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THE MODEL OF THE SYSTEM OF SCHEMES OF ACTIONS AND OPERATIONS ON SYMBOLS AND SIGNS: 10 YEARS LATER

Ricardo Pereira Tassinari[1]

Abstract

The objective of this paper is to introduce, into the English scientific-philosophical literature of Genetic Epistemology, a model called the *Model of the System of Schemes of Actions and Operations on Symbols and Signs* (*MoSSAOSS*), and summarize its results, so far. MoSSAOSS articulates some of the principal theoretical and experimental results obtained by Piaget and his coworkers, in a systemic, systematic and synthetic view. Here, the term *model* means a schematic representation of experience, in which the relation of elements can be explored by means of logic and mathematics in order to deduce properties that correspond to direct observable empirical properties, in a sufficiently accurate form. Through explicit definitions and hypotheses, MoSSAOSS intends to reveal, in an abstract and simplified form, the general structure and functioning of the System of Schemes of Actions and Operations on Symbols and Signs. This system makes explicit the necessary elements for the acquisition of knowledge by the epistemic subject (the knower subject), allowing for the explanation of its developmental stages, as well as the attribution of meaning to objects and situations, and to the subject's actions and operations. Conceived in 1992 and introduced in Portuguese scientific-philosophical literature in 2014, MoSSAOSS has been used in a research program to study the necessary structures for knowledge acquisition, particularly scientific-philosophical knowledge. Its main results are summarized here.

Key-words: Jean Piaget; Genetic Epistemology; Genetic Psychology; Model; System of Schemes

[1] São Paulo State University (UNESP), Department of Philosophy, *Campus* Marília.E-mail: ricardotassinari@gmail.com – Orcid: orcid.org/0000-0003-1026-730X

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O MODELO DO SISTEMA DE ESQUEMAS DE AÇÕES E OPERAÇÕES SOBRE SÍMBOLOS E SIGNOS: 10 ANOS DEPOIS

Resumo

O objetivo deste artigo é introduzir, na literatura científico-filosófica inglesa sobre Epistemologia Genética, um modelo, denominado *Modelo do Sistema de Esquemas de Ações e Operações sobre Símbolos e Signos* (*MoSEAOSS*), e resumir seus resultados até o momento. O MoSEAOSS articula alguns dos principais resultados teóricos e experimentais obtidos por Piaget, suas colaboradoras e colaboradores, numa visão sistêmica, sistemática e sintética. Aqui, o termo *modelo* significa uma representação esquemática da experiência, na qual a relação dos elementos pode ser explorada por meio da lógica e da matemática, a fim de deduzir propriedades que correspondam a propriedades empíricas diretamente observáveis, de forma suficientemente precisa. Através de definições e hipóteses explícitas, o MoSEAOSS pretende revelar, de forma abstrata e simplificada, a estrutura geral e o funcionamento do Sistema de Esquemas de Ações e Operações sobre Símbolos e Signos. Esse sistema explicita os elementos necessários à aquisição do conhecimento pelo sujeito epistêmico (o sujeito do conhecimento), permitindo a explicação de seus estágios de desenvolvimento, bem como a atribuição de significação a objetos e situações, e a ações e operações do sujeito. Concebido em 1992 e introduzido na literatura científico-filosófica portuguesa em 2014, o MoSEAOSS tem sido utilizado em um programa de pesquisa para estudar as estruturas necessárias à aquisição do conhecimento, particularmente, o conhecimento científico-filosófico. Seus principais resultados são aqui resumidos.

Palavras-chave: Jean Piaget; Epistemologia Genética; Psicologia Genética; Modelo; Sistema de Esquemas.

1. Introduction and Method

A great concern in understanding Genetic Epistemology, created by Jean Piaget, and Genetic Psychology, deeply influenced by him, is the great complexity and extension of the work of Piaget and his collaborators, almost a hundred books and hundreds of articles (cf. Tassinari, 2024).

Despite the various efforts to systematize Piaget's thought, the originality and complexity of his work is still, in general, little understood, even in scientific circles, especially due to the complexity and extension of his work, as highlighted. In this context, a systematization of some theoretical and experimental results in a model, as generally done in science, would enable a systemic, synthetic and systematic view of such results and their interrelations. In addition, it can provide a renewed interpretation of them, as they can be re-signified due to this very synthesis.

Here, the term *model* means an abstract and schematic representation of experience whose relations between its elements can be explored, through Logic and Mathematics, to deduce properties that correspond to directly observable empirical properties, with sufficient precision. (cf. Granger, 1995, p. 70)

Therefore, the objective here is to introduce, into the English scientific-philosophical literature of Genetic Epistemology, a model called *Model of the System of Schemes of Action and Operations on Symbols and Signs*, or more briefly, MoSSAOSS, and summarize its results, so far. MoSSAOSS articulates, in a systemic, systematic and synthetic view, some of the main theoretical and experimental results obtained by Piaget and its collaborators.

As Ramozzi-Chiarottino emphasizes, the construction of models can be considered one of the main methods adopted by Piaget for the constitution of his theory:

> […] facing the *intelligent behavior* phenomenon, Jean Piaget proposes to himself proceeding like a physicist facing atomic and electronic phenomena. Not being able to observe the phenomenon, except by its effects, he seeks to explain it by creating a model of its structure (Ramozzi-Chiarottino, 1972, pp. 4-5).

Regarding models in the human sciences, it will be assumed here, in agreement with Gilles-Gaston Granger, that:

> […] such a radical simplification [of a model in human sciences…] can only represent concrete phenomena very partially, and under conditions that are not very feasible. What justifies it, however, is the hypothesis that it reveals fundamental mechanisms (Granger, 1995, p. 90).

In this sense, MoSSAOSS was elaborated to reveal, even in an abstract and simplified way, a general structure named *System of Schemes of Actions*, and its functioning. Those actions are (initially) external and (later) internal. How this System makes possible the construction of the necessary elements for the acquisition of knowledge will be shown here, in general terms.

Conceived in 1998 ([14]), and introduced in scientific-philosophical literature in 2014([1]), MoSSAOSS has been used in a research program to study the necessary structures for knowledge acquisition, particularly scientific-philosophical knowledge. To make explicit the relationship with this research program, a list of the main publications based on the MoSSAOSS was introduced at the end of this paper. As well, some references to them were made in the text by their number in the list between square brackets, as at the beginning of this paragraph. Of course, the framework explained here permeates all those publications. Particularly, this article is, to some extent, an improved translation of some essential parts of [1].

