

The Problem of Closure and Questioning Attitudes

Richard Teague

Abstract

The problem of closure for the traditional unstructured possible worlds model of attitudinal content is that it treats belief and other cognitive states as closed under entailment, despite apparent counterexamples showing that this is not a necessary property of such states. One solution to this problem, which has been proposed recently by several authors (Schaffer 2005; Yalcin 2018; Hoek *forth.*), is to restrict closure in an unstructured setting by treating propositional attitudes as *question-sensitive*. Here I argue that this line of response is unsatisfying as it stands because the problem of closure is more general than is typically discussed. A version of the problem recurs for attitudes like wondering, entertaining, considering, and so on, which are directed at questions rather than propositions. For such *questioning attitudes*, the appeal to question-sensitivity is much less convincing as a solution to the problem of closure.

1 Introduction

When we wonder about some question, we entertain possible answers, acquire and notice evidence that bears upon them, and refine our questioning in various ways that advance our inquiry. One thing we also do is *overlook* possibilities. When wondering where my keys are, I might overlook the possibility that they are already in my pocket. When wondering what hand my opponent has in poker, I might overlook the possibility that they have four aces. Or when wondering how the cosmos originated, I might overlook the possibility that it was caused by some strange phenomenon hitherto undreamt of.

Superficially then, it seems we can overlook possibilities for a variety of reasons. For example, simple forgetfulness (the keys), the intrinsic difficulty of entertaining many different possibilities at once (the poker game), or because of a simple failure of imagination (the origin of the cosmos). Whatever

our reasons for overlooking possibilities while we wonder, the fact that we do so tells us that there are all sorts of questions related to the ones we wonder about that we fail to think of, notice, or consider. Thus, while wondering where my keys are, I fail to consider the question of *whether they are in my pocket*. While wondering what hand my opponent has, I fail to consider the question of *whether they have four aces*. And while wondering how the cosmos originated, I fail to consider *whether it originated in a strange phenomenon hitherto undreamt of*.

This paper argues that these failures to consider, attend to, or wonder about questions related to those we do wonder about constitute a significant phenomenon for our understanding of our content-directed attitudes. These failures are significant because they can be understood as an unexplored instance of a problem already familiar in epistemology, namely, the problem of closure. Moreover, I argue that when it comes to questioning attitudes like wondering or considering, a recent and otherwise plausible approach to the analysis of content-directed attitudes (the question-sensitive approach) doesn't offer clear and satisfying answers about this.

For readers who wish to skip ahead, the structure of the paper is as follows: Section 2 introduces the problem of closure for content-directed attitudes. Section 3 follows up with a brief overview of the question-sensitive approach to this problem. Section 4 illustrates how the problem of closure can recur for an analysis of questioning attitudes that mirrors the Hintikka-style analysis of propositional attitudes and argues that the question-sensitive approach is much less helpful and convincing in this context. And section 5 concludes with some brief and tentative suggestions about where to go from here.

2 The problem of closure

On the traditional unstructured possible worlds approach to propositional attitudes, an agent is said to bear a propositional attitude to some proposition just if that proposition is true in all members of some designated set of possible worlds (Hintikka 1962; Stalnaker 1984; Lewis 1986):

Unstructured Attitudes (UA)

An agent S bears the propositional attitude A to the proposition p iff p is true in all of S 's A -worlds.

As a familiar example of this, letting A be the attitude of knowledge, and the A -worlds be the *epistemically* possible ones (i.e., the worlds compatible with what S knows), S knows p whenever p is true throughout these

epistemically possible worlds. However, this approach famously runs into difficulties. Given that a proposition p entails another proposition q iff q is true at a world whenever p is, an immediate consequence of UA is the following:

Attitudinal Closure (AC)

If an agent S bears the propositional attitude A to a proposition p , then S bears A to all of p 's consequences.¹

For the case of knowledge, the relevant instance of AC is commonly known as “logical omniscience”:

Logical Omniscience (LO)

If an agent S knows p , and p entails q , then S knows q .²

Ordinarily, AC is taken to be a problem for our attempts to represent the logic of propositional attitudes by means of an unstructured possible worlds approach to attitudinal content.³ One reason for this is that many (perhaps all) epistemic agents are logically *non-ideal*. We humans, for example, often seem not to know, believe, or desire the consequences of what we know, believe, or desire. At first glance, there are a couple of reasons why this might be the case. Most straightforwardly, we might lack the *attentional* and *computational* resources (e.g. the time, memory, and focus) required to *deduce* the consequences of our attitudes; in fact, the basic UA account seems to leave no room for deduction in the first place (Stalnaker 1984, ch. 5). We might also lack the *conceptual* resources required to hold attitudes towards entailments, as suggested by the following example from Stalnaker (1984):

William III of England believed, in 1700, that England could avoid a war with France. But avoiding a war with France entails avoiding a nuclear war with France. Did William III believe England could avoid a nuclear war? It would surely be strange to say the he did. (p. 88)

¹There are, in fact, *multiple* “problems of closure”, e.g. the problem of closure under necessary equivalence, closure under conjunction, and closure under believed/known/desired implication. Here I stick with closure under entailment, though the points I make apply *mutatis mutandis* to other forms. See Yalcin (2018), pp. 4-5 for an overview of these problems.

²Note that logical omniscience implies that every epistemic agent knows all logical truths. For a survey of logical omniscience and various responses to it see ch. 9 of Fagin *et al.* (1995).

³As well as the logic of some modalities, e.g. deontic modality. A classic version of the problem as it arises for the logic of obligation is Ross’s paradox (Ross, 1941): if you ought to post the letter then you ought to post it or burn it.

