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# Paraconsistency

Paraconsistency is the study of logical systems with a non-explosive negation such that a pair of contradictory formulas (with respect to such negation) does not necessarily imply triviality, discordant to what would be expected by contemporary logical orthodoxy. From a purely logical point of view, the significance of paraconsistency relies on the meticulous distinction between the general notions of *contradictoriness* and *triviality* of a theory—respectively, the fact that a given theory proves a proposition and its negation, and the fact that a given theory proves any proposition (in the language of its underlying logic). Aside from this simple rationale, the formal techniques and approaches that meet the latter definitional requirement are manifold. Furthermore, it is not solely the logical-mathematical properties of such systems that are open to debate. Rather, there are several foundational and philosophical questions worth studying, including the very question about the nature of the contradictions allowed by paraconsistentists. This entry aims to advance a brief account of some distinct approaches to paraconsistency, providing a panorama on the development of paraconsistent logic.

# Paraconsistent Logic

From a structural standpoint, it can be said that paraconsistency is the property of a consequence relation $\vdash$ for which the principle *ex contraditione [sequitur] quodlibet* (from a contradiction, anything [follows]) does not hold in general; that is,. $A, \neg A \vdash B$ is not always the case (for a given negation $\neg$ and arbitrary propositions A and B). Accordingly, one can simply say that a logic is paraconsistent if and only if its consequence relation is not explosive.

Some other definitions of paraconsistent logic can be found in the literature. However, all of them can be shown to be equivalent to the latter when appropriate qualifications about the properties of the underlying inference relations are considered. Thus, a brief account of the development of such systems is appropriate.

## The Beginnings (Modern Era)

The study of logical systems underlying possibly contradictory theories had risen about the same time as the constitution of the logicist and formalist schools in the philosophy of mathematics, at the end of the 19th century and the beginning of the 20th century. Indeed, already in 1910, Jan Łukasiewicz proposed a criticism of distinct formulations of Aristotle's views on contradiction. Following an equivalent route, in the same year Nicolai A. Vasiliev advanced a kind of reasoning free from the laws of excluded middle and contradiction—called *imaginary logic* as an analogy with Nikolai Lobachevsky's *imaginary geometry* (a non-Euclidean geometry aiming to investigate the independence of the postulate of the parallels). Albeit endowing characteristics of a paraconsistent logic, namely criticizing the principle of non-contradiction, both works would mark the birth of two other kinds of well-studied nonclassical logics—notably many-valued logics and dialectical logics, respectively.

Other works that pioneered the development of paraconsistent logic also adhere to a many-valued approach. The *logic of nonsense*, introduced by Sören Halldén in 1949, aimed at studying logical paradoxes by means of 3-valued logical matrices, closely related to the *nonsense logic* introduced by Dmitrii A. Bochvar in 1938. An analogous approach was made in 1966 by Florencio G. Asenjo in his *Calculus of Antinomies*, which introduces a formal framework for studying antinomies by means of 3-valued Kleene's truth-tables for negation and conjunction, where the third truth-value is distinguished.

## The Rise of Paraconsistent Logic as a Discipline

The contemporary growth of paraconsistent logic came about with translations of the first papers into English and an increase in interest in them. Some advancements that marked the emergence of paraconsistent logic as a discipline, pushing forward novel results, include the works of Stanisław Jaśkowski and Newton da Costa, who independently developed comprehensive systems explicitly focusing on avoiding triviality by restraining the explosive character of contradictions, precisely the major characteristic of a paraconsistent logic—a term coined by Francisco Miró Quesada in a personal letter to da Costa in 1975, and then made public at the Third Latin America Conference on Mathematical Logic in 1976. The term *paraconsistency* is formed by the prefix *para-*, meaning "further than," "beyond," or even "similar to," or "quasi"; and *consistency,* meaning the property of a system that is non-trivial or non-contradictory—two classically equivalent notions whose separation lies at the heart of the paraconsistentist agenda.

According to Jaśkowski, there are three main conditions that a contradictory yet non-trivial theory must satisfy: (1) it must be non-explosive; (2) it should be rich enough to enable practical inference; and (3) it should have an intuitive justification. These latter two conditions, less formal than the first one, capture the biggest challenge to a paraconsistentist—namely, the necessity to show that a certain account of logical consequence is somehow concerned with actual situations of reasoning. Jaśkowski's motivation for advancing in 1948 the so-called *discussive logic* was a puzzling situation posed by Jan Łukasiewicz (of whom he was a disciple): which logic applies in the situation where one has to defend some judgment A, also considering its negation for the sake of the argument? The strategy followed by Jaśkowski was to avoid the combination of conflicting information by precluding the rule of adjunction, making room for A and $\neg A$ without entailing the conjunction of both ($A \wedge \neg A$) since the classic explosion actually still holds in the form of $A \wedge \neg A \vdash B$. In terms of reasoning, it has a straightforward meaning: each agent must still be individually consistent.

For da Costa, the focus is the development of systems that, on the one hand, could be strong enough to capture most of mathematics and, on the other hand, could circumvent some of the well-known paradoxes that historically marked studies in logic. Da Costa's main intuition in the so-called *C-systems*, advanced in his 1963 paper "On the Theory of Inconsistent Formal Systems," is that it is possible to differentiate inconsistent sentences from consistent ones or, in his own words, from those with a "well-behavior"—this latter being a sufficient requisite to guarantee the explosive character of a given formula.

