

FOR AN EMPIRICAL READING OF PHYSICS

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Abstract

This essay invites the reader to interpret physics from a radically empirical standpoint, both diachronic and relative. We start with some criteria of the theory of knowledge, the basis for interpreting the fundamentals of mathematics and physics.

Then we present some expositions of physics, including a new characterization of time, space and movement, with reference to classical mechanics, relativity and quantum mechanics.

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Preliminary considerations

“We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after another, to account for its origins. At last, we have succeeded in reconstructing the creature that made the footprint. And Lo! it is our own.” (EDDINGTON)

Physics does not often make any reference to the principles of the theory of knowledge that are the foundation of its expositions: it takes for granted that they are unquestionable and universally accepted. But in fact, physics has maintained and still maintains different criteria about the nature of reality and about our knowledge of it. Therefore it is convenient to present in advance some underlying considerations which sustain our proposed empirical reading of physics.

We consider that empirical reality is made up of all the observable things or references, and that they are distributed between two different scenarios. In the exterior scenario are the things that are jointly visible to all people such as objects, events and the relationships between them. In the interior scenario, that we call the mind, are the things than can only be observed individually, each person perceiving his/her own - such as his/her memories, thoughts and sentiments. There are as many interior scenarios, all of them private, as there are human beings, but there is only one universal and public exterior scenario and we will call this our world, this world, or simply the world.

Here we refer exclusively to the exterior empirical reality, giving two different meanings to the term *observations*: the first is that of actions that allow us to obtain empirical information about reality, and the second is the

actual information we obtain from these actions. With this second meaning we understand that observations are collections of sensations integrated in accordance with experience and interpreted according to knowledge already elaborated and to socially shared held convictions.

We will use the term *ways of being* of things to refer to their empirical characteristics: these characteristics arouse physiochemical reactions of our senses, which act on the brain through the sensory nerves originating in it the physiological phenomena linked to the reactions. The sensations that produce in each observer the way of being of things are the mental manifestations that accompany these cerebral phenomena, to the extent that if the phenomena were produced by artificial means the same sensations would be aroused in the mind of the observer.

It results from the above that the ways of being of things provoke in us two simultaneous and interdependent empirical manifestations, each one located in a different scenario. In the brain they provoke external physiological phenomena that can be observed by everyone, and in the mind they provoke sensations that can be perceived only internally, each individual perceiving his/her own sensations.

The physiological phenomena linked to the sensations leave lasting biological marks in the brain, and it is by these marks that the phenomena can be reproduced, wholly or partially, with varying degrees of precision. The sensations linked to the reproduced phenomena are the memories of the ways of being we had observed. By combining memories we can conceive things that had never been observed, such as flying horses or talking trees, but we cannot naturally conceive ways of being that we had never observed, since we have no memory of them.

Observations are always accompanied by interpretations that give them meaning. As Kant remarked on the subject of concepts and perceptions, we may regard observations without interpretations as inarticulate, and interpretations without observations as empty.

Interpretations of reality are arranged in systems of propositions classified as myths and theories; myths mainly contain convictions without any empirical evidence, whereas theories start from observable references. The collection of myths and theories shared by a society forms its system of ideas

and beliefs, what we will call the *cultural awareness* of such society.

It is important to note that the cultural awareness of a society contains convictions considered indisputable and which are deeply rooted and widely spread. In many cases these convictions are articles of faith as regards reality, and are never questioned.

We are going to establish that, in general, any theory concerning reality and empirical knowledge has to be compatible with three basic considerations:

1. The real things of our world can be perceived by any normal observer in the right circumstances for this perception;
2. All observations are interpreted within the framework of the cultural awareness in force, which changes with the passing of time and with the increase of experience;
3. The effective extensions of empirical knowledge test the validity of the interpretations that are included in the already elaborated theories.

With these considerations we propose a radically empirical interpretation of physical reality, which rests in a main postulate: *although the ways of being of things are not dependent on our will, they do depend on the sensations they arouse in us, given that we characterize them or 'dress' them with these sensations.* And we understand at the same time that *the logical structures adopted by empirical theories are always relative because they depend on the development of reason and on the usefulness of the established empirical knowledge.*

In consequence, our sensations are the raw material we use to characterize empirical reality. The ways of being associated to it would not exist in the absence of one or other of our senses. For example: without the sense of sight empirical reality would not have visible ways of being. It would have no sonorous ways of being without hearing, nor hard or soft or cold or hot ways of being without touch; there would no be tastes or smells without the appropriate senses.

Since visual observations, very rapid and much more comprehensive than the rest, predominate in this sphere, we will refer to them from now on, unless the contrary is stated.

The senses are not the only channel for the empirical characterization of reality, all the other human characteristics come into play. The size of our body determines the structure and the spatial discernment we make of large objects; if our body were gigantic, earth-like, or microscopically small, then our characterization of spatial structures would be very different in these extreme cases.

In a similar way, the rhythm of the biological processes of our body defines the speed with which we observe things and record memories of them, and this determines our discrimination in characterizing the temporal sequence of events. If we could either observe or remind millions of things per second or, on the contrary, our mind were extremely slow in either observing or processing memories, in each of these extreme cases our ability to discriminate events by their successive parts would be very different.

As to the objectivity of the ways of being of things, we understand this must be taken as relative since it is always referred, implicitly or explicitly, to a certain group of observers.

Therefore we consider there is no single empirical characterization of reality; different characterizations may coexist due to different observers, all of them equally *real*. Our world is single but its empirical identity is not as it may be characterized simultaneously as visible or invisible (by observers gifted with sight or by the blind), sonorous or silent (by those with or without the sense of hearing), and so on.

When we suppose that the world would exist even without human beings, this might mean either that the world could continue even though humans became extinct or that the world would have existed without the presence of human beings. The first of these suppositions might come about one day, but the second is a contradiction in terms since all the things and all the events of all the worlds imaginable are characterized by our sensations. There is no empirical sense in imagining a world existing without us, it is we who imagine it and characterize it.

In short, the radical empirical reading of physics that we propose is based on three basic considerations:

1. All the things of our world *are* in it, but *not existent* of themselves. When we observe things we don't simply see them but we characterize their ways of being.
2. The ways of being of things are always relative to our natural attributes and to the content of our cultural awareness.
3. Our world and our cultural awareness are endlessly and interdependently subject to change.