In a case like this our default judgment is typically supposed to be that if we *were* to conclude that William III believed England could avoid nuclear war, simply because this is entailed by what he did believe, then we would be making some sort of mistake about the nature of belief. And the same point applies, *mutatis mutandis*, to other propositional attitudes.⁴ Thus, the unstructured possible worlds account of attitudinal content, when applied to propositional attitude reports, seems like a failure in characterizing anyone but the most logically perfect angels. By subscribing to this account, we end up representing ordinary agents as having knowledge, beliefs, or desires that they ostensibly do not have.

But the unstructured approach to the contents of our attitudes has proven useful for thinking about intentionality and content in a broad sense, and has advantages that some more structured approaches do not.⁵ As a result, there have been many attempts to hold on to as much of the unstructured approach as possible while nevertheless altering it to avoid a full commitment to AC. One way of doing this is to add slightly more structure to the account by introducing some additional parameter to our analysis of propositional attitudes, effectively *relativizing* these attitudes to the parameter. Examples of such additional parameters include sets of contextually relevant alternatives (as in Dretske 1970, 2005; Lewis 1996), topics or subject matters (e.g., Yablo 2014, 2017, Hawke 2016), or questions (Schaffer 2004, 2005; Yalcin 2018; Hoek *forth.*). Adding more structure or additional parameters allows us to avoid AC by denying that all there is to bearing a propositional attitude towards some proposition is for that proposition to be true in all of some designated set of possible worlds, though the details will vary based on the proposed relativity to a particular parameter. In this paper, I focus on proposals that respond to the problem of closure by taking belief and other propositional attitudes to be *sensitive to a question*.

⁴Stalnaker himself is openly ambivalent about this example, however (*locus cit.*). This is, I take it, because his dispositional conception of propositional attitudes calls the problematic status of closure itself into question.

⁵It is beyond the scope of this paper to discuss these; the literature on the issue is large. A useful starting point is Stalnaker (1984) ch.2, which explores the benefits of the unstructured approach in part by contrasting it with the drawbacks of a more structured approach, specifically a *linguistically structured* approach of the sort discussed in Field (1978). Yalcin (2018, p. 3) also discusses the advantages of the unstructured approach in capturing *holism* about attitudinal content. Fodor and Lepore (1992) is a classic examination and criticism of holism, though see Perry (1993) for a rebuttal.

3 Question sensitivity

What exactly is it for a given propositional attitude to be sensitive to a given question? We might say: to believe, know, or desire p is not merely to stand in a two-place cognitive relation to p but to believe, know, or desire p *as an answer to a particular question*. As Yablo (2014) puts it, the contents of our propositional attitudes are “directed” at questions or subject matters. Yalcin (2018) provides the following characterization:

To be sensitive to a question seems at least to be equipped with possible states that distinguish possible answers to the question, and to be receptive to information which speaks [to] the question...Understood in this way, question-sensitivity is the sort of thing simple measuring devices [in addition to propositional attitudes] can manifest. My thermostat is equipped with possible states that distinguish possible answers to the question, *within what range is the temperature in this room?*, and it is receptive to information which speaks [to] that question. It is not equipped with possible states that distinguish possible answers to the question *how is the weather in Topeka?*; it is not receptive to information which speaks [to] that question. (p. 13)

As another example, consider the proposition that cicadas are making that loud noise. On a question-sensitive account, one might believe or know this proposition as an answer to at least two different questions:

Q1. What is making that loud noise?

Q2. Are cicadas making that loud noise?

Even in a scenario where a true answer to both of these questions is indeed that cicadas are making that loud noise, it nevertheless seems plausible that someone might believe or know the answer to one of these questions without necessarily believing or knowing the answer to the other. As one manifestation of this, an agent might have the ability to truly and reliably answer Q2 when this question is asked without having a similar ability with respect to Q1.

Taking beliefs and other attitudinal contents to be answers in this way involves treating these contents as question-sensitive propositions; not mere unstructured propositions (i.e. sets of possible worlds), but propositions *qua* answers to questions (Schaffer’s 2005 “contrastive” structures’, Yablo’s 2014

“directed contents”, Hoek’s 2019 “quizpositions”). The full technical details of how we might flesh out a question-sensitive account of attitudinal content are interesting and there is scope to develop the notion in different ways, e.g. depending on exactly how one models questions. However, here, for the sake of simplicity, I assume that questions can be modeled as *partitions* of logical space (following the influential work of Groenendijk & Stokhof 1984), and I take answers (or question-directed propositions) to be ordered pairs of the form $\langle p, Q \rangle$ consisting of a set of possible worlds p and partition Q , where p is a union $p_1 \cup \dots \cup p_n$ of cells from the question Q :

Questions and answers

A question Q is a partition of logical space and a (directed) answer to this question is a pair $\langle p, Q \rangle$ where p is a union of cells in Q .⁶

This apparatus also allows us to distinguish between *complete answers* and *partial answers*: a complete answer to Q is a pair $\langle p, Q \rangle$ where p is an element of Q , while a partial answer is an answer that is not complete.⁷ What I have to say about questions here primarily concerns approaches on which they are unstructured entities, i.e. sets of sets of unstructured propositions. Restricting myself to a model on which the sets identified with questions are specifically *partitions* is thus only a cosmetic issue; my arguments are intended to carry across to any similarly unstructured approach.

However one models questions or the contents of questioning attitudes on an unstructured view, what matters is that if one’s attitudes are question-sensitive, we can arguably make some progress towards a substantiated denial of AC. Going back to Stalnaker’s William III example, a question-sensitive account can begin to explain why the eighteenth century monarch did not believe England could avoid nuclear war despite his believing that England could avoid war. Roughly, the story goes as follows: although the proposition that England can avoid war truth-conditionally entails the proposition that England can avoid nuclear war, such propositions are only

⁶While the partition approach goes back to Groenendijk and Stokhof (1984), Lewis (1988) also models questions in this way. Those interested in relevant applications can explore Yalcin (2018) or Hoek (2019) both of whom take a similarly unstructured approach to questions; though whether they wish to identify these unstructured entities as the contents of questioning attitudes is unclear. Another central theory comes from Ciardelli *et al* (2019) who develop a theory of *issues* covering both informative and inquisitive content. On this latter account, attitudinal contents are modeled as downward closed sets of information states (coarse-grained propositions), allowing that the same information state may constitute a different content depending on which issue/question it resolves.