2. The Self-Organization-Adaptation Process and the Action Scheme System

This section introduces MoSSAOSS hypotheses and definitions related to the Piagetian hypotheses of the existence and construction of a biological mental structure which is a condition for the acquisition of knowledge (cf. Ramozzi-Chiarottino, 1984, pp. 32-33, 1988, Chpt. 2).

Hypothesis 1. Knowledge is expressed through the behavior of the organism of the *epistemic subject* (i.e., the subject who knows).

Hypothesis 1 makes it possible to investigate the manifestation of various structures necessary for knowledge acquisition from the conducts of the organism of the epistemic subject (cf. Piaget, 1950, p. 13, 1971, pp. 6-8). The following definition is proposed in order to always keep this hypothesis in mind:

Definition 1. The expression *epistemic subject organism* will refer to the epistemic subject insofar as the investigation of the structures necessary for its knowledge is performed through the investigation of the structures of its behavior.

The following hypothesis describes the general main aspects of the organism and its functioning which will be considered here:

Hypothesis 2. The organism has two inseparable and complementary aspects: a structure and a functioning. The behavior of the epistemic subject organism results from (open) cycles of functioning of the organism's structures. The functioning of the structure is performed through an *organization-adaptation process*, the *organization* being its internal aspect, and the *adaptation* its external aspect. The adaptation process has two inseparable and complementary poles: *assimilation* and *accommodation*. In organism-environment interaction, *assimilation*

is the subprocess of incorporating elements in the functioning of the organism structures. *Accommodation* is the subprocess of modification of the structures for assimilating new elements.

According to Hypothesis 2 (cf. Piaget, 1952, 1951, 1954), the organism is therefore considered as a totality, with internal self-regulations, in constant interaction with its environment, the organism and its environment being codependent parts of a larger unit.

Considering recent studies in Systems Theory or Systemics, the adaptation-organization process can be characterized as a self-organized process, i.e., according to Debrun (2019; cf. also Bresciani F. & D'Ottaviano, 2019, pp. 59-60), a process that produces itself. In particular, Debrun (2019, pp. 14, 16) considers Piaget one of the heralds of self-organization. In this sense, the organization-adaptation process may be referred to as a *self-organization-adaptation process*.

Notably, the subject's knowledge is expressed by particular moves of the subject's organism: actions. Action is defined here, as well as by Piaget himself with collaborators, as follows:

Definition 2. "*Action* is any conduct (externally observable, including by psychological interrogation) aiming at a goal from the point of view of the subject considered." (Apostel et al., 1957, p. 43)

In this sense, the subject's actions are the main object of analysis in order to investigate the subject's knowledge and the necessary structure for knowledge acquisition. An action is a determined event in space and time, and, rigorously speaking, it occurs only once. Nonetheless, analogous actions may be performed in similar situations. The subject performs analogous actions whenever they have the same interest in similar situations. The next definition,

from Piaget himself, introduces and characterizes what is common between various repetitions of the analogous action, and allows for the following hypothesis (cf. also Piaget, 1952, pp. 244-245, 1971, p. 7; Piaget & Inhelder, 1969, p. 4 n. 2; Apostel et al., 1957, pp. 45-46):

Definition 3. "The *scheme of an action* is, by definition, the structured group of the generalisable characteristics of this action, that is, those which allow the repetition of the same [analogous characteristic of the] action or its application to a new content." (Beth & Piaget, 1966, p. 235)

Hypothesis 3. From the psychological point of view, i.e., from the analysis of behavior, the action schemes constitute atom-structures of the functioning of the epistemic subject's organism, and the several action schemes are coordinated in a whole, in a system.

According to von Bertalanffy, founder of General Systems Theory:

> A system can be defined as a set of elements standing in interrelations. Interrelation means that elements, *p*, stand in relations, *R*, so that the behavior of an element *p* in *R* is different from its behavior in another relation, *R'*. If the behaviors in *R* and *R'* are not different, there is no interaction, and the elements behave independently with respect to the relations *R* and *R'* (von Bertalanffy, 1973, pp. 55-56).

According to this definition of *system*, the whole, constituted by the action schemes (*p* elements) and their coordinations (*R* relations), may be considered as a system. In this sense, as highlighted by Bresciani F. & D'Ottaviano:

> A system may be initially defined as a unitary entity of a complex and organized nature, made up of a set of active elements which maintain partial relations between themselves; a system also has characteristics of invariance in time that guarantee its identity. Thus, a *system* is a non-empty set of elements which form a partial structure, with functionality (Bresciani F. & D'Ottaviano, 2019, p. 48).

Therefore, the unitary entity constituted by the set of action schemes and their coordination makes it possible to introduce the following definition:

Definition 4. The *system of action schemes* of an epistemic subject organism is the system constituted by the set of their action schemes (active *p* elements) and by the coordinations between them (partial relation *R* maintained between them).

Note that, from the point of view of the behavior of the epistemic subject organism, its system of action schemes is *the set of all possible actions, immediately available to the subject, to be performed by them in different situations*.

The system of action schemes originates itself from hereditary reflexes (cf. Piaget, 1952, pp. 23-46) and becomes more and more complex. Through the process of self-organization-adaptation between the subject and their environment, the organic structure itself is modified, by its functioning, by assimilation and accommodation. The subject gradually increases both their set of action schemes and the coordination thereof, and becomes, by this very process, capable of performing more distant actions in space and in time.

The previous hypotheses and definitions enable us to introduce the following hypothesis:

Hypothesis 4. Concerning the system of action schemes, as an outcome of the self-organization-adaptation process, in organism-environment interaction: (1) *assimilation* constitutes the subprocess of incorporating objects or situations, using them in the functioning of an action scheme or a coordination of action schemes; (2) *accommodation* constitutes the subprocess of modification of the structure of the system of action schemes, either: (2.1) by the constitution of a new scheme, by *differentiation* of the previous schemes, or (2.2) by a new *coordination* of old schemes in a new composed action. The accommodation

subprocess by *differentiation* is performed when the subject's activity, trying to execute an old action scheme in similar regular situations, generates a new action, no more analogous to the old ones, but with a new different scheme for those situations. The new *coordination* of old schemes may be: (2.2.1) *synchronic*, by a simultaneously *reciprocal assimilation* of the objects or situation by the old schemes components, or (2.2.2) *diachronic*, when the subject composes a new action from actions with old schemes, the final situation of one being the initial situation of another.

Note, therefore, that the accommodation subprocess is not passive, despite its name. On the contrary, it consists of an active modification, by the epistemic subject, of their forms (schemes) of actions, by active differentiation or active coordination. The subject is doubly active! They *actively* modify their own forms (schemes) of *action*.

