Trends in Logic XVI

Consistency, Contradiction, Paraconsistency and Reasoning – 40 years of CLE

Book of Abstracts

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Preface

Trends in Logic XVI: Consistency, Contradiction, Paraconsistency, and Reasoning - 40 years of CLE

"Trends in Logic XVI: Consistency, Contradiction, Paraconsistency, and Reasoning - 40 years of CLE" is being organized by the Centre for Logic, Epistemology and the History of Science at the State University of Campinas (CLE-Unicamp) from September 12th to 15th, 2016, with the auspices of the Brazilian Logic Society, Studia Logica and the Polish Academy of Sciences.

The conference is intended to celebrate the 40th anniversary of CLE, and is centered around the areas of logic, epistemology, philosophy and history of science, while bringing together scholars in the fields of philosophy, logic, mathematics, computer science and other disciplines who have contributed significantly to what Studia Logica is today and to what CLE has achieved in its four decades of existence. It intends to celebrate CLE's strong influence in Brazil and Latin America and the tradition of investigating formal methods inspired by, and devoted to, philosophical views, as well as philosophical problems approached by means of formal methods. The title of the event commemorates one of the three main areas of CLE, what has been called the "Brazilian school of paraconsistency", combining such a pluralist view about logic and reasoning.

CLE was idealized by the philosopher Oswaldo Porchat Pereira, who proposed in the 1970s the creation of a research center at the University of So Paulo[1]. This did not occur, according to Porchat, for reasons having to do with political-ideological resistance at USP. The Centre for Logic, Epistemology and the History of Science (CLE), organized in 1976, was then officially installed in 1977 at the newly created University of Campinas as a result of the efforts of Porchat and the physicist Rogério César de Cerqueira Leite of that University

In the following years, CLE acted as a pole of attraction for highly qualified scholars, and trained many individuals who are today engaged in teaching and research. Besides, CLE fomented logical research in the country in a way that was unique and unprecedented in Brazilian academic history. CLE was conceived with the mission to promote research in the areas of logic, episte-

[1] Following D'Ottaviano, I. M Loffredo and Gomes, E. L. On the development of logic in Brazil I: the early logic studies and the path to contemporary logic. Revista Brasileira de História da Matemática , v. 11, p. 133-158, 2011.

mology, and the history of science, as well as interdisciplinary work, the organizing of seminars and scientific meetings, the publication of research, and the maintenance of academic exchanges with other research groups and institutions in Brazil and in other countries. CLE constituted the first interdisciplinary academic institution in its areas of research in Brazil, and possibly in all of Latin America, having the objective of bringing together scholars from various branches of scientific and philosophical knowledge. CLE currently has over 100 members, including researchers from various institutes and departments at Unicamp and other Brazilian, American, and European universities. In addition to logicians and philosophers, CLE's associates include social scientists, linguists, mathematicians, physicists, biologists, psychologists, and teachers of the arts. The first Director of CLE was Oswaldo Porchat Pereira (19771982), followed by Zeljko Loparić (19821986), Itala M. Loffredo D'Ottaviano (19861993), Osmyr Faria Gabbi Jr. (19931999), Walter A. Carnielli (19992005), Itala M. Loffredo D'Ottaviano (20052009), again Walter A. Carnielli (20092015) and currently Marcelo E. Coniglio (2016-present).

In the 40 years since its inception, CLE has maintained an intense program of scientific exchange and academic cooperation with other academic institutions, both in Brazil and abroad, that are known for excellence in teaching and research. With the support of foreign and Brazilians institutions which promote research and teaching, CLE has sponsored more than 100 large and medium-sized academic events, as well as numerous conferences, seminars, and courses. CLE has also received over 500 well known researchers as visitors. Among these, the following may be mentioned: Alfred Tarski (Berkeley, USA), Andrés Raggio (Buenos Aires, Argentina), Carlos Di Prisco (Caracas, Venezuela), Cecilia Rauszer (Warsaw, Poland), Claudio Pizzi (Siena, Italy), Daniel Isaacson (Oxford, England), Daniel Vanderveken (Montreal, Canada), Daniele Mundici (Florence, Italy), David Miller (London, England), Diego Marconi (Torino, Italy), Don Pigozzi (Iowa, USA), Edgard G.K. López-Escobar (Maryland, USA), Eduardo Rabossi (Buenos Aires, Argentina), Ezequiel Olasso (Buenos Aires, Argentina), G. Malinowski (Lodz, Poland), Gonzalo Reyes (Montreal, Canada), Gottfried Gabriel (Konstanz, Germany), Helena Rasiowa (Warsaw, Poland), Jaakko Hintikka (Boston, USA, and Finland), Jeff Paris (Manchester, England), John Corcoran (Buffalo, USA), John Lucas (Oxford, England), Justus Diller (Münster, Germany), Maximiliano Dickmann (Paris, France), Michal Krynicki (Warsaw, Poland), Raymundo Morado (Mexico City, Mexico), Richard L. Epstein (Berkeley, USA), Richard Routley (Canberra, Austrália), Rizsard Wojcicki (Warsaw, Poland), Roberto Cignoli (Buenos Aires, Argentina), Rolando Chuaqui (Santiago, Chile), Saul Kripke (New York, USA), Xavier Caicedo (Bogotá, Colombia).

The Michel Debrun Library at CLE has one of the best specialized collections in Latin America in the areas of logic, epistemology, and the history of science. Also located at CLE are the Historical Archives of the History of Science, a collection of over 100,000 manuscripts and documents stored in different media. CLE gave academic and administrative support to the specialization courses offered by Unicamp until early in the last decade, as well as to the Graduate

Program in Logic and Philosophy of Science at Unicamp's Department of Philosophy. The latter was a pioneering course in Brazil that was created almost simultaneously with CLE and was the forerunner of the current Graduate Program in Philosophy. The logicians teaching in the current program are also part of the Group for Theoretical and Applied Logic at CLE.

The event includes lectures by distinguished scholars as well as by contributors from several areas of knowledge and is part of the celebrations of the 50 years of Unicamp.

Past conferences in the line of Trends in Logic were held since 2003 in the Netherlands Belgium, Poland, USA, Germany, Georgia, Argentina, China and Denmark, and in Brazil for the first time. , A special volume of the collection "Trends in Logic", a series with 45 titles published so far, is planned to be edited with a selection of the papers presented.

The event counted, and is still counting, with the invaluable help of many people, in several roles:

Scientific Committee

- Walter Carnielli (Campinas, SP, Brazil)
- Marcelo Coniglio (Campinas, SP, Brazil)
- Newton da Costa (Florianópolis, SC, Brazil)
- Ítala D'Ottaviano (Campinas, SP, Brazil)
- Hannes Leitgeb (Munich, Germany)
- Jacek Malinowski (Toruń and Warsaw, Poland)
- Daniele Mundici (Florence, Italy)
- Heinrich Wansing (Bochum, Germany)
- Ryszard Wójcicki (Polish Academy of Sciences, Warsaw)

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- Fábio Bertato (CLE, Campinas, SP, Brazil)
- Juliana Bueno-Soler (FT & CLE, Limeira, SP, Brazil)
- Walter Carnielli (CLE, Campinas, SP, Brazil)
- Rodolfo Ertola (CLE, Campinas, SP, Brazil)
- Gabriele Pulcini (CLE, Campinas, SP, Brazil)
- Rafael Testa (CLE, Campinas, SP, Brazil)
- Giorgio Venturi (CLE, Campinas, SP, Brazil)

Staff at CLE

- Geraldo Alves
- Fábio Basso
- Roney Haddad
- Rovilson Pereira

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Trends in Logic XVI @ CLE-Unicamp

Sponsors

- FAPESP - the State of São Paulo Research Foundation, Brazil.
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- SBL - Brazilian Logic Society.
- PAN - Polish Academy of Sciences.
- UNICAMP - State University of Campinas.

Keynote Speakers

Proof-theoretic semantics and the semantical principle of CUT

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Abstract.

Proof-theoretic semantics is an attempt of explaining the meaning of logical constants in which the basic notion adopted is the notion of proof or construction. It has been influenced by constructivist tenets, in particular intuitionist conception of what is proof.

Two authors deserve to be remembered in connection with proof-theoretic semantics and they are Dummett and Prawitz. In both cases we find the attempt of explaining the notion of validity as a substitute notion for the predicate of truth. While from a tarskian more ontological point of view the meanings of the logical constants is to be explained by appeal to the notion of truth, from a proof-theoretical point of view a more epistemological approach should be preferable, and the notion to be placed at the focus is, as we said above, the notion of proof.

However, this notion of proof has some drawbacks. Many times an attempt at giving a proof-theoretic semantics for intuitionist logical constants validates some non-intuitionistic logical principles. From our point of view, there a natural explanation to this state-of-affairs and it is the fact that the notion of proof employed is a categorical notion, while the concept of hypothetical proof or deduction under hypothesis should be taken as a primary semantical notion.

In our exposition we are going to argue that CUT is indeed a semantical principle essencial for an intentional correct formulation of a constructivist semantics, which we call Hypo. We intend also make some remarks concerning one of the most well succeeded semantics for the intuitionist propositional logic, in order to better locate our proposal: Kripke semantics.

Quantitative Logic Reasoning

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Abstract.

We present a research program which investigates the intersection of deductive reasoning with explicit quantitative capabilities. These quantitative capabilities encompass probabilistic reasoning, counting and counting quantifiers, and similar systems.

The need to have a combined reasoning system that enables the a unified way to reason with quantities has always been recognized in modern logic, as proposals of logic probabilistic reasoning are present in the work off Boole [1854]. Equally ubiquitous is the need to deal with cardinality restrictions on finite sets.

We actually show that there is a common way to deal with these several deductive quantitative capabilities, involving a framework based on Linear Algebras and Linear Programming, and the distinction between probabilistic and cardinality reasoning arising from the different family of algebras employed.

The quantitative logic systems are particularly amenable to the introduction of inconsistency measurements, which quantify the degree of inconsistency of a given quantitative logic theory, following some basic principles of inconsistency measurements.

Thus, Paraconsistent Quantitative Reasoning is presented as a non-explosive reasoning method that provides a reasoning tool in the presence of quantitative logic inconsistencies, based on the principle that inference can be obtained by minimizing the inconsistency measurement.

Entanglement vs. Contradictions, or why neither Schrödinger's cat is simultaneously alive and dead, nor a particle can be in two different places at once

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Abstract.
This is a talk on logic. Logic as applied to science, logic as applied to quantum mechanics. We take the standard notion of contradiction and analyze some quantum situations where it has been said there are inconsistencies and contradictions. We show that this is not the case, that this kind of talk is based on a misunderstanding of the concept of quantum superposition. The conclusion is that if there are contradictions in the quantum realm, they are not related to superpositions.

Studia Logica – past, present and future

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Abstract.
The paper presents the journal Studia Logica. Its history, its present activities and its plan for future.

Consistency and Inconsistency in Probability theory

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Abstract.

I will deal with the notions of consistency and inconsistency in Probability Theory, explaining the deep reasons of the additivity law for the disjunction of incompatible events, and, more intriguingly, of the multiplicativity law for the conjunction of independent events. I will make use of various results in my paper [1] and other, more recent results. I will try to make the exposition as non-technical as possible.

Keywords: Consistency, inconsistency, probability theory

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The Logicality of Frege's Definition of Real Numbers

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Abstract.

The purpose of the talk is investigating Frege's definition of real numbers as it is envisaged in part III of "Grundgesetze der Arithmetic" (1903), and wondering whether this definition (once reconstructed in order to avoid contradiction) can be taken as a logical definition, and, then, as a ground for a logicist view about real analysis. The talk will proceed in three stages:

- A consistent reconstruction of Frege's definition of domains of magnitudes within a third-order predicate theory (this is the only part of Freges definition he provided in detail within his inconsistent system);
- A discussion of how real numbers can be recovered from domains of magnitudes (as ratios on them) by remaining close to Frege's informal (and quite broad) indications;

- A further discussion of how non-empty domains of magnitudes (and ratios on them) can be proved to exist, and of whether the necessity of independently proving their existence affects the logicality (and non-arithmeticality) of the whole definition.

Superficially and deeply contingent a priori truths

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Abstract.
In this paper, I review some standard approaches to the cases of contingent a priori truths that emerge from Kripke's (1980) discussion of proper names and Kaplan's (1989) theory of indexicals. In particular, I discuss Evans' (1979) distinction between superficially and deeply contingent truths. I shall raise doubts about Evans' strategy in general, and also about the roots and meaningfulness of the distinction.

Contributed Talks

Ontoprolog: A language for discourses based on ontologies

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Abstract.

Conceptual modeling languages focus in the expression of the essence of a particular domain of reality for the purpose of understanding and communication, promoting agreements among those interested. In this context, diagrammatic languages, such as ER, UML or OntoUML, are usually preferred instead of textual languages, due to the adequacy of visual notation for communication among human beings. However, textual conceptual modeling languages should not be dismissed out of hand, for they are still better suited to many tasks, such as for quick note, often without computer assistance or drawing tools, for use by visually impaired people, for systems interoperability and also for formal analysis, such as proof demonstrations. Moreover, it is easier to find or develop support tools for textual languages, due to the simpler and well established formal treatment, when compared to diagrammatic languages. Different logical aspects can be syntactically described and combined without necessarily having to deal with representational details, as combination of symbols, figures, or shapes. Therefore, a textual conceptual modeling language – such as a textual version of OntoUML – can expand the conceptual modeling based on OntoUML/*Unified Foundational Ontology* to an even broader scope. To explore these possibilities, we have developed Ontoprolog: a formal language for ontological well-founded conceptual modeling using a metamodeling approach for representing and a Logic Programming approach for reasoning about conceptual models and their foundational ontologies. Based on this, we have proposed Ontoprolog as an application of Classical and Non-Classical Logics for support discourses made by humans about a conceptualization regarding aspects or portions of reality. This paper presents the Ontoprolog language, showing the general strategy adopted to define the syntax, semantics and some functional characteristics of the language, as the integration capability with existing Prolog programs. Usage examples are also provided. We also present an analysis of the relevance of Ontoprolog to the integration of Information Architecture, Conceptual Modeling and Logic Programming.

Keywords: Information Architecture; Ontology; (Modal) Logic Programming; Unified Foundational Ontology; Formal language.

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The price of true contradictions about the world

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Abstract.

Dialetheias are defined as true contradictions, that is, formulas of the form α and $\neg\alpha$ both of which are true (see Priest [2], p. 1 and p.75). While it is controversial even whether there are dialetheias, there is ready to hand one favorite example of contradiction that is a most plausible candidate for being a dialetheia, the sentence comprising the conclusion of the Liar paradox. A feature of the Liar is that it is a semantic paradox: it is clear that it does not infect the concrete world, in the sense that it does not concern concrete objects in space-time. So, even assuming that dialetheias are possible, there is no easy example of a contradiction in the concrete world. Is there any prospect for dialetheism to reign in the concrete world?

In this work, we shall focus precisely on this problem. We stress that we shall deal with the concrete world, because here we shall not be concerned with possible contradictions in an abstract realm, such as a Cantorian universe of naïve set theory containing Russell's set and things like that. We shall focus our discussion on a fairly neglected argument advanced by da Costa [1] chap.3 to the effect that there may be true contradictions about the real (concrete) world. As far as we know, this is one of the few arguments that attempt to present the idea that the actual world may be contradictory in the context of a scientifically-oriented philosophy (that is, avoiding direct appeal to speculative or religious elements to ground the contradiction).

Roughly put, the argument states that contradictions in the world may manifest themselves precisely there where science meets contradictions in successful theories and where the elimination of such contradictions requires radically revisionary moves, resulting most of the times in (what may be regarded at first as) artificial amputation of widely held scientific tenets. The difficulty in obtaining consistent theories in theses cases acts as a sign that the world may be contradictory, that a contradiction in reality is a source of the problem; or at least so the argument goes. Examples of such cases are the wave-particle nature of quantum entities, the possibility of many incompatible interpretations of quantum mechanics, and the nature of movement, whose contradictory facets are shown by Zeno's paradoxes.

Our main claim, however, will be that even though it is not logically forbidden to think that the world may exemplify some true contradictions, the price of doing so for our own world is just too high; there are many difficulties with the strategy advanced by da Costa that seem to point to the fact that consistent solutions are preferable, even at their seemingly exorbitant costs. At least on pragmatic grounds then, there are good reasons to prefer to avoid commitments with true contradictions. The main reason, as we shall see, is that accepting true contradictions puts pressure for adjustments on our overall system of

knowledge that are at odds with our best current scientific and philosophical practice. Indeed, the amputation of currently held scientific tenets is performed by the friend of contradiction, so it is acceptance of true contradictions that will damage much more of the established canons of rationality.

We present three main arguments that seem to point to the fact that acceptance of true contradictions in reality comes at a just too heavy price. The arguments attempt to establish the following facts: i) by following the argument for contradictions in our world, acceptance of true contradictions in reality may lead us to a uncomfortable position of having to judge science from the armchair, imposing a philosophical preference for contradictions over the development of actual science; that is, it puts methodological challenges that are very substantial; ii) there are problems, as we shall argue, with true contradictions being restricted to unobservable objects; in fact, the main cases for true contradictions in the world concern unobservable entities. It is then plausible to argue that the troubles with those entities come much less from their contradictory nature than from their remoteness from observation and the intricate nature of the theories they feature in; iii) we shall present arguments to the fact that keeping with the contradictions will not save any of the main virtues of science that the resolution of such contradictions is said to mutilate (that is, there is no clear benefit in keeping the contradiction, rather the other way around, contradictions seem to trouble the investigation).

As a conclusion, it seems, it is better to keep regarding contradictions as signs that something went wrong with our best theories rather than admit that the world itself may be contradictory. Even if it is not forbidden to adopt the view that the world itself may be contradictory, there seem to be better reasons to prefer **not** to endorse the thesis that the world indeed is contradictory.

Keywords: Non-classical logics, Contradictions.

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Filter functors in logic and applications to categorial analysis of meta-logic properties

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Abstract.

Abstract Algebraic Logics (AAL) can nowadays be seen as the discipline that studies the connections between logic and algebra. These links allow one to use the powerful tools of universal algebra to study metalogical properties. On the Lindenbaum's idea of viewing the set of formulas as an algebra with operations induced by the logical connectives, Tarski gave the precise connection between classical propositional calculus and Boolean algebras. This method to connect logic and algebra is the so-called *Lindenbaum-Tarski method*. Generalizing those ideas, Blok and Pigozzi [2] introduced the notion of algebraizable logic. Superficially speaking, an algebraizable logic consists of a set of formulas in two variables such that expresses the logical equivalence between two formulas and a set of equations whose solutions in the algebras collectively play the role of `truth` in classical logic. For any algebraizable logic a there is a quasivariety $QV(a)$ associated. This quasivariety $QV(a)$ keeps the semantic information of a. Unfortunately, for an arbitrary Tarskian logic no class of algebras endows alone this semantic information. To an arbitrary Tarskian logic $l = (\Sigma, \vdash)$, the set of filters $Fi_l(M)$ for an arbitrary algebra M of Σ-Str, in a certain way, has the semantic information of the logic l. That was the primary motivation to start the study of the notion of filter pairs and its associated logics.