⁷A partial answer can thus be modeled as a union of complete answers.

ever believed relative to questions that they answer, i.e., they are only believed *as* answers to questions. But any two questions, Q and Q' , are not the same question if one has a possible complete answer that the other does not (formally, this just amounts to Q and Q' being different partitions). And the question of *whether England can avoid nuclear war* has a possible complete answer that the question of *whether England can avoid war* does not, i.e., the answer that England can avoid nuclear war. Hence these questions are distinct, and so their complete answers—thought of as question-directed propositions—are also distinct. Given that beliefs are answers to questions, then, it follows that one can believe that England can avoid war (relative to Q) without *ipso facto* believing that England can avoid nuclear war (relative to Q' or some other question).

In general, when Q and Q' are distinct questions like this, the claim is that it is possible to believe a given proposition as an answer to one without believing it as an answer to the other. Furthermore, when one proposition that constitutes an answer to Q entails another that constitutes an answer to Q' , one can believe the former proposition as an answer to Q without necessarily believing the latter as an answer to Q' . More concretely, William III might believe that England can avoid war without thereby believing that England can avoid nuclear war, because those beliefs are complete answers to different questions. The former completely answers the question of *whether England can avoid war* whereas the latter completely answers the question of *whether England can avoid nuclear war*. On the question-sensitive view, the distinctness of these two questions—visible in their different answerhood conditions—underlies the distinctness of belief contents.

Clearly there is much more to be said about question sensitivity. For example, how should we characterize cases where one's propositional attitudes are entirely unsettled on some question? What should we say, in other words, when one has thought long and hard about a question Q but one's attitudes simply remain undecided with respect to Q in that one doesn't believe, desire, or know any of its answers to be true? In such unsettled cases, to echo Yalcin (*ibid*, p. 13), one's attitudes are still receptive and responsive to information relevant to settling Q , so it seems natural to hold that one's attitudes *are* sensitive to Q , despite being unsettled in this way.

One way of cashing this out further is to note that, in the case of William III, in addition to not having any beliefs about nuclear war, his propositional attitudes weren't receptive or responsive to information relevant to settling questions of nuclear war. I take it that this lack of receptivity and responsiveness at least implies that William III was not in a position to *consider*, *entertain*, or *wonder* about any questions concerning nuclear war.

Thus, what “being in a position” to consider, entertain, or wonder about Q amounts to at least involves having the concepts required to explicitly consider, entertain, or wonder about Q , though it might also involve having the quantitative resources (time, memory, focus) required to do so. This supports the following partial explication of question-sensitivity:

Question sensitivity (QS)

For any agent S , S 's propositional attitudes are sensitive to Q only if S is in a position (e.g. has the relevant concepts) to consider, entertain, or wonder about Q .

Given these considerations, the fact that the set of worlds in which England can avoid war is a subset of the set of worlds in which England can avoid nuclear war no longer guarantees that someone who believes the proposition identified with the former also believes the proposition identified with the latter. The distinctness of the questions involved, as well as the possibility that one is in a position to consider, entertain, or wonder about one of those questions without necessarily being in a similar position with respect to the other, underpins the separability of belief contents. And this is so despite the fact that the coarse-grained propositions that partly constitute those belief contents are such that one truth-conditionally entails the other. The same conclusion can then be applied, *mutatis mutandis*, to other propositional attitudes. Consequently, it really looks like AC can be avoided thanks to the question-sensitive approach, though we might still hold on to a restricted version of closure requiring that attitudes directed at *any particular question* be closed under entailment.

4 Questioning attitudes and the logic of questions

Despite the apparent advances of the question-sensitive approach to propositional attitudes, when it comes to questioning attitudes like wondering, considering, and entertaining, a version of the problem of closure resurfaces, and it's not clear how the appeal to question-sensitivity can help. I assume that the contents of such questioning attitudes are simply questions (see Friedman 2013 for extensive argument for this claim), i.e. that questions are whatever play the role of the contents of questioning attitudes.⁸ And, in much the same way as the traditional unstructured possible worlds model

⁸In linguistic circles, there is a classification of attitude verbs as being *rogative*, *anti-rogative*, or *responsive* depending on whether they license interrogative complements only, prohibit interrogative complements altogether, or allow either interrogative or non-interrogative complements, respectively (see for instance Lahiri 2002). For example ‘won-

of attitudinal content brings with it a commitment to AC, analyzing the contents of questioning attitudes in a similarly unstructured setting, e.g. by modelling questions as partitions, results in a version of closure for these attitudes as well.

Questions, like propositions, have a logic all of their own.⁹ And just as we speak of knowing, believing, and desiring propositions to be *true*, we speak of *resolving*, *settling* and, most commonly, *answering* the questions we wonder about, consider, or entertain. In line with this, we not only have propositional attitude reports to contend with, we also have questioning attitude reports such as the following:

- (i) Alice is wondering what's making that loud noise.
- (ii) Bob is considering whether cicadas are making that loud noise.

So, along with the project of determining the truth conditions and logic of propositional attitude reports, there is the related project of determining the truth conditions and logic of questioning attitude reports. Indeed, given the central role that questioning attitudes play in inquiry, this project seems especially important from an epistemological point of view.¹⁰

The unstructured possible worlds account of the contents of our propositional attitudes offers ways to interpret logical relations among those contents in terms of set theoretic relations among sets of possible worlds. Similarly, a partition-based interpretation of the contents of questioning attitudes, on which they are *sets* of sets of possible worlds, offers ways to interpret various logical relations among them. For example, on the partition approach, we can define an analogue for questions of the relation of propositional entailment (which I here call 'q-entailment'):

der' is rogative, 'believe' is anti-rogative, and 'know' is responsive; one can say, e.g., 'I know that it's raining' as well as 'I know whether it's raining'. Since I take knowledge to be a *propositional* attitude however, it is not clear that these syntactic categories perfectly match the distinction between propositional and questioning attitudes.