In particular, if we project our past into the future, we can imagine an unlimited succession of human generations that will continue to develop their empirical knowledge and their cultural awareness, interactively entwined. The human beings of the future will change accordingly, and will become as strange to us as we would be to our prehistoric ancestors if they could see us.

Note that the realistic empirical interpretation of our world, strongly rooted in our cultural awareness, is based on a shared universal conviction that is regarded as unquestionable: *the way of being of things and the relationships between them have an objective empirical identity that is independent of their observation*. Let us agree on calling the set of theories based on this belief *primary realism*.

Another four main convictions of this primary realism can be remarked:

1. Empirical knowledge is worked out by a generic subject, *the human being*, who discovers reality through the appropriate natural faculties, among which are the senses, the reason and the memory.
2. The task of the observer is basically passive and receptive as it consists in *reflecting* reality –and the objective order that governs it– in his/her mind.
3. Equal ways of being provoke equal sensations in different observers, although these may integrate them and react to them in different ways.

4. The number of ways of being of things is limited. The observation and definitive interpretation of all of them and of their relationships is the aim of empirical knowledge in which it will reach its final completeness.

From our point of view, these four realistic convictions are not consistent. We consider that the subject of knowledge is not the generic human being in general but the variable cultural awareness of each society. It also seems to us incorrect to view our task of observers of empirical reality as a mere reflection; we regard it as creative also because we assign to the things identities deriving from our observations and interpretations. And then the idea that identical ways of being provoke identical sensations in different observers has no empirical justification, given that no observer has access to the sensations of others for purposes of comparison.

In addition, since our knowledge of the world grows indefinitely, revealing to observation new fields of reality whose interpretation test the already built theories, it seems unlikely that empirical knowledge is moving towards a final goal. It is more consistent to consider that there is no goal in this sense, but rather a course to be run under the direction of developed knowledge and in the circumstances and with the intentions of the society the develops it.

According to other theories, *true* reality, or at least the *ultimate* reality of things, always lies hidden from empirical observation. These speculative theories have no empirical signification because what cannot be observed cannot be proved.

“What we see of things are the things. Why should we see one thing if there were another? Why should seeing and hearing be deceptive if seeing and hearing are seeing and hearing?”

(PESSOA)

We are convinced that sight, for example, tells us the colours that things *have*, but the truth is that in the absence of the sense of sight colours would not exist. It may seem absurd to deny the independent existence of the visible ways of being of things, that are ‘there outside’ for anyone who looks at them. But we should acknowledge that all visible forms are the result of our seeing them. In the phrase of Berkeley, “nothing is visible except what we perceive by sight”.

Comment on reason and experience

“The light dove cleaving in free flight the thin air, whose resistance it feels, might imagine that her movements would be far more free and rapid in airless space. Just in the same way did Plato, abandoning the world of sense because of the narrow limits it sets to the understanding, venture upon the wings of ideas beyond it, into the void space of pure intellect.” (KANT)

Kant intended to place the mechanics of Newton on a new philosophical foundation. He began by analyzing the significant judgements of the kind subject-predicate whose expressions are known as propositions. As regards their content, these judgements are called *analytical* when their meaning is derived from the predicates that define the subject and *synthetic* when they add predicates to the subject that are not contained in its definition. With regard to their validation, this can be *a priori* or *a posteriori* to the experience.

The idealistic and the empirical philosophers before Kant had considered all the analytical judgements as *a priori* and all the *a posteriori* judgements as synthetic, but they had disagreed as to whether there could be synthetic judgements that were *a priori*, or in other words, whether empirical knowledge could be obtained by reasoning, without the need to confirm it by experience. They had disagreed particularly about the bases for the validity of the principles of mathematics and physics.

In the idealism of Leibniz, the principles of mathematics and of physics are those of reason, which can be validated *a priori*. In the empiricism of

Hume, however, this was meaningless, even though he found no empirical foundation of their validity.

Kant rejected both the rationalistic idealism of Leibniz, which he found lacking in empirical content and therefore ineffective to grant validity to scientific principles, and also the sceptical empiricism of Hume which he found incapable of granting this validity. To overcome this double incapacity, Kant considered that empirical knowledge starts with experience, but that not all of it comes from experience. In particular, Kant characterized space and time as *a priori* forms of our external and internal sensitivity, and this led him to consider that the principles of mathematics and of physics contain *a priori* components which give them the necessary validity, which allows to consider them as *synthetic a priori* judgements.

Kant's exposition was subjected to later debate and finally surpassed in the XIX century, largely as a consequence of the critical thrust that Kant himself had given to the theory of learning. Bertrand Russell remarked this in his personal style: "Kant's sincerity drove him to fulfil more completely than did many other philosophers the obligation to demonstrate his own failure".

There is still controversy over whether there are synthetic *a priori* judgements. The possibility is accepted by the Neo-Kantian philosophers but rejected by positivists and empiricists. We are going to maintain a radically empirical criterion, stating that there is no synthetic *a priori* judgement nor any empirical reference that would endorse such a conjecture. We understand, therefore, that synthetic propositions can be validated only by experience. We wish to underline that the existence of something cannot be substantiated by reasoning but only by observation, which means that existence cannot be demonstrated but only confirmed.

It is important to differentiate the adjectives that define the validity of propositions, giving –as a convention– a single exclusive meaning to each one. So we consider that according to the validity conferred by reasoning, it is only analytical propositions that can be true or absurd (or contradictory). And according to the validity conferred by experience, it is only the synthetic propositions that can be certain or false. If a proposition admits a double reading, analytical and synthetic, then its validity requires a double

verification, and it could be absurd and false, absurd and certain, true and false, or true and certain.

This conventional distinction between meanings makes more complicated the normal use of language in which rational is mixed with empirical, that is to say: true with certain, and absurd with false. With the distinction we establish, truth is purely a matter of reason and certainty is exclusively a matter of experience.

When the validity of a proposition is confirmed by the logical or by the empirical references of the case in point, we say that it is consistent. The consistency of the propositions is relative because it is related to logical systems or to empirical theories into which they are integrated. We will say that analytical propositions rationally consistent are coherent – or that they are cohesive – and that synthetic propositions empirically consistent are consequent.