Ergo, in general, the epistemic subject-organism performs an active process of self-organization-adaptation, in a process of equilibration with the environment (cf. Piaget, 1957, 1977, 2001), in which its system of action schemes becomes more and more complex, with the emergence of new action schemes by differentiation of the old schemes, or by actively coordinating them.

With such preliminary hypotheses and definitions, it is possible to analyze now, some structural aspects of the systems of action schemes and their constitutions, aiming to study, from there, some of the necessary structures for knowledge and their genesis. The next MoSSAOSS hypotheses will be introduced, therefore, in order to enable such analysis.

3. The MoSSAOSS and the Representation of the Action Schemes System

As said, the term *model* here means an abstract and schematic representation of experience whose relations between its elements can be

explored, through Logic and Mathematics, to deduce properties that correspond to directly observable empirical properties with sufficient precision. This section introduces a representation of this type, in relation to the systems of action schemes of the epistemic subject-organism.

Initially, note that Figure 1 may represent the possibility of application of the action scheme *s* (of epistemic subject-organism) to the initial state *a* resulting in the final state *b*.

Figure 1 – Representation of the possibility of an application by an epistemic subject-organism of the action scheme *s* to the initial state *a* resulting in the final state *b*.

$$a \overset{s}{\rightarrow} b$$

For example, according to Piaget and Inhelder (1969, p. 16), *AB* represents a displacement performed by the child (of themself or of an object) from position *A* in space to position *B*. Therefore, *AB* refers to the scheme of a displacement action from *A* to *B*, and the possibility of its application may be represented as in Figure 2.

Figure 2 – Representation of the possibility of application of the displacement scheme *AB* from the initial position *A* to the final position *B* performed by the child (on himself or an object).

$$A \overset{AB}{\rightarrow} B$$

In this sense, from a logical and mathematical point of view (as argued in [8], p. 260), it is possible to see an action (with scheme *s*, initial state *a* and final state *b*):

(A1) as associated with an ordered pair (*a*, *b*); and

(A2) as a partial operation *s* (in a logical and mathematical sense) that applies to element *a* only, and results in element *b*.

MoSSAOSS explores these two action characteristics in the study of the structures necessary for knowledge and their genesis. A1 is analyzed in this section, and A2 is examined in the next section.

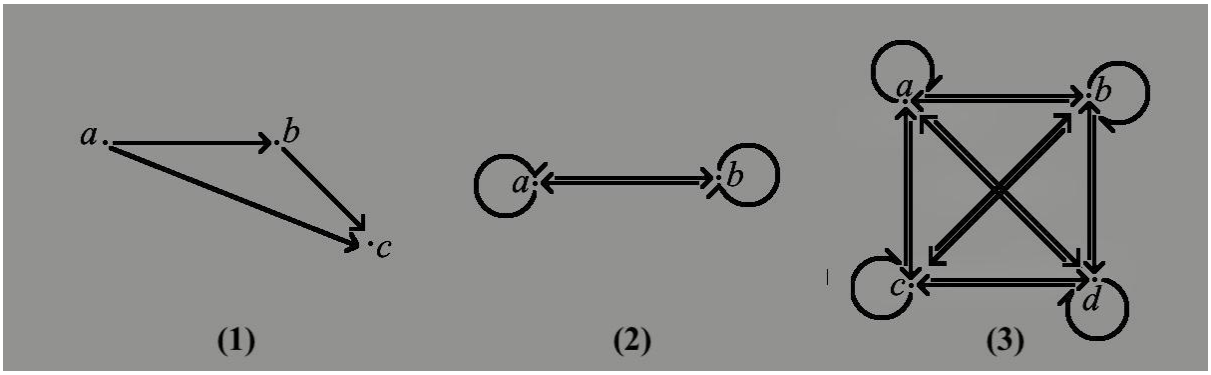
Characteristic A1 makes it possible to consider digraphs as forms of the systems of action schemes of an epistemic subject-organism (as argued in [8]).

Consider the following definition (according to [8], p. 264):

Definition 5. A *digraph D* consists of a set V_D of any elements, referred to as *vertices* of *D*, and a set A_D of ordered pairs of V_D , referred to as *arrows* (or *edges*) of *D*.

Generally speaking, digraphs may be represented through drawings, in which the vertices correspond to points and the edges correspond to drawn arrows that connect points with each other, as in Figure 3.

Figure 3 – Representations of digraphs: the first digraph, represented on the left, has three vertices *a*, *b* and *c* and three arrows (*a*, *b*), (*b*, *c*) and (*a*, *c*); the second, represented in the center, has two vertices *a* and *b* and four arrows (*a*, *b*), (*b*, *a*), (*a*, *a*) and (*b*, *b*); and, finally, on the right, the represented digraph has four vertices *a*, *b*, *c* and *d*, and sixteen arrows (*a*, *a*), (*a*, *b*), (*a*, *c*), (*a*, *d*), (*b*, *a*), (*b*, *b*), (*b*, *c*), (*b*, *d*), (*c*, *a*), (*c*, *b*), (*c*, *c*), (*c*, *d*), (*d*, *a*), (*d*, *b*), (*d*, *c*) and (*d*, *d*).



From this definition (as argued in [8], p. 264), the next proposition follows:

Proposition 1. Every system *A* of action schemes constitutes a digraph *D*, in which each possibility of application of an action scheme of *A*, that can change a situation from state *x* to one of state *y*, is associated with an arrow (*x*, *y*) of *D*, and each situation state (like the said elements *x* and *y*), that *A* relates, is a vertex of *D*.

Therefore, digraphs can be used to represent and study some of the forms of the structures of the systems of action schemes of the epistemic subject-organism, and, from there, also the structures necessary for knowledge. According to Definition 4, a system of action schemes is constituted by the set of action schemes and their coordinations, and, according to Proposition 1, there is a digraph in which the action schemes determine arrows (as described in Figure 1), and its general structure makes explicit the coordinations of these schemes in terms of their initial and final states.

For example, concerning action scheme systems, the drawings in Figure 3 represent, respectively: (1) the possibility of coordinating two actions (*a*, *b*) and (*b*, *c*) into one action (*a*, *c*); (2) for an action (*a*, *b*), the possibility of performing an inverse action (*b*, *a*), as well as the possibility of returning to the initial situation, (*a*, *a*) and (*b*, *b*); and, finally, (3), in a set of situations (*a*, *b*, *c* and *d*), the possibility of the subject reaching a situation from any other.

