjunctions clash with commanding each disjunct (from $\vee R_1$ and $\vee R_2$). Commands of disjunctions importantly do not command either disjunct in particular. Instead, a command of $\Phi \vee \Psi$ rules out the prohibition of both disjuncts together (from $\vee L$).

Imperative bilateralism provides a good grasp on the meaning of imperative negation. A negated imperative $\neg\Phi$ should be read as ‘Don’t Φ ’ rather than ‘It is not the case that Φ ’.

$$\frac{\Gamma \vdash \Phi, \Sigma}{\Gamma, \neg\Phi \vdash \Sigma} \neg L \quad \frac{\Gamma, \Phi \vdash \Sigma}{\Gamma \vdash \neg\Phi, \Sigma} \neg R$$

$\neg L$ tells us that if a position that prohibits Φ clashes then so does one that swaps the prohibition of Φ for a command for $\neg\Phi$. Read contrapositively, if there is no clash in a position that commands $\neg\Phi$ then neither is there in prohibiting Φ in the same context. $\neg L$ allows us to derive $\Phi, \neg\Phi \vdash$, meaning that commands of imperatives clash with commands of their negations. Similarly, $\neg R$ has the result that $\vdash \Phi, \neg\Phi$ meaning that prohibitions of imperatives clash with prohibitions of their negations. This is because $\neg R$ says that if

commanding Φ clashes then prohibiting $\neg\Phi$ also clashes, and contrapositively, that if prohibiting $\neg\Phi$ doesn't clash then neither does commanding Φ . In essence, the two rules make equivalent the force of commanding an imperative and prohibiting its negation and vice versa. Together with the rules for conjunction, disjunction and contraction, the two negation rules allow for the following two derivations:

$$\frac{\frac{\frac{\Phi \vdash \Phi}{\Phi, \neg\Phi \vdash} \neg L}{\Phi \wedge \neg\Phi, \neg\Phi \vdash} \wedge L}{\Phi \wedge \neg\Phi, \Phi \wedge \neg\Phi \vdash} \wedge L \quad \frac{\frac{\frac{\Phi \vdash \Phi}{\vdash \neg\Phi, \Phi} \neg R}{\vdash \Phi \vee \neg\Phi, \Phi} \vee R}{\vdash \Phi \vee \neg\Phi, \Phi \vee \neg\Phi} \vee R$$

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$\Phi \wedge \neg\Phi$ and $\Phi \vee \neg\Phi$ are examples of imperative contradictions and tautologies. An imperative contradiction is an imperative whose command always clashes. One can, of course, command $\Phi \wedge \neg\Phi$, just not coherently. Similarly, imperative tautologies are not imperatives that are “always commanded”, but rather whose prohibition always clashes. This is directly analogous to the declarative case, where it is always incoherent to deny $A \vee \neg A$ without it being the case that every declarative position does in fact assert $A \vee \neg A$.

$$\frac{\Gamma \vdash \Phi, \Sigma \quad \Gamma, \Psi \vdash \Sigma}{\Gamma, \Phi \supset \Psi \vdash \Sigma} \supset L \quad \frac{\Gamma, \Phi \vdash \Psi, Y}{\Gamma \vdash \Phi \supset \Psi, \Sigma} \supset R$$

English, at least, lacks conditionals with imperative antecedents. For example, ‘If go to the beach, put on sunscreen’ is ungrammatical. If one wished, one could exclude such conditionals from the syntax of \mathcal{L}_I because they have no intuitive interpretation.⁵ If these conditionals are kept in the language, $\supset R$ tells us that if a position that commands Φ and prohibits Ψ clashes, then so does one that prohibits $\Phi \supset \Psi$ in the same context. Read contrapositively, $\supset L$ says that if a position that commands $\Phi \supset \Psi$ does not clash, then neither does at least one of the positions that prohibit Φ or command Ψ in the same context.⁶

⁵I am here remaining neutral on whether the lack of imperative antecedents is a mere quirk of grammar or an indication of deeper features of imperatives and conditionals.