We consider that propositional logic is exclusively an abstract science that develops the varieties of redundances. In the words of Planck, “In its purest form, mathematics, only coordinates and gives coherent relationship of one certainty to another”. We must recognize, however, that our cultural awareness still has blind faith in the capacity of reason to provide a priori knowledge of the world, knowledge that is necessarily certain. This belief has been and continues to be a burden that hinders the development of empirical knowledge.

Empirical knowledge and mathematics

“When mathematical propositions are related to reality, they are not true; when they are true, they make no reference to reality.” (EINSTEIN)

Mathematics can be read in two different ways: one logical and abstract in which all the propositions are analytical and are validated *a priori*, and the other empirical and concrete in which all the propositions are synthetic and are validated *a posteriori*. The same mathematical proposition can be read as analytical and as synthetic, in accordance with the meaning – abstract or concrete – assigned to its terms.

Taking account of the exclusive meanings we have assigned conventionally to truth and certainty, the above quotation from Einstein may be expressed as follows : «When concrete meanings are assigned to the terms of the true analytical propositions of pure mathematics, we turn them into synthetic propositions whose certainty is not guaranteed by the truth of the former terms; when analytical propositions of abstract mathematics are said to be true, this statement is not referring at all to reality ».

The conviction that the truth of pure mathematics is a *necessary* guarantee of the certainty of applied mathematics must be rejected outright. The analytical proposition of pure mathematics ‘seven plus five equals twelve’ is true because it says nothing that is not contained in the abstract definition of its terms (‘seven’, ‘five’, ‘twelve’, ‘equality’). However, the corresponding proposition of applied mathematics ‘seven things plus five things are equal to twelve things’ is synthetic and it has to be validated by experience.

With the background of our cultural awareness it is difficult to accept that the sum of five things and seven things may differ from twelve things, since twelve was the result of all our observations. But no empirical reference affirms that this has to be the same in the future, nor in any other possible reality. We cannot reject empirically the existence of another reality – or that this may come to exist –, a reality that would involve the disappearance of some of the things added together or the appearance of new ones, making the sum of five things and seven things different from twelve things.

Starting from a few axioms that seemed to be unquestionably certain *a priori*, Euclid developed geometry as an empirical deductive science, which made it appear both true and certain. With this double validation, geometry was long considered an uncontested example of an *a priori* synthetic science.

But in the XIX century it was shown that the so-called ‘postulate of parallels’ (‘in the plane, given any straight line and a point not on it, there exists one and only one straight line which passes through that point and is parallel to the first line’) could not be deduced from the other Euclidean postulates. In a vain attempt to reduce to the absurd any other number of parallels, two new geometries were drawn up, as coherent as that of Euclid: one the so-called elliptical – or of Riemann – according to which no parallel can be drawn, and the other known as hyperbolic – or of Lobachevsky or Bolyai – which postulated that more than one and even an infinite number of parallels can be drawn.

To confirm which of these three true analytical geometries is certain, one must first transform them into synthetic geometries, assigning precise empirical meanings to their terms. The result of this is that the certainty of any one of them becomes relative since each could be certain respect the different empirical meanings allotted to it. If the terms of the three geometries are given the traditional empirical meanings and the normal spatial fields of our observations are taken into account, then the Euclidean geometry is found to be the only one that is perceptibly certain.

The sum of the three angles of a triangle is a relevant example of the discrepancy between the three geometries: in the Euclidean, the sum is equal to two right angles, whereas in that of Riemann it is more and in that of Lobachevsky it is less. Gauss is said to have measured (in secret, so as not to

scandalize the physicists of his time) the three angles of a nocturnal triangle of light signals. He found that the sum of the angles formed by the visual signals differed from two right angles by less than the foreseeable error in the measurement of the sum. This meant that the empirical space is perceptibly Euclidean in relation to the means of observation available to Gauss and for distances similar to those of the three visual signals.

The development of the two new geometries different from the Euclidean led mathematicians of the second half of the XIX century to enlarge the logical field of geometry, extending its abstract components and generalizing its definitions, axioms and postulates. These analytical geometries, without figures or other visual references, define abstract spaces and must be seen as special branches of logic.

Arithmetic was also considered an *a priori* synthetic science. Followers of Pythagoras even considered numbers to be the *true* reality of things and Kant was still convinced that arithmetical statements were *a priori* synthetic. Nowadays two aspects are considered in arithmetic –as in geometry – inter-related but of a different kind: while pure arithmetic is made up of abstract definitions and postulates, in applied arithmetic these are empirical. A close formal relationship exists between these two aspects, but not between their validations.

Hilbert took the abstract characterization of mathematics to extremes in considering that by means of mathematical definitions and axioms, abstract deductive systems can be set up in which all mathematical propositions are meaningful and can be validated in a single way (which in terms of logic means that deductive mathematical systems are complete and consistent). This view was widely accepted, and in fact is still accepted, by mathematicians even though it was soon shown by Gödel to be untrue.

Empirical mathematics retains the deductive arrangement of pure mathematics and this arrangement facilitates practical applications and guides their development; so that pure mathematics is therefore an indispensable tool for the formal structuring of empirical science.

“How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?.” (EINSTEIN)

This question has been answered in various ways. From the formalist standpoint, that of the majority, pure mathematics is a series of empty logical structures in which experience finds an ordered arrangement. For us, all logic in general and mathematics in particular proceed from experience, so it is normal they accord so well with it. Galileo considered that “the book of nature is written in the language of mathematics”; we think that this should be turned round to say that «the book of mathematics is written in the language of nature».

“I do not think that technical conquests are simply secondary precipitates of natural science; they are its logical proofs. If we had not set out to achieve these conquests, we would not have known how to reason. The only correct reasoning is that which has practical results.” (BOLTZMANN)

“Some thinkers state that the principles of logic are basically ontological principles since if they were not somehow founded in reality they would not prevail.” (FERRATER MORA)

Mathematics and physical knowledge

We will use the term *physical reality* to name the empirical reality dealt with by physics. Physical reality is made up of all the things of our world, but is not made up of all the ways of being of the things of our world; physics deals with a selection of ways of being which it defines and interprets in a specific manner.