An example of this study is what may be referred to as the *Displacement Digraph* established by the Practical Group of Displacements in the Sensorimotor Period:

> Psychologically speaking, this group has the following characteristics: (a) A displacement *AB* and a displacement *BC* may be coordinated into a single displacement *AC*, which is still part of the system. (b) Every displacement *AB* may be reversed to *BA*, whence the behavior pattern of "return" to the point of departure. (c) The combining of the displacement *AB* with its reverse *BA* gives the null displacement *AA*. (d) The displacements are associative; that is, in the series *ABCD*, *AB* + *BD* = *AC* + *CD*. This means that by starting from *A* an identical point *D* may be reached by different paths (if the segments *AB*, *BC*, etc., are not along a straight line). When the child understands this property of space he can begin to solve "detour" problems (Piaget & Inhelder, 1969, pp. 16-17; cf. also [12]).

The Practical Group of Displacements defines a Displacement Digraph in which the points of this digraph are the possible spatial positions (of the objects or of the subject), as *A*, *B*, *C* and *D*, and the arrows of this digraph are the possible *XY* displacements between the possible *X* and *Y* positions, such as *AB*, *BC*, *AC*, etc… The Displacement Digraph is, therefore, the system of spatial positions (of objects and subject) and of possible spatial displacements, considered by the epistemic subject-organism. Two visual examples can be considered here from Figure 3: (1) the represented digraph (2) makes explicit the form of a part of the Displacement Digraph with two spatial positions, *A* and *B* (respectively, *a* and *b* in Figure 3), and all possible displacements among them (the 4 arrows); and (2) the represented digraph (3) makes explicit the form of a

part of the Displacement Digraph with four spatial positions, *A*, *B*, *C* and *D* (represented in Figure 3 by *a*, *b*, *c* and *d*), and all possible displacements among them (the 16 arrows).

Having analyzed Characteristic A1 of the scheme of action system, Characteristic A2 is examined in the next section.

4. Operations, in Piaget's Sense and in the Sense of Logic and Mathematics.

In this section, the notion of operation used by Piaget and the notion of operation in Logic and Mathematics are introduced; the aforementioned Characteristic A2 is explored; and the relationship between the two notions is established to, in the next section, address the issue of operations under MoSSAOSS.

At the end of the Sensorimotor Period, according to Piaget and Inhelder (1969, p. 51), the semiotic function emerges, and "It consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a *signifier* which is differentiated and which serves only a representative purpose".

Therefore, from the emergence of the semiotic function, the epistemic subject can represent the meanings, recognized by them, through signifiers differentiated by them.

In the use of the semiotic function, Piaget considers two types of signifiers, differentiated from their meanings by the epistemic subject: the symbol and the sign.

> The *symbol* and the *sign* are the signifiers of abstract meanings, such as those which involve representation. A *symbol* is an image evoked mentally or a material object intentionally chosen to designate a class of actions or objects. So it is that the mental image of a tree symbolizes in the mind trees in general, a particular tree which the individual

> remembers, or a certain action pertaining to trees, etc. [...] The *sign*, moreover, is a collective symbol, and consequently "arbitrary". It also makes its appearance in the second year, with the beginning of language and doubtless in synchronism with the formation of the symbol. Symbol and sign are only the two poles, individual and social, of the same elaboration of meanings (Piaget, 1952, p. 191).
>
> With regard to [*signs*] *meanings* [which are] of a higher order, which are also collective meanings, the distinction is clear: the signifier is the verbal expression, that is, a certain articulated sound to which one has agreed to attribute a definite meaning, and the signified is the concept in which the meaning of the verbal sign consists (Piaget, 1952, p. 189).

Finally, Piaget established the following definitions of internalized action and of operation:

> *Def*. We will use the term *internalized* in reference to an action performed in thought on symbolic objects, either by representation of its possible unfolding and application to real objects evoked by mental images (it is then the image which plays the role of the symbol), or by direct application to symbolic systems (verbal signs, etc.). *Def*. We will use the term *operations* in reference to interiorized actions, or those that can be interiorized, which are reversible and coordinated in total structures (Piaget, 1957, pp. 44-45).

The following definition of MoSSAOSS is introduced in order to summarize those previous definitions:

Definition 6. *Operations in Piaget's sense* are inner actions on signifiers (symbols or signs) and, with them, also on their meanings.

This definition makes it possible to state the fourth hypothesis of MoSSAOSS, concerning operations (in Piaget's sense):

Hypothesis 5. Operations (in Piaget's sense) and their coordination have an analogous form to that of actions and their coordination, as previously described (as in Figure 1 and Figure 4). The inner actions come to be coordinated with the outer actions in order to constitute a unique system of schemes of outer and inner actions.

According to this hypothesis, analogous to outer actions, Figure 4 may represent the possibility of the application of the operation scheme *s* (that is, a scheme of an inner action) to the initial state *a* resulting in the final state *b*; *a* and *b* represent signifiers (symbols or signs) that the epistemic subject-organism uses to designate the initial and final states of the meanings of its operation.

Figure 4 – Representation of the possibility of an application of operational scheme s, as a scheme of an inner action, in which a and b represent (in the model) the signifiers (symbols or signs) that the epistemic subject-organism uses to designate the initial and final states (signifier meanings) of the operation.

$$a \xrightarrow{s} b$$

Note that Piaget uses the term *operation* in a restricted way only to refer to inner actions. For outer actions (such as of the Sensorimotor Period), Piaget does not use the term *operation* related to them. However, as mentioned in the previous section, according to Characteristic A2, even an outer action may be considered a partial operation *s* (in a logical and mathematical sense) that applies to element *a* only, and results in element *b*. The logical and mathematical definition of the operation does not use, and then does not depend on, the concepts of inner or outer, as can be seen in the following definition. Perhaps, one of the most important contributions of MoSSAOSS is, from that distinction, to give a new perspective about the relationship between Genetic Epistemology, Mathematics and Logic, which enables one to coordinate these domains and overcome past difficulties in this context.

Definition 7. In Logic and Mathematics, given a set *A*, an *n-tuple in A* is a sequence of *n* (not necessarily distinct) elements of *A*. The expression (a_1, a_2, …, a_n) refers to the n-tuple in *A*, whose elements are, respectively, a_1, a_2, …, a_n . An *n-ary operation* on a set *A* is an association of every n-tuple in *A* with a single element of *A*.

For example, considering the set of natural numbers N ={*0*, *1*, *2*, *3*, …}. The *addition operation* associates each pair of natural numbers with a natural number: (*0*, *0*) with *0*; (*0*, *1*) with *1*; (*1*, *0*) with 1; (*1*, *1*) with *2*; etc. Usually, the result associated with the pair (x, y) is denoted by $x+y$.