⁶The syntax of \mathcal{L}_I treats imperatives as syntactic primitives, as in Fine (2018), Fox (2012) and Mastop (2011). This differs from approaches that start with a declarative language whose vocabulary is extended with an imperative forming operator $!$. The operator $!$ takes a declarative p and forms an imperative $!p$, where the declarative p will often state the fulfillment conditions

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The two advantages of Restall’s bilateralism identified in the previous section carry over naturally to imperative bilateralism. First, as in the declarative case, the notion of clash between commands and prohibitions doesn’t draw on notions of truth and falsity. Because of this, imperative bilateralism is able to remain neutral regarding Claim 3 (Relata); hence, also Claim 2 (Truth Apt) and debates about cognitivism. Second, the central notion of clash involves discursive norms, rather than inference as an activity, and therefore does not require taking a stand on the debate about imperative inference.

We finish with two example derivations, using the incompatible and equivalent sentences from Example 1.

$$\frac{\frac{\frac{\phi \vdash \phi}{\phi \wedge \neg\psi \vdash \phi} \wedge L}{\phi \wedge \neg\psi, \neg\phi \vdash} \neg L \quad \frac{\frac{\psi \vdash \psi}{\psi, \neg\psi \vdash} \neg L}{\psi, \phi \wedge \neg\psi \vdash} \wedge L}{\neg\phi \vee \psi, \phi \wedge \neg\psi \vdash} \vee L$$

In the above derivation read ϕ as ‘Buy jam’ and ψ as ‘Buy marmalade’. What results is a sequent telling us that it is incoherent to jointly assert Example 1a ‘Both buy marmalade and don’t buy marmalade’ and Example 1b ‘Either don’t buy jam or do buy marmalade’. In the derivation below read ξ as ‘Buy quinoa falafel mix’ and ρ as ‘Buy soy falafel mix’. The derivation shows that it is incoherent to jointly assert 2a ‘Don’t buy quinoa falafel mix and also not the soy’ and deny 2b ‘Neither buy quinoa nor soy falafel mix’.

$$\frac{\frac{\frac{\xi \vdash \xi}{\xi, \neg\xi \vdash} \neg L}{\xi, \neg\xi \wedge \neg\rho \vdash} \wedge L \quad \frac{\frac{\rho \vdash \rho}{\rho, \neg\rho \vdash} \neg L}{\rho, \neg\xi \wedge \neg\rho \vdash} \wedge L}{\xi \vee \rho, \neg\xi \wedge \neg\rho \vdash} \vee L}{\neg\xi \wedge \neg\rho \vdash \neg(\xi \vee \rho)} \neg R$$

of the imperative (Charlow, 2018; Parsons, 2013; Vranas, 2019). For example, from ‘The door is shut’ could be formed ‘Make it the case that the door is shut’. This approach allows for a syntactic difference between sentences where the imperative operator takes wide versus narrow scope, such as $!(p \vee q)$ and $!p \vee !q$, and opens up the question of whether there should be a semantic difference in such cases. For example, is there a semantic difference between ‘Make it the case that either the door is shut or the window is shut’ and ‘Either make it the case that the door is shut or make it the case that the window is shut’. For better or worse these sorts of questions do not arise in \mathcal{L}_I .

3.3 Mixed inference rules

So far, Restall's bilateralism has been generalised to a purely imperative logic, but not to one that includes both imperatives and declaratives. It would, however, be desirable to have a general account of logical consequence covering declaratives and imperatives. For it seems that logical consequence (or logical relations) is a general notion and this is required to explain mixed arguments, such as Example 3 Umbrella, that feature both imperatives and declaratives.

To be clear about what kind of sentences we are talking about, say that a sentence is *purely declarative* or *purely imperative* if all its subformulas are, respectively, declaratives or imperatives. If a sentence is neither purely declarative nor imperative we say that it is *mixed*. We keep the previous convention of using standard Latin to represent sentences that are purely declarative and Greek for ones that are purely imperative. For example, Φ may be of arbitrary logical complexity but is purely imperative. To represent sentences in general we use lower case Latin maths sans serif: p and q for atomics; upper case A and B for arbitrary sentences; and upper case X and Y for multisets of sentences. For example, A is a sentence of arbitrary complexity that may be mixed, purely declarative or purely imperative.