Several other works in the literature arguably contributed to the development of paraconsistent logic as a mature discipline, most of them serving as a formal basis for particular research programs around which distinguished schools of paraconsistency have been organized. A brief account of those works is given in the next section from the standpoint of their respective approaches to paraconsistency.

# Main Approaches to Paraconsistency

## Preservationism

Out of the first works of Raymond Jennings and Peter Schotch on modal semantics, the early 1980s saw the organization of the preservationist school of paraconsistency, focused on advancing formal tools and intuitive justifications to capture the reasoning of human beings when faced with inconsistent data. Some of the core motivations include dealing with the concepts of belief and obligation, as well as C.I. Lewis' notion of *strict implication*. Roughly speaking (apart from technical details and philosophical questions), given an inconsistent collection of sentences (in an already defined logic, usually classical logic), one should not try to reason about that collection as a whole but rather to focus on internally consistent subsets of premises. For introductory readings on the subject, see Schotch, Brown, and Jennings 2009.

## Adaptive Logics

A logic is said to be adaptive if it adapts itself to the specific premises to which it is applied. Developed out of the early work of Diderik Batens, adaptive reasoning recognizes some so-called abnormalities to develop formal strategies to deal with them: for instance, an abnormality might be an inconsistency (inconsistency-adaptive logics), or it might be an inductive inference, and a strategy might be to exclude a line of a proof (by marking it), or to change an inference rule. Thus, the paraconsistent character of the abnormality is not the main focus. Rather, the intuitive idea to be captured is that human reasoning can be better understood as endowed with many dynamic consequence relations, inconsistency-tolerant reasoning being one of them. Some papers on the subject can be found in a 2000 work by Batens and colleagues.

## Relevant Logics

Mainly concerned in avoiding the paradoxes of material and strict implication, *relevant logics* are substructural logics that demand a meaningful connection between the premises and the conclusion of an argument. This strategy induces a paraconsistent character in the resulting deductions since A and ¬A, as premises, do not necessarily have a meaningful connection with an arbitrary conclusion B. Relevant logic evolved in the 1960s out of some early works by Alan Anderson and Nuel Belnap, themselves extending ideas of Wilhelm Ackermann, Alonzo Church, and others. For a comprehensive technical introduction, see Anderson, Belnap, and Dunn 1992.

## Dialetheism

*Dialetheia* is a neologism formed by the Greek prefix *di[a]-*, meaning "in opposite or different directions"; and *aletheia,* a word for "truth." Accordingly, a dialetheia is a sentence that is both true and false. Developed out of the original works of Richard Routley and Graham Priest in the 1970s, mostly motivated by the advancement of tools to deal with the liar paradox and set theoretic antinomies, dialetheism is the ontological view that there are dialetheia; that is, the view that some contradictions are true. Therefore, as expected, the preferred logic for a dialetheist should be paraconsistent, although not all paraconsistentists are compelled to be dialetheists. For a comprehensive philosophical introduction, see Priest 1995.

## Logics of Formal Inconsistency (LFIs)

A key idea here is that there are situations in which contradictions can, at least temporarily, be admissible if their behavior can be somehow controlled, as da Costa has it. Further generalizing and extending the advancements of the latter, the LFI (as it would be christened by Walter Carnielli and João Marcos in early 2000s) are a family of logics that can encode [in]consistency within their object language, allowing an explicit separation between contradictoriness from inconsistency, inconsistency from triviality, consistency from non-contradictoriness, and non-triviality from consistency. For a state-of-the-art account of the subject, see Carnielli and Coniglio 2016.

## Other Approaches

There are many ways a logic can be contradictory yet non-trivial. Jaśkowski's discussive logic, for instance, not only followed a *non-adjunctivist* approach but also endowed a *modal* character—in fact, it was a fragment of the well-known modal logic S5. Another significant program is the *many-valued* one, for which the first contemporary comprehensive system with an explicit paraconsistentist agenda is Asenjo's calculus of antinomies, the same strategy followed by Priest's *logic of paradox,* advanced in first works on dialetheism.

The fact is that there is no unique way to divide the advancements made in the literature into specific approaches, since they intersect and complement each other. Paraconsistency, as a property of logical systems, is negatively defined—it encompasses every system where *ex contraditione quodlibet* does not hold in general. The division of those systems into distinct schools of paraconsistency, highlighting particular properties and motivations, can thus be understood solely as a pedagogical tool to introduce the rich and fruitful plurality of paraconsistent logics.

In addition to the cornerstone logic-philosophical debate, the study of paraconsistency from the perspective of finite models of arithmetic as well as the applications of paraconsistent logic in some computational areas have provided a new dimension to the ongoing debate. The fact is that since the first works in the area, paraconsistency has turned out to be a remarkably fertile research field that provides us with new ways to deal with contradictory yet non-trivial scenarios, including inconsistent theories, paradoxes, dialectics, ontology, belief dynamics and many more.

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*See also* Logic; Consistency; Paradoxes; Rationality; Reasoning

**Further Readings**


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