Physical theories are formally structured into deductive systems that start from principles or axioms to which the character of natural laws is usually attributed. So physics is an empirical science because of its content and a rational science on account of its form, and therefore the valid propositions of physics have to be both certain and true.

We will assign qualitative definitions to physical entities by means of empirical references that allow them to be observed directly. The qualitative definition of a particular physical characteristic can be extended to a general collection of characteristics, taking all of them as qualitatively equal or as being of the same physical nature. In comparing physical characteristics of the same nature it may be that no empirical difference is found between them, as for example in comparing the straightness of segments. But in most cases empirical differences can and may be observed between them, which allow an apportionment of relative physical values, as occurs with the lengths of segments. We will term physical *amounts* the characteristics that can be valued relatively, and physical *magnitudes* the sets of qualitatively equal amounts.

To form a quantitative definition of physical amounts is to specify a procedure for establishing the relative values among them. The qualitative defi-

dition together with the quantitative one of a physical amount jointly form its empirical definition. To measure amounts consists in determining their relative values and to measure magnitudes consists in measuring their amounts.

Some physicists consider that to make a qualitative definition of physical magnitudes is unnecessary – that it is enough to know how to measure them. They think that the lack of a qualitative definition of magnitudes is not a barrier to operating with them or to develop the physical expositions in which they are included, thus establishing that qualitative knowledge of the magnitudes comes implicitly from the practise of operating with them. As a consequence, throughout the development of physics some magnitudes that are difficult to identify qualitatively, but easy to measure (such as time and space), remain empirically undefined. These magnitudes are normally considered to be ‘primary physical notions, undefinable, intuitive and universally well known’.

“I do not define time, space nor place because these words are universally well known.” (NEWTON)

“Time is one of the things we may not be able to define, but merely say that it is what we know it is.” (FEYNMAN)

We think it paradoxical not to be able to define something that is universally well known. In any case we understand that to operate with magnitudes that have not been identified empirically is a limitation and a clouding over of the meaning of the operations. In this we follow physicists who take an opposite view:

“Nothing that cannot be recognized, nothing that cannot be characterized by the senses, has any significance in science.”
(MACH)

“A concept does not exist for a physicist until the possibility arises to ascertain in a given case whether it is certain or not.”
(EINSTEIN)

There are substantive differences between mathematics and physics that should be stressed. Infinite numbers, accurate results and perfect figures receive rigorous rational treatment in abstract mathematics, whereas in physics there are only finite quantities, approximate results and imperfect figures. Some terms, such as ‘all’ or ‘always’ which have meanings of extremes, have no rigorous physical meaning although they do have it in the expositions of abstract mathematics. We will use these terms with approximate meanings, sufficient for each case we consider, but emphasizing that the rigorous accuracy of mathematical expositions gives rational support to the approximate accuracy of physical expositions; therefore these have no need of a logic of their own to give them coherence.

“From the point of view of physics, in which it is not possible to find physical means of differentiation, there can be no empirical objection to the simplest of mathematical suppositions, that of continuity.” (B. RUSSELL).

It may occur that a proposition admits a rational meaning and an empirical one, and that the two have contrary validations. An example is the Euclidean postulate : “the whole is greater than one of its parts”, a statement that is still physically certain but no longer mathematically true since Cantor distinguished the different classes of countable infinities.

The abstract division of quantities into a number of parts is an operation that is always possible, no matter how many parts. But the division of amounts into smaller and smaller parts always reaches a bottom limit below which it is not feasible. Before the division becomes impracticable, parts are often obtained that are so small that it would have no physical meaning to continue the division. These parts, which have the same physical meaning as their own parts, form the elemental amounts of the magnitudes.

The elemental amounts are relative since they depend on the physical expositions in question. Special mention should be made of the size of the points and the duration of the instants. The size of the earth is insignificant in relation to the scale of the solar system but it is enormous if one considers what is contained on its surface. In a similar way, an earthquake is of significant duration in terms of the succession of its manifestations, but it is

no more than an instant of the geological history of the territory in which it occurs.

Sometimes physics presupposes the existence of empirical things, or of relationships, which have not yet been observed but presumably will be observed to give coherence to the arrangement of other observations and of their interpretations. We will call *observable* to an empirical reference than can be observed and *presumable* to that one than can only be presumed. The interpretations of some of the observables became incompatible with the development of empirical knowledge and for this reason had to be discarded. As for the presumables, they are destined to be converted into observables or else to be abandoned in accordance with the development of physical science and of cultural awareness.

Among the observables that were discarded are found the rotation of the sun around the earth and the constancy of the mass of a body. The planets Neptune and Pluto were converted from presumable to observable, and the phlogiston theory and the existence of ether were among those discarded.

The evolution of the knowledge of physics shows that the interpretation of observables can never be considered as definitive but, in spite of this, in our common cultural awareness remains the idea that the world is governed by laws of nature that are definite, unchangeable and binding. We consider the laws of nature as merely general and provisional, based on a definite number of observations and interpretations, although sometimes this number is very high.

“Belief in a natural necessity arises only when our concepts fit so well to nature that they make their consequences conform with the facts. But the assumption of this satisfactory adaptation of our concepts can be invalidated at any moment by experience.”
(MACH)

We emphasize finally that the theories of physics are considered the more in accordance with reality the more rationally simple their arrangement of known observations and of their interpretations.

Time and memory

Throughout the development of mechanics, various qualitative definitions of time have been proposed. We will take that of Leibniz who attributed to time a relational empirical identity, considering that “instants without things are nothing at all because they consist in the successive arrangement of things”, and in line with this consideration he defined time as “the order of existence of things that are not simultaneous”. We agree with this approach but we find that the Leibniz definition leads to a vicious circle because “the order of existence of things that are not simultaneous” derives from the temporal order.

Newton found it unnecessary to define time because it is “a word that everyone knows”, but he established the existence of two times: the first, “absolute, true and mathematical” in which “in itself and of its own nature it flows uniformly and with no relation to anything external, and is also termed duration”; the second, “relative, evident and vulgar”, which is “a sensitive and external (precise or unequal) measure of duration by movement”. We find no empirical reference that would endorse these two classes of time, and in addition we think that the action of flowing and its uniformity are determined by time, which means that these terms lack empirical meaning before defining time.