⁹Indeed, as with propositions, there are various *logics* (plural) of questions that have been extensively explored. See Hamami and Roelofsen (2015) for an overview.

¹⁰Hintikka's (1981, 1999, 2007) work on his interrogative model of inquiry, which describes inquiry as an essentially questioning activity (e.g. a question-and-answer game between an inquirer and nature) is a leading example of this perspective. In some places (e.g. 2007, pp.24-28), Hintikka explicitly characterizes questions in terms of the attitude of desiring to know their answers. Within inquiry, then, Hintikka's view is that a question is determined by a pair, consisting of a presupposition and a desideratum, with the latter being the state of knowledge that the agent posing the question thereby desires to be in.

q-entailment

A question Q q-entails a question Q' iff any proposition that constitutes a complete answer to Q entails some proposition that constitutes a complete answer to Q' .

Taking questions to be partitions of logical space, q-entailment is the relation between two partitions, Q and Q' , that holds just when any $p \in Q$ is such that there is some $p' \in Q'$ for which $p \subseteq p'$. For example, the question of *what is making that loud noise* q-entails the question of *whether cicadas are making that loud noise*, because a complete answer to the former entails a complete answer to the latter.¹¹ As another example, take the question *who is the murderer* as it arises in the context of the board game *Clue*, which q-entails the question of *whether Professor Plum is the murderer*.

Another way of thinking about this is that Q entails Q' whenever completely answering Q' would at least partially answer Q . Thus for any question, Q , whose complete answers are $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$, and $\neg p \wedge \neg q$, Q q-entails any question whose complete answers are p and $\neg p$ as well as any question whose complete answers are q and $\neg q$. Suppose you ask the following question with rising intonation on each name: *will Alice be at the party, or Bob?*, then your question q-entails both *whether Alice will be at the party* and *whether Bob will be at the party*. Thus, relations of q-entailment are also of clear relevance to the project of understanding the norms of inquiry; to answer a difficult question, it is often a good idea to first try and answer the questions that it q-entails.

With this notion of q-entailment in place, one can then offer a formal treatment of questioning attitudes that is the natural parallel of the earlier Hintikka-style treatment of propositional attitudes. By analogy with UA, which associates a set of *possible worlds* with each propositional attitude the agent has, we can associate a set of *propositions* with each questioning attitude an agent has.¹² This set of propositions corresponds to the conjunction of all the questions (i.e. the union of all partitions) the agent is wondering about/entertaining/considering. This is much the same as some set of possible worlds corresponding to the conjunction of all propositional attitude contents (i.e. the intersection of all the sets of possible worlds) to

¹¹Recall that a complete answer to a question rules out all possible answers incompatible with it, and so all other possible complete answers

¹²Or, if we insist that the contents of propositional attitudes shouldn't simply be identified with propositions, we can think of this as a set of propositional attitude contents rather than a set of propositions. Thanks to an anonymous reviewer for *Synthese* for encouraging me to emphasize this distinction.

which an agent bears a propositional attitude.¹³

Hence, just as bearing a propositional attitude towards some proposition was interpreted on the unstructured account as that proposition's being true in every member of some select set of possible worlds (e.g. the epistemically possible ones), bearing a questioning attitude towards some question can be interpreted as that question's being *answered* (or resolved) by all members of a select set of propositions, e.g. those such that, if the agent *were* to know them, then what she is wondering about, considering, or entertaining would be resolved.

For a given agent, *s*, we can call these propositions that would resolve her inquiries *s*'s *inquiry relevant propositions*, and denote the set of them I_s for convenience. In effect, I_s is one big question/partition, i.e. the one that, if completely answered, would resolve every question that *s* is wondering about. Given this set up, a first pass at an analysis of questioning attitude reports might go something like this (here I focus on wondering as my chief example, but I assume similar analyses apply to other questioning attitudes, e.g. entertaining and considering):

Unstructured Wondering (UW)

An agent, *s*, wonders *Q* iff every member of I_s entails a complete answer to *Q*¹⁴

UW is the counterpart of UA for questioning attitudes, and amounts to the claim that one wonders *Q* just if whatever one is wondering about could be answered by something that would also answer *Q*. Because this analysis directly parallels the UA analysis of attitudinal content, its patent circularity is to be expected. For example, just as one believes *p* on the unstructured account whenever *p* is true in every world consistent with what one *believes*, one wonders *Q* on this account whenever *Q* is answered by every proposition that completely answers what one *wonders* about.¹⁵

But now we have a problem. Putting UW together with the notion of q-entailment, we have the following consequence:

¹³This is the approach taken to questioning attitudes in Ciardelli and Roelofsen (2015) and van Benthem and Minică (2012)

¹⁴For wondering, it might also be important to include that the agent does not *know* *Q*, where this means that there is no complete answer to *Q* that *s* knows. Whether and in what sense one can wonder about what one knows is an interesting issue (See Archer 2018). Ciardelli and Roelofsen (2015, pp. 1659-1660) build an ignorance component into their analysis of wondering. But for simplicity I ignore this complication as it will not bear significantly upon my arguments.

¹⁵Cf. Ciardelli and Roelofsen (2015, pp.1648-60) for similar analyses of entertaining and wondering.

Wondering and q-entailment

An agent, s , wonders Q iff I_s q-entails Q

Return to the William III counterexample to attitudinal closure. The question-sensitive explanation of why he believed that England can avoid war without thereby believing that England can avoid nuclear war was that these beliefs are answers to distinct questions, one about war, the other about nuclear war, and while William III was in a position to wonder about whether England can avoid war it seems fair to say that he simply lacked the concepts required to consider, entertain, or wonder whether England can avoid nuclear war.