Note that, by definition, a *unary operation* on a set A associates each element of A with a single element of A. For example, the *successor operation* s on the set of natural numbers N associates a number with its successor (that is, $s(0)=1$, $s(1)=2$, $s(2)=3$, etc.) and thus it is a unary operation.

Finally, in general, the notion of operation, in Mathematics and Logic, presupposes that it is defined for all elements of the considered domain. This is the case of the addition function, which associates any pair (x, y) of numbers with a number (denoted by $x+y$), or of the successor function, which associates each number x with its successor (denoted by $s(x)$). However, operations that are not defined for their entire domain are also considered in Mathematics and Logic. They are called *partial operations*. For example, in the case of real numbers or rational numbers, the usual *division operation* x/y is not defined for the case where y is equal to *0*, since it makes no sense in these sets to divide any of its elements by *0*. Because of that, the following continuation of Definition 7 is introduced, allowing for a definition of logical and mathematical operation in a broader sense:

Definition 7 (Continuation). An *n-ary partial operation* on a set A is an association of every n-tuple in a subset B of A with a single element of A. *Operations in the sense of Logic and Mathematics* are n-ary partial operations.

Note that an n-ary operation on a set A is a particular case of n-ary *partial* operation on a set A, in which the subset B of A is A itself, i.e., $B=A$. Thus,

both are comprehended by the previous definition of operations in a logical and mathematical sense.

Furthermore, any two elements *a* and *b* of any set *C* define a unitary partial operation *t*, such that *t* applies only to the element *a*, and the application of *t* to *a* results in an element *b*, i.e., $t(a) = b$.

Considering now Figure 1 and the previous definitions, an action may be considered an operation in a logical and mathematical sense, because it defines a partial and unary operation *s*, such that the operation *s* is defined only for a single element, the state *a* and, applied to that state *a*, results in state *b*; in this case, one can write, therefore, $s(a) = b$.

More generally, any ordered pair of the form (x, y) may be considered as defining a unary partial operation: the unary partial operation that applies to the element *x* only, and results in the element *y*. As said, an action with scheme *s*, as represented in Figure 1, is associated with the pair (a, b), as pointed out by Characteristic A1; in this sense, the unitary partial operation defined by the pair (a, b) is precisely the unitary partial operation *s*, such that $s(a) = b$, related to Characteristic A2.

For example, the aforementioned displacement *AB* by the subject (of an object or of themself) from position *A* to position *B* is a operation in a logical and mathematical sense, because it defines a partial and unary operation *s* that associates point *A* with point *B*, that is, $s(A)=B$.

In this sense, any action can be seen as an operation in the sense of Logic and Mathematics, including outer actions (as that of the Sensorimotor Period) and inner actions (as with the operations in Piaget's sense). Thus, every operation in Piaget's sense may be considered as an operation in the sense of Logic and Mathematics, but the converse is not valid: an outer action (without a

correlative inner action) by the subject is not an operation in Piaget's sense, but may be considered a logical-mathematical operation.

Remark. Notice that, from this point forth, the term *operation* (without any complement) will be used to refer to *operation in Piaget's sense*.

Having clarified the different notions of operation, Piagetian and logical-mathematical, and their correlation, a more detailed study of operations can be carried out to complete MoSSAOSS hypotheses, which is done in the next section.

5. MoSSAOSS and Operations

In this section, operations are discussed according to MoSSAOSS, regarding the system of (inner and outer) action schemes of the epistemic subject-organism. Particularly, the fifth and last hypothesis of MoSSAOSS is introduced, as well as the required definitions for it. These definitions make possible a new characterization for the Concrete Operational Period and for the Formal Operational or Hypothetical-Deductive Period. Some of their consequences are analyzed.

The following definitions specify the previous Definition 6 of operation:

Definition 8. *Operations on symbols* are inner actions (as well as the application of their schemes) on states (such as *a* and *b* in Figure 4) represented with symbols (particularly, mental images) by the subject.

Definition 9. Operations on signs are inner actions (as well as the application of their schemes) on states (such as *a* and *b* in Figure 4) represented with signs (particularly, words and phrases) by the subject.

Note that these definitions of operations (in the Piagetian sense) are still operations in the logical and mathematical sense, since they establish a unitary partial operation *s* (in the logical-mathematical sense) whose application results $s(a) = b$, as pointed out. Therefore, these definitions still hold the reconciliation of the two uses of the term operation (in the Piagetian sense, and of Logic and Mathematics).

The sixth and final hypothesis of MoSSAOSS makes it possible to establish a direct relationship between the previous definitions and concrete operations and formal operations.

Hypothesis 6. *Concrete Operational Period* is characterized by the subject having the ability to operate on symbols (particularly on mental images), and *Formal Operational Period* or *Hypothetical-Deductive Period* is characterized by the subject having the ability to operate strictly on signs themselves (especially on words and phrases).

According to Hypotheses 5 and 6, with the emergence of operations, the subject also starts to act internally. Therefore, the schemes of inner actions are also integrated into the system of action schemes of the subject, being coordinated with the outer action schemes, forming a single system of (outer and inner) action schemes of the subject.

From Hypotheses 5 and 6, operations on symbols and signs, and their coordination, have a similar form of outer actions and their coordinations (as in Figure 1 and Figure 4). Those operations on symbols and signs are coordinated with outer actions in order to constitute a unique system of (outer and inner) action schemes or, more explicitly, *a system of schemes of (outer) actions and operations on symbols and signs*; its expression is precisely what motivates the model's name, here described (with the suppression of the term *outer* to shorten

an already long expression). That name also aims to give immediate access to the list of the main concepts (and definitions) involved in the Model: action, action scheme, system of action scheme, operation, symbol, sign, operation on symbol, and operation on sign. Moreover, those can be remembered for its acronym: MoSSAOSS.

According to Hypothesis 6: firstly, there is the emergence of the system of outer action schemes; then, operations on symbols appear, and are coordinated with outer actions in the Concrete Operational Period; and, finally, in the Formal Operational or Hypothetical-Deductive Period, operations on signs appear, and are coordinated with the system constituted by the outer actions and the operations on symbols, and the coordination between them.

Notice that, as the subject can verbally express (with signs) the initial and final states of an outer action or of an operation on symbols, this can lead the observer to presume that the subject would have the ability to operate on signs, as defined here, before the Formal Operational Period or Hypothetical Deductive. However, this is not the case, since the characterization of this Period here (and the precise definition of operations on signs) is "the ability to operate strictly on signs themselves", which means only operations on what is properly signified by signs (as a general concept, for example, which cannot be represented by an action scheme or a symbol). The inexistence of operation on signs in this sense, before the Formal Operational Period or Hypothetical Deductive, is attested by the child's inability to reason by purely verbal means before that period (cf. Inhelder & Piaget, 1958, p. 251; Piaget, 1968, pp. 61-64).