The great majority of our observations are compound in the sense that they can be divided into parts – which are also observations – by using the memory. If we continue to divide these parts, each physical exposition becomes a series of minimal significant observations, each one is temporarily equivalent to any one of its parts. The duration of these elemental parts is known as an instant.

Observations are recorded in our memory while they are occurring, so

observing something and memorizing it are the two sides of the same mental operation. From this operative dualism of our mind we are able to attribute an empirical identity to the temporal order *before/after*: when we can recall an instantaneous observation while making another, we consider the first to come before the second, and when we cannot tell whether one or other came first we consider them to be *simultaneous*, which means that we are giving an empirical characterization to the temporal order by means of the memory.

This characterisation of the temporal order could appear to be merely a recognition of the obvious fact that *before* remembering something we must have observed it, but this triviality either lacks empirical meaning when the term *before* also lacks it, or it becomes a redundancy if the term *before* does have empirical meaning.

A compound observation has two extreme instantaneous parts: the beginning is the instantaneous part before all the others and the end is the instantaneous part after all the others. The temporal order between compound observations is established from that of their extremes; in particular, two compound observations are simultaneous when their extremes are simultaneous.

We can now give an empirical definition to *physical time*, which is a specific magnitude represented by the mathematical variable t in the equations and formulae of physics. The amounts of this magnitude are the durations of events, so in defining the durations we will have defined time. Accordingly we will try to make the empirical definitions as simple as is reasonably possible.

“Time must be defined in such a way that the equations of mechanics are the simplest possible.” (POINCARÉ).

It is an observable fact that if two instantaneous events, located in the same place, provoke two simultaneous observations in an observer, they do provoke also two simultaneous observations in all the observers; therefore the simultaneousness of instant events, located together, is an objective empirical reference. For example, if two cars collide the arrivals of the two at the place of the collision are two objectively simultaneous events.

A compound event can be said to be at rest when all its instantaneous parts occur in the same place. The duration of an event at rest is the empirical characteristic it shares with all the events at rest located in the same place and that are simultaneous with it. This definition is very limited as it excludes events in movement or which do not coincide in the same place, and these are the most numerous.

To define the duration of events in all possible cases, we use as a base the composite events known as periodical. A periodical event consists of a succession of consecutive parts that our memory assumes as equal. Periodical events may be due to natural or to artificial phenomena, the latter provoked by instruments we call clocks.

As the periods of a single clock are not simultaneous, the equality of its durations cannot be confirmed. But we can observe that the relationship between the numbers of periods of different clocks in the same place remains noticeably constant, no matter how many periods are considered. The simplest interpretation of this observable constancy is to consider that the durations of the periods are equal in each of the clocks.

The frequency of a clock is the inverse of the duration of its period. The lag between two clocks of the same frequency is equal, by definition, to the duration of the interval between the end of the period of one of them and the beginning of the following period of the other. When the lag between two clocks shows no variation, they are said to be *synchronized*.

Let us consider that the duration of an event at rest is equal to the duration of a simultaneous interval of a clock located next to it. The duration of the interval is equal to the number of clock periods contained in it, multiplied by the duration of the period. If the duration of a clock period is taken as unity, it is possible to measure the durations of all the events at rest.

The quotient between the duration of any interval of time and the increase in the time-lag between two equal clocks – originated in such interval – can be confirmed to be perceptibly constant; this quotient is equal to the number of periods that have to pass until the time-lag between the two clocks increases by one period. The inverse value of this quotient is known as the *regularity* of the two clocks; the regularity of a group of clocks is equal to the lowest regularity in two of them.

Placing a clock at each point of a physical space, we can define the durations of all the events occurring at this point. The set of these durations form a magnitude known as the *punctual time* in the space in question. The maximum simplicity of the mechanics is reached by assigning a single time to each space, which is known as the *local time* of that space. A definition of the local time implies an attribution of a physical meaning to the instant *now*, when it refers to different places. We will consider that when the clocks of the same space are synchronized, *now* is by definition the indication of any one of them. This is equivalent to giving physical identity to the synchronisation of separate clocks, and for this it is necessary to give physical meaning to the simultaneousness of two instant events that do not occur at the same point of a physical space.

While the speed of light was considered infinite, the indications of all the clocks were assumed to be seen simultaneously, so to synchronize them was an immediate operation. When it was found that light had a finite speed, it was thought sufficient to assign to the clocks the time-lags that were equal to the durations of the time it took the light to pass from them to the observers. This became part of our cultural awareness and lasted until the theory of relativity.

Many physicists entertain the possibility of giving a qualitative definition of physical time from the second law of thermodynamics, making the before/after relation that of the amount of the lesser/greater entropy of an isolated physical system. But it is not possible to define time quantitatively in this way, because the variation of the entropy is not periodical but increases monotonically.

In view of the above, we make the final point which is that clocks do not only measure the duration of events but they also characterize them physically.

Space and sight

“It may be that in the future we may have other ideas, at present beyond our reach, about the nature of space. Until then we will have to consider geometry as being at the level of mechanics, which is empirical.” (GAUSS).

The straightness of a line, the length of a segment and the distance between two points are for us the three basic observables from which we characterize space. We consider that the empirical identity of each of the three derives from the one that preceded it in the order given above, so we begin by attributing empirical identity to straightness, which is equivalent to giving a qualitative definition of the straight line.

The primary component of physical geometry is the *point*, an elementary observable geometrically equivalent to any of its parts. A point is said to be simple when it does not coincide with any other; simple lines contain only simple points. Compound lines, however, contain points that are not simple, but as these can be decomposed into simple lines, we will refer only to lines and points that are simple. We will also assume that all lines are material and rigid.

Lines are either straight or curved. Curved lines are those that are not straight, so a qualitative definition of straight lines defines the curved lines by exclusion.

Throughout the history of geometry, the straight line has been given different empirical qualitative definitions, of which the Euclidean is one the first usually quoted: “the straight line is that which rests equally on its points”. The rest of a line now seems an empirical reference of uncertain significance, since we are separated from the language of Euclid by twenty-

three centuries.

Hero of Alexandria defined the straight line as “the line that remains immobile when its ends are kept fixed”. Other definitions (by Leibniz, Gauss and Poincaré) also refer to a mechanical axis, or are based in some way on movement or repose. For Lobachevsky, the straight line is “the line that coincides with itself in all its positions”. From our point of view, all these definitions of the straight line include what they define in the definition, since the physical characterisation of movement is based on the measurement of distances and all distances are measured with units that are (*straight*) segments.