It is well-known that every Tarskian logic gives rise to an algebraic lattice contained in the powerset $\mathcal{P}(F_\Sigma(X))$, namely the lattice of theories. This lattice is closed under arbitrary intersections and filtered unions. We observe that the structurality of the logic just defined is equivalent to the *naturality* (in the sense of category theory) of the inclusion of the algebraic lattice into the power set of formulas with respect to endomorphisms of the formula algebra: Structurality means that the preimage under a substitution of a theory is a theory again or, equivalently, that the following diagram commutes:

$$\begin{array}{ccccc} F_{\Sigma}(X) & & L & \xhookrightarrow{\;i\;} & \mathcal{P}(F_{\Sigma}(X)) \\ \sigma \big\downarrow & & \sigma^{-1}|_L \big\uparrow & & \big\uparrow \sigma^{-1} \\ F_{\Sigma}(X) & & L & \xhookrightarrow{\;i\;} & \mathcal{P}(F_{\Sigma}(X)) \end{array}$$

Further, it is equivalent to demand this naturality for all Σ-algebras and homomorphisms instead of just the formula algebra.

We thus arrive at the definition of *filter pair*: A filter pair for the signature Σ is a contravariant functor G from Σ-algebras to algebraic lattices together with a natural transformation $i\colon G \to \mathcal{P}(-)$ from G to the functor taking an algebra to the power set of its underlying set, which preserves arbitrary infima and directed suprema.

We consider the special case of filter pairs where the functor $G = Co_K$ is given by congruences relative to some quasivariety K, and give criteria when the associated logic is protoalgebraic, equivalential, algebraizable, truth-equational, congruential or Lindenbaum algebraizable. Also we consider the standard way of producing as logic from a quasivariety Q and a given set τ of equations (namely, the so called τ-assertional logic of Q). The notion of filter functor is useful to the analysis of meta-logical properties: we apply this notion to establish a correspondence between Craig interpolation and the amalgamation property which goes beyond the known cases of protoalgebraic logics. We introduce a notion of morphism of filter functors and show that it encodes translations between their associated logics. Moreover, we show that the category of abstract logics is isomorphic to a full and reflective subcategory of the category of filter pairs.

Keywords: Categories; Filters in Logic; Craig interpolation.

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Recovering safe instances of cut in non-transitive theories with a consistency operator

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Abstract.
We will present a way to expand non-transitive theories (in the vein of the work in p-matrices started by [5]), in order to recover *gentle* versions of Cut, which is the following meta-rule:

$$\text{Cut}\ \frac{\Gamma, A \Rightarrow \Delta \qquad \Gamma' \Rightarrow \Delta', A}{\Gamma, \Gamma' \Rightarrow \Delta, \Delta'}$$

We will show how to do this for the non-transitive logic **ST** (presented in [6, 7] among others) in a direct way, with the help of a recovery operator that we will symbolize as $\circ$. We will do this, drawing inspiration from [3, 4, 2, 1] where 'recovery' of *gentle* versions of classically valid principles –like *Explosion*– is carried out with the help of the consistency operator. We will obtain the logic $\mathbf{ST}^{\circ}$, i.e. **ST** with a recovery operator $\circ$. In $\mathbf{ST}^{\circ}$, even though Cut is still invalid, a rule that we will call *Gentle Cut* is valid.

$$\text{Gentle Cut}\ \frac{\Gamma, A \Rightarrow \Delta \qquad \Gamma' \Rightarrow \Delta', A \qquad \Rightarrow \circ A}{\Gamma, \Gamma' \Rightarrow \Delta, \Delta'}$$

Later on, we will analyze the case of the non-transitive formal theory of truth **ST+**, which is **ST** with a transparent truth predicate and a standard, *strong*, way to generate self-referential sentences, through identities of names of sentences (in this case, the identities are been established in the metalanguage). We will show both that **ST+** with the addition of a recovery operator is trivial, but that a particular theory that uses instead a *weak* self-referential procedure, is *non-trivial*. We will call these alternative theory $\mathbf{ST}^{\circ}_{\mathbf{w}}+$.

With a 'weak' way to achieve self-referential sentences, we will refer to a method that uses biconditionals instead of identities. But in order for it to be successful, the conditional involved in obtaining the self-referential sentences needs to invalidate *Modus Ponens*. This, in fact, is achieved with the ordinary material conditional of **ST**, v.g. $A \supset B =_{def} \neg A \vee B$. The formal theory of truth $\mathbf{ST^{\circ}_{w}}+$ will help us recover not only gentle versions of Cut -i.e. instances of the Cut rule where the cut formula is a well-behaved sentence– but also gentle versions of other invalid meta-rules, that are valid in classical logic. Rules of this sort are, for example, *Meta Modus Ponens* or *Meta Explosion*. In each case, the corresponding *gentle* version of the invalid meta-rule adds a third premise sequent, that says that the formula A is well-behaved, meaning that Cut can be applied without generating any problems.

$$\text{Gentle Meta Modus Ponens}\ \frac{\Rightarrow A \supset B \qquad \Rightarrow A \qquad \Rightarrow \circ A}{\Rightarrow B}$$

$$\text{Gentle Meta Explosion}\ \frac{\Rightarrow A \qquad \Rightarrow \neg A \qquad \Rightarrow \circ A}{\Rightarrow B}$$

In addition to the already mentioned advantages, the resulting theory $\mathbf{ST^{\circ}_{w}}+$ will enjoy a gain in expressive power, because it will be able express two key notions that **ST+** cannot express on pain on triviality. First, it will be able to express that a certain sentence A is 'strictly true' (i.e., $Tr(\langle A\rangle) \wedge \circ A$), secondly, it will be able to express that a certain sentence A is 'strictly false' (i.e., $\neg Tr(\langle A\rangle) \wedge \circ A$).

To conclude the present essay, we will present a non-triviality proof for $\mathbf{ST^{\circ}_{w}}+$ and soundness and completeness results with respect to a suitable three-side disjunctive sequent system that we will call $\mathbf{LST^{\circ}_{w}}+$.

Keywords: Substructural Logics; Non-Transitive Logics; Recovery Operators; Consistency Operators; Logics of Formal Inconsistency.

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A Yabloesque paradox in epistemic game theory

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Abstract.

In this paper, I present an application of Yablo's Paradox in epistemic game theory using the language of propositional bimodal logic. Yablo's paradox, according to its author, is a non-self-referential paradox. The paradox created a significant attention in the literature, especially from the perspectives of truth theory and fixed-point logics. Here, I extend the discussion to game theory and give a yabloesque, non-self-referential paradox within the context of epistemic game theory.

I have two goals in this work. The first is to *apply* Yablo's argument to a field which can provide some further intuition for the discussions regarding the self-referentiality of Yablo's paradox. My second goal is to give a broader reading of the paradoxes of epistemic game theory and their possible solutions. Studying game theoretical paradoxes is the first stepping stone in the study of games with inconsistencies. Therefore, this paper aims at understanding paraconsistent games - games that can have non-trivial inconsistent models where agents may possess inconsistent knowledge or may make inconsistent moves. Moreover, as the standard language of epistemic game theory is modal, this analysis can be viewed as a bimodal extension of Yablo's paradox.

Paradoxes are not foreign to epistemic game theory where a self-referential paradox, due to Brandenburger and Keisler, was suggested earlier. The Brandenburger - Keisler paradox is a two-person self-referential paradox in epistemic game theory [3]. The paradox arises when the following sentence is considered for two players Ann and Bob:

> Ann believes that Bob assumes that Ann believes that Bob's assumption is wrong.

and the question if "Ann believes that Bob's assumption is wrong", where Bob's assumption is the sentence that "Ann believes that Bob's assumption is wrong" is considered.

Both answers generate a contradiction, thus the paradox. The paradox is indeed self-referential, expressible with a fixed-point operator, thus can be seen as a two-person Russell's paradox [1].

On the other hand, Yablo's Paradox, according to its author, is a *non*-self referential paradox [5, 6]. Yablo considers the following sequence of sentences.

$$S_1 \quad : \forall k > 1, S_k \text{ is untrue,}$$
$$S_2 \quad : \forall k > 2, S_k \text{ is untrue,}$$
$$S_3 \quad : \forall k > 3, S_k \text{ is untrue, } \ldots$$

By using *reductio*, Yablo shows that the above set of sentences is contradictory. The scheme of this paradox, however, is not new. To the best of our knowledge, the first analysis of a similar paradox was suggested in 1953 [7].

In what follows, I present a yabloesque version of the Brandenburger and Keisler paradox offering a *non-self-referential* paradox in epistemic games.

Consider the following sequence of assumptions for ω-players where numerals represent the players.

$$\begin{aligned}
A_1 &: 1 \text{ believes that } \forall k > 1, k\text{'s assumption } A_l \text{ about } l > k \text{ is untrue,}\\
A_2 &: 2 \text{ believes that } \forall k > 2, k\text{'s assumption } A_l \text{ about } l > k \text{ is untrue,}\\
A_3 &: 3 \text{ believes that } \forall k > 3, k\text{'s assumption } A_l \text{ about } l > k \text{ is untrue, } \ldots
\end{aligned} \tag{1}$$

Such a situation is not difficult to imagine. Let me give an example illustrating the above statement, similar to Sorensen's [4]. Imagine a queue of players, where players are conveniently named after numerals, holding beliefs about each player behind them, but not about themselves. In this case, each player i believes that each player $k > i$ behind them has an assumption about each other player $l > k$ behind them and i believes that each k's assumption is false. This statement is perfectly perceivable for games, and involves a specific configuration of players' beliefs and assumptions, which can be expressible in the language. Is it then consistent?

Theorem *The set of sentences in Statement* (1) *is inconsistent.*

This is a non-self-referential yabloesque paradox for ω-players in epistemic games.

In conclusion, paradoxes of game theory are important both for philosophical logic and epistemic game theory. The current work relates to philosophical logic by extending the discussion on *interactive* paradoxes, the theory of truth and modal logic, and to epistemic game theory by presenting an interesting epistemic game theoretical paradox.

Keywords: Yablo's Paradox; The Brandenburger - Keisler paradox; Epistemic game theory; Paraconsistent logic.

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Paraconsistent logic and contradiction

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Abstract.

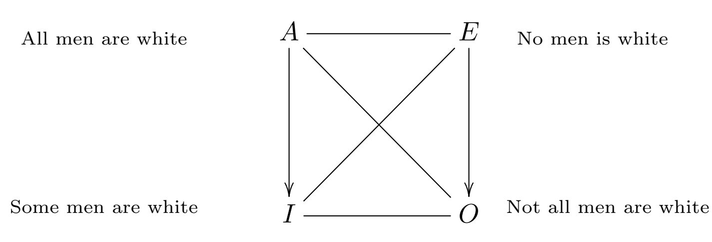
In PROPOSITION IV of Chapter III of the *Laws of Thought* of Boole states the following: "the principle of contradiction is a consequence of the fundamental law of thought, whose expression is $x^2 = x$." At the time (1854) this may have looked a bit strange similarly to the famous controversy in St Petersburg in the 1770s when Euler told Diderot: "Sir! $(a + b^n)/n = x$, therefore God exists, go back to France!"

What is the relation between a mathematical formula and a metaphysical principle? Today, after more than 150 years of development of modern logic, one may think that there is no problem to relate both sides and that everything is crystal clear. Is it really the case? There is still much confusion around the formalization of the notion of contradiction as it can be seen through the philosophical debate surrounding paraconsistent logic.

In modern logic, the principle of contradiction has at least two formulations: $F, \neg F \vdash G$ and $\vdash \neg(F \wedge \neg F)$. These two formulations are not equivalent as it can easily be seen using three-valued matrices. Which one is the correct formulation of the principle of contradiction, if any?

Paraconsistent logic is generally based on the rejection of the first formulation, called the *law of explosion* or *ex-contradictione sequitur quod libet* (from contradiction everything follows). If from $\{F, \neg F\}$ we cannot derive anything can we still call $\{F, \neg F\}$ a contradiction?

A reason to say "no" is based on the theory of valuation and the square of opposition. According to the former if $F, \neg F \nvdash G$ it means F and $\neg F$ can be true together. According to the theory of the square of opposition, in this case F and $\neg F$ are not contradictory, they are at best subcontraries. The diagram below is a standard representation of the square of opposition with contradiction in red, contrariety in blue, subcontrariety in green and subalternation is black.



It seems that one of the reasons to insist on calling $\{F, \neg F\}$ a contradiction despite the invalidity of the law of explosion is based on a confusion between negation and contradiction. It is important to stress that one can defend the idea that a paraconsistent negation is a negation without calling $\{F, \neg F\}$ a contradiction.

We can call a formula such that F and $\neg F$ are both true a **paraconsistent formula**. Among paraconsitent logics there are systems in which all formulas are paraconsistent. This is the case of the three-valued paraconsistent logic of Asenjo (rebaptised LP by G.Priest). By contrast the paraconsistent logic C_1 of Newton da Costa is a logic in which some formulas are not paraconsistent, they are called well-behaved formulas. The well-behaviour of these formulas is characterized within the language and has been generalized by Carnielli with the introduction of a consistency operator in the so-called LFIs. In these logics all atomic formulas are paraconsistent (but not all molecular formulas are classical, like in Sette's three valued logic P^1). We have developed the paraconsistent logic Z according to which given a formula F, its negation $\neg F$ is false iff F is true from all viewpoints. In Z tautologies and antilogies are classical and all other formulas are paraconsistent. Other paraconsistent logics having a similar feature can be developed. In this talk we will argue that this situation makes good sense from a philosophical point of view.

Keywords: Paraconsistency; Contradiction.

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Volutionary foundational visions

paradox, provability, truth and irrefutability

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Abstract.

We expose some central ideas of *volutionary* or *volutionistic* mathematics, which suggests that we should shift attention to the set of sentences whose negation are not theses of the formal mathematical system as traditionally focused upon. Volutionism may alter how we think about foundational matters e.g. in that Gödelian incompleteness phenomena can be taken as paradoxicalities; the induced truth-like predicate in the volutionistic dual mirror of the original formal system gives us pause to contemplate that the standard Gödel sentence of the original formal system is rather like a textbook *liar sentence.* Moreover, volutionism may give some occasion to reinterpret issues concerning decidability and computability. Volutionary systems should not be subsumed under traditional paraconsistent approaches as classical logic is contained in and never contradicted by volutionary versions of formal systems. However, the volutionary resolution of paradoxes has some similarities with that of the author's *librationist* set theory £ developed in former publications and philosophically clarified most substantially in [1].[2] Given terminology in the latter publication one may consider both librationism and volutionism *bialethic* as opposed to *dialethic* points of view. The author does not commit to volutionism, but here offers the ideas for deliberation and possible furtherance.

Keywords: Incompleteness; Paradox; Refutationalism; Volutionism.

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[2] I now further think of £ as an *incohesive* system, and I have come to prefer the term *paracohesionism* to the term *paracoherentism* used in [1].

On quasi-classical probability measures: a paraconsistent probability theory based on LFI1

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Abstract.

Paraconsistency is the investigation of logic systems endowed with a negation $\neg$ such that not every contradiction of the form p and $\neg p$ leads to deductive triviality. In other terms, a paraconsistent logic is free from trivialism in the sense that a contradiction does not necessarily entail everything. The reason is that the following principle, called Principle of Explosion, is not valid in paraconsistent logic:

(PEx) $\alpha, \neg\alpha, \vdash \beta$, for arbitrary β.

Consequently, a paraconsistent logician is more cautious then a classical logician when reasoning, because he is free of the burden of (**PEx**). In the presence of a contradiction he pauses his reasoning to investigate the causes for it, instead of deriving everything from it, as a classical reasoner would be.

The Logics of Formal Inconsistency (LFIs) are a well-recognized tool to formalize such a reasoning paradigm, as they contain linguistic resources to express the notions of consistency (and inconsistency as well) within the object language by employing a connective $\circ$, reading $\circ\alpha$ as "α is consistent" (and $\bullet$, reading $\bullet\alpha$ as "α is inconsistent") that realize such an intuition.

LFIs extend classical logic, in the sense that classicality may be recovered in the presence of consistency: consistent contradictions involving consistent sentences will lead to explosive triviality. Consistency in the LFIs is not regarded as synonymous with freedom from contradiction (as it happens with the traditional notion of consistency of a theory T, where consistency is taken to mean that there is no sentence α such that $T \vdash \alpha$ and $T \vdash \neg\alpha$, where $\vdash$ is a specific consequence relation in the language of T). The distinguishing feature of the LFIs is that the principle of explosion **PEx** is not valid in general, although this principle is not abolished, but restricted to consistent sentences. Therefore, a contradictory theory may be non-trivial unless the contradiction refers to something consistent.

Such features of the LFIs are condensed in the following law, which is referred to as the "Principle of Gentle Explosion":

(PGE) $\circ\alpha, \alpha, \neg\alpha \vdash \beta$, for every β, although $\alpha, \neg\alpha \nvdash \beta$, for some β.

It was shown in [1] that the system **Ci**, a particular system of the family of the **LFIs**, can be quite naturally used to base an extension of the notion of probability able to express probabilistic reasoning under contradictions by means of appropriate notions of conditional probability and paraconsistent updating, via a version of Bayes' Theorem for conditionalization.

Our intention here is to start from the system LFI1 instead of **Ci** in order to obtain a paraconsistent notion of probability. The gain is that LFI1 is much closer to classical logic than **Ci** and can mo re naturally encode a quasi-classical notion of paraconsistent probability. The motivations to consider a probability theory based on LFI1 are the following:

- In LFI1 most of the De Morgans' laws are valid;
- LFI1 is a three-valued logic, and other probability theories based on many-valued logics have been proposed (see [3]), so it worth to compare them;
- LFI1 is maximal with respect to classical logic and thus is in some sense the closest companion to standard probability that can be considered (see [2]);

The most interesting use of probability in paraconsistent logic is to define a new form of Bayesian conditionalization. The well-known Bayes rule permits one to update probabilities as new information is acquired, and, in the paraconsistent case, even when such new information involves some degree of contradictoriness. This makes sense, since one of the most important topics in statistics is how to ensure reliable uncertain inferences, and consequently the notion of probability and its connection to logic is of fundamental importance. Although the questions about the relationship between logic and probability are certainly acontroversial, I believe that proposing a well-founded alternative to standard probability based on a paraconsistent logic such as LFI1 can only be helpful to this question. As a secondary topic, I intend to start a discussion on potential applications of such quasi-classical probabilities.