Notice that the systems of (outer and inner) action schemes of each developmental stage are necessarily a subsystem of the system of action schemes of the next stage, according to Piaget's requirements (1974, pp. 47-53) on stage

characterization: (1) constant succession order of acquisitions, (2) integrative characteristic, (3) whole structure, (4) preparation on the one hand and completion on the other, and (5) formation processes and final equilibrium forms (in the relative sense).

Concerning the analysis of the structure of the system of scheme actions, after all: the operations (on symbols and signs) are coordinated with the outer actions forming a single system of schemes of outer and inner actions; and the form of the operations on symbols and signs (application of the scheme *s* to the initial state *a* resulting in the final state *b*, as represented in Figure 4) is the same as the outer actions (application of the scheme *s* to the initial state *a* resulting in the final state *b*, as represented in Figure 1). Therefore, it enables one to study the coordination structure of the initial states and of (outer and inner) actions through digraphs (like those represented in Figure 3).

In this sense, an object or a situation can be simultaneously the (initial or final) state of an outer action and the (initial or final) state of an operation (on a symbol or on a sign). The emergence of operations (on a symbol or on a sign) makes it possible for the subject to re-signify the object or situation in terms of the new possibilities of inner actions, that is, an object or situation is re-signified in terms of the possibilities of a child's imagination (operations on symbols) or an adolescent's imagination and theorization (operations on symbols and signs).

Furthermore, according to Hypotheses 4 and 5, the complexification of systems of action schemes and operations on symbols and signs, as a system of outer and inner action schemes, occurs in a similar way to the system of outer action schemes studied in detail by Piaget (1952); that is, by a process of self-organization-adaptation with its two inseparable poles, assimilation and accommodation. Therefore, concerning the system of action schemes and

operations on symbols and signs, as established in Hypothesis 4: (1) there is *assimilation* when an object or situation is incorporated into an application of a scheme of action or operation (on a symbol or on signs) or a coordination of these schemes; and (2) there is accommodation when there is modification in the structure of the system of schemes of actions and operations. The accommodation occurs either: (2.1) by the constitution of a new scheme, by *differentiation* of the previous schemes, or (2.2) by a new *coordination* of old schemes in a new composed action. The new *coordination* of old schemes may be: (2.2.1) *synchronic*, by a simultaneously *reciprocal assimilation* of the objects or situation by the old scheme components, or (2.2.2) *diachronic*, when the subject composes a new action from actions with old schemes, the final situation of one being the initial situation of another.

Hypothesis 6 allows one to explain, in general, why the subject decreases their external activities (running, jumping, hitting, etc.) in middle childhood or in the Concrete Operational Period: they do not become less active, they shift their activities into their interior performing operations on mental images. In a similar way, the adolescent or adult, when quiet in a waking state, has an uninterrupted mental activity (operations on mental images and on signs).

The difference between symbols and signs, especially between mental images and signs, establishes a fundamental difference between what operations on mental images enable the epistemic subject-organism to perform, and what operations on signs enable the epistemic subject-organism to perform. That is discussed in the following section.

6. MoSSAOSS, Mental Images, Transfigurations and transsignations.

In this section, the central role of mental images in operations on symbols is discussed, and the definitions of transfiguration and transsignation, according to MoSSAOSS, are introduced.

Mental images play an essential role in operations on symbols: as seen, according to Piaget, operations on symbols are internalized actions; the mental image is the only internal symbol; thus, actions on external symbols (such as deferred imitation, symbolic play, or drawing) are operations only if they have a mental image corresponding to each external symbol in such a way that the action can also be carried out internally (on the corresponding mental images).

In this sense, Hypothesis 5 allows one to explain the simultaneous arising of the various concrete operations, including the *grouping* structures for classification and seriation, as described by Piaget (1942, 1972). They result from operations on mental images and their coordination with external actions (*cf*, [7], [14]), and thus, they emerge, making all the concrete operations possible.

The essential role played by mental images, as a privileged kind of symbol, motivates the following definition:

Definition 10. *Transfigurations* are operations on mental images, that is, inner actions performed on states (such as *a* and *b* in Figure 4) represented by the subject with mental images.

Note that this definition is in accordance with the definition initially introduced by Tassinari ([14], p. 6):

> A *transfiguration* is, by definition, a virtual reversible action, realizable in thought (endogenously) by the subject, which makes it possible for the subject to compare two representations of objects or situations — the mental image thus having the role of the symbol that allows them to be evoked — through the passage from one of the representations

> (which we will call state 1) to another representation (state 2), without merging them into a single image representation, that is, being aware that they are two different objects or situations that are linked by the very action that compares them.

The term *transfiguration* allows one to refer to the inner actions, belonging to the operative aspect of knowledge, on the elements of the figurative aspect of knowledge (*trans* = move across, *figure* = image).

From Definition 10 and the previous analysis, each operation on symbols (including on internal ones) has a corresponding transfiguration, where each external symbol corresponds to a mental image, and thus, each concrete operation has a corresponding transfiguration scheme.

Therefore, according to Hypothesis 5, Concrete Operational Period may be characterized by the existence of systems of transfiguration schemes.

To confirm this characterization, it is necessary (and sufficient) to show how the children's capacity for action in that period can be completely explained by assuming that the child acts internally on mental images. To do that, the following method is proposed.

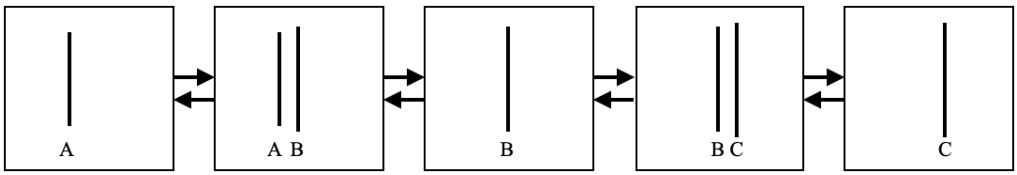
The method of the *Transfiguration-System-Drawing Digraph* consist in representing, through a digraph, a transfiguration scheme system of the subject (using drawings as vertices representing subject's mental images, and arrows representing possible transfigurations) which are necessary and sufficient for the subject to realize the presented task.