The same objection can be made to the commonest definition of the straight line as ‘the shortest distance between two points’. Proclus defined the straight line as “a tensed line between two fixed points”, but this amounts to the same: in order to tense a material line physically between two fixed points its length has to be reduced to a minimum.

Another definition of the straight line is based on its being the only line that is determined from two of its points. However this is only a property that gives no indication of how the remaining points are determined from these two.

“We usually consider that three points belong to a straight line when, with a correct disposition of the visual standpoint, the line of sight common to two of them passes through the third.”
(EINSTEIN)

We take this remark of Einstein as the basis of our definition of empirical straightness. Which observation allows us to prove that a ruler, a rod, a tensed cord or indeed any material line is straight? We answer the question describing what to we do: place each line so that we see the two extremes superimposed and then look to see whether in this position all the other points are also superimposed on these two. So we will consider a line as straight when we *see* it as though it were a single point, or in other words when all its points hide together behind either of its ends. This visual characterization of straightness is shown explicitly in astronomy, geodesy and topography, in which straight lines are essentially visual. In shorter distances, straight lines

can consist of tensed cords or of rigid poles, but in these cases we assume that they all represent materialized lines of sight. The visual nature of straightness was stated implicitly by Plato when he referred to it as “the line whose centre eclipses both its ends”.

Light is reflected and is fractured when it passes through the separation surfaces of different transparent media, curving in optical media that are inhomogeneous, partly surrounding opaque obstacles and dividing into different paths when it propagates in some transparent media. This variable propagation of light is due to the action of the matter, so straight lines coincide exactly with the paths of light only in a vacuum (or, perceptibly, in the air around us). Therefore the statement that light travels in a straight line under a vacuum does not state a property of light but it expresses the redundancy of a propagation of light under vacuum along its own path.

It is not possible to compare the straightness of some segments with others because they are all perceptibly equal, which means that straightness is not a magnitude. For comparison of the segments we will use their length, the other empirical characteristic that they have in common.

When all the points of two material segments at rest can be placed in contact *simultaneously*, these two can be defined as congruent. We will define qualitatively the length of a segment at rest as the empirical characteristic it shares with all the segments that are congruent with it, and only with these. As for the distance between two points at rest, we define it as the length of a segment whose ends coincide *simultaneously* with these two points. Since we have characterized straightness in terms of sight, as a result the length and the distance are also characterized by this sense.

The length of curved lines at rest cannot be defined qualitatively from the congruence between them since a congruence of curved lines is exceptional: no arc is totally nor partially congruent with all the arcs of all the curved lines. For a qualitative definition of the length of an arc of a curved line at rest, a series of polygonal lines with an increasing number of sides must be inscribed in it. It can be shown empirically that the length of these polygonal lines increases with the growing number of sides, until finally, from one of the polygonal lines on, no increase is perceived. This empirical reference allows the length of an arc of a curved line to be defined as the length of any one

of the successive polygonal lines. The outcome of this is that the lengths of curved lines at rest *consist in* lengths of straight lines, which means that the empirical definition of straightness must come before that of length.

We are convinced that when a line moves, it remains equal to itself, for the simple fact of moving. But this conviction has no empirical foundation other than the fact that the line is equal to itself before and after moving. How can a line be defined qualitatively *while* it is moving?

Physically, a moving line *consists* empirically, at each instant, in a line at rest which is congruent with itself at that instant, which means that all the points of the two coincide *simultaneously*. A moving line is therefore a sequence of successive congruent lines at rest, which leave ‘fixed traces’ in the space through which they move. From this definition it follows that the length of a moving line differs in general at each instant of its movement.

We will identify empirically *physical bodies* (or simply *bodies*) as visible material objects that resist being penetrated or deformed, which means that we are characterising physical bodies by means of the visual or tactile sensations they cause on us. The material points of a body are their least physically significant parts.

A geometrical point is place so small that it can be filled entirely by a material point. The large areas of a physical space are a collection of geometrical points in contact with one another, that can be filled by bodies with more than one material point.

“Any spatial localization of the place of an object or of an event is reduced to giving the point of a rigid body of reference that coincides with that object.” (EINSTEIN)

To characterize physical space, we assume that a body extends virtually, without limits, throughout the whole empirical exterior scenario. This unlimited virtual prolongation of the body, similar to a ‘rigid vacuum’ linked to it, is an empirical representation of the physical space referred to that body.

The empirical identity of the geometrical points is determined by rigid surfaces, lines or angles of reference, which form a *system of coordinates*; the terms ‘space’ and ‘systems of coordinates’ are often used without distinc-

tion. When the systems move at constant velocities among them, they are described as ‘inertial’.

We tend to interpret that we see objects in the place they occupy at the moment we see them, but this is true only when they are at rest. When they are moving, as is more frequent, the shape and the location of the objects depends on the place from which they are observed, and they are never where we see them because the speed of light is not infinite. So the general geometrical configuration of the empirical exterior scenario is not an observable unique and absolute, but plural and relative.

Among all the virtual geometrical transformations of bodies and spaces, we consider as most important the symmetry of a space A respect to a plane, with which another symmetrical space A' is formed, with a sense of turn in rotations and a distinction of left from right that are contrary to those of A .

An example of the consequences of this symmetry is found in a famous story. When Alice goes through the looking-glass, she does it as though she were passing from space A to space A' through a window, keeping her corporal integrity and hence her geometrical references of space A . So Alice notices for this reason that beyond the mirror the clocks C' go in the opposite direction to clocks C and that the difference between right and left is also the opposite, except in her own body. But if she had passed symmetrically through the mirror, decomposing and recomposing her body point by point, her new body would have been symmetrical to the former one and she would have found no change in the geometrical references in space A' since they would then have had the same relation to her body.

Then there is the example of two observers living in different realities but able to communicate with each other. *These observers would not be able to interchange any information that would allow them to know whether or not they lived in symmetrical spatial realities.* It is true that modern physics quotes phenomena that are always manifest objectively as either left or right, but this empirical reference is related to *our reality* which we suppose applicable to all possible worlds. However this assumption has no empirical sense as it cannot be proved.