Standard Bayesian networks are directed acyclic graphs whose nodes consist of random variables that represent observable quantities, hypotheses or unknown parameters in general. Edges represent conditional dependencies, while unconnected nodes represent variables that are conditionally independent of each other. Each node is associated with a probability function having as inputs sets of values for the node's parent variables, and as output the probability of the variable represented by the node. Bayesian network are useful devices used to answer probabilistic queries about relationships. One of the offsprings of this work is to define non-standard Bayesian networks based on LFI1 probability distributions, and to access their applicability in causal networks.

Keywords: Probability theory; Paraconsistent logic; Many-valued logic.

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A new algebraic semantics for first-order logic built with polynomials

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Abstract.

Our intention in this paper is to suggest that the disjuncture between first-order logic and its intended algebraic counterpart may be due to the reluctance of modern logicians in using infinitely long expressions, as exemplified by expressing existential quantifiers (essentially infinitary objects) in cylindric algebras by means of the operations Cx. Our second purpose is to take profit of this intuition by presenting a treatment of first-order logic by means of formal *infinitary* polynomials, in the intuitive spirit of Taylor series. We then show how this polynomial representation of first-order sentences could be seen as a legitimate algebraic semantics for first-order logic, alternative to cylindric and polyadic algebras an with a higher degree of naturalness.

1. Introduction

Starting from the algebraic method of theorem-proving based on the translation of logic formulas into polynomials over finite fields, and by adapting the case of first-order formulas by employing certain rings equipped with infinitary operations, this article exposes, from an arbitrary set M, the following definitions:

- M-ring, a kind of polynomial ring defined for each first-order structure, with infinitary operations;
- M- homomorphism;
- M-congruence, denoted by C or $\approx$;
- Definition of $F(\mathcal{M})$;
- Definition of $\mathcal{M}$-ring: $R(\mathcal{M}) = F(\mathcal{M})/C$;
- Definition of a $\mathcal{M}$-homomorphism coherent.

From this definitions, we have that:

$$\tau_{\mathcal{M}} : Form(L) \to F(\mathcal{M})$$

and

$$\_\tau\mathcal{M} : Form(L) \to F(\mathcal{M})/C = R(\mathcal{M})$$

In resume:

$$(M - structure) \longrightarrow F(\mathcal{M}) \longrightarrow R(\mathcal{M})$$

$$(v : Form(LPO) \to \mathbb{Z}_2) \longrightarrow (h_v : F(\mathcal{M}) \to \mathbb{Z}_2) \longrightarrow (H : R(\mathcal{M}) \to \mathbb{Z}_2)$$

Now we define the adequate notion of translation of first-order formulas into M-rings.

2. First Order Logic in polynomial format

The problem relating logic and algebra is to specification in which sense a certain class of algebras corresponds to a given logical system. This seems to be more problematic for FOL than to other cases and as it is well known, the process of algebraization of FOL is intrincate, and not thought to be really natural.

As a result, the first-order logic requires infinite polynomials, based on rings with infinitary operations. So, let M be an arbitrary set, L be a first-order language (with equality) and M be an arbitrary set.

Definition 1: *Let M-ring: $\left(R, +, \cdot, -, 0, 1, (A_i)_{i\in\mathbb{N}}, (E_i)_{i\in\mathbb{N}}\right)$ such that:*

$$A_i : \mathbb{Z}_2^M \to \mathbb{Z}_2,\ (s_a)_{a\in M} = \vec{s} \mapsto \bigwedge_{a\in M} s_a$$

In order that have rings associated with first order structures, consider the $\mathcal{M}$-ring, $F(\mathcal{M})$, such that: $|F(\mathcal{M})| := \bigcup_{n\in\mathbb{N}} F_n$, where:

1. $F_0 = \{0\}\cup\{1\}\{X_{t_1=t_2}\}\cup\{X_{r(t_1,\ldots,t_n)}$: some $n \in \mathbb{N}$ and r a n-ary relational symbol$\}$, where t_i is a closed term in the language $L \cup \{a : a \in M\} \cup \{k_i : i \in \mathbb{N}\}$.

2. $F_{n+1} = F_n\cup$
   $\left\{\langle -, p\rangle, \langle +, p, q\rangle, \langle \cdot, p, q\rangle, \left\langle A, \langle S_{i,a}(p)\rangle_{a\in M}\right\rangle, \left\langle E, \langle S_{i,a}(p)\rangle_{a\in M}\right\rangle\right\}$

Definition 2. *For each $i \in \mathbb{N}$ and $a \in M$ ($M = |\mathcal{M}|$), the family of functions $p \in F_n \mapsto S_{i,a}(p) \in F_n$, with $n \in \mathbb{N}$ - the "substitution of the individual variable x_i by $a \in M$" – is defined by:*

1. $S_{i,a}(A, \langle S_{j,b}(p)\rangle_{b\in M}) := (A, \langle S_{i,a}(S_{j,b}(p))\rangle_{b\in M})$

2. $S_{i,a}(E, \langle S_{j,b}(p)\rangle_{b\in M}) := (E, \langle S_{i,a}(S_{j,b}(p))\rangle_{b\in M})$

For each L-structure, we define the $\mathcal{M}$-ring $R(\mathcal{M})$ as the following quotient set: $R(\mathcal{M}) := F(\mathcal{M})/C$[3] such that:

(i) For each $i \in \mathbb{N}$, let $\bar{D}_i = \{\langle [S_{i,a}(p)]_{a\in\mathcal{M}}\rangle : p \in F(\mathcal{M})\} \subseteq (R(\mathcal{M}))^M$, then:

$$\begin{gathered} A_i, E_i : \bar{D}_i \to R(\mathcal{M}), \\ \langle [S_{i,a}(p)]\rangle_{a\in\mathcal{M}} \mapsto [(A, \langle S_{i,a}(p)\rangle_{a\in\mathcal{M}})] \\ \langle [S_{i,a}(p)]\rangle_{a\in\mathcal{M}} \mapsto [(E, \langle S_{i,a}(p)\rangle_{a\in\mathcal{M}})] \end{gathered}$$

Definition 3. Let $\mathcal{M}$ be a L-structure.

(a) The proto-$\mathcal{M}$-translation is the function defined below by recursion on complexity of L-formulas

$$\tau_{\mathcal{M}} : Form(L) \to F(\mathcal{M})$$

1. $\tau_{\mathcal{M}}(u_1 = u_2) = X_{u_1^\sharp = u_2^\sharp}$[4]
2. Translation for classic operators is analogous to that performed in [3]
3. $\tau_{\mathcal{M}}(\forall x_i \varphi) = A_i\left(\langle S_{i,a}\left(\tau_{\mathcal{M}}(\varphi)\right)\rangle_{a\in M}\right)$
4. $\tau_{\mathcal{M}}(\exists x_i \varphi) = E_i\left(\langle S_{i,a}\left(\tau_{\mathcal{M}}(\varphi)\right)\rangle_{a\in M}\right)$

(b) $\tau_{\mathcal{M}} : Form(L) \to R(\mathcal{M})$ is the $\mathcal{M}$-translation iff $\tau_{\mathcal{M}} = q_{\mathcal{M}} \circ \tau_{\mathcal{M}}$ for the proto-$\mathcal{M}$-translation $\tau_{\mathcal{M}} : Form(L) \to F(\mathcal{M})$.

The details of all definitions, theorems and examples of first-order logic in a polynomial version as well as a demonstration of how this polynomial representation of first-order sentences could be seen as a legitimate algebraic semantics for first-order logic, will be performed in the conference and the full article to be submitted to the appropriate deadline.

Keywords: Proof method; Algebraic semantic; Polynomials.

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[3] We define a C-congruence, $\approx$, as: C is a congruence for the structure $(R, +, -, ., 0, 1)$; and for each $i \in \mathbb{N}$ and $\vec{s}, \vec{t} \in D_i, D_i$ is a domain, such that $(s_a, t_a) \in C$ for every $a \in M$,$(A_i(\vec{s}) \approx A_i(\vec{t}))and(E_i(\vec{s}) \approx E_i(\vec{t}))$

[4] Let $v : \{x_i : i \in \mathbb{N}\} \to M$ be a valuation on $\mathcal{M}$; denote $v_{\mathcal{M}} : Terms(L) \to M$ be the unique extension (defined by recursion on term complexity) of $v : \{x_i : i \in \mathbb{N}\} \to M$; extend $v_{\mathcal{M}}$ to $\tilde{v} : Terms(L \cup \{a : a \in M\}) \to M$ by $\tilde{v}(a) = a$, for each $a \in M$; now consider the identifications (= inverse bijections) $Terms(L \cup \{a : a \in M\}) \overset{\sharp}{\to} \underset{\flat}{\to} \leftrightarrows ClosedTerms(L \cup \{a : a \in M\} \cup \{k_i : i \in \mathbb{N}\})$ given by $x_i \leftrightsquigarrow k_i$, $i \in \mathbb{N}$, and denote $\hat{v} : ClosedTerms(L \cup \{a : a \in M\} \cup \{k_i : i \in \mathbb{N}\}) \to M$ the corresponding function.

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Cuts and cut elimination for complementary classical logic

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Abstract.

Introduction

Two deductive systems $\mathscr{S}$ and $\overline{\mathscr{S}}$, sharing a same language, are said to be *complementary* in case that:

$$\vdash_{\overline{\mathscr{S}}} \alpha \text{ if, and only if, } \nvdash_{\mathscr{S}} \alpha.$$

In other words, a system $\overline{\mathscr{S}}$ turns out to be complementary with respect to another system $\mathscr{S}$ if it exactly proves the nontheorems of $\mathscr{S}$. The informal idea underlying the study of complementarity is that of characterizing a system $\mathscr{S}$ by taking, so to speak, its picture in the negative. In this way, theorems of the positive part can be proved by excluding the possibility of their refutation. Needless to say, the most interesting complementary systems are those complementing well-know logical systems, classical propositional logic *in primis.*

In case of classical propositional logic, whereas the semantical characterization is straightforward (just consider all the formulas for which there is at least one falsifying valuation), the proof-theoretical grasp has been a more challenging task. Łukasiewicz's calculus of refutations can be seen as the first proof system complementing classical logic [7]. More than twenty years later, Caicedo provided the first Hilbert calculus for complementary classical logic in [3]. Another Hilbert calculus was proposed by Varzi at the beginning of the 90s [10, 11], the term 'complementarity' is due to him. Almost in the same years, Tiomkin issued the first sequent system for complementary classical logic with rules for negation and disjunction [8]. This system was independently extended by Bonatti and Goranko so as to include rules for the whole spectrum of classical connectives [1, 6, 2]. These calculi, however, do not consider cut rules and so they *ipso facto* exclude the possibility of implementing a cut elimination algorithm.

Indeed, in complementary classical logic, the cut rule in its standard Gentzen's formulation is not admissible:

$$\frac{\Gamma, \alpha \vdash \Delta \qquad \Gamma' \vdash \Delta', \alpha}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,cut$$

The following example shows how a (classically) valid sequent can be obtained by cutting two (classically) non-valid sequents:

$$\frac{p \nvdash p \rightarrow \qquad p \rightarrow q \nvdash p}{p \vdash p}\,cut$$

In [8], Tiomkin reports a couple of 'hybrid' rules that he calls "cuts for the unprovability" since they are obtained by 'reversing' the (additive) standard cut rule:

$$\frac{\Gamma \nvdash \Delta \qquad \Gamma \vdash \Delta, \alpha}{\Gamma, \alpha \nvdash \Delta} \qquad\qquad \frac{\Gamma \nvdash \Delta \qquad \Gamma, \alpha \vdash \Delta}{\Gamma \nvdash \Delta, \alpha}.$$

However, such a denomination turns out to be proof-theoretically improper since both these rules preserve the subformula property.

In this contribution, we consider Bonatti's calculus as it appear in [2] (see Table 1) and we enrich it with some versions of the cut rule which prove admissible in $\overline{\mathsf{LK}}$ (see Table 2). Then, we design a very simple and effective cut elimination procedure. Such a procedure is shown to be strong normalising (in the sense that any reduction strategy terminates) and strongly confluent [4]. As is well-known, this latter fact implies the uniqueness of the normal form [5]. We observe that, unlike in LK, normalisation in $\overline{\mathsf{LK}}$ always induces a remarkable simplification of proofs.

Keywords: Cut elimination; Complementarity.

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Axiom:

$$\frac{}{\Gamma \mathrel{|\!\sim} \Delta}\,ax \qquad \{\Gamma\}, \{\Delta\} \text{ disjoint sets of atoms}$$

Structural rules:

$$\frac{\Gamma, \alpha, \beta, \Lambda \mathrel{|\!\sim} \Delta}{\Gamma, \alpha, \beta, \Lambda \mathrel{|\!\sim} \Delta}\,exch \mathrel{|\!\sim} \qquad \frac{\Gamma \mathrel{|\!\sim} \Delta, \alpha, \beta, \Lambda}{\Gamma \mathrel{|\!\sim} \Delta, \beta, \alpha, \Lambda}\,\mathrel{|\!\sim} exch$$

Logical rules:

$$\frac{\Gamma, \alpha, \beta \mathrel{|\!\sim} \Delta}{\Gamma, \alpha \wedge \beta \mathrel{|\!\sim} \Delta}\,\wedge \mathrel{|\!\sim} \qquad \frac{\Gamma \mathrel{|\!\sim} \Delta, \alpha}{\Gamma \mathrel{|\!\sim} \Delta, \alpha \wedge \beta}\,\mathrel{|\!\sim} \wedge_{\mathcal{R}} \qquad \frac{\Gamma \mathrel{|\!\sim} \Delta, \beta}{\Gamma \mathrel{|\!\sim} \Delta, \alpha \wedge \beta}\,\mathrel{|\!\sim} \wedge_{\mathcal{L}}$$

$$\frac{\Gamma, \alpha \mathrel{|\!\sim} \Delta}{\Gamma, \alpha \vee \beta \mathrel{|\!\sim} \Delta}\,\vee_{\mathcal{R}} \mathrel{|\!\sim} \qquad \frac{\Gamma, \beta \mathrel{|\!\sim} \Delta}{\Gamma, \alpha \vee \beta \mathrel{|\!\sim} \Delta}\,\vee_{\mathcal{L}} \mathrel{|\!\sim} \qquad \frac{\Gamma \mathrel{|\!\sim} \alpha, \beta, \Delta}{\Gamma \mathrel{|\!\sim} \alpha \vee \beta, \Delta}\,\mathrel{|\!\sim} \vee$$

$$\frac{\Gamma \mathrel{|\!\sim} \alpha, \Delta}{\Gamma, \alpha \to \beta \mathrel{|\!\sim} \Delta}\,\to\mathrel{|\!\sim}_{\mathcal{R}} \qquad \frac{\Gamma, \beta \mathrel{|\!\sim} \Delta}{\Gamma, \alpha \to \beta \mathrel{|\!\sim} \Delta}\,\to\mathrel{|\!\sim}_{\mathcal{L}} \qquad \frac{\Gamma, \alpha \mathrel{|\!\sim} \beta, \Delta}{\Gamma \mathrel{|\!\sim} \alpha \to \beta, \Delta}\,\mathrel{|\!\sim}\to$$

$$\frac{\Gamma \mathrel{|\!\sim} \alpha, \Delta}{\Gamma, \neg\alpha \mathrel{|\!\sim} \Delta}\,\neg \mathrel{|\!\sim} \qquad \frac{\Gamma, \alpha \mathrel{|\!\sim} \Delta}{\Gamma \mathrel{|\!\sim} \neg\alpha, \Delta}\,\mathrel{|\!\sim} \neg$$

Table 1: The $\overline{\mathsf{LK}}$ sequent calculus.

$$\frac{\Gamma, \alpha \mathrel{|\!\sim} \Delta \qquad \Gamma \mathrel{|\!\sim} \Delta, \alpha}{\Gamma \mathrel{|\!\sim} \Delta}\,additive\ cut$$

$$\frac{\Gamma, \alpha \mathrel{|\!\sim} \Delta}{\Gamma \mathrel{|\!\sim} \Delta}\,1\text{-}cut_{\mathcal{L}} \qquad \frac{\Gamma \mathrel{|\!\sim} \Delta, \alpha}{\Gamma \mathrel{|\!\sim} \Delta}\,1\text{-}cut_{\mathcal{R}}$$

$$\frac{\Gamma, \alpha \mathrel{|\!\sim} \Delta \qquad \mathrel{|\!\sim} \alpha}{\Gamma \mathrel{|\!\sim} \Delta}\,cut_{\mathcal{R}} \qquad \frac{\Gamma \mathrel{|\!\sim} \Delta, \alpha \qquad \alpha \mathrel{|\!\sim}}{\Gamma \mathrel{|\!\sim} \Delta}\,cut_{\mathcal{L}}$$

Table 2: Admissible cut rules.

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A hyper-paraconsistent theory for arithmetical truth

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Abstract.
According to Tarski ([4] & [5]), a definition of truth for a language L is materially adequate if it implies all instances of the T-schema for the sentences of L, and formally correct if it conforms to the usual logical rules (classical logic) and to the rules for constructing definitions. Hence, the definition cannot yield a contradiction. These requirements are extended to a theory of truth. Tarski presents three conditions that, taken together, lead to a trivialization of a theory of truth:

(1) Semantically closed languages;

(2) Laws of classical logic;

(3) The unrestricted validity of T-schema.

Feferman returns to the issue:

> there are three possible routes that may be taken [by a theory of truth] in the face of the paradoxes, namely by restriction of (1) language, (2) logic, or (3) basic principles. [3, p. 206].

Tarski rejected semantically closed languages and developed a hierarchical definition/theory of truth – route (1). Feferman restricted the T-schema and based his system on classical logic – route (3). Feferman mentions that a paraconsistent logic may be successful in dealing with paradoxes, but remarks that

> So far as I know, it has not been determined whether such logics account for 'sustained ordinary reasoning', not only in everyday discourse but also in mathematics and the sciences. If they do, they deserve serious consideration as a possible route under (2). *Ibidem.*

Our goal is to take the route (2): to develop a materially adequate theory of truth for arithmetic whose underlying logic is paraconsistent. We will also see that the fear that paraconsistent logics may not account for sustained ordinary reasoning is not well founded.