But another question, one that, *prima facie*, it seems William III *could* have wondered about, is: *which kinds of war can England avoid?* Here it seems plausible to imagine him wondering about the avoidance of land or naval wars, for example, or wars fought on domestic or foreign territory. Thus he might have wondered whether England can avoid a land war on the continent. And yet, on the current definition of question entailment, the question *which kinds of war can England avoid*, q-entails the question *whether England can avoid nuclear war*. In other words, any proposition that constitutes a complete answer to the former entails a proposition that constitutes a complete answer to the latter.

The problem currently before us is that an unstructured analysis of questioning attitudes like UW, built by analogy with UA, implies that if an agent, s , wonders Q , then s wonders Q' , for any Q' such that Q' is q-entailed by Q .¹⁶ Assuming that a similar analysis is given for other questioning attitudes (e.g. considering, entertaining and so on¹⁷) we will end up with the following:

Questioning Attitude Closure (QAC)

If an agent, s , bears a questioning attitude A to Q , then s bears A to Q' , for any Q' such that Q' is q-entailed by Q .

In other words, a theory of questioning attitudes developed by analogy with the unstructured possible-worlds account of propositional attitudes, one on

¹⁶The proof of this is immediate and trivial given the analysis of wonder reports, UW, and the definition of q-entailment. Suppose s wonders Q , where Q q-entails Q' . Then it follows from UW that any proposition that constitutes a complete answer to what s wonders about also constitutes a complete answer to Q . But, since Q q-entails Q' , it follows that any proposition that constitutes a complete answer to Q also constitutes a complete answer to Q' . Thus it follows from UW that s wonders Q' .

¹⁷Ciardelli and Roelofsen 2015(p. 1654) propose an unstructured analysis of this sort for entertaining a question.

which their contents are coarse-grained partition-like entities, results in the closure of these attitudes under q-entailment.¹⁸

But, just as AC is a problem for the traditional possible worlds account of propositional attitudes, I think QAC is a problem for the parallel coarse grained treatment of questioning attitudes, in which case it is also a problem for epistemologists aiming to understand the role these attitudes play in inquiry. It is an odd result indeed to suggest that William III, in the eighteenth century, was wondering about nuclear war. More generally, it is also an odd result for a theory of questioning attitudes to suggest that *anyone* wondering, considering, or entertaining Q (whatever their conceptual repertoire) is thereby wondering, considering, or entertaining every q-entailment of Q . This is because a question's q-entailments (its 'coarsenings' or 'sub-questions') are not wondered about, considered, or entertained *trivially* as a consequence of wondering, considering, or entertaining Q .

Anyone who has played the game *Twenty Questions*, in which one aims to answer the question of what person, place, or thing one's opponent is thinking of, knows that it is a non-trivial exercise to wonder about the many 'yes-no' subquestions that eventually allow one to narrow in on the correct answer. Arguably, a great deal of inquiry just *is* the strategic and temporally extended 'divide-and-conquer' work involved in coming to wonder about, consider, or entertain easier and 'smaller' questions whose complete answers take one closer to answering one's initial 'big' question. Hence, QAC, which trivializes all of this non-trivial work, must be false. And so, any unstructured account of questioning attitudes like UW, which leads to a result like QAC, seems like it too will have to be revised, much as we had to revise UA for propositional attitudes.

Notably, some question-sensitive accounts of propositional attitudes are already susceptible to a related closure worry. This arises due to a commitment in these accounts to a *question restricted* version of closure:

Restricted Attitudinal Closure (RAC)

If an agent s bears A to $\langle p, Q \rangle$, where p truth-conditionally entails p' and Q q-entails Q' , then s bears A to $\langle p', Q' \rangle$ ¹⁹

¹⁸Indeed, it is enough that the account of questioning attitude contents at work in question-sensitive epistemology be coarse-grained (as is assumed by Schaffer (2004,2006), Yalcin (2018), Ciardelli and Roelofsen (2015), and Hoek (forth.)), the restriction to partitions specifically, rather than sets of *overlapping* unstructured propositional attitude contents, isn't essential.

¹⁹cf. Yalcin's 2018 (p.19) 'closure under visible consequence' and Hoek's 2019 (p.16) 'closure under parthood'. Hawke (2016, pp. 2778-81) proposes restrictions on closure based on subject matters determined by atomic predications occurring within sentences.

William III again provides a counterexample to RAC: as an answer to the question *which kinds of war can England avoid* he believes that England can avoid any kind of war. But he does *not* believe that England can avoid nuclear war, despite this apparently being an answer to the very same question. But proponents of question sensitivity have a plausible response to this sort of counterexample. They can argue that the question William III is actually wondering about, and thus the question to which his propositional attitudes are in fact sensitive, is not the unrestricted constituent question *which kinds of war can England avoid*, but rather a restricted version of this question: *which kinds of war (e.g. land, naval, domestic, foreign, brief, protracted, ...) can England avoid?*²⁰

That is, the questions to which William III's attitudes are in fact sensitive are those whose answers are restricted to the concepts he in fact possesses.²¹ Since he lacks the concept NUCLEAR WAR, his propositional attitudes are not sensitive to the unrestricted question: *which kinds of war with France can England avoid?* Furthermore, this domain-restricting of questions to those sets of answers the agent has the concepts to entertain might also serve in response to QAC.

Here the thought is that just as an agent's question sensitive beliefs are not necessarily sensitive to every q-entailed question, her questioning attitudes themselves may not be sensitive to every q-entailed question. An agent who wonders, considers, or entertains Q , where Q q-entails Q' need not wonder, consider, or entertain Q' when she lacks the concepts required to do so. This seems quite intuitive. When one lacks the concept of nuclear war, one's cognitive state, whether a propositional or questioning attitude simply isn't defined on questions of nuclear war.