Movement

We say that a body moves in the given physical space when there is a change in the places occupied by its material points. So the movement of a body is a relative characteristic, referred to the space in which it moves.

From different spaces the movement of a body can be interpreted in different ways, all of them consistent with observation, even though some of them are hardly compatible with already elaborated physical knowledge and for this reason must be rejected. When walking, I think it obvious that I move in relation to the ground, and not the ground that moves in relation to me. Again, if a stone falls, I find it obvious that it is the stone that comes down to hit the ground, and not the ground that rises to hit the stone. In both these cases I take it for granted that the movements I consider obvious are the *real* ones.

Cinematic evidence can vary with time, thus changing a movement considered as *real*. For centuries it was thought that the earth remained at rest in the centre of the universe while the sun moved round it in an annual periodic course (geocentric system). In addition, the firmament was thought to be a dome that turned daily around the earth. Later it was found more consistent to assume that all the planets, including the earth, moved round the sun (heliocentric system) while turning on their axes.

The heliocentric system, in stating that the earth was not a privileged planet in the centre of divine creation, questioned ecclesiastical authority. The Catholic hierarchy of the time rejected it, a rejection that was not contrary to observation since Galileo could not prove with his telescope that the heliocentric system expressed the *real* movement of the earth. Nobody can prove it since no movement has a way of being that can be considered its real way of being. The heliocentric system was only a consistent interpretation

simpler than the geocentric.

As for the daily turn of the heavenly vault, it became incompatible with the development of classical mechanics, whereas the turn of the earth around its axis is compatible, besides simpler, and no other interpretation can be conceived today.

After Galileo, Newton based classical mechanics on very simple laws that provided a simple interpretation of dynamic phenomena. For that reason our cultural awareness has made Newton something of a myth, a brilliant visionary who *discovered* the *natural laws* that govern mechanical phenomena.

As we mentioned already, we consider the natural laws – and among them the laws of physics – to be no more than general rules or postulates although we will keep calling them laws, as it is customary. We see the genius of Newton in the sphere of mechanics as the systematization he achieved of all the accepted concepts, specifying three general postulates from which observable dynamic phenomena can be deduced, and a fourth postulate that defines and quantifies the forces of remote attraction between material bodies.

Note that Newton characterized forces not as physically observables but as presumables to be defined according to their observable effects (the accelerations and deformations of a body). The fact that forces are not observable was a disappointment for many physicists, and some of them developed mechanical systems that omitted them. Kirchhoff and Hertz elaborated mechanical systems made up of differential equations without reference to forces. But the mechanics that don't mention forces are less intuitive and mathematically more complicated, so ordinary mechanical expositions keep mentioning forces.

Newton's fourth postulate was the law of universal gravitation. The existence of forces of attraction between material bodies separated by a distance, under vacuum, has been an empirical question highly disputed from the beginning. Newton himself wrote "the fact that gravity might be innate, inherent and essential to matter, so that one body would act on another at a distance, across vacuum and with no other interference, is so absurd to me that I can imagine no one capable of thinking in terms of philosophy who would accept it". This fourth postulate has now been surpassed by the theory of general relativity as is reported here below.

Modern mechanics

“Many physicists find it unthinkable that the most remote experience can ever change any of the unshakeable principles of mechanics. And yet what comes from experience can always be rectified by experience.” (POINCARÉ)

At the end of the XIX century most physicists shared the conviction that mechanics was founded definitively on *the objective laws that regulate mechanical phenomena*. But the interpretation of the constant of the speed of light and the behaviour of sub-atomic particles brought new theories of mechanics that caused a revolution in the characterization of reality: the theory of relativity and quantum mechanics.

The theory of relativity are two theories: the special and the general, both successively formulated by Einstein. The special theory mainly deals with the relativity of distances and of durations (i.e., of space and of time) in inertial reference systems. And the general theory extends this relativity to any system and interprets gravity in terms of the structure of space.

At the end of the XIX century Maxwell had predicted theoretically that light is an electromagnetic radiation whose velocity, c , is the same in all inertial systems, and Michelson confirmed experimentally the certainty of this prediction. Einstein raised the constancy of the speed of light in vacuum to the category of law of physics and revised mechanics from this standpoint, which led him to some surprising theoretical conclusions: the relative character of distances and durations, the impossibility that anything could move faster than light, the slowing of clocks in motion, the shortening of bodies in the direction of their movement, the increase of the mass of a body with speed, the equivalence between energy and mass, the precession of the or-

bits of the planets and so on. All these conclusions have been confirmed in practice.

To expound his findings in order, Einstein began by a physical definition of the simultaneousness of two instantaneous events occurring in different places, for which he set up a method of synchronizing two separate watches.

“A definition of simultaneousness is required that by its own nature provides the method of decision in given cases.”

(EINSTEIN)

He imagined two separate observers, O and O' , beside the two clocks, C and C' . If observer O sends a luminous signal to observer O' at an instant (t of his clock C) and O' returns this signal to O at the instant of its reception (t' of his clock C') then O , on receiving the return signal (at the instant $t + \Delta t$ of C), considers that clock C' did show the instant $t' = t + \frac{\Delta t}{2}$ when O' returned the signal. If observer O transmits this information to O' , the latter can synchronize his clock with that of O . This would mean that simultaneousness is a physical characteristic of each of the physical spaces, which is founded on the constant of the speed of light.

If all the clocks in a physical space are assumed to be synchronized in this way, the punctual time indicated by any one of them is, by definition, the *local* time of that space. So two separate instantaneous events are simultaneous when they occur at the same instant of the local time of the space in which they are placed.

If the distance between two points is characterized as the length of a segment whose ends coincide *simultaneously* in these two points, and the duration of an event as the difference between the indications of a clock that are simultaneous with the ends of the event, then physical distances and durations can be defined empirically considering a constant the speed of light in vacuum. Therefore speed of light in vacuum is measured by means of itself, so its constancy can be interpreted as a redundance rather than as a natural law.

The transformation of the three coordinates of space and time, from an inertial system S to another S' , provided that c remains constant in the transformation, shows that each of the new coordinates in S' depend on c

and on the coordinates in S , so the constancy of c implies that physical space and time are relative and interdependent. One possible interpretation of this proposes the existence of an absolute space-time continuum of four dimensions: the three traditional spatial dimensions (x, y, z) and time (t) which would be a fourth. This continuum generalises the mathematical treatment of the relativistic theories and allows the establishment of physical invariants that are maintained in a change of physical space.