The underlying logic of our truth theory will be a Logic of Formal Plenitude (*LFP*). *LFP*s are a generalization of the Logics of Formal Inconsistency (*LFI*s), a family of paraconsistent logics developed in [1]. *LFI*s have three main features:

i. The notion of consistency is expressed inside the object language by an unary connective: $\circ A$ means that A is (in some sense) consistent;

ii. The consistency and the non-contradictoriness of a formula A may be non-equivalent;

iii. They are gently explosive, that is, they restrict the principle of explosion: $A, \neg A \nvdash B$, while $\circ A, A, \neg A \vdash B$.

An *LFI*, thus, restricts the application of explosion to some formulas. In other words, an *LFI* divides the propositions of a given language into those for which the logical property of explosiveness holds, from those for which such property does not hold. Nevertheless, an *LFI* is not enough to solve the problem, since it cannot avoid Curry's paradox.

Our plan is to control the paradoxes extending the basic idea of *LFI*s: instead of restricting the principle of explosion to consistent formulae only, we restrict inferences that can be made from what we call suspicious formulae. Let $\star A$ means that A is *unsuspicious*, that means, roughly speaking, that there is no fear that A may be false or contradictory. $\star A$ is defined as $\circ A \wedge A$. A restriction is made on *modus ponens* that holds only if the minor premise is not suspicious, hence *full* with respect to implication. Thus, we substitute *modus ponens* by the rule

$(MP^{\star})$ $\star A, A \to B \vdash B$.

The system so obtained is a Logic of Formal Plenitude (*LFP*). An *LFP* intends to describe contexts of reasoning in which inferences are not made with respect to one or more premises that are suspicious for some reason. Notice that the notion of a suspicious formula is clearly epistemic. We say that an *LFP* so defined is a *hyper-paraconsistent* logic.

Call *Verum* (V) the theory of arithmetical truth we want to build. Suppose V is an extension of Peano Arithmetic (PA), that is, PA plus a truth predicate *T(x)* and the T-schema as an axiom schema. The theory V so obtained has a semantically closed language, and by the diagonal lemma it can be proved that there is a sentence λ such that

$V \vdash \lambda \leftrightarrow T(\#\lambda)$, ('$\#\lambda$' denotes the Gödel number of λ; '#' is omitted from now on).

If the logic is classical, we get a contradiction in a few steps and trivialize V. Now, it would seem that a paraconsistent logic could avoid triviality. Unfortunately, this is not the case because Curry's paradox is awaiting. Indeed, let P be any sentence you want. Again by diagonal lemma, there is a sentence C such that

$V \vdash C \leftrightarrow (T(C) \to P)$

Now, the T-schema plus a few logical resources (e.g. a logic as weak as positive intuitionistic logic), lead to the trivialization of the theory, even without the principle of explosion.

The first-order logic of formal plenitude $QmbC^\star$ is obtained from $QmbC$ (a first-order logic of formal inconsistency presented in [2]) replacing modus ponens by $MP^\star$. Now consider that the underlying logic of V is $QmbC^\star$. We add to V the following *fullness schema*,

(FS) if $\vdash_{PA} A$, then $\vdash_V \star A$,

to be read as 'if A is proved in *PA*, then A is implication-full in V'. We also add to V a truth predicate and the following schemas:

(T1) $T(A) \to A$;

(T2) $A \to T(A)$.

The truth theory so obtained has some interesting results. Clearly, from *FS* and $MP^\star$, V proves *T(A)* for every theorem A of *PA*. The truth predicate applied to the Gödel sentence G expresses a notion of truth outside *PA*. The instances of *T1* and *T2* for G holds in V. However, $V \nvdash G$, $V \nvdash \neg G$ and $V \nvdash T(G)$. Thus, V does not clash with Gödel's first incompleteness theorem. V is also materially adequate, since all instances of T-schema hold. But since the sentences λ and C do not belong to *PA*, V does not prove $\star\lambda$ nor $\star C$. So, *prima facie*, V avoids the Liar and Curry's paradoxes, since the usual arguments do not apply.

Keywords: Paraconsistency; Truth, Curry's paradox; Logics of Formal Plenitude

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A case for a philosophy of Logics of Formal Inconsistency

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Abstract.

A logic is *paraconsistent* in which contradiction does not imply triviality, or (what amounts to the same) in which one cannot validly infer from contradictory claimsclaims one ofwhich is the negation of another any claim whatsoever. Of course, if one accepts a logic that is *not* paraconsistent as standard of argumentative rationality, then some (indeed, all) contradictions *are* absurd, for triviality is itself absurd. However, acceptance of a paraconsistent logic as standard of argumentative rationality does not by itself entail that it would be rational to accept contradictions at all (for, obviously enough, there may be more to contradiction and absurdity than triviality only).

Let us call the belief that a paraconsistent logic is the standard of argumentative rationality *paraconsistentism*, and let us call *proinconsistentism* the belief that it is rational to accept some contradictions. Then proinconsistentism and non-trivialism (the quite natural belief that it is irrational to accept any claim whatsoever) together entail paraconsistentism.

Dialethism is the exceptionally controversial belief that (meaningful, unambiguous) contradictory claims can all be *true*. Classical standards of rational acceptance of claims seem to imply that it is rational to accept a claim only if it is true. Let us call the belief that it is so *weak classicalism* (as opposed to *strong classicalism*, which would also take truth to be *sufficient* for rational acceptance). Thus proinconsistentism together with weak classicalism entail dialethism. Priest's famous defence of dialethism itself amounts to no more than an involved, life-long argument for proinconsistentism under the assumption that weak classicalism is true.(Actually, he also seems to argue for proinconsistentism *from* strong classicalism, but never mind that.)

Most paraconsistent logicians that are not also dialethists reject proinconsistentism but implicitly accept weak classicalism. A case in point are relevance logicians. Although they deny that the truth of the premises implying the truth of the conclusion is *sufficient* for the premises to support the conclusion (that is: they deny strong classicalism), they usually still *do* agree with classical logicians in that truth is necessary for support, and thus, if they are coherent enough, they also believe it to be necessary for rational acceptance of claims. As a matter of fact, weak classicalism seems to be such a widespread tenet in logic that few if any logicians ever bothered to argue for it at all, and in any case it has for sure been far less disputed by logicians than its converse. But since almost all logicians, and even most paraconsistent logicians, take weak classicalism to be as uncontroversial as non-trivialism, they also believe (rightly, under such assumptions) that proinconsistentism cannot but amount to dialethism:

and thus their rejection of dialethism is inescapably bound to be a rejection of proinconsistentism altogether.

This, I take it, has been *the* major philosophical hindrance to the understanding and general acceptance of *logics of formal inconsistency* as paradigms for (reasonable) proinconsistentism. Logics of formal inconsistency are paraconsistent logics enhanced with the formal means to explicitly distinguish trivializing from non-trivializing contraditions. If such distinction is to be of any relevance at all, there must be different standards of rational acceptance for trivializing and for non-trivializing contradictions; and, evidently so, the difference must consist in that, while trivializing contradictions cannot in any case be rationally accepted, non-trivializing ones *might* turn out to be acceptable, at least in some cases. But this simply amounts to proinconsistentism. Therefore, if weak classicalism were true, there could be not much more to being a *formal inconsistentist* (a parainconsistentist that accepts a logic of formal inconsistency as standard of argumentative rationality) other than being a dilethist. (At most, the formal inconsistentist would be a dialethist with a somewhat more ingenious logic.)

In sum: if logics of formal inconsistency are to be *philosophically* relevant at all *apart* from dialethism and its overwhelming conceptual drawbacks, its proponents ought to *deny* weak classicalism; and they ought to do so in order that they can be *proinconsistentists* as they ought to be − without sharing the same fate of the dialethist.

In my talk, after a brief discussion of what *acceptance* and *rejection* amount to as propositional attitudes, I will argue *against* weak classicalism, and thus for the possibility of holding proinconsistentism without holding dialethism. Actually, I will do more than that, and go on to provide a concrete example of a situation in which both weak classicalism is false and proinconsistentism is true, − that is, a case in which it is rational to accept possibly *false* claims, and (consequently) also contradictory ones. Moreover, it will be shown that the logic that is the standard of argumentative rationality in such a situation is a logic of formal inconsistency. This result will then be used as ground for arguing that logicians working in the tradition of logics of formal inconsistency should seriously take what we might call *strong non-classicalism* − the belief that truth is neither sufficient nor *necessary* for rational acceptance of claims − to be the philosophical position about argumentative rationality underlying their work, and thus should also try to explore its many possible consequences for a case against dialethism.

Keywords: Logics of Formal Inconsistency. Philosophy of paraconsistency.

The possibility and fruitfulness of a debate on the Principle of Non-Contradiction

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Abstract.
Up to now, five major stances on the two long-standing interconnected problems of the possibility and fruitfulness of a debate on the principle of non-contradiction (PNC henceforth) can be recognized, namely:

- "Detractors" are ready to give PNC up; its relatively straightforward failure would be enough ground to the possibility of disputing it. Aristotle in *Metaphysics* construed several ancient thinkers as detractors of PNC, notably Heracliteans, and today the most visible efforts are by dialetheists as in Priest (2006a).

- "Demonstrators" such as Kant (1755), Boole (1854), Russell (1903) or Priest (2006) are more open to debate, not necessarily because they are ready to give PNC up, but because they think it can be demonstrated in terms at least as secure as the PNC itself.

- Others, let us call them "methodologists", say that PNC can be not only discussed, but accepted or rejected just as any other claim, namely using methodological principles of rational choice. This view is espoused for example by Bueno and Colyvan (2004) and Priest (2006).

- "Calm supporters" say that PNC has usually been formulated in some strong ways and that detractors are rightly attacking those formulations yet they should accept a very basic form of PNC as to ensure the intelligibility of their proposals and criticisms. This is basically the proposal recently outlined by Berto (2008, 2012) and practiced e.g. by Tahko (2014)

However, in the emergence of the last stance the focus has been on how they interact with detractors. In this paper I show what calm supporters have to say on the other parties wondering about the possibility and fruitfulness of a debate on PNC. The main claim is that one can find all the elements of calm supporters already in Aristotle's works, and that his way of dealing with detractors of PNC in *Metaphysics* has wider implications for the possibility and fruitfulness of a debate on PNC. Aristotle's way to refute detractors not only would do that, but also shows how to conduct a debate about PNC even if it is certain and holds universally, against fierce supporters; why it is not demonstrable, against demonstrators, and not even subject to settlement or rejection

through methodological principles, against methodologists. I want to emphasize that I will not attempt a defense of calm supporters, but merely want to show how the Aristotelian refutative strategy can be used beyond its original target, detractors, and that it succeeds at least in exhibiting some serious difficulties for the other parties. A more thorough examination of each of them is left for further work.

The plan of the paper is as follows. In sections 2 to 4 I introduce some basic "Aristotelian" terminology which serves as background for what follows. And although at some points I do not steer clear from exegetical discussion, my main interest lies on suitable logical reconstructions of Aristotle's views useful for the overall issue about the debate on PNC than in an exposition completely sound to the ears of a scholar on them. In Section 2 we distinguish several kinds of principles of non-contradiction present in Metaphysics Γ and some of its properties. Section 3 is devoted to make more precise about the semantic version of PNC which will be discussed throughout the paper. In Section 4 we discuss Aristotle's notions of demonstration and refutation and show how they help to deal with the anti-debate stance of fierce supporters like Lewis. In Section 5 we reconstruct one of Aristotle's refutations of Heracliteans. This will prove useful for showing what is wrong with the approaches of both demonstrators − issue dealt with in Section 6 − and methodologists − analyzed in Section 7.

Keywords: The Principle of Non-Contradiction.

Inconsistent countable set in second order ZFC and the nonexistence of strongly inaccessible cardinals

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Abstract.

In this article we derived an important example of the inconsistent countable set in second order ZFC (ZFC_2) with the full second-order semantic.Main results are:

(i) $\neg Con(ZFC_2)$,(ii) let k be an inaccessible cardinal and H_k is a set of all sets having hereditary size less then k, then $\neg Con(ZFC + (V = H_k))$.

Keywords: Gödel encoding; Completion of ZFC_2; Russell's paradox, ω-model; Henkin semantics; Full second-order semantic; Strongly inaccessible cardinal.

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A weak form of the constructibility axiom

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Abstract.

It is well-known that the Constructibility Axiom, $\mathbf{V} = \mathbf{L}$, is inconsistent with important large cardinal axioms. Since this axiom has many desirable features, it would be good to formulate a weak form of it that preserves these features and is consistent with large cardinals. In order to do that, first we need examine the inconsistency in $\mathbf{V} = \mathbf{L}+$ large cardinals and show that it can be localized in an "avoidable" component of $\mathbf{V} = \mathbf{L}$. The usual proofs of this inconsistency rely on definability and absoluteness aspects of $\mathbf{V} = \mathbf{L}$, which suggests a strategy for localizing the referred inconsistency.

In this paper we will consider one such weak form of $\mathbf{V} = \mathbf{L}$, the Minimal Ordinal-Connection Axiom, which can be obtained by abstracting, in an entirely definability-free way, the *coarse* ordinal pattern given by the constructible rank when the fine behaviour of its ordinal values is covered up. More precisely, let ZF^-_ρ be the following theory: Its language has, in addition to $\in$, an unary function symbol ρ, and the axioms include all axioms of ZF^- along with Replacement and Separation Axioms for formulas containing ρ. Now, consider the following axiom, the Minimal Ordinal-Connection Axiom, in ZF^-_ρ:

- $\forall x, (\rho(x) \textit{is an ordinal})$.
- $\forall \alpha, (\rho(\alpha) = \alpha)$.
- $\forall x, y, (x \in y \to \rho(x) < \rho(y))$.
- $\forall \alpha\, \exists f; (f : \alpha \cup \omega \to \{x : \rho(x) < \alpha\}\ \textit{ is surjective})$.
- For every set x, (i) if $x \in V_\omega$, then $\rho(x) = rk(x)$, and (ii) if $x \notin V_\omega$, then given a transitive set T containing x and $r : T \to T$ satisfying $1-4$ above, $\rho(x) < r(x)^+$.

It turns out that the most important consequences of $\mathbf{V} = \mathbf{L}$, such as the Axiom of Foundation, the Axiom of Choice, the Generalized Continuum Hypothesis, and the fact that if κ is inaccessible then $L_\kappa = V_\kappa$, where $L_\kappa = \{x : \rho(x) < \kappa\}$, remain valid in ZF^-_ρ with the Minimal Ordinal-Connection Axiom. Also, if θ is another function symbol satisfying $1-5$ above, then $|\theta(x)| = |\rho(x)|$, which means that this axiomatization of the constructible rank is categorical with respect to the cardinal of $\rho(x)$.

Furthermore, this theory is consistent with very large cardinals. This follows from the fact that the natural $\mathbf{L}[A]$-rank is a minimal ordinal-connection in $\mathbf{L}[A]$, provided the J^A_α-structures are acceptable, for every α, in the sense of [**?**]. Indeed, the extender models $\mathbf{L}[\vec{E}]$ satisfy an appropriate acceptability condition.

The Minimal Ordinal Connection Axiom can also be independently motivated as the statement of a *similarity with respect to cardinalities* between the ∈-structure of a universe of sets and the well-ordered class of its ordinals. An interesting conception of similarity cannot, of course, be that of isomorphism, which is too strong and inconsistent, and, also, cannot be that of homomorphism from sets to ordinals, which is just the Axiom of Foundation. Instead, an intermediate notion was defined, and it consists in the existence of a minimal ordinal-connection between sets and ordinals. It is just the formalization of the guiding thought according to which the ∈-structure of a set-theoretic universe and the well-ordered class of its ordinals are cardinality similar if and only if sets in this universe can be organized in a membership hierarchy given by an ordinal rank, such that (a) its stages Z_α grow, in terms of infinite cardinality, at the same rate as α and (b) the growth rate of this ordinal rank is minimal in terms of cardinality.

Keywords: Constructibility; Large Cardinals.

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Inconsistency in Mathematics and inconsistency in Chemistry

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Abstract.

In mathematics theories are rigidly and precisely delineated, or individuated. There are two levels of inconsistency that mathematicians face. One is inconsistency internal to a proposed mathematical theory. The other is an inconsistency between theories. The first, can lead to I) abandonment of the theory, II) theory revision or III) adoption of a paraconsistent underlying logic. The last option is rarely adopted, but they all belong to coherent mathematical practice.

The second is dealt with i) by monism: abandoning one of the (each separately consistent or paraconsistent theories) theories, usually on grounds of intuition about the ultimate and single truth of the matter in mathematics, ii) by careful cordoning off of formal theories from one another and then moving to the meta-level to compare theories, or iii) by pluralism which adopts a paraconsistent device or underlying logic at the meta-level of the theories in question. The choice between these options is informed by deep metaphysical intuitions such as: essentialism, pragmatism, scepticism, anti-realism or quietism. These will be discussed in turn.

In contrast, in chemistry, theories are not so systematically individuated. It follows that inconsistencies in chemistry are not always blatant or clear. We might have a theory of carbons or a theory of electrochemistry, one is based on a particular set of materials/elements, the other on types of reactions and subatomic or submolecular theories. Contradictions might appear within a theory, or between general metaphysical conceptions, for example.

In the past, inconsistencies in chemistry have spurred research and have sometimes led to ideas being abandoned or in theory revision. Often what we find in chemistry is a pluralist attitude towards inconsistencies. We might hold two mutually contradictory theories as hypotheses, and defer deciding between them until such time as it becomes important, or until such time as we can think of an experiment that will determine which of the two theories supports the observation. Of course in chemistry, theory determination rarely happens so simply. The added complexity will be discussed.

Having discussed inconsistency in mathematics and inconsistency in chemistry separately, I shall then ask the question whether one set of reactions by mathematicians or chemists can inform the others practice and theory development. I shall be restricting my investigations to Friend and Byers for the account of contradiction and pluralism in mathematics, and to Chang and Schummer for the account of contradiction and pluralism in chemistry.

The mathematicians strategies are rarely known to chemists, but are each available. However, the drawback is that they depend on being able to indi-

viduate theories in chemistry in a way that is not disloyal to the practice but is formal enough that derivations to contradictions can be recognised. I am sceptical about both necessary conditions except in some limit cases.