Nevertheless, the problems posed by QAC are not limited to those of conceptual poverty. What examples of inquiry like *Twenty Questions* show, is that inquiry, like deduction, is a non trivial task even when the inquirer possesses all of the concepts required to succeed. It requires effort to wonder about, consider, or entertain the q-entailed questions that allow one to further the goals of one's inquiries. And, for non-ideal agents, as my opening examples show, it is common enough to *fail* to wonder about, consider, or

But I think this approach to subject matters can be re-interpreted in terms of question-based restrictions too, albeit on a slightly more syntactic model of questions.

²⁰More abstractly, thinking of concepts as sets of entities to which they apply, and letting C_W denote the intersection of William's 'war concepts' (e.g. NAVAL WAR) he is wondering a question of the form: *for which values of $x \in C_W$ is x a war that England can avoid?*

²¹See Yalcin *ibid* p. 12

entertain the q-entailments that advance one's inquiries – as Hoek (*forth.*) puts it: a good question can be hard to find. This sort of failure is also shown in cases like the following (adapted from Stalnaker 1984):

Absent Minded Detective

A detective has been investigating a murder at the manor, spending her time wondering *who the murderer is*. After some time she narrows in on the butler as a key suspect. Accordingly, she begins to wonder *whether the butler is the murderer*. However, while wondering this, she overlooks the chauffeur, whom she had only briefly considered at the outset of her investigation. Thus, because she is not wondering *whether the chauffeur is the murderer* she fails to realize that the evidence points to him even more convincingly than it points to the butler.

Here we have a scenario in which an agent wondering *Q* (*who is the murderer*) isn't wondering one of *Q*'s q-entailments (*whether the chauffeur is the murderer*) despite clearly having the concepts required to do so. Contrary to this, QAC implies that the detective *is* wondering whether the chauffeur did it. A puzzle for question sensitivity as a response to QAC is that it's not clear why we should think that the agent's questioning attitudes *aren't* sensitive to the question of *whether the chauffeur is the murderer*.

Intuitively, while wondering who the murderer is the detective is disposed in some way or other to respond to evidence pointing to the guilt of the chauffeur (for example she might revise her beliefs or modify her inquiry if the possibility of the chauffeur were explicitly brought to her attention). It's just that she isn't disposed to do this in precisely the same way that she is disposed to respond to evidence bearing on the butler (she is already paying attention to him).²² Consequently, this situation suggests a conflicted picture: on the one hand the detective's questioning attitudes are sensitive to the question of *whether the chauffeur is the murderer*, but on the other hand they aren't.

As per QS in the last section, there is a clear way in which the detective *is* in a position to wonder, consider, or entertain *whether the chauffeur is the murderer* – i.e. she possesses the necessary concepts and has even been recently aware of the chauffeur as a suspect. Thus, her propositional attitudes are sensitive to this question in at least some way. It's just that,

²²Yalcin *ibid.* (p. 13) also notes this fact about the detective, but leaves it unexplored as to the precise sense in which the detective's cognitive state is sensitive to the overlooked question.

despite this sensitivity, the detective simply doesn't wonder about, consider, or entertain this question. Thus, even modulo her concepts, the detective's questioning attitudes are not trivially closed under q-entailment.

I think that examples like the absent minded detective bring us to the crux of the problem with developing an account of questioning attitudes in an unstructured setting of the Hintikka variety I consider here. Even allowing for questions within the agent's conceptual remit, and controlling for questions that are q-entailed by those she is already wondering about, considering, or entertaining, it simply doesn't follow that her questioning attitudes are closed under q-entailment. Moreover, it isn't clear how question-sensitivity can help.

With propositional attitudes, the intuitive appeal of question sensitivity derives from the fact that the contents of these attitudes can be understood as *answers* to questions. Thus propositional attitudes are indirectly about or directed at questions. AC is then blocked because we can distinguish answers to questions based on the distinctness of the questions themselves. But given an unstructured approach to questions, this distinctness is partially lost; that is what the existence of logical relations among the contents of questioning attitudes (like q-entailment) implies. Thus, on an unstructured approach to questioning attitudes – i.e. attitudes that are *directly* about or directed at questions – it is not clear how an appeal to question sensitivity allows us to avoid QAC.

One way of appreciating the problem is to note that, if the absent minded detective's inquiry led her to believe that the butler is the murderer, we can then explain why she doesn't thereby believe that the chauffeur isn't the murderer by saying that she simply didn't *consider* the chauffeur. In other words, she didn't consider the question of whether the chauffeur is the murderer.²³ In effect, we explain a failure of propositional attitude closure by citing a failure of questioning attitude closure. To me, this seems the natural move to make on a question sensitive account of propositional attitudes. But I think we cannot then explain failures of questioning attitude closure in precisely the same way.

How might a question-sensitive response to the problem of closure for questioning attitudes work in detail? Mimicking a question-sensitive account for belief, we could say that, just as agents only believe propositions relative to questions, modelled as partitions, agents only wonder about questions

²³Stalnaker and Yalcin both explicitly present the absent minded detective as failing to *consider* the chauffeur in this way. I take it we can read this as a failure to consider, entertain, or wonder about the question of whether the chauffeur is the murderer. I.e. as a failure to close one's questioning attitudes under q-entailment.

relative to some partition too:

QS-Wondering

An agent, s , wonders Q relative to a partition π , where Q q-entails π , iff I_s q-entails the question corresponding to π

The idea is that π is some partition, some resolution of logical space, on which some distinctions are visible and others aren't. In the case of the absent minded detective, π could be the partition whose cells are just those possible answers the detective is considering, e.g. that the butler is the murderer, the cook is the murderer, or someone other than the butler or the cook is the murderer. It is only relative to this partition then that the detective is wondering who the murderer is. Consequently, she fails to count as wondering whether the chauffeur is the murderer because this possibility is not among the possibilities in π , i.e. those to which her cognitive state is sensitive.²⁴

This sort of account looks initially promising. But I do not think it can adequately capture our default judgments and intuitions about wondering and question-sensitivity in relation to closure. Note that, while aiming to successfully predict failures of closure for wondering, this account must not simply treat the partition π as itself the question being wondered. In other words, if the detective is wondering *who is the murderer* relative to the partition { *It's the butler, it's the cook, it's someone else* }, we can't plausibly say that the question being wondered is in fact the question corresponding to this partition, i.e. *whether the butler, the cook, or someone else is the murderer*. Why not? Because insisting upon this would make a mystery of all sorts of ordinary and intuitively felicitous wonder reports. Here is an example:

Sergeant: Still working on the manor case?