That method can be applied to explain concrete operational behavior (as in [1], [3], [4], [5], [7], [8], [9], [14], [16], [17], [18], [20], [21]). For example, consider the following experimental situation described by Piaget (1968, pp. 49-51) on qualitative seriation:

> A particularly clear example [... is] the qualitative seriation: A < B < C ... etc. During early childhood, a youngster can distinguish two sticks by their length and can judge that B is larger than A, but at this age level [in the Preoperational Period, before the subject is capable of performing transfigurations, according to MoSSAOSS] it is merely a conceptual or intuitive relationship and not a logical operation. If the young child is first shown sticks A < B, and then sticks B < C while A is hidden under the table, and he is asked if A (which has just been compared with B) is larger or smaller than C (which is on the table with B), the child refuses to make a decision and asks to see the two together because he cannot deduce A < C from A < B and B < C. This occurs, naturally, only when the differences are not so great as to remain in the memory as picture images. When will the child be able to make this deduction? Not until he can construct a series or scale of sticks on the table, and, curiously enough, he will not be able to do so before six or seven years of age [during the Concrete Operational Period]. [...] It is also immediately apparent that this construction presupposes the inverse operation (operational reversibility). Each element is conceived both as smaller than all the following elements (relation <) and as larger than all the preceding elements (relation >). This allows the subject to discover a method of construction as well as to intersperse new elements after the total initial series has been constructed.

In this case, the Transfiguration-System-Drawing Digraph, in Figure 5, makes it possible to understand that a subject with a transfiguration scheme system is capable of solving the presented task. It represents a series of situations with three sticks A, B and C (such that A is smaller than B and B smaller than C), in which, respectively: A stands alone, A and B are compared, B stands alone, B and C are compared, and C stands alone.

Figure 5 – The Transfiguration-System-Drawing Digraph representing the transfiguration schemes for understanding the whole seriation of three sticks A<B<C without a direct perception of them.



The represented transfiguration system makes it possible for the subject to compare the sticks without having the actual perceptions of them, because the comparison is made internally by transfiguration and the mental

images of the sticks alone or with one another. On the other hand, its absence explains why, before the transfiguration system, the subject depends on the actual perception of the sticks and cannot establish the total series A<B<C.

Finally, the following definition may be introduced to refer to operations on signs and its schemes:

Definition 11. *Transsignations* are operations on signs, that is, internalized actions performed on states (such as *a* and *b* in Figure 4) represented by the subject with signs.

The emergence of transsignations and their schemes produces a profound change in the subject's possibilities of actions. Before the transsignations, the subject was capable of operating only on what may be represented by mental images. As far as anything may be referred to by a sign, the transsignations allow the subject to operate on anything they recognize. It allows one to understand (from Hypothesis 6 and Definitions 9 and 11) why the Formal Operational or Hypothetical-Deductive Period is the last period. Particularly, words and sentences are signs, and so transsignations enable the subject to reason from mere hypotheses, regardless of whether they are true or false, which is a capability of the Hypothetical-Deductive Period, giving it its name.

The next section summarizes the previous content and establishes its correlations with the operative and figurative aspects of knowledge.

7. MoSSAOSS and the Operative and Figurative Aspects of Knowledge

Generally speaking, the digraphs of the system of schemes of actions and operations on symbols and signs keep the same division established by Piaget between the figurative and operative aspects of knowledge. The digraph

analyses establish a vertices-arrows dichotomy. Vertices and arrows are respectively related to the "states" and "transformations", and they respectively concern figurative and operative aspects of knowledge.

Remembering that, according to Piaget and Inhelder, the *operative aspect*:

> [...] characterizes the forms of knowledge which consist in modifying an object or an event so as to grasp the actual transformations and their results, and not merely as before the static configurations corresponding to the "states" linked by these transformations. These forms of knowledge include[:] (a) sensori-motor actions (except imitation), the only instruments of the sensori-motor intelligence which becomes organized before language; (b) internalized actions which are an extension of the above. They first appear at the pre-operational level (age two to seven); and (c) operations that are properly attributable to intelligence. These are actions which are internalized, reversible and co-ordinated into integrated structures bearing on transformations [...] (Piaget & Inhelder, 2014, p. 87).

The *figurative aspect*:

> [...] characterizes the forms of cognition which, from the subject's point of view, appear as "copies" of reality although, from the objective point of view, they offer only an approximate correspondence to objects or events. But this correspondence relates to the figural aspects of reality, that is to configurations. It is possible to distinguish three fundamental varieties of figurative knowledge: [(1)] perception, which functions exclusively in the presence of the object and through the medium of a sensory field; [(2)] imitation in the broad sense (by means of gestures, sounds or drawing, etc.), functioning in the presence or absence of the object but through actual or manifest motor repro-duction; and [(3)] mental images, functioning only in the absence of the object and by internalized reproduction (Piaget & Inhelder, 2014, p. 87).

Therefore, the vertex-arrow digraph dichotomy (related to the state-transformation dichotomy) is directly related to the figurative-operative dichotomy of aspects of knowledge:

> Now the figurative aspects of cognition bear more particularly on the "states" of reality, though transformations may be perceived, imitated or imagined, in which case, however, they are given a character which is either figural, direct (movement, *Gestalt*, etc.), or symbolic

> (transformation images). The operative aspects, for their part, relate in particular to transformations, although a state may be taken operationally in so far as it is a result of anterior transformations, a starting point of subsequent transformations, or a nullified or compensated transformation (Piaget & Inhelder, 1971, p. 12).

From MoSSAOSS, Table 1 summarizes the constitution of the system of schemes of actions and operations on symbols and signs, its relationship with the figurative and operative aspects of knowledge, and with the vertices and arrows of its representative digraphs.

Table 1 – General representation of the construction of the system of schemes of actions and operations on symbols and signs, and its relationship with the digraphs and the figurative and operative aspects of knowledge, according to the periods.

Period	**Figurative Aspect (States: Objects and Situations)**	**Operative Aspect (Transformations)**
Sensorimotor	Perception (Sensory Images)	Schemes of Actions
Preoperational	+ Symbols (Deferred Imitation, Symbolic Play, Drawing and, especially, Mental Images) + Signs	
Concrete Operational		+ Imagination: Schemes of Operations on Symbols (Transfigurations)
Formal Operational or Hypothetical-Deductive		+ Theorization: Schemes of Operations on Signs (transsignations)
Digraphs	**Vertices**	**Arrows**

8. MoSSAOSS and the Meaning Systems

In this section, the meanings (*significations* in French) of an object, a situation, an action and an operation in relation to the system of action and operation schemes on symbols and signs are defined. As well, the resignifications they come to have according to their modifications are discussed.