The other way around: from how inconsistencies are handled in chemistry to how they could be handled in mathematics is more interesting. First, there is an ignoring of the classical logical consequences of inconsistency. But there is no conscious replacement of this with a formal paraconsistent framework for reasoning. Instead, second, inconsistencies are the source of disagreement in the community, and they are also the source of innovation. Moreover, as Chang argues, keeping them around, and alive for longer what he calls a pluralism in chemistry, leads to more fruitfulness in chemistry, not less. Thus, on the part of Chang and Schummer, there is a deliberate recognition of the important role of inconsistency in the science of chemistry. Interestingly, when there is disagreement, this takes a political and institutional turn. But for the discipline as a whole, it is better to keep the inconsistency alive. Thus, with inconsistency we have local and individual discomfort, but from the point of view of the discipline, a flourishing.

The discipline of mathematics can learn from this. Some paraconsistent approaches encourage the flourishing, others do not. Some examples will be discussed.

Keywords: Contradiction; Paraconsistent logic; Philosophy of chemistry; Philosophy of mathematics; Pluralism in mathematics.

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A paraconsistent non monotonic frame to analyze explanation-context interaction: towards an answer to philosophy of science's challenge

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Abstract.

Scientific explanation is a phenomenon studied by at least two different and interesting disciplines of knowledge: Artificial Intelligence (AI) and Philosophy of Science. Both perspectives can be viewed as complementary alternatives in order to clarify this basic function of science. In the context of Artificial Intelligence, explanations have been seen mainly as a process to infer a hypothesis. From Philosophy of Science, explanations are assumed to be a kind of product of scientific activity. In this tradition of research, the central idea is to construe a theory or a formal model that defines adequately explanations in Science. When the theoretical choice is the construction of a formal model, the classical proposal is to see explanations as argumentative structures. In this line of thought the central issue is: What kind of reasoning can clarify the notion of an explanation? Can we construct a model of reasoning that defines adequately an explanation? The reached model is expected to be a criterion to identify explanations, a criterion to recognize explanations from others things in Science: indentify arguments as explanations trough the model structure. Is important to say that, in its kernel, this line of research in Philosophy of Science is expected to help drawing a clear idea of the notion of scientific rationality. In this theoretical context, we will present a model, in reference with the interesting hard objections derived of the criticism against the classical models of explanation (Hempel & Oppenheim 1948, and Hempel 1965). These objections was discussed in the second half of the twenty century. Some examples of this criticism are Scriven 1962, Achinstein 1981, Salmon 1984, Lewis 1976, Railton 1981, and van Fraassen 1980. This criticism, at least from the theoretical context of Philosophy of Science, did shake the very foundations of any tentative of formal representation of scientific explanation.

From our point of view, the problems about explanation in Philosophy of Science can be classified in three types:

I) Problems against the virtues of the inference of the model.

II) Problems about the relevance relation.

III) Questions about the context of explanation.

In this work we will present very briefly a formal model of explanation that helps solving these Philosophy of Science's challenge by formally representing the following four items:

A) Explanation as a final product of a certain kind of reasoning.

B) Interaction between explanation and a part of its involved theoretical context.

C) Explicative change.

D) Explanation in inconsistent theoretical contexts.

We begin assuming an analysis of the philosophical debate about explanation presented in Gaytán (2014), showing that, if we abandon the pretention of capturing formally the best explanation in each case in question, and we focus on the minimal aspects of an explanation, then the main problems disappear. The idea is to take distance from some philosophical ideals about explanations. This strategy goes beyond the mere disarticulation of the problems, helping us to represent explanation in a more dynamic way. A philosophical cost of this strategy is to abandon the ideal of good explanation and see explanations as epistemic proposals to elucidate the world, far away of deductivism and focused on the pragmatic representation of this function of Science: a fallible, flexible and retractable construction of science invented to contribute to the comprehension of reality. It was defended (in philosophical contexts), that this approach let us a flexible connection among explanations, scientific theories and underlying logics, and helps solving central problems in the philosophical debate about explanation. An important part of this proposal is the construction of a non monotonic and paraconsistent model of explanation, basically proposed in Gaytán (2014), that would solve several of the typical philosophical problems put in the way of the argumentative representation of explanation. The formal background of the model is a combination of the logic of default reasoning of Raymond Reiter (1981) and the hierarchy C_n of paraconsistent calculi of Newton da Costa (1974). An important part of the proposal consists in distinguishing three basic concepts: explanation, explanation relation and explicative relevance relation. The aim of that distinctions is to clearly integrate the different constituents of explanation and their functions, in the context of different scientific theories. Finally, the basic strategy for combinating non monotonicity and paraconsistency in this work is a modification of Reiter notion of extension plus a modification of possibility notion in default rules. All this is aimed at a construction of a modular formal frame to analyze explanans in the context of different theories and different underlying logics.

Keywords: Philosophy of Science; Argumentation; Paraconsistency.

Non-involutive bilattices

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Abstract.

Nuel Belnap [2] gave a famous philosophical justification for considering two orders on truth value spaces, the information order and the logical order. In this respect he suggested that, in addition to the classical logical values *true* and *false*, it would be useful to have values $\top$ and $\bot$ for the information order, corresponding to the situation when there is contradicting information ($\top$) and lack of information ($\bot$).

Belnap's approach was generalized by Matthew Ginsberg [7], who introduced as a uniform framework for inference in Artificial Inteligence. Since then, the Belnap-bilattice formalism has found a variety of applications in quite different areas from the original ones. Nowadays the interest in bilattices has thus different sources: among others, computer science and A.I. [7], [1], logic programming [6], lattice theory and algebra [11], algebraic logic and topological duality theory [3], [4], [10], [5].

One of the main intuitions behind bilattices is to view truth values as split into two components, representing respectively positive and negative evidence concerning a given proposition. Since positive and negative evi- dence need not be the complement of each other, this framework allows one to deal with partial as well as inconsistent information. On an algebraic level, this intuition is reflected in the fact that every bilattice can be represented as a special product $L_1 \times L_2$ (called *bilattice product* or *twist-structure* in the literature) of two lattices (L_1 being the positive-evidence lattice and L_2 the negative-evidence lattice). In principle L_1 and L_2 need not be related, that is, the domains of positive and negative evidence need not coincide. How- ever, all bilattice-based logics considered in the literature so far (Ginsberg, Fitting, Arieli-Avron) rely on the assumption that L_1 and L_2 be isomorphic. T his structural constraint is imposed by the presence of an involutive negation in the logical language, that is a negation that behaves classically in that any proposition φ is equivalent, in the strongest possible sense, to $\neg\neg\varphi$.

In this contribution we look at algebraic structures having a *pre-bilattice* reduct (see e.g. [4]) and a negation operator that is no longer required to be involutive, which we call *non-involutive bilattices.* We believe these to be natural structures to consider from the point of view of the the Belnap-Ginsberg original motivation, for there is no reason to assume that the domain of positive and that of negative evidence must coincide. Furthermore, non-involutive bilattices allow us to rigorously formulate a very natural and expected connection between bilattice-based logics and the topological setting of *d-frames* and *bitopological spaces* [9].

We show that non-involutive bilattices are a general framework that en- compass many of the above-mentioned structures: namely, pre-bilattices, bilattices with an involutive negation, bilattices with implication [3] and d-frames. We provide equational presentations for the class of all non- involutive bilattices and the subclasses corresponding to bilattices with an involutive negation, bilattices with implication etc. For each of these we prove a representation theorem that allows us to view any algebra in the class as a bilattice product of two lattices. The key to our generalized prod- uct bilattice construction is to consider pairs of lattices L_1, L_2 together with maps $n : L_1 \to L_2$, $p : L_2 \to L_1$ between them. These maps allow us to turn positive into negative evidence and vice versa, without requiring the two domains to be isomorphic. By imposing additional properties on the maps n and p (e.g. being meet-preserving) we are then able to recover various bilattice-type structures considered in the literature as special cases of our non-involutive bilattices.

This work is a generalization of [8], to which we also refer for further technical details on the product construction of non-involutive bilattices.

Keywords: Bilattice.

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Sequent calculi for four-valued logics

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Abstract.

On the basis of correspondence analysis for many-valued logics, we present a general method to generate cut-free sequent calculi for paraconsistent truth-functional four-valued logics that are close to *first-degree entailment* (*FDE*). A four-valued logic L_4 evaluates arguments consisting of formulas from a propositional language $\mathcal{L}$ built from a set $\mathcal{P} = \{p, p', \ldots\}$ of atomic formulas, using negation ($\neg$) and finitely many additional truth-functional operators of finite arity. In L_4, a valuation is a function v from the set $\mathcal{P}$ of atomic formulas to the set $\{\emptyset, \{0\}, \{1\}, \{0,1\}\}$ of truth-values 'none', 'false', 'true', and 'both'. We use the following shorthands: $\boldsymbol{n}$ abbreviates $\emptyset$, $\mathbf{0}$ abbreviates $\{0\}$, $\mathbf{1}$ abbreviates $\{1\}$, and $\boldsymbol{b}$ abbreviates $\{0,1\}$. A valuation v on $\mathcal{P}$ is extended recursively to a valuation on $\mathcal{L}$ by the truth-conditions for $\neg$ and the truth-conditions for the finitely many additional operators of finite arity. The truth-conditions for $\neg$, which is a paraconsistent four-valued negation, are as follows:

$$\begin{array}{lll} 0 \in v(\neg A) & \text{iff} & 1 \in v(A) \\ 1 \in v(\neg A) & \text{iff} & 0 \in v(A). \end{array}$$

An argument from a set Π of premises to a set Σ of conclusions is L_4-*valid* (notation: $\Pi \models_{L_4} \Sigma$) if and only if for every valuation v it holds that if $1 \in v(A)$ for all A in Π, then $1 \in v(B)$ for some B in Σ.

First, we show that for every truth-functional n-ary operator $\star$ every truth-table entry $f_\star(x_1, \ldots, x_n) = y$ can be characterized in terms of two sequent rules. For instance, the truth-table entry $f_\star(\boldsymbol{b}, \mathbf{1}) = \mathbf{0}$ for a binary operator $\star$ is characterized by the sequent rules $L^{\star+}_{b10}$ and $R^{\star-}_{b10}$:

$$\frac{\Gamma/\Delta, A \qquad \Gamma/\Delta, \neg A \qquad \Gamma/\Delta, B \qquad \Gamma, \neg B/\Delta}{\Gamma, \star(A,B)/\Delta} L^{\star+}_{b10}$$

$$\frac{\Gamma/\Delta, A \qquad \Gamma/\Delta, \neg A \qquad \Gamma/\Delta, B \qquad \Gamma, \neg B/\Delta}{\Gamma/\Delta, \neg \star (A,B)} R^{\star-}_{b10}$$

Consequently, every truth-functional n-ary operator can be characterized in terms of 2×4^n sequent rules. We use these characterizing sequent rules to generate cut-free sequent calculi and prove their completeness with respect to their particular semantics. Lastly, we show that the 2×4^n sequent rules that characterize an n-ary operator can be systematically reduced to at most four sequent rules. We conclude with some straightforward complexity results about these logics.

Keywords: Sequent calculi; Many-valued logic.

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Consistent merge of inconsistent norms

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Abstract.

The need for merging directives coming from different sources is quite common in social life. We may have, for instance, state laws, corporation rules of conduct, religious regulations, orders, requests or expectations com- ing from different people that apply to the same situation. Usually the different directives can be harmoniously combined. In contemporary Euro- pean countries state law allows for the freedom of religion, most corporations do not regulate what employees do in their free time so when a person is obliged by the rules of his or her religion to participate in a religious service on Sunday (or another day free from work) they can easily comply with such a regulation.

However, sooner or later, one can face conflicting regulations, impulses or motivations. It is enough to add to the example the factor that the partner of our agent wants to go hiking for the whole Sunday to have a conflict.

In many cases such a conflict can be quite easily resolved. Several possible ways of solving norm conflicts have been presented, including preferences on norms or norm sources (see e.g. [6]) or Rabbis' decision in the Talmudic system (see e.g. [1]). Applying a game theoretical approach in which an agent gets penalties and payoffs depending on the importance of the norm and the level of violation or compliance would be another one (see e.g. [3]).

Sometimes, however, an agent cannot resolve the conflict. Such situations, especially when they apply to existentially important matters, are recognized in the literature as moral dilemmas and have been extensively discussed in ethics.

We will limit ourselves to the situations in which we deal with clearly defined normative systems in which specific actions are obligatory, forbidden or unregulated (indifferent). The systems do not have to be codified, we just assume that there is no doubt how to classify an action within a given system. Loosely speaking we can say that the justification for such norms lies in the fact that actions are regarded, from some point of view, as good, bad and neutral respectively. We will, however, not consider the rationale of norms but accept them as they are.

That allows us to use three/four-valued logic as a technical tool. Multi-valued logic has been present in deontic logic from the 1950s [7, 5, 2], more recent works include [8, 4, 9]. The biggest advantage of many-valued logic is its conceptual simplicity and efficient decidability. The latter feature is especially important for applications in artificial systems making many-valued logic popular among researchers in computer science.

In the present paper we use a many-valued logic approach for deontic logic focusing on merging norms. The cases of normative conflict, especially dilemmas are most interesting and challenging so we put most of our effort to model these cases. The presented systems, however, can be used also to model merging of non-conflicting normative systems. Finally, we want to obtain the general normative (legal, moral or social) evaluation of actions undertaken in a complex environment consisting of many, possibly inconsis- tent, normative sub-systems.

As we have mentioned above the idea of multivalued deontic logic is not new. Our contribution lies in providing a new reading of action operators within the logic, making it suitable for dealing with normative conflicts. In the paper we discuss three systems. The first of them is based on the matrices introduced in [7] and complemented with more operators on actions in [5]. The other two systems are original. All of them are presented in a unified way slightly different from the earlier formalizations.

The first system, which we call pessimistic, reveals the tragic character of dilemmas. Any action of an agent facing conflicting norms is treated as forbidden. The second system, which we call optimistic, makes us treat any decision in the case of normative conflict as good, provided that at least one obligation is fulfilled. The third and last system, which we call 'in dubio quodlibet', reflects the intuition that conflicting norm derogate one another and the situation of normative conflict is treated as unregulated. Thus any decision in the presence of a dilemma is treated as deontically neutral.

After defining formal tools of multivalued deontic action logic and explaining the intuitive meaning of the main operators we present the three systems in a form of respective matrices. We complement this presentation by sound and complete axiomatizations of the systems.

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Keywords: Inconsistent norms; Merging; Consistency.

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A local-global principle for the real continuum

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Abstract.

Introduction

The logical foundations of mathematical analysis were developed in the 19th century by mathematicians such as Bolzano, Cauchy, Weierstrass, Dedekind, Cantor, Heine, Borel, and Cousin. They established rigorous proofs based on "completeness" axioms that characterize the real number continuum. As noticed in [16], rigor was not the most pressing question, one of the major motivations being *teaching*. Today, the foundation is still recognized as satisfactory, and all classical textbooks define R as any ordered field satisfying a "completeness" axiom. Here is a list of equivalent such axioms[5]:

[SUP] (L.u.b. Property) Any set of reals has a supremum (and an infimum)[6];

[CUT] (Dedekind's Completeness) Any cut defines a (unique) real number;

[NEST+ARCH] (Cantor's Property) Any sequence of nested closed intervals has a common point + Archimedean property;

[CAUCHY+ARCH] (Cauchy's Completeness) Any Cauchy sequence converges + Archimedean property;

[MONO] (Monotone Convergence) Any monotonic sequence has a limit;

[BW] (Bolzano-Weierstrass) Any infinite set of reals (or any sequence) has a limit point;

[BL] (Borel-Lebesgue) Any cover of a closed interval by open intervals has a finite subcover[7];

[COUSIN] (Cousin's partition, see e.g., [6]) Any gauge defined on a closed interval admits a fine tagged partition of this interval;

[IND] (Continuous Induction, see e.g., ([11], [9]).

[5]Some require the Archimedean property: Any real is upper bounded by a natural number.
[6]Possibly infinite. (For example, $sup\mathbb{R} = +\infty$ and $sup\emptyset = -\infty$).
[7]This is Borel's statement, also (somewhat wrongly) attributed to Heine, and later generalized by Lebesgue and others.

It is somewhat striking that all these equivalent properties look so diverse. This calls for a need of a unifying principle from which all could be easily derived. In this work, we introduce and discuss yet another equivalent axiom in two equivalent versions (definitions to be given below):

[**LG**] (Local-Global) Any local and *additive* property is global;

[**GL**] (Global-Local) Any *global* and *subtractive* property has a *limit point*.

The earliest reference we could find that explicitly describes this principle is Guyou's little-known French textbook [8]. It was re-discovered independently many times in many various disguises in some American circles ([5], [12], [17], [19], [15]).

Present Situation

We have studied in detail the logical ow of proofs in graduate textbooks that are currently most influential in the U.S.A. ([18], [1], [2]), France ([3], [14]) and Brazil ([7], [13]) – not only proofs of the essential properties of the reals, but also of the basic theorems for continuity (intermediate value theorem [**IVT**], extreme value theorem [**EVT**], Heine's theorem [HEINE]) and differentiation (essentially the mean value theorem [MVT]).

It appears that [**SUP**] is by far the preferred axiom, [**NEST+ARCH**] being the only considered alternative in ([3], [2])[8]. Other axioms ([**CAUCHY, ARCH**], [**MONO**], [**BW**], often [**BL**], and sometimes [**CUT**]) are derived as theorems. In contrast, [**COUSIN**] and [**IND**] are never used[9]. [**BW**] is often central to prove the basic theorems of real analysis (particularly [**IVT**], [**EVT**], [**HEINE**]) with sometimes [**BL**] as "topological" alternative. In our opinion, several classical proofs are difficult and subtle for the beginner (e.g., proofs of [**EVT**] or [**HEINE**] using [**BW**]). There have been recent attempts to improve this situation by advocating the use of [**COUSIN**] [6] or [**IND**] ([11], [9]), although using these methods can also be cumbersome at times.

Our Proposal

Let us explain the above [**LG**] and [**GL**] principles by defining the following intuitive notions. To simplify the assertions we consider any $[a, b] \subseteq [-\infty, +\infty]$ and assume that all closed intervals $[u, v] \subseteq [a, b]$ are nondegenerate ($u < v$). We shall always consider properties $\mathcal{P}$ of such intervals and write "$[u, v] \in \mathcal{P}$" if $[u, v]$ satisfies the property $\mathcal{P}$.

[8] Some textbooks also mention the possibility of "proving" the fundamental axiom by first constructing the reals from the rationals – themselves constructed from the natural numbers – the two most popular construction methods being Dedekind's cuts and Cantor's fundamental sequences. While this approach is satisfactory for logical consistency, the details are always tedious and not instructive for the student or for anyone using the real numbers, since the way they can be constructed never in uences the way they are used.