General logical consequence will be defined for a mixed language \mathcal{L}_M .

Definition 7 \mathcal{L}_M is the language whose vocabulary is made up of denumerably many atomic declaratives p_1, p_2, p_3, \dots ; denumerably many atomic imperatives $\xi_1, \xi_2, \xi_3, \dots$; the one-place connective \neg and the two-place connectives \wedge, \vee and \supset ; and whose sentences are all and only those generated recursively from the following rule: all atomic declaratives p and atomic imperatives ξ are sentences, and if A and B are sentences then so are $\neg A$, $A \wedge B$, $A \vee B$ and $A \supset B$.

Note that $\mathcal{L}_M \neq \mathcal{L}_D \cup \mathcal{L}_I$. \mathcal{L}_D and \mathcal{L}_I consist respectively of purely declarative and purely imperative sentences. Whereas, while \mathcal{L}_M is built up from purely declarative and purely imperative atomics, it contains mixed moods sentences such as the conditional $p \supset \phi$.⁷

The notion of mixed positions and mixed inference rules will now be introduced. In the previous section, the declarative nature of assertion and denial was abstracted from, to thinking of them as forms of ruling-in and

⁷In addition to lacking conditionals with imperative antecedents, sentential negation, in English at least, appears not to scope over mixed-mooded sentences. For example, '# It is not the case that, go to the party and I'll see you later' and '# Don't, go to the party and I'll see you later'. See Boisvert (1999) for an analysis of mixed-mooded sentences, arguing that many possible combinations are absent for pragmatic rather than semantic or syntactic reasons.

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ruling-out, and that commanding and prohibiting were their respective imperative forms. Assertions and commands are both instances of ruling-in, with denials and with prohibitions instances of ruling-out. However, to allow for uses of mixed-mooded sentences, such as conditionals with declarative antecedents and imperative consequents, then some rulings-in and rulings-out will themselves be mixed.

Definition 8 (Mixed Positions and Clash) *If X and Y are both multisets of sentences, then $X : Y$ is the mixed position that rules in all $A \in X$ and rules out all $B \in Y$. $X \vdash Y$ expresses the claim that the position $X : Y$ clashes.*

General logical consequence can now be defined as expected.

Definition 9 (General Consequence) *Y is a consequence of X iff the mixed position $X : Y$ clashes.*

We can represent structural rules and those for logical connectives in a sentence-type neutral way.

$$\begin{array}{c}
 \frac{}{A \vdash A} \text{Id} \quad \frac{X \vdash A, Y \quad X, A \vdash Y}{X \vdash Y} \text{Cut} \\
 \\
 \frac{X \vdash Y}{X, A \vdash Y} \text{KL} \quad \frac{X \vdash Y}{X \vdash A, Y} \text{KR} \quad \frac{X, A, A \vdash Y}{X, A \vdash Y} \text{WL} \quad \frac{X \vdash A, A, Y}{X \vdash A, Y} \text{WR} \\
 \\
 \frac{X \vdash A, Y}{X, \neg A \vdash Y} \neg\text{L} \quad \frac{X, A \vdash Y}{X \vdash \neg A, Y} \neg\text{R} \\
 \\
 \frac{X \vdash A, Y \quad X, B \vdash Y}{X, A \supset B \vdash Y} \supset\text{L} \quad \frac{X, A \vdash B, Y}{X \vdash A \supset B, Y} \supset\text{R} \\
 \\
 \frac{X, A \vdash Y}{X, A \wedge B \vdash Y} \wedge\text{L}_1 \quad \frac{X, B \vdash Y}{X, A \wedge B \vdash Y} \wedge\text{L}_2 \quad \frac{X \vdash A, Y \quad X \vdash B, Y}{X \vdash A \wedge B, Y} \wedge\text{R} \\
 \\
 \frac{X, A \vdash Y \quad X, B \vdash Y}{X, A \vee B \vdash Y} \vee\text{L} \quad \frac{X \vdash A, Y}{X \vdash A \vee B, Y} \vee\text{R}_1 \quad \frac{X \vdash B, Y}{X \vdash A \vee B, Y} \vee\text{R}_2
 \end{array}$$