Detective: Yeah, still wondering who did it.

Sergeant: Well, allow me to answer your question, then. Both the cook and the butler provided air-tight alibis this morning, so it cant have been them.

Detective: Ah, well that certainly is helpful, Sergeant, but I'm still wondering who did it.

²⁴My thanks to two anonymous reviewers for *Synthese* for pushing me to consider this approach further

I take it that the detective's final response here makes perfect sense. She is wondering who the murderer is, and her question hasn't been answered merely by ruling out the butler and the cook. Yet, if the question she were wondering about could be interpreted as the partition whose cells were merely the propositions that *the butler did it*, *the cook did it*, and *someone else did it*, i.e. a question that does not q-entail *whether the chauffeur is the murderer*, then her question would be completely answered by the sergeant's information, and so her final response wouldn't make sense.

Instead, what this sort of exchange suggests to me is that it remains more plausible that the question the detective is in fact wondering about is one whose possible answers *do* include the answer that the chauffeur is the murderer. That is, if she were to learn that the chauffeur is the murderer, this would resolve the question she is wondering about. But this leaves us to conclude that the question the detective is wondering about does indeed q-entail *whether the chauffeur is the murderer*.

Informally, there are all sorts of clear ways in which the detective's cognitive state is sensitive to questions about the chauffeur. When asked about the chauffeur she is disposed to accept that conclusive evidence of his guilt would indeed answer her question. And, even when she isn't explicitly asked about it, she is receptive to evidence pertaining to his guilt; she need only encounter such evidence for this receptivity to manifest itself. Formally, then, it is entirely plausible that the set of the detective's inquiry relevant propositions, I_s , her 'big question', q-entails *whether the chauffeur is the murderer*. If this is so, then on the QS-Wondering analysis, the agent is represented as wondering *who the murderer is* relative to the partition $\{it's\ the\ chauffeur,\ it's\ not\ the\ chauffeur\}$. But, in the spirit of the QS analysis, this allows us to conclude that the detective *is* wondering whether the chauffeur is the murderer – precisely the conclusion we were hoping to avoid.

This puzzle also persists if we think of the question the detective is wondering about as changing depending on context. We might be tempted to say that in the context in which the detective is operating the interrogative phrase 'who the murderer is' as it occurs in 'the detective is wondering who the murderer is' actually means *whether the butler, the cook, or someone else is the murderer*. But this too would leave it mysterious as to why the detective *still* counts as wondering who the murderer is after learning of the alibis. This change in context does not seem to automatically change the question being wondered.

That being said, there is good reason to think that wh-interrogatives receive their interpretations relative to contextual parameters such as the

concepts under which the agent identifies different possible answers²⁵. However, note that even if we specify a set of concepts under which the alternative possible answers to *who is the murderer* are identified in a context, we can still get unwanted closure results. The sentence ‘The detective is wondering who the murderer is’ could be true relative to a conceptual cover on which the suspects are identified by profession, so that the possible answers are: *the butler is the murderer, the cook is the murderer, the gardener is the murderer, the valet is the murderer, the chambermaid is the murderer, the chauffeur is the murderer, etc.* Nevertheless – assuming these professions are taken to be necessarily distinct in this context, so that necessarily no one person fulfills more than one of them – a question identified with this partition would still q-entail *whether the chauffeur is the murderer*. And this is so even though the detective who is wondering who the murderer is relative to this conceptual cover of professions might still momentarily be overlooking the chauffeur.²⁶

In general, a question-sensitive approach to the problem of closure for questioning attitudes like wondering attempts to explain the closure failure by saying that the agent’s cognitive state simply isn’t sensitive to the questions q-entailed by what she’s wondering about. In other words, that the reason the detective doesn’t wonder whether the chauffeur is the murderer is because her cognitive state isn’t sensitive to this question: if one’s cognitive state isn’t sensitive to Q , then one doesn’t wonder Q . But, given that there are clear ways in which the detective’s cognitive state *is* sensitive to questions about the chauffeur, as evidenced by her dispositions to notice and respond to information about the chauffeur when this is brought to her attention, the remaining notion of question-sensitivity available to the QS approach seems fairly thin.

²⁵See Aloni 2008

²⁶Or suppose someone is wondering what the square root of 324 is, relative to the conceptual cover of natural numbers. This person need not be wondering whether 18 is the square root of 324; if they were they’d presumably answer their question much more quickly. Another worry with relativizing interrogative interpretations to contextually supplied conceptual covers is what to say about cases of conceptual poverty and conceptual change. Regarding conceptual change, if the detective happens to learn that the butler and the valet are actually the same person, would this mean that now, having revised her conceptual cover, she is wondering about a different question when she continues to wonder who the murderer is? I think this is unclear. Regarding conceptual poverty, it is plausible that, for example, Aristotle might have wondered what the origin of the universe is without having the concepts to consider a possible answer in terms of quantum mechanics. But would that mean that such an answer, if true, would not have answered the question posed by the Stagirite? I think that is also unclear. My thanks to an anonymous reviewer for *Synthese* for prompting me to address this.