[9] Cousin's [**COUSIN**], although proposed at the same time (1895) as Borel's [**BL**], has been largely overlooked since. It was only recently re-exhumed as a fundamental lemma for deriving the gauge (Kurzweil-Henstock) integral (see e.g., [6]). [**IND**] is much more recent, and in fact inspired from [**LG**] ([4], [10]).

Definition 1. *$\mathcal{P}$ is additive if $[u, v] \in \mathcal{P} \wedge [v, w] \in \mathcal{P} \Longrightarrow [u, w] \in \mathcal{P}$. $\mathcal{P}$ is subtractive if $\neg\mathcal{P}$ is additive, i.e., $[u, w] \in \mathcal{P} \Longrightarrow [u, v] \in \mathcal{P} \vee [v, w] \in \mathcal{P}$.*

A useful alternative definition can be given with overlapping intervals (this would not change the method).

Definition 2. *$\mathcal{P}$ is local at x if there exists a neighborhood $V(x)$ in which all intervals $[u, v]$ containing x satisfy $\mathcal{P}$.*

$\mathcal{P}$ has a *limit point* x if $\neg\mathcal{P}$ is not local at x, i.e., any neighborhood $V(x)$ contains an interval $[u, v]$ containing x and satisfying $\mathcal{P}$.

$\mathcal{P}$ is *local* if it is local at every point in $[a, b]$; $\mathcal{P}$ is *global* if $[a, b] \in \mathcal{P}$.

From these definitions it is immediate to see that [**LG**] $\Leftrightarrow$ [**GL**]. Interestingly, many usual properties/objects can be identified as local/limit points. For example, a function f is continuous iff for any $\epsilon > \emptyset$, "$\|f(u) - f(v)\| < \epsilon$" is local; a sequence x_k converges iff "$x_k \in [u, v]$ for sufficiently large k" has a limit point. We feel that local/global concepts are central in real analysis. Thus, taking [**LG**] or [**GL**] as the fundamental axiom for the real numbers it becomes easy and intuitive to prove all the other completeness properties, as well as all the above mentionned basic theorems of real analysis. Due to lack of space we provide only three exemplary proofs.

Proof of [BW]. Let $A \subset [a, b]$ be infinite: the property that "$[u, v]$ contains infinitely many points of A" is global, and evidently subtractive. By [**GL**], it has a limit point, i.e., A has a limit (accumulation) point. $\square$

Proof of [BL]. Let be given a covering of $[a, b]$ by open intervals: the property that "$[u, v]$ has a finite subcover" is local (with only one open interval), and evidently additive. By [**LG**], it is global. $\square$

Proof of [IVT]. Let $f : [a, b] \to \mathbb{R}$ be continuous and let y be lying between $f(a)$ and $f(b)$: the property that y lies between $f(u)$ and $f(v)$ is global, and evidently subtractive. By [**GL**], it has a limit point x, which by continuity of f satisfies $y = f(x)$. $\square$

The aim of this work is to draw attention to such local/global concepts in order to reform teaching of real analysis at undergraduate and graduate levels.

Keywords: Continuity; Local/Global principles; Real Analysis.

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Characterizing finite-valuedness

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Abstract.

It is well known since the work of Lindenbaum that structural Tarskian logics fit exactly with the semantical consequence relation arising from a family of logical matrices. Later, building on the partial results obtained by Łoś and Suszko (1958), and by Wóojcicki (1968), on the abstract notions of uniformity and couniformity, Shoesmith and Smiley (1971) presented the cancellation property capturing exactly the class of structural logics that can be characterized by a single (possibly infinite) logical matrix. In this talk we report on our ongoing effort in developing this track of research. We tackle the problem of describing relevant subclasses of broadly truth functional logics by means of abstract properties regarding their underlying consequence relations.

Besides cancellation, we consider two other properties: *Fm_n-determinedness* a logic is Fm_n-determined for a natural number n, when for all $\Gamma \cup \varphi$ we have $\Gamma \vdash \varphi$ if and only if $\Gamma^\sigma \vdash \varphi^\sigma$for every substitution σ mapping variables to formulas having (at most) $p_1, \cdots, p_n$ as variables $-$ and *local tabularity* $-$ a logic is locally tabular when the Frege equivalence relation divides in finitely many equivalence classes any set of formulas over a finite number of variables. We will study the relative independence between these properties, and ultimately show that combinations of these properties can be used to describe precisely when a logic is characterizable by a family of (at most) n-generated matrices, by a finite family of finite matrices, or by a single finite matrix. These results provide abstract characterizations of different notions of finite-valuedness in logic, that is, logics whose semantics involve a finite number of truth-values.

Although the abstract property present in the above mentioned results is *local tabularity*, the operational notion that we use in the proofs when building finite characteristic matrices is *local finiteness*. A logic is locally finite when the Tarski congruence relation partitions any set of formulas over a finite number of variables in a finite number of equivalence classes. Whereas Frege relation only separates non-equivalent formulas, that is, formulas distinguished by the empty context, Tarski congruence, separates formulas that can be distinguished by some context. From the definition it immediately follows that *local finiteness* implies *local tabularity*. It is not hard to see that these are actually equivalent for a very broad class of logics (e.g. self- extensional, algebraizable). More importantly, in the context of these results, this equivalence also holds in the presence of Fm_n-determinedess, so we are able to invoke local tabularity but actually use local finiteness. However, these notions do not coincide in general, and we shall show an ad-hoc example separating them.

Joint work with Carlos Caleiro and Umberto Rivieccio.

Keywords: Many-valued truth values.

Inconsistencies between theory and observation and the limits of chunk and permeate

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Abstract.

In recent years, much attention has being paid to the role and authenticity of inconsistency in science. A recent view from the philosophy and logic of science has claimed that inconsistency in science can be actually tolerated, and that as a matter of fact, do to it is quite common in scientific activity. This perspective has been enriched by the study of paraconsistent logics and the emergence of case studies from the philosophy of science that seem to illustrate how the presence of some contradictions do not necessarily mean the explosion of the theory in question. The main assertion behind this standpoint is that, contrary to what the traditional view might suggest, inconsistent theories do not always have to be rejected (Lakatos 1970, Laudan 1977, Smith 1988, da Costa 2000, Meheus 2002, Priest 2002).

Some of the most popular examples of this are the early calculus (cf. Brown and Priest 2004), Bohrs Hydrogen Atom (see Brown and Priest 2015), the Dirac Delta function (cf. Benham et al. 2014) or inconsistencies related to Carnots theorem (cf. Meheus, 2002). However, while internal inconsistencies are well-documented in the literature and have been successfully explained by making use of some paraconsistent reasoning strategies, this does not happen with other types of inconsistencies in science, for instance, inconsistencies between theory and observation or inconsistencies between theories. And this is not a trivial issue. As a matter of fact, no one can deny that empirical sciences legitimize, through their methodologies, the role of observation as fundamental for the construction, choice and application of scientific theories. Therefore, if one wants to analyze inconsistencies in empirical sciences, the aspects linked to observation should not, in any sense, be marginalized. Said otherwise, it is necessary to pay special attention to conflicts between theory and observation while looking at inconsistent empirical theories (even from a formal point of view).

In addition, if one aims at embracing an inconsistency toleration thesis, it seems necessary to offer an explanation about how a scientific theory can be inconsistent and not become trivial, especially on the face of the classical assumption that an inconsistent set of premises leads to assume any (well-formed) formula as a result of it. In this direction, some philosophers and logicians of science have addressed this problem by offering reasoning strategies that aim at explaining how it is possible sometimes to reason sensibly from inconsistencies without necessarily arriving to arbitrary conclusions. As a matter of fact, it has been suggested at least, by the paraconsistent tradition that some of such strategies could give a good explanation about how scientists have reasoned from

or with inconsistencies in both, formal and empirical sciences. In this sense, it has been suggested that inconsistencies in science could be successfully modeled through the use of some paraconsistent strategies, such as Chunk and Permeate (Brown and Priest 2004, 2015; Benham et al. 2014, Priest 2014).

Chunk and Permeate is a (mainly) paraconsistent reasoning strategy that was first proposed by Priest and Brown (2004) in order to model the reasoning (that could have been the one) employed in the original infinitesimal calculus. The basic idea behind this strategy is to separate a given set of sentences into consistent fragments (henceforth, chunks) and let specific information to flow between these chunks. The underlying mechanism is a non-adjunctive one that makes each chunk to remain consistent all time, even though the general set of sentences seems to be inconsistent. Even though Chunk and Permeate has been used by its main authors to illustrate three different case studies from inconsistent science (the early calculus, the Dirac delta function and Bohr's Hydrogen Atom), it is expected to be consider as a serious candidate for wider application.

That said, here I will submit the thesis that Chunk and Permeate faces a serious difficulty when recuperating complex cases from inconsistent empirical science (especially the ones that involve observational issues). In order to do so I will proceed in three steps. First, I will characterize Chunk and Permeate as it was presented in (Brown and Priest 2004). Second, I will introduce a case study from neutrino physics (an anomaly from neutrino physics) and I will argue that it illustrates an inconsistency between theory and observation, making it a possible candidate to be modeled with C& P. Third, I will explain why some of the peculiarities of empirical theories, illustrated by this example, cannot be taken into account by using Chunk and Permeate and explore one of the limitations of this formal strategy when dealing with inconsistencies between theory and observation. Finally, I will draw some conclusions on the formal elements and requisites that a paraconsistent strategy has to satisfy if one wants to give an account of more complex kinds of inconsistencies that might not be rare in empirical sciences at all.

Keywords: Chunk and Permeate; Contradictions in Science.

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A Fraïssé theorem for LFIs

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Abstract.

In this talk we show that in QmbC, the minimal system of *logics of formal inconsistency* (Hereafter only LFIs, see [1]), it holds a result analogous to Fraïssé theorem for classical logic. LFIs form a family of paraconsistent logics that restrict the validity of the principle of explosion. According to this principle that holds in classical logic, a contradiction implies everything. By their turn, LFIs restrict the validity of that principle only to *consistent* formulas. We say that a formula φ is consistent if and only if the assumption of φ together with $\neg\varphi$ is sufficient to derive any formula. In LFIs not every formula is assumed consistent. In effect, the assumption of consistency of φ is done explicitly by the assumption of $\circ\varphi$, in which $\circ$ is a primitive logical symbol denoting "consistency". In sum, LFIs are logics that internalize the talk on consistency and inconsistency of formulas of its language in order to satisfy only a weaker version of the principle of explosion.

Fraïssé theorem states a correspondence between syntax and semantics of classical logic: it says that a certain mapping between models can be characterized by the common satisfaction of a certain set of formulas. The mapping that is relevant here is *partial isomorphism*: roughly speaking, two models M and N are partially isomorphic if and only if there is an isormorphism between finite parts of M and N that can be infinitely extended to greater and greater finite parts of those models. The relevant set of formulas is the collection of all formulas of the given language that have at most an arbitrary quantifier rank that is fixed by the condition of the theorem.

The model-theoretic study of the minimal LFI system QmbC started in [2]. In this work, the authors define a Tarskian semantics and prove completeness, compactness and Löwenheim-Skolem theorems for QmbC. This result immediately puts as an agenda the searching for more advanced modeltheoretic results that could be at least partially preserved in QmbC. Our work is set within that agenda. Furthermore, this work is motivated by the following philosophical goal. In classical logic, Fraïssé theorem implies the existence of Hintikka normal forms [3] which can be used for providing a semantical classification of inconsistent formulas of first order logic [4]. We hope to achieve something similar for LFIs through our analogous result. To have such a classification would be specially interesting in order to be able to define finer-grained $\circ$ operators that could verify stricter classes of consistent formulas.

Our talk is organized in the following way. Firstly, we present an overview of [2]. Next, we turn to our aim result by defining partial isomorphism between models of QmbC. Further, we introduce the notion of quasi-prenex normal form which will be useful in the context of our proof. In broad terms, by quasi-prenex normal form we denote a formula φ with a prefix of quantifiers and such that

any quantifier of φ occurring outside of its prefix is in the scope of either a $\neg$ or a $\circ$ operator. Then we prove there is a partial isomorphism of lenght k between two models of QmbC if and only if both agree in any quasi-prenex normal form φ with a prefix of quantifiers of lenght k.

Finally, we draw a comparison between classical Fraïssé theorem and its analog that we present here, calling attention to a more general comparison between QmbC (and LFIs, in general) and first order classical logic. More specifically, we want to adress the following question: what is the semantic relationship between LFIs and classical logic? Until this moment this is a difficult issue. On one hand, LFIs seem to semantically encapsulate classical logic, in the sense that there is an interpretation of classical logic in those logical systems. On the other hand, LFIs seem to characterize an independent class of logics since the notion of isomorphism that they satisfy is slightly different from the classical notion.

Keywords: Model theory; Fraïssé theorem; Logics of Formal Inconsistency.

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Philosophy of information: a philosophy for nowadays?

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Abstract.

In this presentation we discuss the following question: Is the Philosophy of Information a philosophy for nowadays? This question arises as relevant due to the accelerated development of Information and Communication Technologies, and their spread among individuals, which is contributing to the establishment of the "Information Society". In such, we have new kinds of issues, especially concerning to the relation between action/technology/environment. As we will argue, it is emerging a new way of understanding the world, the beings, and the relationship between them. Due to the new informational context, we will analyze the thesis according to which it would be required a Philosophy of Information to comprehend current phenomena (FLORIDI, [8], [10], [12]). We will analyze the core assumptions of this new area of philosophy and some problems that make up its research agenda. One of the first philosophers to propose a characterization of Philosophy of Information (PI) was Luciano Floridi ([8] - revisited in 2011). We believe that such a proposal was due to the development of the "Informational Turn in Philosophy" (Adams, [1]; Gonzalez et al, [13]), from which it was constituted a philosophical scenario around the concept of information. As argued by Adams ([1]), an informational turn would have occurred in Philosophy in 1950, year of publication of the seminal article of Alan Turing entitled Machinery and Intelligence. This turn began a rapprochement between the studies of Philosophy and Science, promoting an interdisciplinary discussion of the ontological and epistemological nature of the information. The impact of the "Informational Turn in Philosophy" during the second half of the twentieth century influenced both the academic and philosophical scope, as the social context in general (MORAES, [14]). The first is evidenced by the large number of philosophical and scientific works developed around the concept of information (eg., WIENER [19] [20], SAYRE [15] [16], Dretske [7] Stonier [17], among others). As for the social sphere, the development of information theory studies promoted technological advancement that we're currently experiencing and that are generating new types of problems, in particular concerning the relationship action/technology/environment. As we will argue, the development of the informational turn has allowed the emergence of the "information society" in which the Information and Communication Technologies (ICT) are widespread in everyday life of individuals. We can understand this "label" of contemporary society in two ways:

Broad: referring to a complex world of innovation and communication, in which the creation of new environments and changes in the social dynamics of individuals;

Narrow: concerns to the change in the world of individuals' biases, changing the way they interact with the world, with others and how they conceive themselves in face to the current reality.

In this context, scholars like Adriaans & Van Benthem ([2]), Allo ([3]), Gleick ([6]), Demir ([5]), Beavers & Jones ([4]), and in particular Floridi ([8], [9], [10]) emphasize the importance and the urgency to develop a PI. Floridi ([10], p. 1) argues that: "Computational and information-theoretical research in philosophy has become increasingly fertile [$\cdots$] It revitalizes old philosophical questions, poses new problems, contributes to re-conceptualization of our world-views." For this reason, in this presentation, we will focus our analysis in the narrow way to conceive the "Information Society". The characterization of contemporary society as "the information society", according to Floridi ([9], [11], [12]), could be regarded as a result of an information revolution, which stresses a relationship of dependency between individuals and ICT. Once resulting from the occurrence of an information revolution, it presents the hypothesis that the PI would be an appropriate philosophy to understand such "new" inherent dynamic of the "Information Society". So our main objective in this presentation is to discuss this hypothesis. Therefore, at first, we made explicit the central bases of PI, identifying its characteristics and elements, which make possible to understand it as an autonomous research area of Philosophy. Then, we analyze the Floridian view that the information revolution would influence the constitution of the "Information Society". Finally, we discuss the question: Is the PI a philosophy for nowadays? As we will indicate, without closing our conclusion in an absolute position, we believe that the PI could be analyzed as a "necessary condition" of a Philosophy for nowadays, but about having a "sufficient" aspect for this purpose it is necessary to confront it to the very development of history. Insofar, we seek to contribute to the understanding of new directions of philosophical research on "Information Society".

Keywords: Philosophy of Information

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Para-disagreement logics

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Abstract.

On closer inspection many apparent contradictions turn out to be mere disagreements between distinct sources of knowledge. For example, if a source s_1 says P and a source s_2 says $\neg P$, their disagreement would only become an actual contradiction if, in our own knowledge base, we naively merged what they say. In this case, our own knowledge base would entail $P \wedge \neg P$ and, therefore, it would be inconsistent. Although we could use traditional paraconsistent logics to avoid the inconsistency's worst consequences, this would be an unsatisfactory approach, because the inconsistency was clearly just a result of our indiscriminate merging.

This paper proposes new logics through which disagreements can be expressed. A *possible worlds* semantics is used, and each source denotes a world. Statements of the form $@_s P$ express that source s claims P and denote that P is true at the world denoted by s. Within these logics, we can merge conflicting knowledge more cautiously. For instance, our knowledge base would entail $(@_{s_1} P) \wedge (@_{s_2} \neg P)$ and, as desired, no inconsistency would be entailed by the disagreement between s_1 and s_2.

The idea of using modal logics to handle (apparent) contradictions can be traced back at least to Jaskowski's discussive logics. However, the para-disagreement logics proposed here use the @ modality, thereby overcoming well-known issues faced by Jaskowki due to his use of the $\Diamond$ modality instead. As in Jaskowki's logics, the $\Box$ modality acts like a consensus operator. $\Box P$ expresses that everybody claims P. Together with the T axiom ($\Box P \rightarrow P$), the behavior of $\Box$ is reminiscent of the $\circ$ consistency operator of logics of formal inconsistency with principles of gentle explosion.

Going further, para-disagreement logics can be extended with operators that allow reasoning about the collective aggregated decisions of a group even when there is no consensus. To illustrate this, a simple *majority* modality is introduced and employed in the context of social preference aggregation through voting.

Para-disagreement logics should be regarded as a complement (but not a replacement) to traditional paraconsistent logics. Not all (apparent) contradictions are disagreements. It is important to choose the right kind of logic for each kind of contradiction.

Keywords: Para-disagreement logics; Paraconsistent logic.

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Institutions for propositional logics and an abstract approach to Glivenko's theorem

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Abstract.