In the general theory of relativity, Einstein defined gravity empirically by the intrinsic curvature of the space-time continuum, following the innovative geometric ideas of Riemann. This definition rejected the existence of distant forces, interpreting gravitational attraction as an effect of the curvature of the continuum space-time originated by material bodies. This new theory of gravity provided an understanding of the physical nature of the universe in terms of coherent geometrical structures and therefore a consequent handling of very intense gravitational fields.

The theory of relativity overcame the contradictions that had arisen in classical mechanics at the end of the XIX century, with a new characterization of space, time and gravity, so it can be considered an innovative culmination of classical mechanics *from within*.

“The theory of relativity has taught us that space and time and also the form and motion of materials in space and time are hypothetical mental constructions, by no means unquestionable.”
(SCHRÖDINGER)

While relativity surpassed the interpretations and postulates of classical mechanics (confirming the open and transitory nature of physical knowledge), an even more radical revolution was to occur in the physics of the XX century. To interpret the physical phenomena that exist on a scale beyond observation, and their manifestations on a scale that can be observed, a new mechanics was developed. Known as *quantum mechanics*, the new system abandoned determinism to interpret reality in probabilistic terms. It attributed a discrete nature to energy, assuming it to be composed of *quanta*, or indivisible elements.

Quantum mechanics states the existence of a discrete lower limit of energy transport from the particles of a process to any instrument of measurement, thus giving statistical character to the observation of the process. The situation of a particle is determined by an associated wave probability function, so the particle can be at any point at which this function is not null.

“The state of a system in quantum mechanics may be characterized by a vector in a multidimensional space, and this vector implies statements about the statistical behaviour of the system in certain conditions of observation. The objective description of the system, in the traditional sense, is impossible, but if experiments are repeated many times we are finally able to deduce statistical distributions from the observations, and if the series of experiments is repeated we can reach objective judgements about these distributions.” (HEISENBERG)

These words of Heisenberg show the great qualitative change operated in classical mechanics by quantum mechanics, the greatest change being the substitution of causal necessity by causal probability, i.e. deterministic physics by random physics.

The definitions and postulates of quantum mechanics, with the radically new interpretation of reality, surpass classical mechanics *from the outside*. It should be noted that the birth and development of quantum mechanics gave rise to great controversy among the physicists of the period and to considerable opposition.

“God does not play dice.” (EINSTEIN)

“It is not for us to tell God how He should govern the world.”
(BOHR)

The example of quantum mechanics led to the development of new theories of physics, coherent and mathematically complicated, that offer abstract definitions of their components and their postulates. To give an empirical interpretation of these theories is often difficult or even impossible so we will not go into them here.

“The theories of superchords and supersymmetries are certainly very fine but we have no experimental proof of them and I do not know what they mean.”

(VELTMAN, Nobel prizewinner of physics, 1999)

Final considerations

The substantial changes that have come about in physics throughout its development are a demonstration of the open nature of this science. Bertrand Russell clearly expressed the succession of these changes : “For Newton, space and time were solid and independent; now they are replaced by space-time, which is not substantial but a system of relationships. Matter has had to be replaced by series of events. Force has been replaced by energy. And energy has turned out to be indistinguishable from the pale ghost that remains of matter.”

With the theories of relativity and quantum mechanics, have we reached the *description* of a part of physical reality *such as it is*? Will these theories come to be definitely incorporated into physics as this continues its development? Our radically empirical standpoint leads us to answer no to these two questions because, on one hand, physical reality has no independent and definitive ways of being that we discover bit by bit, and on the other, physical knowledge is transient and changeable, like the cultural awareness that develops it and is developed by it. Much of what is observed may be definitive, but this is not true of its interpretations, so to characterize physical reality is an endlessly task. It is like trying to catch our own shadow.

One of the most firmly-held beliefs in our cultural awareness is the conviction that the nature of physical reality is always the same in whatever time or place (in this or in another possible world). But this has no empirical meaning as we cannot observe any other different from ours (if such exists), nor can we observe how our future reality will be.

This raises the particularly important question as to whether physics is an essentially unique series of theories becoming simpler and wider, or is it an open series of consistent theories that have been interpreting – in the form

of logical deductive systems – the physical observables known at each time. We think that nothing *necessarily* leads physics to continue as it has been throughout its historical development, which means that present-day physics could be qualitatively different from what it is.

In addition, we think that the cause/effect relationship is no more than a useful rule that simplifies the deductive ordering of empirical references. But the certainty of this relationship cannot be based on experience, even though we have seen this always fulfilled, we cannot see that it may not be fulfilled at some time. *Causal necessity* has no empirical meaning; *causal probability* does have this meaning, even though this probability is usually high enough to be equivalent to a necessity.

“We have often experienced the fall of a stone to the ground
but we have never experienced that it will always fall.” (HUME)

The question ‘What is physics?’ leads to the more general question ‘*What is anything?*’ What empirical meaning is there in asking what something is, or more simply what is *what is*? From our point of view, this question is the same as asking what we can expect from something, and in particular how we should react in its presence. We understand, then, that the *being* that we attribute to any empirical item is a set of data for our use. In computer terminology we can say that all the information that enters the human brain is programmed.

We regard reason as our most valuable and most characteristic attribute with two different interrelated components: a biological *hardware* (the neurological network of the brain) and a mental *software* (the programs recorded in our mind by our instinct, personal experience and cultural awareness). The two components have been developed together, interrelated and modelled by genetic mutations and by changes in the empirical circumstances we had to face. This portrayal of reasoning allows us to conjecture the existence of a formal link between the abstract structure of logic and the material structure of the brain.

And so we understand that reason is not a single independent human feature, bestowed on us by nature, but a developed faculty which could well

be otherwise, in different ways beyond the understanding of our cultural awareness. In empirical terms we should speak not of *reason* but of *reasons*.

And finally we mention a consequence of the interactive development of reason and the characterization of reality. Taking into account the fundamental, simplifying and effective role of reason in the empirical interpretation of our world, we can consider that the most real at each moment is the most rational then, and vice versa.