To be explanatory, the relevant sense of question-sensitivity and the account of wondering it supports clearly needs to be less thin than one on which we merely say that the agent fails to wonder about the chauffeur because she isn't wondering about the chauffeur. In other words, the required notion of an agent's cognitive state being insensitive to Q , which will serve as the explanans for closure failure, needs to be sufficiently distinct from the explanandum, i.e. that the agent fails to wonder Q . But is there a notion of question-sensitivity and a corresponding analysis of questioning attitudes like wondering that can satisfy this demand? That seems unclear at best.

5 Taking stock

We have seen how a question sensitive account of propositional attitudes, as an upgrade to UA, offers some promising inroads towards a response to the problem of attitudinal closure as stated in AC. But, given the existence of questioning attitudes, we can see that the problem of closure is in fact more general than AC; the problem *recurs* for questioning attitudes as well, resulting in QAC.²⁷

One way to go at this juncture is to argue that questioning attitudes must have even more finely structured contents than those presupposed by the question sensitive approach explored here.²⁸ Adding domain restrictions to the contents of questioning attitudes – as motivated by the conceptual limitations of the agent, e.g. William III – is one way of adding more structure. But can we motivate even further structure, e.g. whatever is required to account for the absent minded detective case, without effectively *leaving* the unstructured setting for attitudinal content?

Leaving the unstructured setting, at least for questions, can also be motivated by considering questions whose answers are necessary truths, as in problems like the following (inspired by an example from Stout (2010)): someone learning arithmetic might master addition before subtraction. Thus, during the learning process, she might know the true answer to the question: *what is five plus three?* That is, she might know that the answer is that *five plus three is eight*. However, still struggling with subtraction, she might *not* know the answer to the question: *what plus three is eight?*²⁹ But

²⁷And if there are content-bearing attitudes in addition to propositional and questioning attitudes, it may very well recur for those attitudes too, assuming that their contents have a logic of their own as questions do.

²⁸*i.e.* those presupposed in the approaches favoured by Schaffer (2005), Yalcin (2018), and Hoek (*forth.*)

²⁹More formally: she might know the answer to the question: *for which x is it the case*

the proposition that *five plus three is eight* is true in *all* possible worlds, and any proposition of the form *x plus three is eight* where *x* is not three is true in *no* possible worlds. Thus the questions *what is five plus three* and *what plus three is eight* correspond to the *same* partition, i.e. (where \mathcal{W} is logical space) they correspond to the trivial partition $\{\mathcal{W}\}$.³⁰ Thus, on an unstructured approach to questions, and contrary to assumption, if the learner knows an answer to the first question, she must also know an answer to the second, since they are the same question.³¹ But this simple arithmetical example is only one such case. The point is that whenever the answer to a given question *Q* is supposed to be true in all possible worlds, inquiry into *Q* is trivialized. With unstructured questions, if one knows *what 3+5 is* then, for any necessary truth, *p*, one knows *whether p*, as these are the same question.

In the face of closure problems like QAC, it may well be that a more structured approach to questions is needed if we are to understand the role questioning attitudes play in inquiry.³² And such structured accounts of questions do exist.³³ They can involve explicit inclusions of concepts in our semantics for questions, or they might even involve structure derived from the syntactic constituents of the interrogative sentences by means of which questions are expressed.

However, if the upshot of taking a question sensitive approach to propositional attitudes is that we end up *deferring* the explanatory burden of rejecting AC to that of rejecting QAC, resulting in the temptation to introduce structured contents for questioning attitudes in response to QAC, then it might look like we should have just introduced more highly structured contents for propositional attitudes in the first place; something many have valiantly sought to avoid. But, even so, it is by no means obvious that an appeal to structured contents will solve all of our problems here either.

that $5+3=x$, without knowing the answer to: for which x is it the case that $x+3=8$.

³⁰Indeed, arguably all logical and mathematical questions correspond to this partition.

³¹Formally, in the apparatus of answers as ordered pairs of coarse-grained propositions and partitions, she knows the answer $\langle \mathcal{W}, \{\mathcal{W}\} \rangle$.

³²Friedman (2013, pp. 167-168) also suggests a more structured account on the basis of what she calls ‘the possibility of radical ignorance in inquiry’. This is a putative scenario in which an agent who is unaware of *any* possible answer to a question can nevertheless wonder about, consider, or entertain that question. Personally, I am skeptical about the possibility of such total answer ignorance, in the sense of an inability to imaginatively generate *any* possible (even partial) answer to the questions one wonders about. But this is not the place to explore that skepticism.

³³See, for example, Krifka (2001). Though perhaps an account of questions similar to Hawke’s (2016) ‘state description’ approach to subject matters might also be promising.

The cognitive state of the detective who isn't considering the chauffeur remains sensitive to whether the chauffeur is the murderer. And a derivation of this fact about her cognitive state from a theory attributing only a structured attitudinal content lacking any explicit representation of the chauffeur seems like a tall order to say the least. Alternatively, we might say that the problem is not with the contents themselves but with the way we analyse the attitudes that have these contents. Perhaps there is something peculiar about questioning attitudes in particular that makes them seem ill-suited to analysis in an unstructured setting. Maybe attitudes like wondering, considering, and entertaining are best understood as *occurrent* attitudes, whereas those of believing, knowing, and desiring can be more fruitfully understood as *dispositional*.³⁴ If this is true, then maybe this distinction can help us substantiate a principled separation of our analyses of questioning and propositional attitude reports without having to entirely leave the unstructured setting.³⁵

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³⁴For more on this distinction see Bartlett (2018a,b). For critical discussion of the idea of occurrent mental states see Crane (2013)

³⁵Sincere thanks to Hüseyin Güngör, Robert Rynasiewicz, and especially Justin Bledin for many hours of helpful discussion. My thanks also to the bright minds of the Hammond Society at Johns Hopkins University for their helpful feedback on earlier versions of this paper.

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