The main motivation to study category of logics are methods of combining logics. Some initial steps on "global" approach to categories of logics have appeared in [1], [4], [6], [8]. In [1] is studied the category of logic with strict morphisms $\mathcal{L}_s$. The object in this category are logics viewed like pairs $(\Sigma, \vdash)$ such that Σ is a signature and $\vdash$ is a tarskian consequence operator. The morphisms $f : (\Sigma, \vdash) \to (\Sigma', \vdash')$ are sequences of functions $f : \Sigma \to \Sigma'$ where Σ is a sequence of pairwise disjoint sets $\Sigma = (\Sigma_n)_{n\in\mathbb{N}}$ and $f = (f_n)_{n\in\mathbb{N}}$ such that $f_n : \Sigma_n \to \Sigma_n$. This functions extend to formulas $\hat{f} : F(\Sigma) \to F(\Sigma')$ and given $\Gamma \cup \{\varphi\} \subseteq F(\Sigma)$ then $\Gamma \vdash \varphi \Rightarrow \hat{f}[\Gamma] \vdash' \hat{f}(\varphi)$, such morphism f is called a logical translation. This category has "good" categorial properties but unsatisfactory treatment of the "identity problem" of logics. On the other hand, in [6] and [4] is explored a category with flexible morphism $\mathcal{L}_f$ having a satisfactory treatment of "identity problem" but it does not have "good" categorial properties. The objects in $\mathcal{L}_f$ are the same in $\mathcal{L}_s$ but the morphisms are logical translations such that $c_n \in \Sigma_n \mapsto \varphi'_n \in F(\Sigma')[n]$ where $F(\Sigma)[n]$ is the set of formulas in $\{x_0, \cdots, x_{n-1}\}$.

▶ *Other categories of logics*

In [8] are considered:

• $\mathcal{A}_s(respect.\mathcal{A}_f)$, the category strict (respect. flexible) of (Blok-Pigozzi) BP-algebraizable logics ([3]):
objects: logic $l = (\Sigma, \vdash)$, that has some *algebraizing pair* $((\delta \equiv \epsilon), \Delta)$;
morphisms: $f : l \to l'$; $f \in \mathcal{L}_s(l, l')(\mathcal{L}_f(l, l'))$ and "preserves algebraizing pair" (well defined).

• $Q\mathcal{L}_f$: "quotient" category: $f \sim g$ iff $\check{f}(\varphi) \dashv' \vdash \check{g}(\varphi)$.
The logics l and l' are equipollent ([4]) iff l and l' are $Q\mathcal{L}_f$-isomorphic.

• $\mathcal{L}^c_f \subseteq \mathcal{L}_f$: "congruential" logics: $\varphi_0 \dashv\vdash \psi_0, \ldots, \varphi_{n-1} \dashv\vdash \psi_{n-1} \Rightarrow c_n(\varphi_0, \ldots, \varphi_{n-1}) \dashv\vdash c_n(\psi_0, \ldots, \psi_{n-1})$.
The inclusion functor $\mathcal{L}^c_f \hookrightarrow \mathcal{L}_f$ has a left adjoint.

• $Lind(\mathcal{A}_f) \subseteq \mathcal{A}_f$: "Lindenbaum algebraizable" logics: $\varphi \dashv\vdash \psi \Leftrightarrow \vdash \varphi\Delta\psi$ (well defined).
$Lind(\mathcal{A}_f) \subseteq \mathcal{L}^c_f$ and the inclusion functor $Lind(\mathcal{A}_f) \hookrightarrow \mathcal{A}_f$ has a left adjoint.

• $Q\mathcal{L}_f^c$ (or simply $\mathcal{Q}_f^c$): "good" category of logics: represents the major part of logics; has good categorial properties (is an accessible category complete/cocomplete); solves the identity problem for the presentations of classical logic in terms of isomorphism; allows a good notion of algebraizable logic.

► *On the categories above presented follows some useful results*

In [9] are analyzed:
•Functors:

Forgetful functors: $U : \mathcal{A} \to \mathcal{L}_s$; $U' : \mathcal{A} \to \mathcal{S}_s$;
The (forgetful) functor $(QV \overset{I}{\to} \Sigma - Str \overset{U}{\to} Set)$ has the (free) functor $(Set \overset{F}{\to} \Sigma - Str \overset{L}{\to} QV)$, $Y \mapsto F(Y)/\theta_{F(Y)}$, as left adjoint. Moreover, if $\sigma_Y : Y \to U \circ F(Y)$ is the Y-component of the unity of the adjunction (F, U), then $(Y \overset{t_Y}{\to} UILF(Y)) := (Y \overset{\sigma_Y}{\to} UF(Y) \overset{U(q_{F(Y)})}{\to} UILF(Y))$ is the Y-component of the adjunction $(L \circ F, U \circ I)$. If a is algebraizable, then $LF(X) = F(X)/\Delta$, for some equivalente set of formulas Δ.

•Let $h \in \mathcal{A}_f(a, a')$, then the induced functor $h^\star : \Sigma' - Str \to \Sigma - Str$ $(M' \mapsto (M')^h)$, "commutes over Set" (i.e., $U \circ h^\star = U'$) and it has restriction $h^\star\restriction: QV(a') \to QV(a)$ (i.e. $I \circ h^\star\restriction = h^\star \circ I'$); moreover, $h^\star\restriction$ has a left adjoint $L_h : QV(a) \to QV(a')$;

•Let $g_0, g_1 : l \to a \in \mathcal{L}_f$, with $a \in Lind(\mathcal{A}_f)$. $[g_0] = [g_1] \in Q\mathcal{L}_f \Rightarrow g_0^\star\restriction = g_1^\star\restriction: QV(a) \to \Sigma - str$.

• $h \in Lind(\mathcal{A}_f)(a, a')$ is dense [10] $\Leftrightarrow h^\star\restriction: QV(a) \to Qv(a')$ is full, faithful, injective on objects and satisfies the heredity condition, i.e., given $M \in QV(a)$ and N' a Σ'-substructure of $h^\star\restriction(M)$, then there is $N \in QV(a)$, a Σ- substructure M such that $h^\star\restriction(N) = N'$. When h is dense, a left adjoint $L_h : QV(a) \to QV(a')$ can be given on $M \in QV(a)$ by taking a certain quotient of M.

► *Categorial relationship between Institution and* Π-*Institution*

The notion of *Institution* was introduced for the first time by Goguen and Burstall in [5]. This concept formalizes the informal notion of logical system into a mathematical object. The main (model-theoretical) characteristic is that an institution contains a satisfaction relation between models and sentences that is coherent under change of notation: That motivated us to consider an institution of a logic, i.e., an institution for a logic l will represent all logic l' such that is *equipollent* with l ([4]). A variation of the formalism of institutions, the notion of π-*Institution*, were defined by Fiadeiro and Sernadas in [7] providing an alternative (proof-theoretical) approach to deductive system. In [7] and [12] was showed an way to relate institutions with π-institutions. To the best of our knowledge, there is no literature on categorial connections between the category of institutions and the category of π-institutions. Here we provide a categorial

[10] I.e., for each $\psi' \in F(\Sigma')$, there is $\psi \in F(\Sigma)$, such that $\check{h}(\psi) \dashv' \vdash \psi'$.

relationship using the known relation between objects of those categories, more precisely, we have determined a pair of adjoint functors between those categories.

► *Institutions for abstract propositional logics*

There is a natural "proof-theoretical" encoding of the category of propositional logics: i.e., an way to provide a π*-institution* for the category of propositional logics. This lead us to search (and provide) an analogous "model-theoretical" version of it $-$*institution* for the category of propositional logics– that is different from the canonical one i.e., that obtained by applying the (left adjoint) functor $\pi - \mathbf{Inst} \to \mathbf{Inst}$. However, our the main motivation for the use of institution theory in this work is because it relates the sentences and models of a logic *independently of its presentations*, retaining only its "essence". More precisely, connecting these abstract logical settings with the notions presented in previous works ([2], [9]), we introduce institutions for each (equivalence class of) algebraizable logic a, $IA(a)$, and Lindenbaum algebraizable logic l, $ILA(l)$: this will enable us to apply notions and results from institution theory to study meta-logic properties of a (equivalence class of) well-behaved logics.

► *The abstract Glivenko's theorem*

Concerning the latter, we present above the definition of a *Glivenko's context* between two algebraizable logics. Recall that the classical Glivenko's theorem, proved by Valery Glivenko in 1929, says that one can translate the classical logic into intuitionistic logic by means double-negation of classical formulas. More precisely, if Σ is a commom signature for expressing presentations of classical propositional logic (CPC) and intuitionistic propositional logic (IPC) – for instance, $\Sigma = \{\neg, \to, \wedge, \vee\}$– and $\Gamma \cup \{\varphi\} \subseteq F(\Sigma)$, then

$$\Gamma \vdash_{CPC} \varphi \text{ iff } \neg\neg\Gamma \vdash_{IPC} \neg\neg\varphi.$$

Definition 1 *A* ***Glivenko's context*** *is a pair* $\mathbb{G} = (h : a \to a', \rho)$ *where* $h \in \mathcal{A}_f(a, a')$ *and* $\rho : h^\star\restriction \circ L_h \Rightarrow Id$ *is a natural transformation that is a* section *of the unit of the adjunction* $(L_h, h^\star\restriction)$.

We prove that for each Glivenko's context relating two algebraizable logics (respectively, Lindenbaum algebraizable logics), can be associated a *institutions morphism* between the corresponding logical institutions. Moreover, as a consequence of the existence of such institutions morphisms, we have established abstract versions of Glivenko's theorem between those algebraizable logics (Lindenbaum algebraizable logics), generalizing the results presented in [11]. We can interpret this as another evidence[11] of the (virtually unexplored) relevance of

[11]Beside the nice approach of the identity problem for (algebraizable) propositional logics: "a logic is an institution, thus manifested through many signatures".

institution theory in propositional logic. In particular, considering the institutions of classical logic and of intuitionistic logic, we build a Glivenko's context and thus an abstract Glivenko's theorem such that is exactly the traditional Glivenko's theorem.

Theorem 2 *Each* $\mathbb{G} = (h : a \to a', \rho)$ ***Glivenko's context*** *between two algebraizable logics (respectively, Lindenbaum algebraizable logics) induces a institutions morphism* $M_G : IA(a) \to IA(a')$ *(respectively,* $M_G : ILA(a) \to ILA(a')$*).*

If $\mathbb{G} = (h : a \to a', \rho)$ is a **Glivenko's context** then h is a $(\Delta-)$dense morphism. However not every dense morphism induces a institution morphism M_G.

Corollary 3 *For each Glivenko's context* $\mathbb{G} = (h : a \to a', \rho)$*, is associated an abstract Glivenko's theorem between* a *and* a' *i.e; given* $\Gamma' \cup \{\varphi'\} \subseteq F'(X)$ *then*

$$\rho_{F(X)}[\Gamma'] \vdash \rho_{F(X)}(\varphi') \;\Leftrightarrow\; \Gamma' \vdash' \varphi'$$

Remark 4

(a) *If* $\mathbb{G} = (h : a \to a', \rho)$ *is a* ***Glivenko's context*** *then* h *is a* $(\Delta-)$*dense morphism. However not every dense morphism induces a institution morphism* M_G*.*

(b) *We can define a category* $G\mathcal{A}_f$ *(respect.* $GLind(\mathcal{A}_f)$*) with objects the (Lindenbaum) algebraizable logics and with morphisms are Glivenko's contexts. The theorem defines, in fact, a functor* $\Phi : G\mathcal{A}_f \to Inst$ *(respect.* $\Phi : GLind(\mathcal{A}_f) \to Inst$*).*

(c) *Each Lindenbaum algebraizable logic* a*, determines a* comorphism of institutions*:* $h^a = (\Phi^a, \alpha^a, \beta^a) : ILA(a) \to IA(a)$*.*

(d) *One can ask "why do use different notion of institution of a Lindenbaum algebraizable logic instead of the restrict the notion of institution of algebraizable logic to the class of Lindenbaum algebraizable logic?" The answer of this question is that these institutions seems not be isomorphic, but there are notions of abstract Glivenko's theorem for both of them. This means that we have two different approaches to abstract Glivenko's theorem: We believe that those two different approaches for the abstract Glivenko's theorem can be applied for special classes of logics, for instance we can use the idea behind of the institution for a algebraizable logic to build the institution for an equivalential logic, on the other hand we can use the idea behind of the institution for a Lindenbaum algebraizable logic to build the institution for a truth-equational logic.*

Keywords: Glivenko's Theorem; Algebraizable logics; Institutions.

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Towards a nonclassical metatheory in substructural approaches to paradox

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Abstract.

It is fairly common to make a distinction between two views about how our semantic theories should be like. One view -the so-called *orthodox* view- is that theories that purport to explain how semantic concepts of a certain language work should be couched in a richer language containing expressions for those semantic concepts. Usually, this is cashed-out in terms of the distinction between an object language and a metalanguage. Alfred Tarski famously pointed out that in order to explain how truth behaves in a certain language we need to (and should) ascend to an essentially richer metalanguage.

A relatively more recent view -usually called the *naive* view- endorses the idea that semantic theories should be couched in a language with enough resources to express its own semantic concepts. In this view there is no need to look for a richer metalanguage or to make an artificial distinction between different languages. The formal languages of our semantic theories should mimic ordinary languages, at least to the extent that in them we can talk about the semantic concepts that apply to the expressions of that same language.

The main idea behind the naive approach is to develop theories about naive semantic concepts. However, although there is a wide consensus on what counts as a naive concept of truth, I think that the situation for the concept of validity is somewhat different. Usually, a validity predicate of a theory $\mathcal{S}$ is said to be naive if the following condition holds:

1. For every $\mathcal{S}$-valid argument from Γ to ϕ, $\mathcal{S}$ should prove the statement expressing that the argument from Γ to ϕ is valid.

Now, although this condition is certainly necessary for the corresponding validity predicate to be naive, it is far from being sufficient. In particular, a naive validity predicate for a theory $\mathcal{S}$ should capture at least one additional feature of $\mathcal{S}$:

3. For every $\mathcal{S}$-*invalid* argument from Γ to ϕ, $\mathcal{S}$ should *disprove* the statement expressing that the argument from Γ to ϕ is valid.

The main goal of this paper is to evaluate whether certain substructural theories are able to characterize what might be viewed as a naive concept of validity in the sense above without falling prey to the any self-referential paradoxes.

Sequents and antisequents

There is a small but very interesting literature on how to provide proof procedures for the set of invalidities of classical propositional logic. These proof procedures are designed to prove a certain argument if and only if the argument is *not* valid. As with sets of validities, this task can be done in different ways. Both Caicedo [1] and Varzi [4],[5] offer an axiomatic calculus, while Tiomkin [3] and Carnielli & Pulcini [2] provide a sequent calculus.

Now, the sort of sequent calculus we need does not only work with sequents properly speaking, for the proof of a sequent is meant to represent that the corresponding argument is valid. What we need are, in addition, antisequents. Following [3] we say that an *antisequent* is an object of the form $\Gamma \nRightarrow \Delta$. Intuitively, we should understand this as 'the inference from Γ to Δ fails'.

What does a sequent calculus for invalidities look like? Since we want to talk about both validities and invalidities, what we need is a mixed system. In such a system, there will be two types of objects, sequents and antisequents, and some of the rules will allow us to go from one type of object to the other. Our full system (which I'll call $\mathbb{M}$ for 'mixed') is then as follows:

Definition 1. (*The system* $\mathbb{M}$) Let Γ, Δ, Π and Σ be (finite) multisets of formulas, and let ϕ and ψ be formulas. The system $\mathbb{M}$ is given by the following initial sequents, initial antisequents and rules:

$$Axioms\,\frac{}{p \Rightarrow p} \qquad Antiaxioms\,\frac{}{p_1, ..., p_n \nRightarrow q_1, ..., q_m}\,\text{where for all } i,\ j\ p_i \neq q_j$$

$$Weak\,\frac{\Gamma \Rightarrow \Delta}{\Gamma' \Rightarrow \Delta'}\,\text{where } \Gamma \subseteq \Gamma' \text{ and } \Delta \subseteq \Delta'^{2} \qquad Antiweak\,\frac{\Gamma' \nRightarrow \Delta'}{\Gamma \nRightarrow \Delta}\,\text{where } \Gamma \subseteq \Gamma' \text{ and } \Delta \subseteq \Delta'$$

$$\mathrm{L}Contr\,\frac{\Gamma, \phi, \phi \Rightarrow \Delta}{\Gamma, \phi \Rightarrow \Delta} \qquad \mathrm{L}AntiContr\,\frac{\Gamma, \phi \nRightarrow \Delta}{\Gamma, \phi, \phi \nRightarrow \Delta}$$

$$\mathrm{R}Contr\,\frac{\Gamma \Rightarrow \phi, \phi, \Delta}{\Gamma \Rightarrow \phi, \Delta} \qquad \mathrm{R}AntiContr\,\frac{\Gamma \nRightarrow \phi, \Delta}{\Gamma \nRightarrow \phi, \phi, \Delta}$$

$$\mathrm{L}\neg^{+}\,\frac{\Gamma \Rightarrow \phi, \Delta}{\Gamma, \neg\phi \Rightarrow \Delta} \qquad \mathrm{L}\neg^{-}\,\frac{\Gamma \nRightarrow \phi, \Delta}{\Gamma, \neg\phi \nRightarrow \Delta}$$

$$\mathrm{L}\neg^{+}\,\frac{\Gamma \Rightarrow \phi, \Delta}{\Gamma, \neg\phi \Rightarrow \Delta} \qquad \mathrm{L}\neg^{-}\,\frac{\Gamma \nRightarrow \phi, \Delta}{\Gamma, \neg\phi \nRightarrow \Delta}$$

$$\mathrm{R}\neg^{+}\,\frac{\Gamma, \phi \Rightarrow \Delta}{\Gamma \Rightarrow \neg\phi, \Delta} \qquad \mathrm{R}\neg^{-}\,\frac{\Gamma \nRightarrow \phi, \Delta}{\Gamma, \neg\phi \nRightarrow \Delta}$$

$$\mathrm{L}\wedge^{+}\,\frac{\Gamma, \phi, \psi \Rightarrow \Delta}{\Gamma, \phi \wedge \psi \Rightarrow \Delta} \qquad \mathrm{L}\wedge^{-}\,\frac{\Gamma, \phi, \psi \nRightarrow \Delta}{\Gamma, \phi \wedge \psi \nRightarrow \Delta}$$

[11]Inclusion between multisets should be understood as usual. Also, in the rules *Weak* and *Antiweak* I assume that at least one of the two inclusions has to be strict.

$$\text{R}\wedge^{+}\ \frac{\Gamma \Rightarrow \phi, \Delta \qquad \Gamma \Rightarrow \psi, \Delta}{\Gamma \Rightarrow \phi \wedge \psi, \Delta} \qquad\qquad \text{R}\wedge^{-}\ \frac{\Gamma \not\Rightarrow \phi, \Delta}{\Gamma \not\Rightarrow \phi \wedge \psi, \Delta}$$

The system so far is nothing more than a mixed version of classical propositional logic. If we take the positive part only, we obtain the set of validities of classical propositional logic, and if we take the negative part only, we obtain the set of invalidities of classical propositional logic.