Interpret these rules as before but in terms of ruling-in and ruling-out. Keep in mind that some ruling-ins will be either assertions or commands but that some will be themselves mixed (and ditto for ruling-outs). The above system has two main upshots. The first is that logical connectives have the same meanings regardless of the sentence types involved. This avoids

the consequence that a connective, e.g., ‘and’, means different things in different sentence types, and allows for a simple way to form “mixed mooded” sentences. The second is that it allows for an account of logical consequence defined in terms of clash that is also completely general regarding sentence type. We then have a valid derivation for Example 3 Umbrella:

$$\frac{\text{It's raining} \vdash \text{It's raining} \quad \text{Bring your umbrella} \vdash \text{Bring your umbrella}}{\text{If it's raining then bring your umbrella, It's raining} \vdash \text{Bring your umbrella}} \supset\text{L}$$

As an example of a more complex derivation, consider an argument taken from Clark-Younger (2014, p. 3). Suppose, before a philosophy exam, your friend advises you ‘If you have a choice, don’t answer the Kant question’. In the exam, the instructions say to choose between a question on Hume and a question on Kant. The derivation below shows that ruling in your friend’s advice and the exam instructions makes ruling out ‘Answer the Hume question’ incoherent. Read p as ‘You have a choice’, ϕ as ‘Answer the Kant question’, and ψ as ‘Answer the Hume question’.

$$\frac{\frac{p \vdash p \quad \frac{\phi \vdash \phi}{\phi, \neg\phi \vdash} \neg\text{L}}{\phi, p \supset \neg\phi, p \vdash} \supset\text{L} \quad \frac{\psi \vdash \psi}{\psi, p \supset \neg\phi \vdash \psi} \text{KL}}{\frac{\psi, p \supset \neg\phi, p \vdash \psi}{p \vee \psi, p \supset \neg\phi, p \vdash \psi} \vee\text{L}} \text{KL}$$

In the above the material conditional has been used. Some may have concerns about the material conditional, specific to modelling conditionals with declarative antecedents and imperative consequents. In model-theoretic approaches that generalise truth and falsity for declaratives to satisfaction and violation for imperatives, if the logic is two-valued and the conditional material, then a conditional imperative with a false antecedent counts as satisfied. Some have argued, *pace* Dummett (1978, pp. 8–9), that this is an unintuitive result and that conditional imperatives require a three-valued logic (Sosa, 1967; Vranas, 2008). The material conditional is adopted here for simplicity’s sake and I make no commitment to it as accurately modelling natural language conditionals. I suspect that imperatives are not a special case and that the reasons that would count against the material conditional for imperatives apply equally well to declaratives and vice versa. Arguing this case is, however, left to future work.

4 Conclusion

This paper has outlined a novel theory of imperative, and general, logical consequence by generalising Restall's declarative bilateralism to one that includes imperatives. Imperative consequence is defined in terms of clash between commands and prohibitions, just as declarative consequence is defined in terms of clash between assertions and denials. As an account of imperatives, it has three main virtues: (1) it provides a proof-theoretic account of imperatives; (2) it does not rely on the controversial notion of imperative inference; and (3) it is neutral regarding cognitivism about imperatives. Commands and assertions are both instances of ruling-in; prohibitions and denials instances of ruling-out. General logical consequence is then defined in terms of clash between ruling-ins and ruling-outs. This has the virtue of (4) being a general account of logical consequence that covers declaratives and imperatives together; and (5) having rules for logical connectives that are neutral regarding sentence type and which allow connectives to have the same meaning regardless of sentence type.

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