According to Piaget (1952, p. 189), "To assimilate a sensorial image or an object [...] is to insert it in a system of schemata [that is, an action scheme system], in other words, to give it a 'meaning'". Or even:

> To say that all knowledge presupposes some assimilation and that it consists in conferring meanings amounts, in the final analysis, to the affirmation that to know an object implies incorporating it into action schemata [that is, into the action scheme system], and this is true from elementary sensorimotor behavior right up to the higher logico-mathematical operations (Piaget, 1971, pp. 7-8).

In this sense, from the point of view adopted here, saying that a subject assigns meaning to an object is equivalent to saying that this object is assimilated by their system of (outer and inner) action schemes, or more explicitly, it is assimilated by their system of schemes of actions and operations on symbols and signs.

According to Piaget and collaborators (Apostel et al., 1957, p. 50), by definition: "The meaning of an object A for a Subject S in a situation T is the set of actions of S that are applicable to A in T".

Considering that actions can be both outer or inner actions (including, operations on symbols and signs), the following definition may be introduced:

Definition 12. The meaning of an object A for a Subject S in a situation T is the set of outer and inner actions (including operations on symbols and operations on signs) of S that are applicable to A in T

The following definition extends the notion of meaning to situations as well:

Definition 13. The meaning of a situation T for a Subject S is the set of outer and inner actions (including operations on symbols and operations on signs) of S that are applicable to T.

According to Hypothesis 2, Definition 4 and Hypothesis 4, there is an increase in the set of actions and, consequently, in the field of action and in the complexity of the system of action schemes. In particular, it increases further with the emergence and coordination of inner actions to the system of outer action schemes and, according to Hypotheses 5 and 6, first, with operations on symbols and, later, with operations on signs. That increase causes a change in the meanings of objects and situations. In this sense, the previous definitions also make it possible to study the resignifications of objects and situations as a result of changes in the system of action schemes and operations on symbols and signs.

As seen, a system is constituted (1) of a set of elements *p* and (2) of relations *R* of interdependence between them. In this sense, the following definition summarizes the considerations about the meanings of objects and situations with regard to the system of schemes of actions and operations on symbols and signs:

Definition 14. The *system of meanings* of states according to MoSSAOSS is constituted (1) by the set of states (objects and situations) of a system of schemes of actions and operations on symbols and signs, and (2) by the relationships established between them by that system.

Since the states (objects and situations) are application points of (outer and inner) actions, they are directly related to the set of applicable actions to

them, that is, to their meanings (according to the last Definitions 12 and 13). That is why the system established by Definition 14 is a system of *meanings*.

In this case, the relationships between the meanings of objects and situations are part of, so called by Piaget, *implication in the wide sense*, which enables the study of the phenomena of consciousness:

> […] either consciousness is nothing, or it arises from original and specific categories which by their very nature ignore material facts. These categories do exist [as the following categories of implication] […] We will use "implication in the wide sense" therefore to characterize these two sorts of connections [(1) the relationship between meanings and (2) the relationship of the significant to the signified object] including the second (which can be distinguished by the term "designation") and we hypothesize that the mode of connection proper to phenomena of consciousness is implication in the wide sense, of which implication in the strict sense is a particular case (Piaget, 2014, pp. 187-188).

In this sense, MoSSAOSS provides a means to reconcile the dichotomy between systems of meanings and logical-mathematical systems (cf. Ramozzi-Chiarottino, 1991). MoSSAOSS makes explicit the logical-mathematical forms underlying the systems of meanings (cf. e.g. [3]). These logical-mathematical forms come to awareness and to the subject's consciousness when the subject themself establishes coordinations of their inner action schemes (their operational schemes) with such logical-mathematical forms. This awareness exists because inner actions are (intentionally) organized and coordinated (with these logical-mathematical forms) by the subject themself, unlike the case of the organization of outer actions, which is also supported in and by the external world. For example, this is the case of the subject awareness of the logical-mathematical order relationship among the sticks A<B<C provided by the transfiguration scheme system represented in Figure 5.

Finally, Piaget and his collaborators define the meaning of an action as follows:

> *Def. 10:* From the point of view of a subject S, the meaning of an action is the set of sub-actions from which this subject S composes it, and the set of actions from which the same subject makes it a sub-action (the words "action" and "sub-action" may be replaced respectively by "coordination of actions" and "partial or coordinated actions") (Apostel et al., 1957, p. 48).

Considering that actions can be both outer and inner (as operations on symbols and signs), the following definition may be introduced:

Definition 15. From the point of view of a subject S, the meaning of an action (both outer and inner, as the operations on symbols and operations on signs) is the set of (outer and inner) sub-actions from which this subject S composes it, and the set of (outer and inner) actions from which the same subject makes it (outer and inner) a sub-action (the words "action" and "sub-action" may be replaced respectively by "coordination of actions" and "partial or coordinated actions").

Therefore, definitions introduced in this section enable one to study the meanings of objects, situations, actions and operations, and their re-significations, correlatively with the studies of constitution of the system of the schemes of action and operations in symbols and signs, which leads to the final considerations of this work.

9. Final Thoughts: A Research Program Based on MoSSAOSS

The introduction of MoSSAOSS ([1]) has enabled a research program which consists of reinterpreting the forms of structures necessary for knowledge (mainly scientific and philosophical) and its genesis, based on MoSSAOSS.

This program has been developed by Tassinari and collaborators, particularly in relation to the necessary structures for: philosophical knowledge ([2]); universal mathematical knowledge ([3], [18]); animism, artificialism and philosophical construction of reality ([4], [21]); the universal predicative capacity

and the propositional function ([5], [17]); formalization in Genetic Epistemology with digraphs ([6]); the structure for concrete operative logic ([7], [14]); the notions of conservation of substance, weight and volume ([8]); the notion of space-time from Special Relativity ([9], [20]); the cognitive reframing in Cognitive Behavioral Therapy ([10], [19]); the significant implication and logical-mathematical structuring of the Real ([11]); the Practical Group of Displacements ([12], [15]); the formation of the action scheme system of the notions of space and object conservation in the Sensorimotor Period ([15]); the notion of meaning, in particular, in relation to the notion of time ([16]); values and their systems, and affective development in general ([22]).

Wittgenstein (1961, p. 115) said that "The limits of my language mean the limits of my world". Parodying Wittgenstein, Ramozzi-Chiarottino rewrites: "the limits of one's schemes are the limits of one's world (Ramozzi-Chiarottino, 1984, p. 64)". According to MoSSAOSS, as a general result of this work, and agreeing with Ramozzi-Chiarottino, it may be concluded that the limit of each person's world is the limit of their system of schemes of actions and operations on symbols and signs, that is, on what they act, imagine and theorize.

10. Productions based on the MoSSAOSS

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