Now, if we want to talk about (and prove things having to do with) the validity and invalidity of inferences, it is not enough to consider purely positive or purely negative rules. As I mentioned before, we want to say, for instance, that if q does not follow from p, then $\neg Val(p, q)$. This motivates the presence of mixed rules. The validity rules are then as follows:

$$\text{R}Val^{+}\ \frac{Val(\Gamma_1, \Delta_1), \cdots, Val(\Gamma_k, \Delta_k), \phi_1, \cdots, \phi_n \Rightarrow \psi_1, \cdots, \psi_m}{Val(\Gamma_1, \Delta_1), \cdots, Val(\Gamma_k, \Delta_k) \Rightarrow Val(\Phi, \Psi)}$$

$$\text{L}Val^{+}\ \frac{\phi_1, \cdots, \phi_n \not\Rightarrow \psi_1, \cdots, \psi_m}{Val(\Phi, \Psi) \Rightarrow}$$

$$\text{L}Val^{-}\ \frac{\phi_1, \cdots, \phi_n \Rightarrow \psi_1, \cdots, \psi_m}{Val(\Phi, \Psi) \not\Rightarrow}$$

$$\text{R}Val^{-}\ \frac{\Gamma, \phi_1, \cdots, \phi_n \not\Rightarrow \psi_1, \cdots, \psi_m, \Delta}{\Gamma \not\Rightarrow Val(\Phi, \Psi), \Delta}$$

I'll call the resulting system $\mathbb{M}^V$. The crucial feature of $\mathbb{M}^V$ is that we can now talk about invalidity quite straightforwardly. Moreover, the resulting theory is complete in two ways: for every $\mathcal{S}$-valid inference from Γ to Δ, we have $\Rightarrow Val(\Gamma, \Delta)$ and for every $\mathcal{S}$-invalid inference from Γ to Δ we have $Val(\Gamma, \Delta) \Rightarrow$.

Notice also that the system lacks a Cut rule. This is no accident. One of the things we would like to know about our system is if it is consistent. Since our proof system has two sorts of objects -sequents and antisequents- we can define a new property which I'll call *external consistency*. I'll say that a system is *externally consistent* if it is not the case that $\Gamma \Rightarrow \Delta$ and $\Gamma \not\Rightarrow \Delta$ are both provable in the system. Observe that in the present context, Cut elimination does not ensure external consistency. So in this framework the latter property seems to be a priority. And in fact, $\mathbb{M}^V$ is externally consistent.

Theorem 2 (External consistency) *$\mathbb{M}^V$ is externally consistent. That is, for any Γ and any Δ it is not the case that $\Gamma \Rightarrow \Delta$ and $\Gamma \not\Rightarrow \Delta$ are both provable in $\mathbb{M}^V$.*

There are still several open questions remaining. Of particular interest is the issue of the self-referential paradoxes and more specifically the so-called v-Curry paradox. This creature reemerges in this setting with a different face. One remarkable feature of this paradox is that it seems to affect non-contractive theories. In this sense, the paradox is quite similar to the paradoxes of logical properties introduced by Zardini in [6] and [7].

In spite of this, the non-contractivist theorists has, in my opinion, a plausible way out. She can embrace a conception of validity where certain inferences are neither valid nor invalid, i.e. a conception of validity where there could be validity gaps. Whether this is an unaffordable cost is a question that is too difficult to discuss here.

Keywords: Substructural logics; Paradoxes; Validity; Antisequents.

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Paraconsistentization of logics via category theory

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Abstract.

Paraconsistentization of logics is a general theory of paraconsistent logics. It means the process of finding, for a given logic, its paraconsistent counterpart. This paper proposes an abstract method for paraconsistentizing a logic. We continue the research developed in [1], and keep the discussion at the abstract level. Formally, as is well-known, a **consequence structure** is a pair $\langle X, Cn\rangle$, such that:

(i) X is a set, called **domain** of the structure;

(ii) Cn is an operation in the power set of X, $Cn : \wp(X) \to \wp(X)$, called **consequence operator** of the structure.

Let $\langle X, Cn\rangle$ be a consequence structure and $A \subseteq X$. A is Cn-**consistent** if and only if $Cn(A) \neq X$. If this is not the case, A is Cn-**inconsistent**.

Let $\langle X, Cn\rangle$ e $\langle X', Cn'\rangle$ be two consequence structures. A **homomorphism** h from $\langle X, Cn\rangle$ to $\langle X', Cn'\rangle$ is an injective function $h : X \to X'$ such that $A \subseteq X$, $h(Cn(A)) = Cn'(h(A))$, that is, h preserves the consequence operator, which means that the following diagram commutes:

$$\begin{array}{ccc} \wp(X) & \xrightarrow{Cn} & \wp(X) \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle h} \\ \wp(X') & \xrightarrow[Cn']{} & \wp(X') \end{array}$$

It is immediate that we can construct the **category of consequence structures**, denoted by CON, whose CON-objects are consequence structures and CON-morphisms are homomorphisms.

The next step is to construct an endofunctor $\mathbb{P}$ on the category CON called **paraconsistentization functor**. Let $\langle X, Cn\rangle$ be a consequence structure and $A \subseteq X$. We define

$$Cn_{\mathbb{P}}(A) = \bigcup\{Cn(A') : A' \subseteq A, Cn(A') \neq X\}.$$

In particular, $a \in Cn_{\mathbb{P}}(A) \Leftrightarrow$exists $A' \subseteq A, Cn-$Consistent, such that $a \in Cn(A')$.

The behavior of $\mathbb{P}$ is given by the following clauses:
(i) For CON-objects (X, Cn), $\mathbb{P}(X, Cn) = (X, Cn_{\mathbb{P}})$;
(ii) For CON-morphisms h, $\mathbb{P}(h) = h$.

With this conceptual framework, it is possible to define when a consequence structure is explosive or paraconsistent. Let (X, Cn) be a consequence structure, and suppose that X has a *negation operation*, denoted by the symbol $\neg$. Then (X, Cn) is **explosive** iff for all $A \subset X$, there is an $x \in X$ such that if $x, \neg x \in Cn(A)$, then $Cn(A) = X$ (i.e., A is Cn-inconsistent). Otherwise, (X, Cn) is called **paraconsistent**.

Next, we discuss sufficient conditions a logic must fulfill so the functor may be applied to it yielding its paraconsistent counterpart. This procedure is very fruitful. Consider a logician trying to formalize some given theory, and she decides to use an explosive logic that seems suitable for the task. Nonetheless, during the formalization, inconsistencies are found which, due to the explosive character of the logic, lead to trivialization. Instead, if she applies the paraconsistentization functor, it is possible to confine the use of the initial logic to its consistent subsets. One of the main advantage of this approach is that it allows maintaining the core features of the original logic. So, for instance, a constructivist logician may continue studying an inconsistent theory with the paraconsistent counterpart of her initial logic which, among other things, refutes the law of excluded middle, thus keeping the central aspects of her original logic but avoiding triviality.

With this construction in hand, it is possible to state and discuss some of the philosophical problems related to paraconsistent logics such as: are contradictions in paraconsistent logic really contradictions? Is there a paraconsistent negation? Does the meaning of logical constants change from one logic to another?

Keywords: Category theory; Paraconsistent logic.

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Paraconsistent infectious logics and bilattices

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Abstract.

As is widely known nowadays, due to work by Ginsberg (in [13]), Fitting (in [12, 10, 11]) and many other authors, *bilattices are nice things*. Later on, work by Avron and Arieli proved that there are certain bilattices that are not only interesting mathematical objects, but also interesting *logical* objects. These are called logical bilattices. In [2] it was proved that Belnap-Dunn's four-valued logic FDE, represents 'the' logic of logical bilattices, i.e. that every argument valid in FDE is valid also in every logical bilattice.

Moreover, from Ginsberg's, Fitting's and Avron and Arieli's work, it is reasonable to conclude that since bilattices are nice things, then operations on bilattices are –if not nice– at least worth of some attention. In fact, Fitting started to explore operations on bilattices that go beyond the operations that are usually taken to interpret logical conjunction and disjunction, v.g. the meet and join operations on the truth-order. His motivation to explore these alternatives was to provide bilattice analogues of the behavior of some 'weak' logics, e.g. of subsystems of Weak Kleene Logic $\mathsf{K}_3^{\mathsf{w}}$. In these peculiarly weak logics, no formula with a subformula that received an indeterminate semantic value (i.e. that is 'neither-true-nor-false') can be assigned other than this same indeterminate semantic value. In that context, the indeterminate semantic value can justifiably be regarded as *infectious* –an approach that has been embraced also by Dimitri Bochvar in [3]. The logic $\mathsf{K}_3^{\mathsf{w}}$ is, in fact, the $\{\neg, \wedge, \vee\}$-fragment of Bochvar's logic of nonsense B_3.

Fitting proposal to connect certain subsystems of $\mathsf{K}_3^{\mathsf{w}}$ to bilattices was to define a 'cut-down' operator $a \oplus \neg a$ that helped to further define 'cut-down' operations, i.e. cut-down variants of the traditional bilattice operations that are taken to interpret logical conjunction and disjunction. Recently, an excellent step forward has been made in this area of research, when it was proved in [7] that a certain four-valued logic represents 'the' logic of cut-down operations on logical bilattices. This logic has received many names, Ferguson calls it $\mathsf{S}_{\mathsf{fde}}$, for it is the first-degree entailment of the relevant logic S (see [6] and [8]). In the present paper we will call this logic FDEwk, for it is a variant of the logic FDE, where the indeterminate value is infectious, as it is in Weak Kleene Logic.

The first aim of the present paper is to analyze another kind of 'weak' logical systems and their relationship with other operations on bilattices. The logics we will be discussing here are subsystems of Paraconsistent Weak Kleene Logic PWK. In the case of these weak logics, no formula with a subformula that received an inconsistent semantic value (i.e. that is 'both-true-and-false') can be assigned a consistent semantic value. In that context, the inconsistent semantic value can justifiably be regarded as *infectious* –an approach that has been

embraced by Sören Halldén in [14]. The logic PWK is, in fact, the $\{\neg, \wedge, \vee\}$-fragment of Halldén's logic of nonsense H_3.

Our proposal to connect certain susbsytems of PWK to bilattices –i.e. to connect infectious paraconsistent logics and bilattices– is to define a 'track-down' operator $a \otimes \neg a$ (indeed, closely related to Melvin Fitting and Thomas Ferguson's notion of a 'cut-down' operation) and to later on define track-down variants of the traditional bilattice operations that are usually taken to interpret logical conjunction and disjunction. In accordance with previous efforts, this attempt will allow us to prove here that a certain four-valued logic represents 'the' logic of track-down operations on logical bilattices. Contrary to the previous case, this logic of track-down operations on logical bilattices has not been independently studied in a systematic way in the literature. In the present paper we will call this logic FDEpwk, for it is a variant of the logic FDE, where the inconsistent value is infectious, as it is in Paraconsistent Weak Kleene Logic. Therefore, it is clear that succeeding at the first aim of this paper will allow us to establish new results regarding the relationship between infectious paraconsistent logics and bilattices.

The second aim of this paper is to draw some connections between the logic of track-down operations and a particular family of *containment logics* (see e.g. [1], [9], [5]), v.g. logics where an argument from Γ to φ holds only if certain containment (i.e. set-theoretic *inclusion*) principle between the set of propositional variables appearing in the premises Γ –referred here as $\mathsf{At}(\Gamma)$– and the set propositional variables appearing in the conclusion φ –referred here as $\mathsf{At}(\varphi)$– is respected.

To be more precise, it is highlighted by Ferguson in [7] that the logic of cut-down operations on logical bilattices is indeed a logic that belongs to a larger family of formal systems called Parry systems, conceptivist logics or alternatively *proscriptivist* logics. It is to be understood that a certain inference $\Gamma \vDash_{\mathsf{L}} \varphi$ carried out in the logic L has the proscriptive property only if $\mathsf{At}(\varphi) \subseteq \mathsf{At}(\Gamma)$. These formalisms are studied by Ferguson in [8], [6] and by many others before, e.g. [15].

To wrap up the dualities deployed in this essay, we will show (following the preliminary results of [4]) that the logic of track-down operations on logical bilattices is a containment logic that reverses the containment direction of Parry systems, giving as a result a family of formal systems that we will call (given the lack of a better alternative), *permissivist* logics. Correspondingly, it is to be understood that a certain inference $\Gamma \vDash_{\mathsf{L}} \varphi$ carried out in the logic L has the permissive property only if $\exists \Gamma' \subseteq \Gamma$, such that $\Gamma' \vDash_{\mathsf{L}} \varphi$ and $\mathsf{At}(\Gamma') \subseteq \mathsf{At}(\varphi)$.

Keywords: Bilattices; Infectious Logics; Logics of Nonsense; Containment Logics.

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A pretty classical tonk for the simple semanticist

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Abstract.

The argument from Tonk was proposed by Prior [8] as a rebuttal of the idea that offering introduction and elimination rules for a symbol is enough to furnish it with meaning and to constitute it as a logical connective. We will cal such a point of view *Simple Inferentialism.* In its original presentation, the operation is characterized by the following natural deduction rules:

$(\otimes\mathbf{In}) \quad A \vdash A \otimes B$ $\qquad$ $(\otimes\mathbf{Elim}) \quad A \otimes B \vdash B$

This problem does not appear for the inferentialist's nemesis, who thinks the meaning of a logical connective is given by a truth function. We will call this position *Simple Semanticism* (of course, the Inferentialism and Semanticism − as full fledged philosophical stances − are much more complex than that, and there are many different variations on them. I nevertheless think there is a point to be made without dwelling on such subtleties).

In order to prove $A \vdash B$, as Belnap [2] already noticed, it is necessary to appeal to the transitivity of $\vdash$. So it seems that not only the inferentialist has a way out of the objection, but also now gets the upper hand, since the only thing she has to do in order to accommodate Tonk is to give up the already questionable rule of Cut. The appeal such an answer has for the inferentialist is that it can switch the burden of the objection: she can express − if she wishes so − something her opponent cannot, since the semanticist cannot assign a truth function to $\otimes$.

There is to date only one complete semantics available, found in Fjellstad citetei6, but which abandons not only transitivity but also reflexivity. My goal is to offer a Cut-free but otherwise pretty classical theory, which does not overgenerate with respect to the Tonk Inferences. I will do so in the context of a Strict-Tolerant theory. (see van Rooij, Cobreros, Egré and Ripley [4]). St-models are the same as three-valued Strong Kleene ones, but with a consequence relation which validates exactly the same sequents as classical logic: $\Gamma \Vdash \Delta$ iff there is no v such that $v(A) = 1$ for all $A \in \Gamma$ and $v(B) = 0$ for all $B \in \Delta$.

If we look at the sequent calculus rules for Tonk, we see that the rule to the right forbids the models where $A \geq 1/2$ and $A \otimes B = 0$, while the rule to the left forbids those in which $B \leq 1/2$ and $A \otimes B = 1$.

$$\frac{B, \Gamma \vdash \Delta}{A \otimes B, \Gamma \vdash \Delta}(\otimes L) \qquad \frac{\Gamma \vdash, \Delta}{\Gamma \vdash A \otimes B, \Delta}(\otimes R)$$

This leaves us with four determined cases: $A \geq 1/2$ & $B \leq 1/2 \Rightarrow A \otimes B = 1/2$. And also with four partially determined cases: $A \geq 1/2 \& B = 1 \Rightarrow A \otimes B \neq 0$ and $A = 0$ & $B \leq 1/2 \Rightarrow A \otimes B \neq 1$. Notice that the validity of no argument will be affected by how those four cases are decided. Lastly, in the case where $A = 0$ and $B = 1$, $A \otimes B$ is left completely undetermined, but now it is indeed important how we decide it. If the output in this case were 1, we would be left without a countermodel for $B \Vdash A \otimes B$, which is unprovable in the system. On the other hand, if the output were 0, it would be $A \otimes B \Vdash A$ − also unprovable − which would become valid.

The meaning of $\otimes$ will then not be given by a single truth function − or a matrix − but by two slightly different ones − or a Non deterministic matrix (see for instance Avron and Lev [1] or Carnielli and Coniglio [3]):

<table>
<tr><td>⊗</td><td colspan="4">B</td></tr>
<tr><td>A</td><td colspan="2">1</td><td>1/2</td><td>0</td></tr>
<tr><td>1</td><td colspan="2">1</td><td>1/2</td><td>1/2</td></tr>
<tr><td>1/2</td><td colspan="2">1</td><td>1/2</td><td>1/2</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
</table>

We then prove that ST-Tonk is complete with respect to the class of ST-Tonk models. The strategy of the proof will be taken from the one in Ripley [9] for ST with some modification in order to accommodate the non-deterministic character of the semantics.

The idea is that there is a reduction procedure which gives, for every sequent, either a proof, or else a recipe to build a countermodel (in case it is unprovable, of course). The result of each stage of the reduction is a tree, whose branches may eventually close − get to an axiom − or remain open. If not every branch closes, one takes the union of an infinite open branch, and the model which assign 1 to every atomic formula in the antesequent and 0 to every formula in the posequent is supposed to be a countermodel for the root of the tree.

The induction in the case of Tonk demands for the strict truth of B to be sufficient for the strict truth of $A \otimes B$, and for the falsity of A to be sufficient for the falsity of $A \otimes B$, which is impossible in the case where B is 1 and A is 0. But since we allow both kinds of valuations, even if we do not have a guarantee that the model we are looking at will behave as we want, we do have the certainty that there will at least be another one one that does the job.

Theorem *(Completeness). For every sequent* $\Gamma \vDash \Delta$*, it has a countermodel or it has a proof.*

One think we lose, when regaining reflexivity, is the uniqueness of the connective defined. Is the possibility to semantically characterize Tonk by means of these kinds of truth functions threatened *defectively characterizing a piece of vocabulary*, and *characterizing a piece of defective vocabulary*. The characterization is not defective, since it is given in terms of truth functions, and that is all it takes for the Simple Semanticist for something to be a logical operator. But it leaves room to incorporate underdetermined vocabulary, understood as either vague or ambiguous.

Keywords: Non-deterministic semantics; Tonk.

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